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Regional Specialization via Differences in Transport Costs: An Economic Geography Approach

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Abstract

The regional specialization via differences in transport costs are observed in Japanese manufacturing industries. Concretely, industries with high transport costs for their products, such as iron and steel, petroleum and coal products, remained close to the core region while industries with low transport costs, such as electrical machinery, precision instruments, have relocated to the periphery region. The purpose of this paper is to give a theoretical foundation for this fact by use of a new economic geography model with multiple industries. The urban costs and congestion are explicitly included in the model. We obtain the following results. First, if congestion does not exist, at most one industry disperses when transport and commuting costs are sufficiently small. Furthermore, regional specialization occurs in which industries having higher adjusted transport costs (which are defined as the ratios of transport costs to the number of varieties) than that of the dispersing industry agglomerate in one region. Second, in the case of strong congestion, plural industries might disperse even if transport and commuting costs are small, but as the degree of congestion decreases, the location will change to complete regional specialization.

Key words: regional specialization, economic geography, transport costs, urban costs, congestion.

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1 Introduction

During the past four decades many Japanese manufacturing industries have spread from the ‘core’ region to the ‘periphery’ region¹. Figures 1a and 1b show changes of regional shares in value of products and in employment, respectively. In 1960 Japanese manufacturing produced (resp. absorbed) nearly 55% (resp. 50%) of product-value (resp. workers) in the core, but in 2000 it only produced (resp. absorbed) nearly 35% (resp. 30%) of product-value (resp. workers) there.

However, we should note that all manufacturing industries have not similarly dispersed but they differ in degree. Concretely, industries with high transport costs for their products, such as transport equipment, printing and publishing, petroleum and coal products, iron and steel, remained close to the core while industries with low transport costs for their products, such as electrical machinery, precision instruments, have considerably relocated to the periphery (see Figure 2).² For example, in precision instruments, 75% was produced in the core in 1960 but only 32% was produced there in 2000, when the periphery share (35%) has exceeded the core share. On the other hand, in petroleum and coal products, for example, regional shares have been almost constant since 1960.

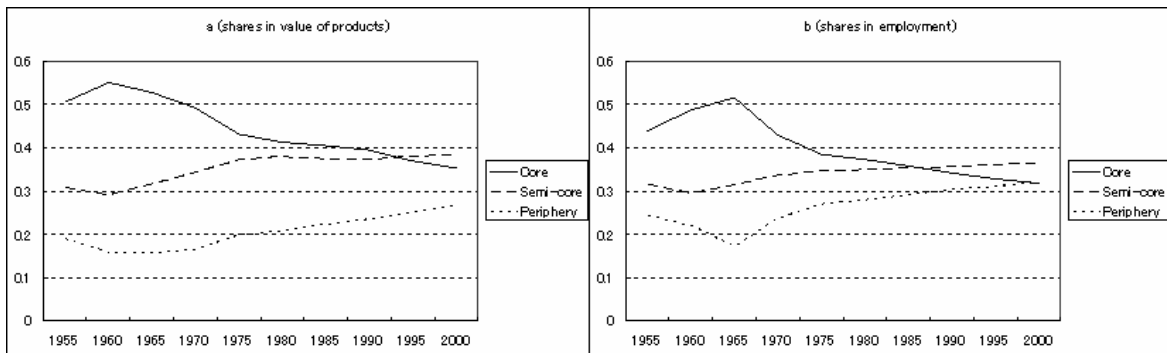


Figure 1: Changes in regional shares [manufacturing total] (Source: Japanese Census of Manufactures)

¹According to Fujita and Hisatake (1999), 47 Japanese prefectures are divided into three macroregions as follows: *Core* consists of Tokyo and Kanagawa (the core of Tokyo @Metropolitan@ area [MA]), Aichi (containing Nagoya MA), Osaka and Hyogo (the core of Osaka MA); *Semi-Core* consists of the Pacific belt zone excluding the Core (18 prefectures) and *Periphery* is the rest of Japan.

²By the regional shares in product-value from the latest Japanese Census of Manufactures in 2000, we divide 21 types of manufacturing industries into following 4 groups: (a) *Core oriented industries* [Core>Semi-Core>Periphery]: General machinery, Transportation equipment, Printing and publishing, Leather and leather products (4 types); (b) *Semi-core oriented industries I* [Semi-Core>Core>Periphery]: Chemicals, Petroleum and coal products, Plastic, Rubber, Iron and steel, Non-ferrous metals, Fabricated metal products (7 types); (c) *Semi-core oriented industries II* [Semi-Core>Periphery>Core]: Processed foods, Textiles, Apparel, Lumber and wood, Furniture, Paper and pulp, Ceramics, stone, clay and glass (7 types); (d) *Periphery oriented industries* [Periphery>Semi-Core>Core]: Electrical machinery, Precision instruments, Beverage, forage and tobacco (3 types).

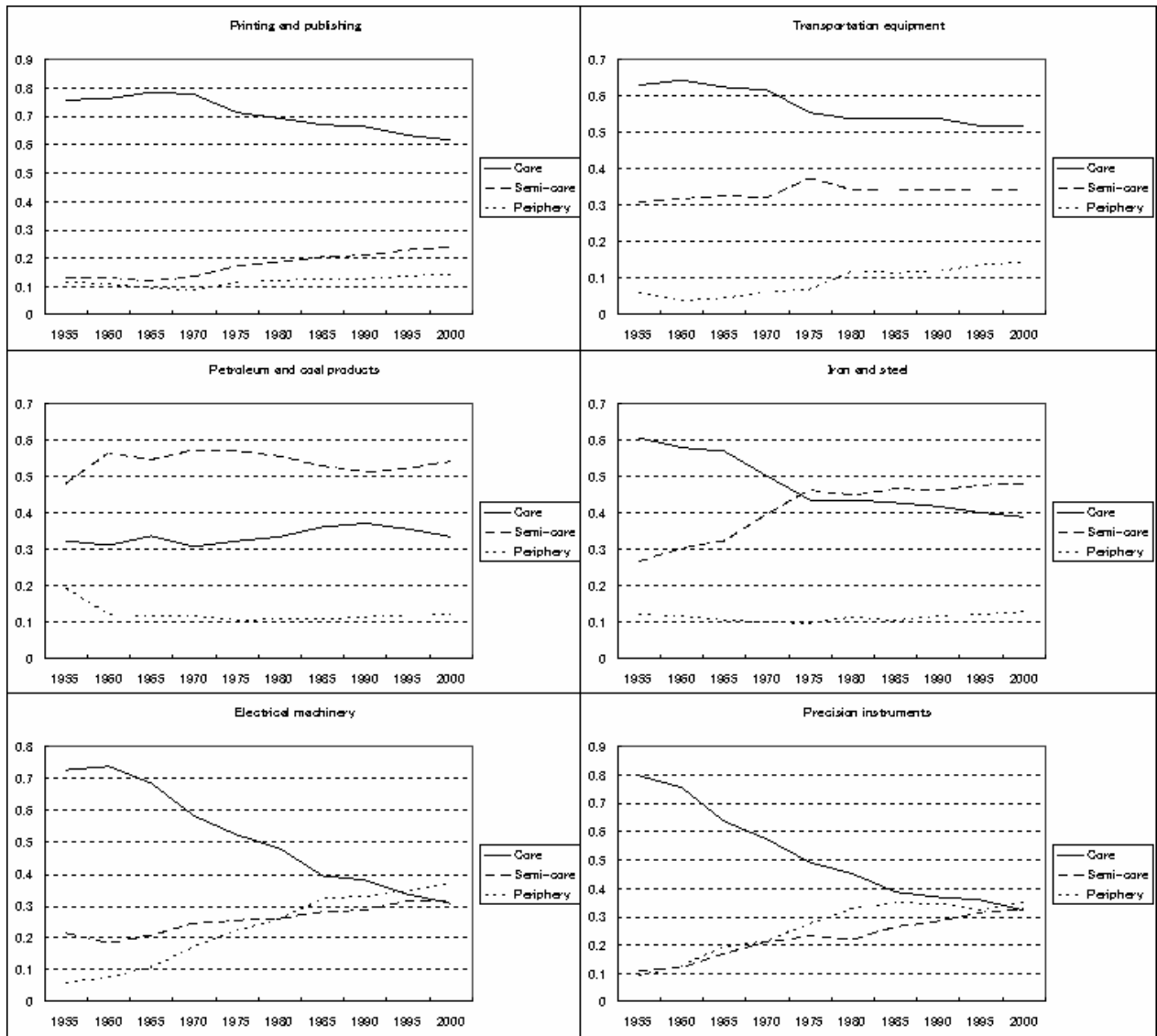


Figure 2: Changes in regional shares in value of products (Source: Japanese Census of Manufactures)

The differences in changes of industrial locations can be viewed as a consequence of regional policies aiming to invite new industries with high value-added, such as electrical and information technology (IT) related industries. Those industries are called ‘close-to-airport industries’ since their products are conveniently transported by airplanes, and some Japanese regional governments (e.g. Ishikawa Prefecture, Chitose City, Kita-kyushu City) are actually inviting such industries by improving facilities of local airports.

We do not deny the possibility that such regional policies in the periphery brought the asymmetric industrial location, i.e., regional specialization via differences in transport costs. In this paper, however, we show that the regional specialization occur even if regions are symmetric

without heterogeneous policies.

Our framework is based on and extends the new economic geography (NEG) model of Krugman (1991). The NEG model has successfully clarified the relation between transport costs and industrial location, however, regional specialization of individual industries has not been explained in the NEG literature yet because most authors assume that there is only one industry in the manufacturing sector for simplicity. To fill this theoretical gap, this paper establishes a NEG model with multiple industries, which are expected to clarify how different industries present different locations when the transportation system improves.

Our model is roughly sketched as follows. The industries are differentiated by their transport costs for their products. Similar to most models of NEG, we assume the consumers' love-for-variety and firm-level increasing returns as the agglomeration force (Krugman, 1991) and urban costs (i.e., housing and commuting costs) as the dispersion force (Tabuchi, 1998; Helpman, 1998). The distribution of industries is determined by the balance of these two forces. The results here show that, in a space with sufficiently developed transportation system, industries with lower transport costs may leave the core for the periphery, since their gains from reducing urban costs become larger than losses from increasing transport costs.

There are some authors who consider similar multi-industry location problems based on NEG. From the viewpoint of international economics, Puga and Venables (1996) and Krugman and Venables (1997) have examined the situation of multiple industries. These studies have succeeded to describe the international spread of industry under increasing in demand of manufacturing goods and industrial clustering under decreasing trade costs, respectively. However, workers are supposed to be immobile in their models, so their results are restricted to international situations. Fujita, Krugman and Mori (1999) first find multiple industries form a hierarchical urban system when the whole population increases. Their model assumes a continuous space and residents are mobile. Although the industries are differentiated by their transport costs, the costs are supposed to be constant there. Recently, Tabuchi and Thisse (2003), have investigated location patterns of industries with different transport costs and urban costs. However, their analysis has been limited to the case of only two industries, and we will find that some properties for multiple industries do not hold for the case of only two industries. Furthermore, they have focused on location changes with only decreasing commuting costs and have not analyzed the case with decreasing simultaneously both transport and commuting costs. A recent paper of Zeng (2003) has also described the regional specialization by a model of multiple industries. There are two main differences between his model and the model here. First, Zeng (2003) differentiates the industries by the numbers of unskilled workers necessary in their productions, and the industries are supposed to have the same transport costs in the framework. In contrast, industries are differentiated by their transport costs in our model. Second, the disper-

sion force of Zeng (2003) is the agricultural sector, while the dispersion force here is originated from urban costs and congestion. Both papers reveal the evolution process of a multi-industry manufacturing sector in a complimentary way.

Each industry requires some skilled workers in its production. This paper supposes that different industries require different kinds of skilled workers. In other words, interindustrial mobility of skilled workers is not allowed here. We have several reasons. First, modern industries indeed depend on peculiar and special technologies and workers choose jobs according to their educational experiences and abilities. Second, our main goal is to describe the regional specialization via differences in transport costs, and we will find that the basic results are true even if there are only three industries. Someone, who rejects our assumption by the fact that some brilliant workers change their jobs quite often, might accept the assumption when we only divide the skills roughly into three categories. Third, the model is analytically solvable by this assumption. In contrast, if skilled workers are mobile across industries, the model becomes intractable. (see Tabuchi and Thisse, 2003).

This paper has two main results. First, if population externality (i.e. congestion) does not exist, at most one industry disperses when transport costs and commuting costs are sufficiently low. Furthermore, industries having larger (resp. smaller) *adjusted transport costs* (defined as the ratio of transport costs to the size of the industry) than that of the dispersing industry agglomerate in one region (resp. the other region). The reason why transport costs are discounted by size of the industry is that demand is less elastic against prices for industries with more firms. Second, in the case of strong congestion, more than one industry might disperse even if transport costs are sufficiently low. However, as the degree of congestion decreases, partial regional specialization, in which multiple industries disperse, emerges first and then complete regional specialization occurs.

The remainder of this paper is organized as follows. In Section 2, we give the NEG model with multiple industries in the manufacturing sector. Then we examine all possible location patterns in the model without congestion in Section 3 and with congestion in Section 4. Section 5 concludes.

2 The model

The economy has two regions, called H and F , and three goods, i.e. an initially endowed homogeneous good which is chosen as the numéraire, a differentiated variety produced by firms in each industry under increasing returns technology, and land. We assume that there are K (≥ 2) types of industries and a continuum N_i of differentiated varieties supplied by industry i ($= 1, \dots, K$). It should be noted that there is also a continuum N_i of firms in industry i since there are no scope of economies due to increasing returns technology.

Firms use only labor for their production and there are K types of workers corresponding to the types of industries because a special training is necessary for working in a particular industry. In other words, for the reasons stated in Section 1, workers are immobile across industries. Therefore, the number (measure) of workers in industry i is fixed to L_i . On the other hand, we assume that workers are mobile between regions. In other words, workers relocate without any costs.

Land is not used by firms but by workers for their housing. Specifically, each region has the central business district (CBD) as a point, and all firms in the region locate there. The space is linearly distributed around the CBD and each location has one unit amount of land. Each worker consumes one unit amount of land for residing and commutes to the CBD. In addition, we assume that the commuting cost per unit of distance is θ units of numéraire, the opportunity cost of land is normalized to zero, and the total land rent of one region is evenly distributed to all residence in the region. Under these assumptions, net urban cost (i.e., land rent payment+commuting cost–land rent revenue) per worker is given by $(\theta/4) \times (\text{population in the region})$, which is independent of workers' locations.

Generalizing the framework of Tabuchi and Thisse (2003) from two types of industries to K types, workers are assumed to hold the same preference, which are described by a quasi-linear utility with quadratic subutility:

$$U(q_0, q_i(j), j \in [0, N_i], i = 1, \dots, K) = \sum_{i=1}^K \left[\alpha \int_0^{N_i} q_i(j) dj - \frac{(\beta - \gamma)N_i}{2 \sum_{k=1}^K N_k} \int_0^{N_i} [q_i(j)]^2 dj - \frac{\gamma}{2 \sum_{k=1}^K N_k} \left(\int_0^{N_i} q_i(j) dj \right)^2 \right] + q_0, \quad (1)$$

where q_0 stands for the consumption of the homogeneous good and $q_i(j)$ is the consumption of variety $j \in [0, N_i]$ in industry i . We assume that $\alpha > 0$, $\beta > 0$, and $\beta > \gamma$, which means that this utility function represents workers' love for variety. We should note that the second term (which represents the degree of love for variety) in the squared parenthesis is weighted by the relative size of its industry ($N_i / \sum_{k=1}^K N_k$). This means that an industry with more firms (varieties) has a greater impact on workers' utility than an industry with fewer firms.³ This utility function also generalizes the one proposed by Ottaviano *et al.* (2002) for one industry only.

Each worker in region r ($= H, F$) maximizes (1) under budget constraint

$$\sum_{i=1}^K \left[\int_0^{N_i} p_{ir}(j) q_{ir}(j) dj + \frac{\theta}{4} L_{ir} \right] + q_0 = w_{ir} + \bar{q}_0,$$

where $p_{ir}(j)$ and $q_{ir}(j)$ are the price and the consumption amount of variety j in industry i for workers in region r , respectively, and where L_{ir} and w_{ir} are the number (measure) of workers

³This weighting also significantly simplifies our mathematical analysis later.

and the wage of workers in industry i and region r , respectively. Finally, \bar{q}_0 is the quantity of the initially endowed homogeneous good.

Workers' utility maximization gives the following individual demand function $q_{ir}(j)$ and indirect utility function V_{ir} :

$$q_{ir}(j) = \frac{\sum_{k=1}^K N_k}{N_i} \left[a - bp_{ir}(j) + c \frac{P_i}{N_i} \right], \quad (2)$$

$$V_{ir} = \left(\sum_{k=1}^K N_k \right) \left[\frac{Ka^2}{2(b-c)} - a \sum_{k=1}^K \frac{\int_0^{N_k} p_{kr}(j) dj}{N_k} + \frac{b}{2} \sum_{k=1}^K \frac{\int_0^{N_k} \{p_{kr}(j)\}^2 dj}{N_k} - \frac{c}{2} \sum_{k=1}^K \frac{\left\{ \int_0^{N_k} p_{kr}(j) dj \right\}^2}{N_k^2} \right] + w_{ir} + \bar{q}_0 - \frac{\theta}{4} \sum_{i=k}^K L_{kr},$$

where $a \equiv \alpha/\beta$, $b \equiv 1/(\beta - \gamma)$, $c \equiv \gamma/[\beta(\beta - \gamma)]$, and $P_i \equiv \int_0^{N_i} p_{ir}(k) dk$, which is the price index of industry i in region r . Since $\beta > \gamma$, we have $2b > b > c$.

The demand function is linear with respect to prices. We should note that, however, coefficients of prices are different between industries. Therefore, the demand is less elastic against price for an industry with more firms (varieties). This is due to the specification of (1), where the second term (the degree of love for variety) is weighted by the relative size of the industry.

Each firm produces a differentiated variety in a monopolistically competitive way. We assume that each firm is negligible so its pricing has not influence on the price index P in (2). It is also assumed that all industries could produce any amount of their varieties by using one unit of worker, thus we obtain $N_i = L_i$. Interregional transport costs of varieties are different between industries and the transport cost of one unit of a variety in industry i is denoted by τ_i . Under these assumptions, all firms in the same industry and the same region are symmetric and a typical firm in industry i and region r maximizes the following profit:

$$\Pi_{ir} = p_{irr} q_{irr} (p_{irr}) \sum_{i=1}^K L_{ir} + (p_{irs} - \tau_i) q_{irs} (p_{irs}) \sum_{i=1}^K L_{is} - w_{ir},$$

where q_{irs} and p_{irs} are the individual demand and the price in region s for firms of industry i and located in region r .

The FOC of the profit maximization and the assumption of free entry give the following equilibrium price and wage:

$$p_{irr}^* = \frac{2a + c\tau_i(L_{is}/L_i)}{2(2b-c)}, \quad p_{irs}^* = p_{iss}^* + \frac{\tau_i}{2},$$

$$w_{ir}^* = \frac{b \sum_{k=1}^K L_k}{L_i} \left[(p_{irr}^*)^2 \sum_{k=1}^K L_{kr} + (p_{irs}^* - \tau_i)^2 \sum_{k=1}^K L_{ks} \right].$$

From these equations, we obtain the utility differential between two regions for workers of industry i , $V_{iH} - V_{iF}$:

$$V_{iH} - V_{iF} = (S_H - S_F) + (w_{iH} - w_{iF}),$$

$$S_H - S_F = \sum_{i=1}^K \left(\frac{1}{2} - \lambda_i \right) \left[\bar{L} \frac{b^2(b-c)\tau_i^2 - 2ab^2\tau_i}{(2b-c)^2} + \frac{\theta}{2} L_i \right],$$

$$w_{iH} - w_{iF} = b\bar{L} \left[\left(\frac{1}{2} - \lambda_i \right) \frac{\{b + \frac{c}{2}(\frac{\bar{L}}{L_i} - 2)\}\tau_i^2 - 2a\tau_i}{(2b-c)} + \frac{(b-c)\tau_i^2 - 2a\tau_i}{(2b-c)L_i} \sum_{j \neq i} L_j \left(\frac{1}{2} - \lambda_j \right) \right].$$

where S_r is the consumer's surplus in region r , λ_i is the share of workers of industry i residing in H , and \bar{L} is the total number of workers, $\sum_{k=1}^K L_k$.

In addition, it is assumed that $\tau_i = \omega_i\tau$, $\theta = \rho\tau$ for simplicity. This means that transport costs and the commuting cost decrease proportionally when τ falls. The assumption is restrictive but it enables us to describe the progress of transportation technology by only one parameter (τ). Under this assumption, the utility differential $V_{iH} - V_{iF}$ is rewritten as

$$\begin{aligned} V_{iH} - V_{iF} &= \left(\frac{1}{2} - \lambda_i \right) \left[\omega_i^2 \nu_{1i} - \omega_i \nu_2 + \frac{\rho}{2} L_i \tau \right] \\ &\quad + \sum_{j \neq i} \left(\frac{1}{2} - \lambda_j \right) \left[\omega_j^2 \mu_1 - \omega_j \mu_2 + \omega_i^2 \xi_1^{ij} - \omega_i \xi_2^{ij} + \frac{\rho}{2} L_j \tau \right] \\ &= \sum_j \left(\frac{1}{2} - \lambda_j \right) \delta_{ij}, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \nu_{1i} &\equiv \frac{b\bar{L}\tau^2}{2(2b-c)^2} \left[6b^2 + 2 \left(\frac{\bar{L}}{L_i} - 4 \right) bc - \left(\frac{\bar{L}}{L_i} - 2 \right) c^2 \right], \nu_2 \equiv \frac{2ab(3b-c)}{(2b-c)^2} \bar{L}\tau, \\ \mu_1 &\equiv \frac{b^2(b-c)}{(2b-c)^2} \bar{L}\tau^2, \mu_2 \equiv \frac{2ab^2}{(2b-c)^2} \bar{L}\tau, \xi_1^{ij} \equiv \xi_1 \frac{L_j}{L_i}, \xi_2^{ij} \equiv \xi_2 \frac{L_j}{L_i}, \\ \xi_1 &\equiv \frac{b(b-c)}{2b-c} \bar{L}\tau^2, \xi_2 \equiv \frac{2ab}{2b-c} \bar{L}\tau, \\ \delta_{ij} &\equiv \begin{cases} \omega_i^2 \nu_{1i} - \omega_i \nu_2 + \frac{\rho}{2} L_i \tau, & \text{if } i = j, \\ \omega_j^2 \mu_1 - \omega_j \mu_2 + \omega_i^2 \xi_1^{ij} - \omega_i \xi_2^{ij} + \frac{\rho}{2} L_j \tau, & \text{if } i \neq j. \end{cases} \end{aligned}$$

Finally, we employ the following dynamic system to describe migration behavior between regions.

$$\frac{d\lambda_i}{dt} = V_{iH}(\boldsymbol{\lambda}) - V_{iF}(\boldsymbol{\lambda}) = \sum_j \left(\frac{1}{2} - \lambda_j \right) \delta_{ij}.$$

3 Regional specialization by the progress of transportation technology

Now, we investigate how the various industries relocate when the transportation technology develops which reduces the transport costs of products and the commuting cost. It will be shown below that the location of industry i depends on the ratio of transport cost to the size of the industry ($= \omega_i/L_i$), which will be called *adjusted transport cost*. For convenience, we give two technical assumptions here.

- Adjusted transport costs are different between any two industries. Namely, $\omega_i/L_i \neq \omega_j/L_j$ for any different i and j .
- The workers cannot be divided into two groups of the same population such that every industry agglomerates. Namely, for any $i_1, \dots, i_l, \dots, i_K \in \{1, \dots, l, \dots, K\}$,

$$\sum_{j=1}^l L_{i_j} \neq \sum_{j=l+1}^K L_{i_j}. \quad (4)$$

Furthermore, without loss of generality, we name the industries such that

$$\frac{\omega_1}{L_1} > \frac{\omega_2}{L_2} > \dots > \frac{\omega_K}{L_K}. \quad (5)$$

Since there are multiple types of industries, someone may worry about that there may exist many kinds of industrial distributions. However, the following proposition tells us that only a few stable location patterns are possible if the transport costs and the commuting cost are low enough.

Proposition 1 *At most one industry disperses if τ is small enough.*

Proof. We show that any distribution pattern in which $k(\geq 2)$ types of industries disperse is unstable for sufficiently small τ . Contrarily, assume a stable equilibrium in which k types of industries disperse and number them by $1, \dots, k$. For these industries, define $\Delta \equiv (\delta_{ij})_{k \times k}$. Then all the real parts of the eigen values of $(-\Delta)$ are negative, and hence, $|\Delta| > 0$.

Let

$$\Upsilon \equiv (v_{ij})_{k \times k}, \text{ where } v_{ij} = \begin{cases} \omega_i^2 \nu_{1i} - \omega_i \mu_2 - \omega_i \xi_2 & \text{if } i = j, \\ \omega_j^2 \mu_1 + \omega_i^2 \xi_1^{ij} - \omega_j \mu_2 - \omega_i \xi_2^{ij} & \text{if } i \neq j, \end{cases}$$

$$\Theta_i \equiv \frac{\rho\tau}{2} L_i \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}_{k \times 1},$$

and $(\Theta_i, \Upsilon^{-i})$ be the matrix Υ where the i th column is replaced by Θ_i . Since $\nu_2 = \mu_2 + \xi_2$, it follows that

$$|\Delta| = |\Upsilon| + \sum_{i=1}^k |(\Theta_i, \Upsilon^{-i})|.$$

The first term is $2k$ -order polynomial of τ and the second one is $(2k-1)$ -order polynomial of τ . We can show that terms with orders below $(2k-2)$ in both polynomials are all zero (see Part (1) of Appendix A). Therefore, the terms of order $(2k-2)$ in the polynomials are significant if τ is small enough. We can also show that the $(2k-2)$ -order term in $|\Upsilon|$ is negative while the $(2k-2)$ -order term in $\sum_{i=1}^k |(\Theta_i, \Upsilon^{-i})|$ is zero (see Part (2) of Appendix A). Therefore, when τ

is small enough, it holds that $|\Delta| < 0$. The contradiction implies that plural types of industries could not disperse simultaneously. ■

This proposition tells us that if τ is small enough, the following two patterns of location are the only possibly stable equilibria:

- (a) Each industry agglomerates in a region;
- (b) Only one industry disperses while other industries agglomerate.

In our model, two regions are supposed to be symmetric. To clarify the location patterns, we focus on the equilibria in which the population in region H is larger than or equal to the population in region F . For $i, l = 1, \dots, K$, let

$$\rho^\dagger(i, l) \equiv \frac{4ab\bar{L}}{2b-c} \left[\frac{\omega_i}{L_i} + \frac{b}{2b-c} \frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j} \right]. \quad (6)$$

Those notations have the following property:

Lemma 1 (i) Function $\rho^\dagger(i, l)$ is strictly decreasing with respect to i . Namely, $\rho^\dagger(i, l) > \rho^\dagger(i+1, l)$ holds for $i = 1, \dots, K-1$.

(ii) If $\sum_{j=1}^{l-1} L_j > \sum_{j=l}^K L_j$, then $\rho^\dagger(l, l) < \rho^\dagger(l, l-1)$.

(iii) If $\sum_{j=1}^l L_j > \sum_{j=l+1}^K L_j$, then $\rho^\dagger(i, l)$ is positive for any i .

Proof. (i) follows directly from (5).

(ii) Since $\sum_{j=1}^{l-1} L_j > \sum_{j=l}^K L_j$, we have

$$\sum_{j=1}^{l-1} \omega_j - \sum_{j=l}^K \omega_j > \frac{\omega_l}{L_l} \left(\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j \right) > 0,$$

where the first inequality is from (5). The first inequality can be rewritten as

$$\omega_l \left(\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j \right) < L_l \left(\sum_{j=1}^{l-1} \omega_j - \sum_{j=l}^K \omega_j \right),$$

and hence

$$\frac{\sum_{j=1}^{l-1} \omega_j - \sum_{j=l}^K \omega_j}{\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j} < \frac{\sum_{j=1}^{l-1} \omega_j - \sum_{j=l}^K \omega_j + 2\omega_l}{\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j + 2L_l} = \frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j},$$

which derives $\rho^\dagger(l, l) < \rho^\dagger(l, l-1)$.

(iii) Since $\sum_{j=1}^l L_j > \sum_{j=l+1}^K L_j$, then

$$0 < \omega_l \left(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j \right) < L_l \left(\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j \right),$$

so that the last term of (6) is positive. Therefore, $\rho^\dagger(i, l) > 0$ holds for any i . ■

The following lemma provides the stability conditions of patterns (a) and (b), where we define $\rho^\dagger(K+1, K) \equiv 0$ for convenience:

Lemma 2 (i) *In any stable equilibrium of pattern (a), there is an industry l such that industries $1, \dots, l$ agglomerate in the larger region, industries $l+1, \dots, K$ agglomerate in the smaller region, and $\rho^\dagger(l, l) \geq \rho \geq \rho^\dagger(l+1, l)$.*

(ii) *If there is an industry l with $\sum_{j=1}^l L_j > \sum_{j=l+1}^K L_j$ and $\rho^\dagger(l, l) > \rho > \rho^\dagger(l+1, l)$, then the equilibrium, in which industries $1, \dots, l$ agglomerate in region H and industries $l+1, \dots, K$ agglomerate in region F , is stable.*

Proof. See Appendix B. ■

Lemma 3 *Let λ^* be an equilibrium of pattern (b), where industry l disperses while other industries agglomerate. Then this equilibrium is stable if and only if*

(i) *either $\lambda^* = (1, \dots, 1, \lambda_l^*, 0, \dots, 0)$ or $\lambda^* = (0, \dots, 0, \lambda_l^*, 1, \dots, 1)$,*

(ii) $\sum_{j=1}^l L_j > \sum_{j=l+1}^K L_j$.

(iii) $\rho \in (\rho^\dagger(l, l), \rho^\dagger(l, l-1))$ if $\sum_{j=1}^{l-1} L_j > \sum_{j=l}^K L_j$,

(iv) $\rho > \rho^\dagger(l, l)$ if $\sum_{j=1}^{l-1} L_j < \sum_{j=l}^K L_j$.

Proof. See Appendix C. ■

These lemmas have the following implication. First, for a sufficiently small τ , industries with larger adjusted transport costs (ω_i/L_i) locate separately from those with smaller adjusted transport costs. In other words, regional specialization based on adjusted transport costs occurs. Second, industries with larger adjusted transport costs locate in the region with larger population. Third, the dispersing industry l is determined by the relative size of the commuting cost to transport costs, ρ .

Let l_\sharp be the industry such that

$$\sum_{j=1}^{l_\sharp} L_j > \sum_{j=l_\sharp+1}^K L_j, \quad \sum_{j=1}^{l_\sharp-1} L_j < \sum_{j=l_\sharp}^K L_j.$$

Then we obtain the next proposition from proposition 1, Lemmas 1, 2 and 3.

Proposition 2 (Regional specialization) *If τ is small enough, the location pattern λ^* is uniquely determined. If ρ is also small enough, all industries agglomerate in a region; otherwise industries with smaller adjusted transport costs locate in the smaller region. Specifically,*

- if $\rho < \rho^\dagger(K, K)$, then $\lambda^* = (1, \dots, 1)$,
- if $\rho^\dagger(K, K) < \rho < \rho(K, K - 1)$, then $\lambda^* = (1, \dots, 1, \lambda_K^*)$ [$0 < \lambda_K^* < 1$],
- if $\rho(K, K - 1) < \rho < \rho(K - 1, K - 1)$, then $\lambda^* = (1, \dots, 1, 0)$,
- if $\rho(K - 1, K - 1) < \rho < \rho(K - 1, K - 2)$, then $\lambda^* = (1, \dots, 1, \lambda_{K-1}^*, 0)$ [$0 < \lambda_{K-1}^* < 1$],
- \vdots
- if $\rho^\dagger(l_\#^*, l_\#^*) < \rho$, then $\lambda^* = (1, \dots, 1, \lambda_{l_\#^*}^*, 0, \dots, 0)$ [$0 < \lambda_{l_\#^*}^* < 1$].

It is intuitively clear that ρ is significant for the location patterns since the urban cost based on the commuting cost is the only dispersion force in our model. In contrast, it is not so intuitive why the adjusted transport cost, ω_i/L_i , is so important in determining the industrial location. To understand it, we first note that the progress of the transportation technology might enable industries with lower transport costs to leave the bigger region. This is because, for such industries, the gain from reducing the urban cost by moving to the smaller region might be larger than the loss from increasing transport costs by leaving the big market. However, why the transport cost should be adjusted? Remember that when a firm relocates from the bigger region to the smaller one, the consumer price of its product in the big market increases and its demand decreases. The demand function (2) shows that the decreases by rising prices are different between industries, i.e., demand is less elastic against prices for industries with more firms (varieties). In other words, even if the consumer price of a variety rises because of the relocation of its producer, the demand does not decrease so much if the industry contains many firms (varieties). Thus finally, the adjusted transport cost is more proper than the transport cost itself for determining the location pattern.

Until now, we have focused our attention on the case of small τ . The cases of intermediate and large τ are also important but there are too many possible equilibria. Therefore, we only provide a conclusion about the stability of the full agglomeration pattern, $\lambda^* = (1, \dots, 1)$.

When $\lambda^* = (1, \dots, 1)$, from (3), the utility differential $V_{iH} - V_{iF}$ is rewritten as

$$V_{iH} - V_{iF} = - \left[A_i \tau^2 + \left(B_i - \frac{\rho}{4} \right) \tau \right] \bar{L}, \quad (7)$$

where

$$A_i \equiv \frac{b}{4(2b-c)^2} \left[\left(\frac{4\bar{L}}{L_i} \omega_i^2 + 2 \sum_{j=1}^K \omega_j^2 \right) b(b-c) + \frac{\bar{L}}{L_i} c^2 \right] > 0,$$

$$B_i \equiv \frac{ab}{(2b-c)^2} \left[(2b-c) \frac{\bar{L}}{L_i} \omega_i + b \sum_{j=1}^K \omega_j \right] > 0.$$

We have $\min_i B_i = B_K = \rho^\dagger(K, K)/4$ from (5). Therefore, we conclude that (i) if $B_K < \rho/4$ (or equivalently, $\rho > \rho^\dagger(K, K)$), then the full agglomeration is never stable, (ii) if $B_K > \rho/4$ (or equivalently, $\rho < \rho^\dagger(K, K)$), then the full agglomeration is stable if τ is not too large.⁴

From the above conclusions, we can not find an evolution pattern ‘from full agglomeration to regional specialization.’ Such a pattern, however, might occur if we consider congestion as we will do in next section.

4 Congestion case

Tabuchi (1998) and Helpman (1998) found that when transport costs of manufacturing goods decrease, the industrial location shifts from agglomeration to dispersion since the urban costs (e.g. housing and commuting costs) become relatively large. By Proposition 2, however, the location pattern does not necessarily shift to dispersion. The difference between these results is due to the different assumptions of urban cost. Our model assumes that the progress of the transportation technology decreases the commuting cost and the urban cost decrease proportionally with transport costs, while Tabuchi (1998) and Helpman (1998) do not assume that the technology development reduces the urban cost.

More specifically, our model assumes the Alonso type city whose residential area changes with population size. However, in the framework of Helpman (1998), the residential area is limited, so that the land per worker is smaller in the region with more population, and the loss from agglomeration is not reduced by the technology progress. Second, our model in the preceding sections does not consider population externality such as congestion and environmental pollution. Those factors are not decreased by the progress of the transportation technology.

To make our analysis more complete, this section introduces population externality into the previous model and investigate the location pattern again. For simplicity, we further specify that $K = 3$, $L_i = L$ ($i = 1, 2, 3$), and $\omega_1 > \omega_2 > \omega_3$. Concerning the population externality, it is assumed that workers’ expenditure inevitably increases with increasing population. Specifically, while the urban cost was given by $(\rho\tau/4) \times (\text{population in the region})$ previously, it is given by $[(\rho\tau + \epsilon)/4] \times (\text{population in the region})$ in this section, where $\epsilon > 0$. Therefore, even if τ is small, the urban cost does not disappear and hence, the dispersion force works significantly. This implies that the population in two regions are almost equalized if τ is small enough. Thus, it becomes impossible for more than two industries to agglomerate in a single region (e.g., location

⁴Let $B_{i^*}/A_{i^*} \equiv \min B_i/A_i$ and $\tau_{trade} \equiv \min 2a/\{\omega_i(2b-c)\}$. For trade to occur in all industries regardless of the location patterns, it should hold that $\tau < \tau_{trade}$, which is assumed to be true here. If $\rho < 4(B_{i^*} - A_{i^*}\tau_{trade})$, then the full agglomeration is always stable. If $4(B_{i^*} - A_{i^*}\tau_{trade}) < \rho < \rho^\dagger(K, K)$, then the full agglomeration is stable when $0 < \tau < (B_{i^*} - \rho/4)/A_{i^*}$.

patterns $\lambda^* = (1, 1, 1)$, $(1, 1, \lambda_3^*)$, and $(1, 1, 0)$ are impossible). In other words, the following three location patterns are the only possible distributions, where $0 < \lambda_1^* < 1$, $0 < \lambda_2^* < 1$, $0 < \lambda_3^* < 1$ if τ is small enough.⁵:

- (A) *full dispersion*: all industries disperse ($\lambda^* = (\lambda_1^*, \lambda_2^*, \lambda_3^*)$);
- (B) *complete regional specialization*: one industry disperses while the others agglomerate in different regions, ($\lambda^* = (\lambda_1^*, 0, 1)$, $(1, \lambda_2^*, 0)$, $(0, 1, \lambda_3^*)$);
- (C) *partial regional specialization*: two industries disperse and the remaining one agglomerates in a region ($\lambda^* = (1, \lambda_2^*, \lambda_3^*)$, $(\lambda_1^*, 1, \lambda_3^*)$, $(\lambda_1^*, \lambda_2^*, 0)$).

The following lemmas give us the stability conditions of patterns (A) and (B), where

$$\begin{aligned}\bar{\epsilon}_0 &\equiv \frac{16a^2b^2}{c(2b-c)^2} \frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2}, \\ \bar{\epsilon}_1 &\equiv \frac{16a^2b^2}{c(2b-c)^2} \frac{(\omega_1 - \omega_2)(\omega_1 - \omega_3)}{\omega_1^2}, \\ \bar{\epsilon}_2 &\equiv \frac{16a^2b^2}{c(2b-c)^2} \frac{(\omega_1 - \omega_3)(\omega_2 - \omega_3)}{\omega_3^2}, \\ \bar{\omega}_2 &\equiv \frac{\omega_3^2}{\omega_1^2 + \omega_3^2}\omega_1 + \frac{\omega_1^2}{\omega_1^2 + \omega_3^2}\omega_3.\end{aligned}$$

It holds that $\bar{\epsilon}_0 > \min\{\bar{\epsilon}_1, \bar{\epsilon}_2\}$.⁶

Lemma 4 *If τ is small enough, $\lambda^* = (1/2, 1/2, 1/2)$ is a unique and stable equilibrium if and only if $\epsilon > \bar{\epsilon}_0$. This is the only possible equilibrium in location pattern (A).*

Proof. See Appendix D. ■

Lemma 5 *If τ is small enough, $\lambda^* = (1, \lambda_2^*, 0)$ is a unique and stable equilibrium if and only if $\epsilon < \min\{\bar{\epsilon}_1, \bar{\epsilon}_2\}$. This is the only possible equilibrium in location pattern (B).*

Proof. See Appendix E. ■

These lemmas tell us that if τ is small enough, the location pattern is uniquely determined if $\epsilon > \bar{\epsilon}_0$ or $\epsilon < \min\{\bar{\epsilon}_1, \bar{\epsilon}_2\}$. When $\epsilon \in (\min\{\bar{\epsilon}_1, \bar{\epsilon}_2\}, \bar{\epsilon}_0)$, a location pattern of (C) occurs, which is also uniquely determined. Specifically, we have the following proposition:

Proposition 3 *(Regional specialization with congestion) For sufficiently small τ , the location pattern λ^* depends on the degree of externality ϵ . All industries disperse evenly for large ϵ , industry 2 disperses while industries 1 and 3 agglomerate in different regions for small ϵ , two industries disperse and the 3rd one agglomerate for intermediate ϵ . Specifically,*

⁵By the symmetry of regions, symmetry patterns like $(\lambda_1^*, 0, 1)$ and $(\lambda_1^*, 1, 0)$ are considered to be the same.

⁶See Appendix F.

- if $\epsilon > \bar{\epsilon}_0$, then $\lambda^* = (1/2, 1/2, 1/2)$.
- (i) When $\omega_2 > \bar{\omega}_2$, we have $\bar{\epsilon}_1 < \bar{\epsilon}_0 < \bar{\epsilon}_2$ and,
 - if $\bar{\epsilon}_1 < \epsilon < \bar{\epsilon}_0$, then $\lambda^* = (\lambda_1^*, \lambda_2^*, 0)$,
 - if $\epsilon < \bar{\epsilon}_1$, then $\lambda^* = (1, \lambda_2^*, 0)$.
- (ii) When $\omega_2 < \bar{\omega}_2$, we have $\bar{\epsilon}_2 < \bar{\epsilon}_0 < \bar{\epsilon}_1$ and,
 - if $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_0$, then $\lambda^* = (1, \lambda_2^*, \lambda_3^*)$,
 - if $\epsilon < \bar{\epsilon}_2$, then $\lambda^* = (1, \lambda_2^*, 0)$.

Proof. See Appendix F. ■

If ϵ is large enough, all industries disperse evenly, but as ϵ decreases, partial regional specialization first emerges and then complete regional specialization occurs. Specifically, (i) when ω_2 is relatively large and close to ω_1 , industry 3 first agglomerates in a region and then industry 1 agglomerates in the other region. On the other hand, (ii) when ω_2 is relatively small and close to ω_3 , industry 1 first agglomerates in a region and then industry 3 agglomerates in the other region. In the both cases, when ϵ is small enough, industry 1 and industry 3 agglomerate in different regions.

The reason why patterns of regional specialization emerge as ϵ decreases is understandable. If ϵ is small, some industries enjoy more gains from market access than suffering the losses from congestion when they locate in the larger region. Such industries are the ones with higher transport costs and they form the larger region. For example, we know that $\lim_{\tau \rightarrow 0} \lambda_2^* > \lim_{\tau \rightarrow 0} \lambda_3^*$ and $\lambda_2^* + \lambda_3^*$ converges to 1/2 from above in equilibrium $(1, \lambda_2^*, \lambda_3^*)$ [see the end of Appendix F.], and that λ_2^* converges to 1/2 from above in equilibrium $(1, \lambda_2^*, 0)$ [see Appendix E.].

The main result of Zeng (2003) and Proposition 1 show that at most one industry disperses if τ is small enough. In contrast, Proposition 3 shows that the result is true only if ϵ is small enough.

Finally, we will consider the case that τ is middle or large, but we will show only a few things about the stability of the full agglomeration pattern, as the previous section.

When $\lambda^* = (1, 1, 1)$, the utility differential of industry i , $V_{iH} - V_{iF}$, is rewritten as

$$V_{iH} - V_{iF} = - \left[A_i \tau^2 + \left(B_i - \frac{\rho}{4} \right) \tau - \frac{\epsilon}{4} \right] \bar{L},$$

where A_i and B_i are the same parameters in (7). Therefore, we conclude that (i) if $\rho > \rho^\dagger(K, K)$, then the full agglomeration is never stable, (ii) if $\rho < \rho^\dagger(K, K)$ and ϵ is small enough, then there exists an interval of τ where the full agglomeration is stable. In other words, with Proposition 3, there exist patterns of ‘from full agglomeration to regional specialization.’

5 Concluding remarks

The regional specialization via differences in transport costs has been observed in Japanese manufacturing industries. Concretely, industries with low transport costs for their goods (e.g. electrical machinery, precision instruments) have relocated to periphery regions while industries with high transport costs for their goods (e.g. iron and steel, petroleum and coal products) remained close to the core. This paper provides a theoretical foundation for this fact, by investigating where various industries tend to locate when the transportation technology develops. To this aim, we have analyzed the location of industries which are differentiated by transport costs and size (number of firms) by use of an analytically solvable model of new economic geography. The urban costs and congestion are explicitly included in the model. The obtained results are consistent with the observed empirical phenomenon. Furthermore, our results support the regional policy of some Japanese local governments, which aims to invite ‘close-to-airport’ industries by improving the facilities in their local airports.

Typical industries with low transport costs are software and information processing. These industries also contain many firms (varieties) so their adjusted transport costs are also low. Our theoretical results predict that they should locate in periphery region. However, until now, we have not observed such a location pattern in Japan. According to the Survey on Service Industries (2000), the revenue share of both Software industry and Information processing & providing industry in the Core are nearly 80%. We guess that this is because these industries strongly require inter-firm communications and market information. Communication and information externality are strong in large cities, which are not included in our model.

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Appendix A. Calculation of $|\Delta|$

(1) Terms of orders below $(2k - 2)$

First, we will consider $|\Upsilon|$. The elements of Υ are composed of the τ^2 -terms $(\nu_{1i}, \mu_1, \xi_1^{ij})$ and τ -terms (μ_2, ξ_2^{ij}) . If $|\Upsilon|$ is decomposed to sum of determinants in which each column is composed of only τ^2 -terms or only τ -terms, then the terms of the order which is below $(2k - 2)$ in $|\Upsilon|$ could be represented as sum of determinants with at least three columns which are composed

of only τ -terms. One of the determinants, for example, is

$$\begin{aligned}
& \begin{vmatrix} -\omega_1\mu_2 - \omega_1 \frac{L_1}{L_1} \xi_2 & -\omega_2\mu_2 - \omega_1 \frac{L_2}{L_1} \xi_2 & -\omega_3\mu_2 - \omega_1 \frac{L_3}{L_1} \xi_2 & v_{14} & \cdots & v_{1k} \\ -\omega_1\mu_2 - \omega_2 \frac{L_1}{L_2} \xi_2 & -\omega_2\mu_2 - \omega_2 \frac{L_2}{L_2} \xi_2 & -\omega_3\mu_2 - \omega_2 \frac{L_3}{L_2} \xi_2 & v_{24} & \cdots & v_{2k} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\omega_1\mu_2 - \omega_k \frac{L_1}{L_k} \xi_2 & -\omega_2\mu_2 - \omega_k \frac{L_2}{L_k} \xi_2 & -\omega_3\mu_2 - \omega_k \frac{L_3}{L_k} \xi_2 & v_{k4} & \cdots & v_{kk} \end{vmatrix} \\
&= \begin{vmatrix} -\omega_1\mu_2 - \omega_1 \frac{L_1}{L_1} \xi_2 & -\omega_2\mu_2 - \omega_1 \frac{L_2}{L_1} \xi_2 & -\omega_3\mu_2 - \omega_1 \frac{L_3}{L_1} \xi_2 & \cdots \\ L_1(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & L_2(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & L_3(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ L_1(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & L_2(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & L_3(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & \cdots \end{vmatrix} \\
&= \begin{vmatrix} -\omega_1\mu_2 - \omega_1 \frac{L_1}{L_1} \xi_2 & (\frac{L_2}{L_1}\omega_1 - \omega_2)\mu_2 & (\frac{L_3}{L_1}\omega_1 - \omega_3)\mu_2 & \cdots \\ L_1(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ L_1(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & 0 & 0 & \cdots \end{vmatrix},
\end{aligned}$$

so we can show that it is zero. Of course, this result holds even if the columns with τ -terms are in different places.

Next, we will consider $\sum_{i=1}^k |(\Theta_i, \Upsilon^{-i})|$. If $|(\Theta_i, \Upsilon^{-i})|$ is decomposed to sum of determinants like above, then the term of the order which is below $(2k-2)$ in $|(\Theta_i, \Upsilon^{-i})|$ could be represented as sum of determinants with at least two columns which are composed of only τ -terms except for i th column. One of the determinant, for example, is

$$\begin{aligned}
& \begin{vmatrix} \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_1 \frac{L_2}{L_1} \xi_2 & -\omega_3\mu_2 - \omega_1 \frac{L_3}{L_1} \xi_2 & v_{14} & \cdots & v_{1k} \\ \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_2 \frac{L_2}{L_2} \xi_2 & -\omega_3\mu_2 - \omega_2 \frac{L_3}{L_2} \xi_2 & v_{24} & \cdots & v_{2k} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_k \frac{L_2}{L_k} \xi_2 & -\omega_3\mu_2 - \omega_k \frac{L_3}{L_k} \xi_2 & v_{k4} & \cdots & v_{kk} \end{vmatrix} \\
&= \begin{vmatrix} \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_1 \frac{L_2}{L_1} \xi_2 & -\omega_3\mu_2 - \omega_1 \frac{L_3}{L_1} \xi_2 & \cdots \\ 0 & L_2(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & L_3(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & L_2(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & L_3(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & \cdots \end{vmatrix} \\
&= \frac{\rho\tau L_1}{2} \begin{vmatrix} L_2(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & L_3(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & \cdots \\ \vdots & \vdots & \vdots \\ L_2(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & L_3(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & \cdots \end{vmatrix} \\
&= \frac{\rho\tau L_1}{2} \begin{vmatrix} L_2(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & 0 & \cdots \\ \vdots & \vdots & \vdots \\ L_2(\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & 0 & \cdots \end{vmatrix},
\end{aligned}$$

so we can show that it is also zero. Of course, this result holds even if the columns with τ -terms and Θ_i are in different places.

Therefore, we have shown that the terms of orders below $(2k-2)$ in $|\Delta|$ are all zero. \blacksquare

(2) The $(2k-2)$ -order term

First, we will consider the $\tau^{(2k-2)}$ -term in $|\Upsilon|$. This term could be represented as sum of determinants which has only two columns with τ -terms by the above decomposition, and one of the determinant, for example, is calculated as

$$\begin{aligned}
& \begin{vmatrix} -\omega_1\mu_2 - \omega_1 \frac{L_1}{L_1} \xi_2 & -\omega_2\mu_2 - \omega_1 \frac{L_2}{L_1} \xi_2 & \omega_3^2\mu_1 + \omega_1^2 \frac{L_3}{L_1} \xi_1 & \omega_4^2\mu_1 + \omega_1^2 \frac{L_4}{L_1} \xi_1 & \cdots & \omega_k^2\mu_1 + \omega_1^2 \frac{L_k}{L_1} \xi_1 \\ -\omega_1\mu_2 - \omega_2 \frac{L_1}{L_2} \xi_2 & -\omega_2\mu_2 - \omega_2 \frac{L_2}{L_2} \xi_2 & \omega_3^2\mu_1 + \omega_2^2 \frac{L_3}{L_2} \xi_1 & \omega_4^2\mu_1 + \omega_2^2 \frac{L_4}{L_2} \xi_1 & \cdots & \omega_k^2\mu_1 + \omega_2^2 \frac{L_k}{L_2} \xi_1 \\ -\omega_1\mu_2 - \omega_3 \frac{L_1}{L_3} \xi_2 & -\omega_2\mu_2 - \omega_3 \frac{L_2}{L_3} \xi_2 & \omega_3^2\nu_{13} & \omega_4^2\mu_1 + \omega_3^2 \frac{L_4}{L_3} \xi_1 & \cdots & \omega_k^2\mu_1 + \omega_3^2 \frac{L_k}{L_3} \xi_1 \\ -\omega_1\mu_2 - \omega_4 \frac{L_1}{L_4} \xi_2 & -\omega_2\mu_2 - \omega_4 \frac{L_2}{L_4} \xi_2 & \omega_3^2\mu_1 + \omega_4^2 \frac{L_3}{L_4} \xi_1 & \omega_4^2\nu_{14} & \cdots & \omega_k^2\mu_1 + \omega_4^2 \frac{L_k}{L_4} \xi_1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\omega_1\mu_2 - \omega_k \frac{L_1}{L_k} \xi_2 & -\omega_2\mu_2 - \omega_k \frac{L_2}{L_k} \xi_2 & \omega_3^2\mu_1 + \omega_k^2 \frac{L_3}{L_k} \xi_1 & \omega_4^2\mu_1 + \omega_k^2 \frac{L_4}{L_k} \xi_1 & \cdots & \omega_k^2\nu_{1k} \end{vmatrix} \\
& = (L_1 \cdots L_k) \\
& \times \begin{vmatrix} -\frac{\omega_1}{L_1}\mu_2 - \frac{\omega_1}{L_1}\xi_2 & (\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\mu_2 & \frac{\omega_3^2}{L_3}\mu_1 + \frac{\omega_1^2}{L_1}\xi_1 & \frac{\omega_4^2}{L_4}\mu_1 + \frac{\omega_1^2}{L_1}\xi_1 & \cdots & \frac{\omega_k^2}{L_k}\mu_1 + \frac{\omega_1^2}{L_1}\xi_1 \\ (\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & 0 & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 & \cdots & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 \\ (\frac{\omega_1}{L_1} - \frac{\omega_3}{L_3})\xi_2 & 0 & \frac{\omega_3^2(\nu_{13}-\mu_1)}{L_3} - \frac{\omega_1^2}{L_1}\xi_1 & (\frac{\omega_3^2}{L_3} - \frac{\omega_1^2}{L_1})\xi_1 & \cdots & (\frac{\omega_3^2}{L_3} - \frac{\omega_1^2}{L_1})\xi_1 \\ (\frac{\omega_1}{L_1} - \frac{\omega_4}{L_4})\xi_2 & 0 & (\frac{\omega_4^2}{L_4} - \frac{\omega_1^2}{L_1})\xi_1 & \frac{\omega_4^2(\nu_{14}-\mu_1)}{L_4} - \frac{\omega_1^2}{L_1}\xi_1 & \cdots & (\frac{\omega_4^2}{L_4} - \frac{\omega_1^2}{L_1})\xi_1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & 0 & (\frac{\omega_k^2}{L_k} - \frac{\omega_1^2}{L_1})\xi_1 & (\frac{\omega_k^2}{L_k} - \frac{\omega_1^2}{L_1})\xi_1 & \cdots & \frac{\omega_k^2(\nu_{1k}-\mu_1)}{L_k} - \frac{\omega_1^2}{L_1}\xi_1 \end{vmatrix} \\
& = (\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\mu_2 (L_1 \cdots L_k) \\
& \times \begin{vmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ (\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\xi_2 & 0 & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 & \cdots & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 \\ (\frac{\omega_1}{L_1} - \frac{\omega_3}{L_3})\xi_2 & 0 & \frac{\omega_3^2(\nu_{13}-\mu_1)}{L_3} - \frac{\omega_1^2}{L_1}\xi_1 & (\frac{\omega_3^2}{L_3} - \frac{\omega_1^2}{L_1})\xi_1 & \cdots & (\frac{\omega_3^2}{L_3} - \frac{\omega_1^2}{L_1})\xi_1 \\ (\frac{\omega_1}{L_1} - \frac{\omega_4}{L_4})\xi_2 & 0 & (\frac{\omega_4^2}{L_4} - \frac{\omega_1^2}{L_1})\xi_1 & \frac{\omega_4^2(\nu_{14}-\mu_1)}{L_4} - \frac{\omega_1^2}{L_1}\xi_1 & \cdots & (\frac{\omega_4^2}{L_4} - \frac{\omega_1^2}{L_1})\xi_1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (\frac{\omega_1}{L_1} - \frac{\omega_k}{L_k})\xi_2 & 0 & (\frac{\omega_k^2}{L_k} - \frac{\omega_1^2}{L_1})\xi_1 & (\frac{\omega_k^2}{L_k} - \frac{\omega_1^2}{L_1})\xi_1 & \cdots & \frac{\omega_k^2(\nu_{1k}-\mu_1)}{L_k} - \frac{\omega_1^2}{L_1}\xi_1 \end{vmatrix} \\
& = -(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\mu_2 \xi_2 (L_1 \cdots L_k) \\
& \times \begin{vmatrix} \frac{\omega_1}{L_1} - \frac{\omega_2}{L_2} & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 & 0 & \cdots & 0 \\ \frac{\omega_1}{L_1} - \frac{\omega_3}{L_3} & \frac{\omega_3^2(\nu_{13}-\mu_1)}{L_3} - \frac{\omega_1^2}{L_1}\xi_1 & -\frac{\omega_3^2}{L_3}(\nu_{13}-\mu_1-\xi_1) & \cdots & -\frac{\omega_3^2}{L_3}(\nu_{13}-\mu_1-\xi_1) \\ \frac{\omega_1}{L_1} - \frac{\omega_4}{L_4} & (\frac{\omega_4^2}{L_4} - \frac{\omega_1^2}{L_1})\xi_1 & \frac{\omega_4^2}{L_4}(\nu_{14}-\mu_1-\xi_1) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_1}{L_1} - \frac{\omega_k}{L_k} & (\frac{\omega_k^2}{L_k} - \frac{\omega_1^2}{L_1})\xi_1 & 0 & \cdots & \frac{\omega_k^2}{L_k}(\nu_{1k}-\mu_1-\xi_1) \end{vmatrix} \\
& = -(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2})\mu_2 \xi_2 (L_1 \cdots L_k) \\
& \times \begin{vmatrix} \frac{\omega_1}{L_1} - \frac{\omega_2}{L_2} & (\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1})\xi_1 & 0 & \cdots & 0 \\ \frac{\omega_1}{L_1} - \frac{\omega_4}{L_4} & (\frac{\omega_4^2}{L_4} - \frac{\omega_1^2}{L_1})\xi_1 & \frac{\omega_4^2}{L_4}\psi_{14} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_1}{L_1} - \frac{\omega_k}{L_k} & (\frac{\omega_k^2}{L_k} - \frac{\omega_1^2}{L_1})\xi_1 & 0 & \cdots & \frac{\omega_k^2}{L_k}\psi_{1k} \end{vmatrix}
\end{aligned}$$

where

$$\psi_{1i} \equiv \nu_{1i} - \mu_1 - \xi_1 = \frac{bc\bar{L}^2\tau^2}{2(2b-c)L_i},$$

$$A \equiv \left(\frac{\omega_1}{L_1} - \frac{\omega_3}{L_3}\right) + \sum_{i=4}^k \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{\omega_1}{L_1} - \frac{\omega_i}{L_i}\right),$$

$$B \equiv \frac{\omega_3^2}{L_3} \psi_{13} + \left\{ \left(\frac{\omega_3^2}{L_3} - \frac{\omega_1^2}{L_1}\right) + \sum_{i=4}^k \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{\omega_i^2}{L_i} - \frac{\omega_1^2}{L_1}\right) \right\} \xi_1.$$

Furthermore, we obtain

$$\begin{aligned} & (L_1 \cdots L_k) \left| \begin{array}{cc} \frac{\omega_1}{L_1} - \frac{\omega_2}{L_2} & \left(\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1}\right) \xi_1 \\ A & B \end{array} \right| \prod_{l \neq 1,2,3} \frac{\omega_l^2 \psi_{1l}}{L_l} \\ &= \left(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2}\right) \prod_{j=1}^k L_j \left[\frac{\omega_3^2}{L_3} \psi_{13} + \left\{ \left(\frac{\omega_3^2}{L_3} - \frac{\omega_1^2}{L_1}\right) + \sum_{i=4}^k \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{\omega_i^2}{L_i} - \frac{\omega_1^2}{L_1}\right) \right\} \xi_1 \right] \prod_{l \neq 1,2,3} \frac{\omega_l^2 \psi_{1l}}{L_l} \\ &\quad - \left(\frac{\omega_2^2}{L_2} - \frac{\omega_1^2}{L_1}\right) \prod_{j=1}^k L_j \left[\left(\frac{\omega_1}{L_1} - \frac{\omega_3}{L_3}\right) + \sum_{i=4}^k \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{\omega_1}{L_1} - \frac{\omega_i}{L_i}\right) \right] \xi_1 \prod_{l \neq 1,2,3} \frac{\omega_l^2 \psi_{1l}}{L_l} \\ &= (\omega_1 L_2 - \omega_2 L_1) \left[\frac{\omega_3^2}{L_3} \psi_{13} + \left\{ \left(\omega_3^2 - \frac{L_3}{L_1} \omega_1^2\right) + L_3 \sum_{i=4}^k \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{\omega_i^2}{L_i} - \frac{\omega_1^2}{L_1}\right) \right\} \xi_1 \right] \prod_{l \neq 1,2,3} \omega_l^2 \psi_{1l} \\ &\quad - (\omega_2^2 L_1 - \omega_1^2 L_2) \left[\left(\frac{L_3}{L_1} \omega_1 - \omega_3\right) + L_3 \sum_{i=4}^k \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{\omega_1}{L_1} - \frac{\omega_i}{L_i}\right) \right] \xi_1 \prod_{l \neq 1,2,3} \omega_l^2 \psi_{1l} \\ &= (\omega_1 L_2 - \omega_2 L_1) \prod_{l \neq 1,2} \omega_l^2 \psi_{1l} \\ &\quad + \xi_1 \prod_{l \neq 1,2,3} \omega_l^2 \psi_{1l} \left[(\omega_1 L_2 - \omega_2 L_1) \left(\omega_3^2 - \frac{L_3}{L_1} \omega_1^2\right) - (\omega_2^2 L_1 - \omega_1^2 L_2) \left(\frac{L_3}{L_1} \omega_1 - \omega_3\right) \right] \\ &\quad + \xi_1 \prod_{l \neq 1,2,3} \omega_l^2 \psi_{1l} \left[(\omega_1 L_2 - \omega_2 L_1) \sum_{i=4}^k \frac{L_3}{L_i} \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\omega_i^2 - \frac{L_i}{L_1} \omega_1^2\right) \right. \\ &\quad \quad \left. - (\omega_2^2 L_1 - \omega_1^2 L_2) \sum_{i=4}^k \frac{L_3}{L_i} \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{L_i}{L_1} \omega_1 - \omega_i\right) \right] \\ &= (\omega_1 L_2 - \omega_2 L_1) \prod_{l \neq 1,2} \omega_l^2 \psi_{1l} \\ &\quad + [\omega_1 \omega_2 (\omega_1 - \omega_2) L_3 + \omega_2 \omega_3 (\omega_2 - \omega_3) L_1 + \omega_3 \omega_1 (\omega_3 - \omega_1) L_2] \xi_1 \prod_{l \neq 1,2,3} \omega_l^2 \psi_{1l} \\ &\quad + \xi_1 \omega_i^2 \frac{L_3}{L_i} \psi_{13} \prod_{l \neq 1,2,3,k} \omega_l^2 \psi_{1l} \sum_{i=4}^k \left[(\omega_1 L_2 - \omega_2 L_1) \frac{L_3}{L_i} \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\omega_i^2 - \frac{L_i}{L_1} \omega_1^2\right) \right. \\ &\quad \quad \left. - (\omega_2^2 L_1 - \omega_1^2 L_2) \frac{L_3}{L_i} \left(\frac{\omega_3 L_i}{\omega_i L_3}\right)^2 \left(\frac{L_i}{L_1} \omega_1 - \omega_i\right) \right], \end{aligned}$$

and the last term of the equation could be written as

$$\xi_1 \omega_3^2 \psi_{13} \prod_{l \neq 1,2,3,i} \omega_l^2 \psi_{1l} \sum_{i=4}^k \left[(\omega_1 L_2 - \omega_2 L_1) \left(\omega_i^2 - \frac{L_i}{L_1} \omega_1^2\right) - (\omega_2^2 L_1 - \omega_1^2 L_2) \omega_3^2 \left(\frac{L_i}{L_1} \omega_1 - \omega_i\right) \right]$$

$$= \sum_{i=4}^k [\omega_1 \omega_2 (\omega_1 - \omega_2) L_i + \omega_2 \omega_i (\omega_2 - \omega_i) L_1 + \omega_i \omega_1 (\omega_i - \omega_1) L_2] \xi_1 \prod_{l \neq 1, 2, i} \omega_l^2 \psi_{1l}.$$

Therefore, the determinant which has columns with τ -terms in the first and second columns could be represented as

$$-\left(\frac{\omega_1}{L_1} - \frac{\omega_2}{L_2}\right) \mu_2 \xi_2 \left\{ (\omega_1 L_2 - \omega_2 L_1) \prod_{l \neq 1, 2} \omega_l^2 \psi_{1l} + \xi_1 \prod_{l \neq 1, 2, i} \omega_l^2 \psi_{1l} \right. \\ \left. \sum_{i=3}^k \left[\omega_1 \omega_2 (\omega_1 - \omega_2) L_i + \omega_2 \omega_i (\omega_2 - \omega_i) L_1 + \omega_i \omega_1 (\omega_i - \omega_1) L_2 \right] \right\}.$$

By the same way, the determinant which has columns with τ -terms in the m th and n th columns could be represented as

$$-\left(\frac{\omega_m}{L_m} - \frac{\omega_n}{L_n}\right) \mu_2 \xi_2 \left\{ (\omega_m L_n - \omega_n L_m) \prod_{l \neq m, n} \omega_l^2 \psi_{1l} + \xi_1 \prod_{l \neq m, n, i} \omega_l^2 \psi_{1l} \right. \\ \left. \sum_{i \neq m, n}^k \left[\omega_m \omega_n (\omega_m - \omega_n) L_i + \omega_n \omega_i (\omega_n - \omega_i) L_m + \omega_i \omega_m (\omega_i - \omega_m) L_n \right] \right\}.$$

The $\tau^{(2k-2)}$ -term in $|\Upsilon|$ is sum of the above determinants for all $(m, n) \in \kappa$, where κ is the set of combinations from $\{1, \dots, K\}$. We should note that

$$\sum_{(m, n) \in \kappa} \left\{ \left(\frac{\omega_m}{L_m} - \frac{\omega_n}{L_n}\right) \sum_{i \neq m, n}^k \left[\omega_m \omega_n (\omega_m - \omega_n) L_i \right. \right. \\ \left. \left. + \omega_n \omega_i (\omega_n - \omega_i) L_m + \omega_i \omega_m (\omega_i - \omega_m) L_n \right] \prod_{l \neq m, n, i} \omega_l^2 \psi_{1l} \right\}$$

is zero, then the $\tau^{(2k-2)}$ -term in $|\Upsilon|$ could be written as

$$- \sum_{(m, n) \in \kappa} \left(\frac{\omega_m}{L_m} - \frac{\omega_n}{L_n}\right) \mu_2 \xi_2 (\omega_m L_n - \omega_n L_m) \prod_{l \neq m, n} \omega_l^2 \psi_{1l} \\ = -\mu_2 \xi_2 \sum_{(m, n) \in \kappa} (\omega_m L_n - \omega_n L_m)^2 \prod_{l \neq m, n} \frac{\omega_l^2 \psi_{1l}}{L_m L_n} \\ = -\frac{a^2 b^{k+1} c^{k-2} \bar{L}^{2k-2}}{2^{k-4} (2b-c)^{k+1} \prod L_i} \left[\sum_{(m, n) \in \kappa} (\omega_m L_n - \omega_n L_m)^2 \prod_{l \neq m, n} \omega_l^2 \right] \tau^{2k-2} \\ < 0,$$

where the last inequality holds from assumption (5).

Next, we will consider the $\tau^{(2k-2)}$ -term in $\sum_{i=1}^k |(\Theta_i, \Upsilon^{-i})|$. This term could be represented as sum of determinants which has only one column with τ -terms except for i th column by the above decomposition, and one of the determinant, for example, is calculated as

$$\begin{aligned}
& \begin{vmatrix} \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_1 \frac{L_2}{L_1} \xi_2 & \omega_3^2\mu_1 + \omega_1^2 \frac{L_3}{L_1} \xi_1 & \omega_4^2\mu_1 + \omega_1^2 \frac{L_4}{L_1} \xi_1 & \cdots & \omega_k^2\mu_1 + \omega_1^2 \frac{L_k}{L_1} \xi_1 \\ \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_2 \frac{L_2}{L_2} \xi_2 & \omega_3^2\mu_1 + \omega_2^2 \frac{L_3}{L_2} \xi_1 & \omega_4^2\mu_1 + \omega_2^2 \frac{L_4}{L_2} \xi_1 & \cdots & \omega_k^2\mu_1 + \omega_2^2 \frac{L_k}{L_2} \xi_1 \\ \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_3 \frac{L_2}{L_3} \xi_2 & \omega_3^2\nu_{13} & \omega_4^2\mu_1 + \omega_3^2 \frac{L_4}{L_3} \xi_1 & \cdots & \omega_k^2\mu_1 + \omega_3^2 \frac{L_k}{L_3} \xi_1 \\ \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_4 \frac{L_2}{L_4} \xi_2 & \omega_3^2\mu_1 + \omega_4^2 \frac{L_3}{L_4} \xi_1 & \omega_4^2\nu_{14} & \cdots & \omega_k^2\mu_1 + \omega_4^2 \frac{L_k}{L_4} \xi_1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho\tau L_1/2 & -\omega_2\mu_2 - \omega_k \frac{L_2}{L_k} \xi_2 & \omega_3^2\mu_1 + \omega_k^2 \frac{L_3}{L_k} \xi_1 & \omega_4^2\mu_1 + \omega_k^2 \frac{L_4}{L_k} \xi_1 & \cdots & \omega_k^2\nu_{1k} \end{vmatrix} \\
& = \left(\frac{\rho\tau}{2}\right)\xi_2 \left\{ (\omega_1 L_2 - \omega_2 L_1) \prod_{l \neq 1,2} \omega_l^2 \psi_{1l} + \xi_1 \prod_{l \neq 1,2,i} \omega_l^2 \psi_{1l} \right. \\
& \quad \left. \sum_{i=3}^k \left[\omega_1 \omega_2 (\omega_1 - \omega_2) L_i + \omega_2 \omega_i (\omega_2 - \omega_k) L_1 + \omega_i \omega_1 (\omega_i - \omega_1) L_2 \right] \right\}
\end{aligned}$$

by the same way as used for the case of $|\Upsilon|$.

Thus, such a determinant is generally expressed as

$$\begin{aligned}
& \left(\frac{\rho\tau}{2}\right)\xi_2 \left\{ (\omega_m L_n - \omega_n L_m) \prod_{l \neq m,n} \omega_l^2 \psi_{1l} + \xi_1 \prod_{l \neq m,n,i} \omega_l^2 \psi_{1l} \right. \\
& \quad \left. \sum_{i \neq m,n}^k \left[\omega_m \omega_n (\omega_m - \omega_n) L_i + \omega_n \omega_i (\omega_n - \omega_i) L_m + \omega_i \omega_m (\omega_i - \omega_m) L_n \right] \right\}.
\end{aligned}$$

The $\tau^{(2k-2)}$ -term in $\sum_{i=1}^k |(\Theta_i, \Upsilon^{-i})|$ is sum of the above determinants for all $(m, n) \in \bar{k}$, where \bar{k} is the set of permutations from $\{1, \dots, K\}$. We should note that only the sign reverses if m and n are replaced each other, then we know the $\tau^{(2k-2)}$ -term is zero.

Therefore, the $\tau^{(2k-2)}$ -term in $|\Delta|$ is negative. ■

Appendix B. Proof of Lemma 2

(i) Let λ^* be an equilibrium of pattern (a), in which l industries i_1, \dots, i_l agglomerate in region H and $K - l$ industries i_{l+1}, \dots, i_K agglomerate in region F . Then for each industry i , it holds that

$$\begin{aligned}
V_{iH}(\lambda^*) - V_{iF}(\lambda^*) & = -\frac{1}{2} \sum_{j=1}^l \delta_{ii_j} + \frac{1}{2} \sum_{j=l+1}^K \delta_{ii_j} \\
& = -\frac{1}{2} \left\{ \frac{\rho}{2} \left(\sum_{j=1}^l L_{i_j} - \sum_{j=l+1}^K L_{i_j} \right) - \frac{2ab\bar{L}}{(2b-c)^2} \left[b \left(\sum_{j=1}^l \omega_{i_j} - \sum_{j=l+1}^K \omega_{i_j} \right) \right. \right. \\
& \quad \left. \left. + (2b-c) \left(\sum_{j=1}^l L_{i_j} - \sum_{j=l+1}^K L_{i_j} \right) \frac{\omega_i}{L_i} \right] \right\} \tau + O(\tau^2).
\end{aligned}$$

For sufficiently small τ , the sign of the above expression is determined by the coefficient of τ . Because of (4), the population of two regions are different, and hence $\sum_{j=1}^l L_{i_j} > \sum_{j=l+1}^K L_{i_j}$,

because of our assumption that H is not smaller than F . Since industries i_1, \dots, i_l are in H , the stability conditions requires $V_{i_j H}(\boldsymbol{\lambda}^*) - V_{i_j F}(\boldsymbol{\lambda}^*) > 0$ for $j = 1, \dots, l$, which implies

$$\rho \leq \frac{4ab\bar{L}}{2b-c} \left[\frac{\omega_{i_k}}{L_{i_k}} + \frac{b}{2b-c} \frac{\sum_{j=1}^l \omega_{i_j} - \sum_{j=l+1}^K \omega_{i_j}}{\sum_{j=1}^l L_{i_j} - \sum_{j=l+1}^K L_{i_j}} \right] \text{ for } k = 1, \dots, l, \quad (\text{A.8})$$

On the other hand, since industries i_{l+1}, \dots, i_K are in F , we have $V_{i_j H}(\boldsymbol{\lambda}^*) - V_{i_j F}(\boldsymbol{\lambda}^*) \leq 0$ for $j = l+1, \dots, K$, which implies

$$\rho \geq \frac{4ab\bar{L}}{2b-c} \left[\frac{\omega_{i_k}}{L_{i_k}} + \frac{b}{2b-c} \frac{\sum_{j=1}^l \omega_{i_j} - \sum_{j=l+1}^K \omega_{i_j}}{\sum_{j=1}^l L_{i_j} - \sum_{j=l+1}^K L_{i_j}} \right] \text{ for } k = l+1, \dots, K. \quad (\text{A.9})$$

The following inequality holds directly from (A.8) and (A.9):

$$\min \left\{ \frac{\omega_{i_1}}{L_{i_1}}, \dots, \frac{\omega_{i_l}}{L_{i_l}} \right\} \geq \max \left\{ \frac{\omega_{i_{l+1}}}{L_{i_{l+1}}}, \dots, \frac{\omega_{i_K}}{L_{i_K}} \right\}. \quad (\text{A.10})$$

From (5), (A.10) tells us that $i_l = l$, and the industries locate in H turn out to be $1, \dots, l$, the industries locate in F turn out to be $l+1, \dots, K$. Therefore, (A.8) and (A.9) are summarized as $\rho^\dagger(l, l) \geq \rho \geq \rho^\dagger(l+1, l)$.

(ii) Let $\boldsymbol{\lambda}^*$ be the equilibrium in which industries $1, \dots, l$ agglomerate in region H and industries $l+1, \dots, K$ agglomerate in region F . Then

$$\begin{aligned} V_{iH}(\boldsymbol{\lambda}^*) - V_{iF}(\boldsymbol{\lambda}^*) &= -\frac{1}{2} \sum_{j=1}^l \delta_{ij} + \frac{1}{2} \sum_{j=l+1}^K \delta_{ij} \\ &= -\frac{1}{2} \left\{ \frac{\rho}{2} \left(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j \right) - \frac{2ab\bar{L}}{(2b-c)^2} \left[b \left(\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j \right) \right. \right. \\ &\quad \left. \left. + (2b-c) \left(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j \right) \frac{\omega_i}{L_i} \right] \right\} \tau + O(\tau^2), \end{aligned}$$

whose sign is determined by the coefficient of τ for sufficiently small τ . From $\rho^\dagger(l, l) > \rho > \rho^\dagger(l+1, l)$, we have

$$\begin{aligned} \rho &< \frac{4ab\bar{L}}{2b-c} \left[\frac{\omega_{i_k}}{L_{i_k}} + \frac{b}{2b-c} \frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j} \right] \text{ for } k = 1, \dots, l, \\ \rho &> \frac{4ab\bar{L}}{2b-c} \left[\frac{\omega_{i_k}}{L_{i_k}} + \frac{b}{2b-c} \frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j} \right] \text{ for } k = l+1, \dots, K. \end{aligned}$$

Furthermore, since $\sum_{j=1}^l L_j > \sum_{j=l+1}^K L_j$, we soon obtain $V_{jH}(\boldsymbol{\lambda}^*) - V_{jF}(\boldsymbol{\lambda}^*) > 0$ for $j = 1, \dots, l$ and $V_{jH}(\boldsymbol{\lambda}^*) - V_{jF}(\boldsymbol{\lambda}^*) < 0$ for $j = l+1, \dots, K$, therefore, $\boldsymbol{\lambda}^*$ is stable. ■

Appendix C. Proof of Lemma 3

We first show that necessity. Suppose that industries i_1, \dots, i_{l_1} locate in region H and industries j_1, \dots, j_{l_2} locate in region F . The remaining industry l disperses with population $\lambda_l^* L_l$ in region H . Then $l_1 + l_2 + 1 = K$ and $V_{lH}(\boldsymbol{\lambda}^*) - V_{lF}(\boldsymbol{\lambda}^*) = 0$, from which we have

$$\lambda_l^* = -\frac{1}{2\delta_{ll}} \left[\mu_1 \Omega_1 - \mu_2 \Omega_2 + \frac{\Lambda}{L_l} (\omega_l^2 \xi_1 - \omega_l \xi_2 + \frac{1}{2} \rho \tau L_l) \right] + \frac{1}{2},$$

where

$$\Omega_1 \equiv \sum_{k=1}^{l_1} \omega_{i_k}^2 - \sum_{k=1}^{l_2} \omega_{j_k}^2, \quad \Omega_2 \equiv \sum_{k=1}^{l_1} \omega_{i_k} - \sum_{k=1}^{l_2} \omega_{j_k}, \quad \text{and } \Lambda \equiv \sum_{k=1}^{l_1} L_{i_k} - \sum_{k=1}^{l_2} L_{j_k}.$$

Since we are interested in the situation of sufficiently small τ , we denote

$$\bar{\lambda}_l \equiv \lim_{\tau \rightarrow 0} \lambda_l^* = \frac{4ab^2 L_l \bar{L} \Omega_2 - (2b-c)^2 (\Lambda - L_l) L_l \rho + 4ab[(2b-c)\Lambda - (3b-c)L_l] \bar{L} \omega_l}{2L_l [(2b-c)^2 L_l \rho - 4ab(3b-c) \bar{L} \omega_l]},$$

which should be in $(0, 1)$. The stability conditions for $\boldsymbol{\lambda}^*$ are

$$\begin{aligned} V_{i_k H}(\boldsymbol{\lambda}^*) - V_{i_k F}(\boldsymbol{\lambda}^*) &> 0 \quad \text{for } k = 1, \dots, l_1, \\ V_{j_k H}(\boldsymbol{\lambda}^*) - V_{j_k F}(\boldsymbol{\lambda}^*) &< 0 \quad \text{for } k = 1, \dots, l_2, \\ \frac{\partial [V_{lH}(\boldsymbol{\lambda}^*) - V_{lF}(\boldsymbol{\lambda}^*)]}{\partial \lambda_l} &< 0, \end{aligned}$$

which are rewritten as

$$\begin{aligned} &V_{i_k H}(\boldsymbol{\lambda}^*) - V_{i_k F}(\boldsymbol{\lambda}^*) \\ &= \frac{16a^2 b^3 \bar{L}^2 \left(\frac{\omega_{i_k}}{L_{i_k}} - \frac{\omega_l}{L_l} \right) \left(\Omega_2 - \Lambda \frac{\omega_l}{L_l} \right) \tau + O(\tau^2)}{4(2b-c) \left[(2b-c)^2 \rho - 4ab(3b-c) \bar{L} \frac{\omega_l}{L_l} \right] + O(\tau)} > 0 \quad \text{for } k = 1, \dots, l_1, \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} &V_{j_k H}(\boldsymbol{\lambda}^*) - V_{j_k F}(\boldsymbol{\lambda}^*) \\ &= \frac{16a^2 b^3 \bar{L}^2 \left(\frac{\omega_{j_k}}{L_{j_k}} - \frac{\omega_l}{L_l} \right) \left(\Omega_2 - \Lambda \frac{\omega_l}{L_l} \right) \tau + O(\tau^2)}{4(2b-c) \left[(2b-c)^2 \rho - 4ab(3b-c) \bar{L} \frac{\omega_l}{L_l} \right] + O(\tau)} < 0 \quad \text{for } k = 1, \dots, l_2, \end{aligned} \quad (\text{A.12})$$

$$\begin{aligned} &\frac{\partial [V_{lH}(\boldsymbol{\lambda}^*) - V_{lF}(\boldsymbol{\lambda}^*)]}{\partial \lambda_l} \\ &= -\frac{1}{2} \left[\rho L_l - \frac{4ab(3b-c) \bar{L} \omega_l}{(2b-c)^2} \right] \tau + O(\tau^2) < 0, \end{aligned} \quad (\text{A.13})$$

respectively. For sufficiently small τ , the signs of the above expressions are determined by their coefficients of τ . Therefore, (A.13) simply implies that

$$\rho > \frac{4ab(3b-c) \bar{L} \omega_l}{(2b-c)^2 L_l}. \quad (\text{A.14})$$

Then the denominators of (A.11) and (A.12) are positive. We consider two cases to further clarify (A.11) and (A.12).

- If

$$\Omega_2 - \Lambda \frac{\omega_l}{L_l} > 0, \quad (\text{A.15})$$

then (A.11) and (A.12) yield

$$\frac{\omega_{i_k}}{L_{i_k}} > \frac{\omega_l}{L_l} > \frac{\omega_{j_k}}{L_{j_k}}, \text{ for any } i_k = i_1, \dots, i_{l_1} \text{ and } j_k = j_1, \dots, j_{l_2}. \quad (\text{A.16})$$

Therefore the industries in region H turn out to be $1, \dots, l-1$ and the industries in region F turn out to be $l+1, \dots, K$. Therefore, the equilibrium takes the form of $\boldsymbol{\lambda}^* = (1, \dots, 1, \lambda_l^*, 0, \dots, 0)$. On the other hand, (A.16) implies that

$$\sum_{k=1}^{l_1} \omega_{i_k} - \sum_{k=1}^{l_2} \omega_{j_k} > \left(\sum_{k=1}^{l_1} L_{i_k} - \sum_{k=1}^{l_2} L_{j_k} \right) \frac{\omega_l}{L_l},$$

which shows that (A.15) is true.

- If

$$\Omega_2 - \Lambda \frac{\omega_l}{L_l} < 0, \quad (\text{A.17})$$

then (A.11) and (A.12) yield

$$\frac{\omega_{i_k}}{L_{i_k}} < \frac{\omega_l}{L_l} < \frac{\omega_{j_k}}{L_{j_k}}, \text{ for any } i_k = i_1, \dots, i_{l_1} \text{ and } j_k = j_1, \dots, j_{l_2}. \quad (\text{A.18})$$

Therefore the industries in region H turn out to be $l+1, \dots, K$ and the industries in region F turn out to be $1, \dots, l-1$. Therefore, the equilibrium takes the form of $\boldsymbol{\lambda}^* = (0, \dots, 0, \lambda_l^*, 1, \dots, 1)$. On the other hand, (A.18) implies that

$$\sum_{k=1}^{l_1} \omega_{i_k} - \sum_{k=1}^{l_2} \omega_{j_k} < \left(\sum_{k=1}^{l_1} L_{i_k} - \sum_{k=1}^{l_2} L_{j_k} \right) \frac{\omega_l}{L_l},$$

which shows that (A.17) is true.

Summarizing the above two cases, we obtain (i) of the Lemma. To derive the others, we focus on the case that $\boldsymbol{\lambda}^* = (1, \dots, 1, \lambda_l^*, 0, \dots, 0)$. The other case can be shown similarly. Contrarily to (ii), we suppose that $\sum_{j=1}^l L_j < \sum_{j=l+1}^K L_j$. Then $\bar{\lambda}_l < 1$ is equivalent to $\rho < \rho^\dagger(l, l)$. However, we have

$$\begin{aligned} \rho^\dagger(l, l) &= \frac{4ab(3b-c)\bar{L}}{(2b-c)^2} \frac{\omega_l}{L_l} \\ &= \frac{4ab\bar{L}}{2b-c} \left[\frac{\omega_l}{L_l} + \frac{b}{2b-c} \frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j} \right] - \frac{4ab(3b-c)\bar{L}}{(2b-c)^2} \frac{\omega_l}{L_l} \end{aligned}$$

$$\begin{aligned}
&= \frac{4ab^2\bar{L}}{(2b-c)^2} \left[\frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j} - \frac{\omega_l}{L_l} \right] \\
&= \frac{4ab\bar{L}}{(2b-c)^2} \frac{\sum_{j=1}^l L_l L_j \left(\frac{\omega_j}{L_j} - \frac{\omega_l}{L_l} \right) + \sum_{j=l+1}^K L_l L_j \left(\frac{\omega_l}{L_l} - \frac{\omega_j}{L_j} \right)}{L_l (\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j)} \\
&< 0,
\end{aligned} \tag{A.19}$$

therefore $\bar{\lambda}_l < 1$ contradicts (A.14), and hence we obtained (ii). In the following, we further consider two cases to obtain the remaining conclusions.

- If $\sum_{j=1}^{l-1} L_j > \sum_{j=l}^K L_j$, then $\bar{\lambda}_l < 1$ is equivalent to $\rho > \rho^\dagger(l, l)$ and $\bar{\lambda}_l > 0$ is equivalent to $\rho < \rho^\dagger(l, l-1)$. Furthermore, since the reverse inequality of (A.19) turns out to be true because of (ii), (A.14) is guaranteed by $\rho > \rho^\dagger(l, l)$. Thus, we obtained conclusion (iii).
- If $\sum_{j=1}^{l-1} L_j < \sum_{j=l}^K L_j$ and $\sum_{j=1}^l L_j > \sum_{j=l+1}^K L_j$, then $\bar{\lambda}_l < 1$ is equivalent to $\rho > \rho^\dagger(l, l)$ again while $\bar{\lambda}_l > 0$ turns out to be equivalent to $\rho > \rho^\dagger(l, l-1)$. Since

$$\begin{aligned}
&\frac{\sum_{j=1}^{l-1} \omega_j - \sum_{j=l}^K \omega_j}{\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j} - \frac{\sum_{j=1}^l \omega_j - \sum_{j=l+1}^K \omega_j}{\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j} \\
&= \frac{(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j)(\sum_{j=1}^{l-1} \omega_j - \omega_l - \sum_{j=l+1}^K \omega_j)}{(\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j)(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j)} \\
&\quad - \frac{(\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j)(\sum_{j=1}^{l-1} \omega_j + \omega_l \sum_{j=l+1}^K \omega_j)}{(\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j)(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j)} \\
&= \frac{2 \sum_{j=1}^{l-1} L_l L_j (\omega_j / L_j - \omega_l / L_l) + 2 \sum_{j=l+1}^K L_l L_j (\omega_l / L_l - \omega_j / L_j)}{(\sum_{j=1}^{l-1} L_j - \sum_{j=l}^K L_j)(\sum_{j=1}^l L_j - \sum_{j=l+1}^K L_j)} \\
&< 0,
\end{aligned}$$

we know that $\rho^\dagger(l, l-1) < \rho^\dagger(l, l)$. Furthermore, since the reverse inequality of (A.19) holds, the conditions can be summarized by $\rho > \rho^\dagger(l, l)$ only, and we obtained (iv).

Finally, the sufficiency holds evidently. ■

Appendix D. Proof of Lemma 4

We first provide the following conclusion:

Claim For any a_i and positive b_i ($i = 1, \dots, n$), it holds that

$$\max \left\{ \frac{a_1}{b_1}, \dots, \frac{a_n}{b_n} \right\} \geq \frac{a_1 + \dots + a_n}{b_1 + \dots + b_n}. \tag{A.20}$$

Proof: If $n = 1$, then (A.20) holds with equality. For $n = 2$, without loss of generality, let $a_1/b_1 \leq a_2/b_2$. Then $a_1b_2 \leq a_2b_1$ so that $a_1b_2 + a_2b_2 \leq a_2b_1 + a_2b_2$, which can be rewritten as

$$\frac{a_1 + a_2}{b_1 + b_2} \leq \frac{a_2}{b_2} = \max\left\{\frac{a_1}{b_1}, \frac{a_2}{b_2}\right\}. \quad (\text{A.21})$$

Suppose that (A.20) holds for k , then

$$\begin{aligned} \max\left\{\frac{a_1}{b_1}, \dots, \frac{a_k}{b_k}, \frac{a_{k+1}}{b_{k+1}}\right\} &= \max\left\{\max\left\{\frac{a_1}{b_1}, \dots, \frac{a_k}{b_k}\right\}, \frac{a_{k+1}}{b_{k+1}}\right\} \\ &\geq \max\left\{\frac{a_1 + \dots + a_k}{b_1 + \dots + b_k}, \frac{a_{k+1}}{b_{k+1}}\right\} \geq \frac{(a_1 + \dots + a_k) + a_{k+1}}{(b_1 + \dots + b_k) + b_{k+1}}, \end{aligned}$$

where the first inequality is from the induction assumption while the second inequality is from (A.21). ■

The only interior equilibrium is evidently $\lambda^* = (1/2, 1/2, 1/2)$. Let $\Delta \equiv (\delta_{ij})_{3 \times 3}$ and the characteristic equation to be $t^3 + At^2 + Bt + C = 0$, then the stability conditions of λ^* are given by

$$\begin{aligned} A < 0 &\Leftrightarrow \text{Tr}(\Delta) > 0 \\ &\Leftrightarrow \frac{3}{2}\epsilon - \frac{6ab(3b-c)(\omega_1 + \omega_2 + \omega_3)}{(2b-c)^2}\tau \\ &\quad + \frac{3b(6b^2 - 2bc - c^2)(\omega_1^2 + \omega_2^2 + \omega_3^2)}{2(2b-c)^2}\tau^2 > 0, \end{aligned} \quad (\text{A.22})$$

$$\begin{aligned} C < 0 &\Leftrightarrow |\Delta| > 0 \\ &\Leftrightarrow \frac{81b^2c}{8(2b-c)^4}\tau^4\{c(2b-c)^2(\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2)\epsilon \\ &\quad - 16a^2b^2[\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2]\} \\ &\quad + O(\tau^5) > 0, \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} AB < C &\Leftrightarrow \text{Tr}(\Delta)(\delta_{11}\delta_{22} + \delta_{22}\delta_{33} + \delta_{33}\delta_{11} - \delta_{12}\delta_{21} - \delta_{23}\delta_{32} - \delta_{31}\delta_{13}) > |\Delta| \\ &\Leftrightarrow \frac{27b\epsilon}{4(2b-c)^3}\tau^2\{c(2b-c)^2(\omega_1^2 + \omega_2^2 + \omega_3^2)\epsilon \\ &\quad - 16a^2b^2\{\omega_1^2 + \omega_2^2 + \omega_3^2 - (\omega_1\omega_2 + \omega_2\omega_3 + \omega_3\omega_1)\}\} + O(\tau^3) > 0. \end{aligned} \quad (\text{A.24})$$

(Samuelson 1945, P. 432).

For sufficiently small τ , the above inequalities are determined by the first terms. Therefore, (A.22) is evidently true and (A.23) is true if and only if $\epsilon > \bar{\epsilon}_0$. Next, we show that (A.24) is also true if (A.23) holds. In fact, since

$$\begin{aligned} &\frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2} - \frac{(\omega_1 - \omega_2)^2}{\omega_1^2 + \omega_2^2} \\ &= \frac{\{\omega_1^2(\omega_2 - \omega_3) - \omega_2^2(\omega_3 - \omega_1)\}^2}{(\omega_1^2 + \omega_2^2)(\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2)} \geq 0, \end{aligned}$$

we have

$$\frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2} \geq \frac{(\omega_1 - \omega_2)^2}{\omega_1^2 + \omega_2^2}.$$

Similarly, the following two inequalities also hold:

$$\begin{aligned} \frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2} &\geq \frac{(\omega_2 - \omega_3)^2}{\omega_2^2 + \omega_3^2}, \\ \frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2} &\geq \frac{(\omega_3 - \omega_1)^2}{\omega_3^2 + \omega_1^2}. \end{aligned}$$

Therefore,

$$\begin{aligned} &\frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2} \\ &\geq \max \left\{ \frac{(\omega_1 - \omega_2)^2}{\omega_1^2 + \omega_2^2}, \frac{(\omega_2 - \omega_3)^2}{\omega_2^2 + \omega_3^2}, \frac{(\omega_3 - \omega_1)^2}{\omega_3^2 + \omega_1^2} \right\} \\ &\geq \frac{(\omega_1 - \omega_2)^2 + (\omega_2 - \omega_3)^2 + (\omega_3 - \omega_1)^2}{2(\omega_1^2 + \omega_2^2 + \omega_3^2)} \\ &= \frac{\omega_1^2 + \omega_2^2 + \omega_3^2 - (\omega_1\omega_2 + \omega_2\omega_3 + \omega_3\omega_1)}{\omega_1^2 + \omega_2^2 + \omega_3^2}, \end{aligned}$$

where the last inequality is from (A.20). In conclusion, (A.24) holds if $\epsilon > \bar{\epsilon}_0$. ■

Appendix E. Proof of Lemma 5

First, we will consider the stability of $\lambda^* = (1, \lambda_2^*, 0)$ ($0 < \lambda_2^* < 1$). From $V_{2H}(\lambda^*) - V_{2F}(\lambda^*) = 0$, we obtain

$$\lambda_2^* = \frac{(2b - c)^2\epsilon + 12ab\{b(\omega_1 - 3\omega_2 - \omega_3) + c\omega_2\}\tau}{2(2b - c)^2\epsilon - 24ab(3b - c)\omega_2\tau + 6b(6b^2 - 2bc - c^2)\omega_2^2\tau^2} + O(\tau^2),$$

where we should note that λ_2^* converges to 1/2 from above as $\tau \rightarrow 0$. The stability conditions for λ^* are given as

$$\begin{aligned} &V_{1H}(\lambda^*) - V_{1F}(\lambda^*) > 0 \\ &\Leftrightarrow \frac{9bL\{16a^2b^2(\omega_1 - \omega_2)(\omega_1 - \omega_3) - c(2b - c)^2\omega_1^2\epsilon\}\tau^2}{4(2b - c)^3\epsilon - 48ab(6b^2 - 5bc + c^2)\omega_2\tau + 12b(12b^3 - 10b^2c + c^3)\omega_2^2\tau^2} \\ &\quad + O(\tau^3) > 0, \\ &V_{3H}(\lambda^*) - V_{3F}(\lambda^*) < 0 \\ &\Leftrightarrow \frac{-9bL\{16a^2b^2(\omega_1 - \omega_3)(\omega_2 - \omega_3) - c(2b - c)^2\omega_3^2\epsilon\}\tau^2}{4(2b - c)^3\epsilon - 48ab(6b^2 - 5bc + c^2)\omega_2\tau + 12b(12b^3 - 10b^2c + c^3)\omega_2^2\tau^2} \\ &\quad + O(\tau^3) < 0, \\ &\frac{\partial[V_{2H}(\lambda^*) - V_{2F}(\lambda^*)]}{\partial\lambda_2} < 0 \\ &\Leftrightarrow -\frac{L\epsilon}{2} + \frac{6abL(3b - c)\omega_2}{(2b - c)^2}\tau + O(\tau^2) < 0. \end{aligned}$$

For sufficiently small τ , all the signs are determined by the first terms in the above expressions. Together with our assumption that $\omega_1 > \omega_2 > \omega_3$, we know that $\boldsymbol{\lambda}^*$ is stable if and only if

$$\epsilon < \min\{\bar{\epsilon}_1, \bar{\epsilon}_2\}.$$

From the symmetry, conditions $\min\{\omega_2, \omega_3\} < \omega_1 < \max\{\omega_2, \omega_3\}$ and $\min\{\omega_1, \omega_2\} < \omega_3 < \max\{\omega_1, \omega_2\}$ are required for the stability of equilibria $(\lambda_1^*, 1, 0)$ ($0 < \lambda_1^* < 1$) and $(1, 0, \lambda_3^*)$ ($0 < \lambda_3^* < 1$), respectively. Since these inequalities contradict to the assumption, these two patterns could not be stable. ■

Appendix F. Proof of Proposition 3

We provide the proof for the case of $\omega_2 < \bar{\omega}_2$. The case of $\omega_2 > \bar{\omega}_2$ can be shown similarly. Above all, we provide some preliminary inequalities. The following inequalities hold immediately from $\omega_2 < \bar{\omega}_2$:

$$\begin{aligned}\bar{\epsilon}_1 - \bar{\epsilon}_0 &= \frac{\{\omega_1^2(\omega_2 - \omega_3) + \omega_2^2(\omega_1 - \omega_3)\}\{(\omega_1^2 + \omega_3^2)(\bar{\omega}_2 - \omega_2)\}}{\omega_1^2(\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2)} > 0, \\ \bar{\epsilon}_0 - \bar{\epsilon}_2 &= \frac{\{\omega_3^2(\omega_1 - \omega_2) + \omega_2^2(\omega_1 - \omega_3)\}\{(\omega_1^2 + \omega_3^2)(\bar{\omega}_2 - \omega_2)\}}{\omega_3^2(\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2)} > 0.\end{aligned}$$

Therefore,

$$\bar{\epsilon}_2 < \bar{\epsilon}_0 < \bar{\epsilon}_1. \tag{A.25}$$

Substituting

$$\frac{a_1}{b_1} = \frac{(\omega_2 - \omega_1)(\omega_2 - \omega_3)}{\omega_2^2} (< 0), \quad \frac{a_2}{b_2} = \frac{(\omega_1 - \omega_3)(\omega_2 - \omega_3)}{\omega_3^2} (> 0)$$

into (A.20), we obtain

$$\frac{(\omega_1 - \omega_3)(\omega_2 - \omega_3)}{\omega_3^2} \geq \frac{(\omega_2 - \omega_3)^2}{\omega_2^2 + \omega_3^2}. \tag{A.26}$$

First, we consider the stability of $\boldsymbol{\lambda}^* = (1, \lambda_2^*, \lambda_3^*)$ ($0 < \lambda_2^* < 1$, $0 < \lambda_3^* < 1$). From $V_{2H}(\boldsymbol{\lambda}^*) - V_{2F}(\boldsymbol{\lambda}^*) = 0$ and $V_{3H}(\boldsymbol{\lambda}^*) - V_{3F}(\boldsymbol{\lambda}^*) = 0$, we have

$$\bar{\lambda}_2 \equiv \lim_{\tau \rightarrow 0} \lambda_2^* = \frac{c(2b - c)^2 \epsilon \omega_2^2 + 16a^2 b^2 (\omega_1 - \omega_2)(\omega_2 - \omega_3)}{2[c(2b - c)^2 \epsilon (\omega_2^2 + \omega_3^2) - 16a^2 b^2 (\omega_2 - \omega_3)^2]}, \tag{A.27}$$

$$\bar{\lambda}_3 \equiv \lim_{\tau \rightarrow 0} \lambda_3^* = \frac{c(2b - c) \epsilon \omega_3^2 - 16a^2 b^2 (\omega_1 - \omega_3)(\omega_2 - \omega_3)}{2[c(2b - c)^2 \epsilon (\omega_2^2 + \omega_3^2) - 16a^2 b^2 (\omega_2 - \omega_3)^2]}. \tag{A.28}$$

The conditions of $\bar{\lambda}_2, \bar{\lambda}_3 \in (0, 1)$ are equivalent to

$$\frac{c(2b - c)^2}{16a^2 b^2} \epsilon > \max\left\{ \frac{(\omega_2 - \omega_3)^2}{\omega_2^2 + \omega_3^2}, \frac{(\omega_1 - \omega_3)(\omega_2 - \omega_3)}{\omega_3^2} \right\},$$

$$\begin{aligned}
& \left. \frac{(\omega_2 - \omega_3)[2(\omega_2 - \omega_3) + (\omega_1 - \omega_2)]}{\omega_2^2 + 2\omega_3^2}, \frac{(\omega_2 - \omega_3)[2(\omega_2 - \omega_3) - (\omega_1 - \omega_3)]}{2\omega_2^2 + \omega_3^2} \right\} \\
& = \frac{(\omega_1 - \omega_3)(\omega_2 - \omega_3)}{\omega_3^2}, \tag{A.29}
\end{aligned}$$

where the last equality holds from (A.26) and

$$\begin{aligned}
& \frac{(\omega_1 - \omega_3)(\omega_2 - \omega_3)}{\omega_3^2} - \frac{(\omega_2 - \omega_3)\{2(\omega_2 - \omega_3) + (\omega_1 - \omega_2)\}}{\omega_2^2 + 2\omega_3^2} \\
& = \frac{(\omega_2 - \omega_3)\{(\omega_1 - \omega_2)\omega_3^2 + (\omega_1 - \omega_3)\omega_2^2\}}{\omega_3^2(\omega_2^2 + 2\omega_3^2)} \geq 0, \\
& \frac{(\omega_2 - \omega_3)\{2(\omega_2 - \omega_3) + (\omega_1 - \omega_2)\}}{\omega_2^2 + 2\omega_3^2} - \frac{(\omega_2 - \omega_3)\{2(\omega_2 - \omega_3) - (\omega_1 - \omega_3)\}}{2\omega_2^2 + \omega_3^2} \\
& = \frac{3(\omega_2 - \omega_3)\{(\omega_1 - \omega_3)\omega_2^2 + (\omega_1 - \omega_2)\omega_3^2\}}{(\omega_2^2 + 2\omega_3^2)(2\omega_2^2 + \omega_3^2)} \geq 0.
\end{aligned}$$

On the other hand, the stability conditions of $\boldsymbol{\lambda}^*$ are given by

$$\begin{aligned}
& V_{1H}(\boldsymbol{\lambda}^*) - V_{1F}(\boldsymbol{\lambda}^*) > 0 \\
& \Leftrightarrow 9bcL \left[\frac{16a^2b^2\{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2\}}{4(2b - c)\{c(2b - c)^2\epsilon(\omega_2^2 + \omega_3^2) - 16a^2b^2(\omega_2 - \omega_3)^2\} + O(\tau)} \right. \\
& \quad \left. - \frac{c(2b - c)^2\epsilon(\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2)}{4(2b - c)\{c(2b - c)^2\epsilon(\omega_2^2 + \omega_3^2) - 16a^2b^2(\omega_2 - \omega_3)^2\} + O(\tau)} \right] \tau^2 \\
& \quad + \frac{O(\tau^3)}{4(2b - c)\{c(2b - c)^2\epsilon(\omega_2^2 + \omega_3^2) - 16a^2b^2(\omega_2 - \omega_3)^2\} + O(\tau)} > 0, \\
& Tr(\Delta_1) > 0 \Leftrightarrow \epsilon + O(\tau) > 0, \\
& |\Delta_1| > 0 \Leftrightarrow \frac{9b\{c(2b - c)^2\epsilon(\omega_2^2 + \omega_3^2) - 16a^2b^2(\omega_2 - \omega_3)^2\}}{4(2b - c)^3} \tau^2 + O(\tau^3) > 0,
\end{aligned}$$

where Δ_i is defined to be $\Delta \equiv (\delta_{ij})_{3 \times 3}$ without the i th row and the i th column. Thus, for sufficiently small τ , $\boldsymbol{\lambda}^*$ is stable if and only if

$$\frac{c(2b - c)^2}{16a^2b^2} \epsilon < \frac{\omega_1^2(\omega_2 - \omega_3)^2 + \omega_2^2(\omega_3 - \omega_1)^2 + \omega_3^2(\omega_1 - \omega_2)^2}{\omega_1^2\omega_2^2 + \omega_2^2\omega_3^2 + \omega_3^2\omega_1^2}, \tag{A.30}$$

and (A.29) hold, which imply that $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_0$. Summarily, $(1, \lambda_2^*, \lambda_3^*)$ is a stable equilibrium if and only if $\epsilon \in (\bar{\epsilon}_2, \bar{\epsilon}_0)$.

Similarly, for sufficiently small τ , the stability conditions for $(\lambda_1^*, \lambda_2^*, 1)$ ($\lambda_1^*, \lambda_2^* \in (0, 1)$) and $(\lambda_1^*, 1, \lambda_3^*)$ ($\lambda_1^*, \lambda_3^* \in (0, 1)$) can be written as

$$\frac{c(2b - c)^2}{16a^2b^2} \epsilon > \frac{(\omega_1 - \omega_2)(\omega_1 - \omega_3)}{\omega_1^2},$$

together with (A.30). These conditions imply that $\bar{\epsilon}_1 < \epsilon < \bar{\epsilon}_0$, which is impossible because of (A.25). In other words, $(\lambda_1^*, \lambda_2^*, 1)$ and $(\lambda_1^*, 1, \lambda_3^*)$ are unstable when $\omega_2 < \bar{\omega}_2$.

Therefore, in the case of $\omega_2 < \bar{\omega}_2$, $\boldsymbol{\lambda}^* = (1, \lambda_2^*, \lambda_3^*)$ is the only stable equilibrium if $\bar{\epsilon}_2 < \epsilon < \bar{\epsilon}_0$, while $\boldsymbol{\lambda}^* = (1, \lambda_2^*, 0)$ is the only stable equilibrium if $\epsilon < \bar{\epsilon}_2$ (by Lemma 5).

Finally, we provide some remarks about the equilibrium $\boldsymbol{\lambda}^* = (1, \lambda_2^*, \lambda_3^*)$. From (A.27), (A.28), and (A.29), we have

$$\bar{\lambda}_2 > \bar{\lambda}_3.$$

Furthermore, we obtain

$$\lambda_2^* + \lambda_3^* = \frac{1}{2} + \frac{12ab^2c\{(\omega_1 - \omega_3)\omega_2^2 + (\omega_1 - \omega_2)\omega_3^2\}\tau + O(\tau^2)}{2\{c(2b - c)^2\epsilon(\omega_2^2 + \omega_3^2) - 16a^2b^2(\omega_2 - \omega_3)^2\} + O(\tau)},$$

so, by (A.29), we conclude that $\lambda_2^* + \lambda_3^*$ converges to 1/2 from above. ■

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