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# Water: Spatial Network Pricing 45th Congress of the European Regional Science Association

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#### Abstract

The paper addresses an important issue of pricing mechanisms is spatially distributed systems with losses, with an application to water supply system. When losses from delivery are high, the asymmetry of spatial location of consumers plays an important role, which is captured by the model. The goal is to compare the efficiency from alternative market structures for water supply. The model can be applied for channels aimed on water redistribution. In particular, the model can be relevant for analysing different market structures, equilibrium water pricing and efficiency for the planned channel between river Ebro (Spain) and communities Valencia and Murcia. While this paper is purely theoretical, it addresses the issues that are still little understood at administrative level. Water market as described is a necessity, but it will not emerge spontaneously and it requires appropriate legislative preparation. Mathematical model is designed in terms of densities and flows of corresponding economic variables.

KEYWORDS: water pricing, networks, spatially distributed systems.

JEL Classification: L95 (water utilities), O13 (agriculture), Q25 (water), D43 (oligopoly and market imperfections).

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# 1 Introduction

The idea of the paper is to study equilibrium pricing and efficiency in different market organizations in industries with physical networks. Water supply belongs to one of them. The basic difference of this market structure from classical approach in microeconomics is explicit accounting for initial heterogeneity in location of producers and consumers that comes from history or geography. Asymmetric access to market takes place only if transmission costs are significant (not negligible) in comparison with other costs. We start from different examples, which might result in completely different papers, since every industry has its particularity.

#### 1.1 Water Channel in a Desert

Cara-cum channel bringing water from Amu-Darja to villages of Turkmenia. Historically it was constructed as a public good financed by state (USSR) and no microeconomic efficiency was considered at that stage. Now, during transition to market economies, in Central Asia, the issue of efficiency becomes important. There is also a negative externality for ecology: Aral Sea is gradually dying as more water is taken out from the river that is sufficient for environmental equilibrium. It also may become a source of cross-country conflict, since benefits are derived by Turkmenistan while losses are mostly for Aral Sea countries: Uzbekistan and Kazakhstan. The channel of the length about 1000 km does not have a perfect engineering design, and losses from evaporation and filtration are substantial. The more distant is location from the river, the higher are losses. Consider alternative possibilities of market organization. The first is public good provided by state for free. In this example, water is not a pure public good. The second possibility is to have a monopolist owning the whole infrastructure. If water provider is a (natural) monopolist, it has a possibility to have or have no spatial price discrimination. Uniform pricing has a clearly negative effect, since marginal cost to supply 1 cubic meter of water for 1000 km is much higher than to supply the same amount at the distance of 100 km. However, spatial pricing requires a perfect accounting system, which might not exist at the present.

#### 1.2 Channel in Spain

There also exists a practical necessity to address the problem of efficient water pricing. When water is scarce, it enters the production function as more important factor then some of the rest factors. Since channel has a particular topology, we are not in the environment of an equal access of all consumers to the market. This gives a possibility of emergence of game between communities with inefficient blocking strategies if not all characteristics of initial endowments and game structure are taken into account. For example, at present water-exporting communities of Aragon and Catalonia are against such a channel while water-importing communities of Valencia and Murcia are in favour of it. This model shows that equilibrium pricing of water should be a function of distance that water passes in a channel. In other words, Murcia should pay more for unit of water in a channel than the price in Southern Catalonia. If such setting would be done by central planner, it can eliminate non-efficient bargaining that now takes place. Also, participants of the market will understand the real value of water and set their production plans taking this into account. As it is shown, water pricing will also have an impact on agricultural land rent in different locations, and this rent will jump to a new equilibrium as soon as pricing rules will be settled.

While it is possible to consider an alternative formulation of the problem, with a finite set of communities as economic actors, the focus of the present model will be on general equilibrium framework, when each participant has no market power and has to accept the rules of the game set by central planner.<sup>1</sup>

# 2 New Type of Market: The Mix of Technological and Legal Issues

#### 2.1 Related Economic Studies

Classical microeconomic theory normally considers a market like something that does not have spatial dimension (think about central square in a me-

<sup>&</sup>lt;sup>1</sup>It is also true that such rules are normally taken as given in economic models and not set by a participant having stronger power, although in real life this can be often the case, especially when law is not set or not respected.

dieval town) and does not address the issue of spatially distributed markets. However, there exist some tradition to approach this issue from different perspectives. At the empirical level, there exist few studies related to construction of maps with price isolines. This approach was popular in the beginning of the 20th century, and the author is familiar only with two maps of this type. The first one presents the isolines of potato pricing in the USA (reproduced in [1]), and another example is related to isoprices of grain in Russia [2]. It is not very clear why such research do not take place nowadays. The author may have several hypothesis:

a) spatial pricing data form strategic information for some multinational companies and they avoid to make it public;

b) while 100 years ago transport costs and geographical latitude really made price of agricultural products a spatial variable, it is no longer the case, because almost perfect competition across small farmers eroded and was replaced by competition across intermediaries, which use local monopoly power to buy inputs and then sell at the prices that include not only transport cost but some information rent<sup>2</sup>;

c) the role of agricultural sector in GDP is declining, and there is less interest to such studies;

d) transport costs are declining with technological development, and economists believe that such effects are no longer important.<sup>3</sup>

As for economic theory, only partial analysis of such distributed systems has been developed. For example, Chamberlin [7] studies monopolistic competition on a line, where producers are discrete and consumers are continuously distributed. In this case, there exist an area near the any producer where it possesses local monopoly power. He may apply different pricing strategies there (see Beckmann, Thisse (1986) for details). In the model of Hotelling the spatial structure is similar, however two firms compete on interval and can choose location. Later d'Aspremont, Gabzewich and Thisse

 $<sup>^{2}</sup>$ The issue of sharp rise of fresh food in Spain in 2003 became a hot political issue, so that some universities in Madrid made case studies and suggested to mark two prices at each product in supermarket, one of producer and second of retailer, to see whether rising production cost really explains the difference

<sup>&</sup>lt;sup>3</sup>On the other hand, it is a well known mathematical result that the limit structure when some parameter vanishes to zero not always coincide with just setting it to zero. Simple example is l'Hopitale rule, but there exist more complex examples with metrics in functional spaces.

(1979) have found a problem with existence of Nash equilibrium in traditional model of Hotelling, suggested a remedy in a form of quadratic cost and thus moved the focus of research of industrial organization literature to product space, away from linear transport costs.

However, linear transport costs are important in many applications, and they are related to physical losses in distributed systems. And there are at least two cases not related to agriculture directly, where losses are almost linear in distance, substantial and the system is continuous in space. The first example is related to electricity transmission. On average, 10 % of it is lost during transmission, and this makes such losses not negligible from economic point of view, although most of the countries have not even thought about such component as market issue<sup>4</sup>. The second example is water pricing in a channel, the problem that this study is addressing.

#### 2.2 Legal Issues and Market Structures

If the marginal price of water in Murcia is let say 2 Euro per cuic meter, while in Catalonia only 1 Euro, there is a room to improve efficient water use by opening trade. However, contrary to financial instruments, it is not sufficient to start trading water assets on stock exchange. The problem is that water has to be delivered first, and here we first face construction costs. But that is not all, to start trading water, it is necessary to set ownership rights for it. While in many states water is virtually free (although water distributors charge price for its delivery from consumers), it is no longer the case in some areas. In the case of Spain, its southern regions has water as scarce product, and thus have to use it efficiently. Efficient use implies price mechanism, and in the case of water its price should differ from one to other geographical point.

If there exist some endowment (supply) and some demand in different geographical points and if delivery is costly and/or includes physical losses (like Samuleson's melting, in our case - evaporation), different methodology for price setting should be used. Partly it was developed in Ph.D. thesis of Yegorov (1999), in the chapter "Equilibrium in continuous space under decentralized production". There each point in continuous space had some

<sup>&</sup>lt;sup>4</sup>For more details of elaboration along these lines, see Yegorov [5]

production and consumption density, varying continuously, and each local producer was facing export flow through his point and had to decide how much to produce and how much to export, taking the local price as given (and it emerged as a continuous function of spatial coordinate).

In the case of water along channel we also can have similar formulation, if we allow to build small plants to produce water in every point. Given the difference in local demand and price across points, the optimal capacity pattern should be chosen. Speaking about Spanish reality, such a formulation is also possible, since there exists a lot of salt water in sea and the technology to transform it in unsalted water, using some energy input. But we will not address this problem at current stage, as there exist more obstacles of mathematically simpler origin.

The first problem is related to ownership. At present, water is owned by state. While different communities have different water endowment, it is central planner (in Madrid) that currently decides how to use this water. Hence, the water does not have market price, but it has value. If central planner would like to transfer some endowment from one community to another without any compensation, such a decision can be potentially blocked by local administration. Thus, an efficient water distribution is impossible without market creation, and this market should legalize possession of initial endowments hold by different communities.

The next problem is the ownership of transmission system. To make things simpler, suppose that it is owned by state but regulated, so that operate as natural monopolist with zero profit, in order to reach more efficiency. The paper will focus mainly on this legal framework.

**Paper structure.** In section 3, simple models without explicit accounting for space and flows are analysed. It is shown that uniform pricing incurs additional losses even for a monopolist. The section 4 is devoted to spatial analysis. First, the local price is obtained taking into account supply cost. Then, consumer (agricultural producer) optimisation problem is solved. Finally, the equilibrium price of land rent and farmers settlement on space is obtained. Note, that the spatial structure is very sensitive to water pricing mechanism, and since water supplier is a natural monopolist, there should be

perfect social regulation and accounting to prevent its negative influence on the market. Finally, the conclusions and policy implications are formulated.

# 3 Simple Models

The problem of water relocation in space sets many sub-problems on different level of economic theory. The goal of this section is to analyse them from the point of view of selecting relevant factors, predicting relevant policy issues and analysing efficiency. These preparations are necessary before introducing more advanced model that treats space continuously.

#### 3.1 Leontieff Production Function

Natural resource economics often uses generalized Cobb-Douglas production function (see for example [4], p.570), with not only capital and labour, but also with term describing energy and material inputs. If some particular material (here water) becomes scarce, it makes sense to focus on its influence rather than on influence of typically considered labour and capital. In the case of agriculture in zones with not sufficient sun (for example, for production of fresh vegetables in cold zones and/or times of the year) such an input can be energy, while in zones with not sufficient rainfall or little access to rivers, water becomes such a natural input.

Assume two geographically different regions, 1 and 2 (one may think about Catalonia and Murcia). They have different endowments of sun (S)and water (W) per unit of territory. Most agricultural plants require both inputs, and for mathematical simplicity we will assume that generalized agricultural product is produced using both of these inputs and is of Leontiefftype:

$$Y_1 = \min\{S_1, W_1\}, \qquad Y_2 = \min\{S_2, W_2\}. \tag{1}$$

Assume also that  $S_1 + S_2 = W_1 + W_2 \equiv E$ , but  $S_1 < S_2$ , while  $W_1 > W_2$ . In other terms, region 1 (Catalonia) has relatively scarse sun and relatively abundant in water, while for region 2 (Murcia) the opposite is true.

#### 3.2 Trade Arrangement

Are these factors geographically mobile? Sun is clearly immobile, while water can be mobile, although transport costs are substantial. Denote region's imbalance by  $\Delta \equiv W_1 - S_1 = S_2 - W_2$ . Pareto efficient allocations would be those with  $\Delta = 0$ . They will allow to produce the total output  $Y = Y_1 + Y_2 = E$ , while the current total production equals to  $E - \Delta$ . That is why reallocation can create surplus, and central planner can do it by channel construction. But is there some decentralized market mechanism for such a trade? If sun would be tradable, the simplest solution would be to trade some sun for water between regions 1 and 2. However, it is not tradable. In this case, the region 1 should get some compensation for transferring some part of its water endowment to region 2. One may think of selling water as of capital investment in region 2, and region 1 can get back some part of additionally produced agricultural product from there.

Clearly, Leontieff production function is a toy model, and its main weakness is non-differentiablity. But it helped us to understand the economic context. If we replace it by other, differentiable, production function, we can come to similar result: marginal value of water in region 1 is lower than its marginal value in region 2:  $MV_1 < MV_2$ . If transport cost (which includes construction cost of channel, spread with discount over exploitation period, operation cost and physical loss (evaporation)) per unit of water is below this difference in marginal values, transfer of water can be mutually beneficial. The different between value differential and transport cost represents the "pie" (term from bargaining theory) that has to be split between two regions. In decentralized framework the outcome depends on regional bargaining powers, but we can also assume that the decision of Central government is taken to give  $\Delta_1$  to region 1 and  $\Delta_2$  to region 2. If such decisions are taken, then the region 1 will sell water at price  $p_1 = MV_1 + \Delta_1$  and region 2 will buy it at price  $p_2 = MV_2 - \Delta_2$ , while the difference between prices should be equal to transport cost  $\tau$ :  $p_2 - p_1 = \tau$ . In this case, supplying company will operate at zero profits. However, such a company is a natural monopoly (building two channels to have oligopoly is too costly) and its should be regulated. Now we would like to see what costs and incentives exist for it, and whether spatial structure could be as simple as we have at the moment.

#### 3.3 Monopolistic Water Pricing, Losses and Efficiency

Assume that the channel is an interval [0, a], and consumers are uniformly distributed around it. Such types of models have been studied, for example, in [6,7]. Suppose that individual demand functions are identical and linear: q = 1 - p. In the simplest case, unsophisticated monopolist charges uniform price p, for any spatial point  $x \in [0, a]$ . The total demand will depend on price p only:

$$D = \int_0^a (1-p)dx = a(1-p).$$
 (2)

**Physical losses.** It is well known from physics, that water can evaporate in an open channel and/or filtrate through imperfections in the channel borders. These losses depend on distance and are typically of Samuelson type. The quantity delivered to point x is linked to the quantity sent at point 0 by the formula:  $q(x) = q(x, 0) \exp(-\gamma x)$ , where  $\gamma$  is coefficient of losses. The easiness of this formulation is that all losses to individual consumers are additive. Then the total loss will be given by the integral

$$L = \int_0^a [q(x,0) - q(x)] dx = (1-p) \left[\frac{e^{\gamma a} - 1}{\gamma} - a\right].$$
 (3)

For  $\gamma \ll 1$ ,  $e^{\gamma a} \approx 1 + \gamma a + \gamma^2 a^2/2$ , and the formula for losses has asymptotical expression:

$$L = \frac{1}{2}(1-p)\gamma a^{2}[1+O(\gamma a)], \qquad (4)$$

where  $O(\gamma a)$  can be infinitely small for sufficiently low  $\gamma a$ . This formula suggests that the share of technological losses is  $L/D = \gamma a/2$ .

Monopolistic uniform pricing. Suppose that unsophisticated monopolist knows the quantity of losses in a channel (they can be calculated by engineers) but does not understand that price can be charged non-uniformly (he needs to know spatial economics, which typically in not studied in economic departments). Suppose he can buy any amount of water at point x = 0 from region 1 at flat price  $p_0$  (which equals to  $p_1$ ), sell water at uniform price p and incur physical losses (which can be calculated using asymptotical formula). Then he solves the following optimisation problem:

$$\max_{p} [a(1-p)(p-p_0) - p_0(1-p)\gamma a^2/2].$$
(5)

The first order condition gives the following optimal price:

$$p = p^* = \frac{1+p_0}{2} + \frac{\gamma}{4}p_0a.$$
 (6)

Note that  $\partial p^*/\partial a > 0$ , i.e. price is increasing with the length of channel.

The issue of efficiency. When economists analyse efficiency in monopolistic environment, they typically compare the sum of producer and consumer surpluses under different market structures. The difference in total welfare is viewed then as "deadweight loss" arising from monopolistic market structure. In this case, the situation is a bit more difficult since we face also physical loss, L. Also, since perfect competition is impossible here, it is useful to select "zero-profit monopolist" as a benchmark model. If consumers buys water at price p and the demand function is q = 1 - p, then consumer surplus (for all consumers) is

$$CS = a \int_0^q p(q) dq = \frac{a}{2} (1-p)^2.$$
(7)

The producer surplus is simply his profit  $\Pi$ , and the social welfare. SW, is the sum of both:

$$SW = \frac{a}{2}(1-p)^2 + a(1-p)(p-p_0) - p_0(1-p)\gamma a^2/2.$$
 (8)

In the benchmark case,  $\Pi = 0$ , and  $p = p_0(1+\gamma a/2)$ . Any profit of monopolist will lead to decline in demand and the decline of physical loss. At the same time, the triangular of the initial consumer surplus will shrink so that part of it will be monopolistic profit and part-deadweight loss. In our case, the deadweight loss will be exactly one half of physical loss. Thus, we have two opposite effects on monopoly: increase in welfare loss in region 2 and decline of physical loss of water. This leads to a necessity to introduce interregional welfare analysis.

How to measure interregional welfare analysis? Suppose that in region 1 the demand for water in similar, but equilibrium price is lower. Any quantity taken out from this market also diminishes the social surplus. But we assume that consumers and producers are compensated their by the revenue from water sale and thus are better off. We will not go into direct calculations, we just mention that direct losses of water can reflect the loss in welfare of region 1, and thus should be also minimized. The problem of optimal balance between welfare gain in region 2 and the physical losses can be better address if we assume that price  $p_0$  is not flat (which is correct only for large region) but depends on quantity of water transferred from region 1 to 2.

### 4 Model in Continuous Space

In general case, we have to consider all productive factors, social (labour, capital) as well as natural (water, sun, land). At the same time, it makes sense to reduce complexity and to keep most relevant factors. While we perfectly understand the importance of such factors like labour and capital, we will not focus much on the them. As for labour, it is assumed that all users of water are individual farmers, supply their labour inelastically and labour endowment is not binding given the size of their farms. The role of capital in traditional agricultural technologies is limited. It is really important in water infrastructure, but here it comes as external factor for agricultural community which pays the price for its services (water).

The role of trade-off between sun and water was considered above. Land is a natural input in agriculture, and land rent is the price of this input. We would like to use production function that uses only land and water. Why? Because capital and labour are much more mobile factors in comparison with land (completely immobile) and water (partially mobile after huge investment in infrastructure). While competition set equilibrium price to mobile factors, immobile factors have the prices that depend on particular location. Since we focus on a model in heterogeneous space, land and water become such factors that are subject to spatial pricing. In other words, instead of a unique price of commodity we have to introduce a continuum of prices that depend on location.

Before doing to such pricing, we have to introduce physical equation of water transportation in a channel.

#### 4.1 Equation for Water Flow

The free flow of water in a condition of a desert resembles the dynamics of mass of an iceberg: water evaporates, while iceberg melts. Under constant physical external condition, the speed of evaporation is proportional to current mass. Hence, we have "iceberg" transport cost of Samuelson type. In conditions with cold climate or delivery in a closed tube, this effect can be neglected.

Let  $\Phi(x)$  denotes the flow of water (in cub.m per sec) at point x along the channel. For simplicity, the speed of water is normalized to 1. If c(x) denotes consumption density (the water consumption on the interval [x, x + dx] is c(x)dx) and  $\beta$  is coefficient of evaporation, then the flow satisfies the following equation:

$$\Phi'(x) = -\beta \Phi(x) - c(x). \tag{9}$$

This is a physical flow of fluid dynamics. However, consumption c(x) is affected by economic forces. This consumption depends on price and represents the balance between demand and supply. Since losses for water delivery in more distant area are higher, with identical demand, equilibrium price there should be also higher. This is an efficient mechanism, but it need to be implemented under particular market structure. The further part of the model will focus on economics.

#### 4.2 Assumptions

1. The channel is formally represented by an interval [0, 1].

2. There are N a priori identical consumers which can choose their location along channel. If agents are located along the channel with density  $\rho(x)$ , then the land endowment per agent is an inverse function of density:  $S(x) = 1/\rho(x)$ .

3. Agents have an agricultural production technology, which involves two inputs: water, c, and land, S. It is assumed to be of Cobb-Douglas type:  $Y(x) = S(x)^{1-a}C(x)^a$ , where  $a \in (0, 1)$ .

4. Agents are identical ex ante, but are located in heterogeneous space. That is why equilibrium land rent should balance price of water in such a way, that every agent is paying the same total cost and produces the same physical output.

The price of water depends on market mechanism. We start from efficient pricing, and then compare the result with monopolistic pricing, with and without discrimination. A possibility of competition in water supply industry as well as regulation of monopoly will also be discussed.

#### 4.3 Solution 1: Water Price is Regulated at Cost Level

Suppose that cost of one unit of water at x = 0 is w. Sometimes water is a free good, but in this model it is both scarce resource and capital good. Price at the beginning of a channel should be at least of the level of its cost. Later along channel some physical losses of water take place, and its cost should be higher. While water supply owner might not account for this heterogeneity, it takes place and can incur inefficiency. We start from the case when water is priced at the level of its cost.

How much water will reach point x? Suppose there is no consumption between points 0 and x. Then the equation  $\Phi'(x) = -\beta \Phi(x)$  has a solution:  $\Phi(x) = \Phi_0 e^{-\beta x}$ . Since agent is free to choose a point of consumption, efficient pricing should correspond to equality of  $p(x)\Phi(x) = const$ , or:  $p(x) = we^{\beta x}$ . This corresponds to the law of preservation of value of what is left from water sent from point x = 0, during its transportation (and evaporation) along the channel.

It is necessary to make a couple of comments. First, such pricing does not involve cost of water delivery, and assumes only cost to build and operate channel, independently on flow structure inside it. If delivery involves cost (like in gas or oil industry, when pumping along the way is necessary), heterogeneity of cost will be even higher. Second, there is additivity of different flows. We can decompose total flow going via cross section into subflow aiming to reach point  $x_1$  and point  $x_2$ . Then each subflow loses a proportional fraction of it along each unit of channel. Hence, there is no externality across different consumers, if pricing is done exponentially. Moreover, we can separate accounting for each consumer, assigning such price differential that takes into account the equal cost of each cub.m of water sent to the consumer from the beginning of a channel.

We will start from consumer optimisation problem. Then aggregation will be done. For any p(x), land rent price R(x) will emerge, so that consumers of water (farmers) will choose to use a particular amount of water C(x) and land slot of particular size S(x), which is optimal for them given prices of both factors. Moreover, they will be indifferent across locations, and exactly Nfarmers will be allocated there, all having different land slots (S'(x) > 0) and different water consumption (C'(x) < 0), but producing the same amount of output and having identical expenditure for land rent and water.

#### 4.4 Consumer Optimisation Problem

A farmer at x solves the following optimisation problem

$$\max_{S,C} [S(x)^{1-a} C(x)^a - p(x) C(x) - R(x) S(x)].$$
(10)

He takes p(x) and R(x) as given. Here p(x) is external function, while R(x) is determined in a general equilibrium framework.<sup>5</sup> The first order conditions,

$$(1-a)(C/S)^a = R, \qquad a(S/C)^{1-a} = p,$$
(11)

lead to an optimal ratio of used inputs as the function of prices:

$$\frac{C(x)}{S(x)} = \frac{a}{1-a} \frac{R(x)}{p(x)}.$$
(12)

Since we have CRS, the equilibrium level of output cannot be determined. But we have already determined the optimal ratio of inputs as a function of prices (which will be different in different locations x). Now we need to turn to market clearing conditions. For given price pattern of water p(x), agents should compete for land. In equilibrium, they will divide it in such slots S(x)and establish such land rent R(x), that: a) agents are indifferent, b) all land used. Next subsection is devoted to mathematical implementation of these ideas.

<sup>&</sup>lt;sup>5</sup>Traditionally for microeconomics, general equilibrium endogeneously determine all functions. This is a slight mathematical modification, where all principles are similar (prices taken as given, market for land clears). We are interested in a response of a complex system on external function p(x).

#### 4.5 Market Equilibrium Conditions

Identical agents should be indifferent. Since agents in equilibrium should be indifferent across locations, they should produce the same output  $Y_0$  (we ignore here transport cost to deliver output to the market and assume identical price for output in all locations). Moreover, they should accrue identical cost, with proportions a and 1 - a going to different factors. Thus,

$$S(x)R(x) = (1-a)Y_0,$$
(13)

$$p(x)C(x) = aY_0. (14)$$

Allocation of land. Since all land along the channel has production value, it should have positive price and all used in equilibrium. We have N agents, and each of them is located in different point  $x_i$ , where he owns land slot of size  $S_i$ . The condition of all land being used is:  $\sum_{i=1}^{N} S_i = 1$ . Moreover, each subinterval of  $x \in [0, 1]$  should belong to one and only one owner. Skipping some details, which are discussed in [3]<sup>6</sup>, we can write:

$$\int_0^1 \rho(x) dx = \int_0^1 \frac{dx}{S(x)} = 1.$$
 (15)

#### 4.6 Solution

Consider  $Y_0$  to be a parameter, and for given p(x), express all unknowns via p(x) and  $Y_0$ . For our case of regulated water price at cost level,  $p(x) = we^{\beta x}$ , and we get the following expressions:

$$C(x) = \frac{aY_0}{w}e^{\beta x},\tag{16}$$

$$S(x) = Y_0 \left[\frac{a}{w}e^{-\beta x}\right]^{-\frac{a}{1-a}},$$
(17)

$$R(x) = \frac{(1-a)Y_0}{S(x)}.$$
(18)

The last parameter,  $Y_0$  is determined from equation (7), now becoming:

$$\left(\frac{a}{w}\right)^{\frac{a}{1-a}} \int_0^1 e^{\frac{\beta a}{1-a}x} dx = Y_0 N.$$
(19)

 $<sup>^6\</sup>mathrm{We}$  go to continuous limit, renormalize measure of farmers to one and use the inverse relation between their density and land use

Finally,

$$Y_0 = \frac{1-a}{N\beta a} (\frac{a}{w})^{\frac{a}{1-a}} [1 - \exp(-\beta \frac{a}{1-a})].$$
(20)

**Results and policy implications.** We obtained equilibrium location of agents, price of land and consumption of land and water by each agent. Note that a change of pricing policy p(x) would change the structure of the whole equilibrium. Land rent will change. It means that value of property of a particular farmer in a desert is vulnerable and depends on the pricing policy for water. If the property rights for land are already established, any change in pricing policy of water supplier would perturb wealth distribution of farmers, making some of them poorer. That is why any changes of pricing policy by suppliers of utilities should be done carefully, with study of all consequences for users. In this case market is not always efficient.

But here we are more concerned with issues of efficiency. In the next sections we will consider an unregulated monopolist owning a channel. He may use flat price or make price discrimination. The efficiencies of different schemes can be measured by total output produced, total water used.

# 5 A Monopolist without Price Discrimination

Suppose now that a channel is owned by monopolist. Assume first that initial land ownership is uniform: each agent owns 1/N units of land, but all in different locations. If price of water is charged uniformly (at level W, which will be chosen by a monopolist to maximize profits), all agents will have the same production plans: to spend share a of expenditures on water, and 1-a on land rent. Again C/S = aR/((1-a)p). Now land rent price R will be uniform, but again depend on water price.

Putting aside some difficulties related to optimal price determination (it depends on cost of water for a firm), let us concentrate on inefficiency that can emerge. Even if water supplier takes into account the cost of lost water during delivery, uniform pricing mechanism does not encourage the remote consumer to save water. Losses of water in the system will be greater than in previous case, and total water used to produced identical output will be also higher. If total supply of water is limited by either river capacity, or environmental restrictions, the mechanism of uniform pricing will generate lower total output.

## 6 Conclusions

1. The new economic question related to water pricing in spatially distributed market is addressed. At the moment such market is not yet created and it cannot emerge spontaneously, without corresponding legal framework.

2. Moreover, the particular infrastructure makes a channel for water distribution a natural monopoly profits of which should be regulated. At the same time, the spatial structure of losses will make this monopoly an incentive to set spatial prices in such a way, that efficiency in consumption is reached.

3. The channel will affect the whole pattern of agricultural production in its neighbourhood. If we assume for simplicity only two production factors - water and land rent, than any water pricing will result in emergence of equilibrium land rent distribution in space.

4. If agriculture is done by small farmers who are the only workers on their land andf if such farmers are equally wealthy ex-ante, then in the new market equilibrium they will settle with a variable population density along the channel, so that the spatial differences in water prices with be compensated by lower land rent in such a way that they produce the same amount of output at their farms, using different factor intensities in production.

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