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Taste Heterogeneity and Substitution Patterns in Models of the Simultaneous Choice of Activity Timing and Duration¹

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Abstract

The recent growth of interest in activity-based methods has focused particular attention on travellers' decision making regarding the timing and duration of their participation in activities. However, to date these two dimensions of activity participation have been largely treated separately. It is clear, however, that in general, the benefit that an individual derives

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from participating in an activity will depend inter alia both upon the time at which the activity is undertaken and the amount of time devoted to the activity. Moreover, it is also clear that this benefit will also depend on a wide range of other factors such as the tastes and preferences of the individual, the perceived quality and other characteristics of the available travel modes, the activity opportunities available at particular destinations as well as the intensity with which the individual undertakes activities. Since many of these factors are inherently difficult or impossible to completely characterise or measure via conventional travel or time use data sources, it is likely that such decisions will also be characterised by significant degree of unobserved heterogeneity.

Based on earlier theoretical work by the authors, this paper proposes a model for the simultaneous choice of the timing and duration of activities and associated travel and uses data from a stated preference experiment to estimate the parameters of this model. The first section of the paper provides a brief review of the existing literature on activity timing and duration choice. The second section introduces the theoretical approach, which assume that the marginal utility derived from activities encompasses two distinct components; one derived from the duration of activity involvement and the other derived from activity participation at a particular time-of-day. The third section briefly describes the stated preference data, which was collected in a survey undertaken in Amsterdam in which respondents were presented with a number of scenarios in which they were asked to choose between alternative tours involving a single destination activity. The timing and duration both of the destination activity and the associated travel varied across scenarios. The fourth section discusses the empirical specification and estimation of the model and presents the estimation results. Particular attention is given to the use of advanced optimisation techniques needed to estimate the nonlinear utility function expressing individuals' timing and duration preferences. The fifth section discusses the significance of the results and their potential application to a number of practical transport and land use planning problems including the prediction of user response to travel demand management policies and accessibility planning. The paper closes with some overall conclusions and a discussion of future research directions.

1. Introduction

With the advance and growing popularity of travel demand and traffic management policies in congested urban areas, the timing of trips is an increasingly important and relevant topic. In particular, the distribution of trips over time (e.g. during a morning peak) may have considerable impact on the degree of congestion, air quality and on overall levels of accessibility. According to activity based travel theory (Ettema and Timmermans, 1997), trips can be regarded as a necessary means to connect spatially remote activities that will logically precede or follow these activities. This implies that the timing of trips not only depends on trip characteristics that vary by time-of-day (such as travel time and delays) but also on preferences with respect to the start time and duration of activities. Consequently, when modelling trip-timing decisions, these should be regarded in the context of the activity-scheduling process (e.g. Ettema and Timmermans, 2003).

With respect to modelling the timing and duration of activities, various approaches have been taken within the activity-based framework. A first group of models (Bowman and Ben-Akiva, 1998; Arentze and Timmermans, 2005), although applying widely different decision-making mechanisms, have essentially treated the timing of activities as being a choice between a limited number of discrete time intervals. For instance, Bowman and Ben-Akiva (1998) conceptualise the timing of activities as the choice between the morning, afternoon or evening. However, if one is interested in the effect of the timing of activities and trips on traffic flows and congestion, dividing continuous time into a number of rather coarse discrete time intervals is far too limited.

Other approaches have indeed treated time as a continuous variable. A first group of studies is concerned with the allocation of time to distinct activities. These studies are based on the seminal work of Becker (1965), who treated time as a finite resource, which could be allocated to activities, resulting in a certain level of utility. Time allocation is in this view regarded as an optimisation problem under the restriction of a fixed amount of resources (time). In particular, the duration of activities is then determined by the allocation of time to activities such that the overall utility is maximised. Becker's model was elaborated by, amongst others, Evans (1971) and De Serpa (1971) in order to account for the consumption of goods given consumption rates, prices and the available monetary budget. Other extensions of

this approach include the modelling of time allocation on the household level (Zhang et al., 2002) and the specification and testing of advanced utility functions (Joh et al., 2003). If the utility derived from an activity can be defined as a log-function of the time spent on an activity, the time allocation model can be formulated as a system to be estimated using for instance seemingly unrelated regressions (Kitamura, 1984). An important property of this type of formulation is that the marginal utility of activities decreases with their duration, representing the onset of activity fatigue.

The Becker-type models are able to describe how individuals maximise utility by allocating time to activities, and what utility they derive from a particular distribution of time to activities. They do, however, not take into account the preferences that individuals have with respect to the timing of activities. For instance, it is assumed that the marginal utility of one time unit of an activity is independent of the time-of-day. However, studies of time-of-day choice of e.g. commute trips (Small, 1982) clearly indicate that the timing of activities affects the utility derived from the activity pattern.

The issue of activity timing has received relatively little attention to date in the literature. In most modelling approaches, timing is a derivative of a number of other interrelated considerations, including scheduling convenience (e.g. minimising travel or costs) or time constraints (e.g. facility opening hours or work hours). Only few exceptions assume the utility of an activity to be directly dependent on time of day. For example, Joh et al. (2002) utilise a scaling factor for the duration dependent utility function, which depends on the start time of the activity. A different approach is taken by Wang (1996), who assumes that the marginal utility of activity participation at a time t equals the observed share of the sample involved in the activity at that time. This is based on the problematic assumption that each activity can be performed at the preferred time, which is unrealistic given the many constraints applying to activity scheduling processes. To overcome this problem, Ettema and Timmermans (2003) propose an alternative model, in which the marginal utility of an activity is a direct function of time-of-day. A similar, marginal utility model formulation was also earlier proposed by Polak and Jones (1994). A problem with the Polak-Jones-Ettema-Timmermans (henceforth, PJET) models, however, is their neglect of the duration component within their marginal utility formulations. Many activities are likely to be subject to fatigue effects, implying that the utility derived from one time unit of activity participation diminishes with increasing duration. The PJET models in contrast, assume that one unit of activity engagement at time-of-day t

will always yield the same utility, irrespective of the duration of activity engagement. Although Ettema and Timmermans (2003) propose a modification allowing for duration effects, the PJET models do not yet offer the full flexibility required.

Recently, Ashiru et al. (2004) and Ettema et al. (2004) have proposed models explicitly accounting for both the duration and timing of activities in the context of full activity patterns. These models (see section 2) assume that individuals derive a utility from each activity in which they participate, which depends on both a time-of-day dependent component and a duration dependent component. Based on the utilities of activities and trips, overall utilities of activity patterns are defined, based on which individuals are assumed to prefer one activity pattern over the other. Ashiru et al. (2004) and Ettema et al. (2004) have defined the basic principles of the models and tested the base formulations empirically. However, these efforts only constitute the first steps towards an operational model of activity timing and duration.

Further work, which will be presented in this paper, concerns two issues. First, sociodemographic variables will be incorporated into the utility functions, to account for heterogeneity in the population with respect to preferences for timing or duration options. This is believed will yield important insights into the activity scheduling considerations of particular population groups. Second, the genetic algorithm based estimation procedure used to date to derive parameter values (Ettema et al., 2004; Ettema and Timmermans, 2003) has been replaced with more efficient optimisation methods, to better deal with the highly nonlinear utility formulations and constraints pertaining to particular parameters and to reach more reliable global optima. In addition, tests have been carried out to test whether the MNL model which has been used to date in the estimation process is better replaced by more advanced models supporting hierarchical decision making processes and taste variations without violation of the theoretical assumptions underlying the choice mechanism.

2. Theoretical framework

2.1. Theoretical model

Our theoretical model follows some basic assumptions put forward by a number of other authors, namely that:

1. Individuals derive a certain utility from allocating time to activities (Becker, 1965; Yamamoto and Kitamura, 1996);

- 2. Individuals derive a certain (dis)utility from the time spent travelling (Ben-Akiva and Lerman, 1985);
- 3. Individuals aim at optimising the utility of their overall activity pattern, being the sum of the individual activity and trip utilities (Becker, 1965; Jara-Diaz; 1998a, 1998b Meloni et al., 2004).

In equation:

$$\max U = \max \left(U^T + U^A \right) \tag{1}$$

where U^T is the total utility derived from trips and U^A the total utility derived from activity participation. These utilities are the sums of the utilities of individual trips and activities:

$$U^{T} = \sum_{m} U^{T}_{m} \tag{2}$$

$$U^{A} = \sum_{n} U_{n}^{A} \tag{3}$$

The individual trip utility is defined as a relatively simple function of travel time $(R_m(t))$ and travel cost $(C_m(t))$ associated with trip *T* made at start time *t*. In addition a constant D_m^l is included to represent the constant utility of a trip made by mode *l*:

$$U_m^T(t) = D_m^l + v_m * R_m(t) + \mu_m * C_m(t)$$
⁽⁴⁾

It is noted that additional trip characteristics can be added without materially changing the approach. However, scheduling costs, which represent the disutility of the diversion of some preferred arrival time for the trip, are not included in the utility of trips. Instead, these are represented in the utilities of activities through the implications for activity duration and timing. To incorporate socio-demographics the utility function is extended to:

$$U_m^T(t) = D_m^l + D_m^{ls} * s + [\nu_m + \nu_m^s * s] * R_m(t) + [\mu_m + \mu_m^s * s] * C_m(t)$$
(5)

where *s* represents a dummy variable indicating whether the respondent falls within some socio-demographic group. D_m^{ls}, v_m^s and μ_m^s represent adjustments to the corresponding parameters for a particular segment *s*.

The utility derived from an activity depends, as noted before, on both the time-of-day and the duration. In other words, the first minute spent on an activity may be valued differently than the 10-th or 50-th minute, but the 10-th minute may be valued differently when engaged in at 7.00 AM or 2.00 PM. Thus, both the history and the timeliness of the activity play a role in this respect:

$$U_n^A(t) = f(t, (t - t_n^0))$$
(6)

where t_n^0 is the start time of activity *n*. In this respect we define the activity utility as a function of two components; namely a duration component U_n^D and a time-of-day component U_n^H :

$$U_{n}^{A}(t) = f(U_{n}^{H}(t), U_{n}^{D}(t))$$
⁽⁷⁾

The time-of-day component is specified as the baseline utility profile, specifying the user benefit of being involved in an activity at a particular time of day. The time-of-day dependent utility is best understood in terms of the marginal utility $U_n^{\prime H}(t)$ specifying the amount of utility gained from participation during one time unit at time *t*. Although alternative specifications are available (see Ettema and Timmermans, 2003), the time-of-day component selected is based on a Cauchy distribution:

$$U_{n}^{\prime H}(t) = \frac{1}{c_{n}\pi \left[\left(\frac{t - b_{n}}{c_{n}} \right)^{2} + 1 \right]} * U_{\max,n}$$
(8)

In this function, *b* defines the optimum location, where the utility is a maximum. *c* defines the width of the curve (which is symmetrical), which defines the time period in which an acceptable level of utility is gained and finally U_{max} , scales the Cauchy distribution (see Ettema et al., 2004 for examples of the effects of the parameters on the utility shape). Sociodemographic variables can principally affect the time-of-day dependent utility through U_{max} , *b* or *c*. If we again define s as a dummy variable representing membership of a particular sociodemographic segment, the utility can be formulated as:

$$U_{n}^{\prime H}(t) = \frac{1}{(c_{n} + c_{n}^{s} * s)\pi \left[\left(\frac{t - (b_{n} + b_{n}^{s} * s)}{(c_{n} + c_{n}^{s} * s)} \right)^{2} + 1 \right]} * (U_{\max,n} + U_{\max,n}^{s} * s)$$
(9)

With respect to the duration dependent utility U_n^D , we assume that utility follows a logarithmic function, as proposed by Yamamoto et al. (2000) and Bhat and Misra (1999):

$$U_n^D(t) = \eta_n \ln(t - t_n^0)$$
(10)

Resulting in the marginal utility:

$$U_{n}^{\prime D}(t) = \frac{\eta_{n}}{(t - t_{n}^{0})}$$
(11)

An important implication of this function is that marginal utility decreases with increasing duration, representing the fatigue effect, which is intuitively plausible. Socio-demographics can affect the duration dependent utility by modifying the constant η , leading to the following formulation:

$$U_{n}^{\prime D}(t) = \frac{(\eta_{n} + \eta_{n}^{s} * s)}{(t - t_{n}^{0})}$$
(12)

where η_n^s represents an adjustment to η_n for a particular segment *s*. Having specified the components $U_n^{\prime D}$ and $U_n^{\prime H}$, the total utility derived from an activity, U_n^A , can be calculated by summing the respective parts:

$$U'_{n}(t) = U'_{n}^{H}(t) + U'_{n}^{D}(t)$$
(13)

Because both components are scaled by U_{max} and γ respectively, it is not necessary to add weights to each component. The marginal utility is thus defined by:

$$U'_{n}(t) = \frac{U_{\max,n} + U^{s}_{\max,n} * s}{(c_{n} + c^{s}_{n} * s)\pi \left[\left(\frac{t - (b_{n} + b^{s}_{n} * s)}{c_{n} + c^{s}_{n} * s} \right)^{2} + 1 \right]} + \left[\frac{\eta_{n} + \eta^{s}_{n} * s}{(t - t^{0}_{n})} \right]$$
(14)

The resulting model is particularly flexible (see examples given by Ettema et al., 2004) and is capable of representing marginal utility curves, ranging from the bell shaped profile to the more commonly used logarithmic functions. The model is relevant to the analysis of trip timing behaviour in that it provides a framework for analysing how individuals decide between alternative activity and trip schedules, based on the total utility as indicated by equation 14. The trip timing decisions are implicitly defined by the start time and duration of the chosen activity schedule.

2.2. Operational model

The operational model is applied to a home-based tour and is operationalised in the current study as follows. Following the approach of Polak and Jones (1994), we assume that travellers choose the departure time of trips from home to work and from work back to the home. This effectively divides the day into three periods (pre-work, work and after-work), which we regard, for simplicity, as each comprising of single activities comprising of all temporally related components associated with the activity. This implies that the total utility of commuters' activity patterns can be formulated as:

$$U = V_1^T + V_2^T + V_1^A + V_2^A + V_3^A + \mathcal{E}$$
(15)

Strictly speaking, the conceptual model outlined in section 2.1. regards timing and duration decisions as the outcome of an optimisation problem, in which time is treated as a continuous variable. In the current study, however, we will assume that an individual chooses between a limited number (say N) of feasible activity patterns $[P_1....,P_N]$ characterised by total utilities $[U_1....,U_N]$. It is recognised that the assumption of time allocation on a continuous scale is not ideally represented as a discrete choice between a limited number of allocation options. However, the data available to validate the model (SP choice data) necessitates this

assumption. Nevertheless, it is felt that the choices made in the SP experiment reflect the preferences for certain time allocation patterns. In particular, the chosen alternative may be considered to be the closest match to an individual's unconstrained allocation outcome. Therefore, the discrete choice data can be used to disentangle the marginal utility functions that guide time allocation on a continuous scale.

Thus, it is assumed that discrete choice theory provides an adequate framework to model the choice of activity patterns, based on utility function (15). Since activity and travel patterns not only involve allocation decisions but also discrete choices of travel mode, destination choice and sequencing, the decision process may be of a non IIA nature, involving unobserved heterogeneity in tastes and for heteroskedascity and complex substitution patterns amongst activity alternatives. Using up to date GEV and mixed logit models (Train, 2003) can account for such effects. As a first test of the non-IIA character of the activity pattern choice model, this paper describes tests of hierarchical nested logit models, based on the hierarchical choice process depicted in Figure 1, suggesting that detailed timing and duration decisions are nested under the more fundamental mode choice decision. Utility functions of the mode and timing/duration alternatives may then be defined as:

$$U_{car} = U_1^{T,car} + U_2^{T,car} + \vartheta_{car} + \varepsilon_{car}$$
(16)

$$U_{PT} = U_1^{T,PT} + U_2^{T,PT} + \vartheta I_{PT} + \varepsilon_{PT}$$

$$\tag{17}$$

Where,

 $U_n^{T,car}, U_n^{T,PT}$ are the utilities of the n-th trip by car and public transport respectively.

$$I_{car} = \ln(\sum_{t \in T_{car}} V_t)$$
⁽¹⁸⁾

$$I_{PT} = \ln(\sum_{t \in T_{PT}} V_t)$$
⁽¹⁹⁾



Figure 1: Hierarchical decision process

Where V_t represents the total utility derived from the activities, as detailed below;

$$V_t = V_t^{pre} + V_t^{work} + V_t^{post} + \mathcal{E}_t$$
(20)

In which,

$$V_{t}^{n} = (\eta_{n} + \eta_{n}^{s} * s) \ln(d) + \int_{0}^{d} \left[\frac{1}{(c_{n} + c_{n}^{s} * s)\pi\left[\left(\frac{x - (b_{n} + b_{n}^{s} * s)}{(c_{n} + c_{n}^{s} * s)}\right)^{2} + 1\right]} * (U_{\max,n} + U_{\max,n}^{s} * s)]dx$$
(21)

It should be noted that equation 21 is the integral of marginal utility function outlined in equation 14.

The formulation (equations 15-21) is partly similar to Bowman and Ben-Akiva's (1998) structure, but a fundamental difference is in the formulation of the utility as a function of the timing and duration of alternatives. Whereas conventional discrete choice models assume each utility to be a linear function of a vector of attributes, the extended JPETA model allows for much more complex functions of the utility of activities, enabling the utility to be sensitive to timing as well as duration on a continuous scale (equation 21). Thus, equation 21 is an extremely flexible specification of activity utility, able to capture many timing and duration effects that are likely to guide activity scheduling decisions in daily life.

3. Stated Preference Data

The model proposed in section 2 was empirically tested using a stated preference data set, collected in the Amsterdam area in 2000 as part of a project to assess commuters' potential responses to various road user charging schemes. Respondents were recruited by means of detailed screening and quota control criteria in which drivers undertaking work, employers business, shopping and social and leisure tours were selected.

The stated preference experiments involved respondents being offered realistic choices between alternative tour patterns. In order to avoid highly unattractive or highly unrealistic SP alternatives, these alternatives were developed based on the characteristics of the individual's current tour, which could include any type of activity.

During the SP experiment respondents were provided with a) re-timing options involving shifts earlier or later relative to the most temporally constrained activity; b) activity duration options; c) total two-way travel time options; and d) total road price charge options. In the survey, a public transport trip, similar to the most attractive existing PT trip, was offered as an alternative for the road pricing options.

Thus, the data set provides data regarding the relevant choice dimensions incorporated in the model: activity timing and duration, trip duration and mode choice and is therefore suitable to test the model. To test the model data for respondents who indicated that their current tour was a work trip was selected, as the resulting home-based tour is considered most likely to represent a daily activity pattern. After tests for data consistency and completeness, this

resulted in some 1,382 observed choices. For each subject, a limited number of sociodemographics were available, along with information regarding their working arrangements.

4. Estimation Procedure & Results

Estimating the model given by equations. 15-21 involves finding the parameters that maximise the goodness-of-fit of the nested logit model. Following Ben-Akiva and Lerman (1985) the log-likelihood function is formulated as:

$$LL(\theta) = \frac{1}{I} \sum_{i=1}^{I} \sum_{m} \sum_{t} y_{mt} \log P_{mt}^{i}$$

$$LL(\theta) = \frac{1}{I} \sum_{i=1}^{I} \sum_{m} \sum_{t} y_{mt}^{i} \log(P_{t|m}^{i} P_{m}^{i})$$
(22)

Where:

- P_{mt}^{i} is the probability that individual *i* chooses mode alternative *m* with timing alternative *t*,
- P_m^i is the probability that individual *i* chooses mode *m*,
- P_{tlm}^{i} is the probability that individual *i* chooses timing option *t* given mode *m*,
- y_{tm}^{i} is a dummy variable indicating whether individual *i* chooses alternative *tm*,
- *I* is the population size.

The parameters θ are then computed by solving the program

$$\max_{\theta} \phi(\theta) = LL(\theta) \tag{23}$$

The highly non-convex character of the log-likelihood (22) leads us to consider nonlinear programming approaches, especially trust-region methods. The main idea of a trust-region algorithm involves the calculation, at iteration k (with current estimate θ_k), of a trial point $\theta_k + s_k$ by approximately maximizing a model m_k of the objective function inside a trust region defined as

$$B_{k} = \{ \boldsymbol{\theta} \text{ such that } \| \boldsymbol{\theta} - \boldsymbol{\theta}_{k} \| \leq \Delta_{k} \},$$
(24)

where Δ_k is called the trust-region radius. We can for instance use a quadratic model:

$$m_k(s) = LL(\theta_k) + s^T \nabla_{\theta} LL(\theta_k) + \frac{1}{2} s^T H_k s, \qquad (25)$$

where H_k is a symmetric approximation of the Hessian $\nabla^2_{\theta\theta} LL(\theta_k)$. The predicted and actual increases in the value of the objective function are then compared by computing the ratio:

$$\rho_{k} = \frac{LL(\theta_{k} + s_{k}) - LL(\theta_{k})}{m(\theta_{k} + s_{k}) - m(\theta_{k})}.$$
(26)

If this ratio is greater than a certain threshold, set to 0.01 in our tests, the trial point becomes the new iterate, and the trust-region radius is (possibly) enlarged. More precisely, if ρ_k is greater than 0.75, we set the trust-region to be the maximum between Δ_k and $2s_k$, otherwise we set $\Delta_k = 0.5\Delta_k$. If the ratio is below the bound, the trial point is rejected and the trust region is shrunk by a factor of 2, in order to improve the correspondence of the model with the true objective function. We have followed Conn et al. (2000) in our choice of the parameters.

We additionally constrain the parameters U_{max} and c of the marginal utility (8) to be strictly positive. The integration of (8) indeed yields the following analytical expression of the utility derived for the time of day:

$$U_{n}^{\prime H}(t_{n}^{0}+d) = \frac{U_{\max,n}}{\pi} \left(arctg\left(\frac{d-b_{n}}{c_{n}}\right) - arctg\left(\frac{-b_{n}}{c_{n}}\right) \right)$$
(27)

which is discontinuous at c_n equal to zero. Moreover we assume that $U_n^H(t)$ is positive, so $U_{\max,n}$ and c_n must be of the same sign. The positiveness constraints are managed by means of log-barrier terms that are added to the objective, leading to the new, unconstrained program

$$\max_{\theta} \phi(\theta) = LL(\theta) - \lambda \log(\theta)$$
(28)

for some strictly positive λ .

Equation 28 can therefore be solved (with some care) by using a standard unconstrained algorithm. Moreover, if the parameter converges to zero, the equation converges to the original problem (equation 23) so for sufficiently small λ , we obtain a good approximate minimiser of our problem. However a small λ leads to numerical difficulties, that can be avoided by solving a sequence of barrier problems, parameterised by the index *j*,

$$\max_{\theta} \phi_j(\theta) = LL(\theta) - \lambda_j \log(\theta)$$
⁽²⁹⁾

with the property,

$$\lim_{j \to \infty} \lambda_j = 0 \tag{30}$$

Under some reasonable assumptions, it can be shown that the algorithm then converges to a solution of equation 23, (Fiacco and McCormick, 1968, and Wright, 1992).

Note however that if $U_{\max,n}$ or c_n converges to zero, both corresponding time of day marginal utility and its integral vanish. Therefore, if some of the positiveness constraints are active at the solution, the associated time of components do not add useful information to the model, and can be excluded from it. The resulting model is then unconstrained, and can be estimated using standard nonlinear programming techniques.

Using the above algorithm, the model specified in eqns. 15 to 21 was estimated. However, the estimation outcomes suggested various changes to the original model specification. First, it appeared that the model significantly improved from adding alternative specific constants to the early and late car trip alternatives, as well as to the public transport alternative. That is to say, up and above the implications for timing and duration of activities and trips, there are inherent preferences for deviating from the standard departure time or switching to another mode. Second, it turned out that the inclusive value parameter ϑ was very close to 1, suggesting that the choice process at hand can be adequately described by a multinomial logit model (while retaining the non-linear time-of-day component). Finally, the time-of-day

component proved to be significant for the work activity, but not for the pre-work and afterwork activities. Apparently, these are less tied to particular times than the work activity, probably due to the general specification of these activities in this study.

These three considerations have led to the base model describing the choice of mode and timing of the out and in bound commute trip, displayed in Table 1. The estimated constants suggest that a priori, travellers dislike deviations from their usual departure time. Late departure is disliked more than early departure. However, the largest disutility is experienced when switching to another mode (public transport). The η s indicate the marginal utility derived from a certain activity (see eqn. 12). The value for η_{pre} thus implies that the highest utility is gained from this activity, whereas the lowest (duration dependent) utility is derived from work. If time-of-day effects are discarded, it can be shown that time is allocated to activities proportional to the η s (Ettema, 2005). However, including the time-of-day component may lead to different outcomes.

	Model 1		Model 2		Model 3	
	estimate	t-value	estimate	t-value	estimate	t-value
$ASC_{early,car}$	-1.13	7.920	-1.14	8.067	-1.14	8.055
$ASC_{late, car}$	-1.85	9.719	-1.83	9.677	-1.83	9.662
ASC _{public transport}	-2.17	9.133	-2.21	9.196	-2.21	9.194
$\eta_{_{pre}}$	7.21	9.027	5.64	6.608	5.71	6.615
$\eta_{\scriptscriptstyle work}$	0.40	1.060	0.36	1.071	0.45	1.236
$\eta_{\scriptscriptstyle post}$	1.09	3.168			0.26	0.514
b_{work}	577.18	63.444	580.27	73.412	579.70	71.171
C_{work}	65.35	3.722	67.36	4.768	66.53	4.567
$U_{max,work}$	5.35	4.338	4.72	4.510	4.65	4.367
V _{car}	-0.0040	2.304	-0.0030	1.753	-0.0028	1.601
V_{PT}	-0.0027	1.240	-0.0604	4.371	-0.0606	4.387
$\mu_{_{car}}$	-0.0569	4.113	-0.0019	0.891	-0.0017	0.788
$\mu_{_{PT}}$	0.0025	0.437	0.0035	0.623	0.0034	0.620
$\eta_{_{pre}}$ * HighEduc			3.99	3.622	3.98	3.593
$\eta_{\scriptscriptstyle post}$ * Male			0.85	3.758	0.71	1.747
$U_{\max,work}$ * HighEduc			2.24	2.230	2.22	2.214
Goodness of Fit	0.9412		0.9347		0.9347	

	Table	1:	estimation	results
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The estimated parameters suggest that the highest TOD dependent utility is derived at 577 minutes (9.37 AM), with the marginal utility being concentrated between 7.30 Am and 1.00 PM. An important implication is that individuals will maximise their utility by adjusting their work start and end time, taking into account both duration dependent and TOD dependent utility.

The overall marginal utilities for each activity are displayed as a function of clock time in Figure 2, assuming regular working hours (8.00-17.00). It is easily seen that the marginal utilities at the activity endings are not equal, as micro-economic time allocation theory would suggest. This finding is likely to be due to the time-of-day dependent utility and constraints with respect to timing of activities may lead to different outcomes. The car time and car cost coefficient are negative as one would expect, and imply a value of time of f 4.22/hour. This figure is significantly lower than official Dutch VOT figures. It should be noted however, that travel times savings in our model also lead to additional utility due to longer activity durations, which would lead to larger VOT values. The coefficient for cost of public transport is found not to contribute significantly to the explanation of the choice behaviour.



Figure 2: Marginal utility of activities

The base model was extended by including socio-economic variables, according to eqn. 21. In this respect socio-economic variables were added in a stepwise manner, including only those

variables that contribute significantly to the explanation of the choice of activity patterns. A first conclusion that can be drawn is that the parameters of the base model are not heavily affected by including the socio-demographic variables. The largest effect is on the time and cost coefficients, now resulting in a VOT of f 2.98/hour for the car. However, this effect may again be modified by the changes in durations of activities. With respect to the sociodemographic variables, we find that males, as compared to females, have a larger $\eta_{\scriptscriptstyle post}$. This suggests that the post-work activity is valued as being more important for males. Highly educated respondents, according to the estimated $\eta_{\it pre} * HighEduc$, value the pre-work activity higher than lower educated respondents, but attach a lower value to the after work period. This can be interpreted as a preference to schedule the work activity interval later during the day. Finally, socio-demographic factors are found to influence the time-of-day dependent utility of work. In particular, we find that highly educated people have a higher $U_{\text{max,work}}$. Apparently, the TOD dependent utility of the work activity is higher for females and highly educated individuals. To illustrate the effects of socio-demographic variables, Figure 3 displays the marginal utilities for a typical working day for a low educated female and a higheducated male. The figure clearly displays the aforementioned effects, which will lead, ceteris paribus, to the work activity starting slightly later, but also ending later for the highly educated male.



Figure 3: Marginal utilities for different population segments

5. Application Of The Model

The model outlined above may have various applications. A first obvious application is to predict responses to road pricing or traffic management strategies. For instance, if we assume that a traveller, when faced with congestion at his usual commute trip, has the option of departing earlier or later (say 30 minutes), changing to another mode or remain travelling at his original time. The estimated model can then be applied to predict the most likely response of this traveller. In the example given in Table 2, tolling the peak period trip leads to a decrease in the probability of travelling in the peak with 7%. However, is the toll accomplished a travel time reduction of 20 minutes for the back and forth trip, this offsets the toll and will create a return to the peak effect. By applying the model to a sample of travellers, estimations can be made of the amounts of travellers shifting to other departure times when travel times, travel costs or the timing of activities changes. Ideally, this should be done iteratively in combination with a traffic assignment model to account for the effect of departure time shifts on travel times.

Page generatie									
Base scenario									
Mode	Car	Car	Car	Public transport					
Start time work	8.45	8.00	9.00	8.45					
End time work	5.15	4.30	5.30	5.15					
Total travel time	90	60	60	90					
Pre-work duration	8.00	7.30	8.30	8.00					
Post-work duration	6.00	7.00	6.00	6.00					
Cost	0	0	0	0					
Market share	58%	23%	11%	7%					
]	Road pricing with	out travel time r	eduction						
Mode	Car	Car	Car	Public transport					
Start time work	8.45	8.00	9.00	8.45					
End time work	5.15	4.30	5.30	5.15					
Total travel time	90	60	60	90					
Pre-work duration	8.00	7.30	8.30	8.00					
Post-work duration	6.00	7.00	6.00	6.00					
Cost	5	0	0	0					
Market share	51%	27%	15%	8%					
Road pricing with travel time reduction									
Mode	Car	Car	Car	Public transport					
Start time work	8.35	8.00	9.00	8.45					
End time work	5.05	4.30	5.30	5.15					
Total travel time	70	60	60	90					
Pre-work duration	8.00	7.30	8.30	8.00					
Post-work duration	6.20	7.00	6.00	6.00					
Cost	3	0	0	0					
Market share	57%	24%	12%	7%					

Table 2: Effect of road pricing policy on time-of-day and mode choice

6. Conclusions

This paper has proposed a utility-theoretic framework for timing and duration preferences embedded in a multi-dimensional choice model, which can be formulated in a flexible way as a GEV-model. Doing so, the proposed framework combines a utility-theoretic underpinning in GEV modelling with a very flexible formulation of time and duration preferences. The model framework is extended to account for socio-demographic and context variables, that may affect individuals' valuation of timing and duration.

An improved estimation methodology was developed for a limited type of activity patterns: a home-work-home sequence, where apart from timing and duration, travel mode was a choice dimension. Models were estimated on a Dutch data set, accounting for context variables such as gender and education level. The estimation results suggest that the estimation methodology is capable of estimating meaningful base models, that provide logical utility functions for timing and duration of activities. The time and cost parameters provided lower VOTs than reported by other authors, but VOTs are in line with other studies if the effect on activity duration is taken into account.

The reported work provides a starting point for further research in various ways. First, more extensive estimation efforts have to be made, including a broader range of sociodemographic and context variables.

Second, as activity patterns do not only entail timing, duration and mode choice, more elaborate models need to be estimated which include additional choice dimensions such as activity choice and destination choice. These models will then constitute a realistic base for estimating space-time accessibility models.

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