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Auction Design without Commitment

Hannu Vartiainen

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Auction Design without Commitment

Summary

We study auction design when parties cannot commit themselves to the mechanism. The seller may change the rules of the game and the buyers choose their outside option at all stages. We assume that the seller has a leading role in equilibrium selection at any stage of the game. Stationary equilibria are characterized in the language of vonNeumann-Morgenstern stable sets. This simplifies the analysis remarkably. In the one buyer case, we obtain the Coase conjecture: the buyer obtains all the surplus and efficiency is reached. However, in the multiple buyer case the seller can achieve more: she is able to commit to the English auction. Typically the converse also holds, the English auction is the only stable auction mechanism.

Keywords: Auction theory, commitment, stable sets

JEL: C72, D44, D78

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Auction Design without Commitment*

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Abstract

We study auction design when parties cannot commit themselves to the mechanism. The seller may change the rules of the game and the buyers choose their outside option at all stages. We assume that the seller has a leading role in equilibrium selection at any stage of the game. Stationary equilibria are characterized in the language of vonNeumann-Morgenstern stable sets. This simplifies the analysis remarkably. In the one buyer case, we obtain the Coase conjecture: the buyer obtains all the surplus and efficiency is reached. However, in the multiple buyer case the seller can achieve more: she is able to commit to the English auction. Typically the converse also holds, the English auction is the only stable auction mechanism.

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1 Introduction

The optimal auctions literature starts by asking how does seller’s revenue maximizing mechanism look like.¹ Cremer and McLean (1988) and McAfee and Reny (1991) provide a largely definitive solution to the problem: they introduce a mechanism which, in almost all scenarios, extracts all the buyers’ surplus. From the viewpoint of the mechanism design theory, this is a negative result. Even if the Cremer-McLean mechanism solves the optimal auctions question, it is only rarely, if ever, used in practice. Why is it that we commonly see mechanisms such as the English auction used instead? This paper provides an answer to the question by appealing to commitment problems.

Full commitment is (one of) key assumptions of the optimal auctions literature. Ability to commit to the auction rules implies that (i) the seller cannot

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¹ Vickrey (1961), Myerson (1981), and Riley and Samuelson (1981) are seminal contributions in the optimal auctions literature. For general survey, see e.g. Klemperer (1999).
change rules of the game in the middle of the play (unilaterally or jointly with buyers), (ii) buyers cannot leave the game once they participate it. Under full commitment one needs not to worry about the dynamics of a bargaining process. However, one can easily imagine why commitment is hard to reach. It may be difficult to prevent seller from ”schill bidding”, and thereby forming artificial competition, or it may be difficult to prevent the seller changing the terms of trade once the auction has been finalized.² It should be particularly difficult to prevent renegotiation under inefficient occurrences. In fact, it is safe to argue that no trading scenario is not subject to potential commitment problems.

We analyse auction design under the hypothesis that parties do not have any commitment power: the seller is allowed to change rules of the auction mechanism at any stage of the game without any cost, and the buyers cannot ever be forced to participate (the value of their outside option is fixed). Thus, we take the opposite view to the commitment issue than the standard optimal auctions literature.

Allowing the seller to reformulate the auction game means that focusing on direct incentive compatible individually rational mechanisms no longer suffices. This is true even if a mechanism only reveals a recommended allocation, given buyers’ messages to the mechanism. Namely a particular recommendation together with the commonly known structure of the mechanism allows players to make inferences about other players’ types. Therefore, the seller typically is tempted to change the rules of the mechanism once a recommendation is revealed. Forward looking buyers of course anticipate this and adjust their play accordingly at the communication stage. Thus, the incentive compatibility of the mechanism breaks down. It can easily be shown that (generically) there is no incentive compatible mechanism whose rules the seller does not want change. Thus, full commitment is critical to the optimal auctions theory.

To analyze auction design without commitment, we focus on an extensive form bargaining game which imposes as little restrictions as possible on the strategic alternatives available to the seller. The grand game: (i) the seller declares an extensive form game, to be played by the buyers, (ii) at any stage of the declared game, the seller can propose an allocation or declare a new game which is to be played thereafter by the buyers (there is no limitations on the number of reformulations), (iii) at any stage of the game, the buyers can choose their status quo payoff rather than participate the game. If the buyers agree on a proposed allocation then this allocation is implemented. There is no discounting (as there is no cost of changing the rules), but if the game continues forever, all parties get their status quo payoff (zero).

It is clear at the outset that the game has many potential (perfect Bayesian) equilibria. Which to focus? Like in the standard optimal auctions analysis, we allow the seller to choose the equilibrium. However, she controls the equilibrium selection at any stage of the game. This implies that any equilibrium selection rule must be dynamically consistent with respect to seller’s own choices. This

²Examples of this are the recent umts-auctions in Europe. In many countries the governments have been trying to change the effective prices of the licences afterwards, depending on the developments of the market situation.
is a very forceful restriction.

We will focus on stationary (or Markovian) equilibria of the grand game. As noted by Chatterjee and Sabourian (1999), stationarity can be justified by appealing complexity considerations. Two nested notions of stationarity are introduced: weak form of stationarity implies that mechanism selection is independent of the history of the game, it only depends on the current beliefs. Strong form of stationarity implies weak stationarity and a property that only differences in terms of expected payoff should affect choices.

Weak stationarity together with the appropriate incentive and participation constraints allows us to characterize the equilibria in the language of von Neumann-Morgenstern stable sets. This approach simplifies the equilibrium identification problem remarkably. Demonstrating the usefulness of this approach in complex game theoretic environments is one of the contributions of the paper.

1.1 Stable set

Stable set consists of two ingredients. First, by the standard revelation argument we note that any equilibrium of the grand game must be implementable by a direct (veto-)incentive compatible mechanism, augmented by an information revealing device. Equilibrium selection problem then reduces to one of identifying direct mechanisms that can be implemented given their informational properties. This observation simplifies the problem considerably but does not affect the set of equilibrium outcome functions.

Second, we argue that the equilibrium can be conveniently analyzed in its reduced form by using stables sets. The graph of mechanisms satisfying veto-incentive compatibility restrictions is called the set of agreeable mechanisms. Over this set we define a dominance relation which we call upsetting relation: an agreeable mechanism is upset by another agreeable mechanism if (i) the latter is defined with respect to prior that is derived by updating on a recommendation of the former mechanism, and if (ii) the latter is mechanism at least as profitable to the seller than the recommendation of the former mechanism. Then, based on

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3For a closely related argument, see e.g. Anderlini-Sabourian (1999) or Chatterjee-Sabourian (2000). For introduction, see Osborne-Rubinstein (1995).

4Greenberg (1990) generalizes this approach, and argues that abstract stable sets can be viewed as a fundamental approach from which many game theoretic solution concepts can be derived from. Recently, several contributors have applied stable sets to noncooperative problems. Kahn and Mookherjee (1992, 1995) use stable sets to characterize coalition-proof equilibria, and to refine equilibria in a class of adverse selection problems. Asheim (1991, 1992) and Tadelis (1996) apply stable sets to refine subgame perfect equilibria in repeated games. Our exposition is most closely connected to Blume and Sobel (1997), who use stable sets to construct a criterion to refine equilibria in a class of cheap-talk games.

5In the mechanism design scenario, Forges (1990, 1994) addresses a closely related question of posterior efficiency. This property requires that once a recommendation of the mechanism has been generated by the mechanism, there should not be a mutually profitable deviation to another mechanism, given the updated posterior beliefs. However, Forges' solution is unapplicable here: in the auction scenario the set of posteriorly optimal mechanisms would typically be empty.

6Veto-incentive compatibility = incentive compatibility + individual rationality after all histories (e.g. Forges, 1998).
the upsetting relation we partition the set of agreeable mechanisms into stable and unstable ones. Two stability criteria are imposed on a partition: any stable agreeable mechanism can only be upset by an unstable agreeable mechanism, and any unstable agreeable mechanism must be upset by a stable one. We argue that a set of stable mechanisms, the stable set, contains the graph of stationary equilibria of the mechanism reformulation game. As described below, the converse is typically true also.

1.2 Results and Connections to the Literature

Much of the discussion on bargaining and commitment has circulated around the famous Coase conjecture saying that as the commitment ability of the seller becomes weak, all the surplus goes to the buyer. Gul, Sommerschein and Wilson (1986) and Fudenberg, Levine and Tirole (1985) confirm this conjecture in the one buyer situation under the "Gap" case. In the "No Gap" case a stationarity assumption is needed to obtain the same result. Analogously, McAfee and Vincent (1997) study in the auction design context seller’s ability to commit to a minimum reservation price. In the limit, as the time period becomes short and a seller’s commitment power vanishes, a version of the Coase conjecture holds true: trade takes place instantaneously with a price equivalent to static mechanism without a reservation price. However, McAfee-Vincent only address the problem of committing to a reservation price, the mechanism itself is fixed (second-price or first-price). Instead, we allow the seller to reformulate also the mechanism.

It is worth sketching our results, and relate them to the discussion on Coase conjecture. In the single buyer situation, we show that in the Gap case the unique stable set is single valued and the stable mechanism allocates the object to the buyer with the price equal to the minimum of buyer’s positive probability valuations. In the No Gap case this is true for any strongly stationary stable mechanisms. Thus the results parallel to those by Gul et.al. and Levine et.al. on the Coase conjecture. Of course, we allow the seller to use arbitrary trading mechanisms, not just simple one sided offers.

In the many-buyer case, we show that a version of the English auction is always strongly stable. The reason is that the English auction effectively reveals two things: (i) the buyer with the highest valuation (the winner), and (ii) the valuations of all but the winner. The seller then faces a similar situation as in the one-buyer case in that now there remains only one relevant bargaining partner whose valuation is unknown. Again, in the absence of commitment the

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7 For other studies on bilateral contracting, renegotiation and commitment, see Freixas, Guesnerie and Tirole (1985), Hart and Tirole (1988), and Laffont and Tirole (1990). For studies on the No Gap case in the durable good monopoly scenario, see Ausubel and Deneckere (1989a,b).

8 In the English (ascending, progressing, open, oral) auction successively higher prices are called until only one bidder remains. He claims the object with the price equal to the last call.

9 Even if the Second Price auction is payoff equivalent with the English auction, the latter is informationally less demanding in the following sense: it does not require winner to reveal his type to the seller.
seller is forced to sell the good to the winner with the price equal to the lowest possible winner’s valuation. As this outcome is equivalent with the original recommendation, she can safely commit to implementing it when designing the mechanism.

The result that the English auction is always strongly stable is used to prove that a strongly stable set of mechanism always exists, and it contains efficient only efficient mechanisms. Efficiency follows from the fact that it is always profitable to reject potentially inefficient recommendation by employing the English auction instead. Finally, and remarkably, if valuations are drawn from a finite set, then the strongly stable the English auction is only stable mechanism under any belief. Discretization can be interpreted as a consequence of the existence of a smallest monetary unit.

It is noteworthy that (in discrete case) the seller cannot commit to any other mechanism that the English auction, even if this auction is typically suboptimal among all auction mechanisms. Hence, this result may partly explain why the English auction is, according to casual empiricism, so commonly used. In more general terms, the Coase conjecture can be rephrased as follows: in the absence of commitment (a version of) the English auction is the only feasible trading procedure. Note that this statement applies also to the one buyer case.

This brings us back to the optimal auctions literature. One unattractive feature of the approach is that optimal auctions are typically difficult to describe in natural economic terms. For this reason, many studies have focused only on mechanisms having natural economic interpretation. Milgrom and Weber (1982) study a class of such mechanisms, and identify rather mild conditions under which the English auction is the optimal within this class. This result has been viewed as one major explanation for the popularity of the English auction. In the broader context, however, it is unclear why the seller would disregard more profitable mechanisms. One could imagine evolutionary or competitive forces that would eventually lead one towards more profitable mechanism if such were available. Thus, what is needed is a more general story of why a seller should select the English auction among all possible mechanisms. Our theory can be considered as such theory.

The paper is organized as follows: Section 2 defines the set-up, and Subsection 2.2 explores the game and our introduces our solution. Subsection 3.1 deals with the one buyer problem, and Subsection 3.2 the many-buyer case. Section 4 discusses about the robustness of the model. In the Appendix, we formalize the notion complexity and derive our stationarity restrictions as a consequence of complexity considerations.

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10 Including the English, the second-price, the Dutch, and the first-price auctions. However, Matthews (1987) and Maskin and Riley (1984) show that risk-aversion would make either the Dutch or the first-price auction more profitable than the other auction forms.

11 Lopomo (1998) shows that the English auction is optimal in a class of “simple” mechanisms.
2 Framework

There is a single seller, selling a single indivisible object, and a set $N = \{1, \ldots, n\}, \ n < \infty$, of buyers with typical elements $i, j$, potentially willing to buy the object. Normalize seller’s reservation value, which is public information, to 0. Buyer $i$’s reservation valuation is represented by random variable $\theta_i$ which is $i$’s private information. and distributed on a bounded set $\Theta_i \subseteq \mathbb{R}_+$. The set of possible valuation profiles is $\Theta = \times_{i \in N} \Theta_i$ with an element $\theta = (\theta_i)_{i \in N}$. Write $\Theta_{-i} = \times_{j \neq i} \Theta_i$ with an element $\theta_{-i} = (\theta_i)_{j \neq i}$. For profile $\theta$, denote the $k$-order statistics by $\theta_{(k)}$, and the vector of all other valuations than the $\theta_{(k)}$ order statistic by $\theta_{-(k)}$. Let $p$ be a probability measure on $\Theta$ with support $S(p)$, and $\mathcal{Y}$ the set of all (Lebesgue) measurable sets in $\mathbb{R}^n$. Let $\Delta^\Theta$ be the set of all probability measures $p$ on $\Theta$. Denote the least upper and greatest lower bounds of $i$’s support by $\underline{\theta}(p_i) = \sup S(p_i)$ and $\overline{\theta}(p_i) = \inf S(p_i)$.

2.1 Direct Mechanisms

Denote by $a_i$ the probability that the object is allocated to buyer $i = 1, \ldots, n$ or to the seller $i = 0$. The set of possible random object allocations is then an $n$-dimensional unit simplex $\Delta^n$. Scalar $t_i \in \mathbb{R}$ represents the pure monetary transfer from buyer $i$ to the seller.\footnote{There is one-to-one mapping from types to equilibrium bids.} Use the vector notation $a = (a_0, a_1, \ldots, a_n) \in \Delta^n$ and $t = (t_1, \ldots, t_n) \in \mathbb{R}^n$. A standard auction mechanism specifies an outcome for all valuation profiles $\theta = (\theta_1, \ldots, \theta_n) \in \Theta$. We define a (Lebesgue) measurable function $r$, augmented direct auction mechanism, which specifies for each valuation profile in the support of $p$ a physical outcome and signal in $\mathcal{Y}$ (hence the attribute "augmented")

$$r(\cdot : p) : S(p) \rightarrow \Delta^n \times \mathbb{R}^n \times \mathcal{Y}. \quad (1)$$

An output of $r$, a recommendation, is written compactly $r = (a, t, Y)$. Denote the family of all $r$ functions by $\mathcal{R}$. Given $p$, the set of recommendations that mechanism $r$ generates with positive probability is $r(S(p) : p)$ or, compactly, $r(S : p)$.

The role of signal $Y$ is to allow revelation of finer information than is contained in $(a, t)$ -part of the recommendation. W.l.o.g., we focus on mechanisms that satisfy $Y = r^{-1}(a, t, Y : p) \subseteq S(p)$, for all $(a, t, Y) \in r(\Theta)$. Thus $r^{-1}(a, t, Y : p)$ generates a partition on the set of types generating allocation $(a, t)$. The aim is to simulate not only the outcome function but also the posterior belief formation of any (non-direct) auction mechanism. E.g. first and second price auctions are fully revealing\footnote{Because of risk neutrality, this is without loss of generality.} and hence the augmented direct mechanisms simulating (equilibria of) these auctions satisfy $Y = \{\theta\}$ for all generated recommendations $(a, t, Y)$. With the English auction $Y = \{\theta_i \in S(p_i) : \theta_i \geq \theta_{(2)} \} \times \{\theta_{-i}\}$, where $i$ has the highest valuation, $\theta_{(2)}$ is the second order statistic and $\theta_{-i}$ the vector types of all buyers but $i$. The auction re-

\footnote{Support of $p$ is a smallest closed set $Y$ such that $p(Y) = 1$.}
veals all but the winner’s exact valuation, and the greatest lower bound of his possible valuations (the second highest bid).

A constant mechanism does not change beliefs with probability one: it sends the same recommendation, say \( r = (a, t, S(p)) \), for all \( \theta \in S(p) \). In such case we write \( r(\cdot : p) = 1_r(p) \). If more than one recommendation is sent with positive probability, then posteriors necessarily change some times, and some information is revealed.

Given \( p \), a posterior belief is unique up to variations in countably many measure zero events. In restrict beliefs in measure zero events, we assume that the conditional measure \( p(\cdot : Y) \) satisfies \( S(p(\cdot : Y)) = Y \), for any state \( Y \subseteq S(p) \). The restriction binds only when \( Y \) is a probability zero event. Also denote conditional measures \( p(\theta : Y) \) for \( Y = \Theta_{-i} \times \{ \theta_i \} \) or \( Y = \Theta_{i} \times \{ \theta_{-i} \} \) by \( p_{-i}(\theta_{-i} : \theta_i) \) and \( p_{i}(\theta_i : \theta_{-i}) \), respectively, for any \( \theta \in S(p) \). Now \( p_{-i}(\cdot : \theta_i) \) is the interim belief of \( i \) given \( \theta_i \), and \( p_{i}(\cdot : \theta_{-i}) \) is the belief concerning \( i \)’s type were all other buyers’ types publicly known. In the case of independent valuations, \( p_{i}(\cdot : \theta_{-i}) = p_i \).

Given prior \( p \), mechanism \( r \), and recommendation \( r \), the conditional measure has the form \( p(\theta : r^{-1}(r)) \) for all \( \theta \in \Theta \). Support \( S(p(\cdot : r^{-1}(r : p))) \) is uniquely determined by our assumption. To simplify notation, write \( p(r, r) \) instead of \( p(\cdot : r^{-1}(r : p)) \) when referring to posterior generated from \( p \) according to mechanism \( r \) and recommendation \( r \). Note that \( \Delta^\Theta \) is closed under updating operation: \( p \in \Delta^\Theta \) implies \( p(r, r) \in \Delta^\Theta \). Further, denote \( p(r, r)(r', r') \) the posterior measure generated from \( p(r, r) \) according to \( r' \) and \( r' \), and so forth for any chain of mechanisms and recommendations.

We analyze the situation where players have no ability to commit to the continuation of a mechanism as planned. This means that the buyers can, if they wish, choose their outside option rather than to complete the trading game as planned. We now describe the constraint that is imposed by the this restriction (the other restriction is due the assumption that the seller can redesign the mechanism).

We assume private valuations and risk neutrality. Denote buyer \( i \)’s payoff, given realized recommendation \( r = (a, t, Y) \) and his type \( \theta_i \) by \( u_i(r, \theta_i) \). This ex post payoff of buyer \( i \) is of the form

\[
u_i(r, \theta_i) = \theta_i a_i - t_i\]

Then, given buyer \( i \)’s privately known valuation and publicly known trading mechanism \( r \), \( i \)’s payoff is a random variable \( u_i(r(\cdot, \theta'_i), \theta_i) \) on \( \Theta_{-i} \) whose ex-

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15Note that any mechanism is defined over the whole domain \( \Theta \) whereas the definition of a constant mechanism only requires that a constant mechanism, say \( \pi \), should be constant over \( S(p) \) (less a null set), given \( p \). When referring to constant mechanisms we assume that \( r_i = (0, 0) \) for all \( i \) and all \( \theta \in \Theta \setminus S(p) \). That is, in all zero probability events \( \pi \) implements the no-trade -allocation. Of course, this restriction neither affects the expected payoffs of any player nor incentives of the buyers (this is easy to check).

16This condition is clearly met by any version of conditional measure whenever \( p(Y) > 0 \) (use Bayes’ rule).

17These assumptions can be weakened significantly. See the remarks in the final section.
expected value at the *interim* stage is\(^{18}\)

\[
\mathbb{E}_{p_i}[u_i(r(\theta_{-i}, \theta'_i : p), \theta_i)] = \int_{\Theta_{-i}} u(r(\theta_{-i}, \theta'_i : p), \theta_i)p_{-i}(\theta_{-i} : \theta_i)d\theta_{-i}.
\]

In particular, given private values,

\[
\mathbb{E}_{p_i}[u_i(1_r(\theta : p), \theta)] = u_i(r, \theta).
\]

The trading mechanism must be (interim) *incentive compatible* (IC):

\[
\mathbb{E}_{p_i}[u_i(r(\theta : p), \theta_i)] \geq \mathbb{E}_{p_i}[u_i(r(\theta_{-i}, \theta'_i : p), \theta_i)], \quad \text{for all } \theta_i, \theta'_i \in S(p_i), \text{ for all } i \in N.
\]

Since participation to the mechanism is voluntary, and buyers lack any commitment power, a feasible mechanism must satisfy *individual rationality* constraint at *any* point of the game (assume the value of the outside option is zero for all buyers), including the *ex post* stage. Note that any *ex post individually rational* mechanism exhibits individual rationality also at the interim and ex ante stages. More formally, given prior \(p\) and an allocation \(r\), ex post individual rationality (EXP-IR) requires that

\[
u_i(r, \theta_i) \geq 0, \quad \text{for all } \theta_i \in S(p_i), \text{ for all } i \in N.
\]

Hence, at the (truthful) equilibrium path, EXP-IR requires that any generated recommendation must be at least as profitable than the outside option in any state for any player. Although EXP-IR is implied by veto right at the ex post stage, but not vice versa, it is useful to introduce the concept for expositional reasons.

One cannot use the combination of interim IC and EXP-IR to obtain a characterization of a mechanism which, on the one hand, can be played with truthful strategies and, on the other, is individually rational at *all* information sets: IC and EXP-IR are not measured at the same information set, and hence they need not be independent.\(^{19}\) Namely, if a player is entitled to use his veto power at any information set, then he should be able do that also outside the equilibrium path and, consequently, interim IC may be violated even if mechanism at hand meets EXP-IR constraint in the truthful path. To cope with the problem, define the following constraint.\(^ {20}\)

**Definition 1** Given \(p \in \Delta^\Theta\), mechanism \(r\) satisfies VETO-IC iff

\[
\mathbb{E}_{p_i}[u_i(r(\theta : p), \theta_i)] \geq \mathbb{E}_{p_i}[\max\{u_i(r(\theta_{-i}, \theta'_i : p), \theta_i), 0\}], \quad \text{for all } i \in N, \text{ for all } \theta_i, \theta'_i \in S(p_i).
\]

\(^{18}\)We use the notation \(\mathbb{E}_\Theta f(\theta) = \int_{\Theta \in \Theta} f(\theta)p(d\theta)\) for any measurable real-valued random variable \(f\) on \(\Theta\) and, similarly, the interim expectation \(\mathbb{E}_{p_i} f(\theta) = \int_{\Theta \in \Theta} f(\theta_{-i}, \theta_i)p_{-i}(d\theta_{-i} : \theta_i)\).

\(^{19}\)In the literature, one typically imposes *interim* IR constraint on mechanism. Relative to EXP-IR, interim individual rationality is easier to analyze but hardly more natural (however, EXP-IR excludes important and quite realistic features of mechanisms, such as entry fees or transaction costs). Exceptions from this practice include Forges (1993, 1998) and Gresik (1991, 1996).

\(^{20}\)VETO-IC is defined by Forges (1998), and is closely related to IC\(^{*}\) of Matthews and Postlewaite (1989).
Thus, VETO-IC requires that truthful reporting forms a Bayes-Nash equilibrium in the mechanism even if untruthful strategies may be followed by the use of the veto option. To interpret VETO-IC, consider a mechanism which, after being played by the buyers and after a recommendation is generated, is subject to vetoing by some of the buyers. If this mechanism is playable in truthful strategies such that no player ever uses his veto right, then it satisfies VETO-IC. The interpretation is that the players can refuse unprofitable transactions and fully anticipate this possibility when they report their type.

**Remark 1** Let mechanism \( r \) satisfy VETO-IC. Then \( r \) satisfies IC and EXP-IR (a.s.).

**Proof.** It is obvious that any \( r \) satisfying VETO-IC also satisfies IC. To verify the EXP-IR part, take \( i \) and choose \( \theta_i = \theta'_i \). Then

\[
\mathbb{E}_{p_{-i}}[u_i(r(\theta:p),\theta_i) - \max\{u_i(r(\theta:p),\theta_i),0\}] = \mathbb{E}_{p_{-i}}[\min\{u_i(r(\theta:p),\theta_i),0\}] \geq 0.
\]

Thus, if \( u_i(r,\theta_i) < 0 \) then \( \mathbb{E}_{p_{-i}}[I(r(\theta:p) = r)] = 0 \) and, consequently, if \( \theta \in S(p) \) then \( r \neq r(\theta) \).

By a version of the ”revelation principle” (Myerson 1982), outcomes of a (pure strategy) equilibrium of any mechanism (without random elements) with veto right can be simulated by a truthful equilibrium of a direct mechanism satisfying VETO-IC. Denote the set of trading mechanisms, given \( p \), satisfying VETO-IC by

\[
R(p) = \{ r(\cdot:p) \in R : r(\cdot:p) \text{ satisfies VETO-IC} \}.
\]

Thus, if buyers lack any commitment ability, we can, without loss of generality, confine our attention to the class \( R(p) \) of mechanisms. Mechanisms outside this set are not playable in truthful strategies, or subject to nonparticipation at the some stage of the game. It should be emphasized, however, that our results below are not dependent on the difference between VETO-IC and EXP-IR + IC. All the arguments remain valid also with the latter restriction. VETO-IC assumption is adopted because it is the right way to model situations where participants do not have commitment ability.

Let us next focus on the seller’s problem. Given recommendation \( r(\theta:p) = (a,t,\gamma) \), seller’s payoff is the sum of transfers specified by \( t \):

\[
v(r(\theta:p)) = \sum_{i \in N} t_i.
\]

Given \( p \) and \( r \), seller’s expected payoff is then

\[
v(r,p) = \mathbb{E}_p[v(r(\theta:p))] = \int_{\Theta} v(r(\theta:p))p(\theta)d\theta \tag{2}
\]

By definition, if \( r(\theta:p) \neq r'(\theta:p) \) implies \( \theta \notin S(p) \), then \( v(r,p) = v(r',p) \).

\[\text{21} \text{Where } I(\cdot) \text{ is an indicator function.}\]
2.2 The Grand Game

In the standard optimal auctions literature, the seller maximizes (2) subject to (interim) IC and IR. There the seller is empowered to choose the mechanism and the equilibrium generated by the mechanism. The mechanism-equilibrium pair solving the optimizing problem can be interpreted as a full commitment benchmark. If, instead, the seller changes the rules after buyers have submitted their announcements, then rational buyers would take the rule change into account in their decisions, and the incentive compatibility of the mechanism would break down. In this section, we construct a mechanisms design game which allows the seller can change the rules of the mechanism infinitely many times.

Consider the following grand game.\(^\text{22}\)

- **Stage 0:** Nature chooses distribution \(p^0 \in \Delta^\Theta\). Buyers’ types are drawn according to \(p^0\). Seller designs a finitely long game ending in an outcome recommendation. The game is played by the buyers, and ends in recommendation \(r^0\).

- **Stage \(t = 1,\ldots\):** Seller either (a) proposes recommendation \(r^{t-1}\), or (b) publicly designs a finitely long game which ends in a recommendation.
  
  (a) If all buyers accept proposal \(r^{t-1}\), then it is implemented. Otherwise the game moves to stage \(t + 1\) with \(r^t = r^{t-1}\).
  
  (b) The game is played by the buyers, and ends in recommendation \(r^t\).

- If the game does not end with finite time, then status quo outcome (payoff 0 for all players) is implemented. There is no discounting.

For simplicity, assume there are no random elements in games and players only use pure strategies. It is clear that the grand game may support many perfect Bayesian equilibria (PBE). We maintain the hypothesis that the buyers are passive in the following sense: at any point of the game the seller chooses her most desirable PBE of the continuation game given that she is empowered to do this in the continuation game. Thus PBE must be dynamically consistent. Note that PBE is defined for any prior \(p^0 \in \Delta^\Theta\).

Let us focus on PBEa that for each \(p\) ends in finite time with probability one. Then, by the revelation principle, there is no loss of generality in focusing on PBE strategies having the property that (i) the game ends in stage 1, (ii) the seller designs a VETO-IC mechanism \(r(\cdot : p^0)\), (iii) \(r(\cdot : p^0)\) is played truthfully by the buyers, and (iv) any recommendation \(r\) of \(r(\cdot : p^0)\) is obediently implemented. Moreover, any belief consistent with the PBE beliefs at a terminal node can be generated by appropriately defining the \(Y\) element of the mechanism. The crucial restriction on feasible mechanism is dynamical consistency: which mechanisms in \(R(p^0)\) can the seller commit to?

\(^{22}\)Details of the game are not crucial. They can be altered as long as the seller is allowed redesign the rules before the outcome is executed, and the buyers have access to their outside option any time.
As in Chatterjee-Sabourian (1999), to narrow further the set of feasible equilibria we appeal to complexity considerations. In the Appendix we argue explicitly that due to (lexicographic) aversion towards more complex strategies, the seller will not condition her actions to the past occurrences when choosing her strategy. Therefore, given that the seller chooses the PBE at each stage, we obtain that any mechanism selection PBE must be stationary in the sense that seller’s choice of a mechanism is dependent only on current beliefs $p$.

Thus stationary PBE can be defined by a function $PBE : \Delta^\Theta \rightarrow R$. When identifying function $PBE$, the central question if of dynamic consistency. In PBE the seller can commit not to change the rules given posterior beliefs generated by the mechanism, whereas outside of PBE the seller cannot commit not to change the rules. More formally, dynamically consistent stationary PBE satisfies the following:

- If $r \cdot p = PBE(p)$ then there is no $r \in r(S : p)$ such that $r' \cdot p(r, r) = PBE(p(r, r))$.
- If $r \cdot p \neq PBE(p)$ then there is $r \in r(S : p)$ such that $r' \cdot p(r, r) = PBE(p(r, r))$.

Let $r = PBE(p)$. By the definition of PBE, the seller obediently implements any outcome $r \in r(S : p)$ given that mechanism $r' = PBE(p(r, r))$ becomes available. This implies that, in fact, $r' \cdot p(r, r) = PBE(p(r, r))$. Conversely, if $r \neq PBE(p)$ then the seller does not obediently implement some outcome $r \in r(S(p))$ implying $r' \cdot p(r, r) = PBE(p(r, r))$.

### 2.3 Stable set

It is convenient to characterize stationary PBE of the grand game directly in the language of stable sets. In order to define a stable set we construct a set of agreeable mechanisms, and establish an upsetting relation over this set. For fixed $p \in \Delta^\Theta$, set $R(p)$ is referred as the set of agreeable mechanisms. If commitment by the seller would not be a problem, then any $r \in R(p^0)$ could be implemented. The set of all agreeable mechanisms is the graph of $R$

$$A = \{ R(p) : p \in \Delta^\Theta \}.$$

Formally, the upsetting relation is defined as follows.

**Definition 2** Mechanism $r \cdot p \in A$ is upset by mechanism $r' \cdot p(r, r) \in A$ if $v(r' \cdot p(r, r)) \geq v(r)$ for some $r \in r(S : p)$, $r' \cdot p(r, r) \neq 1_r(p(r, r))$.

23 Von Neumann and Morgenstern (1944) introduced the idea of stable sets. Greenberg (1990) later used the idea to develop a unifying Theory of Social Situations. For applications of the stable set approach, see e.g. Kahn and Mookherjee (1993, 1995). Blume and Sobel (1997) employed stable sets in a closely related manner to characterize communication in a cheap-talk game.

24 Assumption (??) is very convenient here since now we can restrict our attention to agreements generated by measures in $\Delta^\Theta$ rather than general measures which would complicate the analysis without changing the main conclusions (see the Remarks -section).
Alternatively, mechanism $r'(\cdot : p(r, r))$ does not upset $r(\cdot : p)$ if $v(r'(\cdot : p(r, r))) \geq v(r)$ implies $r'(\cdot : p(r, r)) = 1_r(p(r, r))$. Note that the upsetting relation is not complete nor transitive. However, the relation is antisymmetric, two agreeable mechanism cannot upset one another.

To describe which agreeable mechanisms are feasible stationary PBE we appeal to von-Neumann-Morgenstern’s notions of external and internal stability. These together define a stable set.

**Definition 3** $G$ is a stable set of $A$ relative to upsetting relation if and only if

1. (Internal stability) $r(\cdot : p) \in G$ implies there is no $r'(\cdot : p(r, r)) \in G$ such that $r'(\cdot : p(r, r))$ upsets $r(\cdot : p)$

2. (External stability) $r(\cdot : p) \notin G$ implies there is $r'(\cdot : p(r, r)) \in G$ such that $r'(\cdot : p(r, r))$ upsets $r(\cdot : p)$

In the sequel, our aim is to characterize a stable set relative upsetting relation.

**Remark 2** Let $G$ be a stable set, and let $r(\cdot : p) \in A$. Then $r(\cdot : p) \in G$ if and only if $1_r(p(r, r)) \in G$ for all $r \in r(S : p)$.

**Proof.** "If": Suppose that $1_r(p(r, r)) \in G$ for all $r \in r(S : p)$, but $r(\cdot : p) \in A \setminus G$. By external stability there is $r \in r(S : p)$ such that $r'(\cdot : p(r, r)) \in G$ upsets $r(\cdot : p)$. But then $r'(\cdot : p(r, r))$ also upsets $1_r(p(r, r))$, violating internal stability.

"Only if": Suppose that $r(\cdot : p) \in G$, but $1_r(p(r, r)) \notin G$ for some $r \in r(S : p)$. Then, as $r(\cdot : p) \in A$ we have that $1_r(p(r, r)) \in A$. By external stability there is $r'(\cdot : p(r, r)) \in G$ which upsets $1_r(p(r, r))$. But then $r'(\cdot : p(r, r))$ also upsets $r(\cdot : p)$, violating internal stability. [Q.E.D.]

Thus, by construction, any recommendation of a stable auction must be itself a stable constant mechanism, given the updated beliefs. We now argue that for any stationary PBE we can construct a stable that contains it.

**Theorem 1** The graph of a stationary PBE is contained by a stable set.

**Proof.** Let function $PBE : \Delta^\Theta \to R$ reflect stationary PBE choices of the seller. We construct a stable set which contains $PBE(\Delta^\Theta)$. Let $G = \{r(\cdot : p) \in A : r' \text{ does not upset } r(\cdot : p), \text{ for } r' = PBE(p(r, r)), r \in r(S : p)\}$.

Let $r(\cdot : p) = PBE(p)$ for any $p$. Then $r'(\cdot : p(r, r)) = 1_r(p(r, r))$ for $r' = PBE(p(r, r), r) \in r(S : p)$. Thus, $r(\cdot : p) \in G$.

Internal stability: Conversely, if $r'(\cdot : p(r, r))$ upsets $1_r(p(r, r))$ for some $r \in r(S : p)$, then $r'(\cdot : p(r, r)) \notin PBE(p(r, r))$. Thus there is $r' \in r'(S : p(r, r))$ such that $r'(\cdot : p(r, r)(r', r')) = PBE(p(r, r)(r', r')) \notin 1_r(p(r, r)(r', r'))$ and $\forall(r', p(r, r)(r', r')) \geq \forall(1_r(p(r, r)(r', r')))$. But then $r'(\cdot : p(r, r)) \notin G$. 

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External stability: By contradiction, \( r'(\cdot : p) \in A \setminus G \) implies there is \( r(\cdot : p(r', r')) \in G \) such that \( r(\cdot : p(r', r')) \) upsets \( r'(\cdot : p) \), for \( r' \in r'(S : p) \). □

As a consequence, whenever the graph drawn by stable set is singleton, we have a unique PBE of the grand game. At this point, it is too early to state the converse result that any stable set contains a graph of a stationary PBE. However, this indeed shows up to be case. The result is proven in the Subsection 3.2

It follows from Remark 2 that \( 1_r(p), 1_{r'}(\cdot : p) \in G \) implies \( r = r' \). Thus the notion of stable set contains some degree of "Markov" flavor in that it does not matter which stable mechanism generates a posterior, stable recommendation depends only on posterior. However, we sometimes want to strengthen this property of the solution.

**Remark 3** Let \( G \) be a stable set. If \( 1_r(p), 1_{r'}(\cdot : p(r, r')) \in G \) for \( r \in r(S : p) \), then \( v(1_{r'}, p(r, r')) \geq v(1_r, p) \).

Note that, by Remark ??), if two constant stable mechanisms meet EXP-IR under two belief systems, then necessarily seller’s payoff is the same under both recommendations. The next condition contains the idea that given such symmetric situation, where the seller is indifferent, she should make the same choice in both cases. Thus any situation seller’s choices should depend only on her feasible payoffs, not what has happened before or the details of the beliefs. Thus the condition strengthens the Markovian flavor of the solution.

**Definition 4 (Strong Stationarity)** Stable set \( G \) is strongly stationary if \( 1_r(p), 1_{r'}(p(r, r')) \in G \) and \( 1_r(p) \neq 1_{r'}(p(r, r')) \) implies \( v(1_{r'}, p(r, r')) > v(1_r, p) \).

## 3 The Results

**Lemma 1** Let \( G \) be a stable set. Then \( r(\cdot : p) \in G \) only if

\[
v(r) = \sup \{v(s) : 1_s \in R(p(r, r))\}, \quad \text{for all} \ r \in r(S : p). \tag{3}
\]

**Proof.** Suppose that \( r(\cdot : p) \in G \). Suppose that (3) does not hold. Then there is \( r = (a, t) \in r(S : p) \), and a constant mechanism \( 1_s \) s.t.

\[
v(s) > v(r) \quad \text{and} \quad 0 \leq u_i(s, \theta_i), \quad \text{for all} \ \theta \geq \theta(p(r, r)), \quad \text{for all} \ i \in N.
\]

Thus, \( 1_s(p(r, r)) \) upsets \( 1_r(p(r, r)) \). By Remark 2, \( 1_r(p(r, r)) \in G \). Thus, by internal stability, \( 1_s(p(r, r)) \notin G \). By external stability there is \( r'(\cdot : p(r, r)) \), which upsets \( 1_s(p(r, r)) \). But then \( r'(\cdot : p(r, r')) \) also upsets \( 1_r(p(r, r)) \), violating internal stability. Thus, (3) must hold. □

The logic of Lemma 1 is straightforward. Recall that a mechanism is constant if it chooses the same recommendation in (almost) all possible states; a constant mechanism does not change beliefs with probability one. If a constant mechanism is upset by some other constant mechanism, then the former must
necessary not belong to $G$, otherwise the latter would either be upset by an agreeable mechanism in $G$, or it would itself be an element of $G$. In both cases it follows that the original constant mechanism does not belong to $G$.

**Corollary 1** Let $G$ be stable set. Then $r(\cdot : p) \in G$ only if $v(r) = \min_{i \in N} \theta(p_i(r, r))$ for all $r \in r(S : p)$.

Of course, the condition is plausible mainly in situations where any form of commitment is absent. When the seller is indifferent between two recommendations she cannot make her decision contingent on the previous agreeable mechanism.

Finally, we can simplify the exposition by observing that typically one only needs to focus on mechanisms that are deterministic almost everywhere. The following corollary is a straightforward consequence of Corollary 1.

**Corollary 2** Let $G$ be a stable set. Then $1_r(p) \in G$ only if $r = (b(1, \theta(p_i)))_{i \in N}$ where $b \equiv \arg \max \{ \sum_{i=1}^n b_i' \theta(p_i) : b' \in \Delta^{n-1} \}$.

Thus, if $\theta(p_i) > \theta(p_j)$ for $j \neq i$, then $r_i = (1, \theta(p_i))$ and $r_j = (0, 0)$.

### 3.1 The $n = 1$ Case

First we focus on the special one buyer case (to ease the notation, drop the subscript $i$). First we discuss about the "Gap" case, and then the "No Gap" case.\(^{25}\)

First we show that the notion of stable sets meeting the stationarity property is not overly strong: there exists a partition which meets the requirement. To do that, define a following simple mechanism. For any $\theta \in S(p)$, $r^*(\theta : p) = (a, t, Y)$ satisfies $(a, t) = (1, \theta(p))$. Thus $r^*$ is efficient and the buyer pays a price equal to maximal price that he can always accept. Assume that $r^*$ reveals as little information as possible: given outcome $(a, t)$, signal $Y$ satisfies $Y = (r^*)^{-1}(a, t : p)$. By abusing notation, we write from now on simply $r = (a, t)$.

Construct set:

$$G^* = \{ r^*(\cdot : p) : p \in \Delta^3 \}.$$  

**Theorem 2** Set $G^*$ is stable and strongly stationary.

**Proof.** Internal stability: Observe that $\{ (1, \theta(p)) \} = r^*(S : p)$ and $p = p(r^*, (1, \theta(p)))$. Thus $v(1_r, p(r^*, r)) = v(r^*, p(r^*, r))$ for $r = (1, \theta(p))$.

External stability: Suppose that there is no $r \in r(S : p)$ such that $r^*(\cdot : p(r, r))$ upsets $1_r(p(r, r))$. Then $a = 1$ and $t \geq \theta(p(r, r))$ for all $r = (a, t) \in r(S : p)$. On the other hand, by EXP-IR, we have $t \leq \theta(p(r, r))$. Thus $r = (1, \theta(p(r, r)))$ for all $r \in r(S : p)$. In particular, $r(\theta(p) : p) = (1, \theta(p))$. Since any $\theta \geq \theta(p)$ can report $\theta(p)$, it follows by IC that $a = 1$ and $t \leq \theta(p)$ for all $r = (a, t) \in r(S : p)$. But this implies that $r = r^*$, and thus $r(\cdot : p) \in G^*$. By Lemma 1, external stability is established. Finally, it is obvious that $G^*$ meets the strong stationarity property.

\(^{25}\)See e.g. Fudenberg et al. (1985), Gul et al. (1986), Ausubel and Deneckere (1989a,b).
In the previous theorem we used the following argument: whenever \( r = (1, \theta(p)) \in r(S : p) \), there cannot be any other \( r' = (a', t') \) such that \( a' = 1 \) and \( t' > \theta(p) \) in \( r(S(p)) \), otherwise \( \theta \) generating \( r' \) would rather imitate type \( \theta(p) \) and IC would be violated.\(^{26}\) Intuitively, the seller cannot commit to any price above the minimum possible valuation by the buyer since in case of a rejection she is always (weakly) tempted to sell the object with the minimum price. But this implies that the buyer is not willing to reveal any information to obtain the object since he is eventually going to get the object with the lowest possible price.

In literature on "durable good monopolies" (e.g. Fudenberg et al. 1985, Gul et.al. 1986), the "Gap Case" \( \theta(p) > 0 \) is well studied. The result is that any \( \theta \) strictly higher than the seller's valuation is always in \( G \), and \( \theta(p) \) cannot be upset by a mechanism in \( G \).

\(^{26}\)Typically the argument is used when we apply the phrase "by IC, it follows that ...".

**Lemma 2** Suppose that \( G \) is a stable set. Take \( p \) such that \( \theta(p) > 0 \). Then \( r(\cdot : p) \in G \) if and only if \( r = r^* \).

Proof. "Only If": Suppose that \( r(\cdot : p) \in G \) for \( \theta(p) > 0 \), and \( r \neq r^* \). By Corollary 2, \( a = 1, t = \theta(p(r, r)) \) for all \( r = (a, t) \in r(S : p) \). In particular, \( r(\theta(p) : p) = (1, \theta(p)) \). Since any \( \theta \geq \theta(p) \) can report \( \theta(p) \), it follows by IC that \( a = 1 \) and \( t \leq \theta(p) \) for all \( (a, t) \in r(S : p) \). Thus, in fact, \( r = r^* \).

"If": Suppose that \( r(\cdot : p) \notin G \), for \( \theta(p) > 0 \). But since \( r \neq r^* \) implies \( r(\cdot : p) \notin G \) by the "only if" part, \( r^*(p) \) cannot be upset by a mechanism in \( G \).

An obvious corollary of the previous result is that if all possible buyer’s valuations are strictly higher than the seller’s valuation, then the Gap Case is always valid and, hence, the Coase conjecture always holds. Thus, for a given probability measure there is a unique stable mechanism and, consequently, there is a unique stable set.\(^{27}\)

**Theorem 3 (Gap Case)** Let \( \Theta \subset \mathbb{R}^+ \). Then \( G^* \) is the unique strongly stationary stable set.

Thus, would it be known that buyer’s valuation is strictly higher than the seller’s valuation, it follows that the seller cannot extract any surplus from the trade. The result is of interest since there may well be an optimal price for the object that the seller would wish to impose, while taking into account that some of the buyer’s types would not be willing to pay the increased price. This phenomenon results from the fact that any "no-trade" situation would induce

\(^{27}\)Recall that \( \Theta \) is closed by assumption.
the seller to propose another offer where some or all of the originally refused buyers would be willing to execute trade. In particular, an offer having a form of a constant mechanism would be profitable from the perspective of the seller and, by Lemma 1, the corresponding agreeable mechanism would be good.

However, in the No Gap case one obtains an completely reversed result: the amount of stable sets is very large (if Θ is infinite, then there are infinitely many stable sets).

**Theorem 4 (No Gap Case)** Let 0 ∈ Θ. Then any price in Θ can be sustained by a stable mechanism.

**Proof.** First, choose λ ∈ Θ. Construct a transfer function

\[ t^λ(p) = \inf \{ \theta : \theta \in S(p), \theta \geq \lambda \}, \]

and a mechanism

\[ r^λ(\theta : p) = (a^λ(\theta), t^λ(\theta)) = \begin{cases} (1, t^λ(p)), & \text{if } \theta \geq \lambda, \\ (0, 0), & \text{if } \theta < \lambda. \end{cases} \]

Now, construct set \( G^λ \) as follows

\[ G^λ = \left\{ r(\cdot : p) : \begin{array}{l} r^λ(\cdot : p) \text{ if } \theta(\cdot) = 0 \text{ and } \bar{a}(p) \geq \lambda, \\ r^* (\cdot : p) \text{ if } \bar{a}(p) > 0 \text{ or } \bar{a}(p) < \lambda \end{array} \right\}. \]

We show that \( G^λ \) is a stable set. Clearly \( G^λ \) is internally consistent, thus it suffices to check it is externally consistent. Suppose there is no \( r^0 (\cdot : p(r, r)) \in G^λ \) which upsets \( r(\cdot : p) \notin G^λ \), for any \( r \in r(S : p) \). There are two cases: (i) Case \( \theta(p) > 0 \text{ or } \bar{a}(p) < \lambda \). For any \( r \in r(S : p) \), \( 1_r(p(r, r)) \) is not upset by \( r^* (\cdot : p(r, r)) \) since necessarily \( \theta(p(r, r)) > 0 \text{ or } \bar{a}(p(r, r)) < \lambda \). Thus it must be that \( r = (1, \theta(p(r, r))) \). By IC it then follows that \( \partial(p(r, r)) = \bar{a}(p) \) for all \( r \in r(S : p) \). But then \( r(\cdot : p) = r^* (\cdot : p) \in G^λ \), a contradiction. (ii) Case \( \theta(p) = 0 \text{ and } \bar{a}(p) \geq \lambda \). By EXP-IR it follows that \( r(\cdot : p) = r^a = (a, 0), a \in [0, 1] \). Since \( 1_r(a(p(r, r^a))) \) is not upset by \( r^λ (\cdot : p(r, r^a)) \) it follows that \( \bar{a}(p(r, r^a)) \leq \lambda \). But then, since \( 1_r(p(r, r^a)) \) is not upset by \( r^* (\cdot : p(r, r^a)) \), it follows that \( a = 1 \). This implies, by IC, that \( r = (1, 0) \) for all \( r \in r(S : p) \). Thus \( r = r^0 \). But then, since \( \bar{a}(p) > \lambda \), \( r(\cdot : p) \) is upset by \( r^λ (\cdot : p) \in G^λ \). Thus, \( r(\cdot : p) \notin G^λ \). Hence, \( G^λ \) is stable. Finally, because \( \lambda \) can have any value on \( Θ \), the theorem is proven.

The claim proven by constructing a partition whose good agreeable mechanisms have the property that first, for any \( p \) such that \( \theta(p) = 0 \) and \( \bar{a}(p) \geq \lambda \) for some number \( \lambda > 0 \), the corresponding stable mechanism \( r^λ \) is a take-it-or-leave-it offer for price \( \lambda \): any valuation above or equal to \( \lambda \) accepts the offer and any valuation below \( \lambda \) rejects it. Second, if \( \theta(p) > 0 \text{ or } \bar{a}(p) < \lambda \), then apply mechanism \( r^* \). To see why this partition is consistent, take the first situation. Suppose there is a good agreement \( r(\cdot : p) \) such that \( r \) is not the take-it-or-leave-it mechanism with price \( \lambda \). Since \( r(\cdot : p) \) is not upset by a good
agreeable mechanism, it is necessarily true that there is a no-trade recommenda-
tion \( r \) generating posterior \( p(r, r) \) such that \( \bar{\theta}(p(r, r)) = 0 \) and \( \overline{\mathbf{r}}(r, r) \geq \lambda \). Otherwise mechanism \( r^*(p(r, r)) \) would upset \( \mathbf{1}_r(p(r, r)) \). On the other hand, since \( r^\lambda(p(r, r)) \) does not upset \( \mathbf{1}_r(p(r, r)) \), it must be that \( \overline{\mathbf{r}}(r, r) < \lambda \). Also, since \( r^*(p(r, r')) \) does not upset \( \mathbf{1}_r(p(r, r')) \) for any other recommendation \( r' \) of \( r \), it follows from VETO-IC that, in fact, \( r = r^\lambda \). Thus, the partition is externally stable. Internal stability is easy to verify. Finally, since \( \lambda \) was chosen arbitrarily, the claim is proven.

The Theorem essentially says that in the No Gap case the seller can charge any price within a stable set. This result is analogous to Ausubel-Deneckere (1989) which proves that almost any outcome can be supported as a sequential equilibrium in the No Gap case. Their argument is based on reputational strategies: the seller can commit to any pricing behavior since deviating from such path would only trigger a subgame whose equilibrium gives the seller zero profit. However, such strategies do not satisfy the stationarity or a Markovian restriction imposed by Gul. et al.. We next establish an analogous result: agreeable mechanism formation in Theorem 2 contains "reputational" features that do not meet our stationarity restriction.

To identify agreeable mechanisms meeting the stationarity restriction, define function \( \alpha : \Theta \rightarrow [0, 1] \) such that

\[
\alpha(\theta) = \arg \max_{a \in [0,1]} a \theta, \quad \text{for all } \theta \in S(p),
\]

determining whether the object is allocated to the seller or the buyer. Note that \( \alpha \) is efficient. Moreover, only in the case \( \bar{\theta}(p) = 0 \) can it be that \( 0 < \alpha(0) < 1 \). Thus, \( \alpha \)'s differ only by how the allocate the object in the case of a tie. Given \( p \) and \( \alpha \), construct mechanism \( r^\alpha \)

\[
r^\alpha(\theta) = (a(\theta), t(\theta)) = \alpha(\theta) (1, \bar{\theta}(p)) \quad \text{for all } \theta \in \Theta.
\]

Note that \( r^\alpha \in \mathbf{R}(p) \), for any \( \alpha \), and that \( r^\alpha = r^* \) if \( \bar{\theta}(p) > 0 \).

**Theorem 5** Let \( \mathbf{G} \) be stable and strongly stationary. Then \( r(\cdot : p) \in \mathbf{G} \) only if \( r = r^\alpha \) for some \( \alpha \), for any \( p \in \Delta^\Theta \).

**Proof.** We show that \( r(\cdot : p) \in \mathbf{G} \) implies \( r = r^\alpha \) for some \( \alpha \). Suppose not. To obtain a contradiction, we proceed in number of steps.

**Step 1:** We claim that \( \bar{\theta}(p) = 0 \). Suppose \( \bar{\theta}(p) > 0 \). But then, by Lemma 2, \( r = r^* \), a contradiction

**Step 2:** We claim that \( \overline{\theta}(p) > 0 \) implies there is \( r = (a, t) \in \mathbf{r}(S : p) \) such that \( \overline{\theta}(p(r, r)) > \overline{\theta}(p(r, r)) = 0 \). Suppose not. Take any \( \theta > 0 \) and \( \mathbf{1}_r = r(\theta : p) \) for \( r = (a, t) \). By assumption \( \overline{\theta}(p(r, r)) > 0 \). By Corollary 2, \( r = (1, \overline{\theta}(p(r, r))) \), \( \theta \geq \overline{\theta}(p(r, r)) \). Since this is true for any \( \theta > 0 \), there is \( r^\theta \) such that \( r^\theta = (1, 0) \). By IC it follows that \( r' = (a', t') \in \mathbf{r}(S : p) \) implies \( a' = 1 \) and \( t' \leq 0 \). However, by Corollary 2, \( t' = \overline{\theta}(p(r, r')) \geq 0 \). But then \( r' = r \) and \( p(r, r) = p(r, r') = p \). Thus \( \overline{\theta}(p) = \overline{\theta}(p(r, r)) > \overline{\theta}(p(r, r)) = 0 \), a contradiction.

**Step 3:** By Steps 1 and 2 there is \( r \in \mathbf{r}(S : p) \) and \( \lambda \) such that \( \overline{\theta}(p(r, r)) \geq \lambda > \overline{\theta}(p(r, r)) = 0 \). Define \( r^\lambda \) as in Theorem 4. Now \( r^\lambda(p(r, r)) \) upsets
Thus, by internal stability, \( r^\lambda(p(r, r) \not\in G \) implying, by Remark 2, that \( 1, p(r, r) \not\in G \) for some \( r^\lambda \in \{(0, 0), (1, t^\lambda p(r, r)\}. If \( r^\lambda = (1, t^\lambda p(r, r) \), then \( 1, p(r, r) \not\in G \) or there is \( r^{\lambda'}(p(r, r)) \in G \), a contradiction. Therefore, \( r^\lambda = (0, 0) \) and \( g(p(r, r)) = 0 \). By external stability, there is \( r^\lambda(p(r, r)) \in G \) which updates \( 1, p(r, r) \not\in G \).

Step 4: Define \( R' = \{1, p : r^\lambda(p(r, r)) \in G \} \). Since \( G \) meets strong stationarity, it follows that \( 1, p \in \{1, p \} \) for all \( r^\lambda \in R' \), thus \( R' = \{1, p \} \). Moreover, since \( r^\lambda \not\in 1, p \) and \( \lambda \), it must be that \( \theta(p(r, r)) \not\in \theta(p(r, r)) \) or \( \theta(p(r, r)) \not\in \lambda \). Construct \( r^\lambda \) as in Theorem 4. Then \( r^\lambda \in R^\lambda(p(r, r)) \). Note that \( p(r, r) \not\in R^\lambda(p(r, r)) \). Thus, \( 1, p \) is a good agreeable mechanism of a stable set meeting the stationarity condition.

Thus, by Theorem 2 to the recommendations \( r^\lambda(p(r, r)) \) in cases \( \theta > \theta(p(r, r)) \) or \( \theta(p(r, r)) \not\in \lambda \). However, \( v(r^\lambda(p(r, r)) \geq v(r) = 0 \) and \( r^\lambda(p(r, r)) \not\in 1, p \). Thus \( r^\lambda(p(r, r)) \) updates \( r \cdot p : p \) in \( G \). But this violates internal consistency.

Roughly, the proof proceeds as follows: for any good agreeable mechanism \( r \cdot p : p \) of a stable set with nondegenerate \( p \) it must necessarily be true that either \( r = r^\star \), or there is \( \lambda > 0 \) and no-trade recommendation \( r \in r(S : p) \) such that \( r^\lambda(p(r, r)) \) updates \( 1, p(r, r) \). The first property follows from Lemma 2, and the latter from VETO-IC. Thus, if \( r \not\in r^\star \), then there is a good agreeable mechanism \( r' \cdot p : p(r, r)(r^\lambda, r^\lambda) \) which updates \( 1, p(r, r) \), for some \( r^\lambda \in r^\lambda(p(r, r)) \). Next we identify a recommendation \( r' \) of mechanism \( r' \) such that \( \theta(p(r, r)) \not\in \theta(p(r, r)) \). By definition, \( r^\lambda(p(r, r)) \not\in \lambda \). Now, if the partition meets the stationarity restriction, then a good agreeable mechanism \( r^\lambda(p(r, r)) \) must be good. But \( r^\lambda(p(r, r)) \) also updates \( r \cdot p : p \) violating internal consistency. Thus, if \( r \cdot p : p \) is a good agreeable mechanism of a stable set meeting the stationarity restriction, then necessarily \( r = r^\star \).

Even though the results in the one-buyer case should not be too surprising, more interesting is that the analysis can be extended to the many-buyer case without complications. It is not clear how one should extend a purely noncooperative framework of durable good monopoly to the many-buyer case.

### 3.2 The \( n \geq 2 \) Case

We now turn to the many-buyer case. For simplicity, assume from now on that \( \Theta \subset \mathbb{R}^n_{++} \). We construct mechanism \( r^\star \) which is a version of the English auction. To do that, we develop some concepts. Recall that \( p_i(\theta : \theta_{-i}) \) is determined for all \( \theta \in S(p) \). For short, write \( p_i(\theta : \theta_{-i}) \) for \( p_i(\cdot : \theta_{-i}) \) and \( p_{-i}(\cdot : \theta_{i}) \) for \( p_{-i}(\cdot : \theta_{i}) \). Denote by \( \theta_{(k)} \) the \( k \)th order statistics of \( \theta = (\theta_1, ..., \theta_n) \). Let \( \theta_{(1)} \) = \( (\theta_{(1)}^1, ..., \theta_{(1)}^n) \). Denote by \( p_{(1)}(\cdot : \theta_{-1}) \) the conditional measure of \( \theta_{(1)} \) given \( \theta_i \) that \( \theta_i = (\theta_{(1)}^i) \) and \( \theta_{-1} \) are known. Then also \( \theta(p_{(1)}(\cdot : \theta_{-1})) = \inf S(p_{(1)}(\cdot : \theta_{-1})) \), for all \( \theta \in S(p) \).

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28 Assuming otherwise would not change the insight.
Thus, \( q(p_1(\theta_{-1})) \) identifies the least upper bound of winner's\(^{29}\) possible valuation above the second highest valuation, given the knowledge of nonwinners’ valuations. Of course, \( q(p_1(\theta_{-1})) \geq \theta(2) \) with equality under symmetrically independent types. As another example, let \( n = 2 \) and suppose \( \theta_1 = c\theta_2 \) for \( c > 1 \) for all \( (\theta_1, \theta_2) \in S(p) \). Then \( q(p_1(\theta_{2})) = c\theta_2 \).

For given \( p \in \Delta^\Theta \), let function \( \alpha : \Theta \rightarrow \Delta^{n-1} \) satisfy

\[
\alpha(\theta) = \arg \max_{\alpha \in \Delta^{n-1}} \sum_{i} \alpha_i \theta_i, \quad \text{for all} \quad \theta \in \Theta.
\]

Thus, \( \alpha \) not only selects a winner in each state but also it specifies a tie-breaking rule for situations when there are more than one highest valuation. Only under tie can it be that \( 0 < \alpha_i(\theta) < 1 \) for some \( i \).

Construct mechanism \( r^\alpha \) as follows. For any \( \theta \in \Theta \), \( r^\alpha(\theta : p) = (a, t, Y) \) satisfies,

\[
(a_i, t_i) = \alpha_i(\theta) \left( 1, q(p_1(\theta_{-1})) \right), \quad \text{for all} \quad i \in N.
\]

Thus \( r^\alpha \) is efficient and the winner pays a transfer equal to his smallest possible valuation above the second highest valuation.\(^{30}\) For any \( \theta \in S(p(r^\alpha, r)) \), we have

\[
(a_i, t_i) = \begin{cases} 
(1, q(p_1(\theta_{-1}))), & \text{if} \quad t_i > \max_{j \neq i} \theta_j, \\
(a, a\theta_i), & \text{if} \quad t_i = \max_{j \neq i} \theta_j, \\
(0, 0), & \text{if} \quad t_i < \max_{j \neq i} \theta_j.
\end{cases}
\]

Thus, recommendation reveals the winner and the price he pays. The price is less or equal to winner’s valuation and at more than equal to any nonwinner’s valuation. Given publicly revealed \( r \), posterior belief \( p(r^\alpha, r) \) then satisfies

\[
q(p_i(r^\alpha, r)) \geq \overline{q}(p_j(r^\alpha, r)) \quad \text{if} \quad a_i = 1,
\]

\[
q(p_i(r^\alpha, r)) = \overline{q}(p_i(r^\alpha, r)) \geq \overline{q}(p_j(r^\alpha, r)) \quad \text{if} \quad a_i \in (0, 1).
\]

Only in the case of a tie we may have \( a_i \in (0, 1) \), revealing that \( i \)'s valuation is \( t_i \).

Assume that \( r^\alpha \) does not reveal finer information that is needed to implement \((a, t)\): signal \( Y \) satisfies \( Y = (r^\alpha)^{-1}(a, t : p) \). Abusing notation, we continue writing simply \( r = (a, t) \).

Note the familiar structure of the mechanism: winner’s announcement does not affect his transfer to the seller as long as he wins. The transfer is determined by other buyers’ announcements. In the case of independent \( p_i \)'s, \( r^\alpha \) coincides with the English or the Second Price auctions in terms of payoffs and allocations. As VETO-IC is more restrictive than IC and interim IR, it may not be obvious that the mechanism meets it. To remove any doubts, the next Lemma argues this is the case.\(^{31}\) For the proof, see Appendix A.1.

\textbf{Lemma 3} \( r^\alpha \in R(p), \) for all \( p \in \Delta^\Theta \).

\(^{29}\)Buyer with the highest valuation.

\(^{30}\)Note that \( r^\alpha \)'s differ only by tie-breaking rule. This does not affect the incentive properties of our mechanisms.

\(^{31}\)See Riley (1988) for a closely related argument.
Note that any mechanisms that is subject to ex post vetoing and whose price is not contingent to winner’s exact valuation. Therefore, overreporting is not dominated since a buyer is entitled to veto if the price is less than his valuation whenever he wins. Thus, a nuisance strategy where a buyer chooses a very high report and then vetoes when EXP-IR is violated is a best response. Nevertheless, also the truthful strategy is a (weakly) dominant strategy. Problems with this multiplicity of equilibria can be avoided by modifying the VETO-IC constraint, e.g. by imposing a fee for vetoing at off-the-equilibrium path.\textsuperscript{32} However, what really really matters in the present paper is the EXP-IR at the equilibrium path.

First we show there exists a strongly stationary stable set that contains \( r^\alpha \).

Construct set \( G^\alpha \) as follows:

\[
G^\alpha = \{ r(\cdot : p) : r(\cdot : p) \text{ is not upset by } r^\alpha(\cdot : p(r, r)), r \in r(S : p), p \in \Delta^\Theta \}. 
\]

**Theorem 6** \( G^\alpha \) is a strongly stationary stable set.

**Proof.** First, note that \( 1_r(p(r^\alpha, r)) = r^\alpha(\cdot : p(r^\alpha, r)) \) for all \( r \in r^\alpha(S : p) \). Thus \( r^\alpha(\cdot : p) \) is not upset by \( r^\alpha(\cdot : p(r^\alpha, r)) \) and, consequently, \( r^\alpha(\cdot : p) \in G^\alpha \) for all \( p \in \Delta^\Theta \).

Internal stability: Suppose that \( 1_r(p(r, r)) \) is upset by \( r'(\cdot : p(r, r)), r \in r(S : p), r(\cdot : p) \in G^\alpha \). It suffices to show that then \( r'(\cdot : p(r, r)) \) is upset by some \( r^\alpha(\cdot : p(r, r)(r', r')) \), \( r' \in r'(S : p(r, r)) \). *Per absurdum*, suppose not.

By contruction, \( 1_r(p(r, r)) \) is not upset by \( r^\alpha(\cdot : p(r, r)) \) for \( r = (a, t) \in r(S : p) \). Then, in fact, \( 1_r(p(r, r)) = r^\alpha(\cdot : p(r, r)) \). Similarly, by supposition \( 1_r(p(r, r)(r', r')) \) is not upset by \( r^\alpha(\cdot : p(r, r)(r', r')) \), and it must be that \( 1_r(p(r, r)(r', r')) = r^\alpha(\cdot : p(r, r)(r', r')) \). There are two cases to consider:

(i) There is winner \( i \) such that \( a_i \in (0, 1) \). Then \( \theta(p_i(r, r)) = \theta(p_i(r, r)) \). But then \( r'(\cdot : p(r, r)) = r^\alpha(\cdot : p(r, r)) = 1_r(p(r, r)), \) for all \( r'(\cdot : p(r, r)) \) does not upset \( 1_r(p(r, r)) \).

(ii) There is winner \( i \) such that \( a_i = 1 \). Then \( \theta(p_i(r^\alpha, r)) \geq \theta(p_j(r^\alpha, r)) \) for \( j \neq i \). Since \( 1_r(p(r, r)(r', r')) = r^\alpha(\cdot : p(r, r)(r', r')) \) we have \( a'_i = 1 \), for all \( r'(r' \in r'(S : p(r, r)) \). Consequently, by IC, \( t' = \theta(p_1(r, r)) \) for all \( r' = (a', t') \). But then \( r'(\cdot : p(r, r)) = r^\alpha(\cdot : p(r, r)) = 1_r(p(r, r)), \) and \( r'(\cdot : p(r, r)) \) does not upset \( 1_r(p(r, r)) \).

External stability: Take \( r(\cdot : p) \notin G^\alpha \). Since \( r(\cdot : p) \notin G^\alpha \), it follows that \( r(\cdot : p) \) is upset by some \( r^\alpha(\cdot : p(r, r)), r = (a, t) \in r(S : p) \). Since \( r^\alpha(\cdot : p(r, r)) \in G^\alpha \), by the first part of the proof, external stability is met.

Finally, by construction \( r^\alpha(\cdot : p(r, r)) = 1_r(p(r, r)) \) for all \( r \in r(S : p) \) such that \( r(\cdot : p) \in G^\alpha \). Thus the partition meets the strong stationarity property.

Thus, \( r^\alpha \) mechanisms have the nice property that they are robust against commitment problems. The intuition behind the argument is exactly the same

\textsuperscript{32}Under such construction one should be able to punish any player who vetoes given recommendation of the mechanism once one has taken the EXP-IR into account when designing the mechanism. Also, one could require EXP-IR to be satisfied only at the *equilibrium path*. This would be perhaps the most straightforward application of the EXP-IR constraint.
as in the one buyer case. The seller can commit to a mechanism which always delivers the good to the winner with a price separating the highest possible valuation of the other buyers from the winner valuation. The reason for this is by now familiar: she cannot commit to raising the price any further since in the case of a rejection in the continuation game she is always (weakly) tempted to sell the object to the winner with the minimum price. This is due the fact in any other reasonable scenario she would need to sell the object with a (weakly) lower price to some other buyer having at most as high valuation as the winner. Thus, if mechanism \( r^* \) is stable in the one buyer case, then also \( r^\alpha \) should be stable in the multiple buyer case.

Furthermore, it is interesting that \( r^\alpha \) mechanisms are the only ones that are robust against commitment problems in our sense. To prove this, we first show that for all priors there is always exists one \( \alpha \) such that \( r^\alpha (\cdot : p) \in G \) for stable set \( G \) which is strongly stationary.

**Theorem 7** Let \( G \) be a strongly stationary stable set. Then \( r^\alpha (\cdot : p) \in G \), for all \( p \in \Delta^\Theta \).

**Proof.** To obtain a contradiction, suppose that there is \( p \in \Delta^\Theta \) such that \( r^\alpha (\cdot : p) \notin G \). By Remark 2 there is \( r^\alpha \in r^\alpha (S : p) \) such that \( 1_{r^\alpha} (p(r^\alpha, r^\alpha)) \notin G \). By external stability, there is \( r (\cdot : p(r^\alpha, r^\alpha)) \in G \) which upsets \( 1_{r^\alpha} (p(r^\alpha, r^\alpha)) \). Thus it suffices to show that existence of \( r (\cdot : p(r^\alpha, r^\alpha)) \) violates strong stationarity.

To proceed, we use the same argument as in the \( n = 1 \) case. We need to replace the seller (with valuation 0) to the buyers \( j \neq i \) with valuation \( \theta_j \leq \theta_i (\cdot : p^\alpha (r^\alpha, r^\alpha)) \). In the \( n = 1 \) case the seller is indifferent between selling or not (= selling it to himself with price 0) the object with price 0 whereas here the seller is indifferent between selling the object to \( i \) with price \( \theta_i (\cdot : p^\alpha (r^\alpha, r^\alpha)) \) or selling it to some \( j \neq i \) with the same price. We proceed in number of steps (analogous to those in Theorem 5). For simplicity, write \( p^\alpha = p(r^\alpha, r^\alpha) \).

**Step 1:** We claim that \( \theta(p^\alpha_j) > \theta(p^\alpha_i) \) for some \( j \neq i \). Suppose \( \theta(p^\alpha_j) > \theta(p^\alpha_i) \) for all \( j \neq i \). Since \( r (\cdot : p^\alpha) \in G \), it follows that \( 1_r (p) \in G \) for all \( r \in r(S : p^\alpha) \). Since \( \theta(p^\alpha_j(r, r)) > \theta(p^\alpha_i(r, r)) \) for all \( r \in r(S : p^\alpha) \) we have, by Corollary 2, that \( r_i = (1, \theta(p^\alpha_j(r, r))) \). But then, by IC it follows that \( \theta(p^\alpha_i(r, r)) \leq \theta(p^\alpha_j(r, r)) \) for all \( r \in r(S : p^\alpha) \). Thus \( 1_r (p^\alpha) = 1_{r^\alpha} (p^\alpha) = r^\alpha (\cdot : p^\alpha) \), a contradiction.

**Step 2:** We claim that \( \theta(p^\alpha_i) > \theta(p^\alpha_j) \) implies there is \( r = (a, t) \in r(S : p^\alpha) \) such that \( \theta(p^\alpha_i(r, r)) > \theta(p^\alpha_j(r, r)) = \theta(p^\alpha_j) \). Suppose not. Then \( \theta(p^\alpha_i(r, r)) > \theta(p^\alpha_j(r, r)) \) for all \( r = (a, t) \in r(S : p^\alpha) \). By Corollary 2, \( r_i = (1, \theta(p^\alpha_j(r, r))) \) for all such \( r \). Again, by IC \( r (\cdot : p^\alpha) = 1_{r^\alpha} (p^\alpha) = r^\alpha (\cdot : p^\alpha) \), a contradiction.

Now, construct mechanism \( r^\lambda \) as follows: for any \( p \) and \( \lambda \in \mathbb{R} \) construct function

\[
t^\lambda_k(p) = \inf \{ \theta_k : \theta_k \in S(p_k), \theta_k \geq \lambda \},
\]

and define the mechanism, for \( k \in N \), by supposing that \( \lambda > \bar{\theta}(p_j) \) for all \( j \neq i \) for some \( i \in N \)

\[
r^\lambda_k(\theta) = (a^\lambda_k(\theta), t^\lambda_k(\theta)) = \begin{cases} (1, t^\lambda_k(p)), & \text{if } \theta_k \geq \lambda, \theta \in S(p), \\ (0, 0), & \text{otherwise.} \end{cases}
\]

(4)
Step 3: By Step 1 and 2, there is \( r \in r(S : p^0) \) and \( \lambda \) such that \( \overline{\theta}(p^0_\lambda(r, r)) \geq \lambda > \overline{\theta}(p^0_i(r, r)) = \overline{\theta}(p^0_j) \). Now \( r^\lambda(p^0(r, r)) \) upsets \( r^\lambda(r, r) \). Thus, by internal stability, \( r^\lambda(p^0(r, r)) \notin G \) implying, by Remark 2, that \( 1, \lambda(p^0(r, r)(r^\lambda, r^\lambda)) \notin G \) for some \( r^\lambda \) such that \( \rho^\lambda(r, r) \in \{ (0, 0), \lambda^1_j(p^0(r, r)) \} \). If \( \rho^\lambda = (1, \lambda^1_j(p^0(r, r)), \rho^\lambda \), then \( 1, \lambda(\overline{\theta}(p^0_i(r, r)(r^\lambda, r^\lambda)) > \overline{\theta}_{\lambda^1_j}(p^0_i(r, r)(r^\lambda, r^\lambda)) \) for \( j \neq i \). Thus, by Corollary 2, \( 1, \lambda(\overline{\theta}(p^0_i(r, r)(r^\lambda, r^\lambda))) \in G \), a contradiction. Therefore, \( \rho^\lambda = (0, 0) \) and \( \theta(p^0_i(r, r)(r^\lambda, r^\lambda)) = \theta(p^0_i(r, r)) = \overline{\theta}(p^0_i) \). By external stability, there is \( r^\lambda(\cdot : p^0(r, r)(r^\lambda, r^\lambda)) \in G \) which upsets \( 1, \lambda(p^0(r, r)(r^\lambda, r^\lambda)) \).

Step 4: Define

\[ R' = \{ 1_{r'} : r' \in r'(S : p^0(r, r)(r^\lambda, r^\lambda)), \theta(p_1(r, r)) = \theta(p_1(r, r)(r^\lambda, r^\lambda)(r', r')) \} \]

Since \( G \) is strongly stationary it follows that \( 1, r' = 1_r = 1_{\alpha} \) for all \( 1_{r'} \in R' \), thus \( R' = \{ 1_{\alpha} \} \). Moreover, since \( r' \neq 1_{\alpha} \), it must be that \( \theta(p_1(r, r)(r^\lambda, r^\lambda)) = \overline{\theta}(p_1(r, r)(r^\lambda, r^\lambda)) \equiv \kappa \). Construct \( r^\kappa \) as in (4). Note that \( p^\kappa(r, r)(r^\lambda, r^\lambda)(r', r^\lambda) = p^\kappa(r, r)(r^\kappa, r^\lambda) \). Thus, \( 1, \lambda(p^\kappa(r, r)(r^\kappa, r^\lambda)) \in G \) and, by applying Corollary 2 to the recommendations submitted in cases \( \theta_i > \overline{\theta}(p_i(r, r)(r^\lambda, r^\lambda)(r', r^\lambda)) \), it follows that \( r^\lambda(p^\kappa(r, r)) \in G \). However, \( v(r^\kappa, r, \kappa(r, r)) \geq v(r) = \overline{\theta}(p_i(r, r)) \) and \( r^\kappa(\cdot : p^\kappa(r, r)) \notin 1_r(p^\kappa(r, r)) \). Thus \( r^\lambda(\cdot : p^\kappa(r, r)) \) upsets \( r(\cdot : p^\kappa) \in G \). But this violates internal consistency. ■

Any recommendation generated by \( r^\alpha \) reveals the buyer with the highest valuation and the greatest lower bound of his possible valuations which is higher than the least upper bound of any other buyer’s valuation. Therefore, the above theorem is analogous to Theorem 2 in that all that needs to be shown is that if there is a good agreeable mechanism which upsets \( r^\alpha(\cdot : p) \), then the stationarity restriction is necessarily violated. One can think the situation as the one where seller’s valuation is upgraded to match the least upper bound of nonwinners’ valuations. In such case, by Lemma 2 we can safely disregard all the other buyers but the winner. But then the problem of the seller is identical with the one buyer case, and one can use the same argument as in Theorem 2 to show that \( 1_r(p) \) is the only agreeable mechanism satisfying the stationarity restriction, for any \( r \in r^\alpha(S : p) \).

The following Corollary follows immediately from the previous theorem. It highlights the "Coasian" flavor of the solution in the multiple buyer case.

**Corollary 3** Let \( G \) be strongly stationary stable set. Then \( r(\cdot : p) \in G \) only if

(i) \( r \) is efficient, and (ii) \( v(r(\theta : p)) \geq \overline{\theta}(S(p^0_j(\theta_{-1}))) \geq \theta(2) \) for all \( \theta \in \Theta \).

To see why \( (r, r) \in G \) implies \( r \) is efficient, note that otherwise there would be agreeable mechanism \( (r^\alpha, p(r, r)) \) which would upset any inefficient \( (r, p(r, r)) \) such that \( r \in r(S : p) \). Since \( (r^\alpha, p) \) is good for some \( \alpha \), an inefficient agreeable mechanism could never satisfy external stability. For the same reason, since buyer can always safely use mechanism \( r^\alpha \), it must be true that her payoff in each state is at least as high as the second highest valuation. The above result strengthens the Coase conjecture in that commitment inability leads to
the efficient allocation of the object. To my knowledge, this result is new in a multi-buyer incomplete information environment.\footnote{Milgrom (1987) analyzes a deterministic auction situation, where all the players know each others’ valuations. He shows that any core outcome in such scenario leads to the efficient allocation of resources. Of course, his analysis does not imply that efficiency is reached also in the incomplete information framework.} Therefore, the analysis provides stronger motivation for the Coase conjecture than has been proposed in the literature.

Furthermore, given (i) and (ii) of the previous corollary, it follows immediately from the Revenue Equivalence Theorem that in the independent values case the English auction is (essentially) the only feasible auction mechanism. However, we can say more. The next theorem says that if the type space (or an infinite set is approximated by a discretized), then the English auction is the only feasible mechanism for all priors. (Alternatively, we could assume the existence of a smallest monetary unit.)

**Theorem 8** Let $G$ be a strongly stationary stable set and $\Theta$ finite. Then $r(\cdot : p) \in G$ if and only if $r(\theta : p) = r^\alpha(\theta : p)$ for all $p$.

**Proof.** We know that $r^\alpha(\cdot : p) \in G$, for all $p$. Now we show the converse: given $p$, if $r(\cdot : p) \in G$ then $r(\cdot : p) = r^\alpha(\cdot : p)$. Take $r(\cdot : p) = (a(\cdot), t(\cdot)) \in G$. Write $\theta_i = \theta(p_i)$ and denote $Y_{-i}(\theta_i, p) = \{\theta_{-i} \in S(p_{-i}(\theta_i)) : \theta_i = \max_{j \in N} \theta_j\}$. By Corollary 3 and EXP-IR we have, for $\theta_{-i} \in Y_{-i}(\theta_i, p)$, that

$$a_i(\theta_{-i}, \theta_i) > 0 \text{ and } \theta_i \geq t_i(\theta_{-i}, \theta_i) \geq \theta(p_{1i}(\theta_{-1})) \quad \quad (5)$$

By the efficiency of $r$, it suffices to show that $t_i(\theta_{-i}, \theta_i) = \theta(p_{1i}(\theta_{-1}))$, for all $\theta_i \in Y_{1i}(\theta_i, p)$.

By (5),

$$\mathbb{E}_p \left[u(r(\theta_{-i}, \theta'_i : p), \theta_i) \right] = \sum_{\theta_{-i} \in \Theta_{-i}} \left[a_i(\theta_{-i}, \theta'_i) \theta_i - t_i(\theta_{-i}, \theta'_i)\right] p_{-i} \left(\theta_{-i} : \theta_i\right)$$

$$= \sum_{\theta_{-i} \in Y_{-i}(\theta'_i, p)} \left[\theta_i - t_i(\theta_{-i}, \theta'_i)\right] p_{-i} \left(\theta_{-i} : \theta_i\right).$$

Let $\theta_i^+ = \min\{\theta_{-i}' \in S(p_{-i}) : \theta_{-i}' > \theta_i\}$. Note that $Y_{-i}(\theta_i, p) \cap S(p_{-i}(\theta_i^+)) \subseteq Y_{-i}(\theta_i^+, p)$. By construction, $\theta(p_{1i}(\theta_{-1})) = \theta_i^+$ for all $\theta_i \in Y_{1i}(\theta_i^+, p) \setminus [Y_{-i}(\theta_i, p) \cap S(p_{-i}(\theta_i^+))]$ and, by (5), for these $\theta_{-i}$,

$$t_i(\theta_{-i}, \theta_i^+) = \theta_i^+. \quad \quad (6)$$

By IC, we have

$$\sum_{\theta_{-i} \in Y_{-i}(\theta_i^+, p)} \left[\theta_i^+ - t_i(\theta_{-i}, \theta_i^+)\right] p_{-i} \left(\theta_{-i} : \theta_i^+\right)$$

$$\geq \sum_{\theta_{-i} \in Y_{-i}(\theta_i^+, p)} \left[\theta_i^+ - t_i(\theta_{-i}, \theta_i)\right] p_{-i} \left(\theta_{-i} : \theta_i^+\right)$$
The initial step which was to be shown. 

\[ \sum_{\theta_{-i} \in Y_{-i}(\theta_i^+)} [\theta_i^+ - t_i(\theta_{-i}, \theta_i^+)] p_{-i} (\theta_{-i} : \theta_i^+) \]

\[ \geq \sum_{\theta_{-i} \in Y_{-i}(\theta_i^+)} [\theta_i^+ - t_i(\theta_{-i}, \theta_i)] p_{-i} (\theta_{-i} : \theta_i^+) \]

\[ + \sum_{\theta_{-i} \in Y_{-i}(\theta_i)} [t_i(\theta_{-i}, \theta_i^+ - \theta_i)] p_{-i} (\theta_{-i} : \theta_i^+) . \]

By (5), \( \theta_i^+ > \theta_i \geq t_i(\theta_{-i}, \theta_i) \). By this and (6) we have (note that \( \theta_{-i} \) has positive probability only in \( S(p_{-i}(\theta_i^+)) \))

\[ \sum_{\theta_{-i} \in Y_{-i}(\theta_i)} [t_i(\theta_{-i}, \theta_i^+) - t_i(\theta_{-i}, \theta_i)] p_{-i} (\theta_{-i} : \theta_i^+) \leq 0. \]

Now, suppose that \( \theta_i \) satisfies

\[ t_i(\theta_{-i}, \theta_i) = \theta(p_{(1)}(\theta_{-i})) \text{, for all } \theta_{-i} \in Y_{-i}(\theta_i, p). \]

That is, the \( i \)-th component of \( r \) agrees with \( r^\alpha \) at \( \theta_i \). As \( Y_{-i}(\theta_i, p) \cap S(p_{-i}(\theta_i^+)) \subseteq Y_{-i}(\theta_i^+, p) \), we have by (5),

\[ \theta(p_{(1)}(\theta_{-i})) \leq t_i(\theta_{-i}, \theta_i^+) \text{, for all } \theta_{-i} \in Y_{-i}(\theta_i, p) \cap S(p_{-i}(\theta_i^+)). \]

Thus, by (7), the weak inequality in (9) must hold as equality for all \( \theta_{-i} \in Y_{-i}(\theta_i^+, p) \). Therefore, (8) implies

\[ t_i(\theta_{-i}, \theta_i^+) = \theta(p_{(1)}(\theta_{-i})) \text{, for all } \theta_{-i} \in Y_{-i}(\theta_i, p) \cap S(p_{-i}(\theta_i^+)). \]

This together with (6) implies

\[ t_i(\theta_{-i}, \theta_i^+) = \theta(p_{(1)}(\theta_{-i})) \text{, for all } \theta_{-i} \in Y_{-i}(\theta_i^+, p). \]

Finally, order the elements of \( S(p_i) \) by \( \theta_i^0 < ... < \theta_i^K \) for some finite \( K \). The initial step \( \theta_i^0 = \theta \) obviously satisfies property (8). Replace \( \theta_i = \theta_i^k \) and \( \theta_i^+ = \theta_i^{k+1} \) for \( k = 0, ..., K - 1 \). By the argument above, use induction to get

\[ t_i(\theta_{-i}, \theta_i^k) = \theta(p_{(1)}(\theta_{-i})) \text{, for all } \theta_{-i} \in Y_{-i}(\theta_i^k, p) \text{ for all } k = 0, ..., K - 1, \]

which was to be shown. \( \blacksquare \)

To get the intuition, let \( \Theta_i = \{1, 2\} \) for all \( i \), and suppose \( p \) has full support, \( S(p) = \{1, 2\}^n \). If \( \theta_i = 1 \), then \( i \) cannot ever gain any surplus. only be winner if \( \theta_j = 1 \) for all \( j \neq 1 \) and then
3.3 Summary and Interpretation

In the light of stable sets, the main problem for the seller is to construct a mechanism which is not destabilized by any good agreeable mechanism. In the standard case, the simplest way to do this is, first, to elicit information regarding who has the highest valuation for the object, say $i$, and, second, elicit information of the exact valuation of the other buyers’. Call mechanism that performs this task by $r^α$. When this information is public the seller can commit to selling the object to the player $i$ since anything else would require her to make an incredible threat of selling the object to some other buyer with a lower price. By the same argument as in the one buyer case, such agreeable mechanism would be upset by a good agreeable mechanism within any stable set meeting the stationarity property. Knowing this, $i$ would not reveal information regarding his valuation since he can always buy the object with the price of his lowest possible valuation. Hence, agreeable mechanism employing $r^α$ is indeed good. Moreover, unless an agreeable mechanism is robust against upsetting by an agreeable mechanism employing $r^α$, then the agreeable mechanism is bad. This restriction in turn implies that any good agreeable mechanism is efficient. In the case of independent absolutely continuous valuations, this result is very forceful, and by the Revenue Equivalence Theorem together with the observation that in each state the price is at least the second highest valuation, it follows that any stable mechanism must be generically equivalent with $r^α$.

What is the interpretation of mechanism $r^α$ which was used as a workhorse in the previous analysis? Interestingly, the informational properties of mechanism $r^α$ cannot be associated to any other standard auction form than the English auction: the English auction reveals the magnitude of the second highest valuation and the player with the highest valuation. For example, in the Dutch or in the First Price auctions bidding strategies in the efficient equilibrium are strictly increasing in buyers’ types. Such strategies necessarily reveal too much information to have desirable stability properties. The Second Price (Vickrey) auction requires that there is an impartial mediator which converts buyers’ bids to recommendations and, in particular, does not reveal players’ announcements to the seller. In the absence of such mediation, the Second Price auction is not applicable. Thus, the English auction is the only standard auction form which has required informational properties and, conversely, any stable auction mechanism must have informational and allocational properties which closely resemble those of the English auction (the only variation can only concern the extent that valuations below the second highest valuation are revealed).

4 Robustness of the Model

Risk aversion. If buyers are risk averse things become more complex in the standard auction design scenario. For example, revenue equivalence breaks

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34 In the Dutch or first price sealed bid auctions, there typically exist also inefficient equilibria. Obviously, they cannot be associated with $π$.

35 For analysis, see e.g. Maskin-Riley (1984) or Matthews (1987).
down and the English auction is not the optimal auction form even in the symmetric case. However, or results are fairly insensitive to risk aversion. To see why, observe that Lemma 1, which was fundamental for the analysis, holds true whenever utility for money is *strictly increasing*. Also, as the take-it-or-leave-it offers and the English auction meet the VETO-IC with any such utility specification, our arguments should be extendable to cover this case as well. It is also interesting that some inconvenient structures of optimal auctions under risk aversion (elaborated entry fee and penalty systems) cannot be entertained by any stable mechanism.

**Other motivations for stationarity** The stationarity restriction is rather demanding: the seller cannot even weakly commit to the imposed mechanism. In a more complete model with repeated interaction this assumption may not be natural. Are there ways recover the results even without the stationarity restriction? There are several slight modifications of the model that allow us to do that. An obvious restriction is to assume that $\Theta \subset \mathbb{R}_{++}^n$ is a discrete set where ties never materialize. In such case the Gap Case is always valid, and stationarity restriction never needed. Another option is to modify seller’s payoff function. By letting the seller be slightly concerned about the general welfare, redefine the seller’s utility as follows: $v(r, \theta, \lambda) = v(r) + \lambda (u_1(r, \theta_1) + \ldots + u_n(r, \theta_n))$, for any small $\lambda > 0$. Defining the upsetting relation with respect to this utility specification would imply that (i) the no-trade agreeable mechanism, (ii) the agreeable mechanism which sells the object to the second highest bidder is always upset by a constant agreeable mechanism. Therefore, such agreeable mechanisms cannot ever be good. Consequently, the stationarity restriction is not needed.

5 Concluding Remarks

In this study, we have focused on auction mechanisms under the hypothesis that no player can commit to the completion of a mechanism. Buyers’ inability to make commitments manifests itself as an assumption that they can choose their outside option at any stage of the mechanism. Seller’s commitment inability in turn gives her the option to reformulate the rules of the auction mechanism at any stage of the mechanism. The first problem is taken care of by appropriately reformulating the incentive and participation constraints. To cope with the latter problem, we have developed a solution concept.

Our solution concept is based on the notion of stable sets, introduced by von-Neumann and Morgenstern (1944) and further developed by Greenberg (1990). Our main findings strengthens the famous Coase conjecture: given the inability to make binding commitments, any acceptable, or stable, mechanism must be *efficient*. Intuitively, the reason for this is that with any inefficient mechanism the seller cannot commit not to exploit further trading opportunities. Therefore, inefficient mechanism are subject to renegotiation at the ex post stage. However, any efficient mechanism which sells the object to a buyer with the highest valuation by price at least as high as the second highest valuation can-
not be credibly destabilized since any profitable deviation from such mechanism at the ex post stage would entail that the object is allocated inefficiently under some states. By the previous argument, such mechanism cannot be stable and therefore the deviation is not credible.

By this argument, it follows also that in the independent valuations case the only stable auction mechanism is a version of the English auction. In particular, no other commonly used auction mechanism have similar stability properties. Thus, our result can be viewed as an explanation of why the English auction is one of the most commonly used auction forms.

Hence, the stable set apparatus seems to be very fruitful tool in analyzing commitment problems in mechanism design scenario. Even though the approach is motivated by noncooperative reasoning, it abstracts us from involved analysis of perfect Bayesian equilibria of any dynamic noncooperative model where the seller cannot make binding commitments. Also, our approach is advantageous relative standard noncooperative approach in that it allows one to focus on the general class of feasible auction mechanisms. Therefore we do not need to specify a priori the actual form of the bidding game that is used.\textsuperscript{36}

It is not obvious why a particular bidding game would be more natural than another.

\textsuperscript{36}Which would be necessary if one would merely replicate the approach of the durable good monopoly literature to this multi-buyer situation.
A Appendix

A.1 Machine

Consider a scenario where the seller in her mind uses a machine (or automaton) to implement a strategy. A machine for the seller consists of the following components:

**Definition 5** A machine is a 6-tuple \((Q, R, \Theta, r, \rho, c)\) where

- \(Q\) is a set of states containing \(q^p\) for each \(p \in \Delta^\Theta\),
- \(R\) is a set of functions from \(\Theta\) to \(\Delta^n \times \mathbb{R}^n\),
- \(r : Q \to R\) such that \(r(\cdot : q) \in R(p(\cdot : q))\),
- \(\rho : R \times \Theta \to \Delta^n \times \mathbb{R}^n\) such that \(\rho(r, \theta) = r(\theta : q)\),
- \(c : Q \times R \times (\Delta^n \times \mathbb{R}^n) \to (\Delta^n \times \mathbb{R}^n) \cup Q\) such that \(c(q, r(q), \rho(r(q), \theta)) \in \{\rho(r, \theta)\} \cup Q\).

Since the game is indexed by the prior distribution, the initial state \(q^p\) needs to be indexed by \(p\). Function \(r\) determines the chosen mechanisms, given state \(Q\). Function \(r\) then gives the recommendation of the mechanism, given the true state in \(\Theta\) and the chosen mechanism, function \(c\) finally decides either to implement the recommendation, in which case the machine halts, or to proceed to another state. Note that \(r \circ r \circ c : Q \times \Theta \to (\Delta^n \times \mathbb{R}^n) \cup Q\).

It is clear that with \(Q\) unrestricted, any strategy of the seller can be implemented by a machine. Thus by focusing on machines we do not artificially restrict seller’s strategic options. Nevertheless, the machine-framework endows us with tools to discuss about complexity issues in more detail. Think of the seller choosing the machine before the prior probability distribution is determined. We assume that (i) the seller can replace the machine at any stage of the game if a replacement is (strictly) profitable, (ii) if two machines are equally profitable, she prefers the less complex machine.

In the literature on complexity of games played by automata, the following naïve but simple and intuitive approach is often adopted: the number of states is used as a measure of complexity. The set of states of the machine defines a partition on the set of histories of the game, and complexity is then measured by the size of the partition.

Of course, in the current set-up this counting-states approach is problematic since we need to allow infinitely many of states. Thus, to compare the sizes of the state sets, we need to use other arguments. To proceed to this direction, identify the set of indexed states \(Q^p \subseteq Q\) where \(Q^p = \{Q^p : p \in \Delta\}\). Any machine necessarily contains states \(Q^p\). When evaluating when a size of a set is bigger than another set, we confine our attention to set \(Q^p\). Assume that players wish to avoid needlessly many states in their machine. If there are two machines such that in the first one the set of states consists only of those in \(Q^p\), and the other some extra states as well, then the implementation of first one requires less complex reasoning.
We say that machine $t = (Q, R, \Theta, r, c)$ is minimal if $Q^P = Q$. However, we can identify a condition which captures situations where one of minimal machines is more complex than another. Let $\bar{Q}$ be a partition of $Q^P$ such that $R \in \bar{Q}$ iff $q^p = q^{p'}$ for all $q^p, q^{p'} \in R$. Thus if $Q^P$ consists of distinguished elements, then the partition $\bar{Q}$ consists of singleton sets. We say that partition $\bar{Q}$ is at least as coarse as $\bar{Q}_0$ if $R \subseteq R_0$ for all $R_0 \in \bar{Q}_0$, for some $R \in \bar{Q}$. When the set inclusion is strict in some case, then $\bar{Q}$ is coarser than $\bar{Q}'$. Moreover, $t$ is equally coarse with $t' = (Q', R, \Theta, r', r', c')$ if $\bar{Q}$ is as coarse as $\bar{Q}'$, and $t$ is coarser than $t'$ if $\bar{Q}$ coarser than $\bar{Q}'$.

**Definition 6**

1. Machine $t$ is more complex than machine $t'$ if $t$ is minimal and at least as coarse as $t'$,

2. Machine $t$ is weakly more complex than machine $t'$ if $t$ is minimal and coarser than $t'$

Now, consider the game situation: first the seller chooses a machine to implement a strategy. As she cannot commit not to replace the machine at any later stage, the corresponding strategy must be Perfect Bayesian Equilibrium of the game. Thus, we focus on strategies that form an equilibrium and have the property that there is no other strategy whose machine is less complex while generating the same payoff.

**Definition 7** Strategy of the seller is said to be stationary if it can be implemented by a minimal machine.

A truth-telling PBE strategy $r$ is defined for each history by the following properties,

1. the game ends in the first stage,

2. the conditional distribution $p(\cdot : q)$ is derived via Bayes rule.

**Definition 8** A PBE strategy is (strongly) robust against complexity considerations if there exist no other PBE strategy which is (i) equally profitable and (ii) can be implemented by a (weakly) less complex machine.

The following simple consequence follows from the this complexity argument.

**Proposition 1** A PBE strategy is robust against complexity considerations only if it is stationary.

**Proof.** Suppose that $t$, which implements PBE strategy $r$, is robust against complexity considerations, but $r$ is not stationary. By the revelation principle, any PBE strategy terminates at the end of the first stage. Thus only states in $Q^P$ are reached with positive probability. Replace any state $q \in Q \setminus Q^P$ with state $q^p$ iff $p(\cdot : q) = p$. Clearly, this operation reduces the complexity of the
machine without affecting the payoffs. Thus \( r \) is not robust against complexity considerations. ■

Thus, any nonstationary strategy uses a nonminimal machine, and any such machine can be replaced by a less complex machine which implements an equally profitable strategy (however, such strategy need not be a PBE).

By applying the stronger complexity criterion we obtain the following strengthen- ing of our criterion.

**Proposition 2** A PBE strategy is strongly robust against complexity consider- ations if it is strongly stationary.

**Proof.** Suppose that \( t \), which implements PBE strategy \( r \), is robust against complexity considerations, but \( r \) is not strongly stationary: there are \( r ( \cdot : p ) , ( r' ( \cdot : p' ) \in G \) such that \( r , r' \in \mathbb{R}(p) \) and \( v ( r ( \cdot : p ) ) = v ( r' ( p ) ) \) but \( r \neq r' \). Necessarily \( q_0 \neq q' _0 \).

Replace state \( q'_0 \) with state \( q_0 \). This operation weakly reduces the complexity of the machine without affecting the payoffs. Thus \( r \) is not strongly robust against complexity considerations. ■

This far, we have argued that different kinds of complexity considerations imply different degrees of stationarity from PBE strategies. As a consequence, if one wants to implement a mechanism without commitment ability in a PBE which is robust against complexity considerations, then one arrives to stable mechanisms. However, this leaves open the question whether the converse is also true, i.e. whether stable mechanisms are robust against complexity considerations. Luckily, it is easy to verify that stable mechanisms are robust against strong complexity considerations (and, hence, against complexity considerations).

### A.2 Proof of Lemma 3

We want to show that, for all \( i \in N \), for all \( \theta_i , \theta'_i \in \Theta_i \),

\[
E_{p_{-i}} u_i ( r^a ( \theta , \theta_i ) , \theta_i ) - E_{p_{-i}} \max \{ u_i ( r^a ( \theta_{-i} , \theta'_i ) , \theta_i ) , 0 \} \geq 0. \quad (10)
\]

Recall that mechanism does not depend on the valuations if the winner, as long as it is publicly known who the winner is: \( r ( \theta ) = \alpha ( \theta ) ( 1 , p_{(1)} ( \theta ) ) \) is constant for all \( \theta_i \geq \theta (2) \) in \( S ( p_{(1)} ( \cdot ; \theta_{(1)}) ) \).

Suppose that \( \theta'_i < \theta_i \). Then \( \theta_i < \theta (2) \) implies \( \theta'_i < \theta (2) \) and \( u_i ( r^a ( \theta , \theta_i ) ) = u_i ( r^a ( \theta , \theta_i ) ) = 0 \). However, if \( \theta'_i < \theta (2) \leq \theta_i \), then \( u_i ( r^a ( \theta , \theta_i ) ) = \alpha_i ( \theta | \theta_i - \underline{p}_{(1)} ( \theta ) ) \geq 0 = u_i ( r^a ( \theta_{-i} , \theta'_i ) , \theta_i ) \). Then (10) yields

\[
E_{p_{-i}} \left[ \alpha_i ( \theta | \theta_i - \underline{p}_{(1)} ( \theta ) ) : \theta'_i < \theta (2) \leq \theta_i \right] - E_{p_{-i}} \left[ 0 : \theta'_i < \theta (2) \leq \theta_i \right] = 0 \geq 0.
\]

Thus, underreporting does not pay: \( i \) would only loose the object exactly in the cases where he would gain by having it without changing the price in those cases when he obtains the object.
Suppose that $\theta'_i > \theta_i$. Then $\theta_i \geq \theta_{(2)}$ implies $\theta'_i \geq \theta_{(2)}$. Thus in those cases $r^a(\theta) = r^a(\theta'_i, \theta_{-i})$. However, if $\theta'_i \geq \theta_{(2)} > \theta_i$, then $u_i(r^a(\theta), \theta_i) = \alpha_i(\theta_{-1})((\theta_i - p_{(1)}(\theta_{-1}))) = 0$. On the other hand, since $p_{(1)}(\theta'_i, \theta_{-i}) \geq \theta_{(2)}$, we have $\alpha_i(\theta'_i, \theta_{-i})((\theta_i - p_{(1)}(\theta'_i, \theta_{-i})) \leq 0$. Thus, (10) yields

$$E_{p_{-i}}[0 : \theta'_i \geq \theta_{(2)} > \theta_i] - E_{p_{-i}}[\max\{\alpha_i(\theta'_i, \theta_{-i})((\theta_i - p_{(1)}(\theta'_i, \theta_{-i})), 0) : \theta'_i \geq \theta_{(2)} > \theta_i\]$$

$$= 0 - 0 = 0$$

Consequently, relative to truthful reporting, overreporting would induce recommendations that would suggest $i$ to receive the object in cases where he would need to pay more than his own valuation. This induces he to employ the veto right resulting the 0 payoff. Thus overreporting cannot be profitable. Hence $r^a$ satisfies VETO-IC given any prior information structure $p \in \Delta^\Theta$. 

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