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Proximity and R&D Cooperation between firms: Location, R&D and Output in an Oligopoly with Spillovers

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Abstract

This paper aims at explaining how proximity between firms affects cooperation in R&D. For that purpose, it is proposed a three-stage game amongst three firms where each firm decides about location, R&D and output. Firms' decision about location determines a R&D spillover, which is inversely related to the distance between firms. R&D output is assumed to be cost reducing and exhibit diminishing returns. Cooperation is only allowed in the R&D stage. Our results allows us to conclude that there is a positive relationship between R&D output equilibrium and the distance between firms when firms acts independently. When firms cooperate in R&D, R&D output for a cooperating firm increases with the degree of information sharing between them, as well as with a reduction of the distance between cooperating firms. Firms' decision about location is also affected by R&D activities: if R&D activities run independently, the clustering of firms only occurs for a convex spillover function; if R&D activities run cooperatively, clustering is always observed if there is an increased information sharing between firms.

Keywords: Location; R&D cooperation; R&D spillovers

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1 Introduction

Ever since Marshall (1920 [1890]), it is widely accepted that firms gain from their joint location because they benefit from economies on the transport of goods, people and ideas. However, if firms are rivals in the product market, geographical proximity makes competition between them fiercer and this acts as a centrifugal force. Obviously, the outcome of both centripetal and centrifugal forces depends on their relative strengths. Even if being strong competitors in the product market, firms frequently adopt a cooperative behavior in what concerns, for instance, Research and Development (R&D) activities (e.g. d' Aspremont and Jacquemin (1988) and Kamien *et al.* (1992)). This is generally justified by the public good nature of R&D activities, as firms cannot fully appropriate the returns of their R&D investments, due to the existence of R&D spillovers([1]).

The purpose of this research is to evaluate wether proximity between firms matters for cooperation in R&D. For that end, we will develop a strategic interaction model that merges the topic of firms' location under spatial competition within a R&D cooperation game.

Strategic interaction models typically assume that firms interact strategically with respect to location, as they encompass oligopolistic rivalry (Fujita and Thisse (1996)). This approach finds its roots in the seminal work of Hotelling (1929), according to whom competition for market areas is a centripetal force that would lead firms to congregate, a result known in the literature as the *Principle of Minimum Differentiation*. In a subsequent paper, d' Aspremont *et al.* (1979) demonstrated that the *Principle of Minimum Differentiation* was invalid, and since then, a considerable effort has been devoted to restoring the validity of that principle. For example, by introducing enough heterogeneity in both consumers and/or firms (e.g. Palma *et al.* (1985)), by considering explicitly price collusion (e.g. Friedman and Thisse (1993)) or by recurring to search models (e.g. Schultz and Stahl (1996)).

[1] R&D cooperation is usually justified by the need to internalize spillovers, to capture of economies of scale or complementarities in R&D, as well as potential beneficial effects coming from firms' coordination of research activities and the diffusion of know-how and R&D output among cooperating firms. Against these advantages is the fear that the participating firms may free-ride on other firms, as well as the possibility of reduction of competition in the product market, which would result in a welfare loss.

Recently, strategic interaction models have been extended to capture the topic of firms' location under knowledge spillovers and market competition, although few attempts were made to capture R&D cooperation agreements. Mai and Peng (1999) presented a very simple model of spatial competition à la Hotelling that introduces an element of tacit cooperation through information exchanges between firms that are distance-sensitive. They have shown that equilibrium location can be achieved in a wide range from minimum to maximum differentiation depending upon the relative strength of the cooperation effect over competitive effect. Additionally, they concluded that the larger the externality between firms, the less will be the location differentiation between them. Long and Soubeyran (1998) conditioned R&D spillovers to firms' decision about location and proposed to evaluate the implications of R&D spillovers in the choice of location by Cournot oligopolists. Recurring to a two stage game where R&D acts as a non-decision variable, they exploited how firms' decision about location are affected by the shape of the spillover function and by the decision of firms to cooperate or not in R&D. They concluded that the only Nash equilibrium for a duopoly with symmetric locations is the agglomeration one, while for a oligopoly with asymmetric locations, agglomeration is only guaranteed if the spillover effect is convex in distance. Finally, they considered the possibility of R&D cooperation within a subset of firms and concluded for several equilibria possibilities.

Research on R&D cooperation is typically apart from firms' decisions about location. Usually, cooperative R&D is identified with research collaboration and it is often investigated in the context of two-stage oligopoly models in which firms make their R&D decisions in a first pre-competitive stage and their quantity/price setting in a second stage. The most influential article on R&D cooperation is due to d' Aspremont and Jacquemin (1988), who assumed that there are spillovers in R&D output and concluded that for a large spillover coefficient, the collusive level of R&D was higher than the non-cooperative one. Another prominent work is own to Kamien *et al.* (1992), who proposed spillovers in R&D expenditures and allowed for different R&D organization models that may involve R&D expenditures cartelization and/or full information sharing. They have shown that Research Joint Venture competition was the least desirable model as it yields higher product prices, while RJV cartelization was the most desirable, because it provides the highest consumer plus producer surplus (see Amir (2000) for an analytical comparison of both models).

Since these starting articles, a lot of scientific models emerged around the topic of R&D cooperation.

Some extensions were made to a oligopolistic scenario. This was the case of Suzumura (1992), who concluded that for large spillovers, neither noncooperative nor cooperative equilibria achieve even second-best R&D levels, while in the absence of spillovers effects, the noncooperative equilibrium seems to overshoot the social optimal level while the cooperative R&D does not reach a social optimum. Also in the context of an oligopolist industry but assuming cooperation within a subset of firms, Poyago-Theotoky (1995) demonstrated that there was an inverse relationship between the development of R&D activities and the degree of an exogenous R&D spillover.

Some authors conditioned the R&D spillover within a R&D cartel as a result of a strategic decision made by firms. Katsoulacos and Ulph (1998) tried to examine the effects of a RJV when R&D spillovers are endogenously chosen and demonstrated that noncooperation can produce maximal spillovers. Also, they concluded that a RJV may behave in an anti-competitive way by choosing partial RJV spillovers or by closing a R&D lab. Poyago-Theotoky (1999) considered a typical R&D-output duopoly game but introduced an intermediate stage where each firm decides about how much of the knowledge created in the first stage she will disclose to the other firm. She concluded that R&D cooperation leads firms to engage in more R&D but also make firms fully disclose their information.

Some extensions introduced specificities or asymmetries in R&D spillovers. Vonortas (1994) considered diverse degrees of spillover between cooperating firms, according to the type of research: generic research would produce higher spillovers than specific research. Kamien and Zang (2000) introduced the concept of firms' absorptive capacity, which implies that each firm needs to conduct its own R&D in order to realize spillovers from other firms' R&D activity. They concluded that when firms form a RJV, they choose identical R&D approaches, whilst if they do not cooperate, R&D approaches will be different. Steurs (1995) tried to evaluate the importance of inter-industry R&D spillovers (in addition to the traditional intra-industry spillovers) and concluded that inter-industry R&D spillovers have a very important effect on firms' incentives to invest in R&D both directly and indirectly,

because of their influence on intra-industry R&D spillovers. Also, R&D agreements that cut across industries may be more socially beneficial than cooperatives whose membership comes from a single industry. Amir and Wooders (2000) assumed one-way spillovers, that is, information flows from the firm with higher R&D activity to its rival (but never vice-versa) through a binomial function. They concluded that no equilibrium can be symmetric even if firms were *ex-ante* identical. Thus, an industry configuration emerges with a R&D innovator and a R&D imitator. Additionally, they compared the performance of a RJV with a join lab and pure R&D competition and concluded for the superiority of the joint lab.

The purpose of this paper is to explain how R&D cooperation affects firms' decision about location in the context of competing firms and knowledge spillovers. For that purpose, we developed a three-stage game amongst three firms where firms decide about location and R&D expenditures and after that engage in Cournot competition. Firms' decision about location determines a R&D spillover, which is inversely related to the physical distance between firms. R&D output is assumed to be cost reducing and exhibit diminishing returns. Cooperation is allowed in the R&D stage through R&D cartelization and an increase of the spillover between firms .

The model we developed is related to Long and Soubeyran (1998), but we extended it by introducing an intermediate stage where firms decide about R&D output. This allows us to evaluate the sensitivity of R&D output to the distance between firms, as well as to consider the strategic R&D decision either if firms run R&D independently or in cooperation. Additionally, Long and Soubeyran evaluated how firms' decisions about location are affected by their decisions on R&D cooperation but assuming that firms cooperate in their location decisions if they collusively decide about R&D expenditures. Unlikely, we study the problem of an entrant firm choosing its location among two incumbent firms in a scenario of independent or R&D cooperation, whilst cooperation is only allowed in the R&D game.

Through the resolution of the model, we intend to evaluate if firms' decision about location change if they develop its R&D activities independently or in cooperation. We then proceed with a sketch of the model, which is presented in two scenarios - independent and cooperation in R&D - and end with some brief concluding remarks.

2 The Model

There are N identical firms that produce a homogeneous output, whose inverse demand function is given by

$$P = a - bQ$$

where Q is total output and q_i is firm i's output ($Q = \sum_{i=1}^{N} q_i$) ($a, b > 0$ and $Q \leq a/b$).

Each firm chooses its location in an open convex space M. As a result of location decisions, firms will benefit from a R&D spillover, $\beta(d_{ij})$, which is inversely related with d_{ij}, where $d_{ij} = d(i,j)$ is a measure of the physical distance between firms i and j.

The spillover function is such that $0 \leq \beta(d_{ij}) \leq 1$ and $\beta'(d_{ij}) < 0$, that is, $\beta(d_{ij})$ is a positive and decreasing function of the distance d_{ij} between firms. For convenience, we will simply denote it by $\beta_{ij} = \beta(d_{ij})$.

As it is typical in R&D cooperation models (e.g. d' Aspremont and Jacquemin (1988)), we will assume that R&D output is cost reducing through an additive formulation, that is,

$$c_i = c - x_i - \sum_{t \neq i}^{N} \beta(d_{it})\, x_t$$

where c accounts for stand-alone marginal costs (identical to all firms) ($0 < c < a$) and x_i measures firm i's R&D output. Additionally, it will be assumed that there are diminishing returns to R&D expenditures, that is, $C'(x_i) > 0$ and $C''(x_i) > 0$. In order to ensure positive quantities, we will impose $x_i + \sum_{t \neq i}^{N} \beta(d_{it})\, x_t \leq c$.

The profit of firm i is then given by:

$$\pi_i = (P - c_i)\, q_i - C(x_i)$$

As we focus on the physical distance between firms and its impact on R&D activities through a spillover function, we will neglected transport costs to the product market.

It is proposed a three-stage game, where firms decide about location, R&D and production. The timing is the following:

1^{st}) Firms choose its location in space M, from which results $d_{ij} \in \Re^{+}$ and $\beta_{ij} \in [0,1]$;

2^{nd}) Firms simultaneously choose the level of R&D output, $x_i \in \Re^{+}$, independently or in cooperation;

3^{rd}) Firms simultaneously choose the level of output, $q_i \in \Re^{+}$, through Cournot competition.

For our purposes, we will assume $N = 3$, whilst it can be extended to a larger number of firms through tedious and complex calculations. Additionally, and as in d' Aspremont and Jacquemin (1988), we will consider a specific functional form for the R&D cost function, $C(x_i) = 0.5\gamma x_i^2$.

The game will be solved by backward induction to ensure subgame perfectness and we will consider two alternative scenarios: *Independent R&D*, where firms choose its R&D expenditures independently, and *R&D Cooperation*, where a subset of firms cooperate and coordinate R&D expenditures in order to maximize joint profits.

2.1 Independent R&D

Each firm's profit function is given by:

$$\pi_i(q, x, d) = (a - bQ - c_i)\, q_i - 0.5\gamma x_i^2$$

where $q = (q_i, q_j, q_k)$, $x = (x_i, x_j, x_k)$, $d = (d_{ij}, d_{ik}, d_{jk})$ and $c_i = c - x_i - \beta_{ij} x_j - \beta_{ik} x_k$.

From the Cournot game it is straightforward to determine output equilibrium:

$$q^* = \frac{a - c + \left(3 - \beta_{ij} - \beta_{ik}\right) x_i + \left(3\beta_{ij} - \beta_{jk} - 1\right) x_j + \left(3\beta_{ik} - \beta_{jk} - 1\right) x_k}{4b}$$

and third-stage profit function comes:

$$\begin{aligned} \pi_i\left(q^*, x, d\right) &= \frac{\left(a - c + \left(3 - \beta_{ij} - \beta_{ik}\right) x_i + \left(3\beta_{ij} - \beta_{jk} - 1\right) x_j + \left(3\beta_{ik} - \beta_{jk} - 1\right) x_k\right)^2}{16b} \\ & \quad -0.5\gamma x_i^2 \end{aligned}$$

After taking first-order condition and as firms make a symmetric choice (that is, $x_i = x_j = x_k = x$), we may determine R&D output equilibrium ([2])([3]):

$$x^* = \frac{\left(3 - \beta_{ij} - \beta_{ik}\right)(a - c)}{8b\gamma - \left(3 - \beta_{ij} - \beta_{ik}\right)\left(2\beta_{ij} + 2\beta_{ik} - 2\beta_{jk} + 1\right)}$$

where $8b\gamma - \left(3 - \beta_{ij} - \beta_{ik}\right)\left(2\beta_{ij} + 2\beta_{ik} - 2\beta_{jk} + 1\right) > 0$ in order to ensure an interior and positive solution for R&D output and quantities ([4]).

Proposition 1 *When firms run R&D independently, there is a positive relationship between R&D output equilibrium and the physical distance between firms.*

Proof. The sensitivity of R&D output equilibrium to the physical distance between firms may be evaluated through:

$$\frac{\partial x^*}{\partial \beta_{ij}} \frac{\partial \beta_{ij}}{\partial d_{ij}} = \frac{-(a-c)\left(8b\gamma - 2\left(3 - \beta_{ij} - \beta_{ik}\right)^2\right)}{\left(8b\gamma - \left(3 - \beta_{ij} - \beta_{ik}\right)\left(1 + 2\beta_{ij} + 2\beta_{ik} - 2\beta_{jk}\right)\right)^2} \beta'_{ij}$$

[2] Second order condition implies $b\gamma > \frac{9}{8}$, $\forall \beta_{ij}, \beta_{ik}, \beta_{jk} \in [0, 1]$.

[3] Sufficient condition for the stability of equilibrium requires $b\gamma > 15/8$, $\forall \beta_{ij}, \beta_{ik}, \beta_{jk} \in [0, 1]$.

[4] An interior solution is guaranteed for $b\gamma > 5/8$, $\forall \beta_{ij}, \beta_{ik}, \beta_{jk} \in [0, 1]$.

$$\frac{\partial x^*}{\partial \beta_{ik}} \frac{\partial \beta_{ik}}{\partial d_{ik}} = \frac{-(a-c)\left(8b\gamma - 2\left(3-\beta_{ij}-\beta_{ik}\right)^2\right)}{\left(8b\gamma - \left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^2} \beta'_{ik}$$

Given our assumptions, we have $\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$ and $\beta'_{ij} < 0$, $\beta'_{ik} < 0$. If we assume $8b\gamma - 2\left(3 - \beta_{ij} - \beta_{ik}\right)^2 > 0$ ([5]), then we will have $\partial x^*/\partial \beta_{ij} < 0$ and $\partial x^*/\partial \beta_{ik} < 0$. As a result, $\left(\partial x^*/\partial \beta_{ij}\right)\beta'_{ij} > 0$ and $(\partial x^*/\partial \beta_{ik})\beta'_{ik} > 0$, which means that there is a positive relationship between firms' physical distance and R&D output equilibrium. ▪

This result accords with intuition: as the distance between firms increases, firms will perform a higher R&D output because a lower proportion of its R&D results will flow over the other firms. Two effects give reason to this result. On one side, the inverse relationship between firms' distance and R&D spillovers, $\partial \beta/\partial d < 0$, which derives from our assumptions and can be ascertained for instance, in Mai and Peng (1999) and Long and Soubeyran (1998). On the other side, the well documented negative effect between R&D spillovers and R&D output, $\partial x/\partial \beta < 0$. In fact, several authors concluded that when firms run R&D independently, its R&D output (or expenditure) is higher for a lower R&D spillover. d' Aspremont and Jacquemin (1988) concluded that, for the non-cooperative solution, R&D output decreases with R&D spillover. In an extensive survey on the topic of spillovers and R&D cooperation, Bondt (1997) concluded that with low spillovers, Nash rivals are supposed to invest more in R&D than with high spillovers. Also, he concluded that positive and symmetric intra-industry spillovers tend to reduce the incentive for non-cooperative investments in R&D. Poyago-Theotoky (1995) demonstrated that, if firms run R&D independently, the incentive to carry out R&D is greatly reduced in the presence of high R&D spillovers because the benefits of R&D are common to all firms.

Second-stage profit function then becomes:

$$\pi_i\left(q^*, x^*, d\right) = \frac{0.5\gamma\left(a-c\right)^2\left(8b\gamma - \left(3-\beta_{ij}-\beta_{ik}\right)^2\right)}{\left(8b\gamma - \left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^2} \tag{1}$$

In the location game, we will consider the problem of a single firm i choosing its

[5] This assumption only recquires that $b\gamma > 2.25$, $\forall \beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$.

location in a convex space where firms j and k were located. So, firm i must choose d_{ij} and d_{ik}, given d_{jk}.

Formally, given other firms' location, firm i must choose its location subject to the following triangle inequality:

$d_{ij} + d_{ik} \geq d_{jk}$

Additionally, and as $d_{jk} \geq 0$, we must have:

$d_{ik} \geq 0$

$d_{ij} \geq 0$

In order to get more interesting results, we will impose, without loss of generality:

$d_{ik} - d_{ij} \geq 0$

If we assume $d_{ik} - d_{ij} \geq 0$ and $d_{ij} \geq 0$, then $d_{ik} \geq 0$ is always true, and so, this restriction may be avoided.

Firm i will then solve the following problem:

$$\max_{d_{ij}, d_{ik} \in M} \pi_i \left(q^*, x^*, d\right)$$

$$s.t. \qquad g^1(d) = \ d_{ij} + d_{ik} - d_{jk} \geq 0$$

$$g^2(d) = d_{ik} - d_{ij} \geq 0$$

$$g^3(d) = d_{ij} \geq 0$$

The Lagrangian function corresponding to the maximization problem is then defined

by:

$$L\left(d,\lambda\right)=\pi_i\left(q^*,x^*,d\right)+\lambda_1\left(d_{ij}+d_{ik}-d_{jk}\right)+\lambda_2\left(d_{ik}-d_{ij}\right)+\lambda_3\left(d_{ij}\right)$$

where $\lambda=(\lambda_1,\lambda_2,\lambda_3)$ and $d=(d_{ij},d_{ik},d_{jk})$.

Through Kuhn-Tucker conditions, we have:

$$\frac{\partial L(d,\lambda)}{\partial d_{ij}}=\frac{\partial\pi_i^*}{\partial\beta_{ij}}\frac{\partial\beta_{ij}}{\partial d_{ij}}+\lambda_1-\lambda_2+\lambda_3=0 \tag{2}$$

$$\frac{\partial L(d,\lambda)}{\partial d_{ik}}=\frac{\partial\pi_i^*}{\partial\beta_{ik}}\frac{\partial\beta_{ik}}{\partial d_{ik}}+\lambda_1+\lambda_2=0 \tag{3}$$

with $\lambda_1\geq 0,\lambda_2\geq 0,\lambda_3\geq 0,g^1\left(d\right)\geq 0,g^2\left(d\right)\geq 0,g^3\left(d\right)\geq 0$ and $\lambda_1\left(d_{ij}+d_{ik}-d_{jk}\right)=0,\lambda_2\left(d_{ik}-d_{ij}\right)=0,\lambda_3\left(d_{ij}\right)=0$.

From (1) and through simple arithmetic, we get:

$$\frac{\partial\pi_i^*}{\partial\beta_{ij}}\frac{\partial\beta_{ij}}{\partial d_{ij}}=\frac{2\gamma\left(a-c\right)^2\left(8b\gamma\left(4-2.5\beta_{ij}-2.5\beta_{ik}+\beta_{jk}\right)-\left(3-\beta_{ij}-\beta_{ik}\right)^3\right)}{\left(8b\gamma-\left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^3}\beta'_{ij} \tag{4}$$

$$\frac{\partial\pi_i^*}{\partial\beta_{ik}}\frac{\partial\beta_{ik}}{\partial d_{ik}}=\frac{2\gamma\left(a-c\right)^2\left(8b\gamma\left(4-2.5\beta_{ij}-2.5\beta_{ik}+\beta_{jk}\right)-\left(3-\beta_{ij}-\beta_{ik}\right)^3\right)}{\left(8b\gamma-\left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^3}\beta'_{ik} \tag{5}$$

Given our assumptions, we have $\beta_{ij},\beta_{ik},\beta_{jk}\in[0,1]$ and $\beta'_{ij}<0$, $\beta'_{ik}<0$. Additionally, if we assume $8b\gamma\left(4-2.5\beta_{ij}-2.5\beta_{ik}+\beta_{jk}\right)-\left(3-\beta_{ij}-\beta_{ik}\right)^3>0$([6]), then $\left(\partial\pi_i^*/\partial\beta_{ij}\right)\beta'_{ij}<0$ and $\left(\partial\pi_i^*/\partial\beta_{ik}\right)\beta'_{ik}<0$([7]).

[6] We have $8b\gamma\left(4-2.5\beta_{ij}-2.5\beta_{ik}+\beta_{jk}\right)-\left(3-\beta_{ij}-\beta_{ik}\right)^3>0$ for $b\gamma\geq 5$ and $\beta_{ij}+\beta_{ik}<1$. 570 8. According Amir (2000) and after checking the consistency between the additive nature of the spillover process, he concluded that the model of d' Aspremont and Jacquemin (1988) would be of questionable validity for large spillover values, so that $\beta^{\max}=\left(\sqrt{n+1}\right)^{-1}$. For our particular case, $\beta^{\max}=0.336$.

[7] Previous research on this topic confirms these results (e.g. Long and Soubeyran (1998) established that $\partial\pi_i/\partial\beta_{ij}>0$ and $\partial\pi_i/\partial\beta_{ik}>0$).

Allowing for different location choices, we will evaluate the best location for the entrant firm, assuming first, that the incumbent firms were agglomerated and second, that the incumbent firms were dispersed. We may then formulate the following propositions:

Proposition 2 *If two firms were close located and no cooperation is allowed, then the equilibrium location for an entrant firm is agglomeration.*

Proof. Assume $d_{jk} = 0$, that is, firms j and k are close located in space M. Under this scenario, we may have agglomeration ($d_{ij} = d_{ik} = 0$) or dispersion ($d_{ij} = d_{ik} > 0$) equilibrium. As $d_{ij} = d_{ik}$, then $\beta_{ij} = \beta_{ik} = \beta$ and we will have:

$$(4)\ (5)\ \left(\partial \pi_i^* / \partial \beta_{ij}\right) \beta'_{ij} = \left(\partial \pi_i^* / \partial \beta_{ik}\right) \beta'_{ik} = \frac{2\gamma(a-c)^2\left(8b\gamma\left(4-5\beta+\beta_{jk}\right)-(3-2\beta)^3\right)}{\left(8b\gamma-(3-2\beta)\left(1+4\beta-2\beta_{jk}\right)\right)^3}\beta' < 0$$

So, if firms j and k were close located and the spillover coefficient is inversely related to the physical distance between firms, then firm i will choose to be as close as possible to incumbent firms. ▪

Example 1 *Suppose* $a = 100, b = 1, c = 50$ *and* $\gamma = 5$. *Then, firm i's profit function becomes:*

$$\pi_i^* = 6250\frac{40-(3-\beta(d_{ij})-\beta(d_{ik}))^2}{\left(40-\left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^2}$$

For simplicity, let's assume $d_{ij}, d_{ik}, d_{jk} \in [0, 1]$. *If firms j and k were close located, then* $d_{jk} = 0$ *and* $d_{ij} = d_{ik}$. *In this case, we could have agglomeration* ($d_{ij} = d_{ik} = 0$) *or dispersion* ($d_{ij} = d_{ik} = 1$). *Let's consider a linear spillover function,* $\beta(d) = 0.75(1 - d)$, *whilst similar results would be gathered with different spillover functions. Then, we have:*

$$\pi_i^*(\ d_{ij} = d_{ik} = 0) = 179.55 > \pi_i^*(\ d_{ij} = d_{ik} = 1) = 112.50$$

So, given that firms j and k were agglomerated, then the best strategy for firm i is to join them.

Note that, as $d_{jk} = 0$ and $\beta'_{ij} < 0$ and $\beta'_{ik} < 0$, the entrant firm will always prefer to reduce its distance to both firms in order to benefit from maximal spillovers. Then, if the incumbent firms were agglomerated, the entrant firm will join them. But what will happen if the incumbent firms were dispersed?

Proposition 3 *If two firms were dispersed in a convex space M and no cooperation is allowed, then the equilibrium location for an entrant firm is in the straight-line between incumbent firms, that is:*

$d_{ij} + d_{ik} = d_{jk}$

Proof. Assume false, that is, suppose that firm i chooses to locate in the vertices of a triangle. In this case, $g^1(d)$ is non-binding and $\lambda_1 = 0$. Then, we may have one of the following situations: either (a) $d_{ik} = d_{ij}$ or (b) $d_{ik} > d_{ij}$.

(a)If $d_{ik} = d_{ij}$, $g^2(d)$ is binding, with the associated multiplier non-zero ($\lambda_2 \geq 0$), while the remaining restrictions are non-binding. Then, Kuhn-Tucker conditions may be resumed to:

(2) $\left(\partial \pi_i^* / \partial \beta_{ij}\right) \beta'_{ij} = \lambda_2 \geq 0$

(3) $\left(\partial \pi_i^* / \partial \beta_{ik}\right) \beta'_{ik} = -\lambda_2 \leq 0$

From (4) and (5), we may conclude that Kuhn-Tucker conditions are not observed.

(b) If $d_{ik} > d_{ij}$, all restrictions are non-binding and Kuhn-Tucker conditions become:

(2) $\left(\partial \pi_i^* / \partial \beta_{ij}\right) \beta'_{ij} = 0$

(3) $\left(\partial \pi_i^* / \partial \beta_{ik}\right) \beta'_{ik} = 0$

From (4) and (5), we have that Kuhn-Tucker conditions are not observed. ∎

This proposition is quite intuitive: as the spillover coefficient is a decreasing function of the physical distance between firms, firm i will proceed to be as close as possible to both firms, and so, she will be located in the straight-line between them. The exact location will be sketch in the following proposition:

Proposition 4 *If two firms were dispersed in a convex space M and no cooperation is allowed, then the equilibrium location for an entrant firm depends on the shape of the spillover function:*

(i) If β is a strictly concave function of the distance between firms, then the entrant firm will be located exactly in between incumbent firms;

(ii) If β is a linear function of the distance between firms, then any location along the straight line is possible;

(iii) If β is a strictly convex function of the distance between firms, then the entrant firm will choose to cluster with one of the firms.

Proof. From previous proposition, we have that $d_{ij} + d_{ik} = d_{jk}$. Then, we may have one of the following mutually exclusive locations:

(a) Firm i locates at the same local of firm j and away from firm k

In this case, $d_{ij} = 0$, and so, $g^1(d)$ and $g^3(d)$ are binding. Kuhn-Tucker conditions then come:

(2) $(\partial \pi_i^* / \partial \beta_{ij}) \beta'_{ij} = -\lambda_1 - \lambda_3 \leq 0$

(3) $(\partial \pi_i^* / \partial \beta_{ik}) \beta'_{ik} = -\lambda_1 \leq 0$

As (4) and (5), Kuhn-Tucker conditions are confirmed. Additionally, through simple arithmetic manipulation, we have:

[(2)-(3)] $(\partial \pi_i^* / \partial \beta_{ij}) \beta'_{ij} - (\partial \pi_i^* / \partial \beta_{ik}) \beta'_{ik} = -\lambda_3 \leq 0$

$$[(4)\text{-}(5)]\ (\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij}-(\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik}=\frac{2\gamma(a-c)^2\left(8b\gamma\left(4-2.5\beta_{ij}-1.5\beta\right)-\left(3-\beta_{ij}-\beta\right)^3\right)}{\left(8b\gamma-\left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^3}\left(\beta'_{ij}-\beta'_{ik}\right)$$

As $d_{ij} < d_{ik}$ and $\beta_{ij} > \beta_{ik}$, we can only have $\beta'_{ij} - \beta'_{ik} \leq 0$ for a convex or linear spillover function.

(b) Firm i locates along the straight line joining j and k but nearer to firm j

In this case, $0 < d_{ij} < d_{ik}$ and so, only $g^1(d)$ is binding. Kuhn-Tucker conditions then come:

$$(2)\ (\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij}=-\lambda_1\leq 0$$

$$(3)\ (\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik}=-\lambda_1\leq 0$$

As (4) and (5), Kuhn-Tucker conditions are confirmed. Additionally,

$$[(2)\text{-}(3)]\ (\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij}-(\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik}=0$$

$$[(4)\text{-}(5)]\ (\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij}-(\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik}=\frac{2\gamma(a-c)^2\left(8b\gamma\left(4-2.5\beta_{ij}-2.5\beta_{ik}+\beta_{jk}\right)-\left(3-\beta_{ij}-\beta_{ik}\right)^3\right)}{\left(8b\gamma-\left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^3}\left(\beta'_{ij}-\beta'_{ik}\right)$$

which are only compatible for a linear spillover function.

(c) Firm i locates exactly at the middle point of the straight line joining j and k

In this case, $d_{ij} = d_{ik} > 0$ and so, $g^1(d)$ and $g^2(d)$ are binding. Kuhn-Tucker conditions then come:

$$(2)\ (\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij}=-\lambda_1+\lambda_2$$

$$(3)\ (\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik}=-\lambda_1-\lambda_2\leq 0$$

From (4) and (5), Kuhn-Tucker conditions are confirmed. Additionally,

[(2)-(3)] $(\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij} - (\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik} = 0$

[(4)-(5)] $(\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij} - (\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik} = \frac{2\gamma(a-c)^2\left(8b\gamma\left(4-5\beta+\beta_{jk}\right)-(3-2\beta)^3\right)}{\left(8b\gamma-(3-2\beta)\left(1+4\beta-2\beta_{jk}\right)\right)^3}\left(\beta'_{ij}-\beta'_{ik}\right)$

which are compatible for any shape of the spillover function as $\beta'_{ij} = \beta'_{ik}$. ▪

Our results allow us to conclude that the clustering of firms is only possible for a linear or convex spillover function. In fact, for the case of a strictly concave spillover function, no clustering is observed, as the entrant firm always choose to stay in between the two incumbent firms. This conclusion is simply justified by the shape of the spillover function: for the convex case, the spillover is decreasing with the distance at increasing rates, which means that the distance between firms is costly at increasing rates, and so, the entrant firm will prefer to cluster with an incumbent firm; if the spillover in concave, then the spillover is decreasing with the distance at decreasing rates, and so, the entrant firm will prefer to locate in between the two incumbent firms.

Finally, let's make use of an example to fully clarify our proposition:

Example 2 *Assume* $a = 100, b = 1, c = 50$ *and* $\gamma = 5$. *Then, firm i's profit function becomes:*

$$\pi_i^* = 6250\frac{40-(3-\beta(d_{ij})-\beta(d_{ik}))^2}{\left(40-\left(3-\beta_{ij}-\beta_{ik}\right)\left(1+2\beta_{ij}+2\beta_{ik}-2\beta_{jk}\right)\right)^2}$$

For simplicity, let's assume $d_{ij}, d_{ik}, d_{jk} \in [0,1]$ *and* $d_{ij} + d_{ik} = d_{jk} = 1$. *In this case, we could have agglomeration* $(d_{ij} = 0; d_{ik} = 1)$ *or dispersion* $(d_{ij} = d_{ik} = 0.5)$. *Considering diverse spillover functions, we will have different results for diverse shapes of the function:*

For a linear spillover function, $\beta(d) = 0.75(1-d)$, *agglomeration and dispersion are equivalent:* $\pi_i^*(\ d_{ij} = d_{ik} = 0.5) = \pi_i^*(\ d_{ij} = 0; d_{ik} = 1) = 184.79$

For a convex spillover function, $\beta(d) = 0.75\left(1-\sqrt{d}\right)$, *agglomeration is the best strategy for firm i:* $\pi_i^*(d_{ij} = d_{ik} = 0.5) = 168.80 < \pi_i^*(\ d_{ij} = 0; d_{ik} = 1) = 184.79$

For a concave spillover function, $\beta(d) = 0.75(1 - d^2)$*, dispersion is the best strategy for firm i:* $\pi_i^*(\ d_{ij} = d_{ik} = 0.5) = 198.35 > \pi_i^*(\ d_{ij} = 0; d_{ik} = 1) = 184.79$.

Our results also accords with some previous works that take under consideration the topic of firms' location within R&D models. Leaving out R&D choice and focusing on the location of firms, Long and Soubeyran (1998) concluded that the location of firms is sensitive to the shape of the spillover function. In fact, they observed that the clustering of firms only emerges with a convex spillover function, which confirms our results.

In the next section, we will sketch a similar problem, but assuming that a subset of firms may cooperate in R&D, whilst cooperation in the location game is avoided. Our purpose is to evaluate if the entrant firm changes its location's decision if she intends to cooperate with one of the incumbent firms.

2.2 Cooperation in R&D

Cooperation in R&D may involve different dimensions and run through different design models. Typically, cooperation involves R&D cartelization, that is, the coordination of R&D expenditures in order to maximize joint profits. Most of the literature usually assumes industry-wide agreements (e.g., d' Aspremont and Jacquemin (1988), Kamien *et al.* (1992), Suzumura (1992), Vonortas (1994)), while others analyze cooperation within a subset of firms of a given industry (e.g., Poyago-Theotoky (1995)). Frequently, the size of the R&D cartel is exogenous, whilst few papers aim at endogenize it (e.g., Katz (1986), Atallah (2001)). Besides R&D cartelization, cooperating firms may jointly agree to internally raise the spillover parameter. In the limit, the sharing of R&D results could be set to its maximal value, a scenario described by Kamien *et al.* (1992) as cartelized RJV. Typically, the degree of information sharing between cooperating firms is assumed to be exogenous, while some articles aim at endogeneize it (e.g., Katsoulacos and Ulph (1998), Poyago-Theotoky (1999), Piga and Poyago-Theotoky (2001) and R. Amir and Wooders (2003)).

In this essay, we will assume that a subset of firms cooperate and form a R&D

cartel, whilst cooperation in the location decision or the production stage is never allowed. In our approach, R&D cooperation involves both R&D cartelization and enhancement of the information sharing between cooperating firms. Formally, this may be modelled through an increase of the spillover coefficient through a multiplier $\delta \geq 1$, where $\delta\beta \leq 1$. Throughout this approach, we intend to separate the effect of the location's decision (β) from the effect of the cooperation decision (δ) on total spillover coefficient, $\delta\beta$. Having in mind Kamien *et al.* (1992)'s typification, we will have, for the case of RJV cartelization, $\delta\beta = 1$, while for the R&D cartelization case, we will have $\delta = 1$ and $\delta\beta = \beta$.

Assumefirms i and j decide to cooperate in R&D. Unit production costs then become:

$$c_i = c - x_i - \delta\beta_{ij}x_j - \beta_{ik}x_k$$

$$c_j = c - x_j - \delta\beta_{ij}x_i - \beta_{jk}x_k$$

$$c_k = c - x_k - \beta_{ik}x_i - \beta_{jk}x_j$$

where $\delta\beta_{ij} \leq 1$.

From the Cournot game we get output equilibrium and throughout firms' third-stage profit function:

$$\pi_i\left(q^*, x, d\right) = \frac{\left(a - c + \left(3 - \delta\beta_{ij} - \beta_{ik}\right)x_i + \left(3\delta\beta_{ij} - \beta_{jk} - 1\right)x_j + \left(3\beta_{ik} - \beta_{jk} - 1\right)x_k\right)^2}{16b} - 0.5\gamma x_i^2$$

where $d = (d_{ij}, d_{ik}, d_{jk})$ and $x = (x_i, x_j, x_k)$.

In the R&D stage, cooperation implies that each firm within the R&D cartel will choose its R&D output in order to maximize joint profits, while non-cooperating

firms will maximize individual profit:

$$\frac{\partial}{\partial x_i}\left[\pi_i\left(q^*, x, d\right) + \pi_j\left(q^*, x, d\right)\right] = 0$$

$$\frac{\partial}{\partial x_k}\left[\pi_k\left(q^*, x, d\right)\right] = 0$$

Imposing symmetry between cooperating firms (that is, $x_i = x_j = x =$ R&D output equilibrium for a cooperating firm) and non-cooperating firms (that is, $x_k = y =$ R&D output equilibrium for a non-cooperating firm) and solving for x and y gives us R&D output equilibrium ([8])([9]):

$$x^* = [(a-c)\left(4b\gamma\left(1-\beta_{ik}+\delta\beta_{ij}\right)-\left(3-\beta_{ik}-\beta_{jk}\right)\left(\beta_{ik}^2-5\beta_{ik}+\delta\beta_{ij}\beta_{ik}+\beta_{ik}\beta_{jk}+\right.\right.$$
$$\left.\left.\beta_{jk}+2\delta\beta_{ij}-3\delta\beta_{ij}\beta_{jk}+2\right)\right)]/[2b\gamma(8b\gamma-13+12\beta_{ik}+8\beta_{jk}-8\delta\beta_{ij}-4\beta_{ik}\beta_{jk}+6\delta\beta_{ij}\beta_{ik}+$$
$$2\delta\beta_{ij}\beta_{jk}-3\beta_{ik}^2-\beta_{jk}^2-4\delta^2\beta_{ij}^2)+(-3+\beta_{ik}+\beta_{jk})(-\beta_{ik}^3-5\delta\beta_{ij}\beta_{ik}^2-2\beta_{ik}^2\beta_{jk}+7\beta_{ik}^2+$$
$$2\beta_{ik}\beta_{jk}+4\delta^2\beta_{ij}^2\beta_{ik}+2\delta\beta_{ij}\beta_{ik}-4\delta^2\beta_{ij}^2\beta_{jk}+2\delta\beta_{ij}\beta_{ik}\beta_{jk}-\beta_{ik}\beta_{jk}^2+7\delta\beta_{ij}\beta_{jk}^2-2\delta^2\beta_{ij}^2-$$
$$2+4\beta_{jk}-4\delta\beta_{ij}-5\beta_{jk}^2-2\beta_{ik})]$$

$$y^* = [2\left(a-c\right)\left(3-\beta_{ik}-\beta_{jk}\right)\left(b\gamma-\left(1-\beta_{ik}+\delta\beta_{ij}\right)\left(\delta\beta_{ij}-\beta_{jk}+1-\beta_{ik}\right)\right)]/[2b\gamma$$
$$(8b\gamma-13+12\beta_{ik}+8\beta_{jk}-8\delta\beta_{ij}-4\beta_{ik}\beta_{jk}+6\delta\beta_{ij}\beta_{ik}+2\delta\beta_{ij}\beta_{jk}-3\beta_{ik}^2-\beta_{jk}^2-4\delta^2\beta_{ij}^2)+$$
$$(-3+\beta_{ik}+\beta_{jk})(-\beta_{ik}^3-5\delta\beta_{ij}\beta_{ik}^2-2\beta_{ik}^2\beta_{jk}+7\beta_{ik}^2+2\beta_{ik}\beta_{jk}+4\delta^2\beta_{ij}^2\beta_{ik}+2\delta\beta_{ij}\beta_{ik}-$$
$$4\delta^2\beta_{ij}^2\beta_{jk}+2\delta\beta_{ij}\beta_{ik}\beta_{jk}-\beta_{ik}\beta_{jk}^2+7\delta\beta_{ij}\beta_{jk}^2-2\delta^2\beta_{ij}^2-2+4\beta_{jk}-4\delta\beta_{ij}-5\beta_{jk}^2-2\beta_{ik})]$$

Literature usually refers that R&D output equilibrium for cooperating firms is higher than R&D output equilibrium for non-cooperating firms when R&D spillovers are high because in this case, it can avoid resources duplication (see, at this purpose, Katsoulacos and Ulph (1998), Bondt and Henriques (1995), Steurs (1995) and Bondt (1997)).

Simple simulations on R&D output equilibrium for cooperating and non-cooperating firms also confirms this conclusion. In fact, assuming $b = 1, \gamma = 5$ and $\beta_{ij} \geq \beta_{ik} \geq \beta_{jk}$ (as results from our assumptions), we have:

[8] Second order condition recquires $b\gamma > \frac{5}{4}, \forall\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$ and $\delta\beta_{ij} \leq 1$.

[9] Sufficient condition for the stability of equilibrium recquires $b\gamma > \frac{5}{2}, \forall\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$ and $\delta\beta_{ij} \leq 1$.

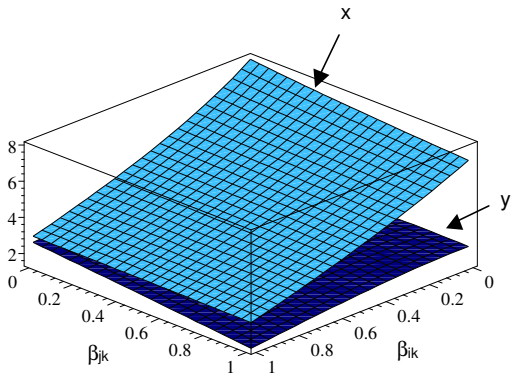

Figure 4: R&D output equilibrium for cooperating (x) and non-cooperating (y) firms when δβ=1.

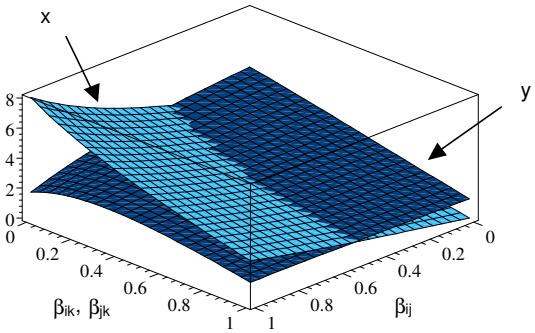

Figure 5: R&D output equilibrium for cooperating (x) and non-cooperating (y) firms when δ=1 and $\beta_{ik} = \beta_{jk}$.

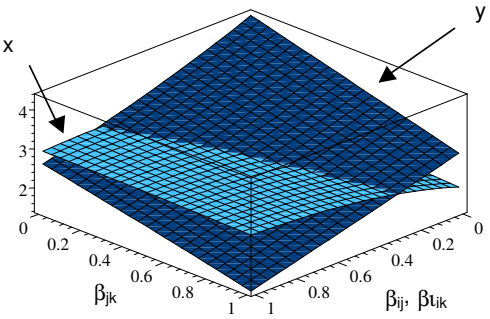

Figure 6: R&D output equilibrium for cooperating (x) and non-cooperating (y) firms when δ=1 and $\beta_{ij} = \beta_{ik}$.

So, we have that for the RJV cartelization case $\left(\delta\beta_{ij} = 1\right)$, R&D output for cooperating firms is always higher than R&D output for non-cooperating firms. However, if there is no increasing of the information sharing between cooperating firms ($\delta = 1$)

and comparing the clustering location $\left(\beta_{ik} = \beta_{jk}\right)$ with the middle point location $\left(\beta_{ij} = \beta_{ik}\right)$, then R&D output equilibrium for cooperating firms is higher than R&D output equilibrium for non-cooperating firms only for high spillovers between cooperating firms.

Let's now evaluate how R&D output is sensitive to the distance between firms.

Proposition 5 *When a subset of firms cooperate in R&D, its R&D output equilibrium will be higher for a higher degree of information sharing and for a lower physical distance between them.*

Proof. Given our assumptions, we have $\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$, $\beta'_{ij} < 0$, $\beta'_{ik} < 0$ and $\delta\beta_{ij} \leq 1$. Then, for $b\gamma > 2.5328$, we have $\partial x/\partial\delta = \partial x/\partial\beta_{ij} > 0$ and then $\left(\partial x/\partial\beta_{ij}\right)\beta'_{ij} < 0$. ▪

These results are quite intuitive and find confirmation in related literature. In fact, and leaving apart the inverse relationship between firms' distance and R&D spillovers, the positive relationship between R&D output equilibrium and the spillover between cooperating firms is well documented in R&D texts. After evaluating different R&D design models, Kamien *et al.* (1992) found that R&D effective output is higher in the RJV cartelization case, where $\delta\beta = 1$, when comparing with simple R&D cartelization, where $\delta\beta = \beta$. Comparing a secretariat RJV (with $\delta\beta = \beta$) with a operating RJV (with $\delta\beta = 1$), Vonortas (1994) concluded that the operating entity is more effective than the secretariat in improving firm's performance over the non-cooperative industry. Particularly, he observed that the operating entity members always invest more in R&D than the members of a secretariat, even when they both spend less than the non-cooperative case. Bondt (1997) reached that cooperative R&D investments are typically stimulated by larger spillovers.

The following corollary completes previous proposition:

Corollary 1 *R&D output equilibrium for cooperating firms increases with the distance to non-cooperating firms.*

Proof. Given our assumptions, we have $\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$, $\beta'_{ij} < 0$, $\beta'_{ik} < 0, \beta'_{jk} < 0$ and $\delta\beta_{ij} \leq 1$. Then, for $b\gamma > 2.6768$, we have $(\partial x/\partial\beta_{ik})\,\beta'_{ik} > 0$. Similarly, for $b\gamma > 2.5328$, we have $\left(\partial x/\partial\beta_{jk}\right)\beta'_{jk} > 0$. ▪

When taking under consideration the distance between cooperating and non-cooperating firms, proposition 1 remains valid: R&D output equilibrium for cooperating firms reduces when the distance from cooperating firms to non-cooperating firms is shorter. Similarly, we expect that R&D output equilibrium for non-cooperating firms increases with the distance from non-cooperating to cooperating firms. This behavior is justified by the need to avoid that R&D output spillovers to competing firms.

Second-stage profit function then becomes:

$$\pi_i\left(q^*, x^*, y^*, d\right) = \frac{\left(a - c + \left(2 + 2\delta\beta_{ij} - \beta_{ik} - \beta_{jk}\right)x^* + \left(3\beta_{ik} - \beta_{jk} - 1\right)y^*\right)^2}{16b} - 0.5\gamma\left(x^*\right)^2 \tag{6}$$

In the location decision game, we will consider again the problem of a single firm i choosing its location in a delimited space where firms j and k were located. Note that cooperation is not allowed in the location problem, and so, the formulation of the problem is very similar to the independent case:

$$\max_{d_{ij}, d_{ik} \in M} \pi_i\left(q^*, x^*, y^*, d\right)$$

$$s.t. \qquad g^1(d) = d_{ij} + d_{ik} - d_{jk} \geq 0$$

$$g^2(d) = d_{ik} - d_{ij} \geq 0$$

$$g^3(d) = d_{ij} \geq 0$$

Applying Kuhn-Tucker conditions to the Lagrangian function gives us similar expression to (2) and (3). Through tedious calculations we have, for $\beta'_{ij} < 0$, $\beta'_{ik} < 0$,

$\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]\,, \delta \geq 1$, $\delta\beta_{ij} \leq 1$ and assuming $b\gamma > 2.5328$, that:

$$\frac{\partial \pi^*}{\partial \beta_{ij}} \frac{\partial \beta_{ij}}{\partial d_{ij}} \beta'_{ij} < 0 \tag{7}$$

$$\frac{\partial \pi^*}{\partial \beta_{ik}} \frac{\partial \beta_{ik}}{\partial d_{ik}} \beta'_{ik} \gtreqless 0 \tag{8}$$

After evaluating different scenarios for location, we may formulate the following propositions:

Proposition 6 *If two firms were close located and cooperation in the R&D stage is allowed, the equilibrium location for an entrant firm is agglomeration.*

Proof. Assume $d_{jk} = 0$. Under this assumption, firm i have two hypothesis: either firm i chooses to locate near j and k ($d_{ij} = d_{ik} = 0$) or firm i chooses to locate away from firms j and k ($d_{ij} = d_{ik} > 0$).

(a) Assume agglomeration equilibrium. In this case, all restrictions are binding with the associated multipliers non negative. Kuhn-Tucker conditions then come:

(2) $(\partial \pi_i^* / \partial \beta_{ij}) \beta'_{ij} = -\lambda_1 + \lambda_2 - \lambda_3$

(3) $(\partial \pi_i^* / \partial \beta_{ik}) \beta'_{ik} = -\lambda_1 - \lambda_2 \leq 0$

From (7), no contradiction is observed.

(b) Assume dispersion equilibrium. In this case, we will have only one binding restriction ($g^2(d) = 0$), with the associated multiplier non negative. From Kuhn-Tucker conditions we get

(2) $(\partial \pi_i^* / \partial \beta_{ij}) \beta'_{ij} = \lambda_2 \geq 0$

(3) $(\partial \pi_i^* / \partial \beta_{ik}) \beta'_{ik} = -\lambda_2 \leq 0$

From (7), we may conclude that Kuhn-Tucker conditions are not confirmed. So, dispersion is not an equilibrium. ▪

Next example will complement previous proposition:

Example 3 *Suppose* $a = 100, b = 1, c = 50$ *and* $\gamma = 5$. *For simplicity, let's assume* $d_{ij}, d_{ik}, d_{jk} \in [0, 1]$. *If firms* j *and* k *were close located, then* $d_{jk} = 0$ *and* $d_{ij} = d_{ik}$. *In this case, we could have agglomeration* $(d_{ij} = d_{ik} = 0)$ *or dispersion* $(d_{ij} = d_{ik} = 1)$. *As before, we considered a linear spillover function,* $\beta(d) = 0.75(1 - d)$, *whilst similar results would be gathered with diverse spillover functions. For different degrees of information sharing, we have:*

Minimum information sharing $(\delta = 1)$*:*

$$\pi_i^*(\ d_{ij} = d_{ik} = 0) = 180.99 > \pi_i^*(\ d_{ij} = d_{ik} = 1) = 126.62$$

Maximum information sharing $(\delta = 1/0.75)$*:*

$$\pi_i^*(\ d_{ij} = d_{ik} = 0) = 192.91 > \pi_i^*(\ d_{ij} = d_{ik} = 1) = 126.62$$

So, given that the incumbent firms were agglomerated, then the best strategy for the entrant-cooperating firm is to join them.

So, if two firms are joint located, the equilibrium location for a cooperating firm is agglomeration. But whatever if they were geographically separated? As in the independent case, the equilibrium location for the entrant firm is in the straight-line between incumbent firms:

Proposition 7 *If two firms were dispersed in a convex space* M *and cooperation is allowed in the R&D stage, then the entrant firm locates in the straight-line between incumbent firms, that is,*

$d_{ij} + d_{ik} = d_{jk}$

Proof. Assume false, that is, suppose that firm i chooses to locate in the vertices of a triangle. In this case, $g^1(d)$ is non-binding. Then, we may have one of the following mutually exclusives situations:

(a) Firm i chooses to locate at the same distance to both firms ($d_{ij} = d_{ik}$). In this case, we will have one binding restriction ($g^2(d) = 0$), while $g^1(d)$ and $g^3(d)$ are non-binding. Then Kuhn-Tucker conditions are resumed to:

(2) $(\partial \pi_i^* / \partial \beta_{ij}) \beta'_{ij} = \lambda_2 \geq 0$

(3) $(\partial \pi_i^* / \partial \beta_{ik}) \beta'_{ik} = -\lambda_2 \leq 0$

From (7), we verify that Kuhn-Tucker conditions are not confirmed.

(b) Firm i chooses to locate closer to firm j ($d_{ij} > d_{ik}$) .In this case, all restrictions are non-binding and the associated multipliers are zero. Kuhn-Tucker conditions come:

(2) $(\partial \pi_i^* / \partial \beta_{ij}) \beta'_{ij} = 0$

(3) $(\partial \pi_i^* / \partial \beta_{ik}) \beta'_{ik} = 0$

From (7), we conclude that Kuhn-Tucker conditions are not observed. ▪

The exact location of the entrant-cooperating firm will be sketch in the following proposition:

Proposition 8 *If two firms intend to cooperate in R&D through joint profit maximization and increased information sharing, then clustering is always observed.*

Proof. Assume false, that is, assume firm i chooses to locate exactly at the middle point of the straight line joining j and k. Then, we have $d_{ij} = d_{ik}$, whilst $d_{ij} + d_{ik} = d_{jk} > 0$.

Kuhn Tucker conditions are resumed to:

(2) $(\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij} = -\lambda_1 + \lambda_2$

(3) $(\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik} = -\lambda_1 - \lambda_2 \leq 0$

From (7), no contradiction emerges. However, through simple calculations, we have:

[(2)-(3)] $(\partial\pi_i^*/\partial\beta_{ij})\beta'_{ij} - (\partial\pi_i^*/\partial\beta_{ik})\beta'_{ik} = 2\lambda_2 \geq 0$

Having in mind that $\beta_{ij}, \beta_{ik}, \beta_{jk} \in [0,1]$ and assuming $b\gamma > 2.5328$, we have, for $\delta > 1$, $\delta\beta_{ij} \leq 1$ and $\beta_{ij} = \beta_{ik}$, $\left(\partial\pi_i^*/\partial\beta_{ij}\right) - (\partial\pi_i^*/\partial\beta_{ik}) > 0$. So,

[(7) - (8)] $\left(\partial\pi_i^*/\partial\beta_{ij}\right)\beta'_{ij} - (\partial\pi_i^*/\partial\beta_{ik})\,\beta'_{ik} < 0$

which contradicts Kuhn-Tucker conditions. ▪

As it was expected, R&D cooperation affects firms' decision about location: if there is an increasing information sharing between firms and joint profit maximization, then the entrant-cooperating firm always prefer to locate near the incumbent-cooperating firm for every shape of the spillover function, and so, clustering is a immediate result from cooperation. However, to achieve this result it is required an increasing information sharing between firms. In fact, if we assume $\delta = 1$, then $\left(\partial\pi_i^*/\partial\beta_{ij}\right) - (\partial\pi_i^*/\partial\beta_{ik}) \gtreqless 0$, and so, we can not eliminate the middle point location. Next example will help us to fully clarify our proposition:

Example 4 *Assume* $a = 100, b = 1, c = 50$ *and* $\gamma = 5$. *For simplicity, let's assume* $d_{ij}, d_{ik}, d_{jk} \in [0,1]$ *and* $d_{ij} + d_{ik} = d_{jk} = 1$. *In this case, we could have agglomeration* $(d_{ij} = 0; d_{ik} = 1)$ *or dispersion* $(d_{ij} = d_{ik} = 0.5)$. *Additionally, we could have a R&D cartel* $(\delta = 1)$ *or a RJV cartel* $(\delta\beta = 1)$.

Considering a linear spillover function, $\beta(d) = 0.75(1-d)$, *agglomeration is the best strategy for firm i, either with a R&D cartel or a RJV cartel:*

$\delta = 1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 184.44 < \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 200.51$

$\delta\beta = 1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 233.3 < \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 242.71$

For a convex spillover function, $\beta(d) = 0.75\left(1 - \sqrt{d}\right)$*, agglomeration is also the best strategy for firm* i*:*

$\delta = 1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 168.72 < \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 200.51$

$\delta\beta = 1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 234.09 < \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 242.71$

For a concave spillover function, $\beta(d) = 0.75(1 - d^2)$*, dispersion is the best strategy for firm* i *if there is an increasing information sharing between cooperating firms:*

$\delta = 1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 201.9 > \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 200.51$

$\delta = 1.1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 205.06 < \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 211.03$

$\delta\beta = 1 : \pi_i^* (\ d_{ij} = d_{ik} = 0.5) = 234.25 < \pi_i^* (\ d_{ij} = 0; d_{ik} = 1) = 242.71$

3 Conclusions

Empirical research usually confirms the strong propensity for the clustering of innovative related activities, which is commonly justified by the existence of knowledge spillovers (Jaffe and Henderson (1993), Audretsch and Feldman (1996), among others). Additionally, proximity is frequently cited as an explanation for the emergence of cooperative behaviors among firms or between universities and firms (e.g., Varga (2000), Arundel and Geuna (2001) and Carrincazeaux *et al.* (2001)).

Inspired in several empirical results, we intend to evaluate if cooperation in R&D affects firms' decision about location. Through a simple game between three firms, from which two of them intend to cooperate in R&D, it was possible to conclude that the clustering of firms is a result of R&D cooperation. In fact, we demonstrated that

if R&D runs independently, the entrant firm will cluster if the R&D spillover function is convex in the physical distance between firms. On the other hand, if R&D runs cooperatively between the entrant and an incumbent firm, then the entrant firm will always prefer to stay close to the cooperating-incumbent firm if there is an increasing information sharing among them. In any case, if the two incumbent firms were close located, an agglomeration equilibrium is always observed.

Our results also concern about R&D equilibrium output. We proved that if R&D runs independently, then R&D equilibrium output is larger as the distance between firms increases. The intuition is simple: as the distance between firms increases, firms will perform an higher R&D output because a lower proportion of its results will flow over the other firms. However, if R&D runs cooperatively, then R&D equilibrium output for cooperating firms increases with the degree of information sharing between them, as well as with a reduction of the distance between cooperative firms. On the other hand, it reduces when the distance from cooperative firms to non-cooperative firms is shorter. With respect to R&D equilibrium output for non-cooperating firms, our results were similar to the independent case: R&D equilibrium output for non-cooperating firms increases with the distance to cooperating firms and between cooperating firms.

Research on the topic of proximity between firms and R&D cooperation may proceed in several directions. One possible line of research is to introduce uncertainty in R&D spillovers and evaluate if it affects firms' cooperation in R&D. Another possible approach is to introduce an intermediate stage where the degree of information sharing between firms is decide by firms and become an endogenous variable.

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