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**Conference Paper**

## Labour Productivity Dynamics in Europe: Alternative Explanations for a Well Known Problem

46th Congress of the European Regional Science Association: "Enlargement, Southern Europe and the Mediterranean", August 30th - September 3rd, 2006, Volos, Greece

**Provided in Cooperation with:**

European Regional Science Association (ERSA)

Suggested Citation: Rodrigues, Antonio Godinho (2006) : Labour Productivity Dynamics in Europe: Alternative Explanations for a Well Known Problem, 46th Congress of the European Regional Science Association: "Enlargement, Southern Europe and the Mediterranean", August 30th - September 3rd, 2006, Volos, Greece, European Regional Science Association (ERSA), Louvain-la-Neuve

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# Labour Productivity Dynamics in Europe: Alternative Explanations for a Well Known Problem

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May 2006

## 1 Introduction

The structure of the spatial surface has direct repercussions on the economic relationships that are undertaken on it. This study aims not only to understand those effects but also to understand the underlying economic relationships. The variable studied is the growth rate of manufacturing labour productivity from a sample of 195 NUTS2 regions of the European Union for the period between 1991 and 2002<sup>1</sup>. Departing from the simple Verdoorn Law, which relates the growth rate of labour productivity with the growth rate of output, two different hypothesis will be tested: first, the significance of the *Marshall's* externalities as well as urbanization economies will be analyzed. The variables will be obtained through the calculation of the weighted densities at the NUT3 level (from a sample of 1044 regions) and aggregated for each of the 195 NUT2 regions of the sample. Secondly, and following the seminal paper by Chinitz (1961), the importance of the productive structure will be tested using three different measures of specialization.

The next section will introduce the nature of the relation studied, giving chief importance to the properties and problems inherent to the study of the economic variables presented. Next, the nature of positive externalities and the measurement of the productive structure will be discussed. This will be followed by a section related to the estimation

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<sup>1</sup>The data used was taken from the Cambridge Econometrics database

of the different specifications. Particular importance will be given to the specification of the spatial structure of the dependent component through a spatial weights matrix. After presenting and discussing the results of the estimations, the paper concludes.

## 2 The Model

This work bestows chief importance to models which assume demand as a growth drive. The empirical relationships identified by Kaldor in mid 60's (McCombie et al. 1976), underlying some of the economic phenomena ignored in the Neo-Classic Economy literature, can be seen as a departure point. Kaldor gives foremost importance to manufacturing as the engine for growth, a sector, which in opposition to the primary sector, is characterized by the existence of increasing returns. The development process of a region implies a transfer of the labor force to the more productive sectors of the economy. These inter-sectorial transfer cycles last until the moment that the marginal product of labour and consequently wages in the primary sector rise as a result of the exodus to the cities.

The second proposition or the second law of Kaldor demonstrates the linear relationship between labour productivity growth ( $p^i$ ) and output growth ( $q^i$ ), associated with the manufacturing industry:

$$p^i = a + bq^i \tag{1}$$

This relationship became known as the *Verdoorn Law*. An alternative specification results from the definition of  $p^i$  as  $q^i - l^i$ , where  $l^i$  is the growth rate of manufacturing labour. Adding this definition to (1) and after manipulation we obtain:

$$l^i = a^* + b^*q^i, \tag{2}$$

where  $a^* = -a$  and  $b^* = 1 - b$ . If  $q = l$ , there are constant returns to scale, which means that  $b^i = 0$  and  $b^* = 1$ . If  $0 < b < 1$  and  $0 < b^* < 1$ , there are increasing returns to scale.

The interpretation of the Verdoorn coefficient becomes clear when studying the relationship between the original expression (1) and a Cob-Douglas production function. This

way it also becomes possible to understand the exclusion of the growth rate of capital in the original Verdoorn Law. If  $\frac{\Delta Q}{\Delta K} \neq 0$ , this omission results in a bias in the Verdoorn coefficient. We will start with the specification of the production function:

$$Q = A_0^{\lambda t} K^\alpha L^\beta \quad , \quad (3)$$

where  $Q$ ,  $K$ ,  $L$  represent respectively output, capital and labour;  $\alpha$  e  $\beta$  represent their respective elasticities. After calculating the logarithms, differentiating in  $t$  and introducing exogenous shocks, we obtain:

$$p = \frac{\lambda}{\beta} + \frac{\beta - 1}{\beta}q + \frac{\alpha}{\beta}k + \xi \quad (4)$$

Kaldor believes that in developed countries the capital growth rate is similar to the growth rate of output in manufacturing. Introducing the  $\Delta Q = \Delta K$  restriction, i.e. accepting the hypothesis that  $\frac{\Delta K}{\Delta Q}$  is equal to one, we obtain:

$$p = \frac{\lambda}{\beta} + \frac{\beta + \alpha - 1}{\beta}q + \xi \quad (5)$$

Replacing in (5)  $\frac{\lambda}{\beta}$ ,  $\frac{\beta + \alpha - 1}{\beta}$  by  $\gamma_0$ ,  $\gamma_1$ , we get:

$$p = \gamma_0 + \gamma_1q + \xi \quad (6)$$

As before, if  $\gamma_1 = 0$ , there are constant returns to scale in manufacturing ; if  $\gamma > 0$ , there are increasing returns.

A second problem associated with the estimation of the Verdoorn Law results from the fact the the regressor may be seen as endogenous. If we accepted that  $\Delta P_t = f(\Delta Q)$ , it is not altogether out of place the idea that  $\Delta P_t$  shocks have a positive effect on output. This problem, though important, will only be examined at a later stage, when the questions concerning the specifications estimated during this work are discussed.

If we Take into account different sources of local returns to scale (*Agglomeration Economies*) identified in the Economic's literature (see McCann 2001, Gordon & McCann 2000), it becomes normal to take the Verdoorn functional form as an incomplete expression. However, it is important to safeguard one aspect related to the interpretation of the Verdoorn coefficient ( $\gamma$ ). Its value, when positive, captures factors normally

identified as agglomeration economies. What the specification of these sources of increasing returns implies is the systematic identification of which centripetal/centrifugal forces contribute to changes in labour productivity growth rates.

Considering the definition of agglomeration economies as those factors which favour growth through the concentration of economic activities in a specific location, we can thus consider them as promoters of technical progress. Following this line of thought it is possible to define  $\lambda$  as a function  $g(\cdot)$  of all factors related with the spatial distribution of activity that function as shocks to the productivity level in the different spatial units. We can thus rewrite (6) as:

$$p = \gamma_0 g(\cdot) + \gamma q + \xi \quad , \quad (7)$$

The formal specification of agglomeration economies as well as other sources of localized increasing returns require a clear and objective exposition of the centripetal economic forces and of the way they condition the activity level of a region, working as location regulators of economic agents. The formal presentation of the hypothesis related to the  $g(\cdot)$  function as well as a brief discussion of their theoretical background will follow.

## 2.1 Hypothesis A: Agglomeration Economies

It is possible to distinguish three types of externalities that influence the location behaviour of economic agents: i, increasing returns associated with one economic unit (i.e. a firm); ii, localization economies; iii, urbanization economies. Type i. of localized externalities is related to all synergies inherent to the agglomeration of various phases of the production chain on a limited geographical area. It is assumed in this study that this phenomenon is captured by the Verdoorn coefficient.

Associated with the localization economies are a group of factors which became known as Marshall's agglomeration economies. Alfred Marshall (1920) emphasized (a) the existence of information spillovers, (b) a specialized pool of labour and (c) the existence of specialized services as specific factors which influence the location behaviour of specific sectors.

The spatial diffusion of technology (or information spillovers) is associated with sectors

characterized by products with short life cycles due to constant technological changes, but also with changes in demand. This type of externality is captured through the spatial weight matrix used ( $W$ ) used in spatial econometric specifications; the explanation lies on the fact that proximity intensifies contact between agents, hence the distance between spatial units captures this contamination effect.

The quantification of specialized labour force and services can be achieved through the measurement of the variable's *density* at an intra-regional level. If we define the set of spatial units defined by  $\{R_1, R_2, \dots, R_M\}$  which form a  $S$  surface and  $R_i = \{r_{i1}, r_{i2}, \dots, r_{iN}\}$ , then the variable's density  $X = \{x_1, x_2, \dots, x_M\}$ , which represents any economic variable can be represented by:

$$D(x_i) = \frac{\sum_{j=1}^N x_{ij}/a_{ij}}{N}, \quad (8)$$

where  $a_{ij}$  represents the sub-unit area (or sub-region)  $r_{ij}^2$ .

The method used in 1996 by António Ciccone and Robert Hall (Ciccone et al. 1996) will be closely followed in this study. Ciccone and Hall studied the changes in labour productivity through the local dimension of the externalities. According to the authors, and following Alfred Marshall (1920), the existence of increasing returns is due to the local geographic externalities and to the variety of intermediate goods, measured by the density of economic activity. Related to this latter aspect is the fact that, as it has been previously stated, the analysis of the life cycle of urban centers, as well as the understanding of those factors which differentiate them, are important aspects towards to the understanding of the nature of externalities.

In the case of the measurement of specialized labour, this will be defined as the density

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<sup>2</sup>This definition of density can be opposed to a more common measure of density as  $d^*(x_i) = \frac{x_i}{a_i}$ . This interpretation is however prone to measurement errors due to the geographic scale of the study. If for instance, the aim is to study productivity changes in the NUTS2 regions, the density of a production factor measured at this level has a limited value, since due to a series of geographic circumstances, outside the urban areas, there are uninhabited/deserted areas which however do not interfere at all with the density of activity in the urban space. This question can however be overcome through the creation of an index of each factor at the NUTS2 regions level, adjusted by the density at a finer level (NUTS3), as it is suggested by expression 8

of industrial employment. Following what has been previously mentioned, the aim in this case is to focus at the NUTS2 regional level, aiming however to introduce the local externalities concept through the analysis at a finer level of aggregation (NUTS3). We can construct a density measure using a production function of the type  $f(L, Q, A)$ . This function describes regional output as a function of labour and density. If  $\alpha$  and  $\frac{\lambda-1}{\lambda}$  represent respectively the elasticities of  $L$  and  $\frac{Q}{A}$ <sup>3</sup>, then the technology can be written as:

$$Q = L^\alpha \left( \frac{Q}{A} \right)^{\frac{\lambda-1}{\lambda}} \quad (9)$$

Accepting that labour is distributed homogenously at the finer aggregation level, then it is possible to define regional output as:

$$Q_r = A_r \left( \frac{L_r}{A_r} \right)^\alpha \left( \frac{Q_r}{A_r} \right)^{\frac{\lambda-1}{\lambda}} \quad (10)$$

The  $r$  index represents regional growth rate at the lowest aggregation level. Solving (10) for  $\frac{Q_r}{A_r}$ , region's  $r$  technology can be represented as:

$$\frac{Q_r}{A_r} = \left( \frac{L_r}{A_r} \right)^{\alpha\lambda} \quad (11)$$

The  $\alpha\lambda$  exponent represents the combination of centrifugal forces or congestion effect ( $\alpha$ ) and the centripetal forces or agglomeration effect ( $\lambda$ ). Following a neo-classic scenario of decreasing returns,  $\alpha\lambda < 1$ . If on the opposite, positive externalities have enough weight to compensate the effect of the centrifugal forces,  $\alpha\lambda > 1$ , which represents localized increasing returns.

If we define  $\gamma = \alpha\lambda$ , then the expression for the highest aggregation level can be written as:

$$Q_R = \sum_{r \in R} L_r^\gamma A_r^{(1-\gamma)}, \quad (12)$$

where  $\gamma = \alpha\lambda$ .

The density index represents the mean productivity of labour which can be represented by:

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<sup>3</sup>  $\frac{\lambda-1}{\lambda}$  represents a distance decay function.

$$D_R^L(\gamma) = \frac{Q_R}{L_R} = \frac{\sum_{r \in R} L_r^\gamma A_r^{1-\gamma}}{L_R}, \quad (13)$$

where  $L_R$  represents labour in region  $R = \{r_1, r_2, \dots, R\}$ .

In the case of specialized services, a similar index will be used. If  $L^s$  represents labour used on the tertiary sector, then this externality will be measured by:

$$D_R^{L^s}(\gamma) = \frac{Q_R}{L_R^s} = \frac{\sum_{r \in R} L_r^\gamma A_r^{1-\gamma}}{N_R}, \quad (14)$$

The last type of agglomeration economy to be modeled concerns advantages shared by all sectors of the economy which result from the size of urban areas and the density of human activity. When studying the relation between cities in terms of their size, it is common to use the Zipf Law (Zipf 1949) as a formal specification for the shape of a urban hierarchy. Following the existing literature on the subject (see for example Gabaix et al.2003), it is normal to observe a regular relationship between size (usually measured by resident population) and the respective ranking of the areas considered. Formally, the Zipf relation can be stated as:

$$P_n = P_1 (n)^{-\alpha}, \quad (15)$$

where  $P_n$  represents the population of the city of rank  $n$  and  $P_1$  represents the population of the largest city. If  $\alpha = 1$ , it means that the hierarchy follows the Zipf Law. The larger  $\alpha$  is, the greater the *dominance* from one or a small number of cities in relation to the rest of the spatial surface. This primacy is a reflection of a city's capacity to attract activity from neighboring areas. Hence, the size of  $\alpha$  can be interpreted as a measure of economies of urbanization, in the sense that when  $\alpha > 1$  there are increasing returns associated with the size of the dominant urban areas. For estimation purposes, the Zipf relation can be transformed into a linear relationship by calculating the logarithms. The resulting expression is:

$$\log(n)_i = \alpha_0 + \alpha_1 \log(p_n)_i + v_i \quad (16)$$

The value of  $\hat{\alpha}$  was used in the calculation of the density measure similar to those previously presented (equations 13 and 14). Formally:



$$D_R^U(\gamma) = \frac{Q_R}{U_R^s} = \frac{\sum_{r \in R} U_r^\gamma A_r^{1-\gamma}}{N_R}, \quad (17)$$

## 2.2 Hypothesis B: The productivity structure of a region

It is generally accepted that the process of growth, stagnation and fall of a region is intimately related with the productive structure of a specific spatial unit; a region is what a region produces. Based on this assumption, this part of the paper will try to understand to what extent specialization or diversification of economic activity contribute to the qualitative improvement of the production processes through an increase in the aggregated productivity level of manufacturing.

An analysis of both the positive and negative aspects of the specialization process of a certain sector presumes the distinction between short-run and long-run effects. On the short-run, a positive shock associated with a specific sector on a certain region has multiplying effects that are a function of the specialization level and the synergies created by the local industry. An essential reference in the study of long-run effects of specialization is the work of Chinitz (1961), which compares the growth trajectories in the metropolitan areas of Pittsburgh and New York. The author concludes that for metropolitan areas of similar dimension, a high specialization level, which means a low level of productive diversification, can make a region sensitive to technological mutations and to demand.

As a way of testing the effect of the specialisation/diversification level in the growth rate of labour productivity, three measures were calculated for each region at the initial period of the sample: These were the Theil Index, the Herfindhal Index and the Blair Index (Godinho Rodrigues, 2000).

The Theil and the Herfindhal Indexes are measures of specialization, which take into account the productive structure of each spatial unit, ignoring however the relative weight of each sector in the Economy as a whole. For  $i = \{1, 2, \dots, M$  sectors and  $j = \{1, 2, \dots, N$  regions, the two indicators are calculated through the following expressions:

$$\begin{aligned} T_j &= - \sum_i^M \left( \frac{x_{ij}}{x_{Mj}} \log \frac{x_{ij}}{x_{Mj}} \right) \\ H_j &= \sum_{i=1}^M \left( \frac{x_{ij}}{x_{Mj}} \right)^2, \end{aligned} \quad (18)$$

where  $X$  is the variable of interest. The Theil Index and the Hefindhal Index tend to one as the level of specialization increases.

The Blair Index is a relative measure of specialization that takes into account the weight of each sector in the Economy. It is given by the following expression:

$$BIS_j = \sum_{i=1}^m \alpha \left[ \left( \frac{x_{iJ}}{x_{IJ}} \right) - \left( \frac{x_{ij}}{x_{Ij}} \right) \right] \quad (19)$$

If  $\left( \frac{x_{iJ}}{x_{IJ}} \right) > \left( \frac{x_{ij}}{x_{Ij}} \right) \rightarrow \alpha = 1$ . If  $\left( \frac{x_{iJ}}{x_{IJ}} \right) < \left( \frac{x_{ij}}{x_{Ij}} \right) \rightarrow \alpha = 0$ . The Blair index tends to one with increasing degree of specialization.

### 3 Estimation

Given the relevance of spatial dependency and heterogeneity associated with spatial data, it is thought as important to analyze the spatial structure of the growth rate of labour productivity ( $p$ ). An adequate starting point to study the spatial structure of  $p$  is the definition of a first order autoregressive stochastic spatial process which allows to test the hypothesis of autocorrelation in the series. Formally, it is written as:

$$p_r = \rho W p_r + \epsilon_r, \quad (20)$$

where  $W p_r$  represents the dependent variable modified by the spatial weights matrix  $W$ ,  $\rho$  is the lagged variable coefficient and  $\epsilon_i$  the error term.

#### 3.1 Spatial Weights Matrix

The study of spatial structure associated with  $p$  is strongly dependent on the form chosen of the spatial weights matrix  $W$ , where each  $w_{ij}$  element represents the proximity between each pair of spatial units. This is a theme of paramount importance, always discussed with more or less emphasis in all econometric studies where the variable *space* is made endogenous (see for example Baumont et al. 1999, Aragon et al. 2003, Ertur et al. 2003).

Probably the two most common forms of imposing a proximity relationship between regions are through a contiguity matrix  $W^f$  and through a nearest neighbors matrix  $W^n$ . In terms of contiguity matrices, it is possible to choose a binary specification, where

$w_{ij} = 1$  when  $i$  and  $j$  are contiguous or zero otherwise (values in the main diagonal are set to zero). A more informative form was chosen for this study, where each element different from zero is equal to the length of the common border between neighbours. Formally, the weights matrix takes the form:

$$W^f = \begin{cases} w_{ij} = 0 & \text{if } i = j \\ w_{ij} = f_{ij} & \text{if } f_{ij} \neq 0 \\ w_{ij} = 0 & \text{if } f_{ij} = 0 \end{cases} , \quad (21)$$

where  $f_{ij}$  represents the common border between regions  $i$  and  $j$ .

The chosen nearest neighbours matrix is a binary matrix, where each element  $w_{ij}$  is equal to one when  $j$  belongs to the set of ( $k$ ) neighbours. Formally:

$$W^k = \begin{cases} w_{ij} = 0 & \text{if } i = j \\ w_{ij} = 1 & \text{if } j \in k \\ w_{ij} = 0 & \text{if } j \ni k \end{cases} , \quad (22)$$

where  $k$  is the set of  $i$ 's nearest neighbors. The decision to choose a binary contiguity matrix in the particular case of the nearest neighbors (as opposed to a more informative form) was due to the desire to compare a specification (21) which has a large amount of information with a more simple expression (22).

To rigorously test the effect of using different specifications of the spatial weights matrix, it is important to estimate the autoregressive process previously mentioned (20) using samples of different size. The initial theoretical hypothesis is that the difference between autoregressive coefficients resulting from the use of different specifications of the  $W$  matrix will be smaller the larger the sample; this is due to the fact that the differences get dissolved as the elements of the matrix equal to zero rise with  $n$ , where  $n$  is the number of spatial units considered in each sub-sample.

Since this study focuses on proximity relationships, it does not make sense to choose smaller random sub-samples in relation to the original sample, since in this case the spatial structure would be lost. The manner chosen to overcome this problem was to select for each sub-sample size the most western  $n$  units; after that another sub-sample was selected by moving one region to the East and continue the process until the most Eastern region

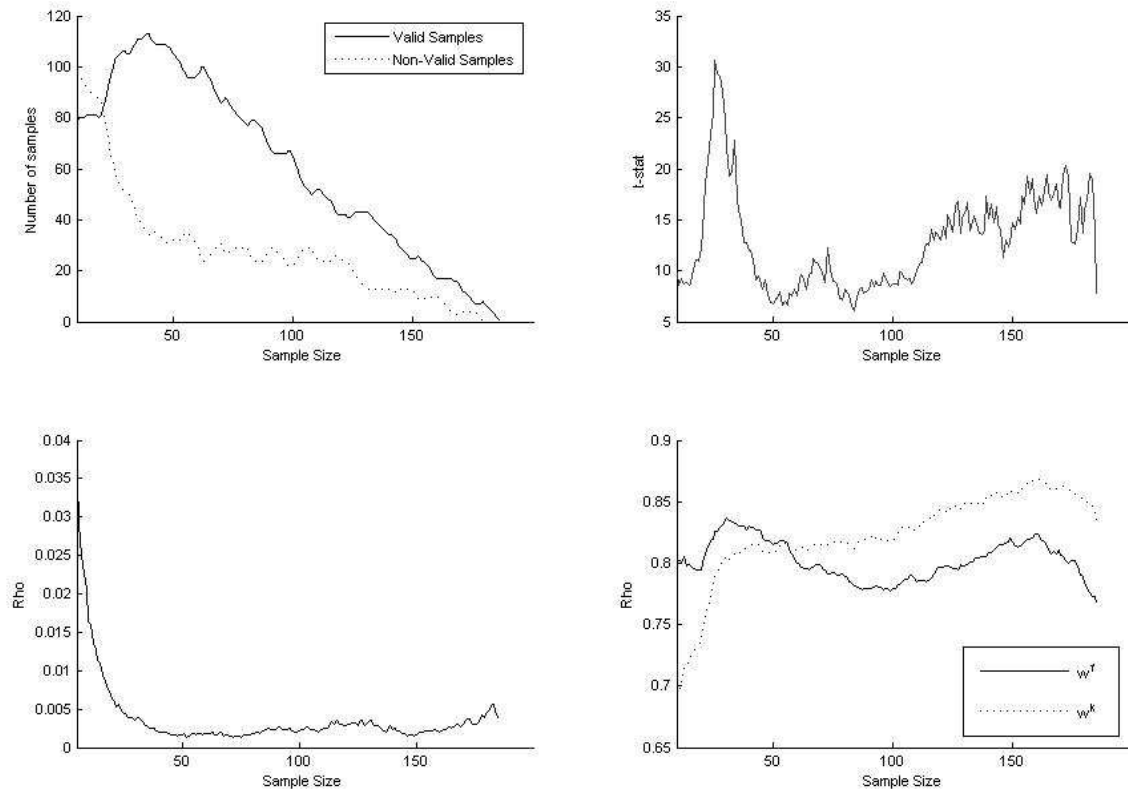


Figure 1: Sub-sample estimations of the first order autoregressive model

is reached. A sub-sample is considered invalid when any of the regions has zero neighbours (figure 1a shows that, as  $n$  increases the number of valid sub-samples rises in relation to the invalid sub-samples). To eliminate any bias caused by structural differences between more western European regions and those more to the East, the number of models equals to the number of valid samples covering all spatial surface.

At this stage, only first order matrices were considered, in relation to the connectivity level. For the  $W^n$  type matrix the number of  $k$  nearest neighbors was chosen according to the mean number of contiguous regions for each sample.

Figures 1c and 1d represent changes in the autoregressive coefficient  $\rho$  as  $n$  rises in relation to models estimated with matrices of type  $W^f$  and  $W^n$ . For each  $n$ , the value presented in figure 1c is the difference between model parameters. Figure 1d shows the value of the autoregressive coefficient for each model. The hypothesis that these differences

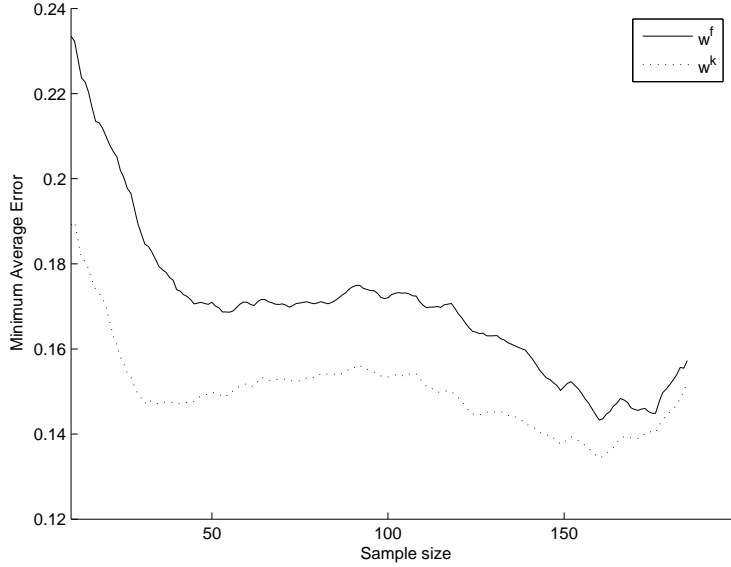


Figure 2: Sub-sample estimations

ease as  $n$  increases is validated, though it is possible to observe a reverse in the tendency of convergence amongst parameters for sub-samples larger than 150 (it would be advisable to undertake tests with different spatial surfaces in order to attain clearer results). It is also interesting to verify that on an initial stage, models estimated with a  $W^f$  type matrix result in higher  $\hat{\rho}$  values; this trend is reversed for sub-samples with  $n > 65$ . The regularity in the relationship of the autoregressive coefficients after this point is also confirmed by the relation between the  $t$  – values showed in figure 1b. Hence, it is possible to interpret  $n = 65$  as a minimum value after which on average the differences between the use of these two types of matrices is not changed in any significant way.

Figure 2 shows, for the same sub-samples, an indicator of robustness. As previously seen, the value put forward represents the weighted average for each  $n$ . The graphic represents the evolution of the Mean Square Error <sup>4</sup>. It is interesting to verify that for  $n = 169$ , it reaches a minimum. As previously, it would be important to test for different samples the existence of a similar pattern.

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<sup>4</sup> $mae = \frac{1}{T} \sum_{i=1}^T (y_i - \hat{y}_i)$ , where  $T$  represents the number of regions in the sample.

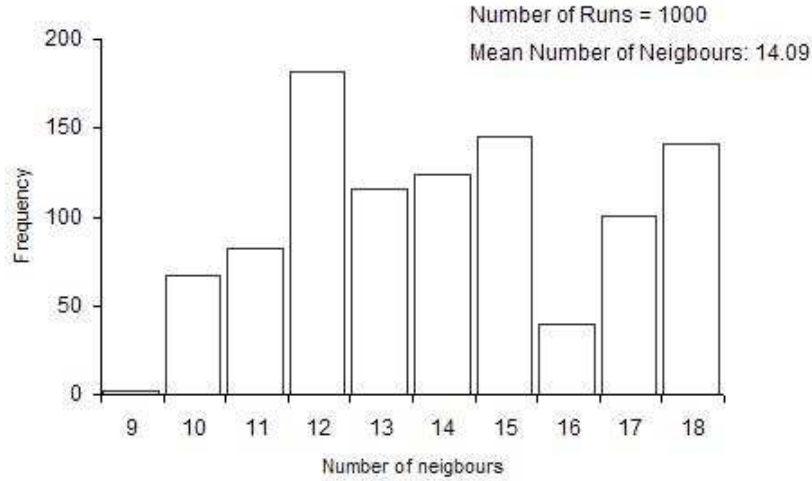


Figure 3: Fases da análise de um sistema espacial

A second pertinent question related to the spatial structure of the variable under scrutiny concerns the *order* of the process used. It is a question with a different type of answer according to the spatial weights matrix previously proposed. In the case of a  $W^k$  type matrix, it is necessary to find the number of neighbors that better depict the interaction between neighboring spatial units; in the case of a  $W^f$  type matrix, the choice is between the set of  $W_i^f = \{W_1^f, W_2^f, \dots, W_N^f\}$  type matrices where  $i$  represents the order considered.

The methodology used in this study had the objective of finding, for matrices of type  $W^k$ , which is the number of neighbors that, for variable  $p$ , maximize the *t-value* associated with the autoregressive coefficient of the stochastic process presented in (20); the second criterium was to find the number of neighbors which minimize the Mean Square Error. In practice, the exercise consisted in the estimation of the model with nearest neighbors binary matrices, with  $k$  varying between given values. In practice, an interval between 5 and 30 was chosen, and the test was performed 1000 times. For each run, the *optimum* number of neighbors was saved for latter analysis.

Figure 3 shows the results obtained with the exercise concerning the  $t$  statistic. The absolute frequencies allow to conclude that according to this criteria, the value of  $k$  which better represents the spatial structure of the  $p$  variable should be between 10 and 18. In relation to the second criteria, the value of  $k = 8$  is the one that minimizes the MAE

statistic in 841 out of the 1000 runs.

Order of contiguity	Mean number of neighbors	Non-zero elements
1 <sup>st</sup>	4.7	892
2 <sup>nd</sup>	14.4	2805
3 <sup>rd</sup>	37.4	7295

Table 1: Mean number of neighbors

The next step was to calculate, for the  $W^f$  type matrices, the mean number of neighbors of each spatial unit. The results (table 1) lead to the conclusion that the mean number of neighbors of a second order contiguity matrix coincides with the mean obtained through the criteria concerning the maximization of the significance of the autoregressive coefficient (see figure 3)<sup>5</sup>.

### 3.2 Estimation of the Verdoorn Law

After verifying the existence of a non-stochastic structure associated with the growth of labour productivity in the sample used, the estimation of the simple Verdoorn Law (see expression 1) was undertaken.  $W^k$  type matrices were used with  $k = 5^6$  and  $k = 14^7$ . First and second order weights matrices of type  $W^f$  were also used. Adding an autoregressive term to the original expression we get:

$$p_r = \rho W p_r^i + \beta q_r^i + \epsilon_r \quad (23)$$

The results presented (table 2) include also the results from the model estimated by Least Squares (with  $\rho = 0$ ); it is possible this way to verify the expected bias in the  $\beta$  parameter (Anselin 1988). In relation to the Verdoorn coefficient ( $\beta$ ), it lies between 0.6 and 0.64, which represents a growth rate of 0.6% in the labour productivity resulting from a 1% change in the output growth rate. The autoregressive coefficient is larger when  $W^k$

<sup>5</sup>the existence of a reasonable number of coastal regions causes generally an *edgeeffect*. As such, the mean number of neighbors is likely to be under-estimated

<sup>6</sup>mean number of neighbors in a first order contiguity matrix

<sup>7</sup>mean number of neighbors of a second order contiguity matrix and mean of the absolute frequencies associated to  $k$  which maximizes the significance of the autoregressive coefficient

type matrices are used. Both types of spatial weights matrices register an increase when second order models are used.

	$W^{k=5}$	$W^{k=14}$	$W^{f=1}$	$W^{f=2}$	OLS
$\hat{\beta}$	0.60	0.61	0.62	0.64	0.70
( <i>sig.</i> )	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$r\hat{h}o$	0.26	0.39	0.19	0.11	-
( <i>tstat</i> )	(3.50)	(3.81)	(2.91)	(3.45)	-
MAE	0.1102	0.1117	0.1114	0.1180	0.1129

Table 2: Estimation results (SAR specification)

After this, the Verdoorn relationship with a spatial component associated with the error term was estimated:

$$\begin{aligned}
 p_r^i &= \alpha + \beta q_r^i + \xi_r \\
 \xi_r &= \lambda W \xi_r + \epsilon_r,
 \end{aligned}
 \tag{24}$$

where  $\epsilon_i$  represents a white noise component.

	$W^{k=5}$	$W^{k=14}$	$W^{f=1}$	$W^{f=2}$
$\hat{\beta}$	0.66	0.66	0.68	0.65
( <i>sig.</i> )	(0.000)	(0.000)	(0.000)	(0.000)
$\hat{\lambda}$	0.27	0.45	0.26	0.27
( <i>tstat</i> )	(2.82)	(3.34)	(2.70)	(7.60)
MAE	0.1136	0.1135	0.1132	0.1171

Table 3: Estimation results (SEM specification)

Comparing the two types of model allows the conclusion that the addition of an autoregressive structure associated with the error term results in higher Verdoorn coefficients, with the exception of the model using a second order contiguity matrix ( $W^{f=2}$ ). It is also this specification that registers the highest significance associated with the autoregressive coefficient ( $\lambda$ ).



### 3.3 Hypothesis

After estimating the simple version of the model, the estimation of the functional relationship  $\Delta P_l = f(\Delta Q) g(\cdot)$  was undertaken. The two hypothesis presented before were tested, one concerning the quantification of the agglomeration economies, and the other concerning the region's productive structure.

#### 3.3.1 Local Externalities

As it has been previously stated, Marshall's agglomeration economies (with the exception of information spillovers) and the Urbanization Economies were quantified through density measures of type  $D(x_i) = \frac{\sum_{j=1}^N x_{ij}/a_{ij}}{N}$  (see equations 13 and 14). The density index are equivalent to Cobb-Douglas production functions; after calculating the logarithms we obtain:

$$q_r^i = \alpha_0 + \alpha_1 l_r^i + \alpha_2 a_r + \epsilon_r , \quad (25)$$

$$q_r^s = \alpha_0 + \alpha_1 l_r^s + \alpha_2 a_r + \epsilon_r , \quad (26)$$

where  $\alpha_1$  e  $\alpha_2$  represent respectively  $\gamma$  and  $(1 - \gamma)$ ;  $q_r^i$  and  $l_r^i$  represent output and manufacturing labour in that order; the interpretation of 26 is the same, but for the service sector. Using the four specifications for the weights matrices used earlier and estimating (13) and (14) with an autoregressive element associated with the dependent variable, and then with the error term, the specification with the highest significance level of the autoregressive coefficient resulted in a value of  $\gamma = 1.017$  in relation to manufacturing labour and  $\gamma = 1.042$  associated with labour in the tertiary sector. Both coefficients show the existence of increasing returns associated with the concentration of specialized labour and services ( $2^{nd}$  and  $3^{rd}$  Marshall's agglomeration economies).

In relation to Urbanization Economies, the expression 16 was estimated using least squares. It is important to keep in mind that when estimating the Zipf Law, data are ordered, thus losing the existing spatial structure. The Zipf coefficient found is equal to 0.96, which shows that, on average, the urban centers' dimension is a source of diseconomies of scale in relation to the sample used.

These auxiliary estimations were then used to aggregate the density indexes for the 195 NUTS2 regions of the sample. The augmented version of the Verdoorn Law estimated is given by the expression:

$$l_r^i = \eta_0 + \eta_1 q_r^i + \eta_2 d_r^L + \eta_3 d_r^S + \eta_4 d_r^U + \epsilon_r , \quad (27)$$

where  $d_i^L$ ,  $d_i^S$  and  $d_i^U$  represent respectively the specialized labour force, local supply of services and urban density (different from population density) in the  $r$  spatial unit.

As previously, the model was estimated by adding an autoregressive component first to the dependent variable and second to the error term. The results presented in the table below show the maximum and minimum coefficients found for each type of model.

	Min(SAR)	Max(SAR)	Min(SEM)	Max(SEM)	OLS
$\hat{\Delta}q^i$	0.62(0.00)	0.66(0.00)	0.62(0.00)	0.64(0.00)	0.68(0.00)
$\hat{d}_i^L$	-0.041(0.00)	-0.046(0.00)	-0.041(0.00)	-0.047(0.00)	-0.048(0.00)
$\hat{d}_i^S$	0.004(0.89)	0.007(0.74)	0.004(0.89)	0.007(0.75)	0.003(0.87)
$\hat{d}_i^U$	-0.007(0.76)	-0.014(0.56)	-0.006(0.80)	-0.015(0.56)	-0.390(0.70)
$\hat{\rho}/\hat{\lambda}$	0.082(0.01)	0.35(0.00)	0.083(0.01)	0.37(0.01)	-

Table 3: Estimation results (local externalities)

The first point to note is that the addition of this set of variables inflated the value of the Verdoorn coefficient. Also, in relation to the SEM specification, the only model with a autoregressive coefficient ( $\hat{\lambda}$  different from zero is the one where a second order contiguity matrix was used. In terms of the estimated coefficients associated with the agglomeration economies, the supply of services ( $d_i^S$ ) is the one with lesser significance. In all specifications, the existence of a specialized pool of labour ( $d_i^L$ ) obtains the highest coefficients, while the urbanization economies ( $d_i^U$ ) also contribute in a positive way to changes in manufacturing labour productivity greater than zero.

### 3.3.2 Productive Structure

The last variation of the Verdoorn Law seeks to test the effect of a region's productive structure on labour productivity growth in the long run. Three specialization/diversification

measures were used: the Theil Index, the Blair Index and the Herfindhal Index. As pointed earlier, it is important to keep in mind that of these three indicators only the Blair Index takes into consideration the relative weight of each sector in the Economy as a whole; the others study the productive structure of each spatial unit independently of other regions. The model takes the form:<sup>8</sup>:

$$\Delta p_r = \gamma_0 + \gamma_1 \Delta q_r^i + \gamma_2 b_r + \gamma_3 h_r + \gamma_4 t_r + \mu_r , \quad (28)$$

where  $\gamma_1$  represents the Verdoorn coefficient,  $b_r$ ,  $h_r$  e  $t_r$ , represent respectively the Blair, Herfindhal and Theil indexes for  $r$ .

	Min(SAR)	Max(SAR)	Min(SEM)	Max(SEM)	OLS
$\hat{\Delta q}^i$	0.62(0.00)	0.66(0.00)	0.62(0.00)	0.64(0.00)	0.68(0.00)
$\hat{B}L$	-0.041(0.00)	-0.046(0.00)	-0.041(0.00)	-0.047(0.00)	-0.048(0.00)
$\hat{H}I$	0.004(0.89)	0.007(0.74)	0.004(0.89)	0.007(0.75)	0.003(0.87)
$\hat{T}H$	-0.007(0.76)	-0.014(0.56)	-0.006(0.80)	-0.015(0.56)	-0.390(0.70)
$r\hat{h}o$	0.082(0.01)	0.35(0.00)	0.083(0.01)	0.37(0.01)	-

Table 4: Estimation results (Productive structure)

As before, the model was estimated using first least squares, then with an autoregressive component associated with the dependent variable (29) and error term (30):

$$\Delta p_i = \rho W \Delta p^i + \gamma_0 + \gamma_1 \Delta q_r^i + \gamma_2 b_r + \gamma_3 h_r + t_r + \mu_r \quad (29)$$

$$\begin{aligned} \Delta p_i &= \gamma_0 + \gamma_1 \Delta q_r^i + \gamma_2 b_r + \gamma_3 h_r + t_r + \epsilon_r \\ \epsilon_r &= \lambda W \epsilon_r + \mu_r , \end{aligned} \quad (30)$$

where  $\mu_r$  represents a white noise component.

The four specifications of the spatial weights matrix were tested. The results show the minimum and maximum value associated to each coefficient for both types of model. It is clearly visible that the estimated parameters of the autoregressive component are

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<sup>8</sup>the specialization coefficient values used in the estimation have been previously transformed to a distribution of zero mean and standard deviation equal to one, in order to make the coefficient values comparable amongst themselves.

significant. Also noticeable is that of the three specialization measures, the Blair Index is the only one with a true value different from zero. The negative value shows that specialization contributes in a negative way to the growth rate of productivity, a statement that supports the Chinitz hypothesis.

## 4 Conclusion

The study of variations in the growth rate of manufacturing labour productivity for a given period across a sample of regions implies the need to consider the spatial structure of the variable studied. The emphasis given to the specification of the spatial weights matrix chosen was due to the acknowledgment that this choice can condition the results obtained and lead to erroneous conclusions if care is not taken. Also, a rigorous analysis of the spatial structure associated with the dependent variable permits the identification of the degree of existing spatial dependency. The introduction of two augmented Verdoorn Law specifications allowed to conclude that first, the existence of a specialized pool of manufacturing labour and the size of urban centres contribute positively to the productivity growth rate of the sector. Second, it was also possible to conclude that diversification also contributes to positive changes in  $p$ . This may be explained by the easier inter-sectoral transfer of labour towards more productive sectors. Future studies should consider samples taken from different geographical scales to confirm the results obtained.

## References

- Anselin, L; *Spatial Econometrics: Methods and Models* (Kluwer Academic Publishers 1988)
- Anselin, L; Florax, R; Rey, S; *Advances in Spatial Econometrics: Methodology, Tools and Applications* (Springer 2004)
- Aragon, Y; Baumont, C; Thomas-Agnan, C; *Spatial Modelling of Urban Housing Values* (2003);
- Baumont, C; Ertur, C; Le Gallo, J; *Spatial Analysis of Employment and Population Density: the case of the Agglomeration of Dijon* (1999)
- Chinitz, B; *Contrasts in Agglomeration: New York and Pittsburgh* (American Economic Review 1961, pp.279-289)
- Ciccone, A; Hall, R; *Productivity and the Density of Economic Activity* (American Economic Review 86.1 1996, pp.54-70)
- Ertur,C; Gallo,J; LeSage,J; Local Versus Global Convergence in Europe: A Bayesian Spatial Econometric Approach (REAL Working Paper Series 03-T-28 2003)
- Fingleton,B; *European Regional Growth* (Springer 2003)
- Fingleton, B; *Externalities, Economic Geography and Spatial Econometrics : Conceptual And Modeling Developments* (International Regional Science Review 26.2, pp.197-207)
- Gabaix, X; Ioannides,Y; *The Evolution of City Size Distributions* (2003)
- Godinho Rodrigues, A; *The State, Market and Accessibility Triangle; Insights into the Manufacturing Landscape of Northern Portugal* (Doctoral Thesis, 2000)
- Gordon, I; McCann, P; *Innovation, agglomeration, and regional development* (Urban Studies 37.3 2000, pp.513-532)
- Hagget, P; *Locational Analysis in Human Geography* (Edward Arnold LTD, 1969)

- Henderson, J; *The Sizes and Types of Cities* (American Economic Review 64.4 1974, pp.640-676)
- LeSage, J; *Spatial Econometrics* (The Web Book of Regional Science, Regional Research Institute, West Virginia University, Morgantown, WV. 1999)
- Marshall, A; *Principles of Economics* (8<sup>th</sup> edition, Macmillan and Co., Ltd. 1920)
- McCann, P; *Rethinking the economies of location and agglomeration* (Working paper of the Centre for Spatial and Real Estate Economics, 1995);
- McCann, P; *Regional and Urban Economics* (Oxford University Press 2001)
- McCombie, J; Thirwall, A; *Economic Growth and Balance of Payments Constraint* (St. Martin's Press 1984)
- Zipf, G; *Human behavior and the principle of least effort: An introduction to human ecology* (Addison-Wesley 1949)