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Meaningful Talk

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Abstract

This paper develops a semiotic-inferential model of verbal communication for incomplete information games: a language is seen as a set of conventional signs that point to types, and the credibility of a message depends on the strategic context. Formally, there is an encoding-decoding step where the receiver can understand the sender's message if and only if a common language is used, and an inferential step where the receiver may either trust the message's literal meaning or disregard it when updating priors. The epistemic requirement that information be transmitted through the literal meaning of the message uttered leads to an equilibrium concept distinct from a Perfect Bayesian Equilibrium, ruling out informative equilibria where language is not used in its ordinary sense. The paper also proposes a refinement by which the sender selects among equilibria if all sender types are willing to play the same equilibrium.

JEL classifications: D83, C72

Keywords: Cheap talk, Language, Literal and equilibrium meaning, Signs, Comprehensibility, Relevance, Trust, Credibility, Equilibrium selection

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1. Introduction

Crawford and Sobel (1982) develop a game-theoretic representation of verbal communication between a sender and a receiver under incomplete information as *cheap talk*, where messages are payoff-irrelevant.\(^1\) Their goal is to analyze the maximal amount of information the informed party may offer the uninformed party when there are incentives to misrepresent information. However, cheap-talk models concentrate on beliefs induced in equilibrium, not on the equilibrium messages that are used (Wang, 2009). Since equilibrium messages are arbitrary, when there is an informative equilibrium, there are infinitely many. We show that *costly talk*, where senders face misrepresentation costs (Kartik, Ottaviani, and Squintani 2007), does not prevent this from happening either.\(^2\)

The root of this multiplicity of informative equilibria is the application of the Perfect Bayesian Equilibrium (PBE) concept to language games. This leads to an epistemic problem because the literal meaning of verbal messages—the only information actually added through verbal communication—is not taken into account in updating priors. Instead, we introduce an

\(^1\) Cheap-talk models set language apart from other signals: while standard signals may be credible because choices are differentially costly, words have no direct payoff consequences so they are credible only if players share common interests (Gibbons, 1992: 210).

\(^2\) Kartik, Ottaviani, and Squintani (2007) show that misrepresentation costs of the sender transform language from cheap talk into a costly signal. Callander and Wilkie (2007) point this out in the context of political campaigns. The meaning correspondence in Demichelis and Weibull (2008)—the relation between the announced message and the intended action—is close to our focus. They introduce lexicographic preferences of the sender for honesty to analyze meaning. However, misrepresentation costs do not capture per se the informative role of natural language we analyze here: it is essential to consider what messages the receiver can understand, not only what the sender chooses to say.
equilibrium concept that takes the literal meaning into account. This is achieved through a semiotic-inferential process of verbal communication.³

First, sparked by Farrell’s (1993: 515) deep insight that, credible or not, natural language has a comprehensible meaning (a sufficient condition for communication), we add the inverse proposition: if a common natural language is not used, the receiver will not be able to understand the sender’s meaning (a necessary condition for communication). This takes place through an encoding-decoding step where the sender’s message to the receiver is comprehensible if, and only if, a common language is used. Without a common language, the speaker cannot verbally communicate meaning, either true or false, because there is no way for the hearer to understand the messages.⁴ In this connection, Bill Murray’s character in Lost in Translation illustrates the problems of comprehension in an unfamiliar language. Second, in the inferential step the receiver must decide whether to trust the sender’s messages, accepting their literal meaning, or disregard them when updating priors.

The resulting equilibrium concept is not a PBE because beliefs on the equilibrium path are not determined by the assignment of messages to each sender type, as in other language games (in all signaling games there is an assignment of signals to sender types in each PBE). Rather, beliefs are determined by the literal content of the equilibrium messages themselves and whether the receiver trusts them or not. This reflects the fact that speakers rely on ordinary words to convey meaning to the hearer, instead of randomly using any word in the dictionary to name something. Besides incorporating natural language into economic theory, we explore the use of explicit communication to select among equilibria. This parallels the idea in Thomas Schelling (1960) of using focal points to achieve implicit coordination among Nash equilibria.

The paper is structured as follows. Section 2 looks at rendez-vous, a simple example of the coordination games that have played an important role in philosophy of language, to illustrate how the PBE concept leads to informative equilibria that ordinary receivers have no way of understanding because they would have to be capable of deciphering encrypted messages. Section 3 proposes a semiotic-inferential model of verbal communication that relies on linguistic

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³ While our approach does not belong to epistemic game theory, the common understandings embodied in common natural language restrict the beliefs that players may entertain in response to verbal communication.

⁴ Jakobson and Halle (1956: 72), for instance, state that “the efficiency of a speech event demands the use of a common code by its participants.” More generally, Sperber and Wilson (1995: 43) point out that communication is an asymmetric process where “it is left to the communicator to make correct assumptions about the codes and contextual information that the audience will have accessible and be likely to use in the comprehension process.”
signs as the vehicle for information transmission. The semiotic step defines *comprehensible* messages. The inferential step defines *relevant* and *trusted* messages in light of the specific strategic context. This two-step epistemic process characterizes *meaningful talk* and leads to an equilibrium concept tailored to language games. In this setting, *credibility* and *trust* are distinguished. We also propose an equilibrium refinement which applies if different sender types are willing to select the same equilibrium. Section 4 presents the main results. As in cheap-talk models, uninformative equilibria are always possible, so meaningful-talk equilibria always exist. With meaningful talk, informative equilibria require that natural language be used in its ordinary sense. The equilibrium refinement implies that informative equilibria are selected only if it is in the interest of at least one sender type to reveal information, and uninformative equilibria are not if it is in the interest of all sender types to reveal information. Section 5 briefly relates our approach to the literature on language. Section 6 contains the closing remarks.

2. Making Sense of Informative Equilibria

We illustrate how the use of PBE leads to informative equilibria that ordinary receivers would be unable to understand. We specifically consider verbal communication in *rendez-vous*, a very simple coordination game. Player one has a type $t$ given by its location. Player two picks a move $a$ by selecting a location. Both players have a payoff of 1 if they meet ($a = t$) and 0 if they do not ($a \neq t$). We distinguish between two planes, those of reality and language. Following the use/mention distinction, quotes distinguish message "$t$" from type $t$. If player one, the sender $S$, picks a message "$m$" = "$t$" and player two, the receiver $R$, picks in response a move $a$, this is a unilateral communication game under incomplete information (Crawford and Sobel, 1982).

The timing is as follows. Nature determines the sender’s type $t$. The receiver has a prior $p(t)$ that the sender’s type is $t$. When communication is possible, the sender $S$ sends a message "$t$" about its type $t$ to the receiver. The receiver observes the message "$t$", if communication is possible, and selects a mover $a$. If the sender’s type and the receiver’s move are the same, both players get a payoff $v^i = 1$, $i = S, R$; otherwise, they each get $v^i = 0$. Hence, there is no incentive for the sender to misrepresent its type.
2.1 Two Types and Two Messages

We first describe the equilibrium of the game without communication, before looking at the cheap and costly talk games. In the game without communication, the solution concept is Bayesian Nash Equilibrium. The priors \( p(t) \) involve exogenous beliefs about the sender’s type which determine the optimal response of the receiver. We specifically assume that the receiver has a prior \( p(l) = 1/2 \) that the sender’s type is \( l \), and a prior \( p(r) = 1/2 \) that is \( r \). In the absence of any new information, the equilibrium is for the receiver to pick a mixed strategy \( \sigma = \left( \frac{1}{2}, \frac{1}{2} \right) \) where both locations are equally likely. The expected payoff is \( v^S = v^R = 1/2 \).5

If, instead, communication is possible and talk is payoff-irrelevant for both players, we are in a cheap-talk game (Crawford and Sobel, 1982). For full disclosure to be possible, at least two messages are required: let the messages be "l" (“left”) and "r" (“right”). An equilibrium is informative if the receiver changes beliefs after some message on the equilibrium path (Sobel, 2011: 5).6 Otherwise, the equilibrium is uninformative or babbling.

There is an informative equilibrium where words are used in their conventional sense: the sender uses "l" to refer to \( l \) and "r" to \( r \), and the expected payoffs for both players are \( v^S = v^R = 1 \). Figure 1 represents instead an informative equilibrium where words are used in a way opposite to their literal meaning: the receiver plays \( l \) after message "r" and \( r \) after message "l". This informative equilibrium is very odd because words are not used in their ordinary sense, as we explain below.

5 If \( p(l) > 1/2 \), the buyer would play \( l \); if \( p(l) < 1/2 \), \( r \). The crucial issue is that, except when \( p(l) = 1 \) or \( p(l) = 0 \), communication can help to improve expected payoffs.
6 Drawing on information theory, Skyrms (2010: 34-5) also distinguishes between informational quantity (how much probabilities change) and informational content (the direction in which probabilities move).
There is also an uninformative equilibrium where both sender types $t = l, r$ have the same probability $\omega^S(t)("m") = \omega("m")$ of sending each message "m"="l","r", and the receiver plays a mixed strategy $\sigma^R("m")(a^R)=1/2$ where moves $a^R = l, r$ are not conditional on message "m", because message "m" is not conditional on the actual location $t$; the payoffs are as in the game without communication.

The setup can be modified by introducing a cost of misrepresentation if message "m'" differs from true type $m$. This nice idea was formalized by Kartik, Ottaviani, and Squintani (2007), and Callander and Wilkie (2007), with misrepresentation costs that depend on the extent of the distortion. Our specific assumption is that disutility depends on the action of misrepresentation itself, as in the fixed honesty costs in Demichelis and Weibull (2008): the difference is that honesty does not enter lexicographically to break ties in material gains; rather, senders have an infinitesimal but constant cost $\epsilon$ from thinking about saying something different from their actual type (Streb and Torrens, 2015).

Costs of misrepresentation eliminate babbling equilibria in rendez-vous: if receivers ignore all messages "ℓ", a seller would rather reveal its true type $t$ because it is the lowest cost message; given that, buyers have an incentive to heed the messages. However, costs of misrepresentation alone are not enough to do away with the excess of informative equilibria.
Figure 2 shows that costly talk does not destroy the informative equilibrium of Figure 1 where words are used arbitrarily: if the sender deviates from the equilibrium message, its payoff would fall from $1 - \varepsilon$ to 0.

Figure 2. Costly Talk: Unnatural Informative Equilibrium in *Rendez-Vous*

![Diagram showing the equilibrium in Rendez-Vous](image)

2.2 Unbounded Messages and Types

Farrell (1993: 515) points out that since meaning cannot be learned from introspection in cheap-talk models, any permutation of messages across meanings gives another equilibrium. But the key stumbling block with existing models of verbal communication turns out to be that the possible equilibrium messages are unbounded. We have assumed that the sender’s location may be communicated by one of two messages, but once we allow any kind of verbal message, countless informative equilibria crop up where words are not used in their ordinary sense.

Furthermore, typically there are more than two types. Our analysis of *rendez-vous* was motivated by the problem of how the buyer and the seller of a used car get together. The market for lemons is a decentralized market: if they successfully meet, they can share the expected gains
from trade. The following information must be transmitted before buyer and seller get together in the second stage: the seller must post an ad saying that a car is for sale, indicating the quality of the car and a phone number; the buyer must call the phone number listed in the ad; and the seller must announce the place and time of meeting. If we ignore the issue of quality, this leaves four pieces of information that must be conveyed from the seller (the sender) to the buyer (the receiver): that what is for sale is a car, the seller’s phone number, the meeting time, and the meeting place. Since the conceptual problem of communication is the same for each piece of information, we have focused on the meeting place. However, think what would happen if the seven digits of the true phone number were randomly scrambled in the newspaper ad (ATMs typically use four-digit passwords). The sheer multiplicity of informative equilibria leaves us where Schelling (1960) left us: focal points. Schelling’s selection arguments suggest that the only focal point is the informative equilibrium where natural language is used in its literal sense.

We pursue a complementary line. Kartik (2009) recognizes that costly talk introduces some standard or norm for saying things, so a message ”t” has a literal or exogenous meaning “my type is t.” Otherwise, senders would not be able to experience a cost of misrepresentation. We now extend the idea of norms to receivers. Though moral codes for senders are important, the most basic feature of language is that we are brought up with a shared social convention that applies both to senders and receivers. The conventional expression for meeting at location left (l) is “left” (“l”) in English, but “izquierda” is used instead in Spanish. Language works only if it is a shared code. This provides an alternative reason for why the only informative equilibria are those where language is used in its literal sense, namely, because otherwise the receiver will not be able to understand the sender’s message correctly.

3. Equilibria

We combine the feature that players rely on a preexisting language with the Crawford and Sobel (1982) framework of strategic information transmission. We start with the definition of PBE used in cheap talk, before defining an alternative solution concept for language games. We then introduce an equilibrium refinement for language games.

If the second stage, the market for lemons, implies market breakdown, high-quality sellers will not participate in the first stage. The coordination problem for sellers and buyers of lemons still remains.
3.1 Perfect Bayesian Equilibria in Language Games

Incomplete information games allow isolating information transmission by the sender to the receiver (Lewis, 1969; Crawford and Sobel, 1982). In a cheap-talk game with incomplete information, the sequence is as follows. First, the set of types \( T = W \) (the game, or little world) and the priors \( p(w) \in P \) about the possible types \( w \in W \) are exogenously given. Second, the sender \( S \) sends a message "\( m \)" (the language). Third, the receiver \( R \) updates its priors. Fourth, the receiver picks \( a^R \in A^R \). Finally, \( v^l: W \times A^l \rightarrow \mathcal{R} \) is the utility function of player \( i = S, R \). While \( W \) and \( A^R \) are finite, the set of messages "\( M \)" might not be. We consider finite message sets because this language is sufficiently rich in our finite world. Strategies and beliefs are given by \( (\omega^S, \sigma^R, \mu) \), where:

- \( \omega^S(w) = (\omega^S(w)("m_1"), ..., \omega^S(w)("m_M")) \), for each type \( w \in W \), is a probability distribution on "\( M \)"; i.e., \( \omega^S(w)("m") \in [0,1] \) and \( \sum_{m \in M} \omega^S(w)("m") = 1 \); a strategy for the sender is a vector of probability distributions \( \omega^S = (\omega^S(w_1), ..., \omega^S(w_W)) \).
- \( \sigma^R("m") = (\sigma^R("m") (a^R_1), ..., \sigma^R("m") (a^R_W)) \), for each message "\( m \)" in "\( M \)"; is a probability distribution on \( A^R \); i.e., \( \sigma^R("m") (a^R) \in [0,1] \) and \( \sum_{a^R \in A^R} \sigma^R("m") (a^R) = 1 \); a strategy for the receiver is a vector of probability distributions \( \sigma^R = (\sigma^R("m_1"), ..., \sigma^R("m_M")) \).
- \( \mu("m") = (\mu("m") (w_1), ..., \omega^S("m") (w_W)) \), for each message "\( m \)" in "\( M \)"; is a probability distribution on \( W \); i.e., \( \mu("m") (w) \in [0,1] \) and \( \sum_{w \in W} \mu("m") (w) = 1 \); a belief for the receiver is a vector of probability distributions \( \mu = (\mu("m_1"), ..., \mu("m_M")) \).

**DEFINITION 1 (cheap-talk equilibrium):** In an incomplete information game, a perfect Bayesian equilibrium satisfies conditions (1) through (4):

1. For each \( w \in W \),
   \[
   \tilde{\varphi}^S(w) = \arg \max \omega^S(w) \sum_{m \in M} \omega^S(w)("m") \sum_{a^R \in A^R} v^S(w, a^R) \tilde{\varphi}^R("m") (a^R) .
   \]
2. For each "\( m \)" in "\( M \)",
   \[
   \tilde{\varphi}^R("m") = \arg \max_{\sigma^R("m")} \sum_{a^R \in A^R} \sigma^R("m") (a^R) \sum_{w} v^R(w, a^R) \tilde{\mu}("m") (w).
   \]
(3) If for a message \( "m" \in \mathcal{M} \), there exists a \( w = w_i \in \mathcal{W} \) such that \( \tilde{\omega}^S(w_i)("m") > 0 \), then
\[
\tilde{\mu}("m") (w_i) = \frac{\tilde{\omega}^S(w_i)("m") p(w_i)}{\sum_w \tilde{\omega}^S(w)("m") p(w)}.
\]

(4) If for a message \( "m" \in \mathcal{M} \), \( \tilde{\omega}^S(w)("m") = 0 \) for all \( w \in \mathcal{W} \), then \( \tilde{\mu}("m") (w) \in [0,1] \) and \( \sum_w \tilde{\mu}("m") (w) = 1 \).

This definition helps to clarify the critique of the multiplicity of informative equilibria in cheap-talk games. Consider the unnatural equilibrium of rendez-vous in Figure 1, where the sender is either at \( l \) or \( r \) with an exogenous probability \( p(r) = p(l) = 1/2 \). Condition (3) determines beliefs by equilibrium strategies, regardless of how that fact might be communicated from sender to receiver by the actual message "m". This is an odd way to model beliefs when the only additional information the receiver gets in each information set is the verbal information "m" on meeting place provided by the sender. In our terms, these informative equilibria rely on encrypted messages that require some meta-message that explains what each message in that equilibrium means. This leads to an infinite regress problem. And what agent sends these meta-messages? This is not a reasonable interpretation for one-shot interactions. This anomaly motivates our model of natural language. However rational players may be, they cannot decipher encrypted messages.

The problem that receivers have no way of deciphering correctly the equilibrium messages is not specific to cheap-talk models, as the unnatural informative equilibrium of the costly talk game in Figure 2 shows. Rather, what is problematic is the application of PBE to language games.

3.2. Language as a Conventional Sign

Natural language can be described resorting to the categories used in semiotics. The basic distinction, which appears in John Poinsot’s 1632 Treatise on Signs, is between conventional signs like the word “fire” or an image of a flame, and natural signs like smoke (Crespo, 2012). Signs typically point to something else.\(^8\)

De Saussure introduces a dyadic model of signs composed of signifier and signified, while Peirce proposes instead a triadic model that is closer to the modern representation of signs as

\(^8\) Self-reference plays no role in the games we analyze. Besides, it can lead to contradictions like the semantic paradox “This sentence is not true.”
composed of three elements (Chandler, 1994). In the specific case of linguistic signs, the components are:

1. The signifier or sign vehicle: a sequence of letters or sounds "m", e.g., “This car is in perfect condition.”
2. The signified, sense, or intension: the concept ♦️ we think about when we read or hear the signifier.
3. The referent or extension: the object ♦️ a signifier refers to, e.g., the used car they are trying to sell us.

We concentrate on full sentences that can express propositions, not on isolated words. Though the signifier "m" is only part of the whole, it is customary to call the signifier “sign.” Following the standard use in economics, we usually employ the term “message” for the signifier. As to the signified ♦️, we distinguish between the literal and equilibrium meanings. In regard to the referent ♦️, we distinguish between the object to which the proposition allegedly applies and the truth-value of the proposition.

3.3 Language as a Means of Communication

In the spirit of Farrell (1993), we assume that if a common pre-existing natural language is used, the receiver will be able to understand the different words the sender utters (this is a sufficient condition for communication). Our addition is the inverse proposition: if a common language is not used, the receiver will not be able to understand the sender’s meaning (this is a necessary condition for communication). The encoding-decoding step is followed by an inferential step.

3.3.1 The Encoding-Decoding Step

The information that natural language conveys is symbolic. The first issue is comprehensibility. The signified is crucial in asymmetric information games: since the referent is unobservable from the receiver’s vantage point, the receiver uses the signified to ascertain the type. More formally, in the encoding stage, the sender S uses the signifier "mS" to express the signified ♦️. In the decoding stage, the receiver R uses the signifier "mR" to recover the signified ♦️. This is the literal meaning. Since thoughts are interior processes, only the signifier or message is manifest:
\[
\begin{align*}
\text{(1)} & \quad "m^{\text{sn}} = e(\hat{m}^S), \\
\text{(2)} & \quad \hat{m}^R = e^{-1}("m^{rn}).
\end{align*}
\]

We assume throughout our discussion that "m" = "m^R" = "m^S", so the message that is heard by the receiver coincides with the message uttered by the sender. We also rule out errors of perception. The issue we analyze is that not all the information may be revealed due to willful distortions of the sender, i.e., "m^S" \neq "w^S" perceived by the sender.

A natural language "L" allows talking about different partitions of the world at large \( \mathbb{W} \), with statements "Q" that point to a subset Q \subseteq \mathbb{W}. Henceforth our analysis is in terms of sets. The finest partition identifies individual elements through singleton sets. Coarser partitions imply more imprecise statements. Consequently, we characterize a natural language as a bijection over the powerset of \( \mathbb{W}, \ e: \mathbb{P}(\mathbb{W}) \to "L\). One direction, "Q" = \( e(\hat{Q}) \), denotes the encoding step by which the sender describes in words a perceived state of the world (a type in our case). The other direction, \( \hat{Q} = e^{-1}("Q") \), denotes the decoding step by which words are interpreted by the receiver in terms of an actual state of the world.

Actual messages "m" can be any linguistic sign at all, they need not belong to a natural language common to the players. This raises the possibility of incomprehensible messages. Meaningful talk is a natural language "M" that is common to the players. Incomprehensible messages are messages "m" \not\in "M". Incomprehensible messages do not enable the receiver to decode the information uttered by the sender, for example “RTL8029AS”—however, an extended zip code with delivery point might work to indicate a location.

ASSUMPTION 1 (comprehensible messages): In the encoding-decoding step, the receiver can understand the literal meaning \( \hat{m}^R = e^{-1}("m^{rn}\) that is being provided verbally if and only if the sender uses a common natural language "M" to utter the message "m^{\text{sn}} = e(\hat{m}^S)."

3.3.2 The Inferential Step

In asymmetric information games, the receiver only has the literal meaning of the utterance to act upon. These common understandings are crucial in explaining why words can be informative despite the fact that they do not provide direct evidence of types. However, the equilibrium meaning depends on the strategic context.
Comprehensible messages need not refer to the actual game $W \subseteq \mathbb{W}$. Let \textit{relevant} messages \("R" \subseteq \mathbb{M}\) be those that refer to strict, non-empty, subsets of $W \subseteq \mathbb{W}$. \textit{Irrelevant} messages are messages that refer to a set $Q$ such that either $Q \cap W = \emptyset$ or $Q \cap W = W$. Irrelevant messages do not add information to the priors, so beliefs cannot be updated from the literal information they provide.\footnote{Keynes (1921: 59) defines irrelevance in terms of new evidence that does not lead to changing a conclusion. Sperber and Wilson (1995) define an input as relevant when, together with available contextual assumptions, it yields positive cognitive effects.}

A key issue is whether the statement implied by the message is true or not, so the point of view of Peirce, Frege, and other logicians comes to the fore: the referent of the message must be taken into account. Truthfulness and trust cannot be defined without the social conventions shared by the players in their common language. While truth and trust are simple to characterize, untruthfulness and mistrust are manifold.

With a two-valued logic, the sender may be either truthful or not. Imprecise messages are less informative than precise messages, but they can be truthful. A sender’s \textit{truth-function} is a function $T^S:\mathbb{M} \times W \rightarrow \{0,1\}$, where for type $w \in W$, $T^S("m",w) = 1$ if and only if "m" = "Q" such that $w \in Q$, $T^S("m",w) = 0$ otherwise. There are many ways of being untruthful. Any degree of truthfulness is potentially possible if we define mixed strategies over truthful and perfectly misleading messages.

On the equilibrium path, the receiver may either trust the literal meaning of the sender’s message or not; messages will affect beliefs only if trusted. A receiver’s \textit{trust-function} is a function $B^R: \mathbb{M} \rightarrow \{0,1\}$, where $B^R("m") = 1$ if message "m" is trusted and $B^R("m") = 0$ if not. There are many ways of not trusting something. Mixed strategies can again lead to intermediate degrees of trust when defined on either trusted or mistrusted messages. However, we impose a restrictive response in case of mistrust. We limit mistrust to ignoring a message’s literal meaning when updating priors. We thus rule out reinterpretations of the message, like interpreting "I" to refer to $r$, and "r" to $l$, in Figures 1 and 2 above. This leads to the second key element in our conceptualization, the assumption that the following inferential step is satisfied.

\textbf{ASSUMPTION 2 (trusted messages):} In the inferential step, the receiver may either trust a relevant message’s literal meaning from the decoding step—$B^R("m") = 1$—and use it to update the priors, or not—$B^R("m") = 0$. 

\footnote{Keynes (1921: 59) defines irrelevance in terms of new evidence that does not lead to changing a conclusion. Sperber and Wilson (1995) define an input as relevant when, together with available contextual assumptions, it yields positive cognitive effects.}
3.4 An Alternative Equilibrium Concept for Language Games

We introduce an alternative notion of equilibrium. Beliefs are not determined by the actual strategies of the sender. Instead, by Assumptions 1 and 2 beliefs are determined by the literal meaning of message that is uttered and whether it is trusted or not. The actual strategies chosen by the sender, however, are crucial to determine whether it is an equilibrium or not, i.e., if trusting a message is indeed warranted for the receiver.

DEFINITION 2 (meaningful talk equilibrium): In an incomplete information game, an equilibrium is given by conditions (1) and (2) of a perfect Bayesian equilibrium, and conditions (3') and (4') below:

(3') If, for a message \( m \in M \), there exists a \( w = w_i \in W \) such that \( \omega^S(w_i)("m") > 0 \), then either all messages on the equilibrium path are trusted or none is:

(i) If message \( m = "Q" \in M \) is comprehensible, relevant, and \( B^R("m") = 1 \), then \( \omega^S(w)("m") = 1 \) for \( w \in Q \), \( \omega^S(w)("m") = 0 \) for \( w \notin Q \), and \( \mu("m")\(w_i) = \frac{\omega^S(w_i)("m")p(w_i)}{\sum_w \omega^S(w)("m")p(w)} \) (trusted messages are true).

(ii) If message \( m \in M \) is incomprehensible, irrelevant, or \( B^R("m") = 0 \), then \( \mu("m")\(w) \in [0,1] \) and \( \sum_w \mu("m")\(w) = 1 \).

(4') If for a message \( m' \in M \), \( \omega^S(w)("m") = 0 \) for all \( w \in W \), then:

(i) For a message \( m' = "Q" \in M \) that is comprehensible, relevant, and \( B^R("m") = 1 \), \( I(w) = 1 \) for \( w \in Q \), \( I(w) = 0 \) for \( w \notin Q \), and \( \mu("m")\(w) = \frac{I(w)p(w)}{\sum_w I(w)p(w)} \).

(ii) For a message \( m' \in M \) that is incomprehensible, irrelevant, or \( B^R("m") = 0 \), \( \mu("m")\(w) \in [0,1] \) and \( \sum_w \mu("m")\(w) = 1 \).

Inferences are based on the whole set of messages related to a given equilibrium because the equilibrium messages are part of a system. In case of trust, condition (3')(i) requires that beliefs be updated according to the message’s literal meaning. In equilibrium, a message that is trusted is correctly considered to be truthful, so Bayes’ rule applies. In case of mistrust, condition
(3')(ii) does not require the message not to be conditional on type, i.e., that \( \tilde{\omega}^S(w_i)("m") = \omega \) for all \( w_i \in W \). The requirement is that the receiver cannot acquire more valuable information than the priors by switching from mistrust to trust. Instead of condition (3')(ii), by which the receiver considers the message uninformative in regard to the priors in case of mistrust, the receiver could assign them an interpretation that differs from what is literally stated. The snag we encounter is that there is no way for coordinating alternative interpretations of the receiver with different misrepresentations the sender may fabricate, at least none that goes beyond the common priors. For instance, in the market for lemons in the next section there is a strategic incentive for owners of lemons to inflate their claims. The strategic context leads the receiver to mistrust the message “This car is in perfect condition,” interpreting it in terms of the priors as a claim that all types make regardless of quality.

Off the equilibrium path, some messages might be trusted and others not. For messages that are not trusted, condition (4')(ii) replicates condition (4) in a PBE, placing no restriction on beliefs. For messages that are trusted, condition (4')(i) restricts the interpretation of a message off the equilibrium path to the ordinary meaning in natural language.

**DEFINITION 3:** An optimistic equilibrium is an equilibrium where \( B^R("m") = 1 \) for some message "m" \( \in \) M such that "m" \( \neq \) W.

Optimistic equilibria are restricted to trusted messages that refer to strict subsets of "W". This avoids irrelevant but true statements like “My type is in W” which do not affect priors, whether trusted or not. The set of optimistic equilibria is broader than the set of informative equilibria, because the messages that are trusted by the receiver might be off the equilibrium path, only predicting what will not be said, as we will see shortly for the phrase “This car is a lemon.”

**DEFINITION 4:** A pessimistic equilibrium is an equilibrium where \( B^R("m") = 0 \) for all messages "m" \( \in \) M.

The set of pessimistic equilibria is a subset of uninformative equilibria where all messages are disregarded, i.e., interpreted in terms of the common priors.

**DEFINITION 5 (credible messages):** A message is credible if there is an equilibrium of the game where the message can be either on the equilibrium path and true, or off the equilibrium path, when it is interpreted in its literal sense.
A credible message must correspond to an equilibrium of the underlying game. Unlike Farrell (1993), where credible messages (his *self-signaling neologisms*) are always trusted by receivers, credibility and trust are here two distinct concepts. Credible messages are trustworthy (or believable), but receivers may trust them or not. When trusted and true, the *equilibrium* meaning of credible messages is given by their literal meaning.

### 3.5 An Equilibrium Refinement

While meaningful talk reduces the number of informative equilibria, requiring that information be conveyed by the ordinary meaning of words, it does not restrict beliefs off the equilibrium path unless a relevant message is trusted. Hence, we adapt a refinement of PBE for signaling games proposed by Streb and Tohmé (2015a). It is inspired by the Schelling (1960) equilibrium selection argument, where the idea is to let the players self-organize from within instead of imposing a structure from without. The key idea is that, since the sender voluntarily picks a signal, the signal must be interpreted in light of the sender’s preferences to reveal information or not (Streb and Tohmé, 2015a).

**DEFINITION 6** *(refinement of equilibria for language games):* In an incomplete information game, a unique equilibrium is selected in the game if all sender types are willing to play the same equilibrium.

### 4. Implications

We first look at some properties of meaningful-talk equilibria. Then we derive implications of the equilibrium refinement.

#### 4.1 Existence

It is trivial to prove that meaningful-talk equilibria exist.

**LEMMA 1:** *In incomplete information games, meaningful-talk equilibria always exist.*

**PROOF:** *Uninformative equilibria always exist:* if the receiver disregards all messages, the sender has no incentive to choose a message that is conditional on its type; if the sender sends a message that is not conditional on its type, the receiver has no incentive to heed the messages.

As in standard cheap-talk games, there are always babbling equilibria. The sender need not actually play strategies that are not conditional on its type, because the receiver has no way of verifying that. What matters is the uncertainty of the receiver about what the sender is actually
doing, as in the epistemic interpretation of Aumann and Brandenburger (1995). This makes perfect sense when the message is incomprehensible or irrelevant. It might be less intuitive when the message is comprehensible, relevant, and credible. However, verbal messages never provide direct evidence of the meaning they convey, so trust implies a leap of faith that might be taken or not.

These results contrast with the Farrell (1993) neologism-proof equilibrium, which may refine away all the cheap-talk equilibria so no equilibrium exists (Sobel, 2011: 11-12). Farrell applies his insight that natural language is comprehensible to refinements of beliefs in response to out-of-equilibrium messages, not to the interpretation of equilibrium messages. Farrell (1993: 519-520) compares the difference between a prearranged set of meanings for anticipated messages and a preexisting natural language as the difference between encrypted codes and ciphers. With encrypted codes, a list of meanings is fixed in advance and cryptic messages are chosen to convey those meanings, such as one light for “by land,” two for “by sea” used during the American Revolution to alert if the Redcoats were coming. In a cipher, messages are instead cryptically isomorphic to a natural language, so unanticipated messages such as “The Redcoats are coming in balloons” can be added. These unanticipated out-of-equilibrium messages are interpreted as meaningful neologisms. These neologisms are credible if they are self-signaling, i.e., if the types \( w_i \in Q \) are the only ones that actually prefer the receiver to believe instead the message "Q" that their type is in set Q.

The non-existence of neologism-proof equilibria can be caused by the assumption that a meaningful neologism is always available for somebody to declare their type. Take Example 3 in Farrell (1993), where the receiver has optimal strategies \( a(A) \) and \( a(B) \) for sender types A and B, but prefers to chose a safe strategy \( a(T) \) if all the information it has are its priors \( (p(A), p(B)) \). B has an incentive for neither player to be distinguished, mimicking A if the message is uninformative, while A wants to separate from B. Farrell’s neologism-proof equilibrium implies that no perfect Bayesian equilibrium exists since A always has the possibility of sending the self-signaling message "A". There is a way out of this. Figure 3 rephrases Farrell’s Example 3 for a prior \( p(A) \in [1/4, 3/4] \).
Since $B$ has an incentive to mimic $A$’s message if it is uninformative, if the equilibrium message in the babbling equilibrium is "A", rather than something else, no self-signaling neologism is available for $A$ to destroy this equilibrium. Hence, self-signaling neologisms may not always be available, contrary to Farrell’s initial assumption. If $A$ were to say something like “I am really, really A” to destroy this uninformative equilibrium, this would not help much, unless, perhaps, this included other types of signs, like body language, something which goes beyond the purely symbolic dimension explored here.

4.2 Optimistic and Pessimistic Equilibria

We return to rendez-vous to illustrate pessimistic and optimistic equilibria. The sender’s announcement "l" may coincide with the actual location $l$ (a truthful announcement) or not (a false announcement). The receiver’s conventional interpretation of message "l" is action $l$. The buyer can either trust the messages literally, interpreting that "l" refers to $l$, or instead continue to have beliefs given by the priors that any move is equally likely. Figure 4 shows this is no longer

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10 To sustain the equilibrium, all we need to assume is that out-of-equilibrium messages, such as "B", are interpreted by the receiver as either denoting $B$, in an optimistic equilibrium, or as uninformative messages that do not affect priors, in the pessimistic equilibrium which is represented in the figure.
a cheap-talk game since messages affect payoffs. The best response to "l" depends on whether the buyer trusts the message or not. Once second-guessing is ruled out, there are no informative equilibria where words are not used in their ordinary sense because payoffs drop to 0 when the receiver trusts these messages.

**Figure 4. Meaningful Talk: No Unnatural Informative Equilibrium in Rendez-Vous**

![Figure 4](image)

Words must be used in their ordinary sense if they are to provide relevant information, and receivers must trust those words. We now provide a sufficient condition for natural language to matter for decision-making.

**LEMMA 2:** In incomplete information games, informative meaningful-talk equilibria exist if there are optimistic equilibria where the sender has an incentive to utter credible messages that are relevant.

**PROOF:** If the receiver trusts all credible messages related to a given equilibrium, any credible messages that the different sender types have an incentive to utter truthfully will be on the equilibrium path. If the credible messages on the equilibrium path are relevant, the priors will be affected by the equilibrium messages.

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11 An informative equilibrium is _influential_ if it affects the decisions of the receiver. Except in degenerate cases where there are multiple best replies, informative and influential equilibria coincide (Sobel, 2011: 5).
An optimistic equilibrium might only help to predict what will not be said in equilibrium. Take the message “This is a lemon” in the Akerlof (1970) model of asymmetric information. Consider two types of quality, \( \theta_i \in \{ \theta_L, \theta_H \} \), where \( \theta_L < \theta_H \). The quality is known to the seller, but not to the buyer. The opportunity cost of a seller is \( \alpha \theta_i \), with \( \alpha < 1 \), and \( \alpha \theta_H > \theta_L \), so market breakdown is possible. Since buyers are willing to pay \( \theta_i \) for a quality \( i \) product and sellers are willing to sell it at \( \alpha \theta_i \), there is a potential gain from trade of \( (1 - \alpha) \theta_i > 0 \). The product is high quality with probability \( 0 < q < 1 \) and low quality with probability \( 1 - q \). Following standard practice, risk-neutral buyers are willing to pay the average quality offered on the market, \( E[\theta] = (1 - q)\theta_L + q\theta_L \), so sellers reap the whole surplus from trade. High-quality sellers will be willing to accept a price equal to the expected quality if and only if the following condition holds:

\[
(1) \quad E[\theta] > \alpha \theta_H.
\]

If condition (1) is not satisfied, only lemons are left on the market, so there is no point in talking about quality. If condition (1) is satisfied, the minimal messages required are "\( \theta_L \)", “This car is a lemon”, and "\( \theta_H \)”, “This car is in perfect condition.”

Consider the price offers \( p_i \in \{ \theta_L, E[\theta], \theta_H \} \). One can rule out separating equilibria because sellers of lemons have an incentive to mimic sellers of high-quality products. A babbling equilibrium exists in which all sellers state "\( \theta_H \)". Though sellers could also pool on message "\( \theta_L \)", that does not correspond to our experience with this market. Why? Figure 5 depicts an optimistic equilibrium where buyers take the message "\( \theta_L \)" at face value (the alternative is to keep the priors that expected quality is \( E[\theta] \), which implies a higher price). If beliefs in response to "\( \theta_L \)" are \( \theta_L \), all sellers have an incentive to say "\( \theta_H \)".

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12 In Buenos Aires, the street expression is “A jewel, never a cab” (“Joya, nunca taxi”).
13 For the proposed message to be part of an equilibrium, the reaction to any other announcement has to be a low price, so the conditional probability of high quality products must be smaller than or equal to \((\alpha \theta_H - \theta_L)/(\theta_H - \theta_L)\).
The only prediction of the model is about what might not be said in equilibrium. The problem of misrepresentation is simpler in the market for lemons because of the incentive to inflate claims (Kartik, Ottaviani, and Squintani, 2007 derive inflated claims with costly talk). In the coordination stage, there is no expected direction for misrepresentation by the seller.

4.3 Equilibrium Selection
The equilibrium refinement is a means of reducing the arbitrariness of beliefs off the equilibrium path. Consider rendez-vous, where an uninformative meaningful-talk equilibrium always exists. Since all sender types have a (weak) preference for the most informative equilibrium, senders have an incentive to choose comprehensible and relevant messages that point to their exact type. Since the receiver knows that no sender type has an incentive to distort the information, if it expects all sender types to coordinate on this equilibrium, it makes sense to trust the messages. More generally:

THEOREM 1: In incomplete information games, if the different sender types are willing to select a common equilibrium, uninformative meaningful-talk equilibria subsist only if not all sender types are worse off than in a given informative equilibrium.
PROOF: Suppose not, so all types of senders are worse off in an uninformative equilibrium. But then all sender types have an incentive to point out to the receiver that they prefer to select a more informative equilibrium where they reveal, in part or in whole, their type.

The equilibrium refinement for language games provides a necessary condition for language to be informative (it is not a sufficient condition, as the market for lemons above shows).

THEOREM 2: In incomplete information games, if the different sender types are willing to select a common equilibrium, informative meaningful-talk equilibria subsist only if not all sender types are worse off than in an uninformative equilibrium.

PROOF: Suppose not, so all types of sender are worse off in an informative equilibrium. If no sender type has an incentive to disclose information and provide relevant information, the receiver will be better off by switching to the priors of the game without communication instead of reacting according to the off-the-equilibrium path beliefs.

By Theorem 2, informative equilibria must make sense for at least one type of sender. Cheap, costly, and meaningful-talk equilibria do not satisfy this criterion. Take the game “I won’t tell,” Example 2 in Farrell (1993), where the receiver has best responses $a(A)$ and $a(B)$ for each sender type, but prefers a safe strategy $a(T)$ if it must act on its priors $(p(A), p(B))$. While the two sender types prefer the uninformative equilibrium, Figure 6 depicts the informative meaningful-talk equilibrium where the receiver plays $a(B)$ unless the message ”$A$" is uttered.14

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14 To not clutter the figure, we leave out the options where A sends message “B” and B sends message “A”; if trusted, they lower the sender’s payoff more than revealing their true type. They might be used in equilibrium only if the receiver disregards all messages.
This out-of-equilibrium message comes out of the analyst’s hat to support the informative equilibrium. Figure 7 shows that this informative equilibrium is impossible once we apply the equilibrium refinement. Since neither type of sender has an incentive to reveal itself, they will select the message “I will not reveal my type,” which is equivalent to “I am either type A or B.” In the absence of new information, the best strategy for the receiver is to play a best response to its priors.\textsuperscript{15}

\textsuperscript{15} We depict an optimistic equilibrium where the off-the-equilibrium path messages about types are trusted.
The equilibrium refinement does not always work. In the uniform-quadratic example in Crawford and Sobel (1982), for example, when the expert has a positive bias there is no partition that is commonly preferred by all sender types. A way out is to consider bilateral communication instead, where the uninformed party proposes a given partition first (Streb and Tohmé, 2015b).

5. Literature on Philosophy of Language

Linguistic signs are a source of information that has not been duly recognized in economics. While Binmore (1994: 3) acknowledges the importance of common understandings, pointing out that common knowledge of these historical data helps to predict the equilibrium on which members of a society will coordinate in a specific game, he does not explore further the role of sharing a common language in sustaining an equilibrium (Binmore, 1994: 140–143).

We briefly relate our approach to the literature on philosophy of language. For Wittgenstein (1953: 21), “The meaning of a word is its use in the language.” In giving the meaning of a word, Wittgenstein (1953: 31) considers that any explanatory generalization should be replaced by a description of its use: “don’t think, but look!” (Biletzki and Matar, 2009). Hence, Wittgenstein (1953) proposes to study language games. We show, at a highly abstract level, that there is no single use; rather, the use (i.e., the equilibrium meaning) varies with the
strategic incentives in each game: words at times are literally true, at others they must be interpreted in terms of the priors of the game.\textsuperscript{16} Since we study utterance comprehension within an idealized strategic context, our approach can be seen as a formal pragmatics.\textsuperscript{17}

In the setting of coordination games with incomplete information, Lewis (1969) models a signal from a sender to a receiver about the state of the world, which is followed by an action taken by the receiver. Lewis finds both informative equilibria, which he calls “signaling system equilibria,” and non-informative equilibria. Since the interpretation of the signals is arbitrary, informative equilibria are interchangeable. This indetermination can explain conventions as something that is arbitrary, but that everyone has an individual incentive to follow when it is the Nash equilibrium.\textsuperscript{18} We build on the idea of natural language as a convention, but the question we ask takes linguistic conventions as something exogenously given. While the indetermination of equilibrium messages plays a key role if one wants to explain the arbitrary nature of the linguistic conventions that arise (Lewis, 1969; Skyrms, 1996), this does not allow answering when a pre-existing language shared by the players helps to achieve informative equilibria, the question we pursue here.

Rescorla (2010, Section 7) discusses how in Lewis (1975) the expectation of conformity to a linguistic convention, which gives everyone a good reason to conform, is based on epistemic reasons (beliefs of others). Lewis (1975) says that a language is used by a population if and only if senders are truthful and receivers are trusting most of the time (if not all the time). The requirement in Lewis (1975) seems unduly stringent. It may clarify matters to distinguish i) understanding a message, which depends on the linguistic conventions shared by the speakers, and ii) being truthful and trusting a message, which depends on the specific equilibrium of each game. While the literal meaning of a word depends on linguistic conventions, and these also apply in equilibrium when there are no incentives for misrepresentation, these conventions are still present in the background even when that is not the case. The distinction between linguistic

\textsuperscript{16} Parikh (2010) discusses the equilibrium meaning of language, but his main concern is about the costs for the sender of being more precise, not about credibility. In this regard, the cost-benefit approach in Sobel (2011: 30-33) offers an interesting approach to describe and interpret information.

\textsuperscript{17} There is now a burgeoning subfield of game-theoretic pragmatics (Franke, 2013).

\textsuperscript{18} Skyrms (1996) shows how one of the multiple informative equilibria in the coordination games studied by Lewis (1969) can be selected through evolutionary stable strategies in a dynamic setup. This approach offers an interesting formalization of the evolutionary ideas on language as a convention established by chance, without any explicit agreement among the players. Skyrms (2010) examines how the results change in response to initial conditions and naïve learning.
conventions and their use helps to understand why conventions are in some instances more honored in the breach than in the observance (Rescorla, 2010): in the market for lemons, the fact that sellers may refrain from saying “This car is a lemon,” lest their words be taken at face value, attests to the underlying linguistic conventions in society.

6. Final Remarks

We strive to close the gap between language in economic theory and daily life. We draw on two traditions. We combine what Lipman (2000) calls the “logical approach,” based on the meaning of a sentence in isolation, and the “equilibrium approach,” which takes into account the context and other extra-logical factors as modeled in game theory. According to the logical approach, sentences carry a literal meaning, a view which can be linked in particular to the ideal language philosophy in the tradition of Frege, Russell, Carnap and Tarski. More generally, according to the semantic approach in linguistics, semiotics, and philosophy of language, words constitute a conventional sign (signal or symbol are sometimes used instead) that, although it provides no direct evidence of types, nevertheless points to them. According to the equilibrium approach developed by Crawford and Sobel (1982), unilateral communication games between a sender and a receiver under asymmetric information must be analyzed strategically. We thus “impose restrictions on the use of messages within a game that capture the way that messages are used outside of strategic interactions” (Sobel, 2011: 13).

To Farrell’s (1993: 515) insight that the meaning of natural language is comprehensible, we add that a common language is a necessary condition for the receiver to be able to understand the sender. Since messages are not always reliable, we couple the encoding-decoding step with an inferential step by which the receiver may either trust the literal meaning of the message or disregard it when updating priors. Trust requires a leap of faith for the receiver, which leads us to talk of optimistic and pessimistic equilibria. The equilibrium concept is specific to language

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19 Wilson and Sperber (2012: 1-10) contrast ideal language philosophy, which treats sentences as encoding something close to full propositions, both to ordinary language philosophy in the tradition of the later Wittgenstein, Austin, and Strawson, which analyzes actual language use in all of its complexity, and to the intermediate position of Grice, Lewis, and Searle, which distinguishes between sentence meaning and speaker meaning (however, sentence meaning, or literal meaning, still is considered to encode something close to a full proposition, with reference assignment being needed to yield a full proposition).

20 Our agents might be thought of as having limited rationality, or limited imagination, because they revert to the priors in case of mistrust. However, fully rational agents cannot coordinate on their own to decipher encrypted messages, at least not through plain language because this leads to an infinite regress problem.
games because linguistic signs carry information precisely through their literal meaning, unlike other signaling games. The equilibrium meaning depends on each specific strategic context.

Explicit communication can complement tacit coordination (Schelling, 1960) as a way to select equilibria under asymmetric information. We explore a refinement that works under unilateral communication if all sender types are willing to select the same equilibrium. This refinement implies that verbal communication provides information only if at least one sender type is better off, a property that standard equilibria do not satisfy.

We are just scratching the surface of verbal communication. Taking the literal meaning as the starting point of the semantic representation of a sentence is more attuned to formal venues.21 Our approach relates to what some call “persuasive communication,” where language is a symbolic instrument to inform about things, as distinguished from “empathic communication” to make known personal thoughts and feelings (Adam Smith’s second view of language; see Alonso-Cortés, 2008: 7).

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21 Take the message “Come at eight,” which means one thing in legal, business, and academic settings, another in informal settings like a dinner invitation, where it may mean “Come no earlier than 8:30” in San Diego, or “Come at nine” in Buenos Aires.
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