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Regional Externalities in the Dynamic System of Three Regions

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Abstract

This study presents a theoretical model of the role of externalities across a three dimensional regional economy. Two decades ago Krugman (1981) developed a two region model of uneven regional development. He showed that initial discrepancy in capital-labour ratios of the two adjacent, competing regions will cumulate over time, and will inevitably lead to the division into the capital-rich and capital-poor regions. Kubo (1995) presented an extension to Krugman's model by incorporating not only scale economies within the regions but also regional externalities across regions. His model provided an explanation for different regional development patterns: uneven, joint and the mix of these two. In this study Kubo's analysis is extended to study the dynamic properties of the development of the three regions instead of two regions.

We characterise dynamics and the stability of steady states in the three-region model. In particular, we show under what conditions steady state is unique, and if there can be multiple steady states. We show that the condition for even regional development in Kubo's model, i.e. that regional externalities are stronger than scale economies in each region, is a necessary, but not a sufficient condition for even regional development in a case of three regions.

Our three region model offers a potentially interesting framework to analyse different regional development patterns and provides some new interesting results about the role of inter-regional externalities on regional development. Our model can be used to analyse, how the domination of the core region affects the growth of peripheral regions and what kind of regional policy should be implemented to promote economic growth in the periphery.

Keywords: scale economies, regional externalities, regional development
JEL classification: C61, R12

1. Introduction

The questions of the existence, the extent, and the nature of external economies of scale have been widely investigated in the literature of economic geography and growth. In location theory, external economies of scale (externalities) within the region have traditionally been used to explain why firms want to be located close to each other, i.e. to agglomerate. In growth theory, external scale economies are an engine of endogenous growth. However, the most important point of view concerning this research is that the introduction of external economies of scale has also made it possible to develop theoretical models of unequal economic development.

Krugman (1981) developed a model of uneven regional development, where he showed that initial discrepancy in capital-labour ratios of the two adjacent, competing regions will cumulate over time, and will inevitably lead to the division into the capital-rich and capital-poor countries/regions, i.e. uneven regional development. An important assumption in his analysis was that the presence of external economies of the industrial sector. Kubo (1995) extended Krugman's model by incorporating not only scale economies within the regions but also regional externalities across regions. An important novelty in his model was to provide an explanation for different regional development patterns: uneven, joint and the mix of these two, within the same framework.

With Kubo's model, we can broaden our concept of inter-regional development and analyse how one region's growth through externalities across the barriers of regional economies can enhance other region's economic development. However, if we want to analyse more precisely how, for instance, increased networking (i.e. increased inter-regional externalities between many regions) will affect regional development, it is necessary to extend the analysis beyond the two-region case. Therefore, we will formulate the most obvious generalisation of the two-region case – the three-region model.

In this study Kubo's analysis is extended to analyse the dynamic properties of the development of the three regions. This extension provides us a framework to analyse such geographical issues e.g. the increased regional networking which cannot be dealt with a two region analysis. On the other hand, it is interesting to find out whether the results of Kubo's two-region model will remain in the three region model.

The study might also shed light on the regional development in Northern Finland. A few years ago the idea of the regional network of Northern Finland was launched by

the Northern Finland Working Group. That idea was meant to spread the economic growth of the city of Oulu – technologically advanced core region - to smaller peripheral areas. This study offer potentially interesting frameworks to analyse different regional development patterns in Northern Finland.

The structure of this study is the following. Section 2 presents the basic structure of the model. In particular, we show under what conditions the steady state is unique, and when there are multiple steady states. In section 3 we study the dynamic properties of the model. In sections 4 and 5 we examine different regional development patterns both in cases of symmetric and asymmetric inter-regional externalities. A concluding discussion is provided in section 6.

2. The model

The economy has three regions denoted by A, B and C (See Figure 1). Each region has an equal amount of labour $L_i = L$, $\forall$ (i = A, B,C) that will not grow over time and is immobile between the regions, i.e. $L_A = L_B = L_C = \overline{L}$. Every region has only two sectors, manufacturing and agriculture. The manufacturing sector uses both labour and capital in producing good M, whereas the agricultural sector uses labour alone. Both commodities, manufacturing and agriculture, are freely traded. The manufacturing sector uses now fixed-coefficient, Leontief technology, but there are increasing returns to scale in production – the accumulation of capital causes reduction of capital and labour coefficients. The assumption of regional externalities implies that not only capital accumulation within the region, but also in other regions, has an effect on production efficiency.

The Leontief type of production function of the manufacturing sector (M_i) in region i is,

$$M_i = (K_i, L_i) = \min\left(\frac{K_i}{L_i}, \frac{L_i}{\ell_i}\right). \tag{1}$$

Analogous to Kubo (1995), capital (k_i) and labour (ℓ_i) coefficients are determined as follows;

$$\mathrm{k}_A = \frac{a}{K_A^{\alpha} K_B^{\mu_B} K_C^{\delta_C}} \qquad \mathrm{k}_B = \frac{a}{K_B^{\beta} K_A^{\mu_A} K_C^{\lambda_C}} \qquad \mathrm{k}_C = \frac{a}{K_C^{\gamma} K_A^{\delta_A} K_B^{\lambda_B}} \tag{2}$$

$$\ell_A = \frac{b}{K_A^{\alpha} K_B^{\mu_B} K_C^{\delta_C}} \qquad \ell_B = \frac{b}{K_B^{\beta} K_A^{\mu_A} K_C^{\lambda_C}} \qquad \ell_C = \frac{a}{K_C^{\gamma} K_A^{\delta_A} K_B^{\lambda_B}},$$

where;

α is the degree of (regional) scale economies in region A

β is the degree of (regional) scale economies in region B

γ is the degree of (regional) scale economies in region C

μ_A is the degree of external effect from region A to region B

μ_B is the degree of external effect from region B to region A

δ_A is the degree of external effect from region A to region C

δ_C is the degree of external effect from region C to region A

λ_B is the degree of external effect from region B to region C

λ_C is the degree of external effect from region C to region B.

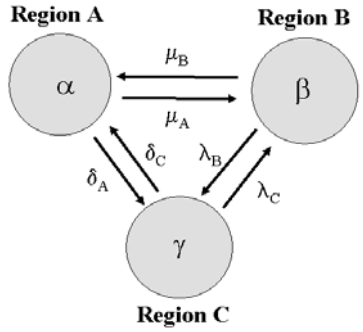

Figure 1. Scale economies and regional externalities in the case of three regions.

We assume that $0< \alpha, \beta, \gamma <1$, $0 < \mu_i, \delta_i, \lambda_i < 1 \ \forall$ (i = A, B,C) and a, b > 0. Neutrality in the scale economies and regional externalities together with the assumption that both regions use the same basic technology means that in each region; k_i / ℓ_i = a / b = constant. So the technologies in all regions are assumed to be the same, but the differences in the strength of scale economies and regional economies are possible.

Production functions in the manufacturing and agricultural sectors are; [1]

$$M_A = \frac{K_A}{k_A} = \frac{K_A^{1+\alpha} K_B^{\mu_B} K_C^{\delta_C}}{a}, \qquad A_A = \overline{L} - \ell_A M_A \tag{3}$$

$$M_B = \frac{K_B}{k_B} = \frac{K_B^{1+\beta} K_A^{\mu_A} K_C^{\lambda_C}}{a}, \qquad A_B = \overline{L} - \ell_B M_B$$

$$M_C = \frac{K_B}{k_B} = \frac{K_C^{1+\gamma} K_A^{\delta_A} K_B^{\lambda_B}}{a}, \qquad A_C = \overline{L} - \ell_C M_C .$$

When we differentiate e.g. manufacturing production of region A with respect to K_A ,K_B or K_C, we note that we have increasing returns to scale within the region and positive externalities across the regions. The wage rate is unity (w = 1) and the price of manufactured good is P_M.[2] Both are expressed in terms of the agricultural good. Furthermore, we assume that a constant fraction τ of the wage income in each region is spent on the manufacturing goods and the rest on the agricultural goods. Assuming free and balanced trade, the price of the manufactured good in equilibrium is determined as,

$$P_M\left(M_A + M_B + M_C\right) = 3\tau\bar{L} \tag{4}$$
$$\Rightarrow P_M = \frac{3\tau\bar{L}}{M_A + M_B + M_C} = \frac{3\tau a\bar{L}}{K_A^{1+\alpha}K_B^{\mu_B}K_C^{\delta_C} + K_B^{1+\beta}K_A^{\mu_A}K_C^{\lambda_C} + K_B^{1+\gamma}K_A^{\delta_A}K_C^{\lambda_B}}.$$

Now the rate of profit in a region i is given by,

$$\rho_i = \frac{P_M M_i - \ell_i M_i}{K_i} = \frac{P_M}{k_i} - \frac{b}{a}. \tag{5}$$

Assuming that profit incomes are saved and reinvested in the respective manufacturing sector to increase capital stock, regional capital stocks evolve over time as a function of all regions' capital according to

$$\frac{\dot{K}_A}{K_A} = \rho_A = \frac{P_M}{\mathrm{k}_A} - \frac{b}{a} = \frac{3\tau\bar{L}}{K_A + K_A^{\mu_A-\alpha}K_B^{1+\beta-\mu_B}K_C^{\lambda_C-\delta_C} + K_A^{\delta_A-\alpha}K_B^{\lambda_B-\mu_B}K_C^{1+\gamma-\delta_C}} - \frac{b}{a} \tag{6a}$$

$$\frac{\dot{K}_B}{K_B} = \rho_B = \frac{P_M}{\mathrm{k}_B} - \frac{b}{a} = \frac{3\tau\bar{L}}{K_B + K_A^{1+\alpha-\mu_A}K_B^{\mu_B-\beta}K_C^{\delta_C-\delta_C} + K_A^{\delta_A-\mu_A}K_B^{\lambda_B-\beta}K_C^{1+\gamma-\lambda_C}} - \frac{b}{a} \tag{6b}$$

$$\frac{\dot{K}_C}{K_C} = \rho_C = \frac{P_M}{\mathrm{k}_C} - \frac{b}{a} = \frac{3\tau\bar{L}}{K_C + K_A^{1+\alpha-\delta_A}K_B^{\mu_B-\lambda_B}K_C^{\delta_C-\gamma} + K_A^{\mu_A-\delta_A}K_B^{1+\beta-\lambda_B}K_C^{\lambda-\gamma}} - \frac{b}{a} \tag{6c}$$

The dynamic properties of the development of three regions are described by this differential equation system. Next we will determine the number of the long-run equilibria, i.e. the steady states of the economy.

Steady state equilibrium

The steady state of the economy is an allocation such that,

$$\frac{\dot{K}_A}{K_A} = \frac{\dot{K}_B}{K_B} = \frac{\dot{K}_C}{K_C} = 0. \tag{7}$$

The relation between K_A, K_B and K_C at the steady state(s) can be expressed as[3]

$$K_B = K_A^{\theta} \quad \text{and} \quad K_C = K_A^{\phi}, \tag{8}$$

where θ and ϕ denote;

$$\theta = \frac{(\alpha-\mu_A)(\gamma-\lambda_C)+(\lambda_C-\delta_C)(\delta_A-\mu_A)}{(\beta-\mu_B)(\gamma-\lambda_C)+(\beta-\lambda_B)(\lambda_C-\delta_C)} \quad \text{and} \quad \phi = \frac{(\alpha-\mu_A)(\beta-\lambda_B)+(\beta-\mu_B)(\mu_A-\delta_A)}{(\beta-\mu_B)(\gamma-\lambda_C)+(\beta-\lambda_B)(\lambda_C-\delta_C)}. \tag{9}$$

Substituting $K_B = K_A^{\theta}$ and $K_C = K_A^{\phi}$ into the equation $\dot{K}_A / K_A = 0$ and rearranging, we get,

$$Z(K_A) \equiv K_A + K_A^{\theta} + K_A^{\phi} = \frac{3\pi\bar{L}a}{b}. \tag{10}$$

Level of K_A at the steady state(s) is determined from (10). Next we study how many solutions there are for (10).

We first assume that both parameters θ and ϕ are positive and study the limit of $Z(K_A)$ as K_A approaches zero and infinity. When both parameters are positive $Z(K_A)$ is an increasing function of K_A and it can be seen that $\lim_{K_A \to 0} Z(K_A) = 0$ and $\lim_{K_A \to \infty} Z(K_A) = \infty$. Now there is a unique K_A that satisfies eq. (10).

The steady state of our model, however, is not necessarily unique. We first consider the case where both parameters in equation (10) are negative. Now we can rewrite function $Z(K_A)$ to the form,

$$Z(K_A) = K_A + \frac{1}{K_A^{\theta}} + \frac{1}{K_A^{\phi}}. \tag{11}$$

It is easy to see that $\lim_{K_A \to 0} Z(K_A) = \infty$ and $\lim_{K_A \to \infty} Z(K_A) = \infty$, which means that function $Z(K_A)$ has at least one minimum. Differentiating $Z(K_A)$ (and setting it equal to zero) we get,

$$Z'(K_A) = 1 - \theta \frac{1}{K_A^{1+\theta}} - \phi \frac{1}{K_A^{1+\phi}} = 0 \Leftrightarrow 1 - \theta \frac{1}{K_A^{1+\theta}} = \phi \frac{1}{K_A^{1+\phi}}. \tag{12}$$

From (12) we see that if there is a solution such that $Z'(K_A) = 0$, there must be only one such solution. The left hand side (LHS) of last expression in (12) is an increasing function of K_A (approaches - ∞ when K_A approaches 0, and 1 when K_A approaches ∞). The right hand side (RHS) of last expression in equation (12) is in turn decreasing function K_A (approaches ∞ when K_A approaches 0, and 0 when K_A approaches ∞). Thus if both exponents in $Z(K_A)$ are negative, it is actually a u-shaped

function, which has only one minimum where $Z'(K_A) = 0$. This means that (11) may have 0, 1 or 2 solutions, i.e. values of K_A where the economy is in steady state.

When one of the exponents in $Z(K_A)$ is negative ($-\theta$) and the other is positive (ϕ), we have three cases. The function $Z(K_A)$ can be written in a form,

$$Z(K_A) = K_A + \frac{1}{K_A^{\theta}} + K_A^{\phi}. \tag{13}$$

The function $Z(K_A)$ approaches again ∞ when K_A approaches 0 or ∞. Differentiating (13) with respect to K_A and by setting it equal to zero we get,

$$Z'(K_A) = 1 - \theta \frac{1}{K_A^{1+\theta}} + \phi K_A^{\phi-1} = 0. \tag{14}$$

We first assume that $\phi > 1$. Setting the $Z'(K_A) = 0$, we get

$$1 + \phi K_A^{\phi-1} = \theta \frac{1}{K_A^{1+\theta}}. \tag{15}$$

The LHS of (15) approaches 1 when K_A approaches 0, and ∞ when K_A approaches ∞. RHS approaches again ∞ when K_A approaches 0, and 0 when K_A approaches ∞. This means that function $Z(K_A)$ has again only one minimum and thereby we can have 0,1 or 2 steady states in the economy.

In the case where $\phi < 1$, $Z'(K_A) = 0$ can be written,

$$1 + \frac{\phi}{K_A^{1-\phi}} = \theta \frac{1}{K_A^{1+\theta}}. \tag{16}$$

Both LHS and RHS of (16) approach ∞ when K_A approaches 0, and 0 when K_A approaches ∞. To study behaviour of $Z'(K_A)$ we define LHS of (16) as $G(K_A)$ and RHS as $H(K_A)$. Differentiating we obtain,

$$G'(K_A) = \phi(\phi-1)K_A^{\phi-2} = \frac{\phi(\phi-1)}{K_A^{2-\phi}} < 0 \tag{17}$$

$$G''(K_A) = \phi(\phi-1)(\phi-2)K_A^{\phi-3} = \frac{\phi(\phi-1)(\phi-2)}{K_A^{3-\phi}} > 0 \tag{18}$$

$$H'(K_A) = \theta(-\theta-1)K_A^{-\theta-2} = \frac{-\theta(\theta+1)}{K_A^{1+\theta}} < 0 \tag{19}$$

$$H''(K_A) = \theta(-\theta-1)(-\theta-2)K_A^{-\theta-3} = \frac{\theta(\theta+1)(\theta+2)}{K_A^{3+\theta}} > 0 \tag{20}$$

Signs of the first and second derivatives show that both functions $G(K_A)$ and $H(K_A)$ are downward sloping and strictly convex. Figure 2 describes curves $G(K_A)$ and $H(K_A)$. $H(K_A)$ (or $G(K_A)$ respectively) can have three different shapes. This means that there

can be 0,1 or 2 solutions for equation (16) depending on the relative convexity of functions $G(K_A)$ and $H(K_A)$. In figure 2 we have described the case for a fixed $G(K_A)$.

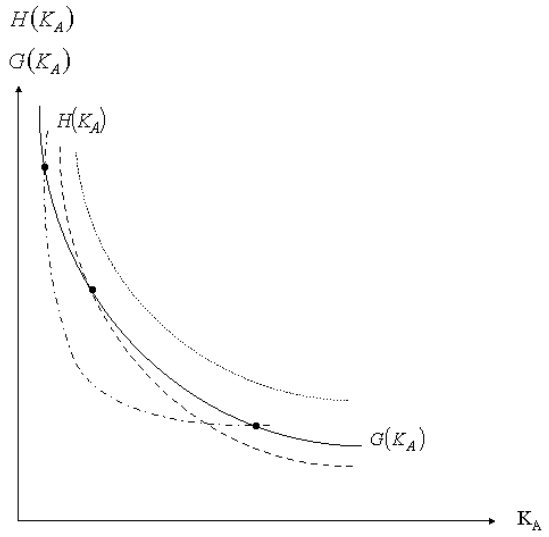

Figure 2. Number of solutions for equation (16).

Thereby we have proved that when one of the parameters of (10) is positive and < 1 and the other is negative, there is possibility for maximum two points of $Z'(K_A) = 0$.[4]

In the case where $\phi = 1$, $Z'(K_A) = 0$ can be written,

$$2 = \theta \frac{1}{K_A^{1+\theta}} \tag{21}$$

Now RHS of (21) approaches ∞ when K_A approaches 0, and 0 when K_A approaches ∞. The LHS is a horizontal line and hence there is only one point where function $Z'(K_A)=0$. So $Z(K_A)$ is a u-shaped function and we may again have 0,1 or 2 steady states in the economy. Table 1 summarises our analysis concerning the number of steady states of the economy.

Table 1. Signs of the parameters in the function of $Z(K_A)$ and the number of steady states of the economy

Signs of the parameters of function $Z(K_A)$	**lim $Z(K_A)$ $K_A \to 0$**	**lim $Z(K_A)$ $K_A \to \infty$**	**Number of steady states**
Both positive	0	∞	1 (unique)
Both negative	∞	∞	0 ,1 or 2
One negative and			
a) One positive and > 1	∞	∞	0 ,1 or 2
b) One positive and < 1	∞	∞	0 ,1 or 2
c) One positive and = 1	∞	∞	0 ,1 or 2

3. Stability of the steady states

To study formally the stability properties of dynamical equilibrium, we form the Jacobian matrix of nonlinear system (6a-6c).

$$J = \begin{bmatrix} \frac{\partial \dot{K}_A}{\partial K_A} & \frac{\partial \dot{K}_A}{\partial K_B} & \frac{\partial \dot{K}_A}{\partial K_C} \\ \frac{\partial \dot{K}_B}{\partial K_A} & \frac{\partial \dot{K}_B}{\partial K_B} & \frac{\partial \dot{K}_B}{\partial K_C} \\ \frac{\partial \dot{K}_C}{\partial K_A} & \frac{\partial \dot{K}_C}{\partial K_B} & \frac{\partial \dot{K}_C}{\partial K_C} \end{bmatrix} \tag{22}$$

We evaluate the Jacobian at the steady state, i.e. we substitute K_A, $K_B = K_A^{\theta}$ and $\mathrm{K_C} = K_A^{\phi}$, and $b/a = 3\bar{tL}/K_A + K_A^{\theta} + K_A^{\phi}$ into the Jacobian matrix. After some manipulation we get,[5]

$$J_E = \begin{bmatrix} \frac{-3\bar{tL}A1}{\mathrm{K}^{*2}} & \frac{-3\bar{tL}A2}{\mathrm{K}^{*2}} & \frac{-3\bar{tL}A3}{\mathrm{K}^{*2}} \\ \frac{-3\bar{tL}B1}{\mathrm{K}^{*2}} & \frac{-3\bar{tL}B2}{\mathrm{K}^{*2}} & \frac{-3\bar{tL}B3}{\mathrm{K}^{*2}} \\ \frac{-3\bar{tL}C1}{\mathrm{K}^{*2}} & \frac{-3\bar{tL}C2}{\mathrm{K}^{*2}} & \frac{-3\bar{tL}C3}{\mathrm{K}^{*2}} \end{bmatrix}_{(K_A^*, K_B^*, K_C^*)} \quad \left(= \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \right) \tag{23}$$

where,

$$\begin{aligned}
A1 &= K_A^* + (\mu_A - \alpha)K_A^{*\theta} + (\delta_A - \alpha)K_A^{*\phi} \\
\mathrm{A2} &= (1 + \beta - \mu_B)K_A^* + (\lambda_B - \mu_B)K_A^{*\phi} K_A^{*-\theta} K_A^* \\
A3 &= (1 + \gamma - \delta_C)K_A^* + (\lambda_C - \delta_C)K_A^{*\theta} K_A^{*-\phi} K_A^* \\
B1 &= (1 + \alpha - \mu_A)K_A^{*\theta} + (\delta_A - \mu_A)K_A^{*\phi} K_A^{*-1} K_A^{*\theta} \\
B2 &= K_A^{*\theta} + (\mu_B - \beta)K_A^* + (\lambda_B - \beta)K_A^{*\phi} \\
B3 &= (1 + \gamma - \lambda_C)K_A^{*\theta} + (\delta_C - \lambda_C)K_A^{*-\phi} K_A K_A^{*\theta} \\
C1 &= (1 + \alpha - \delta_A)K_A^{*\phi} + (\mu_A - \delta_A)K_A^{*\theta} K_A^{*-1} K_A^{*\phi} \\
\mathrm{C2} &= (1 + \beta - \lambda_B)K_A^{*\phi} + (\lambda_B - \mu_B)K_A^* K_A^{*-\theta} K_A^{*\phi} \\
C3 &= K_A^{*\phi} + (\delta_C - \gamma)K_A^* + (\lambda_C - \gamma)K_A^{*\theta} \\
\mathrm{K}^* &= K_A^* + K_A^{*\theta} + K_A^{*\phi} \equiv \text{ The sum of regional capital stocks at the steady state.}
\end{aligned} \tag{24}$$

To study the stability of the steady state(s) we utilise Routh-Hurwitz theorem.[6] First we form the characteristic matrix of the Jacobian determinant (D). The linear homogenous system has (according to a well-known theorem of elementary algebra) non-trivial solutions (in addition to the trivial one) if and only if its determinant is zero. In our case this means that the determinant of the characteristic matrix of D must vanish, i.e. characteristic matrix of D is required to be singular.[7] The determinant of

characteristic matrix of the Jacobian determinant will yield 3rd–degree polynomial in r, i.e. 3rd–degree polynomial equation with real coefficients.

$$a_0 r^3 + a_1 r^2 + a_2 r + a_3 = 0 \qquad (a_0 > 0) \tag{25}$$

The Routh-Hurwitz theorem states that necessary and sufficient stability conditions to have a stable root are given by the following inequalities; [8]

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1 a_2 - a_0 a_3 > 0, \tag{26}$$

where;

$$a_0 = 1$$
$$a_1 = -(F_{11} + F_{22} + F_{33})$$
$$a_2 = F_{11}F_{13} + F_{22}F_{33} + F_{11}F_{22} - F_{13}F_{31} - F_{12}F_{21} - F_{32}F_{23}$$
$$a_3 = F_{13}F_{31}F_{22} + F_{12}F_{21}F_{33} + F_{11}F_{32}F_{23} - F_{11}F_{22}F_{33} - F_{12}F_{23}F_{31} - F_{13}F_{32}F_{21} \ .$$

Next we will examine different regional development patterns both in cases of symmetric and asymmetric inter-regional externalities.

4. Regional development with symmetric regional externalities

First we study the case, where each regions' externalities are symmetric, i.e. $\mu_A = \delta_A$, $\mu_B = \lambda_B$, $\lambda_C = \delta_C$, and they are stronger than scale economies within the regions.[9] This assumption means that θ and ϕ parameters in equations (10) can be written as;

$$\theta = \frac{(\alpha - \mu_A)}{(\beta - \mu_B)} \quad \text{and} \quad \phi = \frac{(\alpha - \mu_A)(\beta - \lambda_B)}{(\beta - \mu_B)(\gamma - \lambda_C)} = \frac{(\alpha - \delta_A)}{(\gamma - \lambda_C)} \quad \text{, when} \quad \mu_B = \lambda_B \text{ and } \mu_A = \delta_A. \tag{27}$$

With these assumptions both parameters θ and ϕ are positive, which in turn implies that we have a unique equilibrium. Furthermore, it is easy to show that if symmetric regional externalities are stronger than scale economies in each region, the unique steady state of the economy is always stable. Inequalities (26) under the assumption of the symmetric externalities are presented in appendix 1. The result is in the line with Kubo's two region model. This yields,

Proposition 1. *When symmetric regional externalities are stronger than scale economies for each three regions, there is unique steady state, which is stable.*

The allocation of manufacturing production among the three regions can be presented as a point of an equilateral simplex triangle, where each point in a triangle presents the relative sizes of regional capital stocks in two dimensions (figure 3).[10] In the corners of the triangle, the manufacturing industry is totally concentrated into one region and other

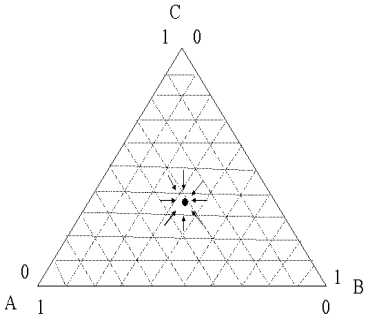

Figure 3. The stable regional development

two regions are de-industrialised. Figure 3 presents the totally symmetric, unique stable steady state case where parameters of the scale economies and externalities are equal in each three regions. From any point of the simplex, system will converge towards the unique (ergodic and predictable) outcome (Arthur 1994). As in Kubo's model, the relative size of the regions in steady state depends on the relative magnitudes of their net externalities. The larger the net externalities of region compared to other region's net externalities, the larger will be the relative size of that region at the steady state.

Next we assume that scale economies are stronger than regional externalities in each region. It is easy to see that parameters θ and ϕ in (10) are again positive, and we have a unique steady state. Whether this steady state is stable or unstable is not so straightforward to answer. However we note that;

$$a_1 = \frac{3\tau\bar{L}}{K^{*2}}\left(K^* + A\right) > 0 \quad \text{if} \quad A > -K^* \tag{28}$$

$$a_2 = \frac{\left(-3\tau\bar{L}\right)^2}{K^{*4}}\left(B + A \cdot K^*\right) > 0 \text{ if } A > -\frac{B}{K^*}$$

$$a_3 = -\frac{\left(-3\tau\bar{L}\right)^3}{K^{*6}} K^* \cdot B \quad > 0 \text{ always}$$

$$a_1 a_2 - a_3 a_0 = \frac{\left(3\tau\bar{L}\right)}{K^{*6}}\left[K A^2 + \left(K^{*2} + B\right)A\right] > 0 \quad \text{if} \quad A < -\left(K^* + \frac{B}{K^*}\right) \text{ or } A > 0 .$$

Combining the inequality conditions in (28) we see that all terms a_1, a_2, a_3 and a_1a_2-a_3 a_0 are now positive if and only if term A (= the sum of net regional externalities weighted by the capital stock of "target regions" at the steady state) denoted in (A2) in appendix 1 is positive. This means that if symmetric regional externalities are smaller than scale economies for each region (i.e. A < 0), the unique steady state is always

unstable. In addition to the unstable interior equilibrium, we have also three unstable equilibria, where manufacturing industry is equally divided between two regions. In that case analysis is similar to uneven regional development in Kubo's model. If we are not exactly on that "saddle path" which leads to the steady state where both regions have a certain share of manufacturing, manufacturing sector always ends up concentrated in the region with the largest capital stock initially (Figure 4).[11]

The assumption is that an increase in a capital stock increases production efficiency and reduces production costs. This in turn increases profits and the rate of capital accumulation in this region. In the case of unbounded agglomeration, the outcome of the locational process will be the monopoly of one location - "Silicon Valley outcome" (Arthur 1994).[12] This result is again in line with Kubo's model. This completes the proof of Proposition 2.

Proposition 2. *When scale economies are stronger than symmetric regional externalities in each three regions, there is unique steady state, which is unstable.*

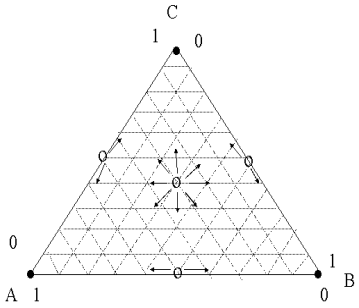

Figure 4. The uneven regional development

In other words, the simplex is divided into three basis of attraction, each of which drains to one of the corners. However, if reservoir of labour or some other factors of production are exhausted in "the leading region" before the upper limit of the regional capital stock has achieved, the manufacturing industry may spread to other regions also. Now the region which will share the manufacturing industry (maybe become partially industrialised) with the leading region is determined by the historical choice of order. If there are upper limits of positive agglomeration effects in each of the regions, it is possible that the manufacturing industry is dispersed to all the regions (Arthur, 1994). This kind of regional development was briefly described in Krugman (1981). [13, 14]

5. Regional development with asymmetric regional externalities

Next we drop the assumption about the symmetric regional externalities. This means that externalities from each region to other two regions are no longer equal (i.e. $\mu_A \neq \delta_A$, $\mu_B \neq \lambda_B$, $\lambda_C \neq \delta_C$). This reflects indirectly regions' capability to adopt externalities from other regions and possible "preference co-operation relationship" between the regions. The stability conditions (26) under the assumption of asymmetric externalities are presented in appendix 2.

In Kubo's two-region model the assumption that externalities are larger than scale economies for each region (i.e. term in parenthesis of equation a_1 in (A3) > 0), implied a unique, stable steady state in the economy. Even regional development was guaranteed, if externalities are stronger than scale economies in each region. But is it a sufficient condition for a unique, stable equilibrium in the three region case also?

Next we assume that externalities from region A to region C are stronger than to region B ($\delta_A > \mu_A$), and externalities from region C to region A are stronger than to region B ($\delta_C > \lambda_C$), respectively. Thus, there is a strong mutual co-operation relationship between regions A and C. For simplicity, we assume that externalities from region B are symmetric (i.e. $\mu_B = \lambda_B$) and regional scale economies are equal in all three regions (i.e. $\alpha = \beta = \gamma$). The aim is to study, how preference co-operation relationship between the two regions of three affects to the nature of steady state(s) of the economy and thereby development of the "regional network" (Figure 5).

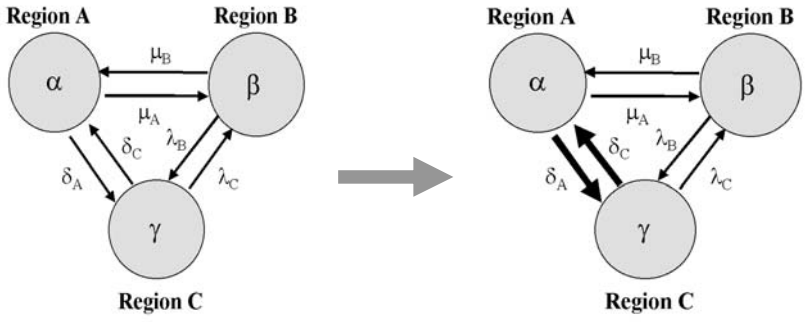

Figure 5. From the symmetric externalities to the asymmetric externalities

The condition for the unique steady state in (10) is that both term θ and ϕ are positive. According to our assumptions, ϕ is now always positive. Respectively θ is positive, if

$$(\alpha-\mu_A)(\gamma-\lambda_C)>(\delta_C-\lambda_C)(\delta_A-\mu_A) \Leftrightarrow (\alpha-\mu_A)(\gamma-\lambda_C)>[(\gamma-\lambda_C)-(\gamma-\delta_C)][(\alpha-\mu_A)-(\alpha-\delta_A)]. \quad (29)$$

Thus a unique steady state condition requires that the net external effects from regions A and C to region B are strong and the externalities of regions A and C are relatively small for each other compared to their external effects to region B. In other words, the unique steady state is possible, if the co-operation between regions A and C is not too strong compared to their co-operation with region B.

Our assumptions imply that "the asymmetric" term C in (A3) is negative if,

$$\{[(\mu_B-\beta)-(\delta_C-\gamma)](\delta_A-\mu_A)\}K_A^{*\phi}+\{[(\lambda_B-\beta)-(\delta_A-\alpha)](\delta_C-\lambda_C)\}K_A^{*}<(\delta_C-\lambda_C)(\delta_A-\mu_A)K_A^{*\theta} \qquad (30)$$

$$\Leftrightarrow \left[\frac{(\mu_B-\beta)-(\delta_C-\gamma)}{(\delta_C-\lambda_C)}\right]K_A^{*\phi}+\left[\frac{(\lambda_B-\beta)-(\delta_A-\alpha)}{(\delta_A-\mu_A)}\right]K_A^{*}<K_A^{*\theta}.$$

This implies that if the co-operation between the regions A and C increases sufficiently, and if at the same time the relative size of the regions B at the steady state is large enough (and the relative size of regions A and C together is small, respectively), the term C can be negative. This, in turn, implies that a stability condition in Kubo's model, i.e. externalities are stronger than scale economies in each region may no longer guarantee stable regional development in our three region model.[15] This is an interesting question because in Kubo's model the steady state was always unique, when symmetric externalities were stronger than scale economies for each region. Furthermore, Kubo (1995) argued that if the regional externalities are stronger than scale economies in both two regions, we have inevitable the unique, stable steady state.

As a matter of fact, it is easy to show that as the asymmetry of the externalities increases, the interior steady state is not necessarily unique and stable. However, as long as the externalities of regions to their neighbouring regions remains enough symmetric, we will have a unique, stable steady state and thus stable regional development. When asymmetry increases, (the preference co-operation between regions A and C becomes stronger than externalities of region B from regions A and C) the stable, unique steady state breaks down. The preference co-operation between two regions (A and C) will cause that one region (B) will not keep up with the growth of the other regions, unless it relative size compared to regions A and C is very large (Figures 6a-6c).[16]

As a result of the increased co-operation between two regions we will have two steady states, stable and unstable. In a stable steady state, the regions having the preference co-operation relationship are very large relative to third region. In the unstable steady state, in turn, the relative sizes of the co-operation regions are extremely small compared to the size of the third region (region B).[17] The figures below show how

the number and the nature of steady states and the relative size of regions change as the co-operation between regions A and C increases (i.e. μ_A and δ_C increases), *ceteris paribus*. This leads to the Proposition 3, which constitutes one of the most important results of our three region model.

Proposition 3. *When regional externalities are stronger than scale economies in each three region, the preference co-operation links between two regions may increase asymmetries in regional externalities and stable, unique steady state breaks down. As a result we will have two steady states, stable and unstable.*

According to the Proposition 3, the assumption that regional externalities are stronger than scale economies in each region is not a sufficient condition for a unique, stable equilibrium and hence for a stable regional development. This is a fundamental difference to Kubo's model.

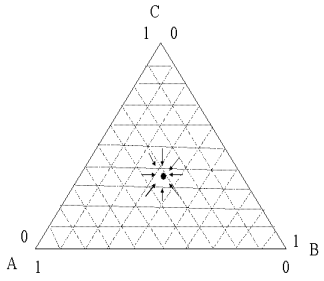

Figure 6a. A unique steady state ($\mu_A=\delta_A= 0.15$)

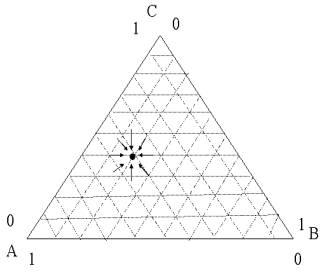

Figure 6b. A unique steady state ($\mu_A=\delta_A= 0.195$)

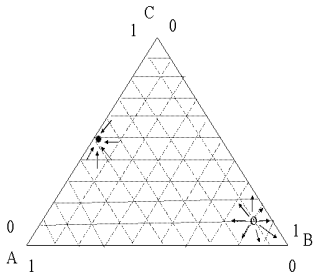

Figure 6c. Two steady states ($\mu_A=\delta_A= 0.40$)

6. Conclusions

Our model provides an interesting framework to analyse the role of intra-regional scale economies and inter-regional externalities in regional economic growth. Model sheds light on the reality and makes it possible to analyse issues that cannot be

examined with the two-region models, the increased regional networking, for instance. It provides theoretical aspects and explanations that are definitely relevant when trying to analyse e.g. the development of Northern Finland and the Multipolis Network. We have proved that much of we learned from the Kubos's model survives a relaxation of the assumption that there are only two regions. Furthermore, we were also able to show some new interesting results provided by the three region case.

From the perspective of the regional policy the most important result or implication yielded by our three region model, is the importance of inter-regional co-operation (although it is not explicitly presented in our model). When regional externalities are stronger than scale economies in each region and symmetric enough, regional development will be even and none of the regions will contract and stagnate. If the case is the opposite, i.e. the scale economies within the regions are stronger than externalities they exert on other regions, the regional development will be uneven. The self-reinforcing mechanism will enhance the domination position of the largest region. In the case of unbounded agglomeration, i.e. there are no diseconomies of agglomeration or "upper limits" for growth, the location process will lock-in to "Silicon Valley" outcome, the monopoly of one location.

Uneven development may be the result also in the case of regional externalities being stronger than scale economies in core region and weaker in other, initially smaller region(s). This may be the case if the relative size of these small regions is not big enough to capture the benefits the core-regions spread to adjacent regions. Furthermore, we were able to show that preference co-operation between the small regions (when regional externalities are stronger than scale economies in each region) may not promote their development, if their relative size is very small compared to the core-region. On the other hand, our analysis showed that this kind of preference co-operation between the core-region and one of the peripheral regions will be detrimental for the development of the third region, even if it receives positive externalities from the core-region.

Moreover, the aim of this research was to consider potential regional development patterns of the Multipolis Network of Northern Finland from theoretical perspectives. According to our theoretical three region model, which can be thought to describe a simplified multi-region network, the role of the Oulu region is decisive for the whole future of the Multipolis Network. The models clearly imply that regional policy, in which only the externalities of the Oulu region to semi-peripheral and peripheral regions

are stimulated through various development programmes, may not guarantee the even regional development in Northern Finland. The growth of small regions requires that their own economy is sufficiently developed and technologically advanced to adopt and utilise the positive spread effects that the economic growth of the core region provides. Selected policy measures which aim the stimulation of positive inter-regional integration, i.e. externalities across regional economies and capital formation in the regions will help the poor, small regions nearby regional centres to get to the favourable growth path. In some cases this may require co-ordinated policy interventions to push the development of the Multipolis Network towards a targeted direction.

Obviously our model of this study is fairly simple in many ways. The certain phenomena like leapfrogging, mobility of labour, and equalisation of factor prices (wages) etc., are not included in the model. These extensions would certainly bring more reality into the models, but also make them considerably more complex. These restrictive assumptions (which have been made to keep the model tractable) are good to keep in mind as we try to draw policy recommendations from the model.

Notes

[1] Agricultural sector uses residual labour in its production (CRS technology), where one unit of labour produces one unit of agricultural good.

[2] The assumption that wage rate is set at unity is crucial. As the manufacturing sector increases its share in a region, increased competition in the local labour market does not increase wages and thereby production costs at the manufacturing sector. Thus one of the major shortcomings in Krugman's and Kubo's models is not corrected in this model.

[3] The equations (6a-6c) can be rewritten as,

$$K_A^{\mu_A-\alpha}K_B^{\lambda_B-\mu_B}K_C^{\lambda_C-\delta_C}\left(K_A^{1+\alpha-\mu_A}K_B^{\mu_B-\lambda_B}K_C^{\delta_C-\lambda_C}+K_B^{1+\beta-\lambda_B}+K_A^{\delta_A-\mu_A}K_C^{1+\gamma-\lambda_C}\right)-\frac{3\bar{L}a}{b}=0$$

$$K^{\lambda_B-\beta}\left(K_A^{1+\alpha-\mu_A}K_B^{\mu_B-\lambda_B}K_C^{\delta_C-\lambda_C}+K_B^{1+\beta-\lambda_B}+K_A^{\delta_A-\mu_A}K_C^{1+\gamma-\lambda_C}\right)-\frac{3\bar{L}a}{b}=0$$

$$K_A^{\mu_A-\delta_A}K_C^{\lambda_C-\gamma}\left(K_A^{1+\alpha-\mu_A}K_B^{\mu_B-\lambda_B}K_C^{\lambda_C-\delta_C}+K_B^{1+\beta-\lambda_B}+K_A^{\delta_A-\mu_A}K_C^{1+\gamma-\lambda_C}\right)-\frac{3\bar{L}a}{b}=0$$

[4] It is interesting to note that if we have two points where $Z'(K_A) = 0$, the other point has to be an inflection point. In other words the derivative does not change its sign from one side of $K_A=K_A^*$ to the other. If it would change its sign we would necessary have at least three zero points of derivative that was not possible as shown in the figure 2.

[4] Now K_A , K_A^{θ} , K_A^{ϕ} in the matrix are the values of regional capital at the steady state and parameters θ and ϕ are functions of scale and externality parameters defined in (9).

[6] Takayama 1994; Gandolo 1980.

[7] Gandolfo 1980.

[8] Unfortunately as the order of the polynomial equation (25) increases, the economic interpretation of the stability conditions (26) becomes extremely complicated. Even in our three region model, where the order of the polynomial equation is only three, it is quite difficult to extract a clear economic meaning for stability conditions.

[9] This strong assumption about the symmetric externalities is based on an idea that externality which region exerts on one region is not excluded from the other regions. Secondly if externality from region A to region C increases, it does not reduce the amount of externality available from region A to region B. So we assume that externalities of regions are "pure public goods" in a sense that e.g. knowledge externalities from the technology advanced region are nonrival and nonexcludable.

[10] As we move from right to left, the relative size of regional capital stock of region A increases. Similarly as we move upwards, the relative size of regional capital stock of region C increases. And finally, as move "down-right", the relative size of regional capital stock of region B increases. See Krugman 1996 and Fujita, Krugman & Venables 1999.

[11] This means that due to the occurrence of different historical events, the economies with identical fundamentals may have a completely different development path. The outcome is both non-ergodic and (slightly) unpredictable. A priori all steady states are attainable, but only one of them will be eventually selected. (Arthur 1994).

[12] In the corners, where manufacturing industry is totally concentrated in one of the regions, the profit rate is the highest in that particular region. Therefore manufacturing industry is locked in that region (if it is once reached that point). See Arthur, 1994, Ch 4.

[13] In the symmetric case it is straightforward to show that capital stocks of regions B and C compared to the capital stock of region A (and to each other) at the steady state depend only on the magnitude of regions' mutual net scale economies. When scale economies are stronger than regional externalities in each region (i.e. we have a unique unstable steady state), the increase in one region's net scale economies increases the possibility of eventual expansion of that region (as in Kubo's two-region case). There is stronger possibility that the whole economy will converge to direction where other two regions will de-industrialised.

[14] Equally well we could simulate the case, where we have regions with positive net externalities and region(s) with positive net scale economies. For simplicity we may assume that symmetric externalities are stronger than symmetric scale economies for each region (i.e. $\alpha = \beta = \gamma$, $\mu_A = \delta_A = \mu_B = \lambda_B = \lambda_C = \delta_C$ and $\mu_A = \delta_A > \alpha$, $\mu_B = \lambda_B > \beta$, $\lambda_C = \delta_C > \gamma$). Then, say scale economies in a region C (i.e. γ) start to increase, *ceteris paribus*. This assumption means that the relative size of region C increases as γ increases. As far as scale economies in a region C remain smaller than its externalities, we will have a unique steady state. However, when γ becomes stronger than regional externalities of region C (i.e. $\lambda_C = \delta_C < \gamma$), we have two steady states in the economy. Now when scale economies are stronger than regional externalities in some region(s) (now region C) and converse in the other regions (regions A and B), the requirement for the steady states to be stable, is that term A in (A2) is positive, i.e.,

$$[(\mu_B - \beta) + (\delta_C - \gamma)]K_A^* + [(\mu_A - \alpha) + (\lambda_C - \gamma)]K_A^{*\theta} + [(\delta_A - \alpha) + (\lambda_B - \beta)]K_A^{*\phi} > 0$$

$$\Leftrightarrow \frac{(\mu_A - \alpha) + (\lambda_C - \gamma)}{(\beta - \mu_B) + (\gamma - \delta_C)} K_A^{*\theta} + \frac{(\delta_A - \alpha) + (\lambda_B - \beta)}{(\beta - \mu_B) + (\gamma - \delta_C)} K_A^{*\phi} > K_A^* .$$

Respectively if the first expression in the above equation is negative, the steady state is unstable. Now the steady state is stable, when the relative size of region C is large enough compared to regions A and B. Respectively, if the size of region C is extremely small compared to regions A and B, the steady state is unstable. In that case it might be unable to utilise externalities from regions A and B and achieve industrialisation. This result is analogous with the case of mixed regional development of Kubo's model. The analysis is very much the same if we assume that scale economies in region B (i.e. β) increases as much as scale economies in a region C, *ceteris paribus***.** However, it is interesting to notice that when unique steady state breaks down, we have two unstable steady states immediately.

[15] As δ_A and δ_C increase, ceteris paribus, the numerators in multipliers of $K_A^{*\phi}$ and K_A^* decrease and the denominators increase in the last expression in 30. This in turn means that multipliers of $K_A^{*\phi}$ and K_A^* decrease quite rapidly as δ_A and δ_C increase.

[16] In figures 6a-6c we have assumed the following initial parameter values: $\alpha=\beta=\gamma=0.1$, $\mu_A=\mu_B=0.15$, $\delta_A=\delta_B=0.15$, $\lambda_A=\lambda_B=0.15$, a=b=1, $\tau = 0.3$, $\bar{L} = 10$. Although we solve the model numerically, the results strongly suggest that they can be treated as analytic ones.

[17] Figure 6a presents "a completely symmetric case", in which scale economies and regional externalities are the same size in all three regions. There is a unique steady state with manufacturing production equally divided among the regions. In Figure 6b regional externalities are no longer symmetric, but the unique steady state of the economy still exists. In Figure 6c, the co-operation between regions A and C has become strong enough to cause two steady states, the unstable and stable, to the economy. As a matter of fact a unique steady states breaks down already with the values of $\mu_A=\delta_A= 0.2$. For simplicity, we assumed that the growth of externalities from region A to region C and vice versa were the same size.

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Appendix 1: Stability conditions in the case of *symmetric externalities*

The assumption of the symmetric externalities implies that inequality conditions (26) can be written as;

$$a_0 = 1 \ > 0 \qquad \text{(A1)}$$

$$a_1 = \underbrace{\frac{3\bar{L}}{\left(K_A^* + K_A^{*\theta} + K_A^{*\phi}\right)^2}}_{\mathrm{K}^*} \left\{ \begin{array}{l} K_A^* + K_A^{*\theta} + K_A^{*\phi} + [(\mu_B - \beta) + (\delta_C - \gamma)]K_A^* + [(\mu_A - \alpha) + (\lambda_C - \gamma)]K_A^{*\theta} \\ + [(\delta_A - \alpha) + (\lambda_B - \beta)]K_A^{*\phi} \end{array} \right\} > 0$$

$$a_2 = \frac{\left(-3\bar{L}\right)^2}{\mathrm{K}^{*4}} \left\{ \begin{array}{l} [(\mu_B - \beta)(\delta_C - \gamma)]K_A^{*2} + [(\delta_A - \alpha)(\lambda_C - \gamma) + (\mu_A - \alpha)(\lambda_B - \beta)]K_A^{*\theta}K_A^{*\phi} + [(\mu_B - \beta) + (\delta_C - \gamma)]K_A^*\mathrm{K}^* \\ + [(\mu_A - \alpha)(\lambda_C - \gamma)]K_A^{*\theta 2} + [(\lambda_B - \beta)(\delta_C - \gamma) + (\delta_A - \alpha)(\mu_B - \beta)]K_A^*K_A^{*\phi} + [(\mu_A - \alpha) + (\lambda_C - \gamma)]K_A^{*\theta}\mathrm{K}^* \\ + [(\delta_A - \alpha)(\lambda_B - \beta)]K_A^{*\phi 2} + [(\mu_A - \alpha)(\delta_C - \gamma) + (\mu_B - \beta)(\lambda_C - \gamma)]K_A^*K_A^{*\theta} + [(\delta_A - \alpha) + (\lambda_B - \beta)]K_A^{*\phi}\mathrm{K}^* \end{array} \right\} > 0$$

$$a_3 = -\frac{\left(-3\bar{L}\right)^3}{\mathrm{K}^{*6}} \cdot \frac{\mathrm{K}^*}{1} \left\{ \begin{array}{l} [(\mu_B - \beta)(\delta_C - \gamma)]K_A^{*2} + [(\delta_A - \alpha)(\lambda_C - \gamma) + (\mu_A - \alpha)(\lambda_B - \beta)]K_A^{*\theta}K_A^{*\phi} \\ + [(\mu_A - \alpha)(\lambda_C - \gamma)]K_A^{*\theta 2} + [(\lambda_B - \beta)(\delta_C - \gamma) + (\delta_A - \alpha)(\mu_B - \beta)]K_A^*K_A^{*\phi} \\ + [(\delta_A - \alpha)(\lambda_B - \beta)]K_A^{*\phi 2} + [(\mu_A - \alpha)(\delta_C - \gamma) + (\mu_B - \beta)(\lambda_C - \gamma)]K_A^*K_A^{*\theta} \end{array} \right\} > 0$$

We can see that when regional externalities are stronger than scale economies, Routh-Hurwitz stability conditions of terms a_0, a_1, a_2, a_3 in (28) are fulfilled, i.e. they are positive. It is easy to show that fifth inequality a_1a_2-a_3a_0 is also positive. We denote the term inside the { }-parenthesis in equation a_3 by B (which is now positive). We get,

$$a_2 a_1 - a_3 a_0 =$$

$$\frac{(3\tau\bar{L})^2}{\mathrm{K}^{*4}}\left\{B+\underbrace{\left\{[(\mu_B-\beta)+(\delta_C-\gamma)]K_A^*+[(\mu_A-\alpha)+(\lambda_C-\gamma)]K_A^{*\theta}+[(\delta_A-\alpha)+(\lambda_B-\beta)]K_A^{*\phi}\right\}}_{=A\ (>0)}\mathrm{K}^*\right\}\cdot$$

$$\left\{\frac{(3\tau\bar{L})}{K^{*2}}(\mathrm{K}^*+A)\right\}-\frac{(3\tau\bar{L})^3}{\mathrm{K}^{*6}}\frac{\mathrm{K}^*}{1}\{B\} \qquad \text{(A2)}$$

$$=\frac{(3\tau\bar{L})^3}{\mathrm{K}^{*6}}\cdot(\mathrm{B}\cdot\mathrm{K}^*+B\cdot A+A\cdot\mathrm{K}^*\cdot\mathrm{K}^*+A\cdot\mathrm{K}^*\cdot A)-\frac{(3\tau\bar{L})^3}{\mathrm{K}^{*6}}\frac{\mathrm{K}^*}{1}\{B\}$$

$$=\frac{(3\tau\bar{L})^3}{\mathrm{K}^{*6}}\cdot[\mathrm{K}^*\cdot\mathrm{A}^2+(B+\mathrm{K}^{*2})\cdot A]>0$$

When regional externalities are stronger than scale economies in each region, terms in (A1) and also a_2a_1-a_3a_0 are clearly positive. Thus, the steady state of the economy is stable.

Appendix 2: Stability conditions in a case of *asymmetric externalities*

In a case of asymmetric externalities the stability conditions (26) can be written in a form;

$$a_0 = 1 > 0 \qquad \text{(A3)}$$

$$a_1=\underbrace{\frac{3\tau\bar{L}}{\left(K_A^*+K_A^{*\theta}+K_A^{*\phi}\right)^2}}_{\mathrm{K}^*}\left\{\begin{array}{l}K_A^*+K_A^{*\theta}+K_A^{*\phi}+[(\mu_B-\beta)+(\delta_C-\gamma)]K_A^*+[(\mu_A-\alpha)+(\lambda_C-\gamma)]K_A^{*\theta}\\+[(\delta_A-\alpha)+(\lambda_B-\beta)]K_A^{*\phi}\end{array}\right\}>0$$

$$a_2=\frac{(-3\tau\bar{L})^2}{\mathrm{K}^{*4}}\left\{\begin{array}{l}[(\mu_B-\beta)(\delta_C-\gamma)]K_A^{*2}+[(\delta_A-\alpha)(\lambda_C-\gamma)+(\mu_A-\alpha)(\lambda_B-\beta)]K_A^{*\theta}K_A^{*\phi}+[(\mu_B-\beta)+(\delta_C-\gamma)]K_A^*\mathrm{K}\\+[(\mu_A-\alpha)(\lambda_C-\gamma)]K_A^{*\theta 2}+[(\lambda_B-\beta)(\delta_C-\gamma)+(\delta_A-\alpha)(\mu_B-\beta)]K_A^*K_A^{*\phi}+[(\mu_A-\alpha)+(\lambda_C-\gamma)]K_A^{*\theta}\mathrm{K}\\+[(\delta_A-\alpha)(\lambda_B-\beta)]K_A^{*\phi 2}+[(\mu_A-\alpha)(\delta_C-\gamma)+(\mu_B-\beta)(\lambda_C-\gamma)]K_A^*K_A^{*\theta}+[(\delta_A-\alpha)+(\lambda_B-\beta)]K_A^{*\phi}\mathrm{K}\\+(\delta_C-\lambda_C)(\lambda_B-\mu_B)K_A^{*2}+(\delta_C-\lambda_C)(\mu_A-\delta_A)K_A^{*\theta 2}+(\delta_A-\mu_A)(\mu_B-\lambda_B)K_A^{*\phi 2}\\+[(\mu_B-\beta)(\delta_A-\mu_A)-(\delta_C-\gamma)(\delta_A-\mu_A)]K_A^{*\theta}K_A^{*\phi}+[(\mu_A-\alpha)(\lambda_B-\mu_B)-(\lambda_C-\gamma)(\lambda_B-\mu_B)]K_A^*K_A^{*\phi}+\\[(\lambda_B-\beta)(\delta_C-\lambda_C)-(\delta_A-\alpha)(\delta_C-\lambda_C)]K_A^*K_A^{*\theta}\end{array}\right\}>0=C$$

$$a_3=-\frac{(-3\tau\bar{L})^3}{\mathrm{K}^{*6}}\cdot\frac{\mathrm{K}^*}{1}\left\{\begin{array}{l}[(\mu_B-\beta)(\delta_C-\gamma)]K_A^{*2}+[(\delta_A-\alpha)(\lambda_C-\gamma)+(\mu_A-\alpha)(\lambda_B-\beta)]K_A^{*\theta}K_A^{*\phi}\\+[(\mu_A-\alpha)(\lambda_C-\gamma)]K_A^{*\theta 2}+[(\lambda_B-\beta)(\delta_C-\gamma)+(\delta_A-\alpha)(\mu_B-\beta)]K_A^*K_A^{*\phi}\\+[(\delta_A-\alpha)(\lambda_B-\beta)]K_A^{*\phi 2}+[(\mu_A-\alpha)(\delta_C-\gamma)+(\mu_B-\beta)(\lambda_C-\gamma)]K_A^*K_A^{*\theta}\\+(\delta_C-\lambda_C)(\lambda_B-\mu_B)K_A^{*2}+(\delta_C-\lambda_C)(\mu_A-\delta_A)K_A^{*\theta 2}+(\delta_A-\mu_A)(\mu_B-\lambda_B)K_A^{*\phi 2}\\+[(\mu_B-\beta)(\delta_A-\mu_A)-(\delta_C-\gamma)(\delta_A-\mu_A)]K_A^{*\theta}K_A^{*\phi}+[(\mu_A-\alpha)(\lambda_B-\mu_B)-(\lambda_C-\gamma)(\lambda_B-\mu_B)]K_A^*K_A^{*\phi}+\\[(\lambda_B-\beta)(\delta_C-\lambda_C)-(\delta_A-\alpha)(\delta_C-\lambda_C)]K_A^*K_A^{*\theta}\end{array}\right\}>0$$

$$a_1a_2-a_0a_3>0$$