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LOCATION DECISION OF POLLUTING FIRMS AND ENVIRONMENTAL POLICY*

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Abstract

The delocation of firms is often viewed as a major outcome of a stiff environmental policy. In this paper, we study the impact of a strict anti-pollution policy pursued by a government on domestic firms locational decisions and determine the main variables that interact with such a policy. Some preliminary welfare implications are also provided.

Keywords: Environmental policy; Plant location; Imperfect competition.

JEL classification: H7; R3; D4.

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1. Introduction

One of the most urgent subjects for international policy is the growing concern that expansion of the world economy will cause irreparable damage to the earth’s environment and a reduced quality of life for future generations.

The external effects of economic activities have turned out to be of such importance that free market forces in the absence of environmental public policy will lead to large scale and irreversible environmental damage. As a consequence there is need for public policy to induce firms to internalise negative external effects. An example of such policies is the introduction of *environmental taxes*.\(^1\) Many authors agree on the use of this measure, even because taxing “bads”, such as pollution, it is considered as an alternative to tax “goods” (work, savings, investment).

As to environmental taxes, a crucial topic is the impact they have on firms’ decision. Firms decide where to localise or delocalise their production by considering the environmental policies among other things. This means that these policies affect both environment quality and production and employment levels within a region (nation). Therefore, national governments could face a potential trade off between production and pollution, which could trigger tax competition phenomena, as well highlighted in the literature (see Oates (2001) for a survey).


In all the papers referred to above the analysis is set up in a two country, or two region, context, with countries using environmental taxation as the policy instrument, either cooperatively or non-cooperatively. The present paper adopts a similar framework, but our approach differs from all these papers in that we assume polluting firms with different technologies (i.e., firms have different emissions abatement costs).\(^2\)

We develop a simple model to explain firms’ location decisions when unilateral environmental policies occur. The hypothesis of different technologies implies that

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\(^1\) Policy instruments to reduce pollution can be divided in two groups: market-based instruments (taxes, subsidies and tradable emission permits) and command-and-control instruments (design standards and performance standards).

\(^2\) Our paper differs also in that we assume the two firms do not take their location decisions simultaneously (see section 4), as, instead, it is commonly assumed in the existing literature on this topic.
firms react in a different way to the introduction of environmental taxes. The expected result is that more polluting firms will have a greater incentive than less polluting ones to delocalise their plants in the region which has no environmental tax or where this tax is lower. By this model it is also possible to study how the welfare of the region, in which the environmental tax has been imposed, changes on the basis of firms and government decisions. We show that unilateral environmental policies can be welfare improving.

The rest of the paper is organized as follows. The next section presents the model. Section 3 derives Cournot-Nash equilibria in the two markets. Section 4 considers the issue of the optimal location choice by firms in response to the environmental policy enforced by government. In section 5 we analyse the alternative strategies government can adopt and the possible effects on welfare. The final section summarizes the results.

2. The model

Consider a two region \((i = A, B)\), two firms \((j = 1, 2)\) economy. Both firms are initially located in region A and produce a homogeneous good. They compete à la Cournot-Nash in the two markets. For both firms production leads to emission of the same pollutant. This pollutant causes only local damage.

Each firm’s cost function can be written as \(c(q, \vartheta)\), where \(q\) is the output and \(\vartheta\) is the chosen level of pollution abatement.\(^3\) We assume that the cost function is linear in output and level of pollution abatement

\[
c_j(q_j, \vartheta_j) = \alpha q_j + \beta_j^2 \vartheta_j q_j
\]

For each firm, marginal costs are given by \(\alpha + \beta_j^2 \vartheta_j\), where \(\beta_j\) is a firm-specific parameter. It is assumed that these marginal costs are constant and proportional to pollution abatement levels.

Firms have different emissions abatement costs. Asymmetric cost structure can be allowed for, as the introduced \(\beta\)-parameters may differ across the firms (see Nannerup,\(^3\) As in Motta and Thisse (1994) we assume that the fixed costs are sunk when the game begins. Hence, firm \(j\) does not have to incur any fixed costs when it operates its domestic plant in A.)
2001). Firm emission of the pollutant is taken to be proportional to production and inversely proportional to abatement effort. Denote emission by $z_j$.\footnote{The specification of abatement costs and emission is based on Kennedy (1994).}

$$z_j = \frac{q_j}{\varphi_j}$$ \hspace{1cm} (2)

When a firm exports it bears a trading cost per unit of output equal to $\tau$, where $0 \leq \tau \leq 1$.

Finally, if firm $j$ decides to establish a plant in region B, it incurs a set-up cost ($F \geq 1$) which is independent of the volume of output.

The market inverse demand functions are linear and symmetric

$$p_i = S_i(1-Q_i)$$ \hspace{1cm} (3)

where $Q_i$ is the total output sold in region $i$ by the two firms, $p_i$ is the corresponding market clearing price and $S_i$ stands for region $i$’s market size.

$D(z(q,\varphi))$ is an environmental damage function and is assumed to have the standard properties $D(0)=0$ and $D'>0$

$$D(z(q,\varphi)) = \eta_i z_j$$ \hspace{1cm} (4)

where $\eta_i > 0$ is a parameter which denotes the effect of one unit of emission in region $i$ on the environment of the region.

\footnote{A “high” $\beta$-value implies “high” expenditures in raising the output-emission ratio in production ($q_j/z_j$) (Nannerup, 2001). Hence, a firm having a “high” $\beta$-value would choose a low level of pollution abatement.}
3. The Cournot-Nash equilibria

In the absence of any environmental regulation, each firm will set \( q_{i,j} = 0 \) i.e. firms will not engage in abatement. In this case, letting \( \Pi_j \) denote firm \( j \)'s profits, the following optimization problem is solved:

\[
\max_{q_{i,j}} \Pi_j = p_i q_{i,j} - \alpha q_{i,j} - \tau q_{i,j} \quad (5)
\]

The following are the Nash equilibrium quantities sold by firm \( j \) on each market:

\[
q_{j,A}^* = \frac{1}{3} \frac{S_A - \alpha}{S_A} \quad (6a)
\]
\[
q_{j,B}^* = \frac{1}{3} \frac{S_B - \alpha - \tau}{S_B} \quad (6b)
\]

The total equilibrium profits earned by firm \( j \) are:

\[
\Pi_j^* = \frac{1}{9} \frac{(S_A - \alpha)^2}{S_A} + \frac{1}{9} \frac{(S_B - \alpha - \tau)^2}{S_B} \quad (7)
\]

We now consider the case when the government of region A decides to impose a tax \( t \) per unit of pollution. Firm \( j \) solves the problem

\[
\max_{q_{i,j}, \vartheta_{i,j}} \Pi_j = p_i q_{i,j} - \alpha q_{i,j} - \tau q_{i,j} - \beta_j^2 \vartheta_{i,j} q_{i,j} - t_j \left( \frac{q_{i,j}}{\vartheta_{i,j}} \right) \quad (8)
\]

The first-order condition for pollution abatement is

\[
- \beta_j^2 q_{i,j} + \frac{t_j q_{i,j}}{\vartheta_{i,j}^2} = 0 \quad (9)
\]

After rearranging first-order condition we obtain
\[ t_i = \beta_j^2 g_{i,j} \]  

Eq. (10) shows that for firm \( j \) the optimal abatement effort is given by the point where marginal abatement costs equal the marginal saving in emission taxes. Moreover, it follows from (2) and (10) that smaller is the value of \( \beta_j^2 \) bigger are the profits for firm \( j \).  

The following are the Nash equilibrium quantities sold by the two firms on each market:

\[
q_{1,A}^* = \frac{S_A + \beta_2^2 \vartheta_{2,A} + \frac{t}{\vartheta_{2,A}^2} - \alpha - 2 \beta_1^2 \vartheta_{1,A} - 2 \frac{t}{\vartheta_{1,A}^2} - \frac{t}{\vartheta_{1,A}^3}}{S_A} 
\]

\[
q_{1,B}^* = \frac{S_B + \beta_2^2 \vartheta_{2,B} + \frac{t}{\vartheta_{2,B}^2} - \alpha - 2 \beta_1^2 \vartheta_{1,B} - 2 \frac{t}{\vartheta_{1,B}^2} - \tau}{S_B} 
\]

\[
q_{2,A}^* = \frac{S_A + \beta_2^2 \vartheta_{2,A} + \frac{t}{\vartheta_{2,A}^2} - \alpha - 2 \beta_1^2 \vartheta_{1,A} - 2 \frac{t}{\vartheta_{1,A}^2}}{S_A} 
\]

\[
q_{2,B}^* = \frac{S_B + \beta_2^2 \vartheta_{2,B} + \frac{t}{\vartheta_{2,B}^2} - \alpha - 2 \beta_1^2 \vartheta_{1,B} - 2 \frac{t}{\vartheta_{1,B}^2} - \tau}{S_B} 
\]

The total equilibrium profits earned by firm 1 are:

\[
\Pi_1^1 = \frac{1}{9} \left( \frac{S_A + \beta_2^2 \vartheta_{2,A} + \frac{t}{\vartheta_{2,A}^2} - \alpha - 2 \beta_1^2 \vartheta_{1,A} - 2 \frac{t}{\vartheta_{1,A}^2}}{S_A} \right)^2 + \frac{1}{9} \left( \frac{S_B + \beta_2^2 \vartheta_{2,B} + \frac{t}{\vartheta_{2,B}^2} - \alpha - 2 \beta_1^2 \vartheta_{1,B} - 2 \frac{t}{\vartheta_{1,B}^2} - \tau}{S_B} \right)^2 
\]

and the profits earned by firm 2 are

\[ ^6 \text{To make this statement more clear, it can be useful to rewrite the Eq. (10) in the following form: } g_{i,j}^2 = t_i / \beta_j^2. \]
We have shown that the smaller is the value of $\beta_j^2$, the bigger are the profits for firm $j$. If we assume that $\beta_2^2 > \beta_1^2$, the profits for firm 2 are lower than the profits for firm 1 $(\Pi_2^* < \Pi_1^*)$. In this case firm 2 could have first an incentive to move in region B, which do not adopt anti-pollution policies. We illustrate this case in the next section.

4. Optimal location choice

Now we consider the issue of the optimal location choice by firm $j$ in response to the environmental policy enforced by region A’s government. As we have seen the effects produced by the introduction of the environmental tax on the two firms are not symmetric, because of the different costs structure. Therefore, we can assume that firm 2 is the first one to assess whether to remain in the region of origin or move to region B.\footnote{The existence of scale economies at plant level makes it unprofitable to open a plant in region B and still operate the plant in region A.} That means to compare the profits earned by firm 2 under the two possible configurations: (i) both firms are in region A, denoted (A, 0) (ii) firm 2 moved to region B, while firm 1 is still in region A, denoted (A, B). Eq. (13) gives the profits in the case (A, 0). In the case (A, B) the total equilibrium profits are:

$$
\Pi_2^*(A,B) = \frac{1}{9} \left( \frac{S_A - \alpha - 2\tau + \beta^2 \partial_{l_A} + \frac{t}{\partial_{l_A}}} {S_A} \right)^2 + \frac{1}{9} \left( \frac{S_B - \alpha + \beta^2 \partial_{l_B} + \frac{t}{\partial_{l_B}} + \tau} {S_B} \right)^2 - (F_A + F_B) \tag{14}
$$

If $\Pi_2^*(A,B) > \Pi_2^*(A,0)$, then firm 2 relocates its plant.\footnote{$\Pi_2^*(A,0)$ is given by the Eq. (13).} Hence, there exists a value of $t$ for which firm 2 is moving to region B. This happens when $t > t_{2e}$, where $t_{2e}$ is the...
value of the environmental tax for which firm 2 is indifferent whether moving or not. The value of $t_{2e}$ is given by the Eq. (15).\(^9\)

\[
t_{2e} = \frac{9S(F_A + F_B) - 4\tau^2}{\frac{19}{8}\beta^2 + \frac{2S - 2\alpha - \tau}{\vartheta_2}}
\]  

(15)

Assume that the value of the tax imposed by region A’s government is slightly higher than $t_{2e}$, then firm 2 decides to shut down its plant in region A and to open a new plant in region B. What happens to firm 1? Is it moving or not? Again, we have to compare firm 1 profits under the two possible configurations: (i) firm 1 is still in region A, denoted (A, B) (ii) both firms are in region B, denoted (0, B). In the configuration (A, B) firm 1 total equilibrium profits are:

\[
\Pi_1(A, B) = \frac{1}{9} \left( \frac{S_A - \alpha - 2\beta_1^2 \vartheta_{1A} - 2 - \frac{t}{\vartheta_{1A}} + \tau}{S_A} \right)^2 + \frac{1}{9} \left( \frac{S_B - \alpha - 2\beta_1^2 \vartheta_{1B} - 2 - \frac{t}{\vartheta_{1B}} - 2\tau}{S_B} \right)^2
\]

(16)

If firm 1 relocates its plant, its profits become

\[
\Pi_1(0, B) = \frac{1}{9} \left( \frac{S_B - \alpha}{S_B} \right)^2 + \frac{1}{9} \left( \frac{S_A - \alpha - \tau}{S_A} \right)^2 - (F_A + F_B)
\]

(17)

If $\Pi_1(0, B) > \Pi_1(A, B)$, then firm 1 relocates its plant, too.

For firm 1 the value of $t$ for which it is indifferent whether moving or not is given by:\(^{10}\)

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\(^9\) To find the value of $t_{2e}$, we put $\Pi_1(A, 0) = \Pi_2(A, B)$, then we solve for $t$. Note that $\vartheta_{1A} = \vartheta_{1B}$ and $\vartheta_{2A} = \vartheta_{2B}$, as only the government of region A has imposed an environmental tax. Moreover, we assume that (i) $\beta_1^2 = \frac{1}{4} \beta_2^2$ (ii) $S_A = S_B$ (i.e. we study the case where the two market sizes of A and B are identical).

\(^{10}\) To find the value of $t_{1e}$, we put $\Pi_1(A, B) = \Pi_1(0, B)$, then we solve for $t$. 

Now what we want to show is that the relation $t_{1e} \geq t_{2e}$ is always true. Rearranging Eq. (10) and substituting into (15) yields:

$$T_{2e} = \frac{-8d_1 + \sqrt{64d_1^2 - \frac{19}{2}d_2 + \frac{19}{2}d_3}}{\frac{19}{4}\beta_2}$$  \hspace{1cm} (19)

Carrying out similar calculations for firm 1, we obtain

$$T_{1e} = \frac{-d_1 + \sqrt{d_1^2 - 2d_2 - 2d_3}}{-4\beta_2}$$  \hspace{1cm} (20)

where

$$T_{2e} = t_{2e}^{1/2}$$
$$T_{1e} = t_{1e}^{1/2}$$
$$d_1 = 2S - 2\alpha - \tau$$
$$d_2 = 4\tau^2$$
$$d_3 = 9S(F_A + F_B)$$

Comparing (19) with the analogous expression for firm 1 (Eq. (20)), we can state that the tax rate $t_{1e}$, for which firm 1 is indifferent whether to relocate or not its plant in region B, is always higher, given a sufficiently large market size, than the tax rate $t_{2e}$, for which firm 2 is indifferent whether to relocate or not its plant.\textsuperscript{11} This point has important policy implications, to which we return in the last section.

Fig. 1 displays the tax rates $t_{1e}$ and $t_{2e}$ with respect to the market size, in the case of low trade costs and low set-up costs. The main results emerging from the study of this

\textsuperscript{11} The detailed demonstration can be found in Appendix.
figure are: (i) there is always a positive value of the equilibrium tax rate for firm 2; (ii) if the market size is not large enough, there is not a tax rate for which firm 1 would stay in region A, after firm 2 moved to region B; (iii) $T_{1e}$ and $T_{2e}$ are both declining in the market size.

Figure 1. Equilibrium tax rate as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_A = 0; F_B = 1.2; \tau = 0.3; \beta_2 = 1$.

We find analogous results in the case of high trade costs and high set-up costs, even if there are some remarkable differences (Fig. 2). Firstly, as we expected, in this costs configuration we have higher value of the equilibrium tax rates. Secondly, the critical size of the market for firm 1 becomes huger.\textsuperscript{12}

\textsuperscript{12} Fig. A1 and Fig. A2, in Appendix, depict the other two possible costs configurations (low trade costs and high set-up costs and high trade costs and low set-up costs).
Figure 2. Equilibrium tax rate as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_A = 0.7; F_B = 1.3; \tau = 0.9; \beta = 1$.

5. The government’s strategies and the possible effects on welfare

In this section we study the alternative strategies region A’s government can adopt and the consequent firms’ location decisions. As shown in Fig. 3, the government can impose a tax at a level (i) lower than $t_{2e}$ (ii) higher than $t_{2e}$, but lower than $t_{1e}$ (iii) higher than $t_{1e}$. There is also a fourth option: to preserve status quo and do not adopt any anti-pollution policy ($t = 0$).
The interpretation of the two extreme cases ($t \leq t_{2e}$ and $t > t_{1e}$) is straightforward: if the tax rate adopted is lower than $t_{2e}$, both firms have not any incentive to relocate their plants (the only consequence is a production reduction); while if it is higher than $t_{1e}$, the optimal location for the two firms is in region B.\textsuperscript{13} More interesting is the case in between ($t_{2e} < t \leq t_{1e}$). For firm 2, the more polluting firm, the best choice is always to move to region B, while for firm 1, the less polluting one, it depends on the market size. Up to a certain critical size, it will relocate its plant, too.\textsuperscript{14} To be precise, we should say that, when market is “small”, $t_{1e}$ does not exist or, better, $t_{1e} = t_{2e}$, since, if firm 2 best choice is to move to region B, this is the best choice for firm 1, too.

Which are the implications of environmental policies on welfare? We assume that the welfare of region A is the summation of consumer surplus, firm $j$ profits, tax revenues and pollution costs. In the configuration denoted (A, 0), the welfare equation is the following:

$$W_A(A,0) = CS_A + \Pi_1 + \Pi_2 + t(z_{1,a} + z_{2,a}) - \eta(z_{1,a} + z_{2,a})$$

(21)

\textsuperscript{13} We assume that if $t = t_{2e}$, firm 2 does not move to region B, even if it would be indifferent towards the two locations. The same assumption holds for firm 1: if $t = t_{1e}$ it stays in region A.

\textsuperscript{14} The critical market size depends on the values of the other variables ($\alpha; \tau; F_A; F_B; \beta^2$).
where the first term is the consumer surplus, the second the profits earned by firm 1, the third the profits earned by firm 2, the fourth the environmental tax revenues, while the last term is the environmental damage caused by production.

In the absence of any environmental tax, we have

$$W_A(\text{notax}) = CS_A + \Pi_1 + \Pi_2 - \eta(z_{1,4} + z_{2,4}) \tag{22}$$

In the case only firm 2 has relocated its plant, welfare is

$$W_A(A,B) = CS_A + \Pi_1 + \lambda \Pi_2 + t(z_{1,4}) - \eta(z_{1,4}) \tag{23}$$

where $\lambda < 1$ is the share of repatriated profits. If both firms have moved to region B, welfare is

$$W_A(0,B) = CS_A + \lambda \Pi_1 + \lambda \Pi_2 \tag{24}$$

If the aim of the government is to maximise the regional welfare, which is the tax rate the regional authority should choose? Fig. 4 displays the welfare levels associated with four particular tax rates ($t = 0; \ t = t_2; \ t = t_1; \ t > t_1$) with respect to the market size.
**Figure 4.** Welfare as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_A = 0; F_B = 1.2; \tau = 0.3; \beta_2 = 1; \lambda = 0.2; \eta = 0.4$.

It is evident that adopting anti-pollution policies increases the regional welfare. Even in the case both firms have moved to region B (corresponding to the curve denoted $W_{(0,B)}$), the level of welfare is higher than the level reached when there is no environmental policy ($W_{(\text{notax})}$).\(^{15}\) The best strategy region A’s government could adopt, is introducing a tax $t = t_{z^e}$ (the top curve in Fig. 4). If we analyse Fig. 5, drawn for high trade costs and high set-up costs, we do not observe any substantial change.

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\(^{15}\) Motta and Thisse (1994) reach the same conclusion.
Figure 5. Welfare as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_d = 0.7; F_b = 1.3; \tau = 0.9; \beta_a = 1; \lambda = 0.2; \eta = 0.4$.

Our conclusions can not be general, since they depend on the particular functional form assumed for $W$ and on the values of the parameters, nevertheless they demonstrate that unilateral environmental policies can be welfare improving.

Clearly, the impact of these policies particularly depends on the effects of the emissions on the environment of the region. Fig. 6 describes the case in which these effects are very negative. It is interesting to note that for a definite range of market sizes, $W_{(0, B)} > W_{(A, 0)}$, which means that region A is better off when both firms relocate their plant.
Figure 6. Welfare as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_\alpha = 0; F_\beta = 1.2; \tau = 0.3; \beta_2 = 1; \lambda = 0; \eta = 2.9$.

6. Conclusions

This paper examines location decisions of polluting firms with different emissions abatement costs when unilateral environmental policies occur.

We show that adopting anti-pollution policies increases the regional welfare. Even in the case both firms relocate their plants, the level of welfare is higher than the level reached when there is no environmental policy. Our conclusions cannot be general, since they depend on the particular functional form assumed for welfare and on the values of the parameters, nevertheless they demonstrate that unilateral environmental policies can be welfare improving.

Furthermore, we show that the market size is a crucial variable which affects firms’ location choices. The tax rate for which the less polluting firm (firm 1) is indifferent whether to relocate or not its plant, is always higher, given a sufficiently large market size, than the tax rate for which the more polluting firm (firm 2) is indifferent whether
to relocate or not its plant; but when market is “small”, if firm 2 best choice is to move, this is the best choice for firm 1, too.

Three other avenues are open for future research. The first is to add some asymmetries between regions in some key variables. The second is to add asymmetry of information between regions and/or firms. Finally, a possible spin off of this research would be to let the governments of the two regions compete in pollution taxes to attract firms owned outside the regions.
Appendix

The purpose of this Appendix is to demonstrate that $t_{te} > t_{2e}$.

The proof is given in a number of steps:

1) Rearranging Eq. (10) and substituting into (15) yields:

$$T_{2e} = \frac{-8d_1 \pm \sqrt{64d_1^2 - \frac{19}{2}d_2 + \frac{19}{2}d_3}}{\frac{19}{4} \beta_2}$$

The zero profit condition yields

$$T = \frac{d_1}{6\beta_2}$$

Eq. (A1) implies $d_1 > 0$, that is $S > \alpha + \frac{1}{2} \tau$.\(^{16}\)

Given $d_1 > 0$, we can reject the solution $T_{2e} = \frac{-8d_1 - \sqrt{64d_1^2 - \frac{19}{2}d_2 + \frac{19}{2}d_3}}{\frac{19}{4} \beta_2}$, since $T_{2e}$ cannot be negative.

2) The condition $S > \alpha + \frac{1}{2} \tau$ implies $d_3 > d_2$; therefore,

$$64d_1^2 - \frac{19}{2}d_2 + \frac{19}{2}d_3 > 0.$$\(^{17}\)

3) For firm 1 we get

\(^{16}\) Note that $T$ cannot be negative. Therefore, since $\beta_2$ is always positive, $d_1$ has to be positive, too.

\(^{17}\) Note that $d_2$ and $d_3$ are always positive.
The zero profit condition yields

\[ T_{1e,2} = -\frac{d_1 \pm \sqrt{d_1^2 - 2d_2 - 2d_3}}{-4\beta_2} \]

The zero profit condition yields

\[ T = \frac{d_1}{4\beta_2} \] (A2)

Given the Eq. (A2), we can reject the solution \( T_{1e} = \frac{-d_1 - \sqrt{d_1^2 - 2d_2 - 2d_3}}{-4\beta_2} \), since \( T_{1e} \) can not be greater than \( T \).

4) The condition \( d_1^2 \geq 2(d_2 + d_3) \) must hold. This condition holds for values of \( S \) sufficiently great. For example:
   i. \( \forall S \geq 6.7 \), for low trade costs (\( \tau = 0.3 \)) and low set-up costs \( (F_A = 0, F_B = 1.2) \);
   ii. \( \forall S \geq 10.3 \), for low trade costs (\( \tau = 0.3 \)) and high set-up costs \( (F_A = 0.7, F_B = 1.3) \);
   iii. \( \forall S \geq 7.4 \), for high trade costs (\( \tau = 0.9 \)) and low set-up costs \( (F_A = 0, F_B = 1.2) \);
   iv. \( \forall S \geq 11.0 \), for high trade costs (\( \tau = 0.9 \)) and high set-up costs \( (F_A = 0.7, F_B = 1.3) \).

5) If \( d_3 = d_2 \), we would obtain:

\[ \sqrt{64d_1^2 - \frac{19}{2}d_2 + \frac{19}{2}d_3} = 8d_1; \]

since \( d_3 > d_2 \), we have \( \sqrt{64d_1^2 - \frac{19}{2}d_2 + \frac{19}{2}d_3} = 8d_1 + \mu \).

Hence,
\[ T_{2e} = \frac{-8d_1 + (8d_1 + \mu)}{19\beta_2} \]  \hspace{1cm} (A3)

A similar line of reasoning gives us the value of \( T_{1e} \):

\[ T_{1e} = \frac{-d_1 + (d_1 - \nu)}{-4\beta_2} \]  \hspace{1cm} (A4)

Under the condition \( d_1^2 \geq 2(d_2 + d_3) \), \( \nu \geq \mu \), which implies \( T_{1e} > T_{2e} \).
Figure A1. Equilibrium tax rate as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_A = 0; F_B = 1.2; \tau = 0.9; \beta_2 = 1$.

Figure A2. Equilibrium tax rate as a function of the market size $S$. The figure is drawn for $\alpha = 0.5; F_A = 0.7; F_B = 1.3; \tau = 0.3; \beta_2 = 1$. 
References