

Economy vs. History

What Does Actually Determine the Distribution of Shops' Locations in Cities?

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Abstract

This study examines in which cases economic forces or historical singularities prevail in the determination of the spatial distribution of retail shops. We develop a relatively general model of location choice in discrete space. The main force towards an agglomerated structure is the reduction of transaction costs for consumers if retailers are located closely, whilst competition and transport costs work towards a disperse structure. We assess the importance of the initial conditions by simulating the resulting distribution of shops for identical economic parameters but varying initial settings. If the equilibrium distributions are similar we conclude that economic forces have prevailed, while dissimilarity indicates that 'history' is more important. The (dis)similarity of distributions of shops is calculated by means of a metric measure.

Keywords: Firm location choice; Discrete space; Path dependency

JEL-codes: C61; L11; R12

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1 Introduction

Founder of a shop have to decide simultaneously on a number of questions. One of these questions, which are crucial for the profitability or even survivability of the firm, is the location. Should it be located where competitors are most distant, or is it profitable to locate where the competitors are and many consumers are attracted? In any case, the locations of the incumbent shops matter for the choice of a newcomer. The locations of these firms in turn have been chosen taking into account the distribution of the then incumbent firms, and so on. This path dependency brings about that the initial situation may determine to some extent the long-run distribution of shops. We refer to the initial situation as 'history' throughout this study, as opposed to economic forces like the demand and the costs structure of firms. Of course, 'history' rarely fully determines the choices of all subsequently entering shops. Rather, the distribution of firms is shaped simultaneously by both, path dependencies and economic forces, 'history' and 'economy', which mutually interact with varying relative weight until a stationary situation is reached.

The aim of this study is to examine in which cases economic forces or historical singularities prevail in the determination of the long-run distribution of retail shops. A glance at the actual distribution of different shops reveals that the factors which decide on the best location may be varying. Therefore, we do not hope to find that one determinant is generally more important than the other, or vice versa. Instead we intend to identify conditions that increase the probability that 'history' or 'economy' decide the final distribution of shops. The reason why we restrict the analysis to final or 'equilibrium' distributions of firms is that the influence of the initial situation may only be temporarily in some cases. Consider for example the simple case, where the stationary long-run distribution of firms is even, with one firm at each discrete location. This situation may be reached from varying initial distributions of firms, hence 'history' has no impact on the firms' long-run distribution. Yet, until the final situation is reached, each newcomer chooses a location that is not already occupied by other firms, i.e. the incumbents' locations matter temporarily.

Furthermore, we restrict our analysis to the distribution of retail shops within cities, i.e. at a relatively low geographic scale. The reason is that at a larger geographic scale the partial equilibrium view we employ would not be appropriate. If, for instance, one is concerned with the concentration of financial institutions in London, or with the concentration of sports car manufacturers around Modena in Italy, the consideration of specialized labor or the self-reinforcing effect of a large market would matter¹. These

¹We do not focus on the question under which conditions agglomerations arise, since both, eco-

'forward and backward linkages' that play such a prominent role in the 'New Economic Geography' (see Fujita, Krugman and Venables, 1999) require a general equilibrium framework. Yet, at a lower geographic scale these effects lose some of their importance, so that a partial equilibrium framework may suffice to explain the location choices. For instance, in many cities antique shops and fashion boutiques are concentrated strongly in one or a few streets. But neither of them requires specialized labor that could make up an advantage for neighboring locations. In the same vein, we question that the income that these shops generate reinforces the demand for their products to such an extent that it is profitable to be located where other shops are. Instead of forward and backward linkages our study considers another positive externality of agglomerations, namely the saving of transaction costs. If similar shops are located close together, it is easier for consumers to gather information on prices and quality of the goods. Therefore, more consumers are frequenting the shops. Ironically, this only works if competition among shops is intense, so that firms are actually attracted by competition instead of fleeing it. Another example for the reduction of transaction costs through concentration is the time and money consumers have to spend searching for a suitable parking lot. These costs are fixed in that they are independent of the amount actually purchased. They bring about that we frequent a large market hall where many retailers sell their vegetables, rather than buying from a single tradesman two corners away. We argue that these fixed costs per firm decrease with the number of frequented shops at the same location. Hence, a firm which shares its location with one or more competitors is more likely to be chosen by a consumer than an otherwise identical firm which is alone at its location. The trade-off between this centripetal and other centrifugal forces determines the long-run distribution of retail shops.

The study is organized as follows: First we develop a theoretical model of the sequential location choice of retailers in discrete space. In section 3 this model is used to simulate the long-run patterns of locations for varying economic parameters. To assess the extent to which economic forces or historic singularities mold the distribution of shops, we carry out this exercise for different initial settings and compare the outcomes. If the distributions are identical, regardless of the initial situation, economic forces prevailed over 'history'. If the distributions are 'very different', 'history' is more important. How different distributions of firms are is quantified by means of a 'measure of dissimilarity', which is built such that it fulfills a number of basic requirements. Section 4 summarizes the main results and concludes.

economic forces and 'history' may operate against or in favor of agglomeration. Yet, because we model the location choice of firms to assess the importance of these determinants, the degree of spatial concentration is determined as a by-product.

2 The model

In order to keep the model manageable, we have to make a number of restrictive assumptions in the following. As Antoine de Saint-Exupéry said: "Perfection is achieved, not when there is nothing more to add, but when there is nothing left to take away." Even though we know that perfection is hardly ever obtainable, we try to take this motto to heart throughout the following analysis, completing it by the appropriate constraint to the minimization problem.

The most famous precursor of our approach is Hotelling (1929). Since then the model has been altered in a number of ways, e.g. by Smithies (1941), Eaton and Lipsey (1975), D'Aspremont, Gabszewicz and Thisse (1979). A recent contribution to this field is Economides, Howell and Meza (2002). This literature focuses on the conditions that cause the so-called 'principle of minimum differentiation' to become or not to become effective, and on the stability of the firms' locational choices.

Assume a linear market of length r , bounded at both ends (see figure 1). Each position of the market is a possible location for the single-outlet shops. The set of possible locations is denoted by $D = \{1, \dots, r\}$. Locations are differentiated by their relative position to the left market boundary, that is, the outmost left location is denoted location 1. Apart from their relative spatial position all locations are equal *ex ante*. In particular, the distance from one location to a neighboring location is always unity, demand is distributed homogenously. The latter is standardized to one without loss of generality, so that buyers $i = 1, 2, \dots, r$ may be identified by the index of their location. For instance, buyer k resides on the k th position of the market, counted from the left boundary. The assumption that consumer locations remain fixed is a severe, but necessary, restriction of the analysis. It implies that we should interpret the emerging pattern of shop agglomeration as being relative to consumer location.

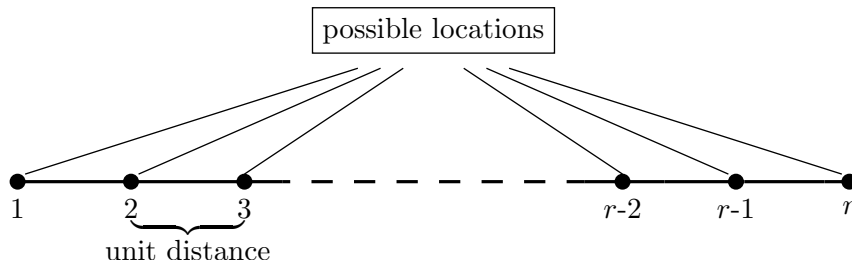


Fig. 1: Linear market with discrete locations

Each firm j produces the amount x_j of a differentiated commodity. The number of firms and heterogenous goods is denoted n , and the set of all firms is $S = \{1, \dots, n\}$. The function $f : S \rightarrow D$ assigns to each firm its respective location, i.e. firm j locates at market position $f(j)$.

2.1 The demand side

Utility of a representative buyer depends upon the consumed amount of the heterogenous goods, $x_{i,j}$. We assume a concave CES utility function, which permits that one or several of the $x_{i,j}$ be zero, thus

$$u_i = \left[\sum_j (x_{i,j})^\rho \right]^{\frac{1}{\rho}}, \quad \text{with } \rho \in (0, 1]. \quad (1)$$

This utility function implies that the elasticity of substitution equals $\sigma = 1/(1-\rho) > 1$. σ does not depend on the consumer prices, which differ among the shops supplying consumer i^2 with goods, due to varying transport costs. The latter brings about that the price index P_i also differs between consumers.

Here, a few more definitions are at order: The subset of S , which contains all shops that actually supply consumer k with goods is denoted by $S_k \equiv \{s \in S : x_{k,j} > 0\}$ with $S = \bigcup_{i=1}^r S_i$, i.e. each variety is purchased by at least one consumer. In analogy, the set of locations of the shops in S_i is denoted by D_i . The number of shops at location $\ell \in D$ is denoted $n_\ell \equiv |\{s \in S : f(s) = \ell\}|$, and the number of shops at location $\ell \in D_i$ from which shopper i buys is $n_{i,\ell} \equiv |\{s \in S_i : f(s) = \ell\}|$.

Buying goods at locations other than the consumer's respective position incurs iceberg transport costs at rate $T > 1$, i.e. if buyer k is willing to consume $x_{k,m}$ unities of firm m 's commodity, she actually would have to buy $x_{k,m} \cdot T^{|k-f(m)|}$ unities. The fraction of goods that melts away during transport is $(T-1)/T$. This implies that from the point of view of this consumer the price of firm m 's commodity is $p_{k,m} = p_m \cdot T^{|k-f(m)|}$, where p_m denotes firm m 's mill price. The price index of shopper i is then defined as

$$P_i \equiv \left[\sum_{j \in S_i} \left(p_j \cdot T^{|i-f(j)|} \right)^{\frac{-\rho}{1-\rho}} \right]^{\frac{1-\rho}{-\rho}} \quad (2)$$

Apart from the mill price plus transport costs, each buyer incurs transaction costs $a_{i,j}$ for each good she actually demands, which are independent of the quantity

²Recall that the number of consumers is standardized to the number of positions of the market, and that consumers are identified by the position they have.

purchased. These costs' effect on the consumers' behavior is that the amount a consumer wishes to buy is subject to a lower threshold below which the advantage of an increased variety does not suffice to outweigh the additional transaction costs. The amount of transaction costs consumer i has to pay for buying from a retailer at location ℓ depends on $n_{i,\ell}$:

$$a_{i,\ell} = \frac{\gamma}{(n_{i,\ell})^\alpha}, \quad \text{with } \alpha \in [0, 1] \quad (3)$$

where γ are the transaction costs if a consumer buys from only one shop at location ℓ . The parameter α represents the advantage of co-location. In the case $\alpha = 0$, transaction costs *per shop* are γ , no matter how many shops at location ℓ a consumer chooses, hence no transaction costs can be saved. In the case $\alpha = 1$, transaction costs *per location* are independent of the number of shops, hence the advantage of buying from several shops at the same location is the greatest. Intermediate values of α imply that, if a consumer buys from an additional shop at location ℓ , total transaction costs increase, but average transaction costs per shop decrease. α thus represents the positive effect of a retail shop choosing a location close to other shops on demand. This effect is partially external to the newcomer, and is a type of economies of localization (see e.g. Anas, Arnott and Small, 1998, p. 1446)

Transaction costs are a crucial feature of our model. They provoke that the model exhibits two most welcome features. First, the reduction in transaction costs that a consumer experiences if two shops are located at the same position constitutes a positive external effect of an agglomeration, i.e. an incentive to co-locate. This effect is much easier to model than the 'forward and backward linkages' that play a similar role in the 'New Economic Geography'. Second, these transaction costs prevent consumers from buying arbitrary small amounts of each variety, which is a property of the basic Dixit and Stiglitz (1977)-model.

Consumers maximize their utility subject to a budget constraint. Their income net of transaction costs y_i is

$$y_i = \psi - \sum_{\ell \in D_i} (a_{i,\ell} \cdot n_{i,\ell}) = \psi - \gamma \cdot \sum_{\ell \in D_i} (n_{i,\ell})^{1-\alpha}$$

where ψ is the exogenous and equal gross income per consumer. Consumers spends all of their net income buying goods at prices $p_{i,j}$:

$$y_i = \sum_{j \in S_i} (x_{i,j} \cdot T^{|i-f(j)|} \cdot p_j) = \sum_{j \in S_i} (x_{i,j} \cdot p_{i,j}). \quad (4)$$

If consumer i 's demand for firm m 's commodity is positive, $m \in S_i$, the amount actually purchased can be found by maximizing utility function (1) under budget

constraint (4) (see Fujita et al. (1999)):

$$\max_{x_{i,j}, \lambda} \mathcal{L} = \left[\sum_{j \in S_i} (x_{i,j})^\rho \right]^{\frac{1}{\rho}} + \lambda \left[y_i - \sum_{j \in S_i} (x_{i,j} \cdot p_{i,j}) \right] \quad (5)$$

The first-order conditions state that the derivatives with respect to all $x_j^i, j \in S_i$ equal zero. From the derivatives with respect to $x_{i,j}$ and $x_{i,m}$ we get

$$\left(\frac{x_{i,j}}{x_{i,m}} \right)^{\rho-1} = \frac{p_{i,j}}{p_{i,m}}.$$

Solving for $x_{i,j}$, and substituting the resulting expression in the budget constraint (4) yields

$$x_{i,m} = \frac{y_i}{p_{i,m}} \cdot \left(\frac{P_i}{p_{i,m}} \right)^{\frac{\rho}{1-\rho}} \quad (6)$$

Other things being equal, this relationship implies that buyer i 's demand for the heterogenous good m depends positively on her net income and the price index, and it depends negatively on the price of the good itself. The price elasticity of demand equals σ , and is independent of the distance between a retailer and a consumer.

Adding one shop to the set of shops S_k from which consumer k buys affects her utility in various ways. On the one hand, for the class of utility functions assumed ($\rho < 1$), she values the increased variety. On the other hand, total transaction costs increase if $\alpha < 1$, which lowers the amount of money she can spend on consumption. In the optimum, she and all other consumers may therefore not buy every firm's good, which limits the number of firms that is actually able to survive in the market. In comparison with related models of monopolistic competition without transaction costs, e.g. by Dixit and Stiglitz (1977), this means that c.p. consumers' demand is concentrated on less retailers.

Consumer i compares every possible *combination* among all shops and locations from which she may buy goods, and chooses the combination that brings about the highest utility. In doing so, she may well leave out a shop that increases utility by more than another one but which has the disadvantage of an isolated location.

2.2 The supply side

Each shop is free to choose the profit-maximizing price for its variety. The maximization problem of a representative retailer thus reads:

$$\max_{p_j} \pi_j = (p_j - 1) \cdot x_j - F \quad (7)$$

where π denotes retailer j 's price, variable costs per unit output are standardized to unity, and fixed costs are denoted by F and equal for all retailers. The first-order condition for a profit maximum is

$$x_j + (p_j - 1) \cdot \frac{\partial x_j}{\partial p_j} = 0 \quad (8)$$

The optimal price p_j depends on the price elasticity of demand. Due to the assumed CES-utility function, the price elasticity of demand is $\sigma = 1/(1 - \rho)$ for all consumers at all locations. Unfortunately, this does not imply that the elasticity of one retailer's aggregate demand is σ as well. For any given number of consumers, the price elasticity of aggregate demand is σ .³ But, for some values of p_j , one or the other consumer will be indifferent between buying from shop j or not. If the price is still higher, she will not buy from this shop at all, and if it is lower, she will buy a discrete positive amount. Mathematically, at these points p_j^s , essential singularities of the individual demand functions emerge, due to the discrete nature of the model. Where the singularities emerge depends, among other things, on the prices of the competitors, which, for their part, depend on the elasticity of demand. We decided to circumvent this problem by assuming that each retailer takes the sets of shops from which 'their' consumers buy as given to them.⁴ Only then, the (conjectured) price elasticity of aggregate demand equals the individual price elasticity, σ .

With a price elasticity of aggregate demand $\sigma = 1/(1 - \rho)$, we obtain for the profit-maximizing price:

$$p_j = \frac{1}{\rho} > 1. \quad (9)$$

Hence the mill price of each firm is the same: $p_j = p = 1/\rho \quad \forall j \in S$. But, of course, the consumer prices $p_{i,j}$ may vary because of transport costs: $p_{i,j} = p_j \cdot T^{|i-f(j)|} = T^{|i-f(j)|}/\rho$. The price index (2) at position i is

$$P_i = \frac{1}{\rho} \left[\sum_{j \in S_i} \left(T^{|i-f(j)|} \right)^{\frac{-\rho}{1-\rho}} \right]^{\frac{1-\rho}{-\rho}} \quad (10)$$

The total amount firm j produces and sells in terms of demand quantities is:

$$x_j = \sum_{i=1}^r \left(x_{i,j} \cdot T^{|i-f(j)|} \right) \quad (11)$$

³By the way, this is the reason why the use of a CES-utility function is so advantageous in most models of the 'New Economic Geography', e.g. the core-periphery model (Fujita et al., 1999).

⁴Our assumption is even more restrictive than the Lösch-conjecture in the theory of spatial competition, according to which each firm considers the *extension* of its market area as being given (see Capozza and Van Order, 1978). Since the price index (2) exhibits jump discontinuities whenever a consumer varies the number of firms she frequents, it does not suffice to assume that firms consider the number of their clients as being stable.

Making use of (6) and (11), profits of firm m read:

$$\begin{aligned}
\pi_m &= (p_m - 1) \cdot \sum_{i=1}^r \left[\frac{y_i}{p_{i,m}} \cdot \left(\frac{P_i}{p_{i,m}} \right)^{\frac{\rho}{1-\rho}} \cdot T^{|i-f(m)|} \right] - F \\
&= (1 - \rho) \cdot \sum_{i=1}^r \left[\frac{\left(T^{|i-f(m)|} \right)^{\frac{-\rho}{1-\rho}} \cdot y_i}{\sum_{j \in S_i} \left(T^{|i-f(j)|} \right)^{\frac{-\rho}{1-\rho}}} \right] - F
\end{aligned} \tag{12}$$

A potential newcomer in the market calculates the maximum profits which would yield at each market position. The position which is associated with the highest positive profits on the drawing board is chosen and becomes the location of the shop. The decision is final because no retailer can move its site once it is settled, e.g. due to high relocation costs. A formal reason to forbear from the possibility of relocation is that otherwise we would run into the quadratic assignment problem, for which Koopmans and Beckmann (1957, p. 69) showed that “no price system on plants, on locations and on commodities in all locations, that is regarded as given by plant owners, say, will sustain any assignment.” Assuming that a newcomer takes into consideration only the potential profits for the given actual distribution of other shops is quite restrictive. In particular, one might expect that an entrant’s prospects about future entries enter into the decision, which would be necessary to derive a subgame-perfect equilibrium (see e.g. Economides et al., 2002). Allowing for such strategic interdependencies would yet complicate the structure of the problem to such an extent that one could not hope to arrive at a conclusion. Formally, this would amount to a location game in which the number of moves and the number of players themselves depend on the moves of the players.

2.3 Equilibrium

As in the classic Dixit and Stiglitz (1977)-model the optimal number of firms / heterogeneous goods is determined by the trade-off between decreasing average costs and the consumers’ preference for variety. In comparison to the continuous Dixit-Stiglitz model, our approach is discrete, which implies that profits are not necessarily zero or even equal for all firms in equilibrium. Decreasing fixed costs do not necessarily come along with a rising number of firms in our model. Instead, the effects of such changes depend on whether specific threshold values are exceeded or not. The discrete structure of the model also renders possible that a small variation causes large-scale modifications of the spatial distribution of retail shops.

A second trade-off decides the spatial distribution of the firms. To discuss the effects on the firms' locational choices it is useful to understand space as a phenomenon that protects firms from competition. A high preference for variety (*low* elasticity of substitution between the goods, ρ close to zero) means that competition is weak. Yet, Koeniger and Licandro (2004) show that this interpretation is wrong in the context of the standard Dixit-Stiglitz general equilibrium model, since variations of the substitutability do not affect the relative price of consumption goods, implying that the decentralized equilibrium is optimal. In our model, though, an increase of substitutability affects output directly. Households reduce the number of consumed varieties because of the transaction costs that come along with each variety. Hence, the degree of competition rises. Firms thus tend to co-locate, if the elasticity of substitution is low. Another force in favor of concentration is caused by the transaction costs. If co-location saves transaction costs, firms which co-locate are more attractive to consumers and thus increase profits. In comparison, high transportation costs make it profitable to be located where demand is, i.e. dispersedly.

A crucial question that has to be answered at this point is: what concept of an equilibrium is appropriate in this model? In comparison to the Dixit and Stiglitz-model of monopolistic competition, our model features complications as discrete choices of locations, a discrete number of firms, and the like. Therefore, it is not applicable to assume that firms enter the market until all firms just break even. Instead, firms may attain varying positive profits, depending on their location. An equilibrium is defined as follows:

Definition 1 *An equilibrium distribution of firms is given if all incumbent firms' profits are non-negative, and if no potential newcomer could enter the market at any location without making a loss, given the incumbent firms' number and locations.*⁵

The restriction 'given the incumbent firms' number and locations' in this definition is an important one. It could be, for instance, that a firm's entry causes other firms to leave the market, which would render the firm profitable. Thus, it could pay to accept losses in the short run to gain a profitable position in the long run. For this to happen, the firms would need to foresee not only consumer behavior but also entry and exit decisions of other firms which may strategically take a loss just as well. On the one hand, the strategic interdependence of potential firms with other newcomers and firms who are already in the market is certainly of great interest from

⁵It was just too hard to overcome the temptation to variegate Saint-Exupéry's quotation: An equilibrium is achieved, when there is nothing more to add, *and* nothing left to take away.

a game theoretic point of view. On the other hand, we do not believe that we can actually deal with this issue. Besides sizeable computational effort even for the case of a given number of players (firms), the main problem is that the number of players itself depends on strategic moves of the players. Therefore, we decided to abstract from this kind of strategic behavior.

3 Simulations

One shortcoming of our approach to deal with specific historic events and discrete choices is that results can only be derived by means of simulation, i.e. exemplarily. To mitigate the disadvantage that these results cannot be generalized because of the specific historic situations they are based on, we simulate the equilibrium distribution of retail shops for a very large number of starting positions and parameter values, a total of 2.100 runs.

Figure 2 shows how the employed algorithm derives equilibria. Starting from a given distribution of existing firms, hypothetic profits of the newcomer are calculated at each of the r market segments, and stored in the vector $\mathbf{\Pi}_h$. The newcomer enters the market at the location that corresponds to the maximum of hypothetic profits, if it is nonnegative. Then, firms that would make losses exit, until the remaining firms' profits are all nonnegative, and the next iteration starts. The process stops when the maximum profits a newcomer could attain are negative. The following subsection specifies which parameter values are being used.

3.1 Specification of variables

To give a precise picture of what happens in the model, we allow the following parameters to vary over a broad range of values. These three parameters are in the following referred to as the 'economic' parameters to distinguish them from the position of the first firm, which is taken as representing the 'history'.

- $\rho \in \{0.4, 0.5 \dots 0.9\}$. This parameter determines how close substitutes the varieties are. The higher it is, the more similar are the varieties perceived by consumers, the more elastic is demand, and the more intense is competition among firms.
- $\alpha \in \{0, 0.1 \dots 0.6\}$. If this parameter is zero, no transaction costs may be saved by buying from several firms at one location. In the case $\alpha = 0.6$, transaction

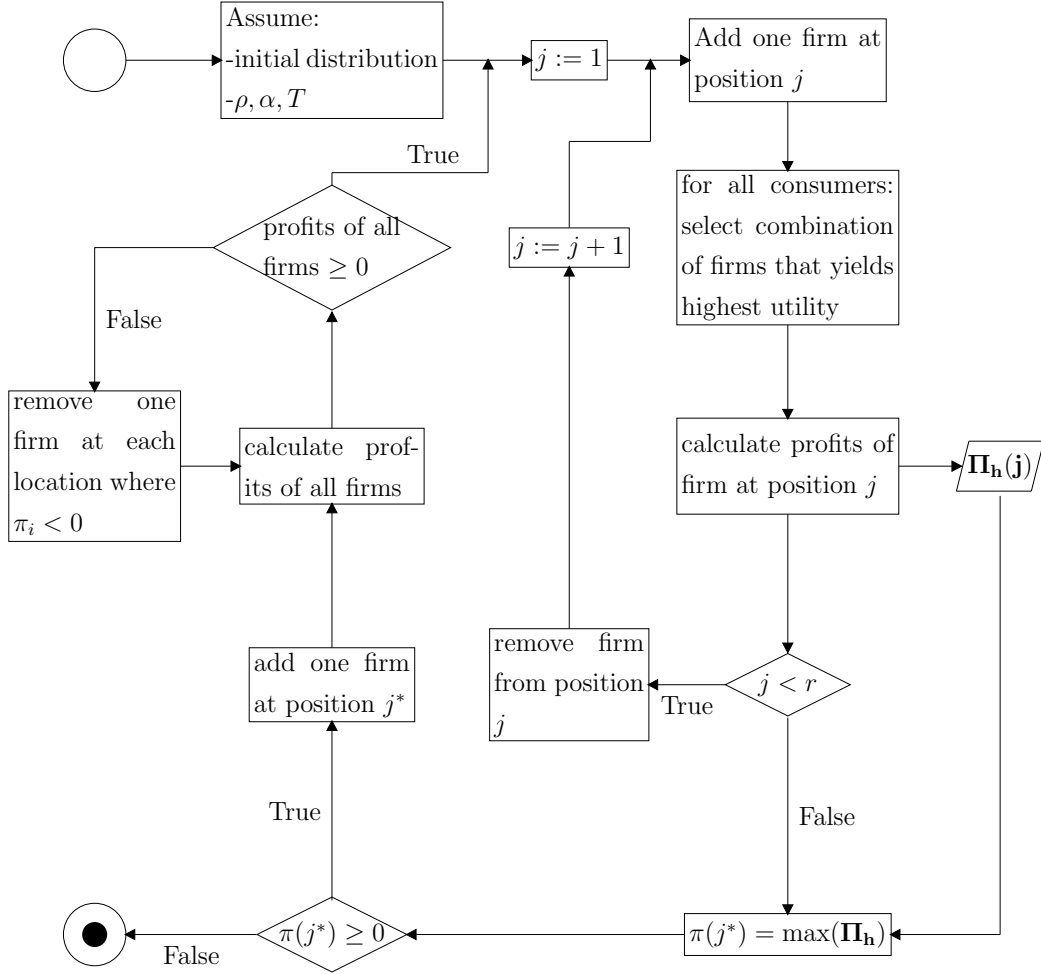


Fig. 2: Inside the black box

costs for two firms at one location are only about 32% higher than for one firm, hence the advantage is quite pronounced.

- $T \in \{1.1, 1.2 \dots 1.5\}$. Transport costs are within the range 10 – 50% of the transported commodity per unit distance. The upper value appears unrealistically high, all the more because our analysis is intended to be related to local agglomerations of businesses. If $T = 1.5$, a consumer one unit distance away from a firm's site would have to buy 50% more of a good than what she is actually willing to consume, i.e. only two third of the amount bought can be consumed. A consumer 2 units of distance away receives only $(2/3)^2 = 4/9$ of what the firm sent off. We chose nevertheless to consider this broad range of parameters, primarily in order to render comparisons to other studies possible.

Parameters whose values are held constant throughout the analysis are summarized in table 1. Gross income ψ is standardized to unity per household for simplicity.

γ , the transaction costs per firm at a sole location, are 0.1, and fixed costs F are 0.5. A market length (number of positions) of about 20 seems to allow for an analysis of edge effects, without making them a dominant factor. Since 20 is even, it would not be possible to set the location of the first firm (the 'pioneer') such that the market is symmetric. Therefore we assume a market length r of 19.

Parameter:	ψ	γ	F	r
Value:	1	0.1	0.5	19

Table 1: Parameters and their values

We are interested in the extent to which the historically determined initial distribution and/or economic forces shape the equilibrium distribution of retail shops. For this aim, we need to define some initial distributions because the number of distributions of firms is unlimited ex ante. We chose to compare the outcome of the model for different locations of the pioneering first firm. Only the pioneer can choose a location without being faced by other firms. Furthermore, this firm is a pure monopolist, which implies that its behavior cannot be described by the above algebra. It knows that, with the assumptions made, all consumers are willing to spend all of their net income $\psi - \gamma$ on its product. Even though consumers are not indifferent regarding the pioneer's location (due to the transport costs they bear), the firm is. Hence, its location has to be assumed anyway. It suffices to consider deviations from the symmetric case (pioneer's location at position 10) in one direction, because of symmetry. Therefore we consider the following 10 cases: $f(1) \in \{1, 2, \dots, 10\}$. If the pioneer's location is a position i that is not included in this set, the outcome is symmetric to the case $r + 1 - i$, which is included.

3.2 Time structure of the model

For simplicity, we assume that firms enter the market one by one. Otherwise, we either would have to cope with incorrect expectations of the firms (if other firms' entrance is neglected), or we would run into the quadratic assignment problem (if firms do anticipate other firms' market entry, see Koopmans and Beckmann, 1957). One period is defined as the time span between the entry of two firms. This period comprises – in this order – the time it takes to calculate and compare potential profits at all locations, the construction of the plant, possibly the exit of one or more firms, production and distribution of all firms' goods, their transport and consumption. All flow figures, like the consumers' income or fixed costs, are related to this time span.

Consistency requires that – as in the definition of an equilibrium distribution of shops – no firm takes losses. Since firms anticipate what their profits would be if the distribution of firms remains the same, and no expectations are made regarding future changes of the firms’ distribution, there is no reason to take a loss. In comparison with market entry, we allow that several firms exit the market simultaneously. Specifically, we assume that at each location, where profits of firms would be negative, one firm abandons. If the remaining firms all attain non-negative profits, the process is finished. Otherwise it is repeated until there no firm is left which would make a loss. It is only then, that the actual distribution of firms is determined, and consumers decide which amount they buy from which of the remaining retail shops.

The latter implies that a firm which enters the market at the beginning of the period may actually be mistaken, since it has not expected that other firms would exit the market. This would not be so much a problem if the ordering of potential profits at different locations would be unaffected by the perishing of firms. Unfortunately, this must not always be the case. Therefore, it might happen that a firm enters the market at location k , other firms perish, and this affects the potential profits such that it would have been advantageous to choose a location different from k . The reason is that the firms’ expectations regarding the distribution of firms is static, while the expectations regarding the behavior of consumers is rational. Note however, that this asymmetry of expectations complies with the asymmetry of these spheres: While the number and locations of the consumers is given to the firms from the first period on, the number of firms and their locations is determined endogenously in a complex and interactive way. Therefore, we see the different assumptions regarding expectations towards firms and consumers as being justified in an admittedly stylized and focused model framework.

3.3 Measuring (dis)similarity

Before we analyze how much the resulting patterns of shop agglomeration differ, depending on differing positions of the pioneering firm, we need to have a measure which is capable to account for differences along several dimensions. Patterns of agglomeration may differ with respect to the size of an agglomeration, the position of an agglomeration, the number of agglomerations, or the total number of firms. Which measure is chosen to compare the patterns that result from varying positions of the pioneering firm is crucial for the question the paper addresses. The most

straightforward measure is the Minkovski-form distance

$$\varphi_{\zeta k} = \left[\sum_{i=1}^r (|\omega_{\zeta}(i) - \omega_k(i)|)^{\mu} \right]^{1/\mu}, \quad \mu > 0 \quad (13)$$

which equals the city-block metric in the case $\mu = 1$, and equals the Euclidean distance in the case $\mu = 2$. One problem with this measure is, however, that it does not account for where the difference occurs. For instance, the value of the measure would be the same if a firm is displaced by one or by five units distance. Also, if two distributions only differ in that one distribution has one more firm at one location, the measured difference, $(1^{\mu})^{1/\mu} = 1$, would be lower than if that firm would be displaced by one position, $(1^{\mu} + 1^{\mu})^{1/\mu} = 2^{1/\mu}$, since in the latter case differences at two positions occur.

Another possible candidate for the measure is the so-called edit (Levinstein) distance. This measure counts the editing effort (symbol deletion, insertion or substitution), needed to transform one pattern into another. The main difference is that the edit distance does not differentiate by how many firms two distributions differ at one segment. All sorts of differences are treated equally.

Since these measures strike as unsatisfactory, a measure that is more 'problem-tailored' is called for. We propose:

$$\varphi_{\zeta k} = \frac{1}{2} \left\{ \sum_{j=1}^r \left| \sum_{i=1}^j [\omega_{\zeta}(i) - \omega_k(i)] \right| + \sum_{j=1}^r \left| \sum_{i=1}^j [\omega_{\zeta}(r+1-i) - \omega_k(r+1-i)] \right| \right\} \quad (14)$$

This measure displays a number of desirable features, which are discussed in detail in the [appendix](#). The basic intuition is that the minimum number of movements of firms is counted that would be necessary in order to transform one distribution into another one, i.e. it measures the 'effort' it takes to produce two identical distributions.

3.4 Results

The following results are based on the measure of dissimilarity (MOD), defined in (14). Table 2 illustrates the dissimilitude of the equilibrium distributions, depending on the locations of the pioneers. Note that each value in table 2 is the average of 210 MOD values. If, for instance, the equilibrium distributions of firms that yield from the pioneer being located at market position 3 are taken as the reference ($f(1) = 3$), and the outcome of the cases with $f(1) = 5$ are being compared to them, the MOD takes values between 0 and 121, depending on the parameters α , ρ and T . The average of these values is 19.16 (7th row, 9th column). Of course, the value is the same, if the distributions that yield from $f(1) = 5$ are defined the reference, and those corresponding to $f(1) = 3$ are compared to it (9th row, 7th column).

The last row of table 2 averages over the average values of the MOD, excluding the 0 which results if the outcomes are compared to themselves, respectively (see axiom 1 in the [appendix](#)). It turns out that the average value of the MOD is relatively low (20.58) if the symmetric cases ($f(1) = 10$) are chosen as reference. If the outcome of other initial distributions is defined as reference, the average value of the MOD first increases with the deviation from the symmetric cases. The distributions of firms with an initial distribution $f(1) = 7$ feature the highest average MOD: 23.78. With a still higher distance of the pioneer's location, possibly surprisingly, lower average values of the MOD yield, so that, with the exception of the special case when the border position is defined the reference, an inverted U-shaped relationship arises. The minimum is reached if the outcomes of the cases with $f(1) = 2$ are used as reference: 20.56. In the case $f(1) = 1$ (the pioneer is located in the left-most position), the average MOD is higher again.

How can we explain the described inverted U-shaped relationship between the position of the pioneer and the value of the MOD of the resulting distributions of shops, i.e. the U-shaped relationship between the position of the pioneer and the similarity of the outcome? To answer this question, it is useful to bear in mind that a location in the center of the market is optimal in that it minimizes average transport costs of all consumers. If the pioneer is located there, and with sizable advantages of a common location (saving of transaction costs), it is likely that an agglomeration arises at the center of the market. If, however, the pioneer is located nearby the center, this may become the location of several retailers, which causes the final distribution to be different. A location in the more peripheral positions in contrast, may not be sustainable in some cases because subsequently entering retailers have a strong incentive to chose a location different from the one of the pioneer, thereby deflecting its demand. Hence, it follows that the final distribution of shops is more similar to the one resulting from the symmetric case precisely because the position of the pioneer is further away. Even though this result may not hold for every single parameter combination, the average values given in table 2 indicate that it indeed holds for a relevant subset of the considered cases. It is summarized in the following proposition:

Proposition 1 *If the pioneers's position is varied, cum grano salis, the resulting distributions of shops are more similar in the cases of a central and a peripheral position, relative to intermediate cases. Excluded from this proposition are the distributions which result from the cases when the pioneer's location is at the endmost position.*

Our goal is to determine the extent to which economic forces or an arbitrary

		Reference distribution yields from pioneer located at position									
		10	9	8	7	6	5	4	3	2	1
Position of the pioneer	10	0	19.83	24.70	25.18	20.35	18.50	16.74	20.38	17.29	22.24
	9	19.83	0	19.41	21.74	24.44	18.97	22.57	17.81	20.98	21.08
	8	24.70	19.41	0	22.75	24.38	24.32	21.71	24.55	18.81	25.15
	7	25.18	21.74	22.75	0	24.92	22.60	25.25	21.63	27.20	22.76
	6	20.35	24.44	24.38	24.92	0	21.92	22.48	24.41	21.03	25.39
	5	18.50	18.97	24.32	22.60	21.92	0	20.43	19.16	20.77	22.16
	4	16.74	22.57	21.71	25.25	22.48	20.43	0	19.42	18.17	22.73
	3	20.38	17.81	24.55	21.63	24.41	19.16	19.42	0	19.82	19.30
	2	17.29	20.98	18.81	27.20	21.03	20.77	18.17	19.82	0	20.99
	1	22.24	21.08	25.15	22.76	25.39	22.16	22.73	19.30	20.99	0
Average		20.58	20.76	22.86	23.78	23.26	20.98	21.06	20.72	20.56	22.42

Table 2: Average MOD values for alternative reference distributions

historic starting position mold an equilibrium distribution of shops. The first step is that we have to define a reference case in terms of where the pioneer’s site is (the ‘initial distribution’). In the next step we use the MOD to assess how similar the resulting distributions are if the position of the pioneer is varied, but the other (economic) parameters are held constant. If the MOD is low, we can conclude that the distributions are relatively similar in spite of the variation of the pioneer’s position: Economic forces prevail over ‘history’. If however a high value of the MOD results although the elasticity of substitution (represented by ρ), the advantage of a common location (α) and transport costs (T) are the same, the historically determined initial position has prevailed over economic forces.

Concerning the determination of the reference distribution, we have ten outcomes, corresponding to alternative positions of the pioneer, for each of the 210 considered combinations of the variables ρ , α and T . It seems advisable to define the outcome of one of these ten cases the standard throughout to guarantee comparability. We chose to define the outcomes of the symmetric cases, where the pioneer is located at market position 10 as reference for three reasons.

First, table 2 shows that the symmetric cases exhibit the second lowest average value of the MOD, which means that the outcomes can justifiably be characterized as typical. Second, it is to be expected that the average MOD would remain constant in the symmetric cases if the mirrored cases $f(1) = 11, \dots, 19$ would be included, while it would increase in all other cases, where the locations of the pioneers are further away in average. Third, the result that the profits of the pioneer are independent from its

location is an artefact of the assumed CES utility function and iceberg transport costs. Only in this special case all market positions are equally likely to be chosen by the pioneer. If, for instance, the price elasticity of demand increases with the price, the retailer would gain from a central location where the average distance from consumers and therefore the elasticity of demand are lowest. Then, it would be straightforward to see consider the outcome of the cases where the pioneer is located in the center as benchmark cases. Therefore, by taking the outcome of the symmetric cases as reference, we reduce the danger that our results depend too much on the assumed functions. In a similar vein, the central position would be more likely to be chosen by the pioneer, if strategic interdependencies would not be disregarded. Even if profits of the pioneer are the same at each position, a central location would reduce the risk that sequencing retailers build an agglomeration which renders the firm unprofitable.

How does the specification of the economic parameters impact on the relationship between 'economy' and 'history'? Figure 3 shows how the value of the MOD depends on α , ρ and T . The results are summarized in propositions 2-4. In calculating these values we average over all MOD values with the respective feature characteristic. For instance, in the part of the figure where the influence of α is examined, the value for $\alpha = 0.4$ is the average of 270 MOD values (6 different values of ρ times 5 values of T times 9 different positions of the pioneer).

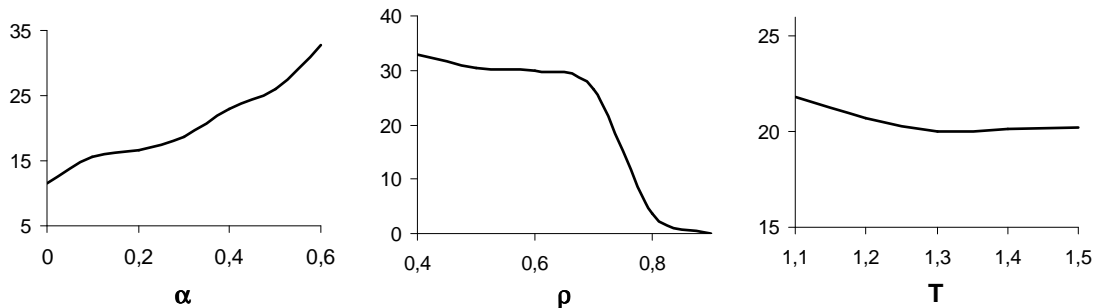


Fig. 3: Measured differences for different settings

Figure 3 reveals that the impact of the pioneer's position on the equilibrium distribution of retail shops is stronger if *i*) the consumers' advantage of a common location of several shops is large, if *ii*) the degree of competition is low, and *iii*) if transport costs are low. The course of the figures and their comparison render possible some more insights which merit our attention. The interpretation of the left-hand part of the figure is quite straightforward: If consumers save much transaction costs by choosing shops that are clustered at a common location, there is a strong incentive for shops to build agglomerations. Hence, the location of existing shops matters strongly for the choice of subsequent firms. If, by contrast, there is less advantage in choosing

the same location as other firms, the trade-off between 'history' and 'economy' is altered in favor of the latter. But even in the extreme case where no transaction costs can be saved ($\alpha = 0$), the distributions of shops that result from different initial settings are not the same because the incumbent firms still have an impact on the location choice of newcomers: Firms avoid locations nearby other firms. Proposition 2 summarizes these results.

Proposition 2 *Positive external effects that arise if several shops share the same location reduce the similarity of the equilibrium distributions of shops from different initial distributions.*

The central part of the figure shows that the effect of an increase in the degree of competition is not steady-going. In general, more competition (a higher ρ) leads to a more disperse structure of the locations. Consumers regard the products as close substitutes, so they are less likely to consume many different varieties because of the fixed costs that come along with each purchase. Therefore, it is profitable for firms to locate nearby consumers, i.e. dispersedly. When ρ approaches one, the elasticity of substitution approaches infinity. For $\rho = 0.9$ there is one shop at each market position in equilibrium, i.e. a perfectly even distribution of retail shops, regardless of the other parameters. In this extreme case, the MOD's value is zero. With $\rho = 0.8$ the same holds true, with the exception of few cases where the advantage of a common location is large enough to offset the disadvantage of extreme competition. Hence, the resulting average value of the MOD is small. With $\rho = 0.7$ there is no combination of the other parameters left which yields an even distribution of retailers. Therefore, the resulting average MOD value is relatively high. Even smaller values of ρ increase the dissimilitude of the resulting distributions further, but to a lesser extent. Proposition 3 recapitulates the main result:

Proposition 3 *If the varieties are close substitutes, the resulting equilibrium distribution of shops is relatively even, and hence less dependent of the initial distribution.*

The right-hand part of figure 3 illustrates that variations of the transport costs parameter, T , have a relatively small effect on the similarity of the equilibrium distribution of firms and the MOD. If transport costs are relatively small, an increase in transport costs lower the value of the MOD, i.e. the resulting distributions of firms become more similar. The reason is again that the distributions of firms become more even if transport costs increase, and the pioneer's location is less important if firms are distributed all over the market in at least one stage of the evolutionary process.

If transport costs are already relatively high, a further increase has only negligible effects on the similarity of the resulting distributions, however. One contrary effect, which may be responsible for the weak impact of transport costs is that they protect firms from competition. Therefore, with sizable advantages of common locations, agglomerations may arise and become sustainable once they have a certain size at peripheral positions of the market, where the pioneer's historically determined location is. This effect works against more similar distributions of firms and may explain why the MOD even rises slightly for very high transport costs. Our result is summarized in proposition 4

Proposition 4 *Variations of the transport costs have only a very limited effect on the heterogeneity of the equilibrium distribution of firms.*

It is unfortunately inevitable that our results hinge to some extent on the employed measure of dissimilarity. Because of this circumstance we put so much effort in explaining the MOD and the intuition behind it. The amenability of our results for straightforward interpretations seems to support its plausibility. Of course, the exact course of the figures must be interpreted with caution, even though we tried to increase the reliability of our results by a large number of runs. For instance, one should not over-interpret the slight increase of the MOD when transport costs are very high. Nonetheless, the simulation of the retailers' location choices enables us to assess not only the direction of a dependency, but also its strength. For instance, we were able to qualify the impact of transport costs on the similarity of the firms' equilibrium distribution as relatively weak. This underlines the adequateness of numerical simulations as a tool of research in this field.

4 Summary and conclusions

The aim of this paper is to examine under which conditions the spatial equilibrium distribution of retail shops is shaped more by historic singularities or by economic forces. We develop a discrete model of location choice with heterogeneous goods and iceberg transport costs. In the simulations of the location choice of subsequent firms we vary the location of the pioneering firm, which is taken as representing 'history'. We then compared the resulting equilibrium distributions of shops by means of a 'measure of dissimilarity' (MOD). This measure indicates how many hypothetical moves of firms would have to be made to transform one distribution into another. We calculated its value respectively for two distributions that result from different positions of the pioneer, but with equal 'economic' parameters. One of the two distributions is the

one that results from a central position of the pioneer, respectively. If the value is low, the two distributions of firms are relatively equal in spite of different initial settings. Hence, economic forces prevailed over 'history'. If however the MOD's value is large in spite of identical economic parameters, 'history' has dominated.

A number of limitations and caveats follow from the assumptions of the theoretical model and from the use of specific functions and parameter values in the simulations. First, we employed a partial equilibrium view, i.e. we abstracted from interdependencies between the goods and inputs markets. For instance, an agglomeration of firms of one branch may lead to a concentration of workers that are specialized in the type of labor that these firms need, which augments the comparative advantage of this location even further. Moreover, an agglomeration of economic activity increases income, which may in turn have a positive impact on demand. All such 'forward and backward linkages' incontestably exist, and they are at issue in a number of recent publications, not only within the 'New Economic Geography'. Ignoring these effects may yet be sensible at a low geographic scale. Explaining the concentration of financial institutions in London may require the consideration of specialized labor. But the latter can hardly explain why we prefer buying fruits at a market with many market stalls, and why e.g. antique shops or fashion boutiques are so much concentrated within a city. This is to say, the model is more appropriate for explaining location choices within cities than at a larger geographic scale.

The combination of iceberg transport costs and a constant elasticity (CES) utility function facilitates the analysis considerably. Since the firms' mill price depends on the elasticity of substitution, which is constant throughout each firm's market area, the price remains constant as well. But this must not be the case. If we had assumed, say, linear demand functions, the elasticity of demand would increase with the distance from the firm. In this case, the equilibrium price of each firm would depend negatively on the extension of its respective market. The opposite holds true if the elasticity of demand decreases with distance. Hence, in the absence of any good reason why the elasticity of demand should increase or decrease with distance, it is possibly not a too bad approximation to assume that it is constant.

Our main results are (see propositions 1-4):

- For given economic conditions, the equilibrium distributions of shops are more similar if the pioneer is located in the center of the market or at peripheral positions, than if the firm is located at intermediate positions.
- Positive external effects of co-location imply that the historical position of the

pioneer is more important for the equilibrium distribution, hence the distributions which result from different choices of the pioneer are less similar if these effects are strong.

- The easier it is for consumers to substitute one variety by another one, the less important is the initial setting.
- Due to mutually opposing effects, transport costs have only a weak impact on the similarity of the distributions of shops resulting from different positions of the pioneer.

Not surprisingly, we found that the equilibrium distribution of firms depends both on the combination of economic parameters and the initial setting. Given that in reality the location pattern of firms from different branches is varying strongly, one would not even appreciate more clear-cut results. If goods in one sector are almost perfect substitutes, the model predicts, the resulting distribution is almost even. This is the case e.g. for tobacco shops and the like. If however the goods are relatively heterogenous, and/ or (reducing) transaction costs is paying for consumers, a more concentrated spatial structure yields. Examples are fashion boutiques and antique shops. Therefore, one would have to assess the relationship of these parameters to predict the spatial distribution of firms in a specific sector of the economy. In those cases, where the positive external effects of choosing the same location are strong, the model predicts that an agglomeration arises where the first firm is located. This common location may be inefficient, though, since only the central position minimizes total transport costs. In this respect, a calibration of the model to a specific situation may provide some guidance to city planners regarding the question whether the location of firms should be influenced at an early stage of the product cycle or an efficient outcome can be expected even without an intervention.

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Appendix

A Some are more equal than others

To decide to which extent the equilibrium distribution of shops depends on the initial distribution or on economic forces we need a measure to quantify differences in several dimensions. If two spatial distributions of firms are equal in spite of different initial conditions, one may conclude that economic forces are dominant in this case. Yet, simply stating that the equilibrium distributions are equal or not equal would abstract from potentially interesting intermediate results and interpretations. If the ultimate distribution of firms in the two cases differs at only one position by only one firm, it would be misleading to conclude that 'history' has prevailed over economic forces. An appropriately defined measure of similarity would render possible to state that one distribution of firms is more similar to a benchmark distribution than another one.

What would such a measure of (dis)similarity have to account for? The measure has to weight the differences between two distributions of firms in several dimensions. The outcome of two simulation runs may differ with respect to the number of firms, the number of locations, the locations themselves, and the distribution of firms between locations. Of course, they may also be different in several or even all of these dimensions. Figure 4 illustrates these cases for the example of a length 5-market ($r = 5$).

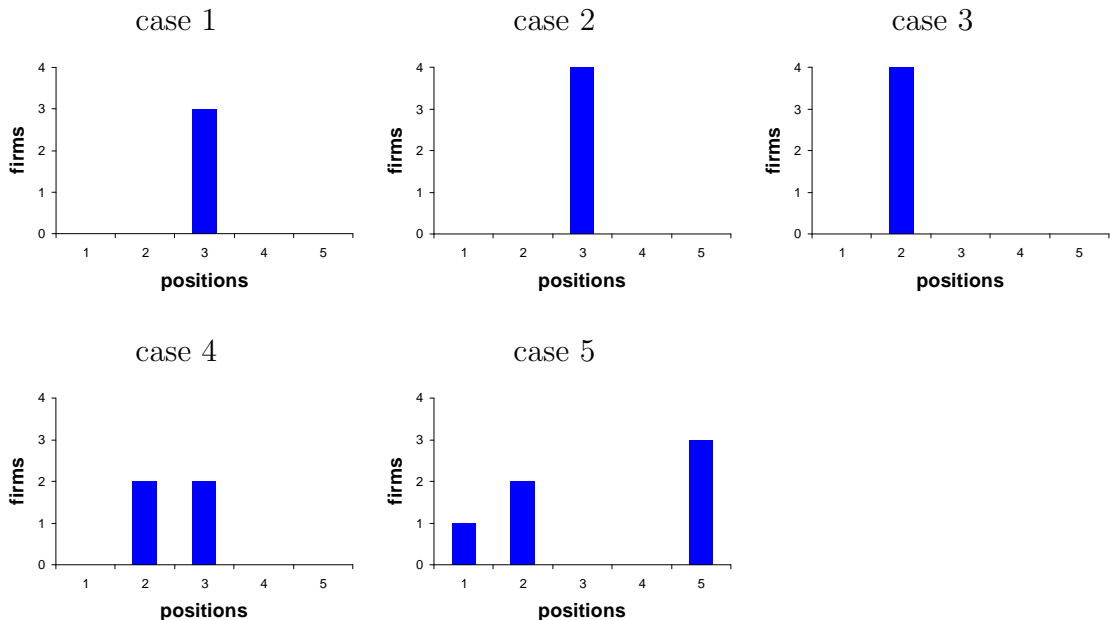


Fig. 4: Example distributions of firms, $r = 5$

Cases 1 and 2 merely differ with respect to the number of firms at the single location at market position 3. In case 3 the number of firms is same as in case 2, but the location is now at position 2. In case 4, there are two locations, positions 2 and 3, where respectively half of the firms reside. Case 5 combines all these differences: It is different from all other cases with respect to the number of firms and locations, where firms locate, and how they are distributed. If one compares the situation in case 2 with all other situations, it is quite evident that case 5 is differing mostly. But it is not as easy to assess which of the other cases is more similar to case 2 because such a judgement necessarily relies on normative weights one attaches to differences in the number of firms, the number of locations or one of the other dimensions.

Even though the weighting will always remain normative, it is possible to identify some desirable features the measure should exhibit. Below, these features are somewhat grandiosely referred to as axioms. Since there is no upper limit of how different two distributions of firms may be, it is difficult to define a point of origin of a measure of similarity, however. Therefore, we reverse the view and construct a measure of dissimilarity, φ . The first 4 axioms are the standard metric axioms (see e.g. Santini and Jain, 1999), which have been discussed in quite detail, and mostly controversial since Tversky (1977). The discussion, whether or not the feature space may be mapped adequately by a metric space is not relevant here, as long as we are in fact considering a metric space. However, if distances are interpreted as product differentiation in a characteristic space, the adequacy of a metric measure becomes questionable. Before we state the first axiom, a precise definition of 'distribution of firms' seems at order:

Definition 2 *A distribution of firms ω is a vector of length r , where the k 's element is n_k , $\omega(k) = n_k$. In other words, the vector assigns to each location the corresponding number of firms. Ω is the set of all distributions.*

The first two axioms define the point of origin of the measure φ .

Axiom 1: Equal distributions

$$\varphi_{jj} \equiv \varphi_{\omega_j, \omega_j} = \varphi_{\omega_{\bar{j}}, \omega_{\bar{j}}} \equiv \varphi_{\bar{j}\bar{j}}$$

where $\omega_j, \omega_{\bar{j}} \in \Omega$ are two different distributions of firms. That is to say, if identical distributions are being compared, the measure φ exhibits always the same value. The second axiom is:

Axiom 2: Minimality

$$\varphi_{jj} \leq \varphi_{j\bar{j}}$$

i.e. the smallest value of φ , corresponding to a maximum of similarity, is assigned to identical distributions of firms. This value may be standardized to zero without further loss of generality. The third axiom reads:

Axiom 3: Symmetry (1)

$$\varphi_{sj} = \varphi_{js}$$

If two distributions are being compared, the order of the comparison should be meaningless. With a higher number of distributions, however, it becomes important, which case is chosen as reference. Imagine, for instance, that two distributions of firms are equal, and the third is very different from them. Then, defining the latter as reference distribution, would yield two high values of φ , but the piece of information that the two are equal would be lost. Thus, it is advisable to define a typical distribution as reference. The fourth metric axiom is:

Axiom 4: Triangle Inequality

$$\varphi_{sj} + \varphi_{j\bar{j}} \geq \varphi_{s\bar{j}}$$

Unlike the other axioms, the triangle inequality cannot be expressed in ordinal terms, which makes it difficult to falsify (Tversky, 1977). Applied to our context, it essentially states that the sum of measured dissimilarity between the reference (ω_j) and two other distributions of firms is greater or equal to what would be measured if one would compare the two other distributions directly.

In addition to these axioms, which are fairly standard in the literature on mathematical psychology, we impose the two following axioms. The first additional axiom is:

Axiom 5: Symmetry (2)

$$\begin{aligned} \varphi_{sj} = \varphi_{\bar{s}\bar{j}}, \quad & \text{with } \tilde{\omega}_j(i) = \omega_j(r - i + 1) \quad \forall i \in D \\ & \text{and } \tilde{\omega}_s(i) = \omega_s(r - i + 1) \quad \forall i \in D \end{aligned}$$

Thus, the measure must have the same value as before, if both distributions are reversed from left to right. If a measure violates this condition, it would matter from which end of the market a comparison of two final distributions of firms would start. By imposing this condition, we are able to restrict the analysis to initial locations in half of the market, because the same results must apply for the remaining half. The second additional axiom is

Axiom 6: Independence from the number of distributions

$$\begin{aligned} & \text{if } \varphi_{\zeta j} = a, \quad \text{with } \omega_{\zeta}, \omega_j \in \Omega, \quad a \in \mathfrak{R}^+ \\ & \text{then } \varphi_{\zeta j} = a, \quad \forall \Omega_h : \omega_{\zeta}, \omega_j \subset \Omega_h \end{aligned}$$

This axiom requires that adding or deleting elements of the set of distributions which are being compared (Ω) does not alter the value of the MOD between any two distributions, provided that these two distributions are still within the set. This requirement ensures that the results of a survey may be carried forward to other issues which are analyzed. One shortcoming of this assumption is that it is not possible to standardize the measure to be in the interval $[0, 1]$, since there is no upper limit of differences between two distributions, which may be assigned the value 1.

The latter assertion points to the fact that there is not only a single measure which fulfills all the requirements given in axioms 1–6. The circumstance that the measure has no upper limit, together with axioms 1, 4 and 5, implies that if there is a measure φ which fulfills the axioms, then the measure $a \cdot \varphi$, $a \in \mathfrak{R}^+$, is appropriate, too. That is, every multiple of the measure would meet with axioms 1–6 just as well.

The MOD between two distributions of firms, ω_{ζ} (the reference) and ω_k , we propose is presented in equation (14). It has all the properties axioms 1–6 call for. The intuition behind our measure is that it calculates the necessary number of hypothetical movements of firms from one location to a neighboring one in order to transform one distribution of firms into the reference distribution. To explain the underlying idea let us compare cases 1 (defined the reference) and 5 in figure 4, and firstly focus on the first double sum within the braces. For $j = 1$ the inner sum simply calculates the difference between the number of firms at market position 1 in the cases of both distributions, which is 1 in our example (imagine, this firm is moved to position 2, which is illustrated by red numbers in figure 5's upper part). For $j = 2$, the number of firms on market positions 1 and 2 for each distribution is subtracted. The difference is now 3, which amounts to moving 3 firms (to position 3, see figure 5). The sum of differences, calculated by the outer sum is 4. To yield the actual number of hypothetical movements, which would be necessary to transform one distribution into the other, the absolute values of these differences have to be added. For $j = 3$, the number of firms in the cases of both distributions is 3, hence the sum of movements remains 4. In the last step, 3 firms that reside on position 5 have to be moved by one step to be outside the market, which yields a sum of 7 movements.

There is one problem with the measure until here, however. Namely, if both distributions are reversed from left to right, the measure takes a different value, in this case 11, as can easily be verified (see the lower part of figure 5). This contradicts

position		case 1	case 5	Transformation					Σ
1	number of firms at positions 1-5	0	1	0	0	0	0	0	
2		0	2	3	0	0	0	0	
3		3	0	0	3	3	3	3	
4		0	0	0	0	0	0	0	
5		0	3	3	3	3	3	0	
	number of moved firms			1	3	0	0	3	7

position		case 1	case 5	Transformation					Σ
1	number of firms at positions 1-5	0	1	1	1	1	3	0	
2		0	2	2	2	2	0	0	
3		3	0	0	3	3	3	3	
4		0	0	3	0	0	0	0	
5		0	3	0	0	0	0	0	
	number of moved firms			3	3	0	2	3	11

Fig. 5: Transforming one distribution into another

axiom 5. The reason is that the supernumerary firms in case 5 have to be moved further, if we start adding up the differences at the right market border. To eliminate this problem, the second double sum calculates the number of hypothetical movements from the right to the left border of the market. The final measure is simply the average of both double sums, in our example $(7 + 11)/2 = 9$. To average over the number of hypothetical movements from both ends of the market brings about that, if the number of firms differs by one, the measure increases by $(r + 1)/2$, in our example by 3, no matter where the difference occurs. Another property is that if two distributions differ from the reference distribution only with respect to the location of one firm, the measure φ is the higher, the more the location of this firm is displaced in relation to the reference distribution.

Table 3 summarizes the values of our measure φ for each case in figure 4. Column 1 respectively gives the reference distribution, the cases in the first row are compared to it. A glance at the table shows that the first two axioms are fulfilled. Comparing one distribution with itself always yields a value of zero, hence axioms 1 and 2 are met. Second, the lower left half and the upper right half of the matrix are

symmetric, which signifies that axiom 3 is met, too. Axioms 4, 5, and 6 are fulfilled by construction.

	case 1	case 2	case 3	case 4	case 5	Σ
case 1	0	3	6	4	9	23
case 2	3	0	4	2	9	18
case 3	6	4	0	2	9	21
case 4	4	2	2	0	8	16
case 5	9	9	9	8	0	35

Table 3: Measure of dissimilarity (MOD), φ , example values

The table shows that case 4 displays the smallest values of the measure in average (the sum of the measures is 16). Compared to the next best case 2, it is less similar to case 1, but this is more than offset by a higher similarity to cases 3 and 5.