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## Price Competition with Information Disparities in a Vertically Differentiated Duopoly

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**Price Competition with Information  
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# Price Competition with Information Disparities in a Vertically Differentiated Duopoly

## Summary

In this paper we extend the model of vertical product differentiation to also consider information disparities about the extent of quality differences. Equilibrium prices turn out to depend not only on the share of informed consumers but also on uninformed consumers beliefs about quality differences. If uninformed consumers overestimate vertical differentiation, informed consumers exert a positive externality on the purchasers of the high quality good as its price decreases when the share of informed consumers decreases. Considering also that the price of the low quality good increases with the share of informed consumers, higher prices can-not signal high quality goods. If uninformed consumers have pessimistic beliefs and underestimate the extent of vertical differentiation, informed consumers can exert a positive externality on firms. In fact either market demands are inelastic to prices and the profits of the high quality firm increase with the share of informed consumers or market demands are elastic to prices and the profits of both firms increase with the share of informed consumers. In the latter case prices are also equal to those that would prevail with perfect information. In the case of optimistic consumers we can then find some theoretical foundation concerning the fact that information undermines brand, while with pessimistic consumers we can explain demand collapses and insensitivity to price changes due to consumer suspicions about product quality.

**Keywords:** Vertical product differentiation, Asymmetric information, Quality uncertainty, Prices as quality signals

**JEL:** L13, L15

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# 1 Introduction

In the last twenty years there has been an enormous growth in the economic literature devoted to product quality issues. There are two main fields of analysis that characterize this literature. One concerns the impact of asymmetric information about quality on firms and markets. The amount of contributions dealing with these issues is huge and it is impossible to quote them here<sup>1</sup>. The second field of analysis concerns vertical product differentiation models in oligopoly theory (Gabsewicz and Thisse (1979) and Shaked and Sutton (1982)).

Until now there have been only a few attempts to consider vertical product differentiation models that also deal with imperfect information about quality, though in real markets we observe firms - competing in quality and prices - that deal with buyers that are often incompletely informed about product quality. Moreover consumers often differ with regard to their information about product quality. Some consumers may be more informed than others about the value of their purchase to the extent that this value is related to the price-quality relationship. Information diversity among consumers may be due to information provision by consumers associations or Internet shopping, but may also derive from word-of-mouth advertising or from direct and indirect information provision by public agencies. Consumers associations provide their members with quality tests that are extremely useful in comparing the value of different brands of the same product. To the extent that the product is a credence good (Darby and Karni, 1973) no information about quality is conveyed by purchase and consumption. Repeat purchases cannot provide the incentive to supply high quality goods and reputation may fail as a self-enforcing mechanism. Even in the case of experience goods that are purchased less frequently by consumers (durables), information about quality may be scarce. Often firms make huge brand investments like persuasive advertising to get consumer loyalty independently of the information they are able to provide. Thus, excluding public institutions that perform quality tests on drugs and other credence goods, only a consumers association has the expertise to ascertain product quality. Access to the worldwide web gives consumers more opportunity to be informed about the price-quality relationship, considering that some consumers associations are publishing on their websites the results of quality tests that were once available only to their associates and other information intermediaries are performing similar functions. Even when information is provided by public agencies, consumers may differ concerning their ability to process information. Thus information disparities occur even in that case.

The economic implications of splitting the market between informed and uninformed consumers has already been analyzed by Chan and Leland (1982) and Cooper and Ross (1984). In their model they show that if the number of informed consumers is high enough firms are incited to produce high quality goods and prices end up being quality signals. However these models suppose there is perfect competition on the product market, imperfect information be-

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<sup>1</sup> However the reader can see Tirole (1989), chapter two, for a review of the main results obtained by this literature until the end of the eighties.

ing the only market failure that drives equilibrium far from the competitive one<sup>2</sup>. Schwartz and Wilde (1985) consider equilibria where all consumers either prefer high quality goods or prefer low quality goods. Wolinsky (1984) is less explicit concerning his assumptions about market structure, but the only market failure he considers is imperfect information about quality. Moreover Wolinsky differentiates consumers along the willingness-to-pay dimension but concerning imperfect information there are no disparities among them. On the contrary Judd and Riordan (1994) consider a new product monopolist where higher prices signal higher quality products. Thus to our knowledge there is no paper in the literature dealing also with strategic issues arising from vertical product differentiation. Bester (1998), for example, considers the issue of quality uncertainty in a vertically differentiated oligopoly but is not concerned with information disparities. On the contrary, we would like to propose a duopoly model where products are vertically differentiated and consumers are heterogeneous both from the point of view of their willingness to pay for quality and their information about product quality. Income differences among consumers can explain both heterogeneity concerning the willingness to pay for quality and diversity concerning information about product quality<sup>3</sup>.

In this model we shall not deal with the decision of consumers to buy information. The number of informed consumers is exogenously given. Consumers will just choose to buy an high quality or a low quality product on the basis of their willingness to pay for quality, the product price and their information about product quality. In spite of our simplifying assumption, introducing the information variable in a vertically differentiated duopoly complicates the analysis a great deal. Thus in this paper we limit our discussion to the price competition stage.

The paper is organized as follows: in section two we present the model. In section three we show the construction of market demands, distinguishing the case in which uninformed consumers overestimate high quality (optimistic consumers in subsection 3.1) from the case in which uninformed consumers underestimate quality (pessimistic consumers in subsection 3.2). Equilibrium analysis is carried out in section four, thus distinguishing price competition when uninformed consumers are optimistic (subsection 4.1) from price competition when uninformed consumers are pessimistic (subsection 4.2). Some general conclusions follow in section five.

## 2 The Model

We consider a market with  $N$  consumers. Each consumer demands one unit of the product (we assume that the market is covered). Consumer preferences can

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<sup>2</sup> Cfr. also the review by Stiglitz (1987) concerning prices as quality signals.

<sup>3</sup> Actually consumers may obtain information about product quality through membership to a consumers association. Despite lack of data concerning associates, membership is said to be mainly related to two variables: income and the degree of scholarship. As the second variable is correlated to the first one, in our model we assume that information about product quality is also related to the willingness to pay for quality.

be represented by the following quasi-linear utility function:

$$U = \theta q - P$$

The willingness to pay for quality is represented by  $\theta$ , which is uniformly distributed between  $\underline{\theta}$  and  $\bar{\theta}$  with  $\bar{\theta} = \underline{\theta} + 1$  and density  $f(\theta) = 1$ .  $P$  is the market price and  $q$  represents product quality, which can be low ( $q^L$ ) or high ( $q^H$ ). We assume that low quality is a minimum quality standard, enforceable by the government; thus  $q^L = q^\circ$  and  $q^\circ$  is common knowledge. High quality is perfectly known to the producer but completely unknown to the consumer, unless it is informed. Thus informed consumers know the true quality  $q^H$  offered by high quality producers. Uninformed consumers have just an expectation concerning high quality: they are only sure that  $q^H \geq q^\circ$ . To simplify the model we assume that each uninformed consumer has the same expectation  $q^E$  concerning high quality. As we do not put any restriction on  $q^H$  and  $q^E$ , we can immediately distinguish two cases: 1) Either  $q^H > q^E$ , i.e. uninformed consumers are **optimistic**, or 2)  $q^H < q^E$  i.e. uninformed consumers are **pessimistic**. In what follows both the definition of market demands and equilibrium analysis will be carried out separately for these two cases. As to the distinction between informed and uninformed consumers we split the market in two parts, following the distribution of  $\theta$ . Consumers with a willingness to pay for quality  $\theta \geq \theta^*$  are also willing to pay the information cost and observe  $q^H$ . Consumers characterized by a willingness to pay  $\theta < \theta^*$  are not willing to pay the information cost and remain uninformed; thus they do not observe  $q^H$  and have only an expectation  $q^E$  concerning quality. Therefore, the greater is  $\theta^*$  and the lower is the portion of informed consumers. In what follows we shall not put any restriction on the value of  $\theta^*$  except that  $\underline{\theta} \leq \theta^* \leq \bar{\theta}$ . It is important to point out that the decision to become informed (becoming for example a member of a consumers association) is separated from the purchase decisions carried out by consumers in each market. Therefore the information cost paid by each consumer can be spread over a wide amount of goods, and though it can affect the decision to become informed it is not related to purchase decisions in each single market.

The timing structure of our model can be described in the following way:

1. In the first stage the market is split between uninformed and informed consumers.
2. In the second stage firms, taking consumers information as given, choose the quality level
3. In the third stage firms, given their decisions concerning quality, compete in prices.

In the market there are two firms that can produce either a good of quality  $q^\circ$  or a good of quality  $q^H$ . Let firm one specialize in the production of the good of quality  $q^\circ$  and firm two specialize in the production of quality  $q^H$ . There are no fixed production cost and we normalize to zero the variable cost of production. Low quality goods are sold at price  $P_L$  and high quality goods are sold at price  $P_H$ .

### 3 Market Demands

In order to define market demand for the low quality and the high quality product we must start from the definition of the marginal consumer, who is indifferent between buying from firm one or from firm two. However the peculiarity of our model is that the market is split in two parts. Informed consumers observe the true quality  $q^H$  while uninformed consumers just have an expectation about quality:  $q^E$ . Thus we are led to define two marginal consumers. The first is the uninformed marginal consumer  $\theta'$ , who is defined by the following equality:

$$\theta q^\circ - P_L = \theta q^E - P_H$$

giving

$$\theta' = \frac{P_H - P_L}{q^E - q^\circ}$$

Let us call  $\Delta_E = q^E - q^\circ$  the expected quality difference perceived by uninformed consumers.

The second is the informed marginal consumer  $\theta''$  who is defined by the following equality:

$$\theta'' = \frac{P_H - P_L}{q^H - q^\circ}$$

and let us call  $\Delta = q^H - q^\circ$  the true quality difference, known only to informed consumers.

Observing the expressions of marginal consumers we are immediately led to distinguish two main cases. In fact either uninformed consumers are optimistic and thus  $q^E > q^H$  or uninformed consumers are pessimistic and  $q^E < q^H$ . Thus in the optimistic case (case A)  $\theta' < \theta''$  while in the pessimistic case (case B)  $\theta' > \theta''$ . In what follows we shall deal separately with these two cases

#### 3.1 Market demands when uninformed consumers are optimistic ( $\theta' < \theta''$ )

As we do not put any restriction on the value of  $\theta^*$ , we can start by distinguishing three main subcases:

**A.1)**  $\underline{\theta} \leq \theta' \leq \theta^* \leq \theta'' \leq \bar{\theta}$ . A graphical example of this sub-case is given in figure 1. From this figure we see that both the demand for the low quality product ( $D_L$ ) and the demand for the high quality product ( $D_H$ ) are given by the sum of the demand of uninformed consumers plus the sum of the demand of informed consumers:  $D_L = \theta' - \underline{\theta} + \theta'' - \theta^*$  and  $D_H = \theta^* - \theta' + \bar{\theta} - \theta''$ .

From the assumption given above we can obtain the following restrictions concerning market prices, which will be useful to define the price domain of demand functions. Concerning  $D_L$  we get

$$\begin{aligned} P_H - \theta^* \Delta_E &\leq P_L \leq P_H - \underline{\theta} \Delta_E \\ P_H - \bar{\theta} \Delta &\leq P_L \leq P_H - \theta^* \Delta \end{aligned} \tag{1}$$

and concerning  $D_H$

$$\begin{aligned} P_L + \underline{\theta}\Delta_E &\leq P_H \leq P_L + \theta^*\Delta_E \\ P_L + \theta^*\Delta &\leq P_H \leq P_L + \Delta\bar{\theta} \end{aligned} \quad (2)$$

**A.2)**  $\underline{\theta} \leq \theta^* \leq \theta' \leq \theta'' \leq \bar{\theta}$  A graphical example of this sub-case is given in fig.2. Given  $\theta^*$ , the demand for the low quality product is the sum of the demand coming from uninformed consumers,  $\theta^* - \underline{\theta}$  and the demand coming from informed consumers,  $\theta'' - \theta^*$ . Thus we have  $D_L = \theta'' - \underline{\theta}$ . Therefore the demand for the low quality good comes only from uninformed consumers. The demand for the high quality product comes only from informed consumers:  $D_H = \bar{\theta} - \theta''$ . Thus increasing the number of informed consumers reduces the demand for the high quality product with respect to the latter. As in case A.1, we can obtain some restrictions characterizing market prices, from the aforementioned assumptions:

$$\begin{aligned} P_H - \bar{\theta}\Delta &\leq P_L \leq P_H - \theta^*\Delta \\ P_L + \theta^*\Delta_E &\leq P_H \leq P_L + \Delta\bar{\theta} \end{aligned} \quad (3)$$

**A.3)**  $\underline{\theta} \leq \theta' \leq \theta'' \leq \theta^* \leq \bar{\theta}$ . A graphical example of this case is given in fig.3. The demand for the low quality product comes only from uninformed consumers and is given by  $D_L = \theta' - \underline{\theta}$ . The demand for the high quality product comes from both uninformed consumers,  $D_H = \theta^* - \theta'$ , and from informed consumers:  $D_H = \bar{\theta} - \theta^*$ . Thus we have  $D_H = \bar{\theta} - \theta'$ . Reducing the number of informed consumers extends the demand for the high quality product and reduces the demand for the low quality product. Case A.3 boils down to the following restrictions characterizing market prices:

$$\begin{aligned} P_H - \theta^*\Delta &\leq P_L \leq P_H - \underline{\theta}\Delta_E \\ P_L + \underline{\theta}\Delta_E &\leq P_H \leq P_L + \theta^*\Delta \end{aligned} \quad (4)$$

In order to define demand functions we have to consider some further restrictions on the ratio  $\frac{\Delta_E}{\Delta}$ . We can then distinguish the following cases:

**A.a)** By assuming:  $P_H - \underline{\theta}\Delta_E \geq P_H - \theta^*\Delta \geq P_H - \theta^*\Delta_E \geq P_H - \bar{\theta}\Delta$  or  $P_L + \Delta\bar{\theta} \geq P_L + \theta^*\Delta_E \geq P_L + \theta^*\Delta \geq P_L + \underline{\theta}\Delta_E$  we obtain the following restriction concerning the relationship between product quality differences and the weight of informed consumers upon the market:

$$1 \leq \frac{\Delta_E}{\Delta} \leq \text{Min} \left\{ \frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*} \right\} \quad (5)$$

Given the last assumption, we can now specify price domains and market demands:

$$D_L(P_L, P_H) = \begin{cases} \theta' - \underline{\theta} & \text{if } P_H - \theta^*\Delta \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta' - \underline{\theta} + \theta'' - \theta^* & \text{if } P_H - \theta^*\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta'' - \underline{\theta} & \text{if } P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \bar{\theta}\Delta \end{cases} \quad (6)$$



$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta'' & \text{if } P_L + \theta^* \Delta_E \leq P_H \leq P_L + \bar{\theta} \Delta \\ \bar{\theta} - \theta'' + \theta^* - \theta' & \text{if } P_L + \theta^* \Delta \leq P_H \leq P_L + \theta^* \Delta_E \\ \bar{\theta} - \theta' & \text{if } P_L + \underline{\theta} \Delta_E \leq P_H \leq P_L + \theta^* \Delta \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_H \leq P_L + \underline{\theta} \Delta_E \end{cases} \quad (7)$$

**A.b)** Either from (1) by assuming  $P_H - \theta^* \Delta \geq P_H - \underline{\theta} \Delta_E \geq P_H - \bar{\theta} \Delta \geq P_H - \theta^* \Delta_E$  or from (2) by assuming  $P_L + \theta^* \Delta_E \geq P_L + \Delta \bar{\theta} \geq P_L + \underline{\theta} \Delta_E$

$\geq P_L + \theta^* \Delta$  we obtain the following restriction concerning the main parameters of the model:

$$\text{Max} \left\{ \frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}} \right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}} \quad (8)$$

Given this restriction, we can specify market demands as follows:

$$D_L(P_L, P_H) = \begin{cases} \theta'' - \theta^* & \text{if } P_H - \underline{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta \\ \theta'' - \theta^* + \theta' - \underline{\theta} & \text{if } P_H - \Delta \bar{\theta} \leq P_L \leq P_H - \underline{\theta} \Delta_E \\ 1 - \theta^* + \theta' & \text{if } P_H - \theta^* \Delta_E \leq P_L = P_H - \Delta \bar{\theta} \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \Delta_E \theta^* \end{cases} \quad (9)$$

$$D_H(P_L, P_H) = \begin{cases} \theta^* - \theta' & \text{if } P_L + \bar{\theta} \Delta \leq P_H \leq P_L + \theta^* \Delta_E \\ \theta^* - \theta' + \bar{\theta} - \theta'' & \text{if } P_L + \underline{\theta} \Delta_E \leq P_H \leq P_L + \bar{\theta} \Delta \\ 1 - \theta'' + \theta^* & \text{if } P_L + \theta^* \Delta \leq P_H = P_L + \underline{\theta} \Delta_E \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_H \leq P_L + \theta^* \Delta \end{cases} \quad (10)$$

**A.c)** In this sub-case we assume  $P_H - \underline{\theta} \Delta_E \geq P_H - \theta^* \Delta \geq P_H - \bar{\theta} \Delta \geq P_H - \theta^* \Delta_E$  and  $P_L + \theta^* \Delta_E \geq P_L + \Delta \bar{\theta} \geq P_L + \theta^* \Delta \geq P_L + \underline{\theta} \Delta_E$  and obtain the following restrictions:

$$\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}}$$

Concerning demand functions and their price domain we get:

$$D_L(P_L, P_H) = \begin{cases} \theta' - \underline{\theta} & \text{if } P_H - \theta^* \Delta \leq P_L \leq P_H - \underline{\theta} \Delta_E \\ \theta' - \underline{\theta} + \theta'' - \theta^* & \text{if } P_H - \bar{\theta} \Delta \leq P_L \leq P_H - \theta^* \Delta \\ \theta' - \underline{\theta} + \bar{\theta} - \theta^* & \text{if } P_H - \theta^* \Delta_E \leq P_L \leq P_H - \bar{\theta} \Delta \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \theta^* \Delta_E \end{cases} \quad (12)$$

$$D_H(P_L, P_H) = \begin{cases} \theta^* - \theta' & \text{if } P_L + \bar{\theta}\Delta \leq P_H \leq P_L + \theta^*\Delta_E \\ \theta^* - \theta' + \bar{\theta} - \theta'' & \text{if } P_L + \theta^*\Delta \leq P_H \leq P_L + \bar{\theta}\Delta \\ \bar{\theta} - \theta' & \text{if } P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_H \leq P_L + \underline{\theta}\Delta_E \end{cases} \quad (13)$$

**A.d)** In this sub case we assume  $P_H - \theta^*\Delta \geq P_H - \underline{\theta}\Delta_E \geq P_H - \theta^*\Delta_E \geq P_H - \bar{\theta}\Delta$  and  $P_L + \Delta\bar{\theta} \geq P_L + \theta^*\Delta_E \geq P_L + \underline{\theta}\Delta_E \geq P_L + \theta^*\Delta$  and obtain:

$$\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}$$

As for price domains and demand functions we get:

$$D_L(P_L, P_H) = \begin{cases} \theta'' - \theta^* & \text{if } P_H - \underline{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta \\ \theta'' - \theta^* + \theta' - \underline{\theta} & \text{if } P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta_E \\ \theta'' - \underline{\theta} & \text{if } P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \bar{\theta}\Delta \end{cases} \quad (15)$$

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta'' & \text{if } P_L + \theta^*\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta \\ \bar{\theta} - \theta'' + \theta^* - \theta' & \text{if } P_H + \underline{\theta}\Delta_E \leq P_H \leq P_L + \theta^*\Delta_E \\ 1 - \theta'' + \theta^* & \text{if } P_H + \theta^*\Delta \leq P_H \leq P_H + \underline{\theta}\Delta_E \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_H \leq P_H + \theta^*\Delta \end{cases} \quad (16)$$

There would be a further case to consider, leading to  $\frac{\Delta_E}{\Delta} = 1$ ; however in this case one can state that even uninformed consumers are randomly led to identify the true quality level of the high quality good. Therefore demand functions turn out to be completely identical to those obtained in a classical model of vertical product differentiation with complete information, whose results are well known.

### 3.2 Market demands when uninformed consumers are pessimistic ( $\theta'' \leq \theta'$ )

As we did in the case of optimistic consumers we shall not put any restriction on the value of  $\theta^*$ . Thus we are led again to consider three main cases:

**B.1)**  $\underline{\theta} \leq \theta'' \leq \theta^* \leq \theta' \leq \bar{\theta}$

**B.2)**  $\underline{\theta} \leq \theta'' \leq \theta' \leq \theta^* \leq \bar{\theta}$

**B.3)**  $\underline{\theta} \leq \theta^* \leq \theta'' \leq \theta' \leq \bar{\theta}$

**B.1)** A graphical example of this case is given in fig.8. We can see that informed consumers only buy high quality goods, while uninformed consumers only buy low quality goods. Thus information disparities create a separation between the two markets. We get:  $D_L(P_L, P_H) = \theta^* - \bar{\theta}$  and  $D_H(P_L, P_H) = \bar{\theta} - \theta^*$ . Thus market demands are affected only by the weight of informed

consumers as they are perfectly inelastic with respect to prices. Concerning price domains we have:

$$\begin{aligned} P_H - \theta^* \Delta &\leq P_L \leq P_H - \underline{\theta} \Delta \\ P_H - \bar{\theta} \Delta_E &\leq P_L \leq P_H - \theta^* \Delta_E \end{aligned} \quad (17)$$

and

$$\begin{aligned} P_L + \underline{\theta} \Delta &\leq P_H \leq P_L + \theta^* \Delta \\ P_L + \theta^* \Delta_E &\leq P_H \leq P_L + \bar{\theta} \Delta_E \end{aligned} \quad (18)$$

**B.2)** This case is shown in fig.9. Informed consumers only buy high quality goods while uninformed consumers buy both low quality and high quality goods. Thus we get  $D_L(P_L, P_H) = \theta' - \bar{\theta}$  and  $D_H(P_L, P_H) = \bar{\theta} - \theta^* + \theta^* - \theta' = \bar{\theta} - \theta'$ . Concerning price domains we have:

$$P_H - \bar{\theta} \Delta \leq P_L \leq P_H - \theta^* \Delta \quad (19)$$

and:

$$P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta} \Delta_E \quad (20)$$

**B.3)** This case is shown in fig.10. Uninformed consumers buy low quality goods while informed consumers buy both high quality and low quality goods. Thus we get  $D_L(P_L, P_H) = \theta^* - \underline{\theta} + \theta'' - \theta^* = \theta'' - \underline{\theta}$  and  $D_H(P_L, P_H) = \bar{\theta} - \theta''$ . Concerning price domains we have:

$$P_H - \theta^* \Delta_E \leq P_L \leq P_H - \Delta \underline{\theta} \quad (21)$$

and

$$P_L + \Delta \underline{\theta} \leq P_H \leq P_L + \theta^* \Delta_E \quad (22)$$

In order to define demand functions we have to put some further restrictions on the ratio  $\frac{\Delta_E}{\Delta}$ . We can distinguish the following four cases:

**B.a)** Let's assume  $P_H - \underline{\theta} \Delta \geq P_H - \theta^* \Delta_E \geq P_H - \theta^* \Delta \geq P_H - \bar{\theta} \Delta$  and  $P_L + \bar{\theta} \Delta_E \geq P_L + \theta^* \Delta \geq P_L + \theta^* \Delta_E \geq P_L + \underline{\theta} \Delta$  to get:

$$Max \left\{ \frac{\theta^*}{\bar{\theta}}, \frac{\underline{\theta}}{\theta^*} \right\} \leq \frac{\Delta_E}{\Delta} \leq 1 \quad (23)$$

As to demand functions and their price domains we get:

$$D_L(P_L, P_H) = \begin{cases} \theta' - \underline{\theta} & \text{if } P_H - \theta^* \Delta_E \leq P_L \leq P_H - \Delta \underline{\theta} \\ \theta^* - \underline{\theta} & \text{if } P_H - \theta^* \Delta \leq P_L \leq P_H - \theta^* \Delta_E \\ \theta'' - \underline{\theta} & \text{if } P_H - \bar{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \bar{\theta} \Delta_E \end{cases} \quad (24)$$

and

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta'' & \text{if } P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta} \Delta_E \\ \bar{\theta} - \theta^* & \text{if } P_L + \theta^* \Delta_E \leq P_H \leq P_L + \theta^* \Delta \\ \bar{\theta} - \theta' & \text{if } P_L + \underline{\theta} \Delta \leq P_H \leq P_L + \theta^* \Delta_E \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_H \leq P_L + \underline{\theta} \Delta \end{cases} \quad (25)$$

**B.b)** Let's assume  $P_H - \theta^* \Delta_E \geq P_H - \underline{\theta} \Delta \geq P_H - \bar{\theta} \Delta_E \geq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \geq P_L + \bar{\theta} \Delta_E \geq P_L + \underline{\theta} \Delta \geq P_L + \theta^* \Delta_E$  to get:

$$\frac{\underline{\theta}}{\theta} \leq \frac{\Delta_E}{\Delta} \leq \text{Min} \left\{ \frac{\theta^*}{\bar{\theta}}, \frac{\underline{\theta}}{\theta^*} \right\}$$

As to demand functions and price domains we get:

$$D_L(P_L, P_H) = \begin{cases} \theta^* - \underline{\theta} & \text{if } P_H - \underline{\theta} \Delta = P_L \leq P_H - \theta^* \Delta_E \\ \theta^* - \underline{\theta} & \text{if } P_H - \bar{\theta} \Delta_E \leq P_L \leq P_H - \underline{\theta} \Delta \\ \theta^* - \underline{\theta} & \text{if } P_H - \theta^* \Delta \leq P_L = P_H - \bar{\theta} \Delta_E \\ \theta^* - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \theta^* \Delta \end{cases} \quad (26)$$

and

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta^* & \text{if } P_L + \bar{\theta} \Delta_E \leq P_H \leq P_L + \theta^* \Delta \\ \bar{\theta} - \theta^* & \text{if } P_L + \underline{\theta} \Delta \leq P_H \leq P_L + \bar{\theta} \Delta_E \\ \bar{\theta} - \theta^* & \text{if } P_L + \theta^* \Delta_E \leq P_H \leq P_L + \underline{\theta} \Delta \\ \bar{\theta} - \theta^* & \text{if } 0 \leq P_H \leq P_L + \theta^* \Delta_E \end{cases} \quad (27)$$

It is interesting to notice that in this case both demands are perfectly inelastic to prices in the entire price domain. Thus market shares do not depend on prices but on the number of informed consumers.

**B.c)** Let's assume  $P_H - \theta^* \Delta_E \geq P_H - \underline{\theta} \Delta \geq P_H - \bar{\theta} \Delta \geq P_H - \bar{\theta} \Delta_E$  and  $P_L + \bar{\theta} \Delta_E \geq P_L + \bar{\theta} \Delta \geq P_L + \underline{\theta} \Delta \geq P_L + \theta^* \Delta_E$  to get:

$$\frac{\theta^*}{\bar{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\underline{\theta}}{\theta^*}$$

As to demand functions and their price domains we get:

$$D_L(P_L, P_H) = \begin{cases} \theta^* - \underline{\theta} & \text{if } P_H - \underline{\theta} \Delta = P_L \leq P_H - \theta^* \Delta_E \\ \theta^* - \underline{\theta} & \text{if } P_H - \theta^* \Delta \leq P_L \leq P_H - \underline{\theta} \Delta \\ \theta'' - \underline{\theta} & \text{if } P_H - \bar{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \bar{\theta} \Delta_E \end{cases} \quad (29)$$

and

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta'' & \text{if } P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta} \Delta_E \\ \bar{\theta} - \theta^* & \text{if } P_L + \underline{\theta} \Delta \leq P_H \leq P_L + \theta^* \Delta \\ \bar{\theta} - \theta^* & \text{if } P_L + \theta^* \Delta_E \leq P_H = P_L + \underline{\theta} \Delta \\ \bar{\theta} - \theta^* & \text{if } 0 \leq P_H \leq P_L + \theta^* \Delta_E \end{cases} \quad (30)$$

**B.d)** Let's assume  $P_H - \underline{\theta} \Delta \geq P_H - \theta^* \Delta_E \geq P_H - \bar{\theta} \Delta_E \geq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \geq P_L + \bar{\theta} \Delta_E \geq P_L + \theta^* \Delta_E \geq P_L + \underline{\theta} \Delta$  to get:

$$\frac{\underline{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\bar{\theta}}$$

$$D_L(P_L, P_H) = \begin{cases} \theta' - \underline{\theta} & \text{if } P_H - \theta^* \Delta_E \leq P_L \leq P_H - \underline{\theta} \Delta \\ \theta^* - \underline{\theta} & \text{if } P_H - \bar{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta_E \\ \theta^* - \underline{\theta} & \text{if } P_H - \theta^* \Delta \leq P_L \leq P_H - \bar{\theta} \Delta_E \\ \theta^* - \underline{\theta} & \text{if } 0 \leq P_L \leq P_H - \theta^* \Delta \end{cases} \quad (31)$$

and

$$D_H(P_L, P_H) = \begin{cases} \bar{\theta} - \theta^* & \text{if } P_L + \bar{\theta} \Delta_E \leq P_H \leq P_L + \theta^* \Delta \\ \bar{\theta} - \theta^* & \text{if } P_L + \theta^* \Delta_E \leq P_H \leq P_L + \bar{\theta} \Delta_E \\ \bar{\theta} - \theta' & \text{if } P_L + \underline{\theta} \Delta \leq P_H \leq P_L + \theta^* \Delta_E \\ \bar{\theta} - \underline{\theta} & \text{if } 0 \leq P_H \leq P_L + \underline{\theta} \Delta \end{cases} \quad (32)$$

With pessimistic uninformed consumers there would also be a further case to consider, leading to  $\frac{\Delta_E}{\Delta} = 1$ ; however, as with optimistic consumers, in such a case consumers are randomly led to identify the true quality differential and demand functions turn out to be completely identical to those obtained in a classical model of vertical product differentiation with complete information, whose results are well known.

## 4 Equilibrium Analysis

In this section we analyse price competition between the two firms, given expected and real quality differences ( $\Delta_E$  and  $\Delta$ ). Firms decide on prices in a non-cooperative fashion: each seller chooses a strategy that is the best reply to the other seller's strategy. Thus let  $\Pi_i(P_i, P_j) = P_i D_i(P_i, P_j)$   $i, j = L, H$  denote the profit function of firm  $i$ , remembering that we have assumed that firm one sells the low quality product and firm two sells the high quality product

**Definition 2** A price (Nash) equilibrium is a pair  $(P_L^*, P_H^*)$  such that no firm has an incentive to change its price unilaterally:

$$\Pi_i(P_i^*, P_j^*) \geq \Pi_i(P_i, P_j^*) \quad i, j = L, H$$

In the following sub-sections we shall look for a Nash equilibrium in prices both in the optimistic and the pessimistic case. For each configuration of the demand function (i.e. cases A.a-A.d if consumers are optimistic and cases B.a-B.d if consumers are pessimistic) we can find the candidate Nash equilibrium prices, considering each price domain for each demand function. For each case we can moreover obtain the restrictions on the number of informed consumers that result from checking that the candidate equilibrium prices actually belong to the price domains in question.

In order to show that these price pairs are indeed a Nash equilibrium we have to check that the last Definition is satisfied. This will be equivalent to checking that the candidate equilibrium prices assure optimisation of the profit functions not only in the price domains considered one at a time, but also in the entire price range characterising each configuration of the demand functions<sup>4</sup>.

<sup>4</sup>For a similar analytical methodology, see Garella and Martinez Giral (1989)

As we did in the last section we shall carry out a separate analysis for the case of optimistic uninformed consumers and for the case of pessimistic uninformed consumers.

#### 4.1 Price competition with optimistic consumers

Let us start with case A.a, as described in section 2.1. In this case demand functions are represented in fig.4. Since demand functions are piecewise linear and considering the assumption that their entmarket is covered we can state that if a price equilibrium exists it is such that either  $P_H - \theta^* \Delta \leq P_L^* \leq P_H - \underline{\theta} \Delta_E$  and  $P_L + \theta^* \Delta_E \leq P_H^* \leq P_L + \bar{\theta} \Delta$ , (sub-case A.a.1) or  $P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \leq P_H^* \leq P_L + \theta^* \Delta_E$  (sub-case A.a.2) or  $P_H - \bar{\theta} \Delta \leq P_L^* \leq P_H - \theta^* \Delta_E$  and  $P_L + \theta^* \Delta_E \leq P_H^* \leq P_L + \bar{\theta} \Delta$  (sub-case A.a.3).

**A.a.1)** In this sub-case profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta')$ . Solving the f.o.c for  $P_L$  and  $P_H$  we find the following candidate Nash equilibrium prices:

$$P_L^* = \frac{\Delta_E(\bar{\theta} - 2\underline{\theta})}{3}; P_H^* = \frac{\Delta_E(2\bar{\theta} - \underline{\theta})}{3} \quad (33)$$

And, given the following price domains characterising this sub-case:  $P_H - \theta^* \Delta \leq P_L^* \leq P_H - \underline{\theta} \Delta_E$  and  $P_L + \theta^* \Delta_E \leq P_H^* \leq P_L + \bar{\theta} \Delta$ , we can obtain the following restriction concerning the number of informed consumers:

$$\theta^* \geq \frac{\Delta_E(\bar{\theta} + 1)}{3\Delta} \quad (34)$$

As for equilibrium profits we get:

$$\Pi_L^* = \frac{\Delta_E(\bar{\theta} - 2\underline{\theta})^2}{9}; \Pi_H^* = \frac{\Delta_E(2\bar{\theta} - \underline{\theta})^2}{9} \quad (35)$$

**A.a.2)** In this sub-case profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \underline{\theta} + \theta'' - \theta^*)$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta'' + \theta^* - \theta')$ . Solving the f.o.c. for  $P_L$  and  $P_H$  we obtain the following candidate Nash equilibrium prices:

$$P_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]}{3(\Delta_E + \Delta)}; P_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)} \quad (36)$$

Given the price domains characterising this sub-case:  $P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \leq P_H^* \leq P_L + \theta^* \Delta_E$  we obtain the following restriction concerning informed consumers:

$$\frac{\Delta(1 + 2\underline{\theta})}{3\Delta_E + \Delta} \leq \theta^* \leq \frac{\Delta_E(1 + 2\underline{\theta})}{\Delta_E + 3\Delta} \quad (37)$$

and equilibrium profits:

$$\pi_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]^2}{9(\Delta_E + \Delta)}; \pi_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]^2}{9(\Delta_E + \Delta)} \quad (38)$$

**A.a.3)** In this sub-case profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta'')$ . Solving the f.o.c. for  $P_L$  and  $P_H$  we obtain the following candidate Nash equilibrium prices:

$$P_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})}{3}; P_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})}{3} \quad (39)$$

with the following restriction concerning informed consumers:

$$\theta^* \leq \frac{\Delta(1 + 2\underline{\theta})}{3\Delta_E} \quad (40)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})^2}{9}; \Pi_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})^2}{9} \quad (41)$$

Let us now consider case A.b. In this case demand functions are represented in fig.5. As in case A.a we are led to consider three sub-cases to find the candidate Nash Equilibrium prices

**A.b.1)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \theta^*)$  and  $\Pi_H(P_L, P_H) = P_H(1 - \theta'' + \theta^*)$  given price domains:  $P_H - \underline{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta$  and  $P_L + \theta^*\Delta \leq P_H \leq P_L + \underline{\theta}\Delta_E$ . Candidate Nash equilibrium prices are:

$$P_L^* = \frac{\Delta(1 - \theta^*)}{3}; P_H^* = \frac{\Delta(1 + \theta^*)}{3} \quad (42)$$

However this sub-case requires that  $\theta^* \leq 0$ , implying either  $\theta^* < 0$  or  $\theta^* = 0$ . The first inequality cannot hold in this model while the second can hold only by assuming that  $\underline{\theta} = \theta^* = 0$ . This implies that in turn that there are no information disparities as all consumers are informed.

**A.b.2)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \theta^* + \theta' - \underline{\theta})$  and  $D_H(P_L, P_H) = P_H(\bar{\theta} - \theta'' + \theta^* - \theta')$  with price domains:  $P_H - \Delta\bar{\theta} \leq P_L \leq P_H^* - \underline{\theta}\Delta_E$  and  $P_L + \underline{\theta}\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta$ . Candidate Nash equilibrium prices are:

$$P_L^* = \frac{\Delta_E\Delta[1 - (\underline{\theta} + \theta^*)]}{3(\Delta_E + \Delta)}; P_H^* = \frac{\Delta_E\Delta[2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)} \quad (43)$$

given the following restrictions about informed consumers:

$$\frac{(\underline{\theta} - 1)}{2} + \frac{3\Delta_E\underline{\theta}}{2\Delta} \leq \theta^* \leq \frac{\underline{\theta} + 2}{2} + \frac{3\bar{\theta}\Delta}{2\Delta_E} \quad (44)$$

Equilibrium profits are:

$$\Pi_L^* = \frac{\Delta_E\Delta[1 - (\underline{\theta} + \theta^*)]^2}{9(\Delta_E + \Delta)}; \Pi_H^* = \frac{\Delta_E\Delta[2 + \underline{\theta} + \theta^*]^2}{9(\Delta_E + \Delta)} \quad (45)$$

**A.b.3)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(1 - \theta^* + \theta')$  and  $\Pi_H(P_L, P_H) = P_H(\theta^* - \theta')$  with price domains:  $P_H - \theta^* \Delta_E \leq P_L = P_H - \Delta\theta$  and  $P_L + \theta \Delta \leq P_H \leq P_L + \theta^* \Delta_E$ . Candidate equilibrium prices are:

$$P_L^* = \frac{\Delta_E(2 - \theta^*)}{3}; P_H^* = \frac{\Delta_E(1 + \theta^*)}{3} \quad (46)$$

with the following restriction about  $\theta^*$ :

$$\theta^* \geq \frac{1}{2} + \frac{3\Delta\bar{\theta}}{2\Delta_E} \quad (47)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E(2 - \theta^*)^2}{9}; \Pi_H^* = \frac{\Delta_E(1 + \theta^*)^2}{9} \quad (48)$$

Let us now consider case A.c. In this case demand functions are represented in fig. 6:

**A.c.1)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta')$  with price domains  $P_H - \theta^* \Delta \leq P_L \leq P_H - \underline{\theta} \Delta_E$  and  $P_L + \underline{\theta} \Delta_E \leq P_H \leq P_L + \theta^* \Delta$ . Candidate equilibrium prices are:

$$P_L^* = \frac{\Delta_E(\bar{\theta} - 2\underline{\theta})}{3}; P_H^* = \frac{\Delta_E(2\bar{\theta} - \underline{\theta})}{3} \quad (49)$$

with the following restriction on  $\theta^*$ :

$$\theta^* \geq \frac{\Delta_E(1 + 2\bar{\theta})}{3\Delta} \quad (50)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E(\bar{\theta} - 2\underline{\theta})^2}{9}; \Pi_H^* = \frac{\Delta_E(2\bar{\theta} - \underline{\theta})^2}{9} \quad (51)$$

**A.c.2)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \underline{\theta} + \theta'' - \theta^*)$  and  $\Pi_H(P_L, P_H) = P_H(\theta^* - \theta' + \bar{\theta} - \theta'')$  with price domains  $P_H - \bar{\theta} \Delta \leq P_L \leq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta} \Delta$ . Candidate equilibrium prices are:

$$P_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]}{3(\Delta_E + \Delta)}; P_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)} \quad (52)$$

with the following restriction on  $\theta^*$ :

$$\theta^* \leq \text{Min} \left\{ \frac{\Delta_E(1 + 2\underline{\theta})}{\Delta_E + 3\Delta}, \frac{\Delta_E(2 + \underline{\theta}) + 3(\underline{\theta} + 1)\Delta}{2\Delta_E} \right\} \quad (53)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]^2}{9(\Delta_E + \Delta)}; \Pi_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]^2}{9(\Delta_E + \Delta)} \quad (54)$$



**A.c.3)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(1 + \theta' - \theta^*)$  and  $\Pi_H(P_L, P_H) = P_H(\theta^* - \theta')$  with price domains  $P_H - \theta^* \Delta_E \leq P_L \leq P_H - \bar{\theta} \Delta$  and  $P_L + \bar{\theta} \Delta \leq P_H \leq P_L + \theta^* \Delta_E$ . Candidate equilibrium prices are:

$$P_L^* = \frac{\Delta_E(2 - \theta^*)}{3}; P_H^* = \frac{\Delta_E(1 + \theta^*)}{3} \quad (55)$$

with the following restriction on  $\theta^*$  :

$$\theta^* \geq \frac{\Delta_E + 3\bar{\theta}\Delta}{2\Delta_E} \quad (56)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E(2 - \theta^*)^2}{9}; \Pi_H^* = \frac{\Delta_E(1 + \theta^*)^2}{9} \quad (57)$$

Finally let us consider case A.d. In this case demand functions are represented in fig. 7:

**A.d.1)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \theta^*)$  and  $\Pi_H(P_L, P_H) = P_H(1 - \theta'' + \theta^*)$  with price domains  $P_H - \underline{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \leq P_H \leq P_L + \underline{\theta} \Delta_E$ . Candidate equilibrium prices are

$$P_L^* = \frac{\Delta(1 - \theta^*)}{3}; P_H^* = \frac{\Delta(2 + \theta^*)}{3} \quad (58)$$

given the following restriction on  $\theta^*$  :

$$\theta^* \leq \text{Min} \left\{ 1, \left( \frac{3\Delta_E \underline{\theta}}{2\Delta} - \frac{1}{2} \right) \right\} \quad (59)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta(1 - \theta^*)^2}{9}; \Pi_H^* = \frac{\Delta(2 + \theta^*)^2}{9} \quad (60)$$

**A.d.2)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \underline{\theta} + \theta'' - \theta^*)$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta'' + \theta^* - \theta')$  with price domains  $P_H - \theta^* \Delta_E \leq P_L \leq P_H - \underline{\theta} \Delta_E$  and  $P_L + \underline{\theta} \Delta_E \leq P_H \leq P_L + \theta^* \Delta_E$ . Candidate equilibrium prices are:

$$P_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]}{3(\Delta_E + \Delta)}; P_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)} \quad (61)$$

with the following restriction on  $\theta^*$  :

$$\theta^* \geq \text{Max} \left\{ \frac{\Delta(1 + 2\underline{\theta})}{3\Delta_E + \Delta}, \frac{\underline{\theta}(3\Delta_E + \Delta) - \Delta}{2\Delta} \right\} \quad (62)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]^2}{9(\Delta_E + \Delta)}; \Pi_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]^2}{9(\Delta_E + \Delta)} \quad (63)$$

**A.d.3)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta'')$  with price domains  $P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E$  and  $P_L + \theta^*\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta$ . Candidate equilibrium prices are:

$$P_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})}{3}; P_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})}{3} \quad (64)$$

with the following restriction on  $\theta^*$

$$\theta^* \leq \frac{\Delta(1 + 2\underline{\theta})}{3\Delta_E} \quad (65)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})^2}{9}; \Pi_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})^2}{9} \quad (66)$$

We now have to check that the candidate Nash equilibrium prices obtained in the last section are indeed a Nash equilibrium price pair once we are not restricted to a single price domain but consider the entire price range defining each of the four demand functions that characterise the optimistic case. The results of this analysis are summarised in the following propositions:

**Proposition 3** *If uninformed consumers are optimistic, then for different shares of informed consumers in the market, the pair of candidate equilibrium prices  $P_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]}{3(\Delta_E + \Delta)}$ ;  $P_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)}$  is a Nash equilibrium of the price competition game.*

Proof: Let us consider as an example the candidate Nash equilibrium prices obtained when analysing sub-case A.a.2. Given definition one, we must then check that:

$$\begin{aligned} (i) \Pi_L^{A.a.2}(P_L^*, P_H^*) &\geq \Pi_L^{A.a.1}(P_L', P_H^*) \\ (ii) \Pi_L^{A.a.2}(P_L^*, P_H^*) &\geq \Pi_L^{A.a.3}(P_L'', P_H^*) \\ (iii) \Pi_H^{A.a.2}(P_L^*, P_H^*) &\geq \Pi_H^{A.a.1}(P_L^*, P_H') \\ (iv) \Pi_H^{A.a.2}(P_L^*, P_H^*) &\geq \Pi_H^{A.a.3}(P_L^*, P_H'') \end{aligned} \quad (67)$$

where  $P_L'$  maximises profits for firm one in the interval:  $P_H - \theta^*\Delta \leq P_L^* \leq P_H - \underline{\theta}\Delta_E$  (sub-case A.a.1) given  $P_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)}$ ;  $P_L''$  maximises profits for firm one in the interval:  $P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E$  (sub-case A.a.3) given  $P_H^* = \frac{\Delta_E \Delta [2 + \underline{\theta} + \theta^*]}{3(\Delta_E + \Delta)}$ ;  $P_H'$  maximises profits to firm two in the interval  $P_L + \theta^*\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta$  (sub-case A.a.1), given  $P_L^* = \frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]}{3(\Delta_E + \Delta)}$ ;  $P_H''$  maximises profits for firm two in the interval  $P_L + \theta^*\Delta_E \leq P_H \leq P_L + \bar{\theta}\Delta$  (sub-case A.a.3). Clearly profits are defined case by case, using the demand functions related to each sub-case.

We have to check inequality (i):  $\frac{\Delta_E \Delta [1 - (\underline{\theta} + \theta^*)]^2}{9(\Delta_E + \Delta)} \geq P_L'(\theta' - \underline{\theta})$ . However before checking this inequality we have to find  $P_L'$  as a profit maximising price. We

then obtain  $P'_L = -\frac{1}{6} \frac{-2\Delta + 2\Delta\Delta_E - \Delta\theta^* + 3\theta\Delta_E}{\Delta_E + \Delta} \Delta_E$ . Then either  $P'_L$  is interior to the price domain concerning sub-case A.a.1 and thus we have to check (i) or  $P'_L$  is not interior to the price domain and then, since (i) is irrelevant, we can consider (ii). Checking (i) we find that it has (significant) solutions for any value of  $\Delta, \Delta_E$  and  $\theta^*$  once the value of  $\underline{\theta}$  is given. Considering inequality (ii) :  $\frac{\Delta_E\Delta[1-(\underline{\theta}+\theta^*)]^2}{9(\Delta_E+\Delta)} \geq P''_L(\theta'' - \underline{\theta})$  we can now compute  $P''_L$  as the profit maximising price, to obtain  $P''_L = -\frac{1}{6} \frac{-2\Delta_E + 2\theta\Delta_E - \Delta_E\theta^* + 3\Delta\Delta_E}{\Delta_E + \Delta} \Delta$  and check also in this case if  $P''_L$  is interior or not to the price domain concerning sub-case A.a.3. Checking (ii) we find that it has significant solutions for any value of  $\Delta, \Delta_E$  and  $\underline{\theta}$ , once the value of  $\theta^*$  is given. A similar reasoning applies to (iii) and (iv). Checking both inequalities one finds that both come true for any value of  $\Delta, \Delta_E$  and  $\theta^*$  once the value of  $\underline{\theta}$  is given.

Similar results of existence are obtained for cases A.b.2; A.c.2 and A.d.2. We do not report proofs concerning the latter cases for reasons of space. However such an analysis shows that the candidate Nash equilibrium prices  $P_L^* = \frac{\Delta_E\Delta[1-(\underline{\theta}+\theta^*)]}{3(\Delta_E+\Delta)}$ ;  $P_H^* = \frac{\Delta_E\Delta[2+\theta+\theta^*]}{3(\Delta_E+\Delta)}$  are indeed a Nash Equilibrium for any shape of the demand function in the optimistic case.

The results of equilibrium analysis summarised in proposition three are quite general, as they hold in each case where uninformed consumers have optimistic beliefs. Such results show that with the increase of informed consumers in the market ( $\theta^*$  decreases) the equilibrium price of the high quality good decreases while the equilibrium price of the low quality good increases. If there are more and more people that realize that the quality gap between low quality and high quality goods is reduced with respect to their expectations, low quality goods can be sold at higher prices and high quality goods should be sold at lower prices than before. Due to strategic interaction between firms, even the price of the low quality good -whose quality is perfectly known to consumers- is affected by information disparities concerning the extent of the quality gap. From this point of view we can say that with the increase in the number of informed consumers higher prices for the low quality good signal that the quality of the other good is not so higher as expected by uninformed consumers. Though the price of the high quality good still remains higher with respect to the price of the low quality good, the firm selling the lower quality good can obtain a higher profit margin with the increase of informed consumers. What happens is that some consumers with a high willingness to pay for quality decide to refrain from buying the high quality good if they become informed, while others with a lower willingness to pay still buy the high quality good because they are uninformed. On the contrary when the number of informed consumers decreases ( $\theta^*$  increases) the low quality firms earn a lower profit margin. Looking at the expression for the equilibrium price of the low quality good we can easily observe that the higher the willingness to pay of the "poorest" consumer ( $\underline{\theta}$ ) and the lower the share of informed consumers (higher  $\theta^*$ ), the lower is the price of the low quality good. Therefore one can imagine that the equilibrium price of the low quality good could shrink to zero or even become negative if the number of informed

consumer becomes very low and/or the lowest willingness to pay becomes very high. In this case the firm selling the high quality good not only obtains higher profits but becomes a monopolist. Furthermore we can also imagine the high quality firms enjoying a monopolistic position simply due to very small share of informed consumers in the market, even when the willingness to pay of the "poorest" consumer ( $\underline{\theta}$ ) is lower.

Moreover, as equilibrium prices depend on both expected and real quality, firms may be able to increase their revenues either by increasing expected quality or by increasing real quality. In our analysis the cost of quality is not considered but one could assume that expected quality can be increased by persuasive advertising - letting consumers become more and more "optimistic"- and real quality could be increased either by R&D expenses or by variable costs. In a model where costs are also considered, the optimal mix between the two kind of quality enhancing strategies could follow from profit maximisation.

**Proposition 4** *If uninformed consumers are optimistic, assuming  $\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}$  and  $\theta^* \leq \text{Min} \left\{ 1, \left( \frac{3\Delta_E\theta}{2\Delta} - \frac{1}{2} \right) \right\}$ , then the pair of candidate equilibrium prices  $P_L^* = \frac{\Delta(1-\theta^*)}{3}$ ;  $P_H^* = \frac{\Delta(2+\theta^*)}{3}$  is a Nash equilibrium of the price competition game.*

Proof: Let us consider the candidate Nash equilibrium prices obtained when analysing sub-case A.d.1. Given definition one, we must then check that:

$$\begin{aligned} (i) \Pi_L^{A.d.1}(P_L^*, P_H^*) &\geq \Pi_L^{A.d.2}(P_L', P_H^*) \\ (ii) \Pi_L^{A.d.1}(P_L^*, P_H^*) &\geq \Pi_L^{A.d.3}(P_L'', P_H^*) \\ (iii) \Pi_H^{A.d.1}(P_L^*, P_H^*) &\geq \Pi_H^{A.d.2}(P_L^*, P_H') \\ (iv) \Pi_H^{A.d.1}(P_L^*, P_H^*) &\geq \Pi_H^{A.d.3}(P_L^*, P_H'') \end{aligned} \quad (68)$$

where  $P_L'$  maximises profits for firm one in the interval:  $P_H - \theta^*\Delta_E \leq P_L \leq P_H - \underline{\theta}\Delta_E$  (sub-case A.d.2) given  $P_H^* = \frac{\Delta(2+\theta^*)}{3}$ ;  $P_L''$  maximises profits for firm one in the interval:  $P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E$  (sub-case A.d.3) given  $P_H^* = \frac{\Delta(2+\theta^*)}{3}$ . Clearly profits are defined, case by case, using the demand functions concerned in each sub-case.

We have to check inequality (i)  $\frac{\Delta_E(1-\theta^*)^2}{9} \geq P_L'(\theta' - \underline{\theta} + \theta'' - \theta^*)$ . However, before checking this inequality, we have to find  $P_L'$  as a profit maximising price. We then obtain  $P_L' = \frac{1}{6} \frac{2\Delta + 2\Delta_E - 2\theta^*\Delta + \Delta_E\theta^* - 3\Delta\theta}{\Delta + \Delta_E} \Delta_E$ . Then either  $P_L'$  is interior to the price domain concerning sub-case A.d.2 and we have to check (i) or  $P_L'$  is not interior to the price domain and then, since (i) is irrelevant, we can consider (ii). Checking (i) we find that it has (significant) solutions for any value of  $\Delta, \Delta_E$  and  $\theta^*$  once the value of  $\underline{\theta}$  is restricted to an interval of parameters values. Considering inequality (ii)  $\frac{\Delta_E(1-\theta^*)^2}{9} \geq P_L''(\theta'' - \underline{\theta})$  we can now compute  $P_L''$  as the profit maximising price, to obtain  $P_L'' = \frac{1}{3}\Delta_E + \frac{1}{6}\Delta_E\theta^* - \frac{1}{2}\underline{\theta}\Delta$ . Checking (ii) we can show that it comes true for any value of  $\Delta, \Delta_E$  and  $\theta^*$  provided that  $\underline{\theta}$  is restricted to an interval of parameter values. Checking (iii) and (iv) we can also show that a restriction on the value of  $\underline{\theta}$  is required in order for both inequalities to be true.

Turning to proposition four, we can observe that, provided the share of informed consumers is high, and given the appropriate restrictions on the other parameters of the model, there can be other equilibrium prices such that prices are not only independent from  $\underline{\theta}$  and merely depend on the number of informed consumers but they can also be said to represent a signal of the real quality differential supplied by the high quality firm, to the extent that only the true quality differential appears in the equilibrium expressions. Therefore in the case of proposition four we can say that prices can be a signal of quality for uninformed consumers. However, as in the case of proposition three, with the increase of informed consumers the price of the low quality good increases while the price of the high quality good decreases. Thus price can be more and more a signal of the real quality differential but we cannot state that a higher price for the high quality good is a signal of quality when uninformed consumers have optimistic beliefs. Not only the price of the high quality good decrease when the number of informed consumer increses, but for any share of informed consumers in the market prices are lower due to the fact that they depend on the real quality differential, which is lower with respect to the expected one.

**Proposition 5** *If uninformed consumers are optimistic, assuming either  $\text{Max} \left\{ \frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}} \right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$  or  $\frac{\bar{\theta}}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\underline{\theta}}$ , if  $\theta^* \geq \frac{\Delta_E + 3\bar{\theta}\Delta}{2\Delta_E}$ , then the pair of candidate equilibrium prices  $P_L^* = \frac{\Delta(2-\theta^*)}{3}$ ;  $P_H^* = \frac{\Delta(1+\theta^*)}{3}$  is a Nash equilibrium of the price competition game.*

Proof: let us consider the candidate equilibrium prices obtained in sub-cases A.b.3 and A.c.3. We give a proof concerning sub-case A.b.3. Given definition one, we must then check that:

$$\begin{aligned} (i) \Pi_L^{A.b.3}(P_L^*, P_H^*) &\geq \Pi_L^{A.b.1}(P_L', P_H^*) \\ (ii) \Pi_L^{A.b.3}(P_L^*, P_H^*) &\geq \Pi_L^{A.b.2}(P_L'', P_H^*) \\ (iii) \Pi_H^{A.b.3}(P_L^*, P_H^*) &\geq \Pi_H^{A.b.1}(P_L^*, P_H') \\ (iv) \Pi_H^{A.b.3}(P_L^*, P_H^*) &\geq \Pi_H^{A.b.2}(P_L^*, P_H'') \end{aligned} \quad (69)$$

We have to check inequality (i)  $\frac{\Delta_E(2-\theta^*)^2}{9} \geq P_L'(\theta'' - \theta^*)$ . However before checking this inequality we have to find  $P_L'$  as a profit maximising price and then obtain  $P_L' = \frac{1}{6}\Delta_E + \frac{1}{6}\Delta_E\theta^* - \frac{1}{2}\theta^*\Delta$ . Then either  $P_L'$  is interior to the price domain concerning sub-case A.b.1 and we have to check (i) or  $P_L'$  is not interior to the price domain and then, since (i) is irrelevant, we can consider (ii). Checking (i) we find that it has no solutions, so that such a Nash equilibrium exists only considering those parameter values consistent with  $P_L'$  not being interior to the price domain concerning sub-case A.b.1: i.e.  $P_H^* - \underline{\theta}\Delta_E \leq P_L' \leq P_H^* - \theta^*\Delta$ . Then (i) becomes in fact irrelevant and we have to check (ii). Considering inequality (ii) :  $\frac{\Delta_E(2-\theta^*)^2}{9} \geq P_L''(\theta' - \underline{\theta} + \theta'' - \theta^*)$  we can now compute  $P_L''$  as the profit maximising price to obtain  $P_L'' = -\frac{1}{3}\Delta_E\Delta\frac{-1+\theta+\theta^*}{\Delta_E+\Delta}$ . Checking (ii) we can show that significant solutions (implying positive prices) require that

$\Delta_E = \Delta \frac{-3-2\theta+2\theta^*+\theta^2+2\theta\theta^*}{(\theta^*-2)^2}$ . We can further show that inequalities (iii) and (iv) come true imposing some further restrictions both on  $\underline{\theta}$  and  $\theta^*$ .

The results considered in proposition five confirm those summarised in proposition four, to the extent that equilibrium prices depend on the share of informed consumers and on the real quality differential. But the price of the low quality good is even higher and the price of the high quality good is even lower with respect to the prices reported in proposition four. However, considering the restrictions imposed on the share of informed consumers, the price of the low quality good remains lower with respect to the price of the high quality good. Equilibrium prices reported in proposition five can be related to market demands, so that informed consumers buy only low quality goods, while uninformed consumers buy both low quality goods and high quality goods. However, the share of informed consumers need not be very high in this case. What matters is that all the richest and informed consumers choose to buy the low quality good. The high quality good can be sold only to optimistic and uninformed consumers with a lower willingness to pay with respect to uninformed consumers. The fact that the lower quality good is sold to the richest consumers can explain the narrower difference between the price of the high quality good and the price of the low quality good, but the restrictions imposed on the share of informed consumers assure that such a difference is never reversed to the detriment of firm two. The share of informed consumers must be narrow enough to enable firm two to keep an high market share in order to compensate for the reduced price it can charge because of the behavior of the richest consumers. Otherwise its profits could not be higher with respect to the profit of the firm selling the low quality good. However the results reported in proposition five depend on very restrictive assumptions concerning the ratio  $\frac{\Delta_E}{\Delta}$  as one can easily observe checking the proof.

**Proposition 6** *When uninformed consumers are optimistic and either  $1 \leq \frac{\Delta_E}{\Delta} \leq \text{Min} \left\{ \frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*} \right\}$  or  $\text{Max} \left\{ \frac{\bar{\theta}}{\theta^*}, \frac{\theta^*}{\underline{\theta}} \right\} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\underline{\theta}}$  then if the share of informed consumers is such that  $\theta^* \geq \frac{\Delta_E(\bar{\theta}+1)}{3\Delta}$ , the pair of prices  $P_L^* = \frac{\Delta_E(\bar{\theta}-2\underline{\theta})}{3}$ ;  $P_H^* = \frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3}$  is a Nash Equilibrium of the price competition game.*

Proof: Let us consider candidate equilibrium prices in sub-cases A.a.1 and A.b.3. Here we present a proof concerning sub-case A.a.1. Therefore, given definition one, we have to check the following inequalities:

$$\begin{aligned} (i) \Pi_L^{A.a.1}(P_L^*, P_H^*) &\geq \Pi_L^{A.a.2}(P_L', P_H^*) \\ (ii) \Pi_L^{A.a.1}(P_L^*, P_H^*) &\geq \Pi_L^{A.a.3}(P_L'', P_H^*) \\ (iii) \Pi_H^{A.a.1}(P_L^*, P_H^*) &\geq \Pi_H^{A.a.2}(P_L^*, P_H') \\ (iv) \Pi_H^{A.a.1}(P_L^*, P_H^*) &\geq \Pi_H^{A.a.3}(P_L^*, P_H'') \end{aligned} \tag{70}$$

We have to check inequality (i)  $\frac{\Delta_E(\bar{\theta}-2\underline{\theta})^2}{9} \geq P_L'(\theta' - \underline{\theta} + \theta'' - \theta^*)$ . However before checking this inequality we have to find  $P_L'$  as a profit maximising price.

We then obtain  $P'_L = \frac{1}{6}\Delta_E \frac{-2\theta\Delta+2\Delta-3\theta^*\Delta+\Delta_E\theta+2\Delta_E}{\Delta+\Delta_E}$ . Then either  $P'_L$  is interior to the price domain concerning sub-case A.a.2 and we have to check (i) or  $P'_L$  is not interior to the price domain and then, since (i) is irrelevant, one can consider (ii). Checking (i) we find that it has (significant) solutions for any value of  $\Delta, \Delta_E$  and  $\theta^*$  once the value of  $\underline{\theta}$  is restricted to an interval of parameter values. Considering inequality (ii) :  $\frac{\Delta_E(\bar{\theta}-2\underline{\theta})^2}{9} \geq P''_L(\theta'' - \underline{\theta})$  we can now compute  $P''_L$  as the profit maximising price, to obtain  $P''_L = \frac{1}{6}\Delta_E\underline{\theta} + \frac{1}{3}\Delta_E - \frac{1}{2}\underline{\theta}\Delta$ . Checking (ii) we can show that it cannot come true for any value of  $\Delta, \Delta_E$  and  $\theta^*$ , so that such a Nash equilibrium exists only for those parameters values consistent with  $P''_L$  not being interior to the price domain concerning sub-case A.a.3, i.e.:  $P_H - \bar{\theta}\Delta \leq P_L \leq P_H - \theta^*\Delta_E$ . Checking (iii) and (iv) we can show that they come respectively true provided that  $\theta^*$  is restricted to a given interval and  $\Delta_E = 9\Delta \frac{(\underline{\theta}+1)^2}{(\underline{\theta}-1)^2}$ .

**Proposition 7** *When uninformed consumers are optimistic and either  $1 \leq \frac{\Delta_E}{\Delta} \leq \text{Min} \left\{ \frac{\theta^*}{\underline{\theta}}, \frac{\bar{\theta}}{\theta^*} \right\}$  or  $\frac{\theta^*}{\underline{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\bar{\theta}}{\theta^*}$ , then, if the share of informed consumers is such that  $\theta^* \leq \frac{\Delta(1+2\underline{\theta})}{3\Delta_E}$ , the pair of prices  $P_L^* = \frac{\Delta(\bar{\theta}-2\underline{\theta})}{3}$ ,  $P_H^* = \frac{\Delta(2\bar{\theta}-\underline{\theta})}{3}$  is a Nash Equilibrium of the price competition game.*

Proof: Let us consider candidate equilibrium prices concerning sub-cases A.a.3 and A.d.3. Here we present a proof concerning sub-case A.a.3. Therefore, given definition one, we have to check the following inequalities:

$$\begin{aligned} (i) \Pi_L^{A.a.3}(P_L^*, P_H^*) &\geq \Pi_L^{A.a.1}(P'_L, P_H^*) \\ (ii) \Pi_L^{A.a.3}(P_L^*, P_H^*) &\geq \Pi_L^{A.a.2}(P''_L, P_H^*) \\ (iii) \Pi_H^{A.a.3}(P_L^*, P_H^*) &\geq \Pi_H^{A.a.1}(P_L^*, P'_H) \\ (iv) \Pi_H^{A.a.3}(P_L^*, P_H^*) &\geq \Pi_H^{A.a.2}(P_L^*, P''_H) \end{aligned} \quad (71)$$

We have to check inequality (i)  $\frac{\Delta(\bar{\theta}-2\underline{\theta})^2}{9} \geq P'_L(\theta' - \underline{\theta})$ . However before checking this inequality we have to find  $P'_L$  as a profit maximising price and then obtain  $P'_L = \frac{1}{6}\Delta a + \frac{1}{3}\Delta - \frac{1}{2}\underline{\theta}\Delta_E$ . Then either  $P'_L$  is interior to the price domain concerning sub-case A.a.1 and we have to check (i) or  $P'_L$  is not interior to the price domain and then, since (i) is irrelevant, we can consider (ii). Checking (i) we find that it comes true for any value of  $\theta^*, \underline{\theta}$  and  $\Delta$  provided that either  $\Delta_E = \Delta$  or  $\Delta_E = \Delta \frac{(\underline{\theta}+2)^2}{9\underline{\theta}^2}$ . Concerning (ii) we have to check that  $\frac{\Delta(\bar{\theta}-2\underline{\theta})^2}{9} \geq P''_L(\theta' - \underline{\theta} + \theta'' - \theta^*)$ . Finding  $P''_L$  as the profit maximising price, given  $P_H^*$ , we obtain  $P''_L = -\frac{1}{3}\Delta_E\Delta \frac{\underline{\theta}+\theta^*-1}{\Delta_E+\Delta}$ . Then either  $P''_L$  is interior to the price domain concerning sub-case A.a.1 and we have to check (ii) or  $P''_L$  is not interior to the price domain and then, since (ii) is irrelevant, we can consider (iii). Checking (ii) we can observe that it is true for any value of  $\theta^*, \underline{\theta}$  and  $\Delta_E$ , provided that  $\Delta = \Delta_E\theta^* \frac{(\theta^*+2\underline{\theta}-2)}{(\underline{\theta}-1)^2}$ . We can further check that there are no parameter values such that (iii) can come true, so that a price equilibrium exists only when  $P'_H$  is not interior to the price domain concerning sub-case A.a.1.

Concerning inequality (iv) we can observe that it is true for any value of  $\underline{\theta}$ ,  $\Delta$  and  $\Delta_E$ , provided that  $\theta^*$  belongs to a given range of parameter values.

Very restrictive assumptions concerning the ratio  $\frac{\Delta_E}{\Delta}$  also characterise the results summarised in proposition six and proposition seven, as reported in the related proofs. Concerning proposition six one we observe that both firm one and firm two can charge a price that depend on the quality differential expected by uninformed consumers, provided the share of informed consumers is lower with respect to a given threshold. The thin share of informed consumers cannot exert any influence on equilibrium prices. The largest share of the market is uninformed and buy the low quality good. On the contrary, in the case of proposition seven the share of informed consumers is high and they all buy the high quality good. Thus both firms are induced to charge a price that depends on the true quality differential. In this case prices become a perfect signal of quality as they are identical to the equilibrium prices that would prevail in the same model without incomplete information and information disparities. However, checking the proof of proposition seven we can observe that either this ratio  $\frac{\Delta_E}{\Delta}$  assumes a specific value or we have to assume that  $\Delta_E = \Delta$ , implying that even uninformed consumers (by chance) capture the real quality differential through the formation of their expectations.

## 4.2 Price competition with pessimistic consumers

Let us discuss the case of pessimistic consumers starting from case B.a, as described in section 2.2. In this case demand functions are represented in fig.11, where the different price domains can also be observed. Since demand functions are piecewise linear, also in this case, continuing to assume that the entire market is covered we can state that if a price equilibrium exists, it is such that either:

$$P_H - \theta^* \Delta_E \leq P_L^* \leq P_H - \Delta \underline{\theta} \text{ and } P_L + \underline{\theta} \Delta \leq P_H^* \leq P_L + \theta^* \Delta_E \text{ (case B.a.1)}$$

$$P_H - \bar{\theta} \Delta_E \leq P_L^* \leq P_H - \theta^* \Delta \text{ and } P_L + \theta^* \Delta \leq P_H^* \leq P_L + \bar{\theta} \Delta_E \text{ (case B.a.2)}$$

$$P_H - \bar{\theta} \Delta_E \leq P_L^* \leq P_H - \theta^* \Delta \text{ and } P_L + \theta^* \Delta \leq P_H^* \leq P_L + \bar{\theta} \Delta_E \text{ (case B.a.3)}$$

**B.a.1)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \underline{\theta})$ , and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta')$  with price domains  $P_H - \theta^* \Delta_E \leq P_L \leq P_H - \Delta \underline{\theta}$  and  $P_L + \underline{\theta} \Delta \leq P_H \leq P_L + \theta^* \Delta_E$ . Thus we get the following candidate Nash equilibrium prices:

$$P_L^* = \frac{\Delta_E(\bar{\theta} - 2\underline{\theta})}{3}, P_H^* = \frac{\Delta_E(2\bar{\theta} - \underline{\theta})}{3} \quad (72)$$

and the following restriction on  $\theta^*$ :

$$\theta^* \geq \frac{1 + 2\underline{\theta}}{3} \quad (73)$$

to get equilibrium profits:

$$\Pi_L^* = \frac{\Delta_E(\bar{\theta} - 2\underline{\theta})^2}{9}, \Pi_H^* = \frac{\Delta_E(2\bar{\theta} - \underline{\theta})^2}{9} \quad (74)$$



**B.a.2)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta^* - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta^*)$  with price domains:  $P_H - \theta^* \Delta \leq P_L \leq P_H - \theta^* \Delta_E$  and  $P_L + \theta^* \Delta_E \leq P_H \leq P_L + \theta^* \Delta$ . In this sub-case uninformed consumers just buy low quality goods while informed consumers just buy high quality goods. Thus information introduces a complete separation between the market for the low quality goods and the market for the high quality goods. In this range of prices demand functions are perfectly inelastic, thus it will be optimal for firms to charge the highest price they can impose (i.e. the upper limit of the interval). However considering simultaneously the upper limits of both price domains:  $P_L = P_H - \theta^* \Delta_E$  and  $P_H = P_L + \theta^* \Delta$ , we can easily find a contradiction concerning the value of  $\theta^*$ . Thus we are not able to define a unique candidate Nash equilibrium in this sub-case.

**B.a.3)** Demand functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta'')$  with price domains:  $P_H - \bar{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta} \Delta_E$ . We get the following candidate Nash Equilibrium prices:

$$P_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})}{3}; P_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})}{3} \quad (75)$$

and the following restriction on  $\theta^*$ :

$$\theta^* \leq \frac{2\underline{\theta} + 1}{3} \quad (76)$$

equilibrium profits are given by:

$$\Pi_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})^2}{9}; \Pi_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})^2}{9} \quad (77)$$

Let us consider case B.b. In this case demand functions are represented in fig.12:

**B.b.1)** In this case demand functions are discontinuous and perfectly inelastic to prices as  $D_L(P_L, P_H) = 0$  if  $P_L > P_H - \theta^* \Delta_E$  and  $D_L(P_L, P_H) = \theta^* - \underline{\theta}$  if  $0 \leq P_L \leq P_H - \theta^* \Delta_E$  while  $D_H(P_L, P_H) = 0$  if  $P_H > P_L + \theta^* \Delta$  and  $D_H(P_L, P_H) = \bar{\theta} - \theta^*$  if  $0 \leq P_H \leq P_L + \theta^* \Delta$ . Thus our conclusions closely follow those reported for case B.a.2.

Let us then consider case B.c- In this case demand functions are represented in fig 13:

**B.c.1)** In this sub-case profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta'' - \underline{\theta})$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta'')$  with price domains  $P_H - \bar{\theta} \Delta_E \leq P_L \leq P_H - \theta^* \Delta$  and  $P_L + \theta^* \Delta \leq P_H \leq P_L + \bar{\theta} \Delta_E$ . Thus we get the following candidate Nash equilibrium prices:

$$P_L^* = \frac{\Delta(\bar{\theta} - 2\underline{\theta})}{3}; P_H^* = \frac{\Delta(2\bar{\theta} - \underline{\theta})}{3} \quad (78)$$

and the following restriction on  $\theta^*$ :

$$\theta^* \leq \frac{2\underline{\theta} + 1}{3} \quad (79)$$

and equilibrium profits:

$$\Pi_L^* = \frac{\Delta(\bar{\theta} - 2\theta)^2}{9}; \Pi_H^* = \frac{\Delta(2\bar{\theta} - \theta)^2}{9} \quad (80)$$

**B.c.2)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta^* - \theta)$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta^*)$  with price domains  $P_H - \theta^* \Delta \leq P_L \leq P_H - \theta^* \Delta_E$  and  $0 \leq P_H \leq P_L + \theta^* \Delta$ . Thus also in this case our conclusions closely follow those reported for case B.a.2.

The last case to be considered is case B.d. Demand functions in this case are represented in fig.14:

**B.d.1)** Profit functions are given by  $\Pi_L(P_L, P_H) = P_L(\theta' - \theta)$ , and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta')$  with price domains  $P_H - \theta^* \Delta_E \leq P_L \leq P_H - \theta \Delta$  and  $P_L + \theta \Delta \leq P_H \leq P_L + \theta^* \Delta_E$ . Thus we get the following candidate Nash equilibrium prices:

$$P_L^* = \frac{\Delta_E(\bar{\theta} - 2\theta)}{3}; P_H^* = \frac{\Delta_E(2\bar{\theta} - \theta)}{3} \quad (81)$$

and the following restriction on  $\theta^*$ :

$$\theta^* \geq \frac{2\theta + 1}{3} \quad (82)$$

equilibrium profits will then be given by:

$$\Pi_L^* = \frac{\Delta_E(\bar{\theta} - 2\theta)^2}{9}; \Pi_H^* = \frac{\Delta_E(2\bar{\theta} - \theta)^2}{9} \quad (83)$$

**B.d.2)** Profit functions are given by:  $\Pi_L(P_L, P_H) = P_L(\theta^* - \theta)$  and  $\Pi_H(P_L, P_H) = P_H(\bar{\theta} - \theta^*)$  with price domains  $0 \leq P_L \leq P_H - \theta^* \Delta_E$  and  $P_L + \theta^* \Delta_E \leq P_H \leq P_L + \theta^* \Delta$ . Thus also in this case our conclusions closely follows those reported for case B.a.2.

We now have to check that the candidate Nash equilibrium prices obtained in each price domain are indeed a Nash equilibrium price pair once we are no longer restricted to this same price domain and consider the entire price range defining the demand functions in cases B.a, B.b, B.c, B.d. The results of our analysis are summarised in the following propositions.

**Proposition 8** *If uninformed consumers are pessimistic and demand is perfectly inelastic with respect to price, then for any share of informed consumers in the market there is a continuum of equilibria where each firm charges the highest price it can impose, given the price charged by its competitor.*

Proof: Just consider cases B.a.2, B.b, B.c.2 and B.d.2. Our conclusions reported for sub-case B.a.2 simply extend to all these cases and sub-cases.

**Proposition 9** *When uninformed consumers are pessimistic and  $\text{Max} \left\{ \frac{\theta^*}{\bar{\theta}}, \frac{\theta}{\theta^*} \right\} \leq \frac{\Delta_E}{\Delta} \leq 1$  or  $\frac{\theta^*}{\bar{\theta}} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta}{\theta^*}$ , the if the share of informed consumers is such that  $\theta^* \leq \frac{2\theta+1}{3}$ , the pair of prices  $P_L^* = \frac{\Delta(\bar{\theta}-2\theta)}{3}; P_H^* = \frac{\Delta(2\bar{\theta}-\theta)}{3}$  is a Nash Equilibrium of the price competition game and prices end up being a perfect quality signal.*

Proof: Let us consider candidate equilibrium prices in sub-cases B.a.3 and B.c.1. Considering sub-case B.a.3, given definition one, we must then check that:

$$\begin{aligned}
(i) \Pi_L^{B.a.3}(P_L^*, P_H^*) &\geq \Pi_L^{B.a.1}(P_L', P_H^*) \\
(ii) \Pi_L^{B.a.3}(P_L^*, P_H^*) &\geq \Pi_L^{B.a.2}(P_L'', P_H^*) \\
(iii) \Pi_H^{B.a.3}(P_L^*, P_H^*) &\geq \Pi_H^{B.a.1}(P_L^*, P_H') \\
(iv) \Pi_H^{B.a.3}(P_L^*, P_H^*) &\geq \Pi_H^{B.a.2}(P_L^*, P_H'')
\end{aligned} \tag{84}$$

where  $P_L'$  maximises profits to firm one in the interval:  $P_H - \theta^* \Delta_E \leq P_L \leq P_H - \Delta\theta$  (sub-case B.a.1) given  $P_H^* = \frac{\Delta(2\bar{\theta}-\theta)}{3}$ ;  $P_L''$  maximises profits to firm one in the interval:  $P_H - \theta^* \Delta \leq P_L \leq P_H - \theta^* \Delta_E$  (sub-case B.a.2) given  $P_H^* = \frac{\Delta(2\bar{\theta}-\theta)}{3}$ ;  $P_H'$  maximises profits to firm two in the interval  $P_L + \theta\Delta \leq P_H \leq P_L + \theta^* \Delta_E$  (sub-case B.a.1), given  $P_L^* = \frac{\Delta(\bar{\theta}-2\theta)}{3}$ ;  $P_H''$  maximises profits to firm two in the interval  $P_L + \theta^* \Delta_E \leq P_H \leq P_L + \theta^* \Delta$  (sub-case B.a.2) given  $P_L^* = \frac{\Delta(\bar{\theta}-2\theta)}{3}$ . Clearly profits are defined case by case, using the demand functions in question in each sub-case.

We have to check inequality (i)  $\frac{\Delta(\bar{\theta}-2\theta)^2}{9} \geq P_L'(\theta' - \theta)$ . However before checking this inequality we have to find  $P_L'$  as a profit maximising price. We then obtain  $P_L' = \frac{1}{6}\Delta\theta + \frac{1}{3}\Delta - \frac{1}{2}\theta\Delta_E$ . Then either  $P_L'$  is interior to the price domain concerning sub-case B.a.1 and we have to check (i) or  $P_L'$  is not interior to the price domain and then, since (i) is irrelevant, we can consider (ii). Checking (i) we find that it has (significant) solutions for any value of  $\theta^*$ ,  $\theta$  and  $\Delta$  if  $\Delta_E$  is either restricted to  $\Delta_E = \Delta$  or restricted to  $\Delta_E = \frac{1}{9}(2 + \theta)^2 \frac{\Delta}{\theta^2}$ . Considering inequality (ii) we must recall that in case B.a.2 the demand for the low quality good becomes completely inelastic to price and is equal to  $(\theta^* - \theta)$ . Thus  $P_L''$ , the profit maximising price is assumed to be the upper extreme of the price interval, i.e.  $P_L'' = P_H^* - \theta^* \Delta_E = \frac{\Delta(2\bar{\theta}-\theta)}{3} - \theta^* \Delta_E$ . Checking inequality (ii) we find that it has significant solutions for any value of  $\theta^*$ ,  $\theta$  and  $\Delta$  if  $\Delta_E = \frac{1}{9}(2 + \theta)^2 \frac{\Delta}{\theta^2}$ . A less significant solution implies that all consumers are informed. Following the same methodology we can check inequality (iii) and (iv) and find that, in this case as well, there are significant solutions for any value of  $\theta^*$ ,  $\theta$  and  $\Delta$  if  $\Delta_E = \frac{1}{9}(2 + \theta)^2 \frac{\Delta}{\theta^2}$ .

**Proposition 10** *When uninformed consumers are pessimistic and either  $\text{Max}\left\{\frac{\theta^*}{\theta}, \frac{\theta}{\theta^*}\right\} \leq \frac{\Delta_E}{\Delta} \leq 1$  or  $\frac{\theta}{\theta^*} \leq \frac{\Delta_E}{\Delta} \leq \frac{\theta^*}{\theta}$  then if the share of informed consumers is such that  $\theta^* \geq \frac{1+2\theta}{3}$ , the pair of prices  $P_L^* = \frac{\Delta_E(\bar{\theta}-2\theta)}{3}$ ;  $P_H^* = \frac{\Delta_E(2\bar{\theta}-\theta)}{3}$  is a Nash Equilibrium of the price competition game.*

Proof: Let us consider candidate equilibrium prices in sub-cases B.a.1 and B.d.1. Considering sub-case B.a.1 and given definition one, we have to check that:

$$\begin{aligned}
(i) \Pi_L^{B.a.1}(P_L^*, P_H^*) &\geq \Pi_L^{B.a.3}(P_L', P_H^*) \\
(ii) \Pi_L^{B.a.1}(P_L^*, P_H^*) &\geq \Pi_L^{B.a.2}(P_L'', P_H^*) \\
(iii) \Pi_H^{B.a.1}(P_L^*, P_H^*) &\geq \Pi_H^{B.a.3}(P_L^*, P_H') \\
(iv) \Pi_H^{B.a.1}(P_L^*, P_H^*) &\geq \Pi_H^{B.a.2}(P_L^*, P_H'')
\end{aligned} \tag{85}$$

where  $P_L'$  maximises profits to firm one in the interval:  $P_H - \bar{\theta}\Delta_E \leq P_L \leq P_H - \theta^*\Delta$  (sub-case B.a.3) given  $P_H^* = \frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3}$ ;  $P_L''$  maximises profits to firm one in the interval:  $P_H - \theta^*\Delta \leq P_L \leq P_H - \theta^*\Delta_E$  (sub-case B.a.2) given  $P_H^* = \frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3}$ ;  $P_H'$  maximises profits to firm two in the interval  $P_L + \theta^*\Delta \leq P_H \leq P_L + \bar{\theta}\Delta_E$  (sub-case B.a.3), given  $P_L^* = \frac{\Delta_E(\bar{\theta}-2\underline{\theta})}{3}$ ;  $P_H''$  maximises profits to firm two in the interval  $P_L + \theta^*\Delta_E \leq P_H \leq P_L + \theta^*\Delta$  (sub-case B.a.2) given  $P_L^* = \frac{\Delta_E(\bar{\theta}-2\underline{\theta})}{3}$ . Clearly profits are defined, case by case, using the demand functions in question in each sub-case.

We have to check inequality (i)  $\frac{\Delta_E(\bar{\theta}-2\underline{\theta})^2}{9} \geq P_L'(\theta'' - \underline{\theta})$ . However before checking this inequality we have to find  $P_L'$  as a profit maximising price. We then obtain  $P_L' = \frac{1}{6}\Delta_E\theta^* + \frac{1}{3}\Delta_E - \frac{1}{2}\theta^*\Delta$ . Then either  $P_L'$  is interior to the price domain concerning sub-case B.a.1 and we have to check (i) or  $P_L'$  is not interior to the price domain and then, since (i) is irrelevant, we can consider (ii). Checking (i) we find that it has (significant) solutions for any value of  $\underline{\theta}$  and  $\Delta$  if  $\Delta_E$  is restricted either to  $\Delta_E = \Delta$  or to  $\Delta_E = \frac{1}{9}(2 + \underline{\theta})^2 \frac{\Delta}{\underline{\theta}^2}$ . Considering inequality (ii) we must recall that in case B.a.2 the demand for the low quality good becomes completely inelastic to price and is equal to  $(\theta^* - \underline{\theta})$ . Thus  $P_L''$ , the profit maximising price is supposed to be the upper extreme of the price interval, i.e.  $P_L'' = P_H - \theta^*\Delta_E = \frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3} - \theta^*\Delta_E$ . Checking inequality (ii) boils down to checking that  $\frac{\Delta_E(\bar{\theta}-2\underline{\theta})^2}{9} \geq (\frac{\Delta_E(2\bar{\theta}-\underline{\theta})}{3} - \theta^*\Delta)(\theta^* - \underline{\theta})$  and finding that it has significant solutions for any value of  $\theta^*$  and  $\Delta_E$ , provided that  $\underline{\theta} = 1$ . Alternatively a less significant solution implies that all consumers are informed. Following the same methodology we can check inequality (iii) and (iv). Concerning (iii) there are significant solutions for any value of  $\Delta$  and  $\Delta_E$  provided that  $\underline{\theta}$  is chosen in a given interval, while concerning (iv) there are significant solutions for any value of  $\theta^*$  and  $\Delta_E$  provided that  $\underline{\theta} = 1$ .

Proposition 8 shows a general result related to the case of pessimistic beliefs, as it holds in all four cases (B.a-B.d). In such cases information disparities can give rise to demand functions that are inelastic to price, as - in a given price interval or in all the price range defining market demands (case B.b) - all uninformed consumers buy the low quality good and all informed consumers buy the high quality good. In this case we could state that information creates a barrier between the market for the high quality good and the market for the low quality good. Thus the two markets are separated. Communication between the two markets requires either that some uninformed consumers buy the high quality good or that some informed consumers buy the low quality good; price variations will then induce demand changes. In the cases we are dealing with,

price variations will not induce any change in both market demands. Moreover in some sub-cases changes in information are associated with a discontinuity in the demand function, meaning that outside the price range giving rise to inelastic demands, a small price change is likely to lead to a null demand. When demand is not discontinuous a small change in prices leads outside of the price interval where the inelasticity holds, meaning that the two markets start to communicate either because some uninformed consumers buy the high quality good or alternatively because some informed consumer buy the low quality good.

In all cases we are dealing with, any increase in the number of informed consumers extends the market share of the firm selling the high quality good and reduces the market share of the firm selling the low quality good. In the price range where demand is inelastic each firm can maximise its profits by charging prices equal to the superior extreme of the price range. However such a price depends on the equilibrium price charged by the other firm. Therefore in the framework of our model a continuum of equilibrium prices exist in these cases. The fact that a unique equilibrium price does not exist highlights a coordination problem for competing firms. The highest price that each firm can charge depends on the price charged by the other firm. Such a statement means that the high quality firm should avoid charging such a higher price to uninformed consumers because this leads some of them (or even all of them in the case of discontinuous demand) to switch to the low quality good. But how much higher this price is depends on the price charged by the other firm. The low quality firm faces the same problem to the extent it should also avoid charging too high a prices to uninformed pessimistic consumers otherwise some of them (or even all of them in case B.b) would consider switching to the high quality firm, in spite of their beliefs. To the extent that the market share of the high quality firms is increasing in the number of informed consumers we can imagine that this same firm may want to subsidise consumer information in order to increase its profits. Of course the low quality firm faces the opposite incentives in this case.

Proposition 9 and 10 summarize less general results. If the number of informed consumers is high enough and given the other restrictions on the parameters illustrated in proposition 4, then equilibrium prices end up being a perfect signal of quality as their expression is identical to the one that would be obtained in the case of a model of vertical differentiation with complete information about product quality. However the proof of proposition 4 points out that such a result depends either on the assumption that the true quality differential is identical to the expected quality differential or on a very specific relationship between these differentials. The first assumption implies that even uninformed consumers by chance are able to identify the true quality differential or that the high quality firm decides to produce just the quality expected by uninformed consumers, thereby perfectly fulfilling consumers expectations. If the number of informed consumer is lower with respect to the threshold of proposition 4 then proposition 5 holds, and equilibrium prices just reflect the quality differential expected by pessimistic uninformed consumers. In this case price reflects consumers expectations, but they may not be fulfilled as the real

quality differential deviates from the expected one.

## 5 Conclusions

In this paper we have analysed the impact of information disparities concerning product quality on price competition, considering the framework of a vertically differentiated duopoly. Even without any initial assumption about the perceptions of uninformed consumers, building such a model of vertical product differentiation with information disparities leads to the consideration of two main types of beliefs characterising uninformed consumers: optimistic and pessimistic beliefs. Two main cases are then considered in our analysis. In the first case uninformed consumers overestimate the differential between high quality and low quality (optimistic consumers). In such a case product differentiation can be "virtual", as far as it is partially or completely based on consumers beliefs concerning a quality differential that either does not exist or is reduced with respect to consumers expectations. As frequently happens in real markets, if consumers do not have credible information concerning product quality, persuasive advertising can be effective in shaping consumers beliefs according to the firms aims. Then product differentiation advantages can accrue to brands that are able to persuade consumers that the quality of their products is higher with respect to other brands sold on the market. As this model shows, introducing a share of informed consumers in the market, can reduce "virtual" product differentiation and intensify price competition between different brands of the same product in the market. The price of high quality goods falls in proportion to the share of informed consumers and, due to strategic interaction, the price of low quality goods increases with this share, though in our model low quality is not only a minimum quality standard but is also common knowledge.

Such a result could be an example of the fact that consumers information can undermine brand. Empirical evidence concerning this phenomenon can be found in Waldfogel and Chen (2003) who analyse information provision concerning retailers reliability through Information Intermediary websites<sup>5</sup>. While consumers who are uncertain about sellers reliability favor recognizable brands, once they are equipped with knowledge about vendor's existence and reliability, they become less loyal to branded retailers by reducing their shopping at them by substantial and statistically significant amount. This results in more competition and a lower market concentration. Another example, concerning the effect of information on brand, and accounting also for the final effect on price competition, is related to the market for drugs. This market is characterised by the existence of well known brands whose market power is temporarily protected by patents. When a generic rival is introduced into the market at a cheaper price a lot of consumers are indirectly informed about product quality simply by the

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<sup>5</sup> According to Waldfogel and Chen the late 1990s saw the appearance of information intermediaries such as DealTime and my Simon, providing price and delivery information, and others like BizRate providing vendors reliability information as well.

approval coming from regulators that enable the purchase of the new product. This product could be perceived as a low quality product to the extent that it is not branded; however it is a drug with the same quality attributes of the well known brand. Usually the introduction of a generic leads to a sharp fall in the price of the well known brand and may imply further price adjustments in the market<sup>6</sup>. To explain price competition in this case we should not only consider that monopoly comes to the end but also the fact that brand loyalty decreases simply because the approval by the regulator carries with it the information that the new generic drug is at least as effective as the new one. Therefore even the richest consumers may decide to switch to the generic together with the poorest, while the old brand could be able to keep just those consumers with an "intermediate" willingness to pay and that remain uninformed. In order to keep a market share the old brand has thus to accept a huge price decrease. If the end of monopoly was accompanied by the persistence of quality uncertainty across all consumers, then the incumbent would be able to keep a positive market share without incurring such a wide price decrease. In many markets where there is no supply of information or just a few consumers are informed, price competition may be less intense when monopoly comes to an end. In fact our results point out that incomplete information can be a specific source of market power, making price competition softer when consumers are both richer and poorly informed about the extent of quality differences between products.

A further result that deserves mention is the function of prices as quality signals due to the externality that informed consumers exert on uninformed ones. Even though in our model prices turn out to be a quality signal for uninformed consumers, when the share of informed consumers is high, we cannot state that high prices signal high quality. On the contrary both the price of the low quality good and the price of the high quality one are reduced due to the fact that firms are both motivated to reveal the true quality differential. However the higher the share of informed consumers, the lower the price of the high quality good and the higher is the price of the low quality good. Therefore it is the growing price of the low quality good that can signal to consumers that the high quality good is not so high as expected by uninformed consumers

If consumer beliefs are pessimistic we come to different conclusions. Not only does information drive consumers away from low quality goods but market demand can also become inelastic to price due to information disparities. In a given price range, all informed consumers will stick to high quality goods while all uninformed consumers will stick to low quality goods. In that case the share of informed consumers becomes crucial for the amount of profits as - in a given price range - no price variation can affect market demand. An

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<sup>6</sup>We can provide further anecdotal evidence concerning this phenomenon considering the recent battle over ulcer treatment in the international market for pharmaceutical: "The battle over ulcer treatment Prilosec intensified when Novartis said it planned the US launch of a cheap generic copy of the AstraZeneca drug, which was until recently, the world's best selling medicine. Novartis is now the third company to sell a copycat version of Prilosec. (...) The launch is likely to lead to a sharp fall in the price of Prilosec in the US. Kevin Scotcher, analyst at SG Cowen in London said: "It is now a pricing game". He said the cost of Prilosec could fall 40-50 per cent." See Dyer (2003), p.19.

example of pessimistic beliefs may be represented by uninformed consumers that are suspicious about the launch of new products that can damage their health or the environment. Unless available information destroys such beliefs they will not consider a change of brand, even if the new product is not expensive. Therefore, the old low quality brand - whose demand also becomes inelastic - can charge higher prices without incurring the risk of losing market shares. Once consumers become informed and realize that new products are less risky than expected, their attitude changes completely and they switch to the new high quality good. This can explain not only inelasticity to price but also the discontinuity that market demands show in some cases. In that case the firm that sells the new product may be willing to subsidise information provision in order to gain a wider market share. Furthermore with pessimistic beliefs prices cannot be a signal of quality: as information separates market and market demands are inelastic to price, informed consumers cannot exert any externality on uninformed ones. Only high quality firms benefit from information provision.

Further analysis that also considers the cost of quality may be necessary to explore firms' incentives to produce high quality goods, when information disparities affect market demands. The model shows that considering both the cost of advertising to influence perceived quality and the production cost of high quality goods to influence real quality differences may also affect equilibrium prices, especially in the optimistic case. Thus a further step in this research will consist in the analysis of a two-stage game where firms first choose quality and then compete in prices, given market demands that we have shaped in this paper.

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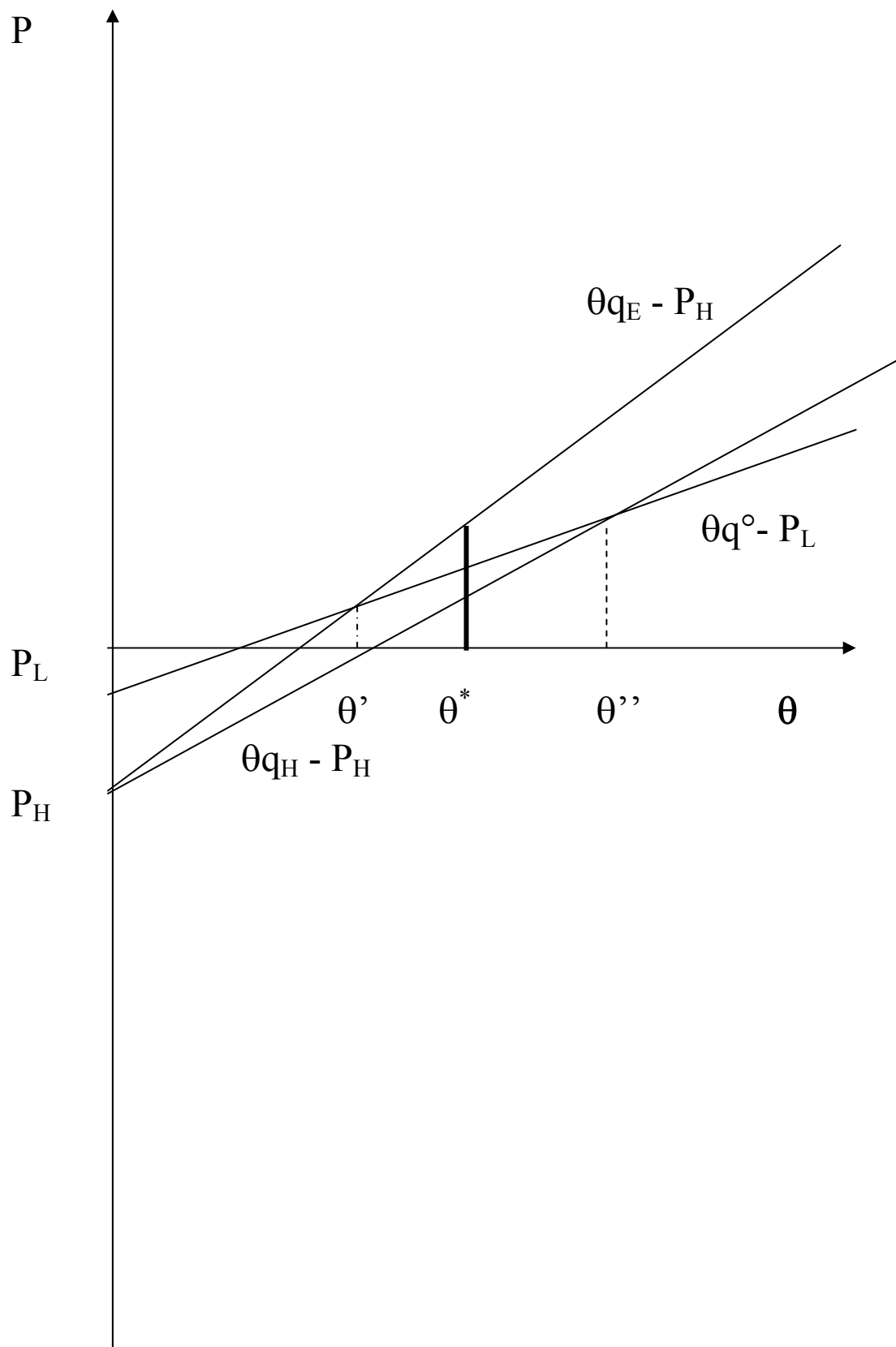


FIG. 1- CASE A.1

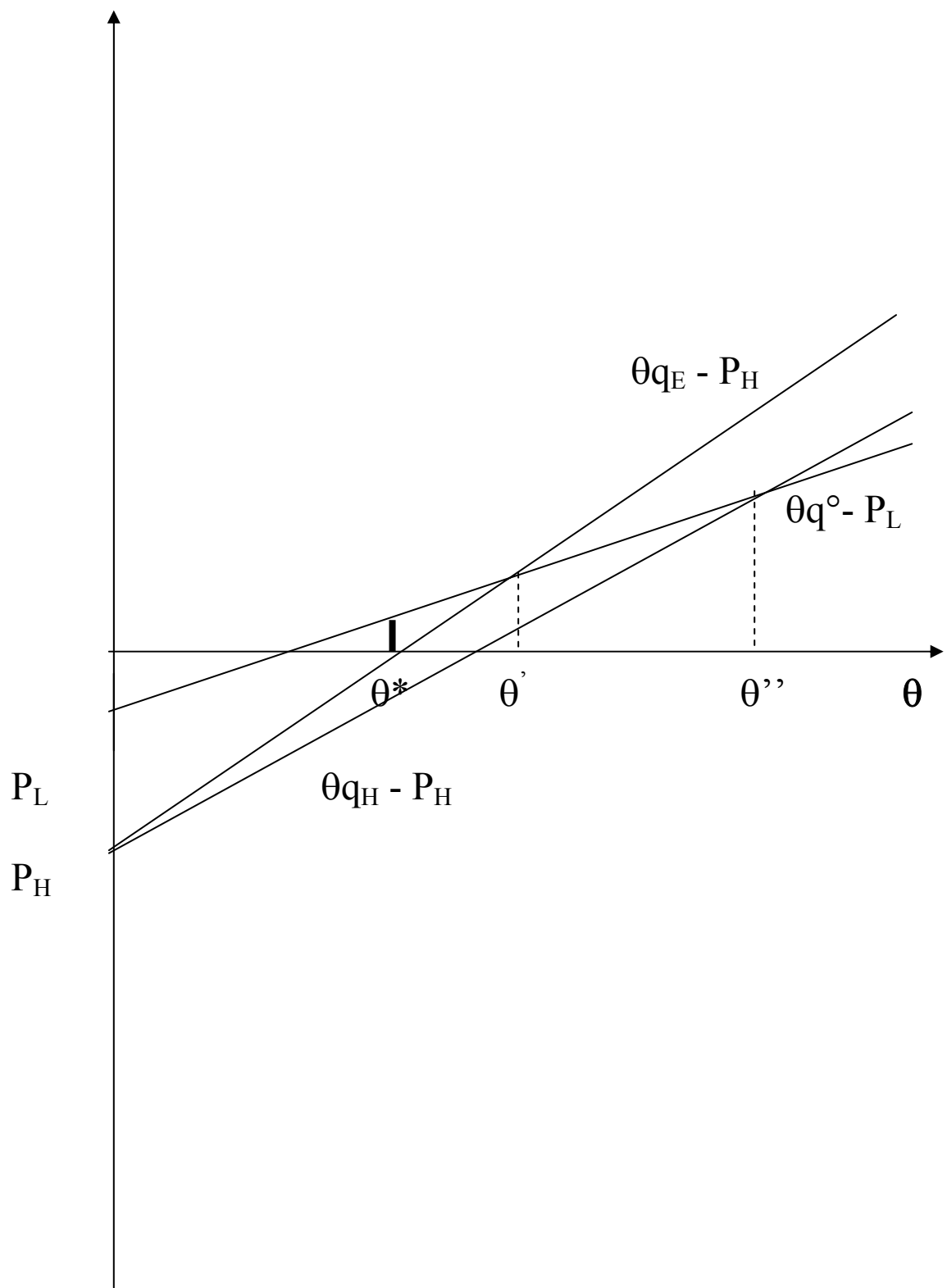


FIG. 2- CASE A.2

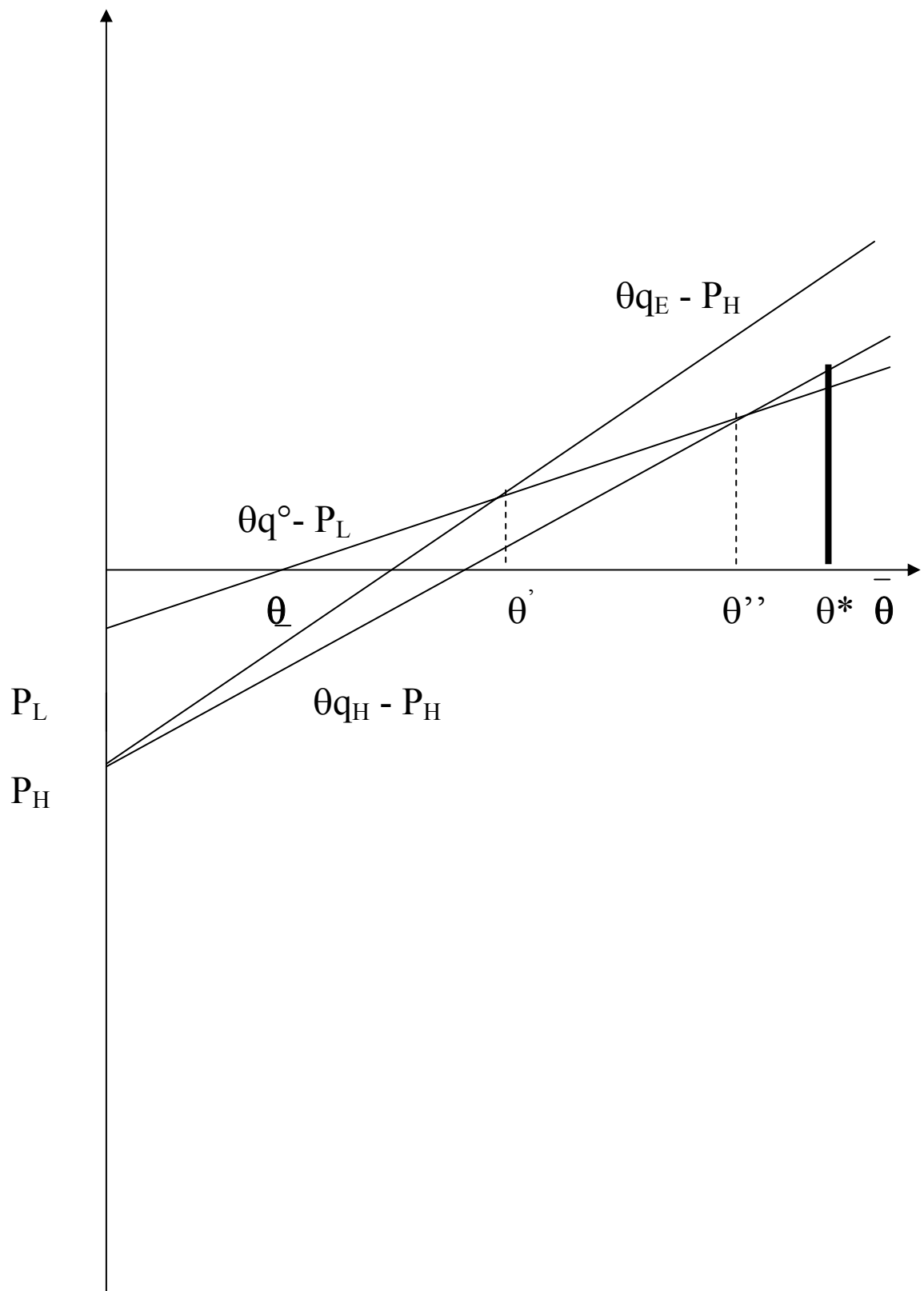
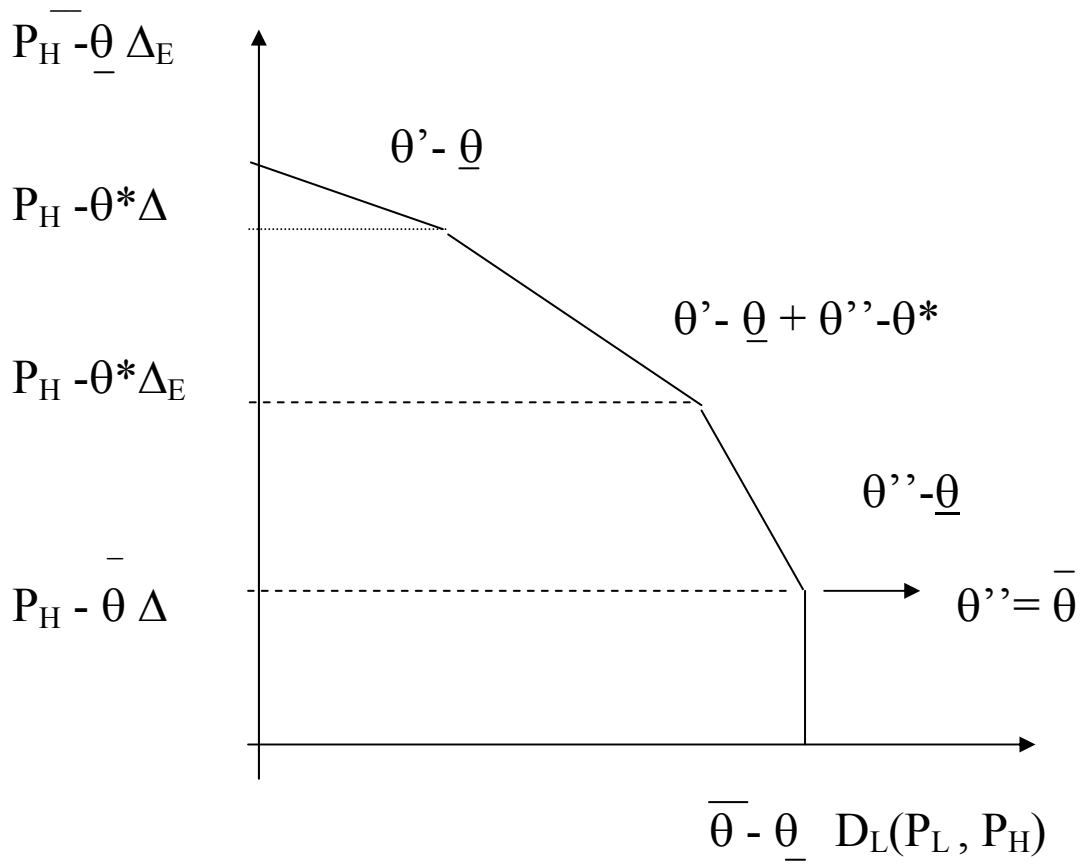


FIG. 3- CASE A.3

X



X

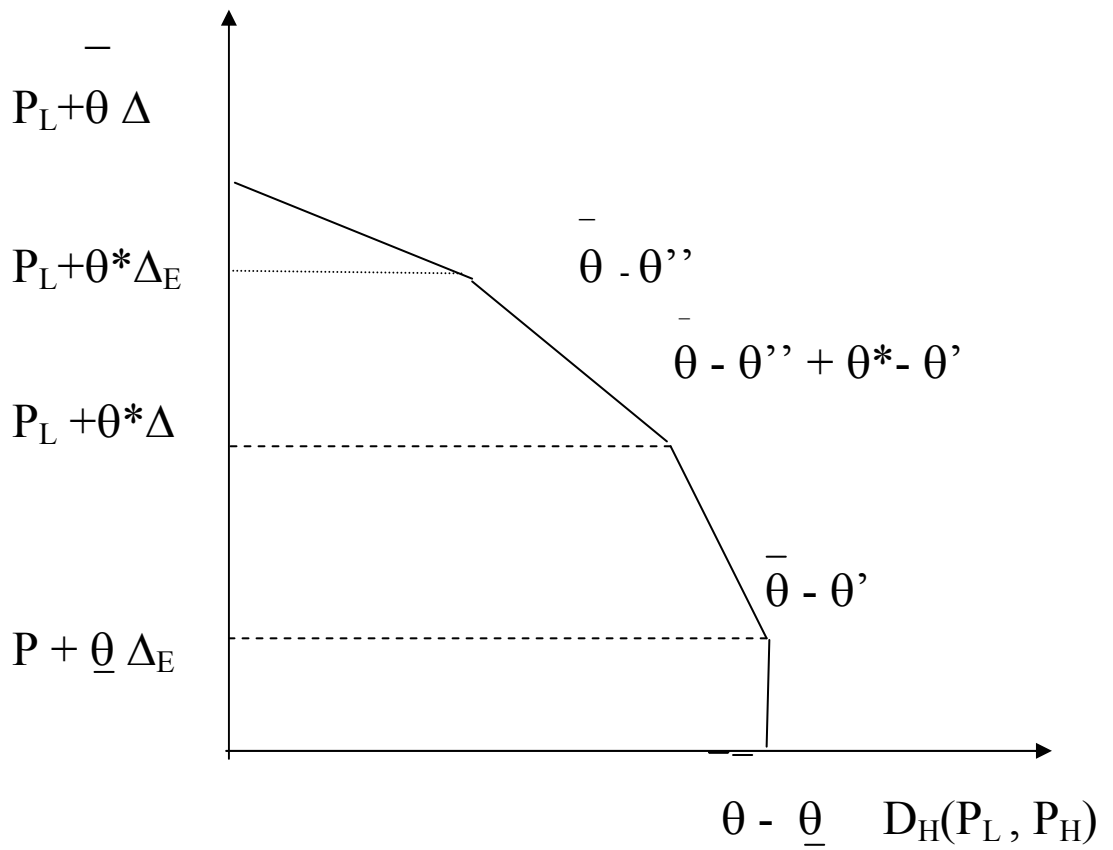
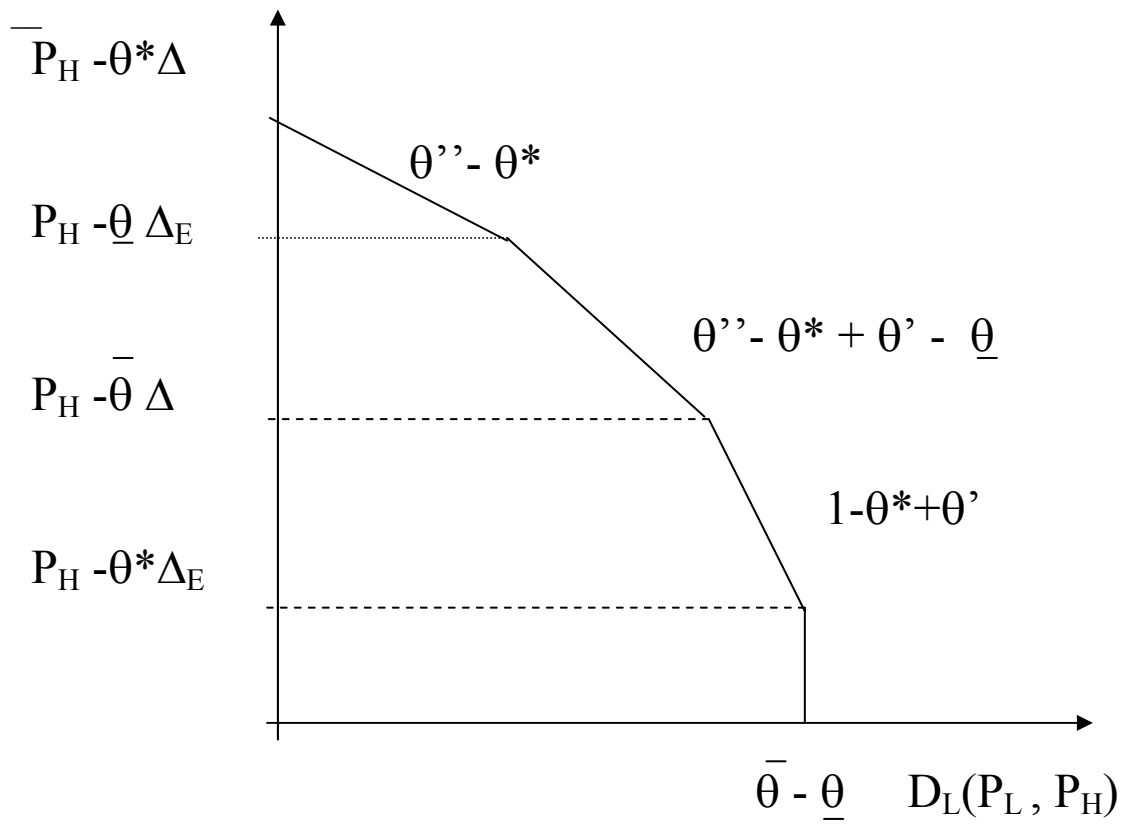


FIG.4 DEMAND FUNCTIONS -CASE A.a

X



X

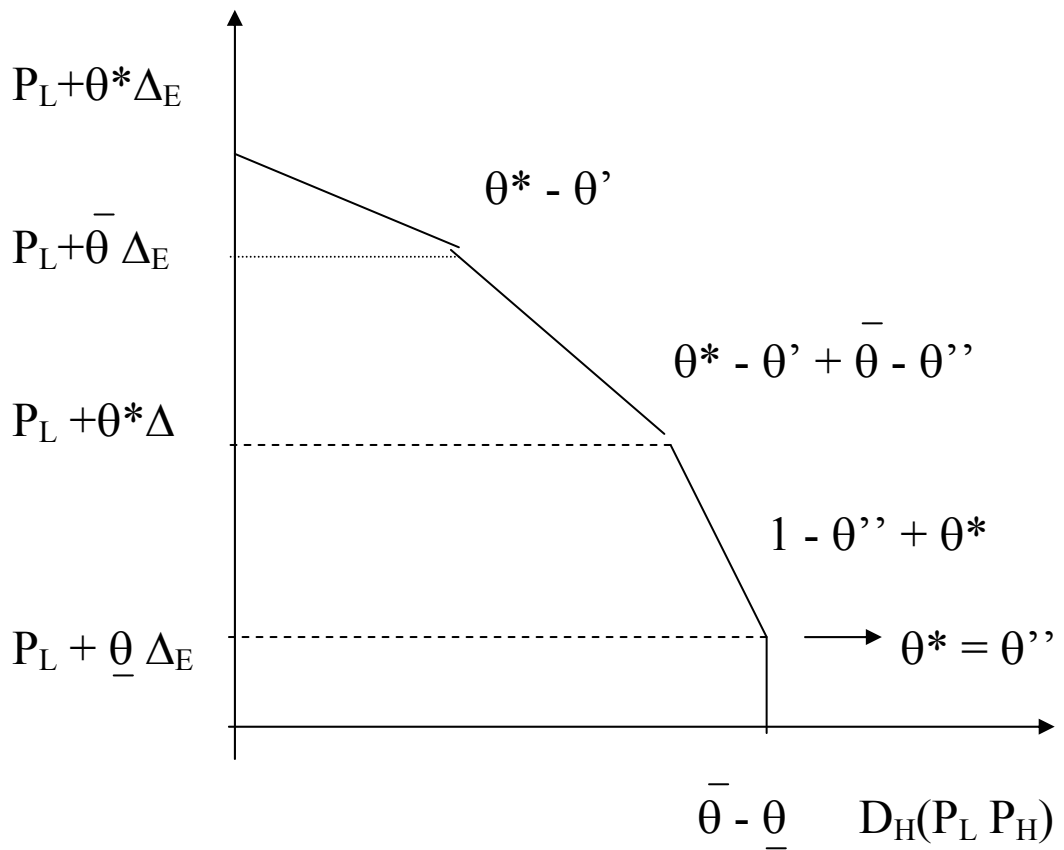
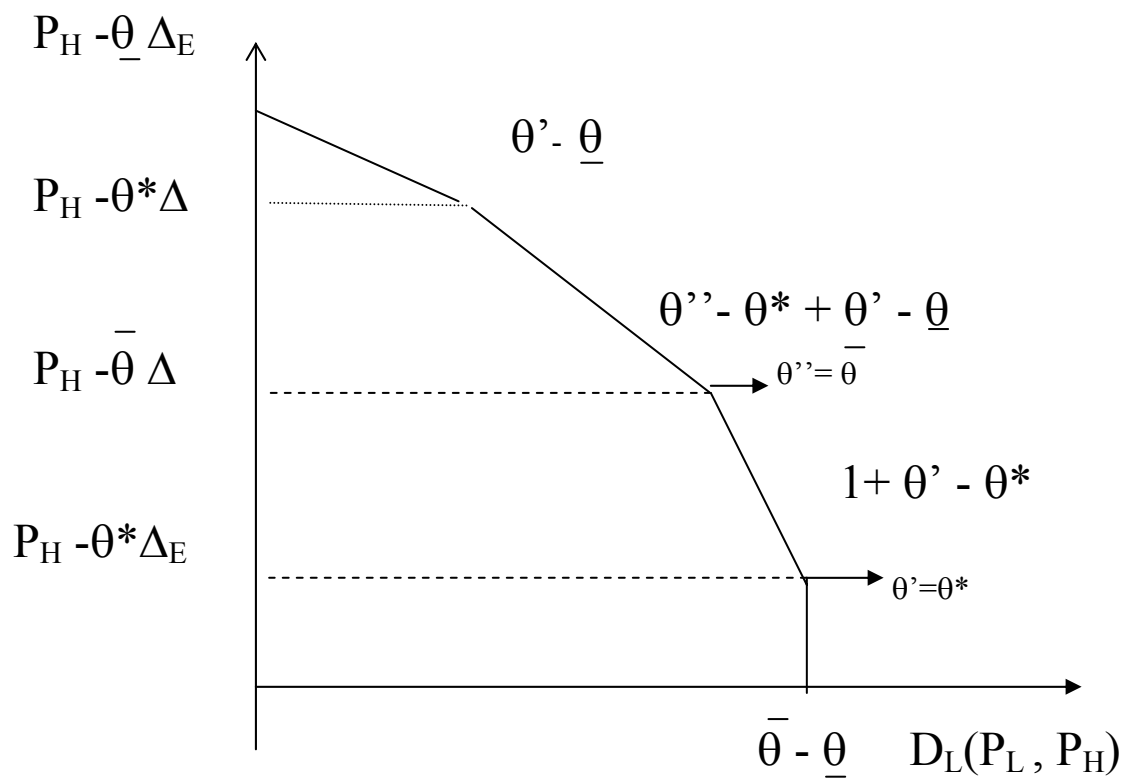


FIG.5 DEMAND FUNCTIONS –CASE A.b

X



X

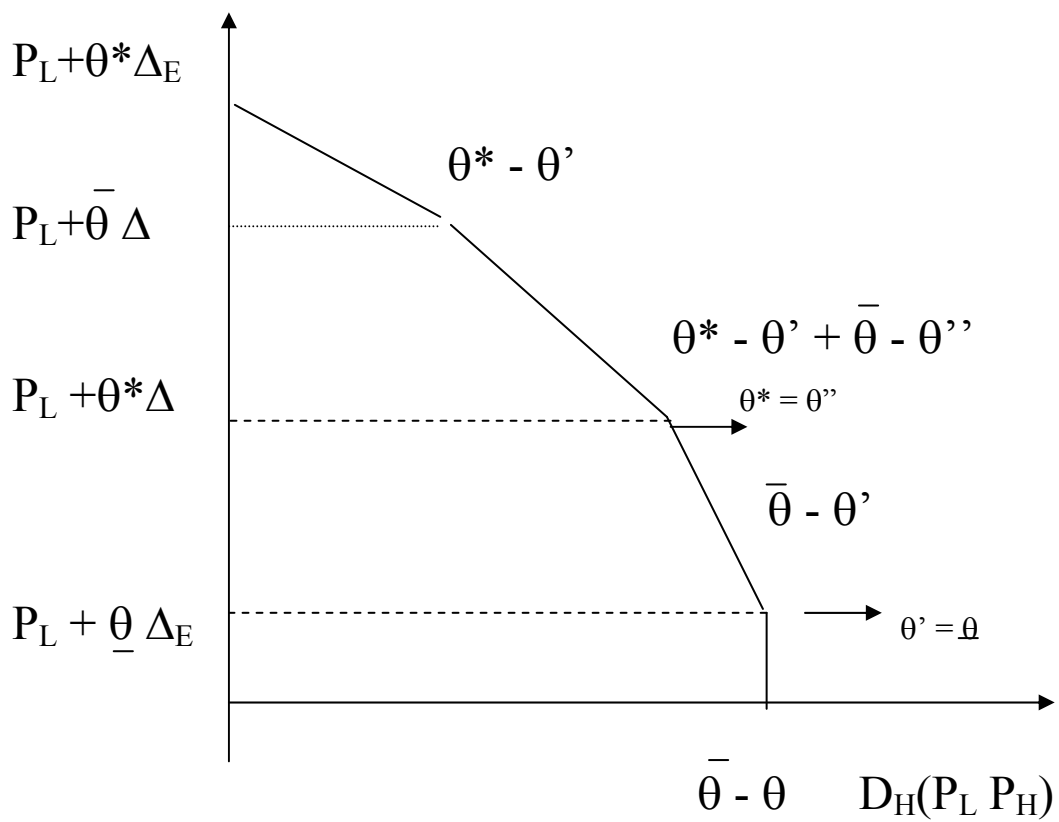


FIG.6 -DEMAND FUNCTIONS: A.c

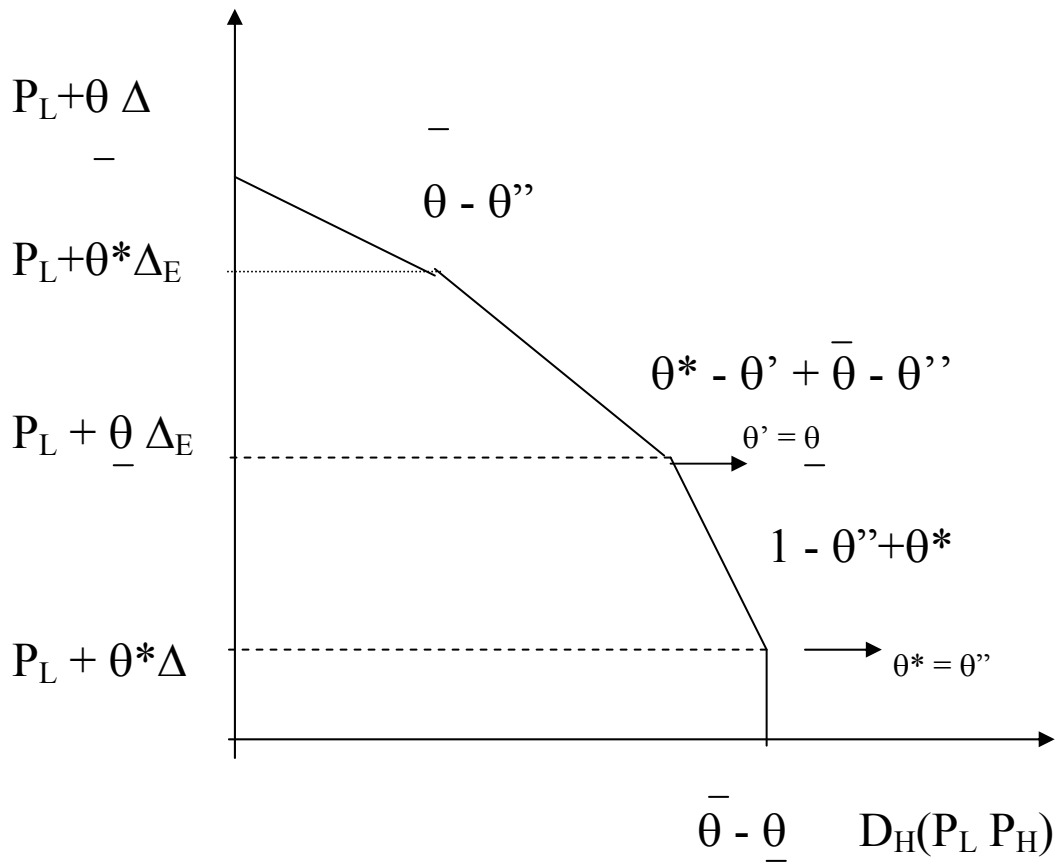
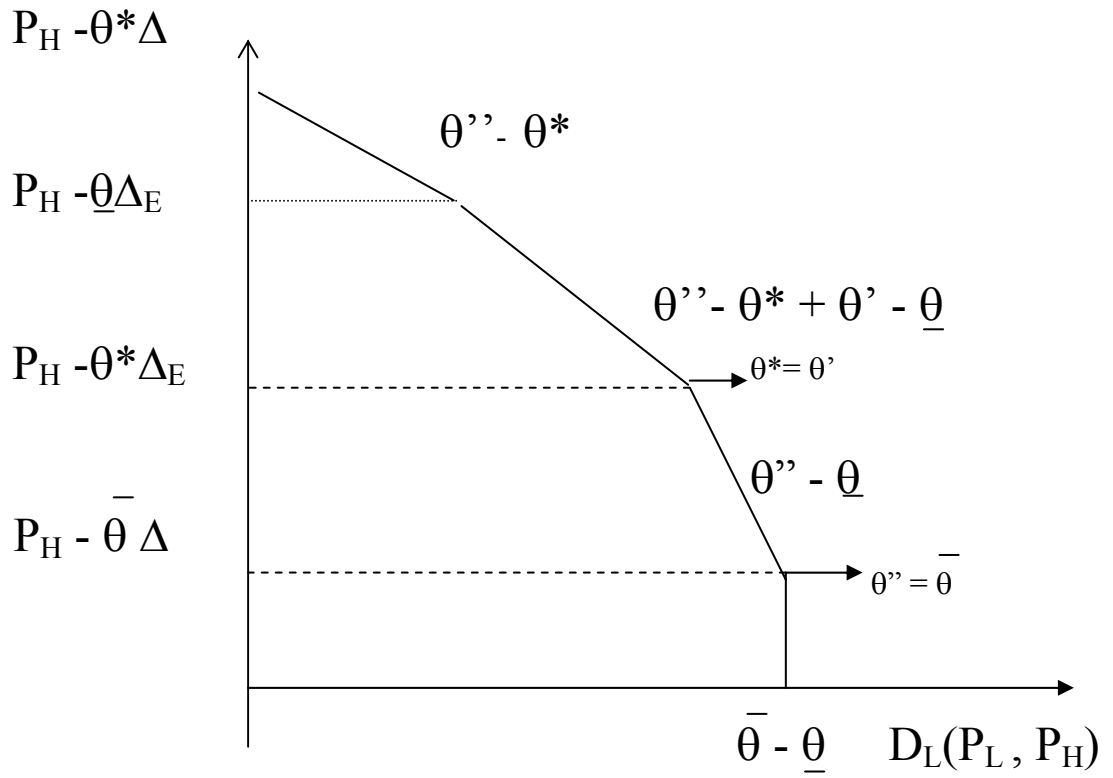


FIG.7 -DEMAND FUNCTIONS: A.d



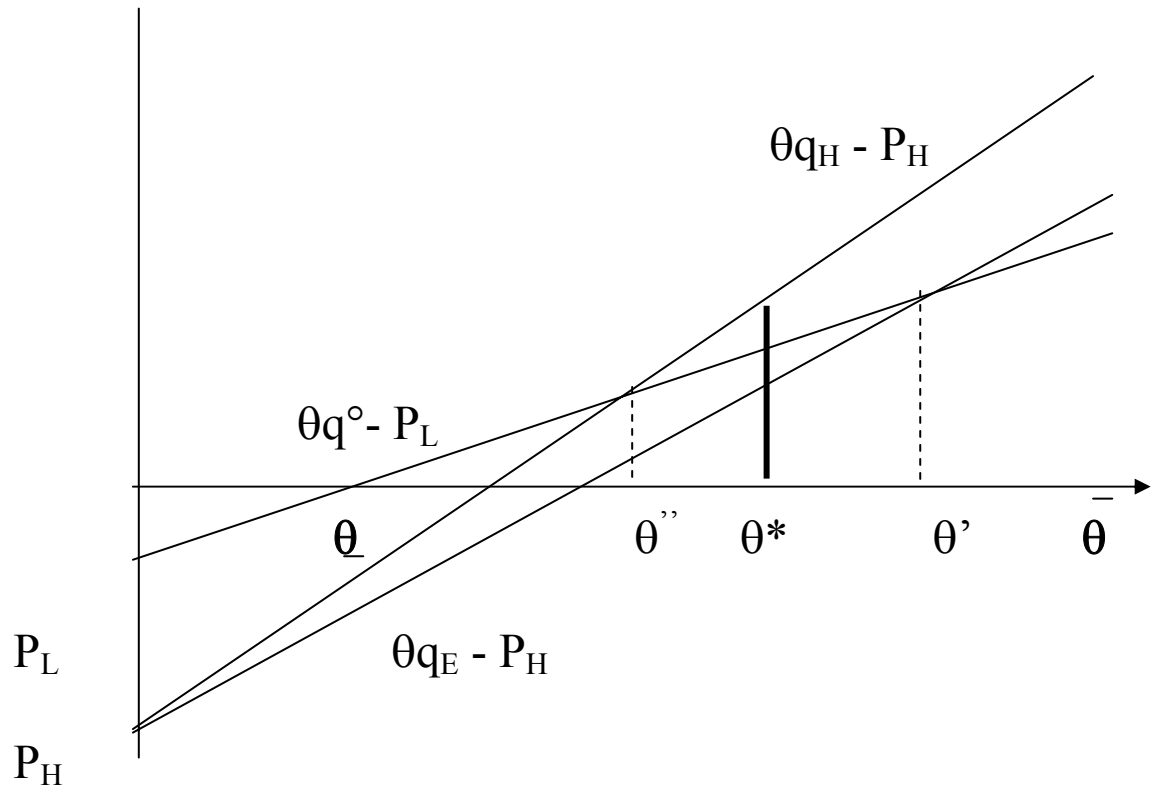


FIG. 8- CASE B.1

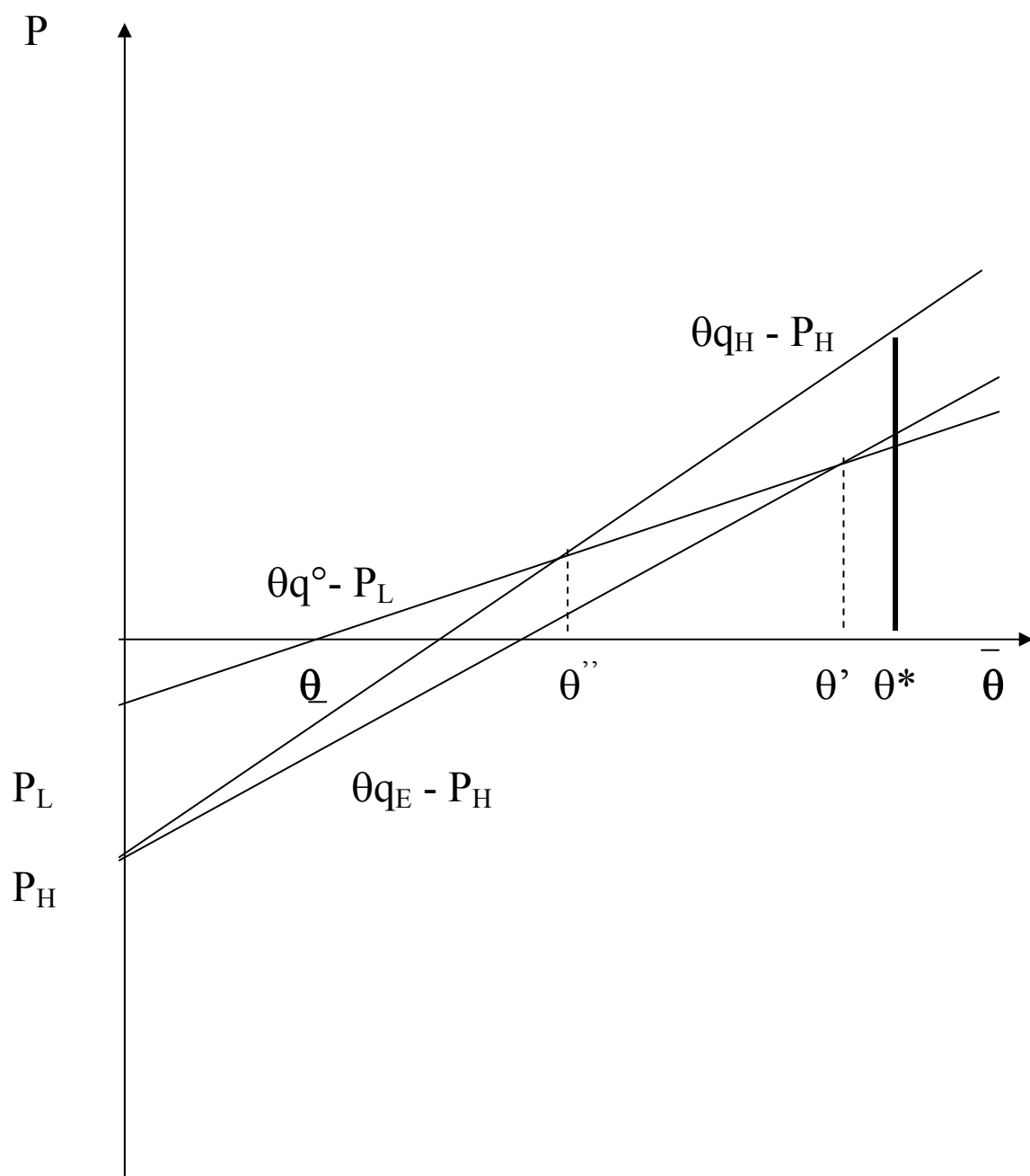


FIG. 9- CASE B.2

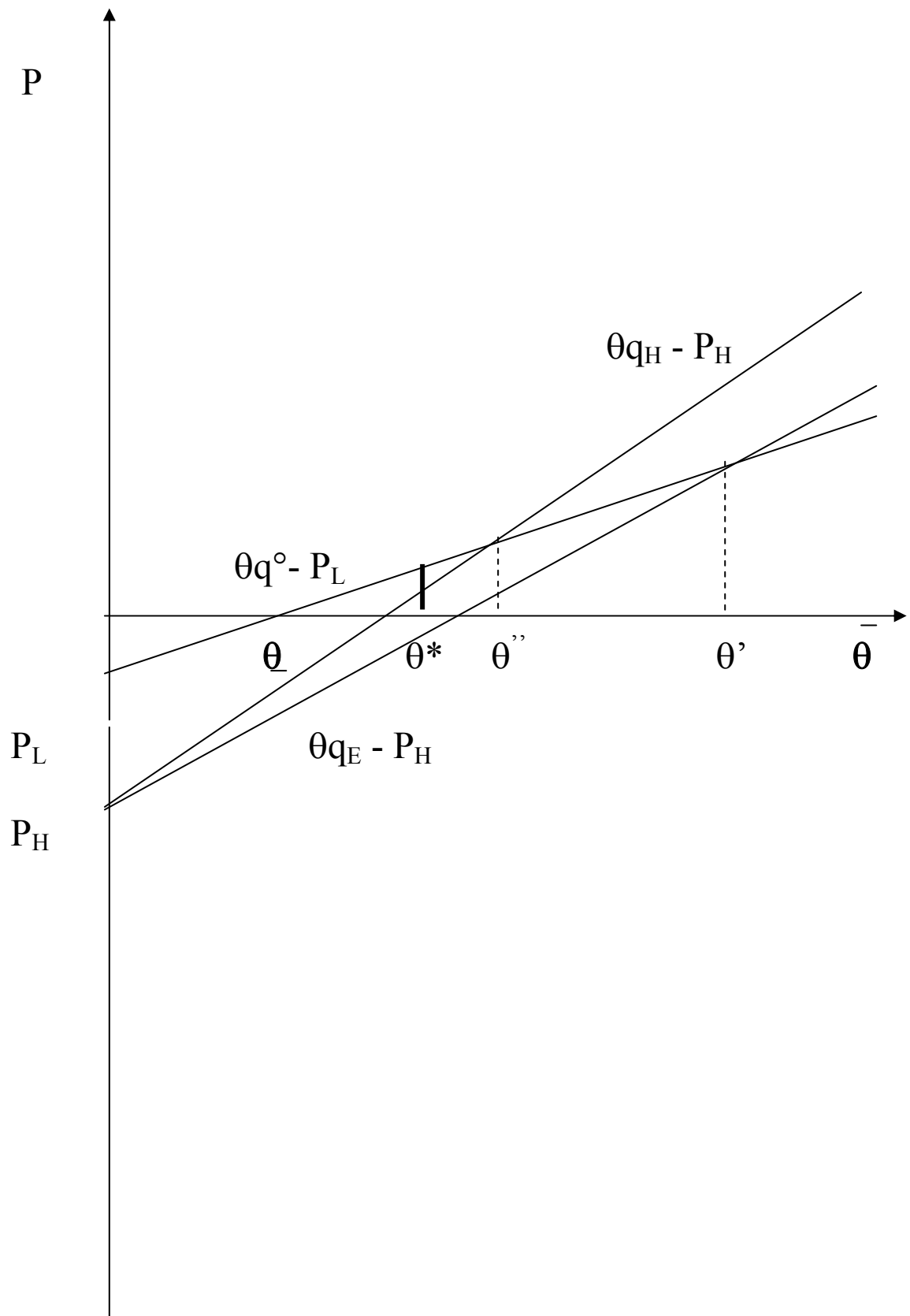
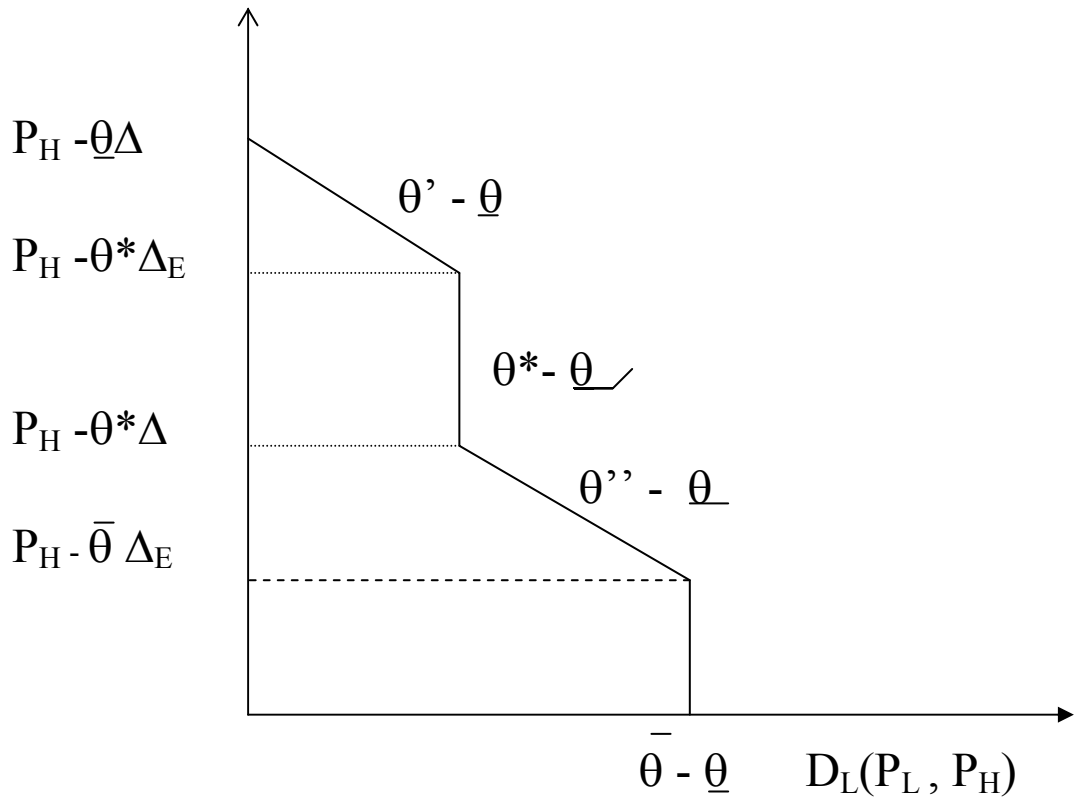


FIG. 10- CASE B.3

X



X

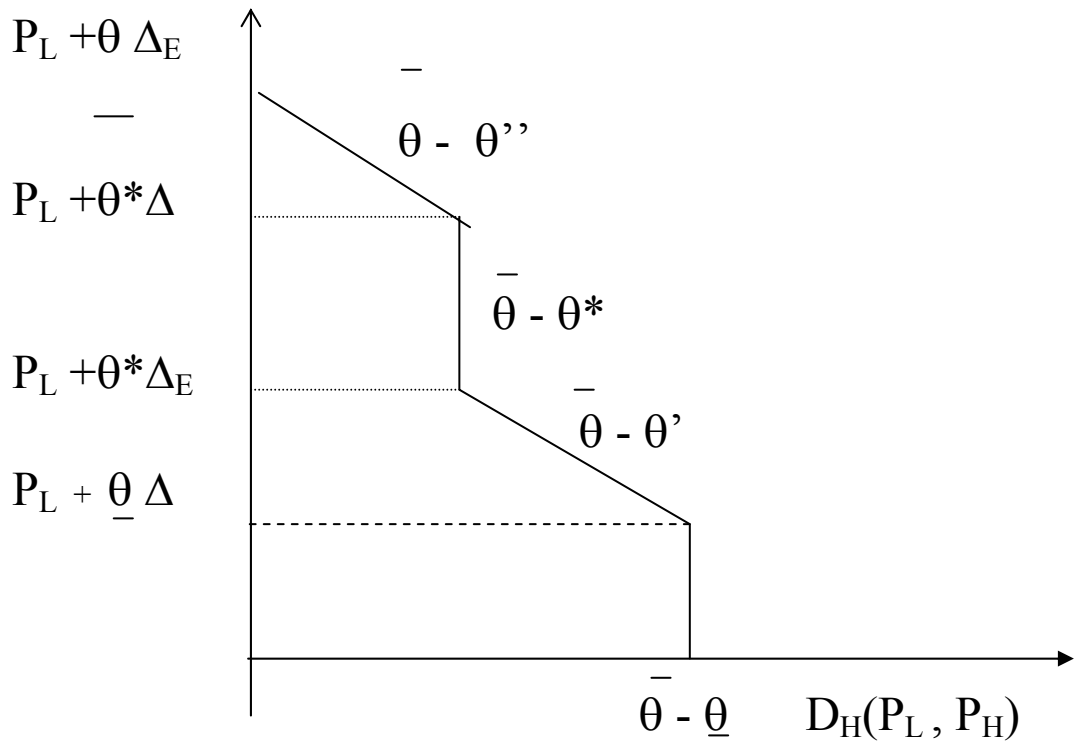
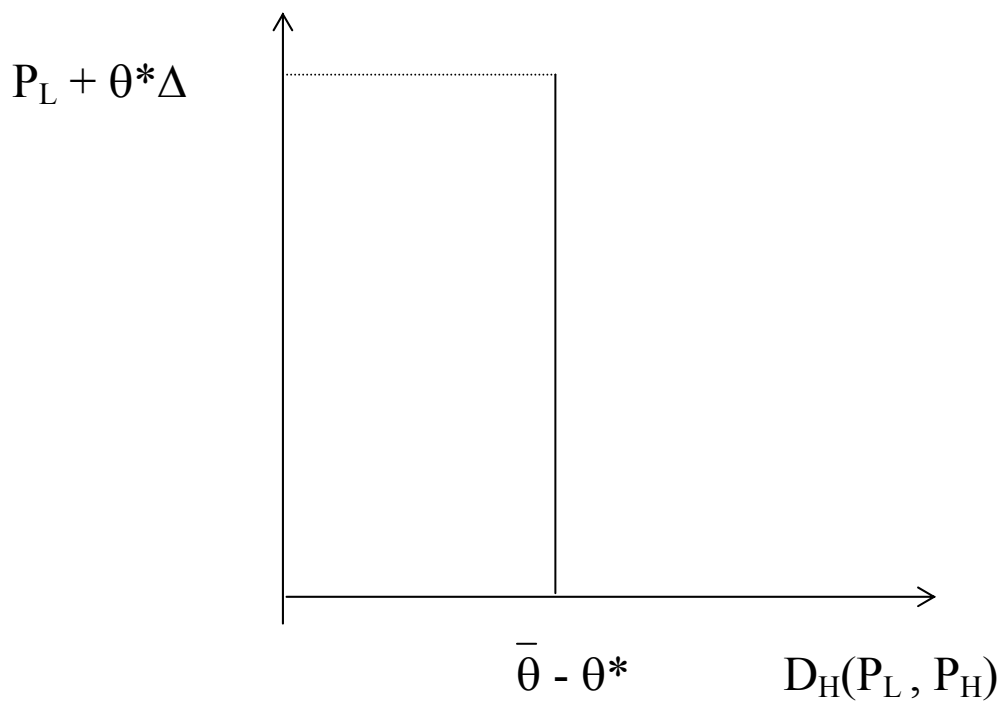
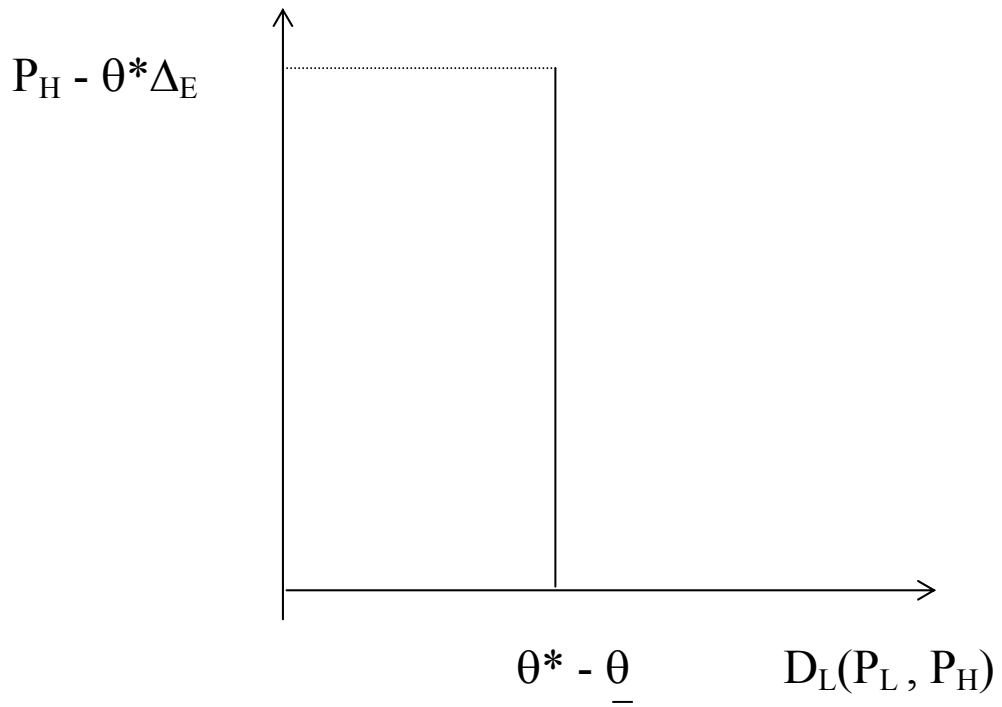
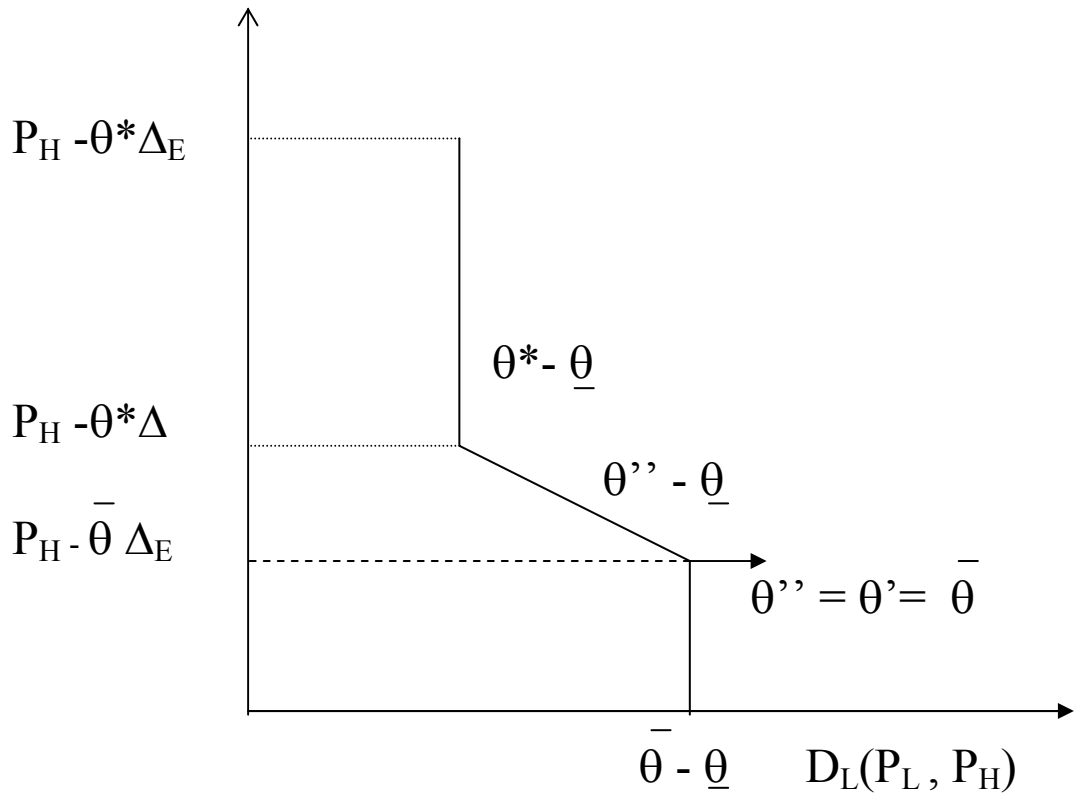


FIG.11 -DEMAND FUNCTIONS: B.a



**FIG.12 -DEMAND FUNCTIONS: B.b**

X



X

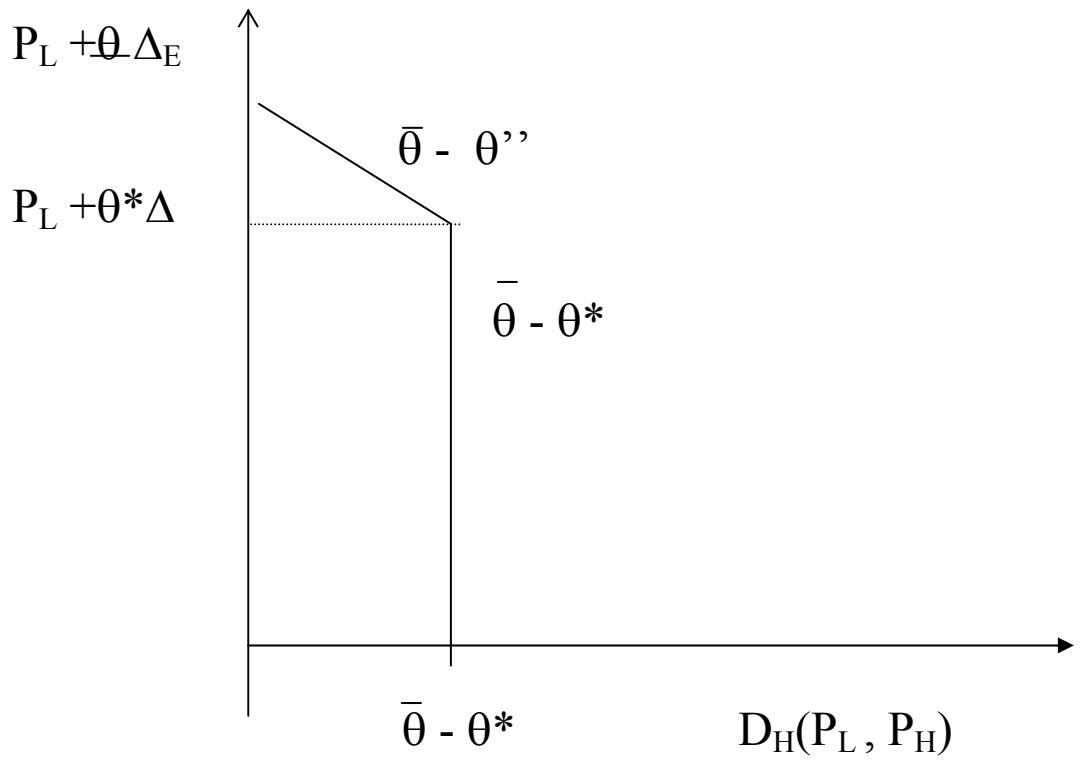
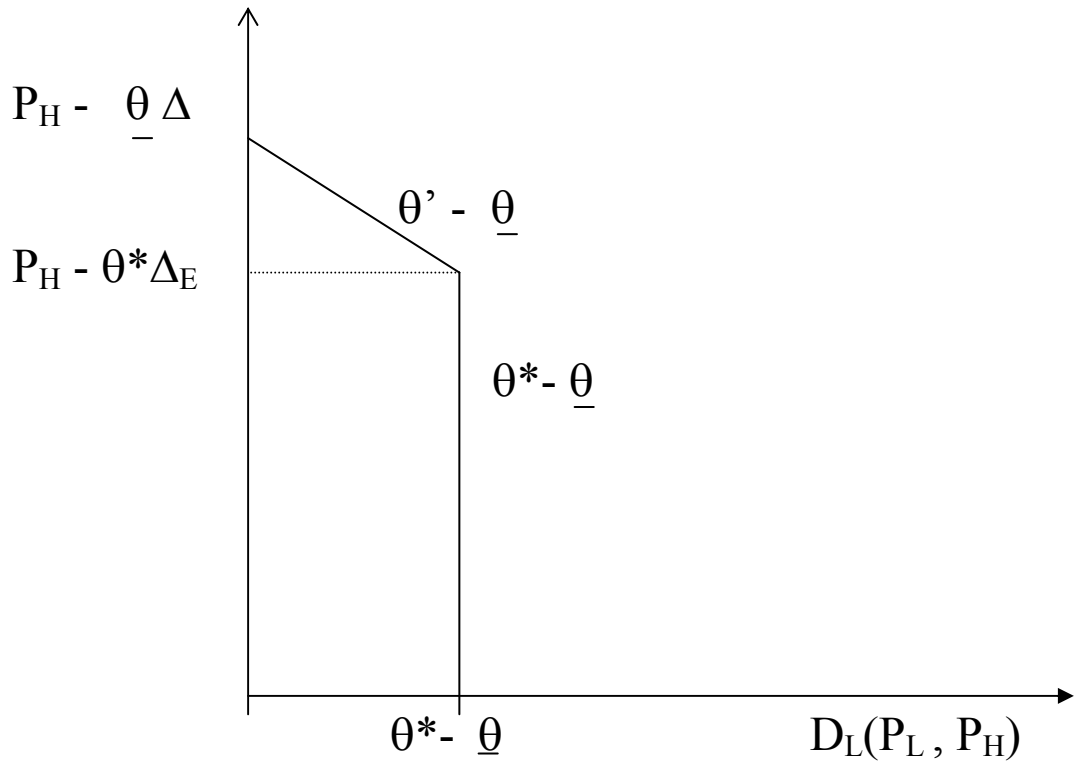


FIG.13 -DEMAND FUNCTIONS: B.c

X



X

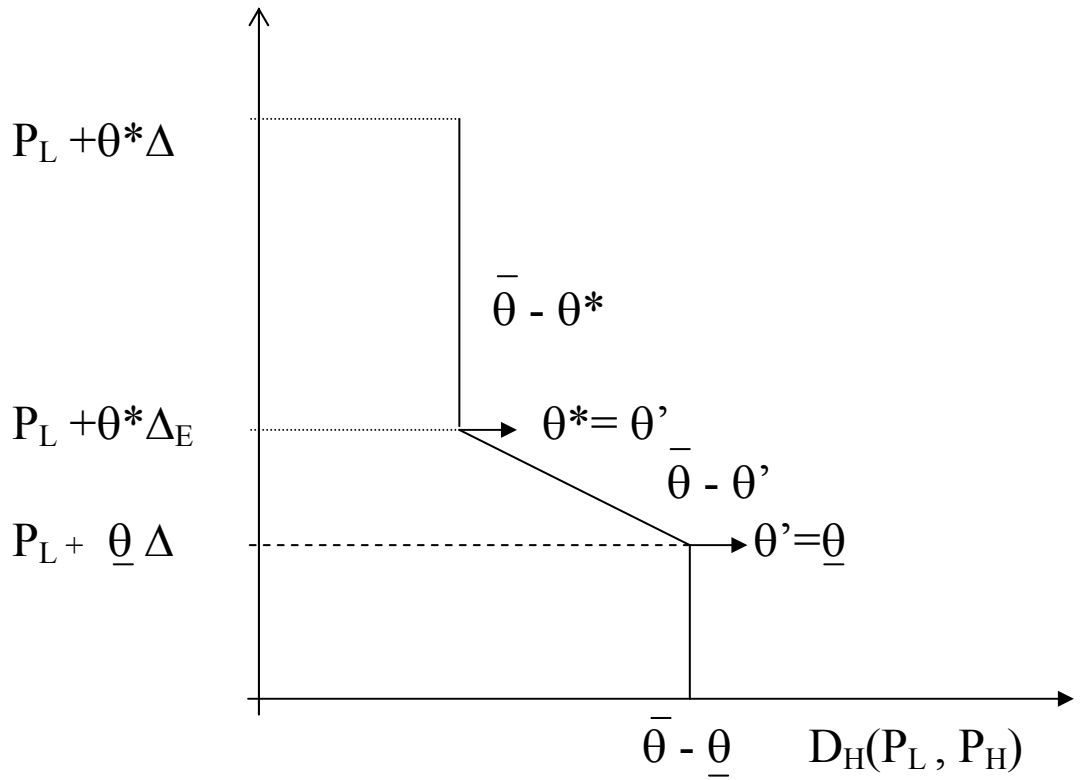


FIG.14 -DEMAND FUNCTIONS: B.d

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- (lix) This paper was presented at the ENGIME Workshop on “Mapping Diversity”, Leuven, May 16-17, 2002
- (lx) This paper was presented at the EuroConference on “Auctions and Market Design: Theory, Evidence and Applications”, organised by the Fondazione Eni Enrico Mattei, Milan, September 26-28, 2002
- (lxi) This paper was presented at the Eighth Meeting of the Coalition Theory Network organised by the GREQAM, Aix-en-Provence, France, January 24-25, 2003
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- (lxv) This paper was presented at the EuroConference on “Auctions and Market Design: Theory, Evidence and Applications” organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003
- (lxvi) This paper has been presented at the 4th BioEcon Workshop on “Economic Analysis of Policies for Biodiversity Conservation” organised on behalf of the BIOECON Network by Fondazione Eni Enrico Mattei, Venice International University (VIU) and University College London (UCL), Venice, August 28-29, 2003
- (lxvii) This paper has been presented at the international conference on “Tourism and Sustainable Economic Development – Macro and Micro Economic Issues” jointly organised by CRENoS (Università di Cagliari e Sassari, Italy) and Fondazione Eni Enrico Mattei, and supported by the World Bank, Sardinia, September 19-20, 2003
- (lxviii) This paper was presented at the ENGIME Workshop on “Governance and Policies in Multicultural Cities”, Rome, June 5-6, 2003

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