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Distribution Network Configuration Considering Inventory Cost

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Distribution Network Configuration Considering Inventory Cost

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Abstract:
Inter-city distribution network structure, which is a very important issue in regional planning, is strongly affected by those firms' logistic management policies to control inventory cost and to meet with expanding variety of demands and needs of customers.

The purpose of the present paper is to analyze the distribution network structure with consideration of inventory cost in each distribution center, following the two-echelon inventory allocation model by Nozick and Turnquist(2001). With contrast to that their work focused on difference of inventory policies between different type of goods, our analysis pays attention on regional differences in demand rates and unit location cost.

For this purpose, this paper proposes alternate calculation process to obtain the solution consistent both with the distribution center location sub model and optimal inventory allocation sub model.

Our paper applies the model to the realistic Japanese transportation network, and show which cities may possess distribution center function in the nationwide distribution network. The results showed that distribution center usually possess safety stock, but in remote location with small demand but with relatively high location cost, distribution centers stand without safety stock and rely on the back stock at the plant.

Key Words: Logistics, Facility Location, Distribution center, Inventory, Safety Stock

1. INTRODUCTION

Type and quantity of economic activities locating in each sub-region is the most important topic in regional planning and has been analyzed through industrial location theory, inter-regional input output model, and other types of theory and models. Such locations, although being affected by public development policies for transportation system and land-use assistance such as industrial park projects, are the collected result of non-centralized decision makings of each firms, who seeks to build an efficient and cheap distribution network.

In the field of operations research, several types of optimal facility location problems and algorithms for them have been proposed. Such problems typically minimize the logistic cost with exogenously given inter-city transportation cost and facility location cost, but usually dismissed inventory cost. But, when we take inventory to coop with fluctuating demands into account, facility size becomes different for each location reflecting the level of uncertainty of demand there. As observed in many developed countries, customers require more variety of commercial goods, and we must prepare more number of commercial goods. Moreover, life length of each product becomes shorter. Without highly organized management, large inventory for many products yields large risk of depreciation of commercial value as well as large cost for floor space for stocking. Considering those, inventory cost should be explicitly considered in distribution network configuration problem. There is an essential trade off between inventory cost and transportation cost; when you set smaller number of distribution centers having thicker demands there, relative stock size to coop with fluctuations become small and then, we need less inventory cost. But such concentrated location pattern results longer transportation to the customers and larger transportation cost.

Nozick and Turnquist(2001) formulated a two-echelon distribution network formation problem considering inventory cost at plant(central logistic center) and distribution center(DC)s. The model was consisted of two sub models for optimal inventory allocation and optimal DC location. They first made assumption for the number of DCs and average demand flow at DCs and calculated the first sub model for optimal inventory assignment, considering the expected penalty of distribution center stock-out and plant stock-out. Stock-out was considered as the situation when Poisson distributed demand exceeded stock size, and the average demand there was given by the assumption above. Inventory size of each distribution center alters the location cost of distribution center, therefore the second sub model for optimal facility location was solved. If the number of DCs derived meets to the assumption, they admitted the solution. Their paper showed that for thick demand goods, safety stock are kept both in each DC and at plant,

while safety stock for thin demand goods is stored only in plant, based on the calculation by the above assume and check algorithm.

The present paper essentially follows their work, but possesses different interest on regional heterogeneity. We admit the difference of unit location cost for distribution centers by geographical locations. By that expansion, the following situations become to be considered; DCs in highly dense metropolitan regions usually support relatively thick demand. According to the original model, those DCs are considered to possess safety stock by themselves, but large stocks usually forbidden by the higher land price, then safety stock would be stored collectively at central plant, instead. If so, advantage of collecting demands in smaller number of DCs disappears, then larger number of DCs will be settled in Metropolitan regions without own safety stock. In that way, the optimal locations can be very different one from the solution of the original model, if we take regional heterogeneity into account.

In order to get solution sufficiently near to the real optimal, calculation procedure must be improved so as to permit the difference of unit intently cost by region, which reflects land price level.

Our paper applies the model to the hypothetical good distribution system in realistic Japanese transportation network, and show which cities may possess distribution center function in the nationwide distribution network. The results show that distribution center usually possess safety stock, but in metropolitan region with high location cost, distribution centers stand without safety stock and rely on the back stock in the plant elsewhere.

This paper is organized as follows. **Sec.2** explains the model with attention to the improvements from the original one by Nozick and Turnquist(2001). **Sec.3** proposes a calculation procedure for the model which is compatible with regional heterogeneity of unit inventory cost as well as location cost. **Sec.4** shows the result of model application to Japanese Transportation network. **Sec.5** summarizes the consideration, and further research issues.

2. TWO ECHELON INVENTORY MODEL

2.1 Two echelon system

Nozick and Turnquist (2001) formulated a two-echelon distribution network formation problem and endogenized optimal inventory allocation between a central logistic center (called as ”plant”) and several number of distribution center(DC)s, selected from the possible sites, as illustrated in **Fig.1**. Although many manufacturing firms are begin to manage total supply chains including parts and material supply process, in reality, their final assembly plants cannot be easily relocated. Then in the application, it seems sufficient to analyze market chain from their plants to the customers. Moreover, many firms try to make the market chain simpler and reduce the number of echelons.

Therefore, we also assume the same two echelon system composed by one ”plant”(indicated by $j = 0$) and DCs (indicated by $j = 1, 2, \cdots, N$), providing the distribution service to the

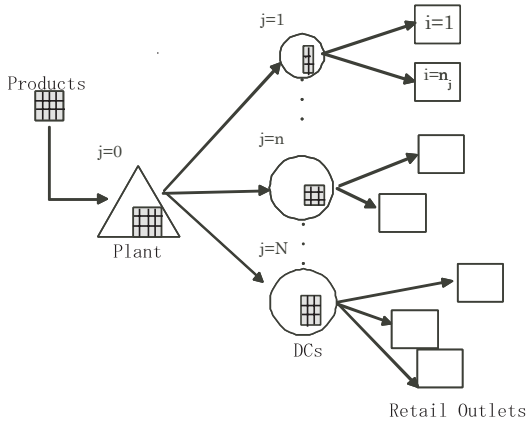

Figure 1. Two echelon inventory system

retail outlets locating all over the nation (indicated by $i = 1, \cdots, I$), replying to the orders from them.

From final manufacturing factories, finished products are sent to the ”plant”, once in the predetermined interval μ_0 to make the plant storage full (s_0). From the plant, several number of products are sent in the given interval μ_1, in order to refill of the DC stock (s_j). DC (indicated by j) have a full stock of s_j just after refill, and send one product when it receives order from the retail outlets under its supervision area (i for $Y_{ij} > 0$). Orders from each retail outlet is assumed to follow mutually independent Poisson distribution with given arrival rate (λ_i). If Y_{ij} be the proportion of demands at retail outlet i supplied by DC j, the aggregated order arrival at DC j is also given by the Poisson distribution, whose arrival rate Λ_j is given by

$$\Lambda_j = \sum_{i=1}^{I} Y_{ij}\lambda_i. \tag{1}$$

If the number of orders in the given refill interval (μ_1) exceeds the storage size (s_j), stock-out occurs and makes the customer wait until the next refill. Possibly some customers prefer canceling to waiting, then make loss of profit. Such loss is evaluated as parameter α.

The probability of DC stock-out $r(s_j)$ is given by the following, when let m_j be number of orders at DC j during the refill interval μ_1.

$$r(s_j) = Prob(m_j > s_j) = \sum_{m_j=s_j+1}^{\infty} \frac{\exp(-\Lambda_j\mu_1)(\Lambda_j\mu_1)^{m_j}}{m_j}. \tag{2}$$

The total demand at the plant is also a Poisson process with mean arrival rate

$$\Lambda_0 = \sum_{j=1}^{N} \Lambda_j = \sum_{i=1}^{I} \lambda_i. \tag{3}$$

The stock-out probability at plant with capacity s_0 and replenishment interval μ_0 is given by the similar equation with *eq.*(2), such as

$$r(s_0) = Prob(m_0 > s_0) = \sum_{m_0=s_0+1}^{\infty} \frac{\exp(-\Lambda_0\mu_0)(\Lambda_0\mu_0)^{m_0}}{m_0}, \tag{4}$$

where m_0 is number of orders at plant during the replenishment interval μ_0.

There is no direct effect of plant stock-out on customers, as long as stock remains at the DCs. However, once stock-out is happened at the plant and backorders accumulate at the plant, the succeeding replenishment to DCs must be postponed. The average additional waiting time at the DCs is given by the expected number of backorders at the plant, divided by the plant demand rate, Λ_0, according to the Little's Law.

$$W_0 = \left\{ \sum_{m_0=s_0+1}^{\infty} (m_0 - s_0) \frac{\exp(-\Lambda_0\mu_0)(\Lambda_0\mu_0)^{m_0}}{m_0} \right\} \Big/ \Lambda_0. \tag{5}$$

The refill interval from the plant to the DCs μ_1 in *eq.*(2) is replaced by the expected replenishment time as,

$$\mu_1' = \mu_1 + W_0. \tag{6}$$

This replenishment postponing violates the assumption of independency between the demand and resupply processes, but the difficulties are considered to be minor (Diks et al., 1996).

In case of simultaneous stock-outs at the plant and at some DC, the customer must wait longer, and the expected profit loss becomes larger than the case of stock-out in DC only. The expected loss is indicated by given parameter β, which is naturally larger than α for stock-out at DC only.

In order to avoid such stock-out losses, larger number of products than the average demand must be stored at DCs and the plant. In the logistic theory, the stock for average demands in replenishment time is called cycle stock, while additional stock over that cycle stock is called safety stock. In this model, we assume that safety stocks are non-negative, then,

$$s_0 \geq \mu_0\Lambda_0, \quad s_j \geq \mu_1\Lambda_j \tag{7}$$

Fig.2 illustrates the typical dynamics of the number of products stored in one DC during the replenishment interval. When excess demand during the interval becomes larger than the safety stock, DC stock-out occurs. Without safety stock, the cycle stock can cover the fluctuating demand with just 50 % of probability. As more safety stock is prepared, stock-out probability becomes smaller.

In order to prepare the stock at plant or at each DC site, corresponding cost is required. Assume that for each site, total inventory cost C_j can be given as linear function of stock size s_j, with certain fixed cost f_j. If h_j be unit cost for storage capacity, then,

$$C_j(s_j) = f_j + h_j s_j \tag{8}$$

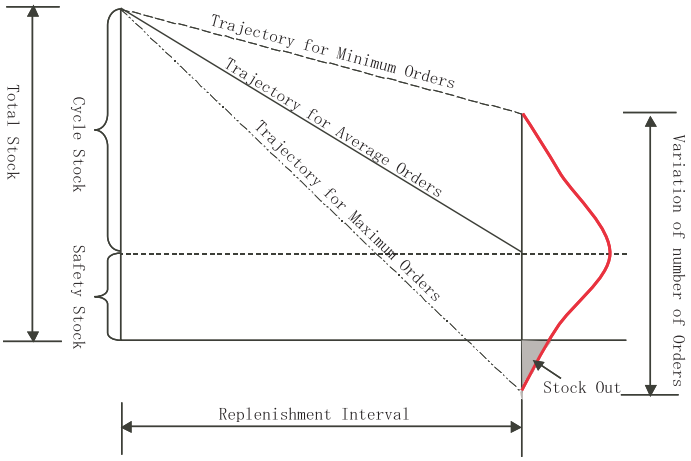

Figure 2. Dynamics of the number of products stored in one replenishment interval

Contrast to the original formulation, we permit the heterogeneity of unit cost h_j, as well as the fixed cost f_j according to the location of DC. In our analysis, those costs are given reflecting the land price of each location.

Similarly, at the plant, storage cost C_0 is given by the following linear function of storage size s_0;

$$C_0(s_0) = f_0 + h_0 s_0 \tag{9}$$

2.2 Optimal stock allocation

In order to know the most efficient level of safety stocks at the plant and the DCs, the following cost, which consists of expected stock-out penalties and inventory costs, must be minimized, with non negative safety stock conditions *eq.*(7).

$$\min_{s_j, s_0} \alpha [1 - r_0(s_0)] \sum_{j=1}^{N} \Lambda_j r_j(s_j) + \beta r_0(s_0) \sum_{j=1}^{N} \Lambda_j r_j(s_j) + \sum_{j=1}^{N} h_j s_j + h_0 s_0. \tag{10}$$

2.3 Optimal DC location selection

To search the efficient number of the DCs N and location of each DC, we can utilize optimal facility location problem minimizing the total cost composed by the location cost and transportation cost, as formulated in the field of operation research. We take the following assumptions in order to simplify the location problem.

1) Consider a firm whose customers are locating all over the country.

2) Products are conveyed one way from the plant to the DCs, and from each DC to retail outlets locations supervised by the DC by trucks.

3) Transportation cost between the plant and DCs is negligible, because those transportation in large lot size require relatively small unit cost, comparing to the lower transportation from DC to the retail outlets.

4) The fixed location cost f_j and unit storage cost h_j are given in proportional with land price of the location j.

5) The location of the plant is exogenously given.

Optimal facility location problem to give the number of DCs and the locations can be formulated as follows, when K be the candidate location set for DCs.

$$\min_{X_k, Y_{ik}} Z^p_{LP} = \sum_k^K C_k X_k + \sum_k^K \sum_i^I g\lambda_i d_{ik} Y_{ik} \tag{11}$$

$$\text{subject to} \quad X_k \in \{0,1\}, \qquad \forall k \in K, \tag{12}$$

$$\sum_k^N Y_{ik} = 1, \qquad \forall i \in I, \tag{13}$$

$$Y_{ik} \leq X_k, \qquad \forall k \in K, \ \ \forall i \in I, \tag{14}$$

where, X_k is integer variable indicating the existence of DC in location $k \in K$, Y_{ik} is the proportion of demand in i supervised by DC k, C_k is location cost of DC at location k, g is unit time period, and d_{ik} is unit transportation cost between location k to i. (13) is a condition to cover all $i \in I$. (14) is a consistency between X_k and Y_{ik}, if customer in i can not be assigned to k without facility (eliminating $Y_{ik} = 1$ when $X_k = 0$).

Because Y_{ik} become binary due to the consistency condition of (14) and binary definition (12) of X_k, this problem is integer programming problem (IP).

3. INTEGRATED MODEL AND SOLVING PROCEDURE

Combining the two minimization problems (10) and (14), we can get optimal number of DCs, location and stock size at each DCs, as well as stock size at the plant.

At first, exogenous parameters, $\alpha, \beta, g, \mu_1, \mu = 0, \lambda_i, f_j, h_j, f_0, h_0$ and d_{ik} are given. Solution of the second location model, Y_{ik} give the demand arrival rate of each DC Λ_j through *eq.*(**??**) as the input for the first stock assignment model.

The first model is a non-linear problem whose solution space has dimension of $N + 1$ over control variables s_0 and s_js. However, there are no interactions between the stock capacities of the different DCs in *eq.*(10), the optimization problem is separable and monotonic for each s_j. The following procedure can be used considering that s_0 and s_j are integer variables satisfying *eq.*(7).

1) Set s_0 be the smallest integer value no less than the cycle stock at the plant, $\mu_0\Lambda_0$.

2) For each DC, j, set s_j be the smallest integer value exceeding $\mu_1\Lambda_j$, calculate the value of the objective function(10).

3) Increase s_j one by one until the total cost begins to increase. Keep s_j^* as the candidate for the optimal solution.

4) After all DC stocks are determined, the value of optimal function for s_0 and s_j^* $\forall j$ is calculated and kept for candidate solution.

5) Unless the function value increase, add one to s_0 and repeat the steps from step **2)**, above.

Using the solution of the first model, s_j, we reflesh the location cost for each DC through *eq.*(8), which is required for the location problem. However, *eq.*(8) give the location cost only for the sites where DC is locating in the present situation.

The original study of Nozick and Turnquist(2001), proposed two alternate ways to give the stock capacity where DC is not locating at present. One way is to determine a critical stock-out level and know the total stock required in total system (plant and all DCs). If we divide that value by N after subtraction of s_0, required stock level is estimated. The other way is to give the average stock of the present DCs for all potential locations. In their work, they neglect any differences in unit stock cost h_j by locations, those two ways give the similar result. But if we introduce the heterogeneous stock cost h_j in each location, both of their approximation of stock level gives a trouble in conversion of iterative process.

We take therefore, a different way to give the stock for potential DC locations, which is compatible when optimal solution is met. The way is to assume that if a potential location is selected, then such new DC takes over the function of the DC now responsible for that location, instead. Then, the same amount of stock must be prepared. This assumption is formulated as following,

$$s_k^* = s_j, \text{ such that } Y_{kj} = 1 \tag{15}$$

Then, we can set the location cost in the location problem is given as,

$$C_k = \begin{cases} f_k + h_k s_k & \text{if DC locates at } k \\ f_k + h_k s_j \text{ such that } Y_{kj} = 1 & \text{unless DC locates at } k \end{cases} \tag{16}$$

With this procedure, all parameter of the optimal location problem (14) are fixed and can be solved by appropriate algorithm for non-capacitated facility location problem.

If binary condition (12) is relieved to positive real, we get a linear programming (LP) and simplex method is applicable to get the optimal solution Z_{LP}^p (Campbell, 1990). Due to a strength of constraint for solution space, Z_{IP}^p is not less than Z_{LP}^p, and equal sign only appears when optimal LP solution is integer. However, simplex method needs a long calculation time for the problem with many constraints. Actually our model includes N (number of DC candidates) $\times$ I (number of demand locations) constraints, and the

constraints matrix are very sparse. Such problem can not be effectively solved even by modern LP and interior point method. Another popular algorithm for IP is branch and bound method, which is an enumeration method using lower bound information of objective function. This procedure makes sub-problems by setting restrictions on some locating candidates k (i.e. $X_k = 1$ or $X_k = 0$ for some k), which is called 'branch', and estimate the lower bound of the branch k. If the lower bound of the branch k is inferior to another branch that is already estimated, we can terminate the branch k and move to further branch, which is called 'bound'. Therefore, the efficiency of branch and bound critically depends on the accuracy of lower bound and calculation time for sub-problems. The algorithm for sub-problem is required accuracy and quickness.

Erlenkotter (1978) proposed an efficient procedure based on branch and bound method. According to duality theorem in LP, the value of dual objective function under a set of feasible dual solution gives a lower bound value of the primal objective function ($Z^d \leq Z^p$). If Z^d is equal to Z^p, the feasible dual solution is optimal. The dual objective function for (14) is formulated as following (17).

$$\max_{v} Z^d(v) = \sum_{i \in I} v_i \tag{17}$$

The objective function will be maximized subject to

$$\textstyle\sum_{i \in I} max\{v_i - D_{ik}, 0\} \leq C_k \quad \forall k \in K \tag{18}$$

where, v_i is dual variable, D_{ik} is $g\lambda_i d_{ik}$

As relationships between optimal primal solutions (X_k^*, Y_{ik}^*) and optimal dual solutions (v_i^*) under LP solution space, complementary slackness conditions are required as following (19) and (20).

$$X_k^*(C_k - \textstyle\sum_{i \in I} max\{v_i^* - D_{ik}, 0\}) = 0 \quad \forall k \in K \tag{19}$$

$$(X_k^* - Y_{ik}^*)(max\{v_i^* - D_{ik}, 0\}) = 0 \qquad \forall i \in I, \forall k \in K \tag{20}$$

When a primal objective function is to be minimized, the corresponding dual objective function is to be maximized. By introducing slack variables (sl_k), we can rewrite (19) into (21)

$$\begin{aligned} \textstyle\sum_{i \in I} max\{v_i - D_{ik}, 0\} + sl_k = C_k \qquad & \forall k \in K \\ if\ sl_k = 0 \Rightarrow X_k = 1, otherwise, X_k = 0 & \end{aligned} \tag{21}$$

Eq.(21) means that v_i can be increased until blocked by one of C_k. Therefore, if we increase v_i with filling the constraint in *eq.*(21), we can maximize the dual objective function (17) and obtain X_k by checking sl_k. Y_{ik} is obtained by checking the minimum D_{ik} among k with $X_k = 1$, then $Y_{ik} = 1$ for such k.

Erlenkotter's procedure consists of three stages. First stage is called *dual ascent procedure*, we increase v_i in stepwise from the lowest D_{ik} among k for each i until all v_i blocked by C_k through *eq.*(21). However, *dual ascent procedure* can not always give a set of optimal solution, because the solution of this procedure depends on the ascending order in v_i. Then secondly, if $Z^p \neq Z^d$, we can check violations in *eq.*(20). Decreasing v_i which violates *eq.*(20), then again v_i are increased with different ascending order, in order to get better solution. That is called *dual adjustment procedure*. Thirdly, in case of $Z^p \neq Z^d$ after *dual adjustment procedure* finished, final stage (*branch and bound*) is required. In this stage, by checking violations in *eq.*(20) again, we can branch for violating k and evaluate the lower bound of the branch, then bound to another violations. In the third stage, *dual ascent / adjustment procedure* are repeatedly called as subroutines in order to estimate a lower bound of the branch.

Through the application test, Erlenkotter reported that even if *dual ascent / adjustment procedure* can not give a optimal solution, these procedure yields the good approximation to optimal (i.e. $\delta = Z^p - Z^d$ is small enough), this procedure can terminate a branch efficiently in most case. We apply this algorithm for our problem.

After the optimal location problem is solved, the number of DCs N and the market assignment for each DC, Y_{ij} are replaced by the new solution, and repeat to solve the first stock allocation problem. Such refreshment is repeated until the solution of the model meet the predetermined level of convergence. If the process gives cyclical solutions, we select the minimum cost solution from them.

As a result, our calculation procedure enable a PC to get the solution of 207 demand sites problems with different stocking cost in 207 possible DC sites, in feasible calculation time, contrasting to the simple procedure by Nozick and Turnquist (2002) applicable for uniform stock cost for all potential DCs.

4. APPLICATION

4.1 Case Setting

As an example of the application of the model system, we consider a distribution system of the finished automobiles, just same as the original study, but our case focus only on the distribution of trucks in Japan, rather than American passenger automobiles.

We consider the distribution from one domestic plant to the 207 regions all over Japan, through highway network in year 2000. Demand arrival rate in each region is given by allocating the annual domestic truck sales in year 1995 (177,264) into each region with proportion of the present number of registered trucks in each region. The arrival rates are distributing as **Fig.3**.

Inter-regional transportation cost d_{ij} is given by the generalized cost including the expressway fare for truck and time value (3,000 yen / hr) of driving time between the regions

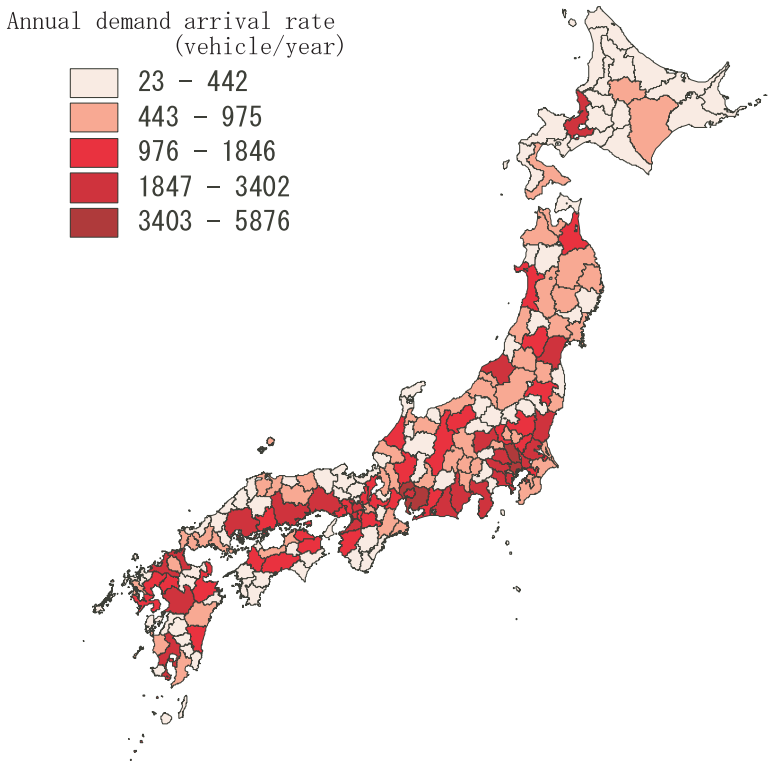

Figure 3. Demand arrival rate of each region

through the shortest time path based on expressway, national road and prefectural road network. Since the target network is inter regional, we can neglect the congestion (transportation time is flow independent). We calculate it by using GIS function (ARC/INFO), for the network in year 2000.

DCs are considered to be locatable at any of 207 regions. Both fixed location cost f_i and unit stock cost h_j are set reflecting the land price level in each location. We assume that each DC requires fixed area for offices ($100m^2$) plus unit parking space ($30m^2$) times the stock capacity, s_k. Assume the business length of each DC be ten years. We consider the firm purchase the land for DC and that cost must be returned by flat payment for the years, with 4% of interest rate. Therefore, the annual payment is given as 12.3% of the land price. Land price data for each region is given as average price of residential and industrial used spots in the region, reported by the Ministry of Land, Infrastructure and Transportation. **Fig.4** shows the distribution of the annual payment for the unit area over the regions. DC location cost is given by the required annual payment for the required space plus fixed cost of setup and maintenance of a DC (5 million yen, annually). As stated before, we consider only transportation cost between DC and retail regions, ignoring that between the plant and DCs. Due to this assumption, we can neglect the location of the plant, because that location has no effects on the locating problem. Although, we must set certain value for unit stock cost at the plant, h_0 to solve the stock allocation problem. We set it as the average value of h_ks over the 207 regions. (Fixed location cost

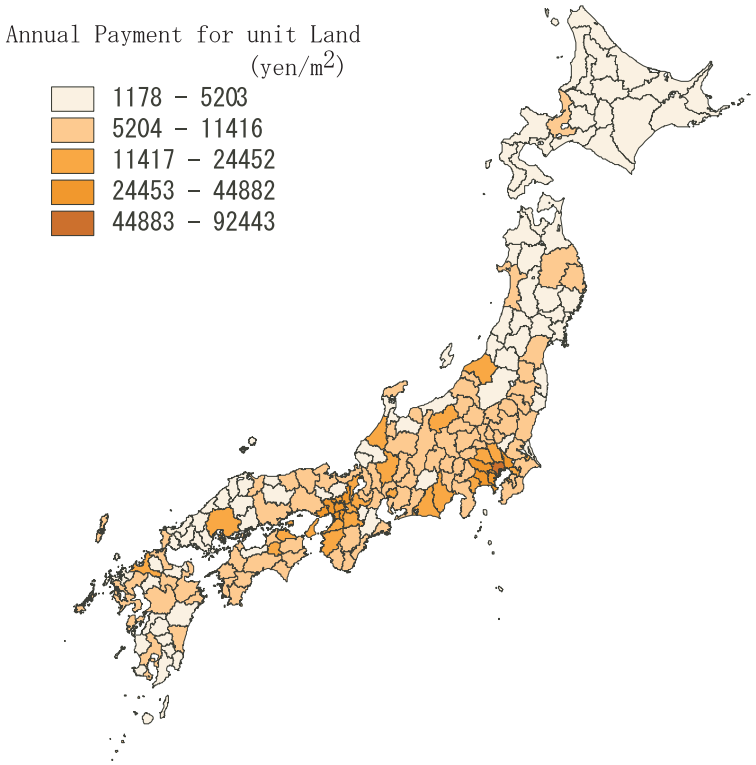

Figure 4. Annual land-use cost per unit area

of plant f_0 has no effect for the optimality of the problem, then we ignore it.)

The other parameters are set as follows; replenishment interval at plant $\mu_0 = 6$(days) and that in DCs $\mu_1 = 3$(days), respectively, stock-out penalties are $\alpha = 600$(yen) and $\beta = 1,200$(yen).

4.2 Result of the base case

Let us call the setting explained above, "Base case," which give the 42 DC locations out of 207 candidates, as shown by **Fig.5**. The territories of DCs are similar to the 46 Prefectures division, but in metropolitan regions such as Tokyo, Nagoya and Osaka, smaller number of DCs collectively provide service for wider area than Prefecture division. Our model permits heterogeneity in location cost for DCs, with contrast to the original model, then successfully describe the following phenomena; In order to reduce the location cost, DC location is usually selected at relatively inexpensive outskirts, rather than nodal city in each territory. However, in peripheral region with sparse transportation network, where accessibility of outskirts locations is much inferior to the nodal city, DC will stand at the nodal city: Such situation is observed in Sapporo, Akita, Okayama and Kumamoto regions.

Concerning safety stock allocated, there are two types of DCs in the solution. One type is DCs with safety stock, the other is without any safety stock. In the base case, 35 DCs are in the first category, and 7 DCs (Hakodate, Tokushima, Nagasaki, Kagoshima

Figure 5. DC locations (42) in the Base Case

and other 3 remote island regions) are classified as type 2. Safety stock has effect on reducing the stock-out probability and expected penalty. Because independent distribution is assumed for arrival in each retail locations, simultaneous demand over-runs seldom occur. Therefore, in order to keep stock-out probability constant, larger DC need not to possess the stock of the same proportion to the demand rate, comparing to smaller DC. This property activates as incentive to collect the territories and let it be served by larger scale DC. This is why type 1 DC with safety stock are dominant.

When the market area locates in less accessible location, such as at remote islands, collective governance becomes expensive considering transportation cost. Even in such case, if that area has large demand enough and inexpensive location cost, stocking its own safety stock becomes affordable. But it is not the case for the type 2 DCs above; these locations are poor accessible from the adjacent regions, land price is relatively high in spite of sparse demand.

4.3 Case neglecting the stock cost at DCs

In order to clarify the effect of inventory cost, we solve the problem ignoring the variable location cost relating to the stock capacity, by setting h_k as 0 for all regions. The solution is shown in **Fig.6**, and 66 DCs are observed with smaller territories. Because we neglect the stock cost, location cost have smaller weight comparing to that of transportation cost. Moreover, the effect of collective safety stocking discussed above disappears. As a result,

Figure 6. DC locations (66) in the case neglecting the stock cost

smaller territories by larger number of DCs become effective policy, and the DC locations become more strongly demand driven.

4.4 Sensitivity to stock-out penalties

Quality of reliable service at DC expected by customers is modelled through the stock-out penalty parameters, α and β. **Table.1** shows the solution of the model when those parameters are changed from the base case, keeping the relationship of $\beta = 2\alpha$. As stock-out penalty parameters increase, safety stock to supress the stock-out probability becoms more important. In order to take the advantage of collective safety stocking effect, larger DCs becoms more efficient, in spite of aditional transportation cost from DC integration. As a result, number of DCs decreases as shown in the third column in **Table.1**.

After $\alpha \geq 3000$, however, number of DCs does not decrease anymore. Stock-out probabilities are surpressed by increasing safety stock in each DC, instead, as shown in the last two colums.

While stock-out penalty is small, each DC can reduce location cost by having no own safety stock. Up to $\alpha \leq 60$, all DC store cycle stock, only, as well as the plant. On the contrary, when $\alpha \geq 1200$, all DC possess own safety stock. If penalty valus sits in between, some DCs have safety stock but the others does not. **Fig.7** shows the two types of DCs on the demand and land cost plate, which teaches that DCs with dense demand and inexpensive location cost possess own safety stock.

Table 1. Sensitivity to stock-out penalty parameters

α	β	N	Location	Transp.	Stock-out	Total	Total Stocks	
(yen)	(yen)		Cost	Cost	Penalty	Cost	at DCs	Plant
0	0	43	58,153	69,683	0	127,836	2051	4087
3	6	43	58,153	69,683	103	127,939	2051	4087
15	30	43	58,153	69,683	517	128,353	2051	4087
30	60	43	58,153	69,683	1,034	128,870	2051	4087
60	120	43	58,153	69,683	2,069	129,904	2051	4087
90	180	43	58,462	69,683	2,816	130,961	2080	4093
150	300	40	57,521	73,364	3,172	134,057	2195	4087
210	20	44	61,671	70,228	3,477	135,376	2257	4087
240	480	43	61,729	69,369	3,358	134,456	2292	4087
270	540	42	61,376	70,131	3,186	134,693	2333	4087
300	600	42	61,725	70,617	3,078	135,420	2354	4087
600	1200	42	64,424	70,220	2,432	137,077	2501	4087
1200	2400	39	63,301	74,406	1,868	139,576	2594	4087
1500	3000	38	63,303	75,198	1,789	140,290	2620	4087
1800	3600	39	63,743	74,179	1,744	139,666	2653	4087
2100	4200	40	65,067	72,560	1,749	139,377	2678	4087
3000	6000	37	63,842	75,760	1,586	141,171	2693	4087
15000	30000	38	65,522	78,538	1,356	145,416	2860	4087
30000	60000	37	63,306	81,093	1,154	145,554	2932	4087
450000	90000	37	66,457	78,063	1,197	145,717	2957	4087

(Costs are $\times 10^4$ yen)

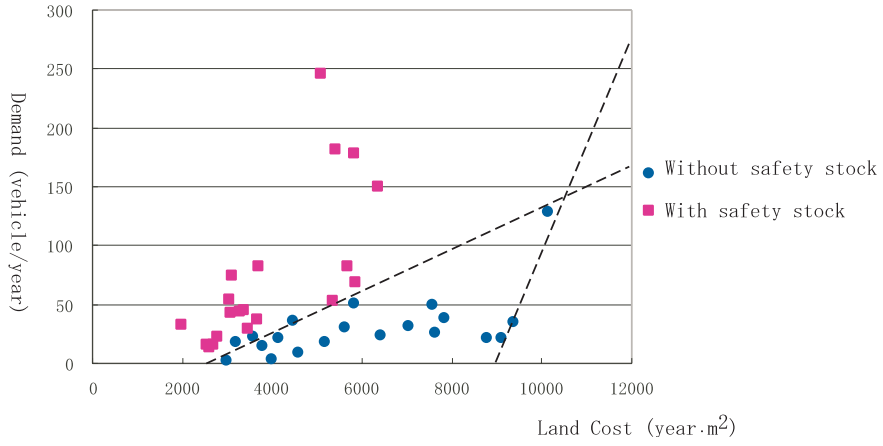

Figure 7. Demand, location cost and type of DCs (α = 150, β = 300)

4.5 Sensitivity to transportation cost

From the local government perspective, who seek inviting DC as economic activity providing local jobs, The following two policy options are typically considered.

1) To improve highway network and to provide inexpensive transportation cost,

Table 2. Effect of general level of transportation cost

Cost modif. rate	N	Location Cost	Transp. Cost	Stock-out Penalty	Total Cost	Total Stocks at DCs	Total Stocks at Plant
0.005	1	9,048	5,172	48	14,268	2209	4087
0.01	2	10,740	7,332	98	18,170	2254	4087
0.03	2	10,740	21,996	98	32,833	2254	4087
0.05	5	18,732	20,731	289	39,752	2304	4087
0.1	6	23,431	27,392	381	51,204	2314	4087
0.15	9	28,027	29,933	544	58,505	2385	4087
0.2	12	35,605	31,328	741	67,674	2378	4087
0.25	11	33,192	40,965	669	74,827	2378	4087
0.3	12	32,982	48,721	651	82,354	2381	4087
0.31	15	32,896	48,608	698	82,201	2408	4087
0.32	16	37,056	43,894	809	81,759	2412	4087
0.32	20	4,890	37,149	1,181	83,220	2439	4087
0.33	20	44,142	37,537	1,125	82,805	2439	4087
0.35	19	39,167	43,860	948	83,975	2439	4087
0.4	20	43,052	45,691	1,083	89,825	2441	4087
0.45	25	49,585	43,982	1,422	94,989	2456	4087
0.5	25	50,086	49,268	1,492	100,847	2454	4087
0.55	30	53,709	50,549	1,675	105,933	2475	4087
0.6	34	57,003	50,819	1,907	109,729	2482	4087
0.8	36	56,448	66,609	1,901	124,958	2495	4087
1.0	42	64,424	70,220	2,432	137,077	2501	4087
1.05	43	65,114	73,070	2,520	140,703	2494	4087
1.11	43	65,151	77,444	2,509	145,104	2503	4087
1.18	6	67,325	78,873	2,675	148,873	2502	4087
1.25	56	86,885	61,196	4,542	152,623	2435	4129
1.43	64	94,389	60,486	5,624	160,499	2406	4141
1.67	71	101,301	58,150	6,455	165,906	2378	4153
2.0	76	98,921	70,706	5,985	175,612	2396	4147

(Costs are $\times 10^4$ yen)

2) To provide inexpensive land near transportation node (port or expressway IC), through industrial park project, for example.

The proposed model include heterogeneous land price and inter-regional transportation cost as exogenous parameters, then we can simulate the case with such policies, one by one, by solving the model under the corresponding adjusted parameter settings. However, in this paper, we only check the sensitivity of the model for the changes in general level of transportation cost.

Table.2 shows the effect of transportation cost change, when all inter-regional transportation costs are proportionally modified from the base case. The first column shows the modification rates increasing from 0.005 to2. As transportation cost becomes more

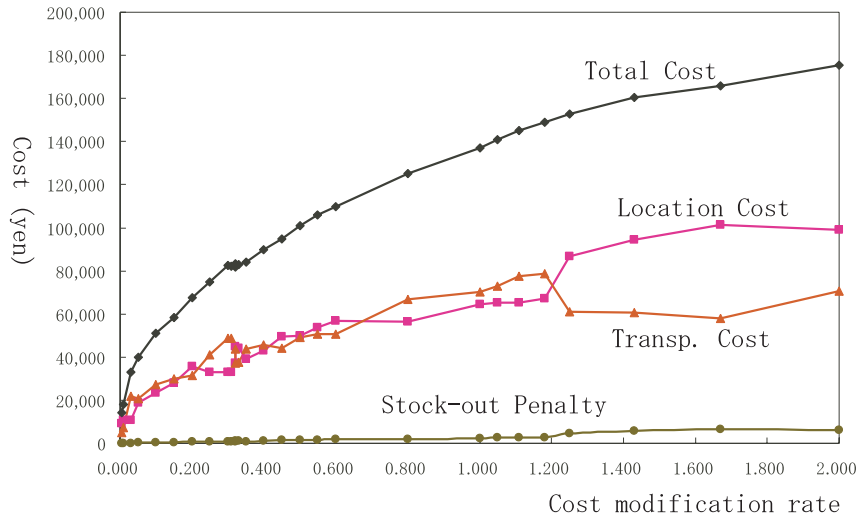

Figure 8. Change in total cost and its components

important, the number of DCs increased as shown in column 2, especially rapidly for the ratio between 0.3 and 0.6, and between 1.25 to 1.43. **Fig.8** shows the change of total cost and the components of it. It is natural that transportation cost inflation makes monotonous increase of total logistic cost, as shown by the first plots. But if we check the cost components, the behavior of the model is not proved so simple. At several points of the modification ratio, location cost crossed with transportation cost. This phenomenon seems to reflect that relative efficiency of the following two policies changes; to decrease transportation cost with sacrifice of location cost increase by opening new DC, and to decrease the location cost by DC unification with sacrifice in transportation cost.

According to the last two columns of **Table.2**, total DC stock is increased up to the modification ratio of 1.2, because collective safety stocking effect become smaller as the number of DCs increased. After the threshold, demand per one DC becomes too small to keep its own safety stock, then collective plant stock takes over the role of the de-centralized safety stock.

5. CONCLUSION

This paper has improved the two-echelon inventory allocation model by Nozick and Turnquist(2001), so as to coop with regional heterogeneity of demand rate and of location cost. The alternate calculation procedure was also developed to obtain the solution consistent both with the distribution center location sub model and optimal inventory allocation sub model.

The model was applied to the distribution system of truck vehicles to 207 regions through the realistic Japanese transportation network, and showed which cities may possess distribution center function in the nationwide distribution network. Comparing to

the result from the model neglecting stock cost at DCs, the model proposed a system with smaller number of DCs at inexpensive locations, in order to enjoy the collective safety stock effect in larger DCs.

Sensitivity analyses were also done for stock-out penalty parameters and general level of transportation cost. Number of DCs and configuration of type of DCs in term of own safety stock are affected by the changes of those settings. Those interesting results became available from our model expansion from the original one.

There are several directions for further research work. Realistic case studies for other products besides the truck vehicles and surveys of real DC location patterns would prove general applicability of the present model. In our model, products flow oneway from plant to DC and DC to retail outlets, but if stock-out occurs at one DC, refill from other DC with stock can be done. We must consider the modifications of the model in order to admit such possibilities. Conceptual expansion of the model such that it can include reverse flow are another possibility, to coop with growing importance of cyclical society.

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