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Conference Paper

An Empirical Study of the Purchasing Power Parity in the European Union

44th Congress of the European Regional Science Association: "Regions and Fiscal Federalism", 25th - 29th August 2004, Porto, Portugal

Provided in Cooperation with:

European Regional Science Association (ERSA)

Suggested Citation: Feijoo, Santiago Rodriguez; Caro, Alejandro Rodriguez; Correa, Carlos Gonzalez (2004) : An Empirical Study of the Purchasing Power Parity in the European Union, 44th Congress of the European Regional Science Association: "Regions and Fiscal Federalism", 25th - 29th August 2004, Porto, Portugal, European Regional Science Association (ERSA), Louvain-la-Neuve

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An Empirical Study of the Purchasing Power Parity in the European Union

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Abstract: This paper studies the convergence in the European Union on Purchasing Power Parity. Firstly, we develop a method for the estimation of PPP series and then, we put it into practice and estimate the monthly Purchasing Power Parity for the period January 1995 to July 2003. Secondly, convergence is studied using distribution dynamics from the continuous and discrete approximations. Results show some slow convergence in the European Union.

Keywords: Purchasing Power Parity, Convergence, Distribution Dynamics, Markov Chains

1. Introduction

Purchasing Power Parity Theory (PPP) was formally enunciated by Gustav Cassel in 1918. This theory is fulfilled when "the equilibrium rate of exchange equalizes the purchasing power of a currency in a country, with what it might buy in the exterior if it was changed into a foreign currency" [mentioned in Spiegel (1996), p. 690]. From this theory, its territorial comparison character in terms of prices is deduced, including the interior price in each territory and the price which they are interrelated: the exchange rate.

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In the last decades, studies on PPP have become more relevant because of several reasons. Monetary policies must take the stability of prices as an objective; due to its movements can produce important imbalances, originating changes in the production and consumption decisions. The globalization process has increased the exchanges volume between countries, which also has multiplied the risk of monetary flows due to the movements of internal and external prices. The increase of these risks can provoke again abnormal market behaviour and realizes an inefficient assignment of resources.

Nevertheless, studies on PPP for the European Union could only be done for the medium term, because periodicity of data is annual, are published with an important delay and are also short series. Previous works have approached to the problem of PPP using Consumer Price Indexes (CPI) instead of PPP's measures [Alberola and Marquess (2001), and Canceló et al (2000)]. Rodríguez, Gonzalez and Rodríguez (2002) and Rodríguez, Gonzalez and Rodríguez (2004) have showed some of the problems that this substitution brings up and propose a new index, which is obtained from CPI and exchange rates. This index measures changes in the Purchasing Power Parity between two areas.

When one PPP data are available and based on it, in the present paper we developed an expression to estimate Purchasing Power Parities using Consumer Price Index and Exchange Rates series. To study convergence on PPP in the European Union, we have estimated monthly PPP for the period January 1995 to June 2003.

From this sample, we analyze convergence using distribution dynamics. This methodology used initially by Quah (1993a, 1993b) in the discrete case for the Gross Domestic Product. This methodology has also been used in Lopez-Bazo and other (1999), Magrini (1999) and La Gallo (2001) for European Union regions, by Pekala (1999) for Finnish provinces, and by Tizonas (2002) for Greek regions. In the continuous case, proposed by Quah (1996a, 1996b, 1996c) has been used by Mancusi (2000) for the evolution of the industrialized countries technology or by Jonhson (2000) for the convergence in levels of relative income in the United States of America.

The structure of this paper is the following. In the second part, the estimation process of the Parity of Purchasing power for the countries members of the European Union is

presented. In the third part, we show the estimations results, from the continuous and discrete viewpoint. Finally, the most relevant conclusions are presented.

2. Estimation of Purchasing Power Parity.

Following Rodriguez et al (2004), Purchasing Power Parity can be estimated using the $B_{0,t}^{iG}$ statistic in [1].

$$B_{0,t}^{iG} = \frac{\frac{e_t^{iG}}{e_0^{iG}} I_t^i - 1}{I_t^G - 1} \quad [1]$$

where G represents a Global Market of n regions or countries represented by i. e_t^{iG} is the exchange rate of region i and the Global Market. I_t^G and I_t^i are the Consumer Price Index (CPI in the following) of G and i, in period t with base period in 0.

Generally speaking, CPI is the quotient of prices in two periods of time, we can write those of G and I as in [2]

$$I_t^i = \frac{P_t^i}{P_0^i}, I_t^G = \frac{P_t^G}{P_0^G} \quad [2]$$

where P_t^i , P_t^G , P_0^i y P_0^G are prices in i and G in period t and 0.

Using [2] in [1] and ordering the resulting expression we have [3].

$$\frac{e_t^{iG} P_t^i}{e_0^{iG} P_0^i} - B_{0,t}^{iG} \frac{P_t^G}{P_0^G} = 1 - B_{0,t}^{iG} \quad [3]$$

Multiplying and dividing the first quotient of [3] by P_t^G y P_0^G

$$\frac{e_t^{iG} P_t^i \frac{P_t^G}{P_t^G}}{e_0^{iG} P_0^i \frac{P_0^G}{P_0^G}} - B_{0,t}^{iG} \frac{P_t^G}{P_0^G} = 1 - B_{0,t}^{iG} \quad [4]$$

PPP of region i and the global market G is defined in [5],

$$PPC_t^{iG} = \frac{e_t^{iG} P_t^i}{P_t^G} \quad [5]$$

Using [5], expression [4] can be finally written as [6].

$$PPC_t^{iG} = PPC_0^{iG} \times \left[\frac{1 - B_{0,t}^{iG} \times [1 - I_t^G]}{I_t^G} \right] \quad [6]$$

Estimation of Purchasing Power Parities of each country in the European Union have been carried out, Based on [6], using Eurostats' PPP for 1995, monthly Harmonized Consumer Price Index and Exchange Rates, for the period January 1995 – June 2003.

This preliminary estimate has been adjusted to the annual data provided by Eurostat for 1996-2002 using the disaggregation method proposed by Boot et al (1967). This methodology has been applied to the annual differences between real data and the preliminary estimates. This methodology is the most appropriate method when no additional information is available, as shown in Rodriguez et al (2003).

3. Distribution Dynamics.

In the continuous framework, estimation of stochastic kernels have been carried out. Quah (1997) defines the stochastic kernel $M_{(\mu,\nu)}$ as a function that satisfies

$$\mu(A) = \int M_{(\mu,\nu)}(y, A) d\nu(y) \quad [7]$$

where μ, ν , are probabilities, A is an state, y is the subset of status, $\mu(A)$ is the fractions of economies that end up in state A regardless their initial ones.

Two graphical representations are commonly used to show results, 3D graphics, and the contour plot. In both plots, X and Y axis represent the different values of the PPP at different periods of time, and in Z axis probability to be in state A k periods later.

Two aspect of this distribution centre our attention, its shape and mobility. The former is about the modality of the function. Only one peak would indicate a convergence in the distribution, on the other hand, more than one peak would show us a divergence process or the presence of “convergence” clubs, i.e., countries would tend to be divided in more than one group. Mobility (or persistence) refers to the position of the function over the 45-degree diagonal. The more concentrated along this diagonal, the less mobility (higher persistence) in the distribution. A counter-clockwise movement would

indicate that countries with higher (lower) level of PPP are more likely to have lesser (higher) level of PPP in k periods ahead, i.e. it would show some kind of convergence.

The advantage of the discrete point of view is that results are clearer and easier to interpret but there are some risks involving the process to discretise a continuous process. First of all, let us define notation. Let be P the transition matrix, whose elements (p_{ij}) are the probabilities of getting to state j from state i after k periods. M is the total number of possible states. Maximum likelihood estimator of any probability in P is [8]

$$\hat{p}_{ij} = \sum_{t \in T} \frac{n_{ij}(t)}{n_i(t)} \quad [8]$$

Where $n_{ij}(t)$ is the number of elements that end up in state j , being in state i in period t . $n_i(t)$ is the total number of elements in state i in period t ($n_i(t) = \sum_{j=1}^M n_{ij}(t)$).

Markov property implies [9]

$$N(t+q) = PN(t+q-1) = P^2N(t+q-2) = \dots = P^qN(t) \quad [9]$$

Where $N(\cdot)$ is the vector of number of elements in each state at time (\cdot) , ie, $N(\cdot) = [n_1(\cdot), n_2(\cdot), \dots, n_M(\cdot)]$

If the Markov chain holds the ergodic property, then the ergodic distribution can be found and satisfies [10]

$$N(\infty) = N(\infty)P \quad [10]$$

4. Convergence on the European Union

Estimation of stochastic kernels for 1 and 12 periods ahead are shown in Figure 1 to Figure 4. These estimations have been carried out using professor Quah's TsRf software, and the Epanechnikov kernel. For every month in the sample, the average of the European Union is equal to 100. In order to analyze the possible effect of the Euro entry in 1999, we have estimated the stochastic kernels for the 1995-1998 and 1999-2003 periods.

The results show some clear aspects. Firstly, two peaks are presented in the estimated distributions with a dip in the middle. These show us the presence of two groups of countries, one in each side of the distribution, with high and low PPP. There is little mobility in the distribution as a large portion of the mass probability is situated along the 45° diagonal line; so countries are more likely to be in the same position k periods later (high persistence), this situation will be also reaffirmed later with discrete Markov chains. Mobility of the distribution has to be studied in order to find the evolution in the long term. As it could be seen, the relative position to the 45° diagonal line is counter-clockwise, revealing some kind of slow convergence because of the diagonal structure with no probability far away from the 45° diagonal line.

Following Jonhson (2000), the ergodic distribution of the one period ahead kernel estimations are computed, for the whole period and the 1995-98 and 1999-2003 subsamples. Results, which are shown in Figure 5, demonstrate the convergence process shown in **¡Error! No se encuentra el origen de la referencia.** to Figure 3. But it also presents the existence of a small peak in the left side of the distribution, showing the possible existence of a trap in low values of PPP.

The discrete approximation has its simplicity and easy to explain and translate short-run dynamics into long-run tendencies its main advantage. The need for a discretisation method comes up to compute the discrete Markov transition probability matrix from a continuous time series as PPP, avoiding potential problems of an inappropriate or subjective discretisation process.

Magrini (1999) suggested the following method to estimate the discrete density function. Two elements need to be defined Ω y h . The former is the origin of the intervals computed using the bin width h in $(\Omega+kh, \Omega+(k+1)h)$, for negative and positive integers k . In our study this origin is 100, which has a clear meaning as the average of the European Union.

Two possible solutions are shown to estimate h . The first one is proposed by Freedman y Diaconis (1981):

$$\hat{h}_2 = 2Rn^{-1/3} \quad [11]$$

Where R is the interquantile range and n is the total number of observations in the sample. The second one is proposed by Devroye y Györfi (1985):

$$\hat{h}_1 = 2.72sn^{-\frac{1}{3}} \quad [12]$$

Where s is the sample standard deviation. Applying these expressions to our data base, we obtain $\hat{h}_2 = 4.46$ y $\hat{h}_1 = 3.71$ and the associated Markov chains (with 15 and 18 intervals each) are in Figure 6 and Figure 7, respectively. Upper limits of each interval are shown in the first row and column of each matrix. Ergodic distribution and recurrence time are presented in the last two rows. These estimated transition matrices show the usual diagonal structure, with high persistence and slow mobility, in our study, tend to convergence.

Nevertheless, we test homogeneity over time as presented in Bickenbach y Bode (2001), taking as break point the birth of the euro in 1999. Estimations of transitions matrices for the whole period and 1995-98 and 1999-2003 sub-samples are done to use the following LR² test

$$\begin{aligned} H_0 &: \forall t : \hat{p}_{ij}(t) = p_{ij} \\ H_1 &: \exists t : \hat{p}_{ij}(t) \neq p_{ij} \end{aligned} \quad [13]$$

$$Q = \sum_{t=1}^T \sum_{i=1}^M \sum_{j=1}^M n_i(t) \frac{(\hat{p}_{ij}(t) - \hat{p}_{ij})^2}{\hat{p}_{ij}} \sim \text{asym} \chi^2_{\sum_{i=1}^M (a_i - 1)(b_i - 1)}$$

where $\hat{p}_{ij}(t), n_i(t)$ are values of each sub-sample, a_i is the number of samples where positive values in row i are available, and b_i in the number of positive probabilities in row i in sub-sample t. Results of this test are shown in Figure 8 and there are evidence to reject the null hypothesis (time-stationarity) in both alternatives.

In this point, given the number of intervals in both solutions, what it involves that the number of elements in each row is not large, we have adopted to divide the whole sample in m quantiles, where $3 \leq m \leq 6$, as proposed in Quah (1993a) and used in many other works as López-Bazo et al (1999), Le Gallo (2001), Neven and Gouyette (1995),

² For details see Anderson y Goodman (1957) y Kullback et al (1962)

Hites (2002). The main advantage of this solution is to avoid non robust estimations because of small number of observations in one or more rows of the estimated transition matrix. In the present paper m is equal to five, being the upper limits (82.885%, 98.854%, 105.504%, 115.548%, 133.39%).

However, the main disadvantage is the different bin width of each resulting interval, so an adjustment is required in the ergodic distribution to keep the histogram interpretation of the long term distribution (Hites (2002), pag 10).

To solve the discretisation problem we apply the solution proposed by Hites (2002). The true mobility from state i to j denoted $m_{ij}(t)$ is observed with an error term $\varepsilon_{ij}(t)$, and resulting the mobility $n_{ij}(t)$ under expression [14]. The latter mobility is then used in the likelihood estimation of the transition matrix:

$$n_{ij}(t) = m_{ij}(t) + \varepsilon_{ij}(t) \quad [14]$$

Using the following probabilities

$$\begin{aligned} P(m_{ij}(t) = 0/n_{ij}(t) = 0) &= 1 - f[\varepsilon(t)] \\ P(m_{ij}(t) = 0/n_{ij}(t) = 1) &= 0 + f[\varepsilon(t)] \\ P(m_{ij}(t) = 1/n_{ij}(t) = 0) &= 0 + f[\varepsilon(t)] \\ P(m_{ij}(t) = 1/n_{ij}(t) = 1) &= 1 - f[\varepsilon(t)] \end{aligned} \quad [15]$$

In the previous empirical studies it is assumed that $f(\varepsilon(t))=0$, so a transition between two states could only be 1 or 0. This binary solution could be relaxed by allowing values of $f(\varepsilon(t)) \neq 0$. This error term would represent, for example, model misspecification or differential data inaccuracy. Hites (2002) proposed to transform the observed points into triangular distributions centered on these points. The base of these triangles is defined by a parameter α . For example, Summers y Heston (1991, 1994, 2002) provided a quality grade on countries income series.

Moreover, each observation could be assigned to more than one state, and the value is the area place on every state. Representing by $P(y_t \in E_i)$ the area of the triangle, which is centered in y_t , in the state i , transitions from t to $t+1$ can be derived using expression [16]

$$\begin{pmatrix} P(y_t \in E_1) \\ P(y_t \in E_2) \\ \vdots \\ P(y_t \in E_m) \end{pmatrix} (P(y_{t+1} \in E_1) \quad P(y_{t+1} \in E_2) \quad \dots \quad P(y_{t+1} \in E_m)) \quad [16]$$

Where the first vector is the probabilities of being in every m state in period t and the second one is in period t+1.

Estimated Markov chain using Hites (2002) methodology are shown in Figure 11, for different levels of α , where $\alpha=0\%$ is the usual methodology. High values of persistence can be seen for low values of α , and mobility is increasing with the values of α . Time-stationary test are again carried out for every single of α . Results for some values of α can be seen in the last two columns of Figure 11 and for all the studied values of α in Figure 10. Results show that the null hypothesis is rejected for values of α between 1% and 7%. The main reason is the different probabilities in the second and third estate. When $\alpha=0\%$ the null hypothesis can not be rejected, one feasible reason could be the small number of observations and so, the high sensitive of the distribution to small changes on them. This fact can be seen in Figure 9 where the elasticities of the elements outside the main diagonal of the transition matrix when $\alpha=0$ to the ergodic distributions are presented. Almost in all cases, the absolute values are greater than one, so small changes in those probabilities amplified its effects on the ergodic distribution. The Rescaled ergodic distributions show a slow convergence process in Purchasing Power Parity in the European Union.

5. Conclusions

In the present paper, a methodology to estimate Purchasing Power Parity using Consumer Prices index, Exchange Rates and one value of PPP is developed. Using this methodology, an estimation of monthly PPP for the countries in the European Union is carried out.

Employing this monthly data-base from January 1995 to June 2003, we have developed a study on the convergence using distribution dynamic methodology.

Results from both points of view, continuous and discrete, show a slow convergence process in the studied period. We also show a significant difference behaviour before and after Euro Entry.

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Figure 1: Stochastic Kernel 1 period ahead. January 1995 - June 2003 Sample

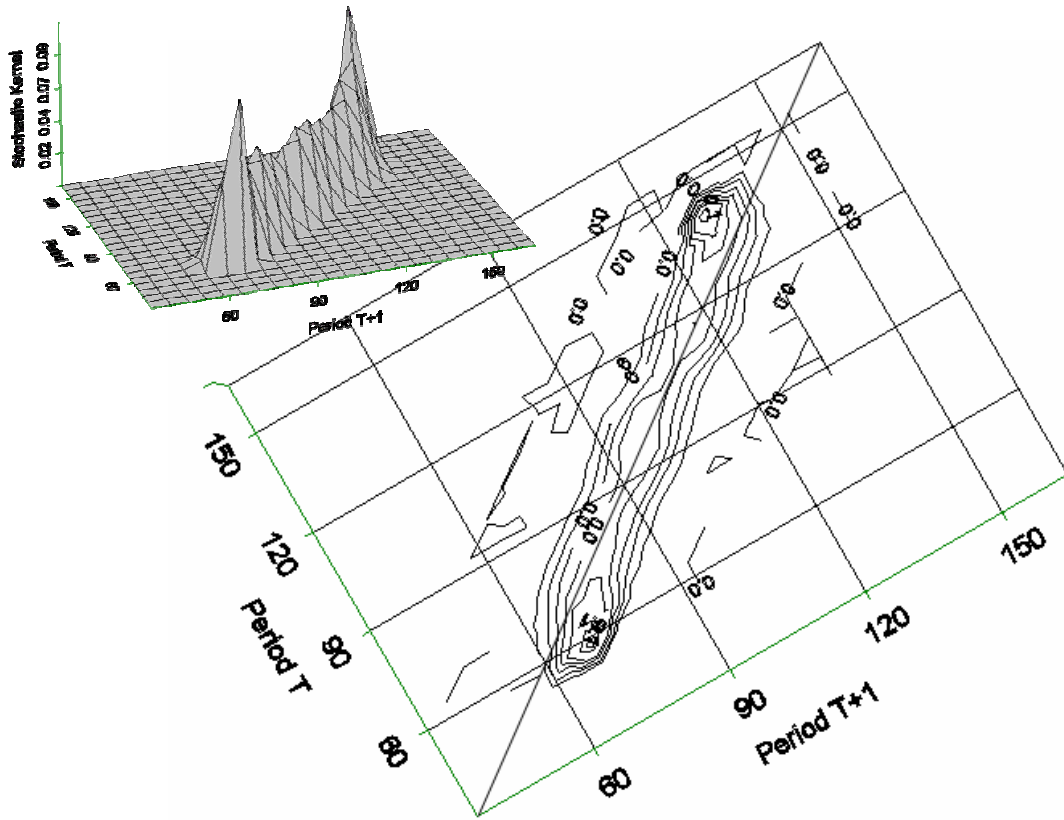


Figure 2: Stochastic Kernel 12 period ahead. January 1995 - June 2003 Sample

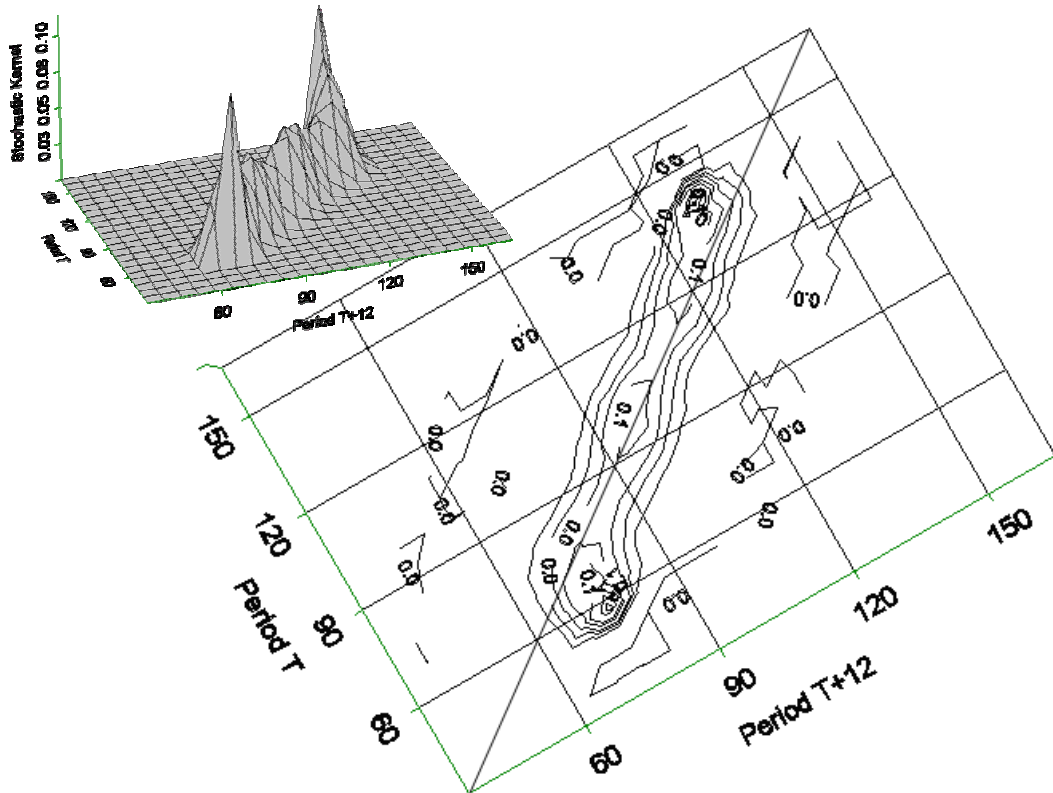


Figure 3: Stochastic Kernel 1 period ahead. January 1995 – December 1998 Sample

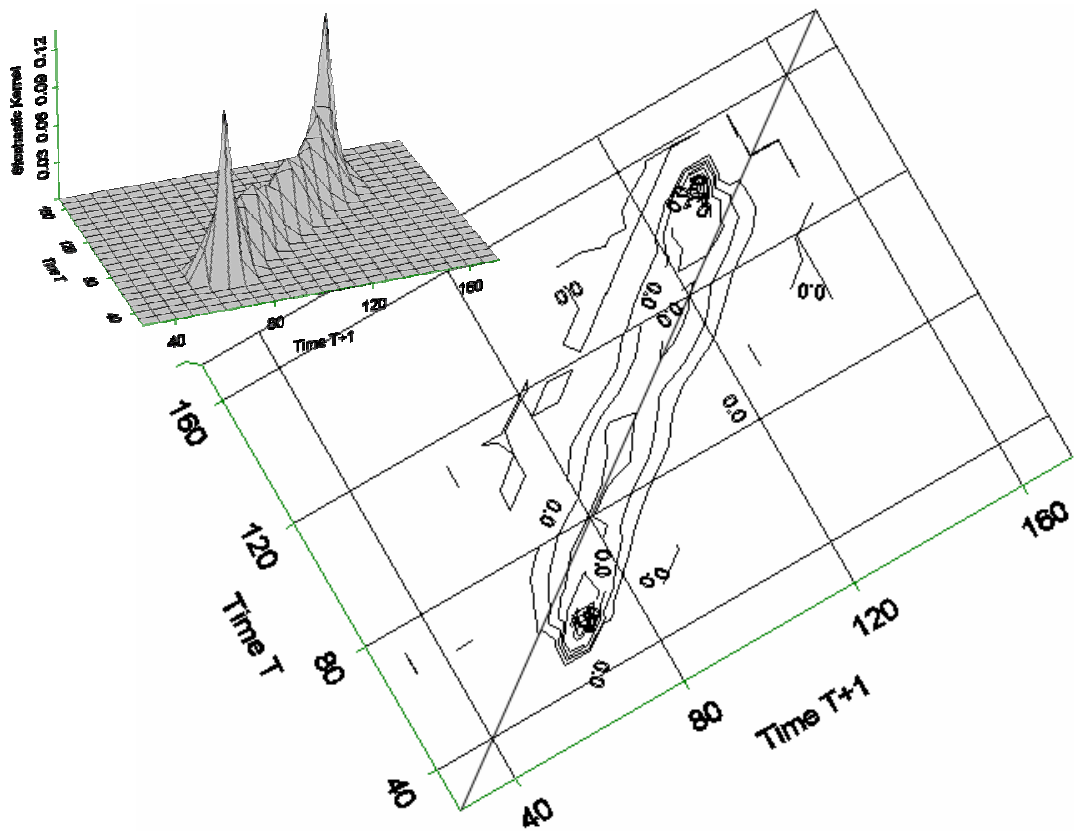


Figure 4: Stochastic Kernel 1 period ahead. January 1999 - June 2003 Sample

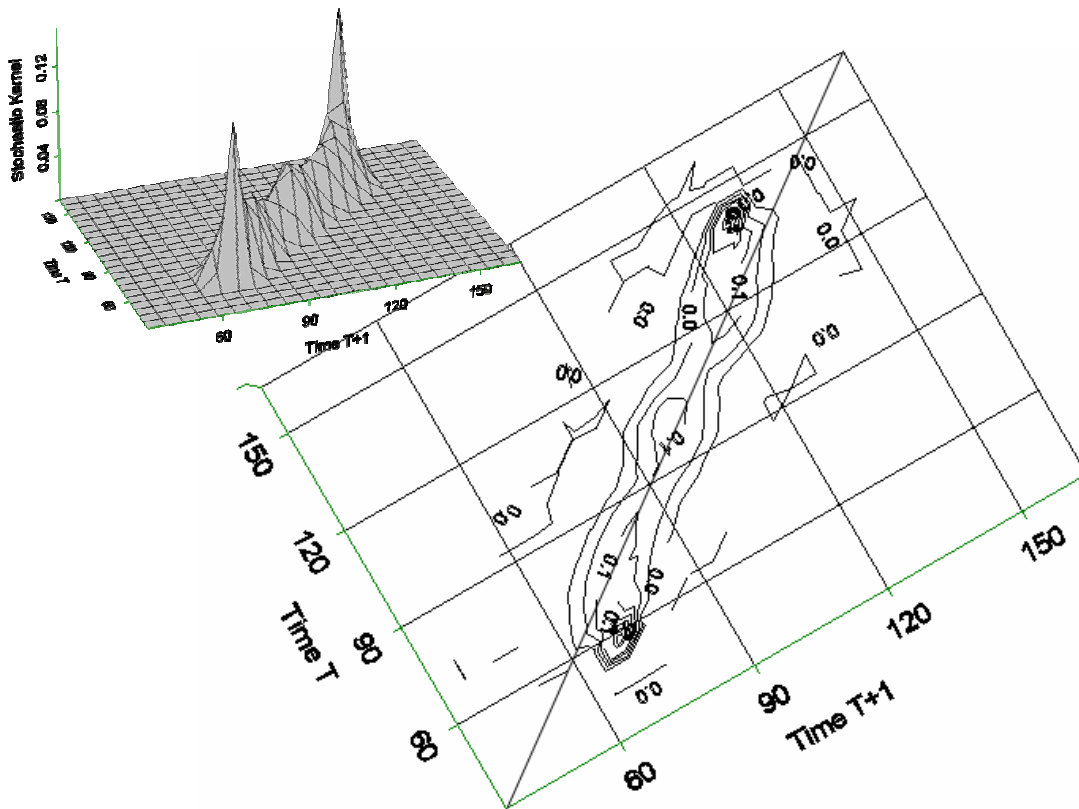


Figure 5: Ergodic distributions one month transitions

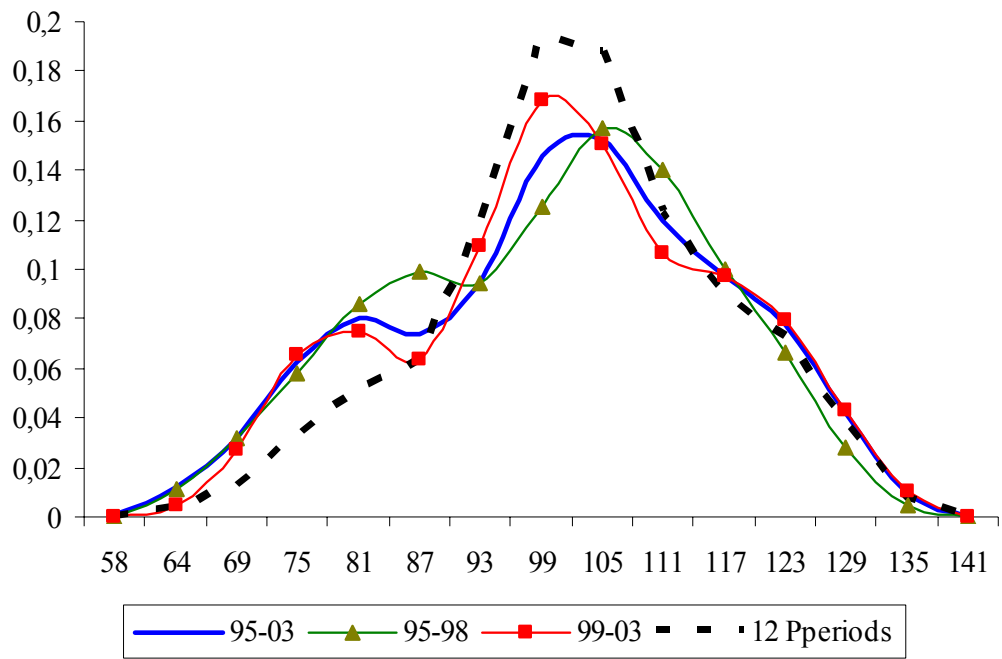


Figure 6: Transition Matriz using Freedman y Diaconis (1981), $h_2=4.46$

ES	68.73	73.19	77.66	82.13	86.60	91.06	95.53	100.00	104.47	108.94	113.40	117.87	122.34	126.81	133.39	n_i
68.73	0.78	0.22	-	-	-	-	-	-	-	-	-	-	-	-	-	23
73.19	0.12	0.85	0.03	-	-	-	-	-	-	-	-	-	-	-	-	40
77.66	-	0.01	0.87	0.12	-	-	-	-	-	-	-	-	-	-	-	78
82.13	-	-	0.04	0.88	0.08	-	-	-	-	-	-	-	-	-	-	129
86.60	-	-	0.01	0.10	0.80	0.09	-	-	-	-	-	-	-	-	-	69
91.06	-	-	-	-	0.06	0.88	0.06	-	-	-	-	-	-	-	-	77
95.53	-	-	-	-	-	0.03	0.86	0.11	-	-	-	-	-	-	-	69
100.00	-	-	-	-	-	-	0.03	0.91	0.06	-	-	-	-	-	-	187
104.47	-	-	-	-	-	-	-	0.08	0.83	0.09	-	-	-	-	-	176
108.94	-	-	-	-	-	-	-	-	0.09	0.85	0.06	-	-	-	-	190
113.40	-	-	-	-	-	-	-	-	-	0.16	0.74	0.10	-	-	-	82
117.87	-	-	-	-	-	-	-	-	-	-	0.05	0.90	0.05	-	-	166
122.34	-	-	-	-	-	-	-	-	-	-	0.02	0.09	0.80	0.09	-	64
126.81	-	-	-	-	-	-	-	-	-	-	-	-	0.05	0.92	0.03	104
133.39	-	-	-	-	-	-	-	-	-	-	-	-	-	0.07	0.93	46
Ergodic	0.002	0.004	0.01	0.03	0.02	0.03	0.06	0.21	0.16	0.16	0.06	0.10	0.05	0.08	0.04	
Recurrence Time	528.32	288.17	96.05	37.84	52.02	34.68	17.34	4.73	6.31	6.31	16.82	9.94	21.86	12.14	28.34	

Figure 7: Transition matrix using Devroye y Györfi (1985), $h_1=3.71$

ES	70.32	74.03	77.74	81.45	85.16	88.87	92.58	96.29	100.00	103.71	107.42	111.13	114.84	118.55	122.26	125.97	129.68	133.39	n_i
70.32	0.96	0.04	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	56
74.03	0.08	0.62	0.30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13
77.74	-	0.03	0.85	0.12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	73
81.45	-	-	0.07	0.83	0.10	-	-	-	-	-	-	-	-	-	-	-	-	-	87
85.16	-	-	0.01	0.07	0.85	0.07	-	-	-	-	-	-	-	-	-	-	-	-	95
88.87	-	-	-	-	0.09	0.83	0.08	-	-	-	-	-	-	-	-	-	-	-	64
92.58	-	-	-	-	-	0.05	0.78	0.17	-	-	-	-	-	-	-	-	-	-	58
96.29	-	-	-	-	-	-	0.10	0.70	0.20	-	-	-	-	-	-	-	-	-	46
100.00	-	-	-	-	-	-	0.01	0.03	0.89	0.07	-	-	-	-	-	-	-	-	180
103.71	-	-	-	-	-	-	-	-	0.10	0.84	0.05	0.01	-	-	-	-	-	-	140
107.42	-	-	-	-	-	-	-	-	-	0.07	0.87	0.06	-	-	-	-	-	-	179
111.13	-	-	-	-	-	-	-	-	-	-	0.14	0.81	0.05	-	-	-	-	-	102
114.84	-	-	-	-	-	-	-	-	-	-	-	0.08	0.67	0.24	0.01	-	-	-	71
118.55	-	-	-	-	-	-	-	-	-	-	-	-	0.12	0.83	0.05	-	-	-	128
122.26	-	-	-	-	-	-	-	-	-	-	-	-	0.02	0.11	0.76	0.11	-	-	55
125.97	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.05	0.91	0.04	-	97
129.68	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.09	0.89	0.02	47
133.39	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.11	0.89	9
Ergodic	0.003	0.002	0.017	0.024	0.030	0.023	0.038	0.043	0.213	0.149	0.128	0.065	0.041	0.079	0.034	0.074	0.033	0.006	
Recurrence Time	301.67	603.38	60.34	41.48	33.19	42.67	26.67	23.53	4.71	6.72	7.84	15.32	24.52	12.60	29.70	13.50	30.38	167.09	

Figure 8: Time-stationary test

Criterion	Q Statistics	Degrees of Freedom	p-value
Freedman y Diaconis (1981)	60.63	29	0.000515
Devroye y Györfi (1985)	73.91	39	0.000618

Figure 9: Ergodic distribution elasticities $\alpha=0\%$.

-16.837	16.837	-2.449	2.449	-1.531	1.276	-1.531	0.765
4.592	-4.592	-2.449	2.449	-1.531	1.276	-0.765	0.765
4.592	-4.592	1.837	-1.837	-1.531	1.276	-0.765	0.765
3.827	-3.827	1.531	-1.531	2.296	-1.913	-0.638	0.638
-3.827	-3.827	1.531	-1.531	2.296	-1.913	2.934	-2.934

Figure 10: Evolution of time-stationary test

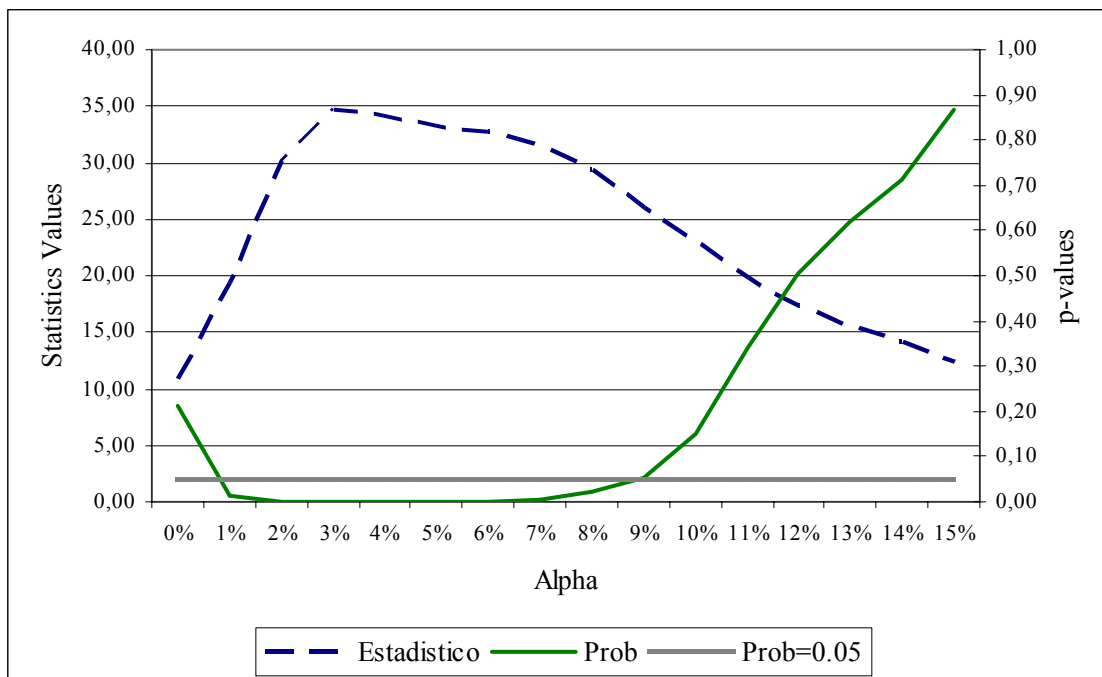


Figure 11: Transition matrices 5 states and different values of α^3

		Sample (1995-2003)					1 ^a Sub-Sample (1995-1998)					2 ^a Sub-Sample (1999-2003)					X ²	Prob
ES	$\alpha = 0\%$	82.885	98.854	105.504	115.548	133.39	82.885	98.854	105.504	115.548	133.39	82.885	98.854	105.504	115.548	133.39		
		0.99	0.01	-	-	-	0.98	0.02	-	-	-	0.99	0.01	-	-	-	1.24	0.27
		0.01	0.94	0.05	-	-	0.01	0.95	0.04	-	-	0.01	0.93	0.07	-	-	1.49	0.48
		-	0.05	0.90	0.05	-	-	0.05	0.85	0.10	-	-	0.05	0.92	0.02	-	7.48	0.02
		-	-	0.06	0.89	0.05	-	-	0.06	0.89	0.05	-	-	0.05	0.90	0.05	0.55	0.76
		-	-	-	0.05	0.95	-	-	-	0.05	0.95	-	-	-	0.05	0.95	0.09	0.77
Total =															10.85	0.21		
		Sample (1995-2003)					1 ^a Sub-Sample (1995-1998)					2 ^a Sub-Sample (1999-2003)					X ²	Prob
$\alpha = 5\%$		0.90	0.10	-	-	-	0.88	0.12	-	-	-	0.91	0.09	-	-	-	1.05	0.30
		0.08	0.75	0.16	0.00	-	0.10	0.81	0.09	0.00	-	0.07	0.70	0.23	0.00	-	12.09	0.01
		-	0.19	0.63	0.19	0.00	-	0.13	0.55	0.31	0.00	-	0.22	0.67	0.11	0.00	17.91	0.00
		-	0.00	0.18	0.67	0.14	-	0.00	0.20	0.68	0.12	-	0.00	0.16	0.66	0.17	2.29	0.52
		-	-	0.00	0.15	0.85	-	-	0.00	0.15	0.85	-	-	0.00	0.14	0.86	0.01	1.00
		Total =															33.34	0.00
		Sample (1995-2003)					1 ^a Sub-Sample (1995-1998)					2 ^a Sub-Sample (1999-2003)					X ²	Prob
$\alpha = 10\%$		0.82	0.18	0.00	-	-	0.81	0.19	0.00	-	-	0.84	0.16	0.00	-	-	0.63	0.73
		0.13	0.63	0.19	0.05	0.00	0.16	0.68	0.13	0.04	0.00	0.10	0.60	0.25	0.05	0.00	10.51	0.03
		0.00	0.27	0.43	0.29	0.01	0.00	0.20	0.40	0.38	0.02	0.00	0.32	0.44	0.22	0.01	9.01	0.06
		0.00	0.06	0.24	0.49	0.21	0.00	0.04	0.24	0.52	0.19	-	0.07	0.23	0.46	0.23	2.72	0.61
		-	0.00	0.01	0.22	0.77	-	0.00	0.02	0.22	0.76	-	0.00	0.01	0.22	0.77	0.14	0.99
		Total =															23.00	0.15
		Sample (1995-2003)					1 ^a Sub-Sample (1995-1998)					2 ^a Sub-Sample (1999-2003)					X ²	Prob
$\alpha = 15\%$		0.76	0.24	0.00	0.00	0.00	0.75	0.24	0.00	0.00	0.00	0.77	0.23	0.00	0.00	-	0.08	1.00
		0.16	0.53	0.19	0.11	0.01	0.19	0.55	0.14	0.10	0.01	0.14	0.51	0.22	0.12	0.01	6.16	0.19
		0.00	0.32	0.30	0.30	0.08	0.00	0.26	0.29	0.34	0.10	0.00	0.35	0.31	0.27	0.06	3.77	0.44
		0.00	0.15	0.24	0.38	0.24	0.00	0.12	0.23	0.41	0.24	0.00	0.17	0.24	0.35	0.24	1.96	0.74
		-	0.01	0.06	0.22	0.71	-	0.02	0.06	0.22	0.70	-	0.01	0.05	0.22	0.72	0.43	0.93
		Total =															12.39	0.87

³ (-) shows no probability, whilst 0.00 shows probabilities greater than zero but less than 0.00.

Figure 12: Rescaled ergodic distribution for 5 estates⁴

α	Sample (1995-2003)					1 ^a Sub-Sample (1995-1998)					2 ^a Sub-Sample (1999-2003)				
	82.885	98.854	105.504	115.548	133.39	82.885	98.854	105.504	115.548	133.39	82.885	98.854	105.504	115.548	133.39
0%	0.150	0.156	0.372	0.207	0.115	0.101	0.164	0.295	0.295	0.145	0.151	0.150	0.440	0.157	0.101
1%	<i>0.136</i>	<i>0.159</i>	<i>0.370</i>	<i>0.214</i>	<i>0.120</i>	<i>0.127</i>	<i>0.160</i>	<i>0.285</i>	<i>0.291</i>	<i>0.137</i>	<i>0.120</i>	<i>0.152</i>	<i>0.444</i>	<i>0.169</i>	<i>0.114</i>
2%	<i>0.135</i>	<i>0.158</i>	<i>0.360</i>	<i>0.222</i>	<i>0.125</i>	<i>0.136</i>	<i>0.164</i>	<i>0.272</i>	<i>0.294</i>	<i>0.136</i>	<i>0.125</i>	<i>0.149</i>	<i>0.431</i>	<i>0.173</i>	<i>0.121</i>
3%	<i>0.133</i>	<i>0.161</i>	<i>0.354</i>	<i>0.225</i>	<i>0.127</i>	<i>0.137</i>	<i>0.168</i>	<i>0.270</i>	<i>0.292</i>	<i>0.133</i>	<i>0.126</i>	<i>0.152</i>	<i>0.420</i>	<i>0.178</i>	<i>0.124</i>
4%	<i>0.132</i>	<i>0.163</i>	<i>0.350</i>	<i>0.228</i>	<i>0.128</i>	<i>0.138</i>	<i>0.171</i>	<i>0.272</i>	<i>0.288</i>	<i>0.132</i>	<i>0.123</i>	<i>0.155</i>	<i>0.411</i>	<i>0.184</i>	<i>0.127</i>
5%	<i>0.130</i>	<i>0.167</i>	<i>0.345</i>	<i>0.230</i>	<i>0.129</i>	<i>0.138</i>	<i>0.172</i>	<i>0.275</i>	<i>0.284</i>	<i>0.131</i>	<i>0.121</i>	<i>0.161</i>	<i>0.402</i>	<i>0.189</i>	<i>0.128</i>
6%	<i>0.128</i>	<i>0.170</i>	<i>0.341</i>	<i>0.232</i>	<i>0.129</i>	<i>0.137</i>	<i>0.173</i>	<i>0.277</i>	<i>0.282</i>	<i>0.130</i>	<i>0.120</i>	<i>0.166</i>	<i>0.392</i>	<i>0.193</i>	<i>0.129</i>
7%	<i>0.128</i>	<i>0.173</i>	<i>0.335</i>	<i>0.234</i>	<i>0.130</i>	<i>0.138</i>	<i>0.174</i>	<i>0.277</i>	<i>0.280</i>	<i>0.131</i>	<i>0.118</i>	<i>0.172</i>	<i>0.381</i>	<i>0.199</i>	<i>0.130</i>
8%	<i>0.127</i>	<i>0.176</i>	<i>0.327</i>	<i>0.239</i>	<i>0.131</i>	<i>0.138</i>	<i>0.176</i>	<i>0.275</i>	<i>0.279</i>	<i>0.131</i>	<i>0.118</i>	<i>0.177</i>	<i>0.369</i>	<i>0.206</i>	<i>0.130</i>
9%	0.128	0.181	0.318	0.242	0.131	0.139	0.178	0.272	0.278	0.133	0.117	0.182	0.357	0.213	0.131
10%	0.127	0.184	0.311	0.245	0.133	0.140	0.181	0.269	0.276	0.134	0.116	0.187	0.345	0.219	0.132
11%	0.127	0.188	0.303	0.247	0.134	0.141	0.184	0.265	0.274	0.136	0.116	0.191	0.336	0.225	0.132
12%	0.128	0.192	0.296	0.249	0.135	0.142	0.188	0.261	0.270	0.139	0.116	0.196	0.326	0.229	0.133
13%	0.128	0.194	0.293	0.247	0.138	0.144	0.190	0.258	0.266	0.143	0.116	0.198	0.321	0.231	0.134
14%	0.128	0.197	0.289	0.246	0.140	0.145	0.191	0.257	0.262	0.145	0.115	0.201	0.315	0.231	0.137
15%	0.129	0.199	0.286	0.239	0.147	0.147	0.195	0.255	0.250	0.153	0.115	0.204	0.310	0.229	0.141

⁴ In italics values which reject time-stationary test (at 5% significant level)