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## Two-dimensional fiscal competition

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# Two-Dimensional Fiscal Competition

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## **Abstract**

This paper analyzes commodity tax competition between two neighboring countries whose governments are tax-revenue maximizers in a two-dimensional market. The results suggest three conclusions in a geographical sense. First, a small country sets a lower tax than does a big country, and per capita revenue of the small country is larger than that of the big country. Second, these two countries are subject to severer competitive pressure in the case of a more curved national border. Finally, the impact of border curvature on tax and revenue differences are always incompatible with the impact of border curvature on tax and revenue ratios.

*Keyword:* tax competition; two dimensions; cross-border shopping; Nash equilibrium

*JEL Classification:* H71; H73; R51

## 1. Introduction

There is a growing literature on commodity tax competition among countries in the European Union (EU). The interest in this topic may be due to the fact that differing value-added tax (VAT) rates without physical border control induces cross-border shoppers who pay the VAT of other countries rather than their own. Since such cross-border shopping among the EU member countries directly affects government revenues, tax competition is an essential economic problem for the EU to solve. Also, there is a relationship between the geographical elements of countries and the present VAT rates in the EU, as pointed out in Kanbur and Keen(1993), and Ohsawa(1999,2002). It is therefore worthwhile to closely examine the tax revenue and cross-border activity caused by commodity tax competition within a geographical setting.

Analytical studies on commodity tax competition in a linear market have already received attention in recent years. Kanbur and Keen(1993) formulated a non-cooperative Nash game between two countries of the same size but with different customer densities, to analyze the impact of the density difference on tax rates and revenues. The underlying mechanism of this model is that, if a country raises its tax rate relative to that of another country, some of its domestic customers would evade tax by shopping abroad. On the other hand, if it cuts its tax rate, tax revenue from its domestic customers would be reduced, but it would capture revenue from its competitor. In this way, each country faces a decision-making problem of choosing its commodity tax rate to maximize its tax revenue.

After the seminal article by Kanbur and Keen(1993), Cremer and Gahvari(2000) introduced a public good into the model. Nielsen(2001) extended the model by addressing two issues: transportation cost of the commodity and border inspection. Pieretti(1999) considered not only tax competition but also commodity price competition between two countries through a two-stage game. To explore the effect of country size and position on tax rates and revenues, Ohsawa(1999,2002) considered a multi-country model in which customers are uniformly distributed over the whole market, but country sizes differ. Trandel(1994) formulated a two-country model where each country has only one firm with a fixed location and customers are distributed non-uniformly over the linear market in order to examine the impact of: (i) whether welfare-maximizing or revenue-maximizing taxes are employed; and (ii) whether two firms make use of marginal-cost or profit-maximizing pricing.

The abovementioned studies clarified the impact of the spatial characteristics such as

country size and position on tax rates, tax revenues, and cross-border demand. However, a limitation that they all have in common is the assumption of a linear market, while countries in the real world represent two-dimensional markets. To the best of our knowledge, researchers have devoted remarkably little attention to analyzing tax competition in two-dimensional markets. This paper is an attempt to overcome this limitation. It aims to set up a Nash game between two neighboring countries in a two-dimensional market, and to analyze the resulting Nash equilibrium. When one turns from a one-dimensional market to a two-dimensional market, the essential difference between the two cases is that in the latter case, country shape such as the length and curvature of the national border matters. In order to pinpoint the effects of such spatial characteristics, we employ the simplest model.

The structure of this paper is as follows. In the next section, a Nash game in a two-dimensional market is formulated and the impact of a difference in country size on the Nash equilibrium is characterized. Section 3 discusses how country shapes affect tax rates, tax differences, and cross-border shopping: we will detail this through simple examples based on a rectangular market. The final section contains our conclusions. For convenience, all proofs have been relegated to the Appendix.

## 2. Model

The model setup is similar to the one of Kanbur and Keen(1993), and of Ohsawa(1999). We consider only two countries in a two-dimensional plane with a given external boundary. As illustrated in Figure 1(a), the boundaries of the two countries are defined by the thin line, and the shared national border by the thick line. It is assumed that the distribution of customers across the territories is continuous. Let  $A_B$  and  $A_S$  ( $0 < A_S \leq A_B$ ) be the number of customers within the big and small countries, respectively. Note that the big country is not necessarily the one with the greater area because our situation includes non-uniform customer distributions. Let  $L$  ( $< \infty$ ) be the border length. In our illustration as shown in Figure 1 (a),  $A_B$  and  $A_S$  are the number of customers within the unshaded and shaded regions, respectively, and  $L$  is the length of the thick line.

Firms that are continuously distributed across the two countries can produce any quantity of a homogeneous commodity at zero marginal production cost. These two countries levy their commodity tax rates, denoted by  $t_B$  and  $t_S$  respectively, on the firms located within the corresponding countries. Each customer has an inelastic demand for the commodity and

buys one unit from the firm that sells at the lowest full price, which is defined by the tax rate plus the travel costs. The travel cost equals  $\gamma(> 0)$  per unit distance. Each firm is non-cooperative, so it would price at its tax-inclusive marginal cost. Thus, all firms in the big (resp. small) country would charge the same and constant mill prices  $t_B$  (resp.  $t_S$ ).

When tax rates are different, some customers near the border shop in another country. If  $t_B > t_S$  (resp.  $t_S > t_B$ ), then the market area of the small (resp. big) country encroaches on the other country by a distance  $|t_B - t_S|/\gamma$  from the border. Accordingly, the total area is divided into two non-overlapping and mutually exclusive markets. In Figure 1(b), the broken line indicates the market boundary for  $t_B > t_S$ . The striped region shows the extended market of the small country. Let  $D_B(t_B, t_S)$  and  $D_S(t_B, t_S)$  be the demand captured by firms of the big and small countries, respectively. We define  $C(t_B, t_S)$  by

$$C(t_B, t_S) \equiv A_B - D_B(t_B, t_S) = D_S(t_B, t_S) - A_S. \quad (1)$$

Hence,  $|C(t_B, t_S)|$  indicates the cross-border demand. Let  $M(t_B, t_S)$  be the demand in the indifference zone where customers are indifferent to purchase in either countries. In Figure 1(b),  $C(t_B, t_S)$  and  $M(t_B, t_S)$  indicate the demand in the striped region and the broken line, respectively.

The *revenue* of a country is defined as the amount of taxes that it collects, herein referred to as  $T_B(t_B, t_S)$  for the big country and  $T_S(t_B, t_S)$  for the small country. Hence,  $T_B(t_B, t_S) = t_B D_B(t_B, t_S)$  and  $T_S(t_B, t_S) = t_S D_S(t_B, t_S)$ . In this paper, these two countries play a Nash game in tax rates to maximize their revenues. As usual, an asterisk (\*) denotes evaluation at the Nash equilibrium. In this way,  $t_B^*$  and  $t_S^*$  are in Nash equilibrium if and only if  $t_B^* \geq 0, t_S^* \geq 0, T_B(t_B^*, t_S^*) \geq T_B(t_B, t_S^*)$  for all  $t_B \geq 0$ , and  $T_S(t_B^*, t_S^*) \geq T_S(t_B^*, t_S)$  for all  $t_S \geq 0$ . For brevity, let  $T_B^*$  and  $T_S^*$  be  $T_B(t_B^*, t_S^*)$  and  $T_S(t_B^*, t_S^*)$ , respectively.

A lemma is formulated below to serve as the basis for our subsequent analysis.

**Lemma 1** *If an equilibrium exists, then*

$$\frac{M(t_B^*, t_S^*)}{\gamma} t_B^* = D_B(t_B^*, t_S^*), \quad \frac{M(t_B^*, t_S^*)}{\gamma} t_S^* = D_S(t_B^*, t_S^*). \quad (2)$$

See Appendix A.1 for the derivation of these equations.

The tax rates and per capita revenues in Nash equilibrium can be characterized as follows.

**Proposition 1** *If an equilibrium exists, then*

$$t_S^* \leq t_B^*, \quad \frac{T_S^*}{A_S} \geq \frac{T_B^*}{A_B}.$$

Each equality holds if and only if  $A_B = A_S$ .

The proof is given in Appendix A.2. The conclusions drawn by Kanbur and Keen(1993), and Ohsawa(1999) are: (i) the small country sets a lower tax rate than does the big country; and (ii) per capita revenue of the small country is larger than that of the big country. Proposition 1 means that these conclusions remain true for any arbitrary distribution of customers in a two-dimensional market. The intuition for the first inequality is simple, as pointed out in Ohsawa(1999). The big country gets most revenue from its domestic market, so it would set its rate for this purpose. On the other hand, the small country finds it relatively more important to attract taxpayers, so it would set a lower rate to penetrate the foreign market.

The cross-border demand  $C(t_B^*, t_S^*)$  measures the extent of tax evasion due to shopping abroad. It should be noted that the revenue difference  $T_B^* - T_S^*$ , the revenue ratio  $\frac{T_B^*}{T_S^*}$ , and the tax ratio  $\frac{t_B^*}{t_S^*}$ , which may be also useful to quantify the intensity of competition, can be expressed analytically using either the tax difference or the cross-border demand as follows:

$$T_B^* - T_S^* = (A_B + A_S)(t_B^* - t_S^*), \quad (3)$$

$$\frac{T_B^*}{T_S^*} = \left( \frac{t_B^*}{t_S^*} \right)^2 = \left( \frac{A_B - C(t_B^*, t_S^*)}{A_S + C(t_B^*, t_S^*)} \right)^2. \quad (4)$$

The derivation of these equations is given in Appendix A.3. Equation (3) states that the revenue difference is directly proportional to the tax difference. The first equality in (4) states that the revenue ratio is equal to the square of the tax ratio. The second equality in (4) shows that both tax and revenue ratios are decreasing in the cross-border demand. Note that  $T_B^* - T_S^* \geq 0$  and  $\frac{T_B^*}{T_S^*} \geq 1$  result from the first inequality of Proposition 1.

### 3. Impact of Country Shape

#### 3.1. Nash Equilibrium

As long as the general country shapes are maintained, no clear conclusions can be derived. In fact, it is very difficult to get exact expressions of the area and length of the extended market for general shapes. To simplify our analysis, we make the following assumptions: (a1) the external boundary is defined by a polygon; (a2) the small country is composed of pairwise disjoint convex regions; (a3) when a national border hits the external boundary, it does so at a right angle; and (a4) customers are distributed at uniform density equal to one.

Various country shapes that meet assumptions (a1)-(a3) are illustrated in Figure 2. For the same rectangular external boundary, small countries of the same size are indicated by the shaded regions, and the borders by the solid lines. Example (a), which is mathematically identical to the one-dimensional model of Ohsawa(1999), is a stylized representation of tax competition between Germany and Denmark. Example (d) resembles a geographical relationship between Spain and Portugal. Example (f) corresponds to a geographical situation of EU countries with a common tax rate and Switzerland.

In order to measure the amount of curvature of the national border, we make use of the total angular turn of the normal vector going around the border. We denote the total angular turn by  $\rho$  and refer to it as the *frontier coefficient*. This is illustrated in Figure 3, where the small country consists of two regions  $\Omega_1$  and  $\Omega_2$ . The amount of rotation of the normal vector traversing along the border within  $\Omega_1$  is given by the angle  $\theta$  formed by two normal vectors  $\vec{n}_1$  and  $\vec{n}_2$ , as illustrated in Figures 3 (a) and (b). The amount of rotation traversing along the border within  $\Omega_2$  is given by  $2\pi$ , as shown in Figures 3 (a) and (c). Hence, the frontier coefficient is  $\theta + 2\pi$ . For special cases such as borders that consist of straight lines in Figures 2(a) and (b), there cannot be any angular turn, so  $\rho = 0$ . If the border consists of a closed curve as in Figure 2(f), then the normal vector would turn a full circle, leading to  $\rho = 2\pi$ . Thus,  $\rho$  is proportional to the number of times the border winds around parts of the small country, whereby the proportionality constant is  $2\pi$  radians. Note that the frontier coefficient is similar to the total curvature in Integral Geometry; see Santaló(1976,pp.112-113).

The eight country shapes in Figure 2 are arranged according to two criteria. First they are sequenced in order of frontier coefficient, and then in order of border length. As shown in the figures, the greater the frontier coefficient, the more curved the border. If we only deal with a rectangular external boundary as in Figure 2, then the frontier coefficient is limited

to a multiple of  $0.5\pi$ . However, it should be noted that the frontier coefficient can take any non-negative real number by considering general polygonal external boundaries.

The market boundary is defined by a curve at a distance  $\frac{|t_B - t_S|}{\gamma}$  from the border. Therefore, when  $\frac{|t_B - t_S|}{\gamma}$  is not too large, the market boundary may have a shape similar to the border. The  $r_{max}$  is defined as  $r_{max} \equiv \min\{r_O, r_I/2\}$  where  $r_O$  is the distance from the border to the edges of the external boundary whose endpoints are both situated in the big country (indicated by the broken line in Figure 3 (a)), and  $r_I$  is the distance between the disjoint regions of the small country. These notations are illustrated in Figure 3 (a). For  $0 \leq \frac{t_B - t_S}{\gamma} \leq r_I/2$ , the small government has the same number of disjoint market regions with its territory. For  $0 \leq \frac{t_B - t_S}{\gamma} \leq r_O$ , if the border meets an edge of the external boundary, then the market boundary meets the same edge. This together with assumption (a3) means that the market boundary also hits the edge at a right angle. Combining these two observations means that if  $0 \leq t_B - t_S \leq \gamma r_{max}$ , then the frontier coefficient of the market boundary coincides with that of the border. The broken lines in Figure 2 indicate the market boundaries that correspond to the  $r_{max}$ . Note that such market boundaries in Figures 2(a) and (b) lie on the edges of the external boundary.

Assumption (a4) implies that the market sizes of countries correspond to their areal extension. Consequently, we get

$$C(t_B, t_S) = L \frac{t_B - t_S}{\gamma} + \frac{\rho}{2} \left( \frac{t_B - t_S}{\gamma} \right)^2, \quad (5)$$

$$M(t_B, t_S) = L + \rho \frac{t_B - t_S}{\gamma}. \quad (6)$$

The derivation of these equations is provided in Appendix A.4. Two points are worth noting. When borders are curved, i) the cross-border demand  $C(t_B, t_S)$  increases at a faster rate than the tax difference  $t_B - t_S$ ; and ii) the market boundary length  $M(t_B, t_S)$  increases in direct proportion to the tax difference. In both cases, the strength of the effects rise as borders become more curved.

Substituting  $C(t_B^*, t_S^*)$  defined by equation (5) and  $M(t_B^*, t_S^*)$  defined by equation (6) into the equalities given in (2), and combining the two resulting equations, with (1) yields the following system of simultaneous equations:

$$\left( L + \rho \frac{t_B^* - t_S^*}{\gamma} \right) \frac{t_B^* + t_S^*}{\gamma} = A_B + A_S, \quad (7)$$

$$\left( 3L + 2\rho \frac{t_B^* - t_S^*}{\gamma} \right) \frac{t_B^* - t_S^*}{6\gamma} = A_B - A_S. \quad (8)$$



For the special case of a straight border line where  $\rho = 0$ , then we get

$$t_B^* = \frac{\gamma}{3} \left( 2\frac{A_B}{L} + \frac{A_S}{L} \right), \quad t_S^* = \frac{\gamma}{3} \left( \frac{A_B}{L} + 2\frac{A_S}{L} \right),$$

which are consistent with the results for a linear market by Kanbur and Keen(1993) and Ohsawa(1999). Solving the quadratic equation (8) for  $t_B^* - t_S^*$ , while taking account of  $t_B^* \geq t_S^*$ , yields

$$t_B^* - t_S^* = \left( \frac{\gamma L}{4\rho} \right) \left( \sqrt{9 + 8\rho(A_B/L^2 - A_S/L^2)} - 3 \right). \quad (9)$$

The insertion of this into (7) gives

$$t_B^* + t_S^* = \left( \frac{\gamma L}{2\rho} \right) \left( \frac{\rho(A_B/L^2 + A_S/L^2)}{1 + \rho(A_B/L^2 - A_S/L^2)} \right) \left( \sqrt{9 + 8\rho(A_B/L^2 - A_S/L^2)} - 1 \right). \quad (10)$$

These equations enable us to derive analytically  $t_B^*$  and  $t_S^*$ . In addition, they indicate that  $t_B^*$  and  $t_S^*$  are expressed in the same unit with  $\gamma L$  since  $A_B/L^2$ ,  $A_S/L^2$ , and  $\rho$  are scale-invariant.

Table 1 shows two tax rates, tax difference, tax ratio, two revenues, revenue difference, revenue ratio, and cross-border demand in Nash equilibrium corresponding to the cases given in Figure 2. Notice that the external boundary is a rectangle with 1.5 units length and 1.0 unit width, whereas the areas of the big and small countries are 1.15 and 0.35, respectively, i.e.,  $A_B = 1.15$  and  $A_S = 0.35$ . The unit transportation cost is normalized by setting it equal to one, i.e.,  $\lambda = 1.0$ . This table shows that the Nash equilibrium depends on country shape, while maintaining the respective size of the big and small countries throughout the various geographical setting. Thus, we see that country shape plays a significant role in fiscal competition.

The rest of the paper focuses on how national border curvature  $\rho$  and length  $L$  affect the equilibrium tax rates, tax difference, and cross-border demand, provided that  $A_B > A_S$ . As shown by equations (3) and (4), the changes in these parameters in turn alter the government revenue difference, revenue ratio, and tax ratio.

### 3.2. Effects of Border Curvature

To focus on the role of the border curvature, we compare country shapes with the same border length but different frontier coefficients.

**Proposition 2** *If  $t_B^* - t_S^* \leq \gamma r_{max}$ , then  $t_B^*$ ,  $t_S^*$ ,  $t_B^* - t_S^*$ , and  $C(t_B^*, t_S^*)$  are all decreasing in  $\rho$  for fixed  $L$ ,  $A_B$ , and  $A_S$ .*

The proof is given in Appendix A.5. This proposition indicates that the tax rates, the tax difference, and the cross-border demand decrease as the border becomes more curved, i.e., a rise in the frontier coefficient boosts tax competition. As shown in Table 1, the country shape (b) has the greatest taxes, the greatest tax difference, and the greatest cross-border demand among (b), (d) and (e), though they have the same border length. The same holds for the relationships among (f), (g) and (h). In this regard, this proposition matches the pattern shown in the numerical results in Table 1.

The effect on the tax rates and the tax difference can be explained as follows: the more curved the border, the larger the region which the small country captures, *ceteris paribus*. Hence, the small country would be able to expand its market area more effectively. So it has more incentive to employ an aggressive policy through cutting its tax rate. Hence, the market is subject to stronger competitive pressure. Thus, the tax rates and the tax difference fall as the frontier coefficient rises.

The intuition for the effect on the cross-border shoppers is not obvious. The area where the small country captures foreign demand increases with the frontier coefficient, *ceteris paribus*. But, the tax difference decreases with the frontier coefficient as just pointed out. However, as we can see in Appendix A.5, the impact of latter the always dominates the former.

Combining the result on  $t_B^* - t_S^*$  in Proposition 2 with equation (3) implies that the revenue difference also falls with the frontier coefficient. Combining the result on  $C(t_B^*, t_S^*)$  with equation (4) shows that both the tax and revenue ratios rise with the frontier coefficient. Thus, the changes in the tax and revenue differences is opposite to that in the tax and revenue ratios. These alternative measures of tax competition, are therefore not affected in the same way.

### 3.3. Effects of Border Length

To focus on the impact of the border length, we compare various country shapes with the same frontier coefficient but different border lengths. Note that a closed curve with the shortest length that encloses a given area is a circle. This is known as the isoperimetric problem: see Guggenheimer(1963, pp.83-84) and Santaló(1976, pp.34-37). As the border becomes shorter, the closed curve approaches a circle, other things remain the same. On the other hand, as the border gets longer, it comes to resemble a straight line.

**Proposition 3** *If  $t_B^* - t_S^* \leq \gamma r_{max}$  for fixed  $\rho$ ,  $A_B$ , and  $A_S$ , then (i)  $t_B^*$  and  $t_B^* - t_S^*$  are decreasing in  $L$ ; and (ii) for  $\rho = 0$ ,  $C(t_B^*, t_S^*)$  is independent of  $L$ , while for  $\rho > 0$ ,  $C(t_B^*, t_S^*)$  is increasing in  $L$ .*

The proof is provided in Appendix A.6. Part (i) signifies that the tax rate of the big country and the tax difference fall with the border length. Part (ii) implies that the cross-border demand does not fall with the border length. These findings are consistent with the numerical relationship between (a) and (b), and the one between (c) and (d) in Table 1.

Let us offer intuition for part (i). This can be explained from two viewpoints: the area near the border; and the effect of the border curvature per border length. From the first viewpoint, a longer border implies that there are a larger number of customers in the vicinity of the border. As a result, there is competition over many places, so the tax rates and the tax difference would fall. From the second viewpoint, as the border length gets longer, it tends to be a straight line, so the impact of the curvature per border length diminishes. This may not stimulate the small government to employ an aggressive policy through cutting its tax rate. Based on these two observations, the impact of the border length is ambiguous. As shown in Appendix A.6, the former always dominates the latter for  $t_B^*$  and  $t_B^* - t_S^*$ . It should be noted that  $t_S^*$  is decreasing in  $L$  for small  $A_B$ , but it can be increasing for very large  $A_B$ : see Appendix A.6.

The intuitive explanation of part (ii) under straight borders is as follows. From the first viewpoint, the small country encroaches less deep on another territory with increasing border length. An increase in the border length is completely offset by a decrease in the depth of the encroachment; there is thus no change in the cross-border demand. To interpret part (ii) under curved borders, the effect of border curvature has to be considered. From the second viewpoint, the competition becomes weaker with the border length, as shown. This means that the tax difference is decreasing with the increasing border length more slowly than under the straight borders situation. This, together with the result under the straight borders situation, would mean that the cross-border demand would rise with the border length.

#### **4. Conclusions**

This paper assumed two-dimensional countries in order to study the relationship between country shape and fiscal competition. Although the analysis was done within the simplest possible framework, this suggests at least three significant implications for designing future European Union tax policy.

First, this paper demonstrates that a small country sets a lower tax rate than does a big country, and per capita revenue of the small country is larger than that of the big country even in two-dimensional markets. This finding holds for any customer distribution and any country shape. Therefore, the model provide a better explanation for the observation that Luxembourg imposes the lowest VAT rate among the EU countries than the models proposed by Kanbur and Keen(1993) and Ohsawa(1999). Second, this paper finds that the more curved the national border, the smaller the tax rates, the tax difference and the cross-border shoppers. The small and landlocked countries like Switzerland, Luxembourg, Andorra and Liechtenstein in Europe have more curved borders. Our results indicate that fiscal competition between such small countries and their neighbors is likely to be severe. Finally, this paper proves that although tax difference, revenue difference, tax ratio and revenue ratio can be considered as measures of the intensity of tax competition, the impact of the border curvature on tax and revenue differences are always the opposite of the impact on tax and revenue ratios. The last two findings are peculiar to the two-dimensional markets, and they cannot be explained by any one-dimensional model. This point deserves explicit emphasis.

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## Appendix

### A.1. Proof of Lemma 1

Suppose we reduce  $t_S^*$  by a sufficiently small amount  $\epsilon > 0$ . This extends the market of the small country by  $\epsilon \frac{M(t_B^*, t_S^*)}{\gamma}$ , and thus, increases its revenue from the newly extended market by  $\epsilon t_S^* \frac{M(t_B^*, t_S^*)}{\gamma}$ . On the other hand, the reduction simultaneously decrease its revenue from the former market by  $\epsilon D_S(t_B^*, t_S^*)$ . Evidently, to make it in equilibrium, the gain has to be equated with the loss, i.e.,  $\epsilon t_S^* \frac{M(t_B^*, t_S^*)}{\gamma} = \epsilon D_S(t_B^*, t_S^*)$ . This establishes the second equation

in (2). The first equation in (2) can be derived similarly.  $\square$

#### A.2. Proof of Proposition 1

Any country that might fail to attract customers can set the same tax as its competitor, and thus, gain by collecting tax revenue. In equilibrium, therefore, both countries attract some customers, so that  $M(t_B^*, t_S^*) > 0$ . Suppose now we let  $t_S^* > t_B^*$ . This together with the equalities in (2) yields

$$t_S^* = \frac{\gamma D_S(t_B^*, t_S^*)}{M(t_B^*, t_S^*)} < \frac{\gamma A_S}{M(t_B^*, t_S^*)} \leq \frac{\gamma A_B}{M(t_B^*, t_S^*)} < \frac{\gamma D_B(t_B^*, t_S^*)}{M(t_B^*, t_S^*)} = t_B^*,$$

which contradicts  $t_S^* > t_B^*$ .

Since  $t_S^*$  is the best response against  $t_B^*$ ,  $T_S^* \geq T_S(t_B^*, t_B^*) = t_B^* A_S$ . Also, it follows from  $t_S^* \leq t_B^*$  that  $T_B^* = t_B^* D_B(t_B^*, t_S^*) \leq t_B^* A_B$ . Combining these result gives  $\frac{T_S^*}{A_S} \geq t_B^* \geq \frac{T_B^*}{A_B}$ , so we have the second inequality in the proposition.  $\square$

#### A.3. Derivation of Equations (3) and (4)

In view of equation (1), equation (2) can be rewritten with  $C(t_B^*, t_S^*)$  as follows:

$$\frac{M(t_B^*, t_S^*)}{\gamma}(t_B^* + t_S^*) = A_B + A_S, \quad (11)$$

$$\frac{M(t_B^*, t_S^*)}{\gamma}(t_B^* - t_S^*) = A_B - A_S - 2C(t_B^*, t_S^*). \quad (12)$$

It follows from (2) that  $\gamma T_B^* = M(t_B^*, t_S^*)(t_B^*)^2$  and  $\gamma T_S^* = M(t_B^*, t_S^*)(t_S^*)^2$ . Subtracting the latter equation from the former, while using (11), yields the equation (3). Dividing the former equation by the latter establishes the first equality in (4). Dividing the first equation in (2) by the second in (2), while using (1), gives the second equation in (4).  $\square$

#### A.4. Derivation of Equations (5) and (6)

Throughout this appendix, we assume that  $0 \leq t_B - t_S \leq \gamma r_{max}$ . From Guggenheimer(1963, pp.80-81) and Santaló(1976, pp.7-8), when the border consists of a closed convex curve, then

$$C(t_B, t_S) = L \frac{t_B - t_S}{\gamma} + \pi \left( \frac{t_B - t_S}{\gamma} \right)^2, \quad M(t_B, t_S) = L + 2\pi \frac{t_B - t_S}{\gamma}.$$

Therefore, if the border consists of  $m$  disjoint closed convex curves, then

$$C(t_B, t_S) = L \frac{t_B - t_S}{\gamma} + m\pi \left( \frac{t_B - t_S}{\gamma} \right)^2, \quad M(t_B, t_S) = L + 2m\pi \frac{t_B - t_S}{\gamma}.$$

Analogous reasoning proves equations (5) and (6) for any country shape that fulfills assumptions (a1), (a2), (a3) and (a4) in Section 3.  $\square$

#### A.5. Proof of Proposition 2

Define  $x$  and  $y$  by  $x \equiv \frac{t_B^* - t_S^*}{\gamma}$  and  $y \equiv \frac{t_B^* + t_S^*}{\gamma}$ . Also, define  $\beta$  and  $\sigma$  by  $\beta \equiv \rho A_B / L^2$  and  $\sigma \equiv \rho A_S / L^2$ , so  $\beta - \sigma > 0$ . The proof proceeds in two stages.

First, in order to prove that  $t_S^*$  and  $t_B^*$  which fulfill (9) and (10) are in equilibrium, it suffices from Lemma 1 to prove that i)  $t_B^*$  and  $t_S^*$  are relative maxima of  $T_B(t_B, t_S^*)$  and  $T_S(t_B^*, t_S)$ , respectively; and ii) the small country cannot obtain higher revenue for  $t_B^* - t_S > \gamma r_{max}$ . In the neighborhood of  $t_S^*$ , employing (1) and (5) yields

$$T_S(t_B^*, t_S) = t_S(A_S + C(t_B^*, t_S)) = t_S \left( A_S + \frac{L}{\gamma}(t_B^* - t_S) + \frac{\rho}{2\gamma^2}(t_B^* - t_S)^2 \right).$$

Taking the second derivative of  $T_S(t_B^*, t_S)$  with respect to  $t_S$ , and evaluating this derivative at  $t_S^*$  gives

$$\frac{d^2 T_S(t_B^*, t_S)}{dt_S^2} \Big|_{t_S=t_S^*} = -\frac{2L}{\gamma} - \frac{2\rho}{\gamma^2}(t_B^* - t_S^*) + \frac{\rho}{\gamma^2}t_S^* = -\frac{\rho}{2\gamma} \left( \frac{4L}{\rho} + 5x - y \right).$$

In order to show that  $\frac{d^2 T_S(t_B^*, t_S)}{dt_S^2} \Big|_{t_S=t_S^*} < 0$ , we verify that  $\frac{4L}{\rho} + 5x - y > 0$ . Direct computation using (9) and (10) yields  $\frac{4L}{\rho} + 5x - y = \frac{L}{4\rho(1+\beta-\sigma)} \left( (5+3\beta-7\sigma)\sqrt{9+8(\beta-\sigma)} + 1+3\beta+\sigma \right)$ . Since  $1+\beta-\sigma > 0$  and  $1+3\beta+\sigma > 0$ , let us show that  $5+3\beta-7\sigma > 0$ . It follows from assumption (a2) and the isoperimetric inequality that  $1 \geq 2\sigma$ : see Guggenheimer(1963, pp.83-84) and Santaló(1976, pp.34-37). Therefore,  $5+3\beta-7\sigma \geq \frac{3}{2}+3\beta > 0$ , as required.

It should be noted that for a given  $t_B^*$ , if  $t_B^* - t_S > \gamma r_{max}$ ,  $D_S(t_B^*, t_S)$  is below  $A_S + L \frac{t_B^* - t_S}{\gamma} + \frac{\rho}{2} \left( \frac{t_B^* - t_S}{\gamma} \right)^2$ . This indicates that  $T_S(t_B^*, t_S)$  cannot exceed the revenue at the tax rate which maximizes  $t_S \left( A_S + L \frac{t_B^* - t_S}{\gamma} + \frac{\rho}{2} \left( \frac{t_B^* - t_S}{\gamma} \right)^2 \right)$ , i.e.,  $T_S^*$ .

Second, let us do comparative static analysis. Note that if we fix the external boundary, then  $\rho$  can take on a very limited number of values for any country shape. However, if we allow also the external boundary shape to change aside from country shape, then  $\rho$  can vary smoothly with fixed  $L$ ,  $A_B$  and  $A_S$ . Therefore, we make use of differential calculus for the proof. By using  $x$  and  $y$ , the equations (7) and (8) can be rewritten respectively as follows:

$$Ly + \rho xy = A_B + A_S, \quad (13)$$

$$3Lx + 2\rho x^2 = A_B - A_S. \quad (14)$$

Taking total differentials gives

$$\begin{pmatrix} \rho y & L + \rho x \\ 3L + 4\rho x & 0 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -y \\ -3x \end{pmatrix} dL + \begin{pmatrix} -xy \\ -2x^2 \end{pmatrix} d\rho. \quad (15)$$

Therefore, we have

$$\begin{aligned} \frac{1}{\gamma} \frac{d(t_B^* - t_S^*)}{d\rho} = \frac{dx}{d\rho} &= \frac{1}{\Delta} \begin{vmatrix} -xy & L + \rho x \\ -2x^2 & 0 \end{vmatrix} = \frac{2x^2(L + \rho x)}{\Delta} < 0, \\ \frac{dy}{d\rho} &= \frac{1}{\Delta} \begin{vmatrix} \rho y & -xy \\ 3L + 4\rho x & -2x^2 \end{vmatrix} = \frac{3Lxy + 2\rho x^2 y}{\Delta} < 0, \end{aligned}$$

where  $\Delta = -(L + \rho x)(3L + 4\rho x) < 0$ . Hence,  $\frac{dt_B^*}{d\rho} = \frac{\gamma}{2} \frac{d(x+y)}{d\rho} < 0$ , and  $\frac{dt_S^*}{d\rho} = \frac{\gamma}{2} \frac{d(y-x)}{d\rho} = \frac{\gamma}{2} \left( \frac{Lx(3y-2x) + 2\rho x^2(y-x)}{\Delta} \right) < 0$ .

In addition, the insertion of (14) into (5) gives  $C(t_B^*, t_S^*) = \frac{1}{4}(Lx + A_B - A_S)$ , so  $\frac{dC(t_B^*, t_S^*)}{d\rho} = \frac{L}{4} \left( \frac{dx}{d\rho} \right) < 0$ .  $\square$

#### A.6. Proof of Proposition 3

It follows from (15) that

$$\frac{1}{\gamma} \frac{d(t_B^* - t_S^*)}{dL} = \frac{dx}{dL} = \frac{1}{\Delta} \begin{vmatrix} -y & L + \rho x \\ -3x & 0 \end{vmatrix} = \frac{3x(L + \rho x)}{\Delta} < 0, \quad (16)$$

$$\frac{dy}{dL} = \frac{1}{\Delta} \begin{vmatrix} \rho y & -y \\ 3L + 4\rho x & -3x \end{vmatrix} = \frac{3Ly + \rho xy}{\Delta} < 0. \quad (17)$$

Accordingly,  $\frac{dt_B^*}{dL} = \frac{\gamma}{2} \frac{d(x+y)}{dL} < 0$ . The proof of part (i) is complete.

The insertion of (14) into (5) gives  $C(t_B^*, t_S^*) = \frac{1}{6}(-\rho x^2 + 2(A_B - A_S))$ . This implies that  $\frac{dC(t_B^*, t_S^*)}{dL} = -\rho \frac{x}{3} \left( \frac{dx}{dL} \right) \geq 0$ , which completes the proof of part (ii).

On the other hand, using the two inequalities (16) and (17) yields

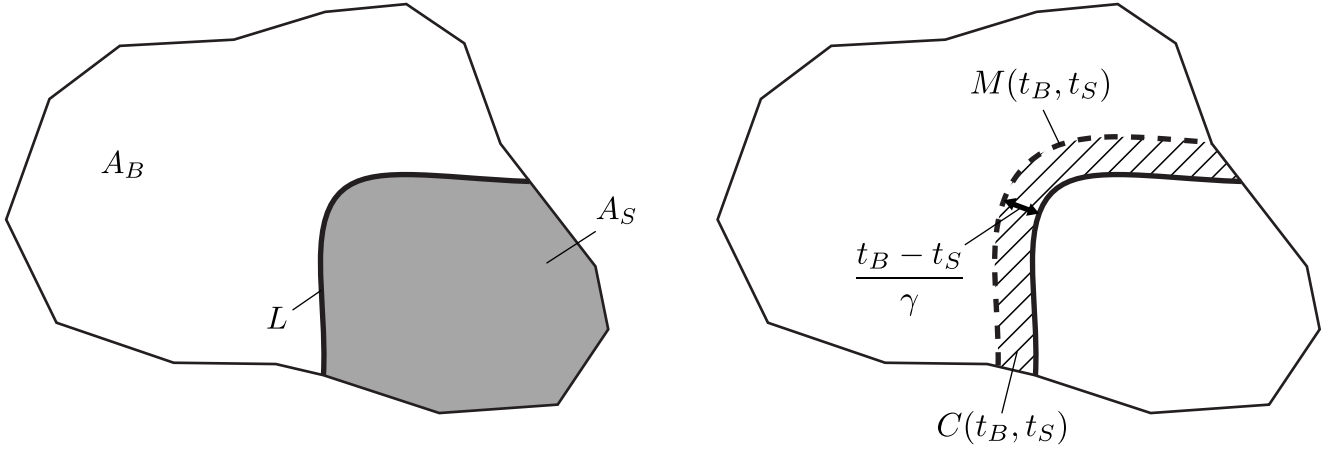
$$\begin{aligned} \frac{dt_S^*}{dL} = \frac{\gamma}{2} \frac{d(y-x)}{dL} &= \frac{\gamma}{2\Delta} (3Ly + \rho xy - 3Lx - 3\rho x^2) \\ &= \frac{\gamma}{2\Delta} \left( 2Ly + A_B + A_S - \frac{3}{2}(A_B - A_S - Lx) \right) \\ &= \frac{\gamma}{4\Delta} (4Ly + 3Lx - A_B + 5A_S), \end{aligned}$$

where the last second equality holds because of (13) and (14). Therefore, for fixed  $L$ ,  $\rho$  and  $A_S$ , when  $A_B$  is significantly large, then  $x \propto \sqrt{A_B}$  and  $y \propto \sqrt{A_B}$ , indicating that  $\frac{dt_S^*}{dL} > 0$ . When  $5A_S > A_B$ , then  $\frac{dt_S^*}{dL} < 0$ .  $\square$



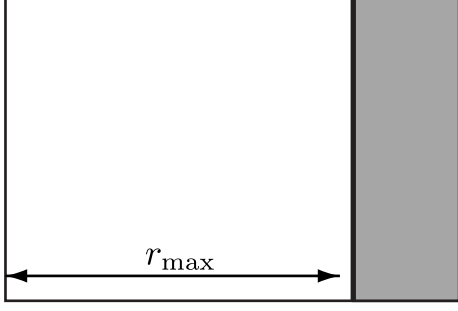
Table 1: Tax Rates and Revenues

	$\rho$	$L$	$t_B^*$	$t_S^*$	$t_B^* - t_S^*$	$\frac{t_B^*}{t_S^*}$	$T_B^*$	$T_S^*$	$T_B^* - T_S^*$	$\frac{T_B^*}{T_S^*}$	$C(t_B^*, t_S^*)$
(a)	0	1.0	0.883	0.617	0.267	1.432	0.780	0.380	0.400	2.052	0.267
(b)	0	1.5	0.589	0.411	0.178	1.432	0.520	0.254	0.267	2.052	0.267
(c)	$0.5\pi$	1.1	0.630	0.427	0.203	1.476	0.563	0.259	0.305	2.178	0.256
(d)	$0.5\pi$	1.5	0.508	0.348	0.160	1.459	0.452	0.212	0.240	2.130	0.260
(e)	$\pi$	1.5	0.456	0.308	0.147	1.478	0.408	0.187	0.221	2.185	0.255
(f)	$2\pi$	2.6	0.282	0.192	0.090	1.467	0.251	0.117	0.134	2.149	0.258
(g)	$2.5\pi$	2.6	0.272	0.185	0.087	1.472	0.243	0.112	0.131	2.168	0.257
(h)	$4\pi$	2.6	0.248	0.166	0.081	1.488	0.222	0.100	0.122	2.215	0.253

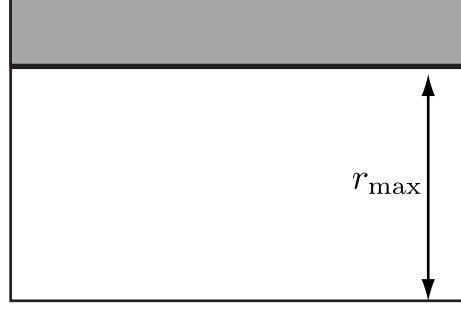


(a) Two Countries and Their Border      (b) Market Boundary

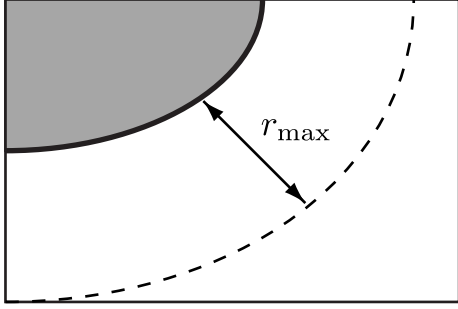
Figure 1: Two-Country Competition



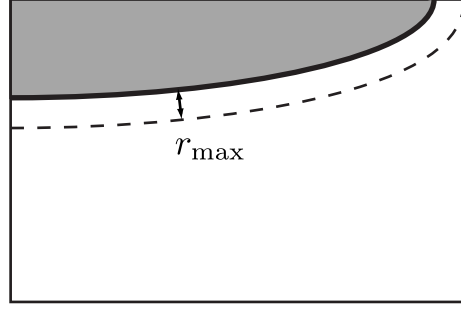
(a)  $\rho = 0$  and  $L = 1.0$



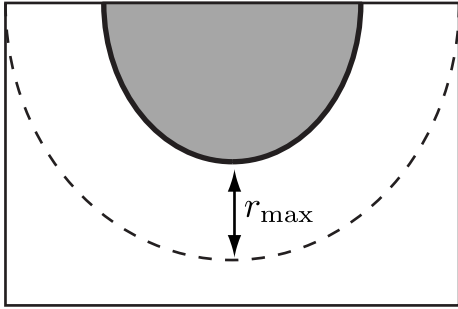
(b)  $\rho = 0$  and  $L = 1.5$



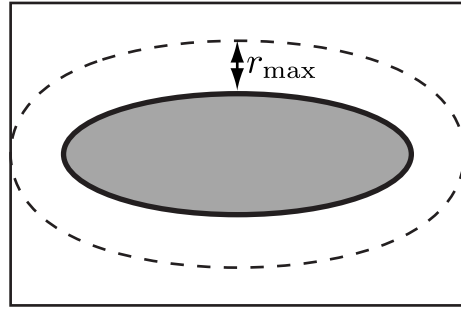
(c)  $\rho = 0.5\pi$  and  $L = 1.1$



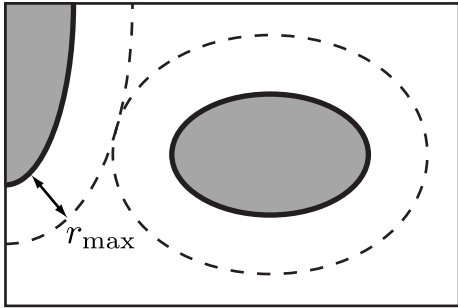
(d)  $\rho = 0.5\pi$  and  $L = 1.5$



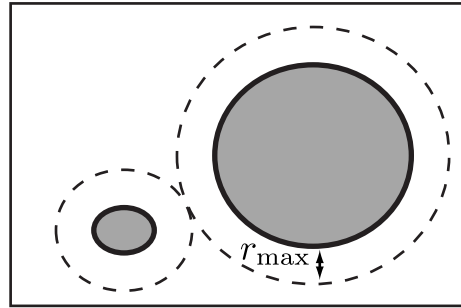
(e)  $\rho = \pi$  and  $L = 1.5$



(f)  $\rho = 2\pi$  and  $L = 2.6$

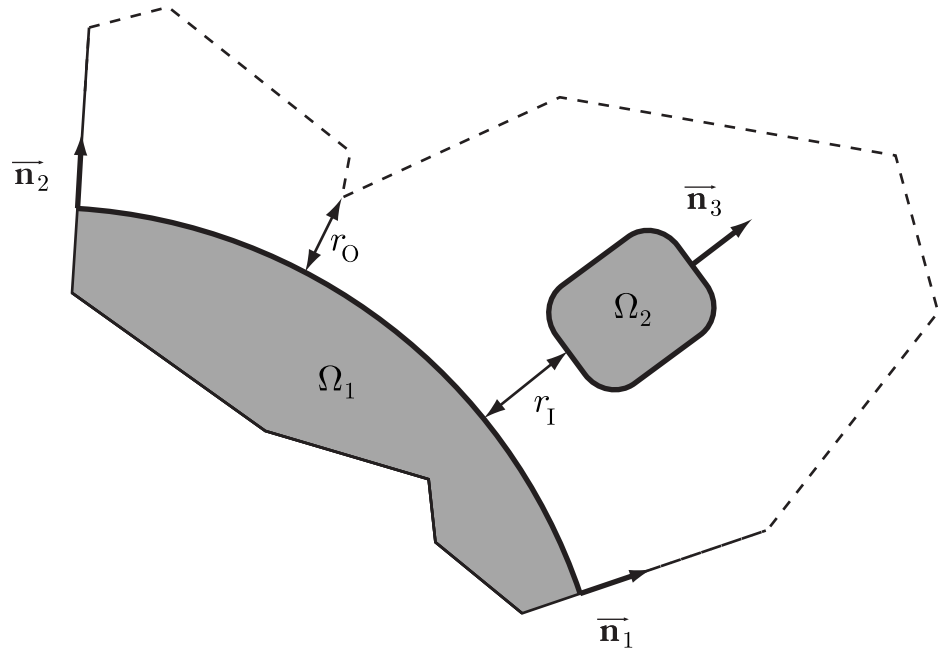


(g)  $\rho = 2.5\pi$  and  $L = 2.6$

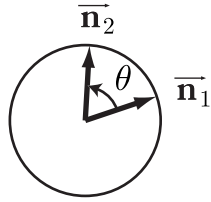


(h)  $\rho = 4\pi$  and  $L = 2.6$

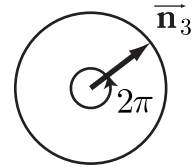
Figure 2: Country Shape and Construction of  $r_{max}$



(a) Example of  $r_O$ ,  $r_I$ ,  $\vec{n}_1$ , and  $\vec{n}_2$



(b) Total Angular Turn in  $\Omega_1$



(c) Total Angular Turn in  $\Omega_2$

Figure 3: Illustration of  $\rho$ ,  $r_O$ , and  $r_I$