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A Test Strategy for Spurious Spatial Regression, Spatial Nonstationarity, and Spatial Cointegration

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**A Test Strategy for Spurious Spatial Regression,
Spatial Nonstationarity,
and Spatial Cointegration.**

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ABSTRACT

A test strategy consisting of a twofold application of a Lagrange Multiplier test is suggested as a device to reveal spatial nonstationarity and spurious spatial regression. It is further illustrated how the test strategy can be used as a diagnostic for presence of a spatial cointegrating relationship between two variables. Using Monte Carlo simulations it is shown that the small sample behaviour of the test strategy is as desired in these cases.

JEL Classifications: C21; C40; C51; J60.

Keywords: Spatial nonstationarity, spatial cointegration, spurious regression.

1. INTRODUCTION

Spatial regression has been discussed widely in books dedicated to developments in spatial econometrics, notably by Anselin (1988a), and Anselin and Florax (1995). The consequences for estimation and inference in the presence of stable spatial processes have been extensively investigated (Haining 1990; Anselin 1988a; Bivand 1980; Richardson 1990; Richardson and Hémon 1981; Clifford and Richardson 1985; Clifford, Richardson and Hémon 1989). A recent study (Fingleton 1999) takes the first steps into analyses of implications of spatial unit roots, spatial cointegration and spatial error correction models. A follow-up to this study is found in Mur (2002), where the concept of spurious spatial regression is established in a framework of spatial trend (non)stationarity. In Lauridsen (2002) estimation of spatial error-correction models using an IV approach is investigated.

The present paper refines the suggestions of Fingleton (1999). Specifically, Fingleton suggests that “very high” values of Morans I test for spatial residual autocorrelation indicate spatial nonstationarity and spurious regression. It is, however, left as an open question how to distinguish between stationary positive autocorrelation and nonstationarity. The present investigation shows that a twofold application of the LM test for residual autocorrelation can provide a better founded basis to separate these two cases. It is further shown that the same procedure works as a diagnostic for spurious regression. Next, it is suggested that the test procedure works well as a test for spatial cointegration, using a specific two-variable data generating process. In all cases, the small-sample properties of the suggested procedures are derived using Monte Carlo simulation. It is concluded that the procedure works well in all cases, even for fairly small sample sizes. Finally, the practical applicability of the suggested approach is shortly illustrated using two

cases from recent empirical research.

2. MODELS WITH SPATIAL DYNAMICS

2.1. The regressive, spatial autoregressive model.

The first order spatial autoregressive model [SAR(1) model] was initially studied by Whittle (1954) and has been used extensively in works by Ord (1975); Cliff and Ord (1981); Ripley (1981); Upton and Fingleton (1985); Anselin (1988a); Griffith (1992); Haining (1990); Lauridsen (2002). The regressive, first order spatial autoregressive model [SARX(1) model] is defined by

$$(2.1) \quad \mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}),$$

in which \mathbf{y} is an $n \times 1$ vector, \mathbf{X} an $n \times K$ matrix of exogenous variables, ρ the autoregressive parameter, \mathbf{I} the $n \times n$ identity matrix, $\boldsymbol{\epsilon}$ an $n \times 1$ vector of white noises distributed with variances σ^2 , and \mathbf{W} an $n \times n$ proximity matrix defined by $W_{ij} = 1$ if observation j is assumed to impact observation i , and $W_{ij} = 0$ otherwise (Footnote 1). \mathbf{W} may be noncircular, which is the case for the time series variant where $W_{ij} = 1$ if $j = i-1$, for $i = 2, 3, \dots, n$. For the general spatial case, \mathbf{W} is generally circular. For example, if the sample consists of a cross-section of n regions, \mathbf{W} is usually defined by $W_{ij} = W_{ji} = 1$ if region i and j are neighbours. As proved by Anselin (1988a), circularity of \mathbf{W} renders OLS estimation of the parameters inconsistent. This is in contrast to the time series case (and any other non-circular cases) where OLS provides consistent (although inefficient) estimation.

2.2. Spurious regression and nonstationarity.

If one or more of the \mathbf{x} variables are generated according to a SAR scheme, a risk of spurious regression occurs. Especially, the case of spatial nonstationarity, where y and one or more of the \mathbf{x} variables have a ρ close to 1, the risk of spurious regression is alarmingly high. This is demonstrated in Fingleton (1999) where extremely high values of the Moran's I test for spatial autocorrelation is shown to indicate spatial nonstationarity. The Moran I test is defined as

$$(2.2) \quad I = (n/S) (\mathbf{e}'\mathbf{W}\mathbf{e} / \mathbf{e}'\mathbf{e}),$$

where S is the sum of the elements in \mathbf{W} and \mathbf{e} the estimated OLS residual vector. It is asymptotically normally distributed with mean

$$E(I) = (n/S) \text{tr}(\mathbf{M}\mathbf{W}) / (n-K)$$

and variance

$$V(I) = (N/S)^2 [\text{tr}(\mathbf{M}\mathbf{W}\mathbf{M}\mathbf{W}') + \text{tr}(\mathbf{M}\mathbf{W})^2 + (\text{tr}(\mathbf{M}\mathbf{W}))^2] / [(N-K)(N-K+2)] - E(I)^2$$

where K is the number of exogenous variables (including the constant term), and $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. Thus, the standardised value $Z = (I - E(I)) / \sqrt{V(I)}$ is asymptotically standard normally distributed under H_0 : ϵ is white-noise. However, Fingleton leaves it as an open question how to separate the case of stationary positive autocorrelation ($0 < \rho_\epsilon < 1$) from the nonstationarity case ($\rho_\epsilon = 1$).

The present study suggests a twofold application of a Lagrange Multiplier test for spatially autocorrelated errors. The LM error statistic (LME) developed in Anselin (1988a, 1988b),

$$(2.3) \quad \text{LME} = (\mathbf{e}'\mathbf{W}\mathbf{e} / \sigma^2)^2 / \text{tr}(\mathbf{W}^2 + \mathbf{W}'\mathbf{W}).$$

is asymptotical χ^2 distributed with 1 degree of freedom under H_0 : $\rho_\epsilon = 0$

In the case of spurious regression, the error term ϵ of the regression

$$(2.4) \quad \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

will contain a unit root, i.e.

$$\boldsymbol{\epsilon} = \rho_{\epsilon} \mathbf{W} \boldsymbol{\epsilon} + \boldsymbol{\mu}, \quad \boldsymbol{\mu} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}),$$

with $\rho_{\epsilon}=1$. Therefore, a large LME value indicates either spatial nonstationarity or stationary (positive or negative) autocorrelation. This result corresponds to the suggestions of Fingleton (1999) with the Moran I test replacing the LM test. Next, under H_0 : nonstationarity, it follows that

$$\boldsymbol{\epsilon} = \mathbf{W} \boldsymbol{\epsilon} + \boldsymbol{\mu} \Leftrightarrow \boldsymbol{\epsilon} = \boldsymbol{\Delta}^{-1} \boldsymbol{\mu}$$

so that

$$(2.5) \quad \boldsymbol{\Delta} \mathbf{y} = \boldsymbol{\Delta} \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\mu}$$

where $\boldsymbol{\Delta} = \mathbf{I} - \mathbf{W}$ denotes the spatial difference operator. Equation (2.5) implies that a regression of $\boldsymbol{\Delta} \mathbf{y}$ on $\boldsymbol{\Delta} \mathbf{X}$ provides a white noise error, so that the LM error test statistic for this spatially differenced model (DLME) will be close to zero. On the other hand, if H_0 : nonstationarity does not hold, then the spatial differencing will bring about a negative (stationary) spatial residual autocorrelation leading to a positive DLME value. Concluding, the test strategy consists of calculating and inspecting the LME and the DLME values, leading to one of three conclusions (Footnote 2): Nonstationary, spurious regression (LME positive, DLME zero); stationary spatial autocorrelation (LME and DLME positive); or absence of autocorrelation (LME zero, DLME positive).

It may be further relevant to investigate whether \mathbf{y} or any of the \mathbf{x} variables are spatially nonstationary. This may be revealed by using the suggested procedure for a regression of the variable in question (i.e. \mathbf{z} being one of $\mathbf{y}, \mathbf{x}_1, \mathbf{x}_2, \dots$) on a constant term. Specifically, the regressions

$$\mathbf{z} = \boldsymbol{\alpha} \mathbf{i} + \boldsymbol{\epsilon}$$

and

$$\Delta \mathbf{z} = \alpha \Delta \mathbf{i} + \epsilon$$

readily provide the LME and DLME test statistics, which lead to one of three conclusions: \mathbf{z} is spatially nonstationary (LME positive, DLME zero); \mathbf{z} represents a stationary SAR scheme (LME positive, DLME positive); or \mathbf{z} is free of any spatial pattern (LME zero, DLME positive).

A further advantage of the LM test strategy is that it is quite flexible. Thus, it is possible to control for omitted model features insofar that these can be incorporated as part of the likelihood function. For example, it is straightforward to account for omitted heterogeneity and an omitted autoregression in the dependent variable, along the lines suggested in Anselin (1988b).

2.3. Spatial cointegration.

Spatial cointegration denotes the case where two or more variables in a regression are nonstationary, while the residual is stationary. A simple data generating process which generates two nonstationary but possibly cointegrating series is the following system:

$$(2.6) \quad \mathbf{x} + \beta \mathbf{y} = \mathbf{u}, \quad \mathbf{u} = \mathbf{W}\mathbf{u} + \mathbf{e}_1$$

$$(2.7) \quad \mathbf{x} + \alpha \mathbf{y} = \mathbf{e}_2$$

where \mathbf{e}_1 and \mathbf{e}_2 are white noise processes. From these definitions,

$$\mathbf{x} = \alpha(\alpha - \beta)^{-1} \mathbf{u} - \beta(\alpha - \beta)^{-1} \mathbf{e}_2$$

$$\mathbf{y} = -(\alpha - \beta)^{-1} \mathbf{u} + (\alpha - \beta)^{-1} \mathbf{e}_2$$

from which it is clear that \mathbf{x} and \mathbf{y} are $SI(1)$ but that they cointegrate for any α different from 0 and certain β values, because $(\mathbf{x} + \alpha \mathbf{y})$ is $I(0)$. Specifically, the relation will be non-integrated if (i)

$\alpha=0$ or (ii) $\alpha>0$ and $\beta>\alpha$.

We suggest that the above LM strategy may apply to this situation. Specifically, a regression of \mathbf{y} on \mathbf{X} represents a cointegrating relation (if LME is zero and DLME is negative) or a non-integrating relation (if LME is positive and DLME is zero). The limiting case of “near integration” ($\alpha>0, \beta>\alpha$) will be also be revealed (if LME and DLME are positive).

3. Monte Carlo simulation studies: Designs and results.

In the following section, the small-sample properties of the above suggested test strategies will be investigated using Monte Carlo simulation studies. The chosen Monte Carlo designs are outlined together with the results.

3.1. Spurious regression.

To investigate the finite sample properties of the suggested LME test strategy for spurious regression, the following Monte Carlo design were investigated. To reflect reality, we assume a multiple regression where some of the explanatory variables are stationary and some nonstationary. Specifically, we employ four explanatory variables, of which two are nonstationary and two are white noise processes. The specific design were as follows:

For specific sample size n : Perform 10,000 iterations:

Generate $\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4$ as independent $N(0,1)$ series.

Let $\mathbf{e} = (\mathbf{I} - \rho_e \mathbf{W})^{-1} \mathbf{e}_0$.

Let $\mathbf{x}_i = (\mathbf{I} - \rho_x \mathbf{W})^{-1} \mathbf{e}_i, i=1,2$.

Let $\mathbf{x}_i = \mathbf{e}_i, i=3,4$.

Let $\mathbf{y} = \mathbf{i} + \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 + \mathbf{x}_4 + \mathbf{e}$.

Regress \mathbf{y} on $\mathbf{X} = [\mathbf{i} \ \mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{x}_3 \ \mathbf{x}_4]$ and $\Delta\mathbf{y}$ on $\Delta\mathbf{X}$. Report LME and DLME.

Report the percentage of cases out of 10,000 where LME, respective DLME exceeds the 5 per cent critical value of $\chi^2(1) = 3.84$.

To investigate the impact of contiguity matrix type, we used rook and queen type contiguity matrices based on an $r \times r$ board (so that $n = r^2$) with r assumed to take the values 5, 10, 15, and 20. In this as well as the following simulations, we followed the usual convention and employed a row-standardized \mathbf{W} matrix, i.e. W_{ij} were replaced by $W_{ij} / \sum_{j=1..n} W_{ij}$. Further, the behaviour of the strategy under H_0 : nonstationarity as well as H_1 : stationarity (including the case of near nonstationarity) were investigated by assuming ρ_x to take the values 0, 0.5, 0.9, 0.99 and 1. For each of these, ρ_x is assumed to take the values 0, 0.5 and 1.

The results are provided in Table 1.

(table 1 around here)

Table 1 shows that the procedure performs well, and that the performance of the procedure is acceptable, even for fairly small sample sizes. That the case of near nonstationarity causes problems in identifying the “true” data generating process is well-known from time series analysis. However, contrary to time series analysis spatial dependence of moderate size (i.e. values of about 0.5) in economic systems seems to be much more reasonable than the case of near nonstationarity (see e.g. Rey and Montouri, 1999; Kosfeld, Eckey and Dreger, 2002). Further, the performance of the procedure seems to be unaffected by the type of contiguity matrix, as the rook and the queen cases provide similar results.

3.2. Test for nonstationarity.

To investigate the finite sample properties of the suggested LME test strategy for nonstationarity of a single variable, the following Monte Carlo design was investigated:

For specific sample size n : Perform 10,000 iterations:

Generate \mathbf{e} as independent $N(0,1)$ series.

Let $\mathbf{y} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{e}$.

Regress \mathbf{y} on $\mathbf{X} = \mathbf{i}$ and $\Delta\mathbf{y}$ on $\Delta\mathbf{X}$. Report LME and DLME.

Report the percentage of cases out of 10,000 where LME, respective DLME exceeds the 5 per cent critical value of $\chi^2(1) = 3.84$.

Again, we used the rook and queen type contiguity matrices based on an $n \times r$ board with r assumed to take the values 5, 10, 15, and 20. Further, the behaviour of the strategy under H_0 : nonstationarity as well as H_1 : stationarity (including the case of near nonstationarity) were investigated by varying ρ between the values 0, 0.5, 0.9, 0.99 and 1.

The results are provided in Table 2.

(table 2 around here)

As Table 2 shows that the procedure performs well even for fairly small sample sizes. This holds true under the assumption of nonstationarity as well as different stationarity cases. The case of near nonstationarity is again included for comparative purposes. Note that performance is independent of contiguity matrix type.

3.3. Test for cointegration.

To investigate the finite sample properties of the suggested LME test strategy for cointegration using the suggested example, the following Monte Carlo design was investigated:

For specific sample size n : Perform 10,000 iterations:

Generate $\mathbf{e}_1, \mathbf{e}_2$ as independent $N(0,1)$ series.

Let $\mathbf{u} = (\mathbf{I} - \mathbf{W})^{-1} \mathbf{e}_1$.

Let $\mathbf{x} = \alpha(\alpha - \beta)^{-1} \mathbf{u} - \beta(\alpha - \beta)^{-1} \mathbf{e}_2$ and $\mathbf{y} = -(\alpha - \beta)^{-1} \mathbf{u} + (\alpha - \beta)^{-1} \mathbf{e}_2$.

Regress \mathbf{y} on $\mathbf{X} = [\mathbf{i} \ \mathbf{x}]$ and $\Delta \mathbf{y}$ on $\Delta \mathbf{X}$. Report LME and DLME.

Report the percentage of cases out of 10,000 where LME, respective DLME exceeds the 5 per cent critical value of $\chi^2(1) = 3.84$.

To investigate the impact of contiguity matrix type, we again used the rook and queen type contiguity matrices based on an $r \times r$ board with r assumed to take the values 5, 10, 15, and 20. Further, the behaviour of the strategy under H_0 : nonstationarity as well as H_1 : stationarity (including the case of near nonstationarity) were investigated by varying α and β between the values shown in Table 3.

The results are provided in Table 3.

(table 3 around here)

Table 3 shows that the procedure performs well, especially for fairly large n , and that the performance of the procedure is acceptable, even for fairly small sample sizes. Especially, in the case of cointegration ($\alpha=1$) and non-integration ($\alpha=0$), the procedure works excellently, while the

greyzone case of near-integration ($0 < \alpha < 1$, $\beta > \alpha$) is characterized by inconclusive test sizes. These conclusions hold for both types of contiguity matrices, with a single exception for the cases where α and β are close to each other. In such cases, the rejection percentages are much higher for the queen than for the rook case.

4. AN EMPIRICAL ILLUSTRATION

To illustrate the above concepts, we provide an empirical example investigated in Lauridsen and Nahrstedt (1999) and Lauridsen (2002). The model is concerned with determination of a regression model for outcommuting ratios as a function of unemployment, participation rate, density of working places and average household size. Data were from a 1994 census for 275 Danish municipalities. See Table 4 for a description of the data.

(table 4 around here)

Table 5 presents the estimated model. In Lauridsen (2002) it was left as an open question whether the unexpected negative sign for the UNEMP coefficient was caused by spuriousness due to spatial nonstationarity. The Lagrange Multiplier tests for spatial nonstationarity, provided in Table 5, points to stationarity of the residuals as well as of the single variables. It is concluded that the single variables as well as the entire regression are stationary. Thus, the negative sign for unemployment is rather due to structural properties than to spatial nonstationarity.

(table 5 around here)

5. CONCLUSIONS

Until now, it has not been well established how to separate the case of spatial nonstationarity from the case of stationary positive autocorrelation. As a consequence, reliable diagnostics for spurious spatial regression and for the existence of spatial cointegrating relations have not been available. The present study aims to contribute to close these gaps by proposing a strategy for detecting spatial nonstationarity. It is shown that the strategy of a twofold application of a Lagrange Multiplier test provides adequate diagnostics for both spurious spatial regression and the presence of spatial cointegrating relations. By means of Monte Carlo simulations it is demonstrated that the finite sample properties of the suggested methodology are as desired.

FOOTNOTES

1. A further generalization, which is not in focus of the present study, is to allow the non-zero units to be different from 1. In this way, it is possible to apply weights to the strength of the impact from observation i onto observation j .
2. The test result is termed to be “positive” if the LM test statistic differs significantly from zero and “zero” otherwise.

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TABLE 1. MONTE CARLO RESULTS FOR SPURIOUS REGRESSION STUDY.
 PERCENTAGE OF CASES WHERE LME / DLME VALUE REJECTS H0 AT 5% LEVEL.
 10,000 ITERATIONS.

ρ_e	ρ_x	-----Rook-----				-----Queen-----			
		r=5 (n=25)	r=10 (n=100)	r=15 (n=225)	r=20 (n=400)	r=5 (n=25)	r=10 (n=100)	r=15 (n=225)	r=20 (n=400)
0.0	0.0	0.040/0.647	0.044/1.000	0.049/1.000	0.047/1.000	0.025/0.302	0.043/0.998	0.045/1.000	0.051/1.000
0.0	0.5	0.051/0.678	0.052/1.000	0.049/1.000	0.050/1.000	0.027/0.298	0.042/0.999	0.046/1.000	0.047/1.000
0.0	1.0	0.069/0.714	0.057/1.000	0.051/1.000	0.051/1.000	0.037/0.289	0.055/0.999	0.057/1.000	0.051/1.000
0.5	0.0	0.293/0.294	0.966/0.977	1.000/1.000	1.000/1.000	0.193/0.111	0.820/0.899	0.991/1.000	1.000/1.000
0.5	0.5	0.230/0.324	0.956/0.981	1.000/1.000	1.000/1.000	0.135/0.115	0.786/0.908	0.988/1.000	1.000/1.000
0.5	1.0	0.241/0.344	0.966/0.982	1.000/1.000	1.000/1.000	0.065/0.118	0.682/0.905	0.978/1.000	1.000/1.000
0.9	0.0	0.900/0.049	1.000/0.292	1.000/0.630	1.000/0.878	0.707/0.022	1.000/0.187	1.000/0.486	1.000/0.773
0.9	0.5	0.826/0.058	1.000/0.300	1.000/0.623	1.000/0.879	0.616/0.021	1.000/0.190	1.000/0.496	1.000/0.775
0.9	1.0	0.883/0.071	1.000/0.319	1.000/0.646	1.000/0.888	0.372/0.025	0.999/0.202	1.000/0.495	1.000/0.780
0.99	0.0	0.986/0.032	1.000/0.049	1.000/0.064	1.000/0.085	0.837/0.033	1.000/0.042	1.000/0.051	1.000/0.074
0.99	0.5	0.946/0.034	1.000/0.049	1.000/0.070	1.000/0.096	0.741/0.034	1.000/0.041	1.000/0.056	1.000/0.078
0.99	1.0	0.992/0.040	1.000/0.056	1.000/0.079	1.000/0.102	0.492/0.029	1.000/0.044	1.000/0.064	1.000/0.076
1.0	0.0	0.736/0.031	1.000/0.044	1.000/0.049	1.000/0.049	0.844/0.043	1.000/0.046	1.000/0.051	1.000/0.047
1.0	0.5	0.691/0.037	1.000/0.042	1.000/0.049	1.000/0.049	0.759/0.037	1.000/0.045	1.000/0.049	1.000/0.048
1.0	1.0	0.572/0.035	1.000/0.044	1.000/0.049	1.000/0.049	0.508/0.025	1.000/0.043	1.000/0.048	1.000/0.047

TABLE 2. MONTE CARLO RESULTS FOR NONSTATIONARY VARIABLE STUDY.
 PERCENTAGE OF CASES WHERE LME / DLME VALUE REJECTS H0 AT 5% LEVEL.
 10,000 ITERATIONS.

ρ_e	-----Rook-----					-----Queen-----				
	0.0	0.5	0.9	0.99	1.0	0.0	0.5	0.9	0.99	1.0
r=5 (n=25)	0.041/0.900	0.442/0.556	0.979/0.127	0.999/0.065	0.921/0.054	0.030/0.425	0.259/0.167	0.811/0.031	0.906/0.028	0.918/0.027
r=10 (n=100)	0.049/1.000	0.971/0.990	1.000/0.380	1.000/0.069	1.000/0.047	0.042/0.999	0.843/0.931	1.000/0.225	1.000/0.046	1.000/0.043
r=15 (n=225)	0.047/1.000	1.000/1.000	1.000/0.690	1.000/0.087	1.000/0.053	0.049/1.000	0.992/1.000	1.000/0.528	1.000/0.063	1.000/0.045
r=20 (n=400)	0.049/1.000	1.000/1.000	1.000/0.897	1.000/0.110	1.000/0.048	0.050/1.000	1.000/1.000	1.000/0.796	1.000/0.083	1.000/0.046

TABLE 3. MONTE CARLO RESULTS FOR COINTEGRATION STUDY.
 PERCENTAGE OF CASES WHERE LME / DLME VALUE REJECTS H0 AT 5% LEVEL.
 10,000 ITERATIONS.

α	β	-----Rook-----				-----Queen-----			
		r=5 (n=25)	r=10 (n=100)	r=15 (n=225)	r=20 (n=400)	r=5 (n=25)	r=10 (n=100)	r=15 (n=225)	r=20 (n=400)
0.0	0.01	0.888/0.047	1.000/0.047	1.000/0.050	1.000/0.049	0.904/0.027	1.000/0.042	1.000/0.050	1.000/0.047
	0.10	0.895/0.046	1.000/0.051	1.000/0.048	1.000/0.052	0.901/0.031	1.000/0.043	1.000/0.046	1.000/0.044
	0.50	0.893/0.046	1.000/0.046	1.000/0.050	1.000/0.052	0.902/0.027	1.000/0.044	1.000/0.049	1.000/0.049
	1.00	0.890/0.044	1.000/0.047	1.000/0.052	1.000/0.047	0.901/0.028	1.000/0.039	1.000/0.049	1.000/0.050
0.5	0.00	0.042/0.875	0.050/1.000	0.048/1.000	0.049/1.000	0.032/0.427	0.042/1.000	0.046/1.000	0.046/1.000
	0.40	0.050/0.340	0.064/0.922	0.058/1.000	0.055/1.000	0.095/0.097	0.130/0.643	0.112/0.966	0.094/0.999
	0.49	0.067/0.248	0.095/0.777	0.077/0.987	0.069/1.000	0.147/0.073	0.024/0.452	0.186/0.851	0.144/0.982
	0.51	0.070/0.224	0.109/0.741	0.086/0.978	0.074/1.000	0.163/0.063	0.264/0.425	0.212/0.804	0.161/0.974
	0.60	0.108/0.169	0.166/0.552	0.124/0.894	0.096/0.992	0.247/0.048	0.386/0.275	0.308/0.626	0.254/0.878
	1.00	0.354/0.078	0.598/0.173	0.463/0.309	0.354/0.493	0.586/0.025	0.836/0.085	0.732/0.157	0.635/0.272
1.0	0.00	0.045/0.876	0.047/1.000	0.051/1.000	0.050/1.000	0.033/0.431	0.042/1.000	0.051/1.000	0.047/1.000
	0.50	0.039/0.602	0.044/0.998	0.047/1.000	0.049/1.000	0.043/0.219	0.054/0.939	0.054/1.000	0.053/1.000
	0.90	0.057/0.274	0.083/0.847	0.067/0.996	0.059/1.000	0.123/0.084	0.185/0.548	0.145/0.910	0.118/0.999
	0.99	0.071/0.232	0.098/0.767	0.079/0.982	0.070/1.000	0.156/0.068	0.239/0.438	0.202/0.842	0.157/0.981

TABLE 4. VARIABLES USED FOR EMPIRICAL STUDY

Variable	Definition	Mean	S.D.	Min	Max
OUTCOM	Number of persons with residence in the municipality and workplace in another municipality in percentage of the number of workplaces in the municipality ^a	58.14	37.79	6.00	237.00
PSH1766	Population share of 17-66 year-olds (%) ^a	65.22	2.85	57.90	74.20
WORKPL	Number of workplaces per 100 inhabitants ^a	43.11	11.63	21.00	100.00
IPHOUS	Number of inhabitants per household ^a	2.39	0.16	1.74	2.77
UNEMP	Number of unemployed per 100 17-66 year-olds ^a	9.37	2.24	5.00	18.70
Proximity matrix:					
W1	Neighbourhood matrix for N=275 Danish municipalities ^b				
	Description of number of links per municipality:	4.59	1.68	1	8
	Density of $W_1 = .017$				
W	Row standardization of W_1				

Data collected 1994, for N=275 Danish municipalities.

Source: a : Statistics Denmark, Copenhagen.

b : Own construction.

TABLE 5. ESTIMATION OF THE COMMUTING MODEL

Dependent variable: OUTCOM.

Variable	Parameter	Standard Error	T value	Probability
Intercept	-264.63	33.60	-7.88	<.001
UNEMP	-3.39	0.53	-6.46	<.001
PSH1766	6.30	0.36	17.49	<.001
WORKPL	-2.33	0.10	-22.99	<.001
IPHOUS	18.79	8.08	2.32	0.021

Tests for nonstationarity:

Variable	LME	Probability	DLME	Probability
OUTCOM	38.27	<0.001	69.62	<0.001
UNEMP	449.77	<0.001	46.53	<0.001
PSH1766	554.34	<0.001	47.12	<0.001
WORKPL	498.51	<0.001	69.25	<0.001
IPHOUS	547.90	<0.001	49.53	<0.001
residual	48.49	<0.001	54.03	<0.001
