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Informal Employment and Business Cycles in Emerging Economies:
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Abstract

This paper documents how informal employment in Mexico is countercyclical, lags the cycle and is negatively correlated to formal employment. This contributes to explaining why total employment in Mexico displays low cyclical and variability over the business cycle when compared to Canada, a developed economy with a much smaller share of informal employment. To account for these empirical findings, a business cycle model is built of a small, open economy that incorporates formal and informal labor markets, and the model is calibrated to Mexico. The model performs well in terms of matching conditional and unconditional moments in the data. It also sheds light on the channels through which informal economic activity may affect business cycles. Introducing informal employment into a standard model amplifies the effects of productivity shocks. This is linked to productivity shocks being imperfectly propagated from the formal to the informal sector. It also shows how imperfect measurement of informal economic activity in national accounts can translate into stronger variability in aggregate economic activity.

JEL classifications: F41, F44

Keywords: Emerging economies, Business cycles, Informal employment

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1 Introduction

A growing literature on the sources of business cycles in emerging market economies (EMEs) has documented how frictions matter when it comes to amplifying the effects of technology or interest rate shocks. Such frictions can either be broadly recovered as permanent shifts to total factor productivity (TFP) or, more narrowly, be associated with financial frictions that amplify interest rate shocks. However, an important observation to be made on this literature is that it has largely abstracted from a closer analysis of the labor market and its link with aggregate fluctuations. And even less attention has been given to informal employment, despite the fact that informality is often a distinctive characteristic of labor markets in EMEs. In Latin America, for instance, recent estimates have found that, on average, one out of every two workers is employed in the informal labor market.

This observation raises both empirical and theoretical questions. On the empirical side, does informal employment exhibit any distinctive pattern across the business cycle of EMEs? If so, is it relevant in shaping the dynamics of aggregate employment? And are those aggregate dynamics different from those in advanced economies? On the theoretical side, how should the current framework for studying aggregate fluctuations in EMEs be modified to account for the large share of informal employment in these economies and their business cycle properties? And how does our understanding of the sources of business cycles in EMEs change from such a modified framework?

This paper aims at providing answers to these questions. Our strategy is divided into two parts. First, we empirically document the link between informal employment and business cycles. To do this we use a dataset of five alternative measures of informal employment

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3Notable exceptions that study labor market frictions in small open and emerging economies are Altug et al. (2011), Boz et al. (2009), Fernández and Meza (2011), Li (2011), and Lama and Urrutia (2013). Also Conesa et al. (2002), Fiess et al. (2010), Restrepo-Echavarría (2011), and Finkelstein (2012) explicitly model an informal sector in a dynamic general equilibrium setting. A comprehensive literature review is given in the Appendix.

4See Powell (2013). This study reports that the (population-weighted) average of informality across Latin American countries is 44.1 percent. Dispersion by income quintiles is also significant within countries: the bottom quintile rarely exceeds 20 percent coverage of formal employment. In this work informality is measured as the percentage of employed workers in each income quintile, aged 20 and older, who are not contributing to social security.
at a quarterly frequency for 23 years in Mexico, a representative emerging economy. We then investigate the business cycle properties of our proxies for informal employment.

This empirical exploration reveals several stylized facts. We start by highlighting the differences in terms of labor market dynamics between Mexico and Canada, the latter often taken as a representative developed small open economy. Relative to output, aggregate cyclical employment in Mexico is nearly half as volatile as it is in Canada. Moreover, in Mexico employment displays a correlation with the cycle that is considerably lower than that of Canada. We then argue that both the large share of informal employment in Mexico and its distinctive cyclical properties play an important role in explaining the different labor market dynamics across the two countries. In particular, we document that in Mexico informal employment is unambiguously countercyclical, lags the cycle and is negatively correlated with formal employment to the point that total employment’s variability and procyclicality are reduced. Finally, self-employment is relatively well synchronized with the other proxies for informality.

Motivated by these new stylized facts, a second part of our work is devoted to building a small open economy (SOE) model with both formal and informal labor markets with which we can rationalize these empirical findings. In the model, households choose how much labor to allocate to each market. They accumulate two different capital stocks, which are sector-specific, and consume formal and informal goods produced in each sector. Formal capital is rented to firms, while informal capital is used in the informal sector. Households can buy or sell one-period non-contingent bonds in foreign capital markets. Production in the formal sector is done by firms. In the informal sector people are self-employed, given the tight link between informality and self-employment that we document for Mexico. Both technologies have constant returns to scale and use capital and labor in production. Formality entails costs and benefits. The formal firm has to pay taxes on the wage bill but enjoys high productivity levels. The informal producer is less productive but does not pay taxes. The productivity gap is motivated by the evidence on the large TFP differential across formal and informal firms in Mexico. The only source of uncertainty in the model is a shock to the growth factor of labor-augmenting productivity in the formal sector. Shocks to the formal sector are passed through to the informal sector. Crucially, however, we allow for an imperfect propagation
of these shocks from the formal to the informal sector, as a reduced-form device to capture institutional or other types of channels that prevent business cycle drivers from spreading uniformly across labor markets. Finally, the government taxes personal income (wages and capital rents) and the hiring of labor by formal firms to finance a stream of government purchases in formal goods.

The model is calibrated to Mexican data so that it matches two empirical targets. In particular, the standard deviation of the productivity shock is set so that formal output volatility matches the one in the data. We additionally set the parameter that measures the propagation of shocks across sectors to match the empirical correlation between informal employment and output. The model is then evaluated in terms of its performance along some of the other unconditional second moments that describe business cycles in Mexico. We further assess its performance conditional on the two largest recessions of our sample. The experiments yield several results of interest. First, the relative size of the shocks needed to account for the observed output volatility is around 10 percent lower than that required if labor informality is absent. Hence a first quantitative result is that of an amplification of shocks, in the sense that the inclusion of informal employment in an otherwise standard SOE model amplifies the effects of growth shocks. Second, this amplification is linked to an imperfect propagation of shocks across sectors. Seen through the lens of the model, the strong countercyclicality of informal labor implies a relatively low pass-through level of shocks across sectors, as roughly 36 percent of a shock in the formal sector is not contemporaneously propagated into the informal sector. The imperfect transmission of shocks generates differences in relative productivity that create more labor reallocation. In equilibrium, this reallocation is accompanied by more macroeconomic volatility, which is at the center of the model’s mechanism for generating business cycles. A third quantitative result of interest is that proper measurement of informal activity matters when quantifying this macro volatility. In particular, we show that the volatility of the aggregate variables falls if informality is properly accounted in national accounts. For instance, in our model total output is less volatile relative to formal output. Our results also show that if one were to measure the ratio of volatilities (e.g., standard deviations) of aggregate consumption to output after properly accounting for informality, then the ratio would fall below one, unlike what we observe in
the data.

The model is capable of generating several other second moments in line with the data. It accounts for a high persistence of output and a strong procyclicality of consumption. It reproduces a stronger procyclicality of formal employment relative to that of aggregate employment, a salient stylized fact in the data, and it captures the fact that both formal and informal employment are more volatile than total employment. Perhaps more importantly, the model matches quite well the dynamic correlations between output and informal employment, which is one of the most distinctive business cycle characteristics of informal employment in Mexico. Indeed, it reproduces informal employment’s lagging property. Finally, the model does a fair job of capturing the dynamics during crisis events such as the Tequila crisis or the more recent Great Recession. Both episodes had a large impact on economic activity, and in both of them cyclical informal employment rose.

We analyze the sensitivity of our results by changing several elements of the model. We compute the model using different values for the elasticity of substitution between formal and informal consumption goods. A higher elasticity of substitution yields more reallocation of labor across sectors, and therefore higher output volatility. Also, we relax some of the simplifying assumptions of our benchmark framework. We consider, for example, the inclusion of stationary shocks, and we allow imperfect pass-through to differ across the two productivity shocks. We also add capital adjustment costs in both sectors. Finally, we explicitly model varying degrees of imperfect measurement of informal economic activity in the model’s macro variables. The results are largely robust in these alternative experiments.

The rest of the paper is divided into six sections including this introduction. In Section 2 we present the empirical findings. In Section 3 we display the model. In Section 4 we calibrate the model and present the main results. In Section 5 we report several alternative experiments and models. Finally, we provide conclusions in Section 6. Further technical details are gathered in a companion Appendix.
2 Stylized Facts

This section documents the link between informal employment and business cycles in Mexico, a representative emerging economy, for which we build a dataset on informal employment. Mexico is among the few countries, if not the only one, where long time series on informal employment at a quarterly frequency exist.\(^5\)

Our starting point is the comparison of Mexican business cycle statistics with those of Canada. Such a comparison was already made in Aguiar and Gopinath (2007), where Mexico and Canada were taken as representative emerging and developed economies, respectively. In Table 1 we report the second moments that were presented in this work (standard deviation and procyclicality of income, consumption, investment and the trade balance), and we add two more on labor market dynamics in the bottom of the table: the volatility of total employment, $\sigma(h)/\sigma(Y)$, and its correlation with the cycle, $\rho(h,Y)$.\(^6\) It is important to note that we are working with data on the number of workers employed, and not with data on hours worked. As already documented in Aguiar and Gopinath's work, Mexican and Canadian business cycles differ in many dimensions: the business cycle in Mexico is relatively more volatile, with investment and, more importantly, consumption being relatively more volatile than output, which gives rise to a strong countercyclicality of the trade balance. But the evidence on $\sigma(h)/\sigma(Y)$ and $\rho(h,Y)$ shows that there are also important differences in terms of labor market dynamics over the business cycle. Relative to output, the volatility of aggregate cyclical employment in Mexico is only 0.42, nearly half as in Canada, 0.74. Furthermore, in Mexico employment displays a correlation with the cycle of only 0.54, which is considerably lower than that of Canada, 0.88.

[1. Business Cycles: Mexico and Canada]

\(^{5}\)To the best of our knowledge, three other emerging economies, Brazil, Argentina and Colombia, also record high-frequency data on informal employment, but the time span they cover is much smaller than that of Mexico. We are unaware of the existence of similar data for other EMEs. Despite these data limitations, some cross-country evidence on the business cycle properties of informal employment is provided in the Appendix.

\(^{6}\)There is a slight discrepancy between the moments in Table 1 and those reported by Aguiar and Gopinath (2007), which comes from the fact that our sample (1987.Q1 to 2003.Q2) starts seven years later than theirs (1980.Q1 to 2003.Q2). We make this choice because 1987 is the year where employment statistics become available. However, the differences are minor.
Our working hypothesis is that both the large share of informal employment in Mexico and its distinctive cyclical properties, particularly its strong countercyclicality, play an important role in explaining labor market dynamics over the business cycle in Mexico. It is well known that emerging economies display high levels of informal employment relative to developed economies (see Powell, 2013, and the cross-country evidence in the Appendix), and Mexico is by no means an exception. Levy (2008), for instance, has argued that Mexican informal employment accounts for more than half of the labor force. However, virtually no attention has been given to the business cycle dynamics of informal employment. As we document below, it also has distinctive dynamics over the business cycle.

Before looking at the evidence we now briefly describe the dataset that we have put together on Mexican informal employment. Building on the National Survey of Urban Employment (ENEU in Spanish), we divide total employment into four different measures of formal and informal employment over the period 1987.Q1 to 2003.Q2. The first measure of informal employment ($h_1$) refers to employment in establishments with 1 to 5 employees; the second ($h_2$) uses employment not covered by labor legislation benefits (remunerated time-off, etc.); the third ($h_3$) is employment from wage earners who do not receive benefits provided by the labor legislation; and the fourth is self-employment ($h_4$). The end of the collection of the ENEU and the creation of a new employment survey, the National Survey of Occupation and Employment (ENOE in Spanish), led us to consider a second sample period ranging from 2000.Q2 to 2010.Q4. Because new information was introduced into the ENOE we are able to compute for this period a fifth measure of informal employment ($h_5$) defined as employment in economic units not distinguished from households (in national accounts these economic units are called private unincorporated enterprises). However, the ENOE

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7 A notable exception is Bosch and Maloney (2008).
8 As mentioned earlier, we are working with number of workers, and not with hours. Other researchers, for example Aguiar and Gopinath (2007), have used average hours in the manufacturing sector to calculate total hours worked. However, the behavior of labor in that sector may not be representative of the rest of the economy, which is much larger.
9 We are guided by the literature when constructing the five measures of informal employment. The first measure of formal employment is suggested by the evidence that establishments in the informal sector are small. Amaral and Quintin (2006) report data for Argentina, and in the Appendix we report evidence for Mexico from Busso et al. (2012) that points in the same direction. These works find that establishments with a small number of employees account for a significantly higher fraction of informal employment. The second measure is similar to one used by Pratap and Quintin (2006). Levy (2008) has also strongly favored the use of labor legislation benefits as a metric to measure labor informality. The third measure captures...
also introduced a different classification which prevents us from keeping track of \( h_1^I \) and \( h_3^I \). The five measures of formal employment \( (h_1^F, h_2^F, h_3^F, h_4^F, h_5^F) \) are the residual obtained after subtracting each measure of informal employment from total employment.\(^\text{10}\)

We now highlight six key stylized facts coming from this dataset. First, the share of informal employment in Mexico is large. The average shares of the five measures of informality are:

\[
\overline{h}_1^I = 0.42, \quad \overline{h}_2^I = 0.36, \quad \overline{h}_3^I = 0.23, \quad \overline{h}_4^I = 0.20, \quad \overline{h}_5^I = 0.28
\]

where \( \overline{h}_x^I \equiv \sum_{t=1}^{T_x} \frac{h_{x,t}^I}{(h_{x,t}^I + h_{x,t}^F)} \) for \( x = 1, \ldots, 5 \), and \( T_x \) is the maximum sample size in measure \( x \). \( I \) stands for informal and \( F \) for formal. Hence, according to our dataset, between 20 and 42 percent of the labor force in Mexico is informal. This is large when compared to developed economies. Using ILO estimates, the average share of self-employment in a pool of EMEs is around 22.2 percent. This is more than double what is found in another pool of developed economies, where the average is 10 percent.\(^\text{11}\)

Second, informal employment in Mexico is strongly countercyclical, which is captured by a negative and statistically significant correlation coefficient between (cyclical) informal employment and output. This is documented in Table 2, where we report the cyclical properties of total employment in Mexico, disaggregated into formal and informal employment. It is important to note that this stylized fact is robust to all five proxies of informal labor in our dataset, regardless of which of the two samples one looks at. The correlations coefficients, \( \rho(h^I_x, Y) \), are all negative across \( x \)'s, statistically significant, and range between \(-0.50\), for \( h_3^I \) (first sample), to \(-0.25\), for \( h_2^I \) (second sample). This stands in contrast to the dynamics

\(^\text{10}\)We did not splice the data from ENEU and ENOE. There are two reasons for this. The first is that ENEU had coverage of urban areas, whereas ENOE has broader, national coverage. The second is that ENEU reported quarterly employment data for a population of 12 years and above. ENOE, on the other hand, reports quarterly employment for a population of 14 years and above. More information on the differences between ENEU and ENOE can be found in INEGI's website. INEGI is Mexico's official statistical agency. We downloaded the ENEU and ENOE data from INEGI's dataset "Banco de Información Económica."

\(^\text{11}\)For developed economies, these are Australia, Austria, Belgium, Netherlands, New Zealand, Norway, Portugal, Spain, and Switzerland. For EMEs, these are Argentina, Brazil, Chile, Colombia, Ecuador, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovakia, South Africa, Thailand and Turkey. The Appendix contains more detail.
of formal employment, which are strongly procyclical, and whose correlations with the cycle range between 0.53 to 0.84 for $h_2^F$ in the second and first samples, respectively.

[Table 2. Second Moments of Formal and Informal Employment in Mexico]

Third, informal employment in Mexico is not only countercyclical, but also a lagging indicator of the cycle. This is substantiated by the evidence in Figure 1, which provides the cross correlation between output’s cycle in $t$ and $h_{x,t+j}^I$, $\text{Corr}(Y_t, h_{x,t+j}^I)$, for $x = 1, 2, 3, 4$ and $j = -4, -3, ..., 4$ using the first sample (upper panel), and for $x = 1, 4, 5$ using the second sample (lower panel). It shows how informal employment is not only countercyclical but also a lagging indicator, because that correlation peaks in $j > 0$, in the two samples studied. For example, if the peak of an economic expansion occurs in $t = 0$, informal employment will have its trough in the subsequent quarters.

[Figure 1. Cross Correlation of Output and Informal Employment in Mexico]

Fourth, the countercyclical nature of informal employment is independent from the phase of the cycle. This is documented in the last four columns of Table 2 where $\rho(h_{x}^I, Y)$ is computed conditional on the phase of the cycle. We use two alternative measures when computing the phase. One relies on the peaks and troughs found using Hodrick-Prescott (HP)-filtering. The other uses INEGI’s official business cycle dating. The main message coming out of these columns is that, just as informal labor is below its trend when the economy is expanding, it goes above its trend when the economy is contracting. All measures of informal employment exhibit correlation coefficients in expansions that are statistically equivalent to those computed during contractions. The only exception is $h_3^I$, for which countercyclicality is stronger in expansions. Figure 2 further illustrates this fact by plotting rolling window correlations between the cycle and, respectively, $h_{x}^F$ (upper two panels) and $h_{4}^I$ (lower two panels), in a centered window of 21 quarters. The panels on the left present the results for the first sample, while those on the right do so for the second. There is not a single period in either of the two samples where the correlations change sign. Throughout both
samples the point estimates show that formal employment is consistently procyclical while informal employment is countercyclical. Moreover, neither correlation exhibits distinctive behavior during the (shaded) periods identified as recessions.

[Figure 2. Rolling Correlations between Output and Formal/Informal Employment in Mexico]

Fifth, informal employment is somewhat more volatile than formal employment, although this is not robust across all measures of informal employment. In two out of the four measures considered in the first sample, $h_I^3$ and $h_I^4$, informal employment is more volatile than formal employment, but in the other two their volatilities are statistically indistinguishable across formal and informal measures. In the second sample all measures considered display higher variance relative to their formal counterparts. Importantly, this relatively higher variance of informal labor does not translate into a higher variance of aggregate total employment. In fact, the opposite occurs. Given the large and negative covariance between formal and informal employment documented above, the volatility of total employment is lower than formal and informal measures taken separately.

Sixth, self-employment in Mexico is strongly correlated with most of the other proxies of informality. As documented in Table 2, in the first sample, the correlation between $h_I^4$ and $h_I^1$, $h_I^2$ and $h_I^3$ is 0.74, 0.72 and 0.37, respectively. In the second sample, the correlation between $h_I^4$ and, respectively, $h_I^2$ and $h_I^5$ is −0.04, and 0.83. This is in line with other studies that have argued that self-employment is a good approximation to informality in developing economies (Loayza and Rigolini, 2006).

Summing up, this section has documented important differences in the aggregate labor dynamics in an emerging economy like Mexico when compared to a similar developed counterpart like Canada, notably a relatively low correlation and variability of total employment over the business cycle. We have argued that such differences are related to the presence of a large share of informal employment relative to total employment and its distinctive business cycle characteristics. This informal labor is unambiguously countercyclical, lags the cycle and is negatively correlated to formal employment to the point that it reduces total employment’s variability. Finally, self-employment is relatively well synchronized with the other
proxies for informality. These stylized facts will now serve as both guidelines and metrics when building and evaluating the model presented in the next section.

3 Model

This section describes our theoretical framework. We build a business cycle model of a small open economy with formal and informal labor markets.\footnote{From now on we will refer to markets or sectors indistinctively. For example, we use “formal sector” or “formal market” interchangeably.} We assume that households in the model choose how much labor to allocate to each market. While probably extreme, this assumption is motivated by the high transition rates across the two markets and the lack of market segmentation in Mexico and other EMEs documented by Levy (2008), Maloney (2004), and Pratap and Quintin (2006), among others (see also the Appendix). Preferences are of the GHH type (Greenwood, Hercowitz and Huffman, 1988).\footnote{In a different work, Fernández and Meza (2011), we have shown that a similar model with Cobb-Douglas preferences produces a negative correlation between output and labor, which is counterfactual.} Households derive utility from leisure and consumption. The latter is a bundle of formal and informal goods that are highly substitutable among themselves. Households accumulate two different capital stocks, which are market-specific, and no capital adjustment costs exist. They can also buy or sell one-period non-contingent bonds in foreign capital markets.

Regarding production, goods in the formal sector are produced by firms. In the informal sector people are self-employed. This is motivated by the fact, documented above, that self-employment is a good proxy for informality in Mexico. Both technologies have constant returns to scale and use capital and labor in production. The formal firm faces frictions that are intrinsic to formality. It pays taxes on the wage bill but enjoys higher productivity levels compared to the informal sector. The informal self-employed producer faces a lower productivity level but does not pay taxes. We are guided here by the empirical evidence of a large productivity gap between formal and informal activities in Mexico as well as other EMEs (see the evidence in the Appendix but also in Pagés, 2010; Powell, 2013; and Busso et al., 2012). Finally, the government taxes personal income (wages and capital rents) and the hiring of labor by formal firms to finance a stream of government purchases in formal
goods.

To keep the model as parsimonious as possible and draw intuition from it, we consider only one driving force, a labor-augmenting productivity shock in the production of formal goods. Crucially for our analysis, we allow for the possibility that this shock is not fully propagated from the formal to the informal market. We do so in order to make the shocks to both markets imperfectly correlated, thereby generating incentives for labor to reallocate across sectors in equilibrium. While clearly a reduced form, we think this assumption captures institutional or other types of barriers that prevent driving forces of business cycles from fully spreading across formal and informal markets uniformly. Possible examples include actual technological innovations that occur in formal markets but take time and resources to be acquired by informal ones; changes in financial conditions that only affect formal firms with access to capital markets; and changes in terms of trade or policy changes in the trade regulation that affect more directly formal firms that trade with the rest of the world.\footnote{To provide an example, consider the price of energy. In the case of Mexico, electricity for industrial use has become more expensive over time. This negatively affects the formal industrial sector. However, walking in areas of Mexico City which are well-known for informal activity, one can observe informal vendors connecting to the electricity network and not paying for the input. While anecdotal, we believe this simple example illustrates well what we want to capture in the model, that the usual business cycle drivers considered in the literature may affect formality and informality in asymmetric ways.}

On some dimensions, the assumptions we make are deliberately simple in order for the model to remain tractable. Some of them will be relaxed later on. For instance, we will consider various elasticities of substitution between formal and informal goods. In addition, we will later include an additional source of uncertainty by introducing shocks to mean-reverting, Hicks-neutral productivity. Finally, we will also consider capital adjustment costs.

Still, our model will continue to abstract from interesting issues. For example, we do not model unemployment. While we make this choice primarily for simplicity, it also reflects the fact that the unemployment rate is low in Mexico, the country to which we calibrate the model. Looking at INEGI data from the first quarter of 1987 to the second quarter of 2003 we find that the average rate is 3.4 percent, a very small number by international standards. We think that this small number reflects precisely what we are trying to model, which is the presence of informal employment.\footnote{Another reason why unemployment affects a small fraction of the active labor force is that there is no national unemployment insurance system in Mexico. There is an unemployment insurance system in Mexico...}另一个角度来说，我们没有考虑失业。我们做出这一选择主要是为了简化模型。这也反映了墨西哥的失业率低的事实，即我们对这一国家进行校准。根据INEGI的数据显示，1987年第一季度至2003年第二季度的平均失业率为3.4%，在国际标准下是一个非常小的数字。我们认为这个小数字反映了我们要建模的精确内容，即非正规就业的存在。\footnote{另一个原因是因为失业影响一小部分活跃劳动人口，是因为在墨西哥没有全国性的失业保险制度。在墨西哥...}
the presence of downward nominal wage rigidities in the formal sector, as these asymmetries would imply that, in a low-inflation environment, informal employment increases only during the contractionary phase as wages do not fall and laid-off workers transit from formal into informal jobs. But the same would not happen in the expansionary phase of the cycle, where wages are flexible. However, as documented above, we do not find evidence of this asymmetry in the data, given that the cyclicity of informal employment is independent of the phase of the cycle.

Last, we do not model non-linearities in the choice of hours in the formal sector as in Conesa et al. (2002), who assume indivisibility. Nor do we model the interaction between formal and informal activities as in Restrepo-Echavarría (2011), who assumes investment goods to be a composite of intermediate formal and informal investment goods. We believe that modelling these two aspects would further reinforce the relevance of informal labor markets, which is the main point of this work, and postpone it for future research.

3.1 The Representative Household

The representative household has a lifetime expected utility

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t u \left( C_t^A, h_t^A \right) \]  

where, as mentioned earlier, we use a GHH utility function for \( u(\cdot) \):

\[ u \left( C_t^A, h_t^A \right) = \frac{\left( C_t^A - \Gamma_{t-1} \left( h_t^A \right)^{\kappa} \right) \left( 1 - \sigma \right)}{1 - \sigma} - 1, \]

\( C_t^A \) is aggregate consumption modeled as a CES aggregator of the formal and informal consumption goods \( C_t^F \) and \( C_t^I \), respectively:

\[ C_t^A = \left( a(C_t^F)^e + (1 - a)(C_t^I)^e \right)^{1/e}, \]
where aggregate labor is denoted by $h^A$, and is defined as the sum of labor in the formal and informal sectors, denoted $h^F_t$ and $h^I_t$, respectively,

$$h^A_t = h^F_t + h^I_t.$$  

(3)

$\Gamma^F_{t-1}$ is (trending) labor productivity in the formal sector. We include it in the utility to achieve a balanced growth path. The discount factor $\beta$ takes values between 0 and 1. Parameters $\kappa > 1$, $\sigma > 0$, and $a \in [0, 1]$ determine the wage elasticity of labor supply, the intertemporal elasticity of substitution $(1/\sigma)$, and the weight of each consumption good in the CES aggregator, respectively. The elasticity of substitution between formal and informal goods is $1/(1 - \epsilon)$.

The sequential budget constraint of the household is

$$q_t D_{t+1} = C^F_t + p_t C^I_t + I^F_t + p_t I^I_t + D_t - (W_t h^F_t + r_t K^F_t) (1 - \tau^Y) - p_t Y^I_t.$$  

(4)

The numeraire is the price of the formal good. The relative price of the informal good is $p_t$. $D_{t+1}$ is the stock of non-contingent debt the household can issue at a price $q_t$ in world markets in $t$ to be redeemed in $t + 1$. $I^F_t$ and $I^I_t$ are the flow of investment goods in the formal and informal sectors, respectively. $W_t$ is the real wage per unit of labor in the formal sector. $r_t$ is the rental rate of the stock of capital, $K^F_t$, paid by the formal firm. $Y^I_t$ is the amount of income generated in the informal sector. $\tau^Y$ is the income tax rate applied to flows of income from the formal sector. Upper (lower) case variables (do not) trend in the balanced growth path.

The representative household has access to a technology in the informal sector given by

$$Y^I_t = (K^I_t)^{\alpha_I} (\Gamma^I_t h^I_t)^{1-\alpha_I}.$$  

(5)

where $K^I_t$ is the stock of capital used by the informal sector and $\Gamma^I_t$ is a labor-augmenting productivity process. The informal capital income share is $\alpha_I$. 

13
There are two capital laws of motion

\[ K_{t+1}^F = I_t^F + K_t^F (1 - \delta^F) \]  \hspace{1cm} (6)

\[ K_{t+1}^I = I_t^I + K_t^I (1 - \delta^I) \]  \hspace{1cm} (7)

where \(0 < \delta^j < 1\), for \(j = F, I\) is the depreciation rate in each of the two types of capital stocks.

The problem of the consumer is to maximize (1) subject to (2) - (7), together with a no-Ponzi condition.

3.2 The Representative Formal Firm

The representative firm that operates in the formal sector maximizes profits \(\Pi_t\) each period \(t\), defined as

\[ \Pi_t = Y_t^F - (1 + \tau^N) W_t h_t^F - r_t K_t^F \]  \hspace{1cm} (8)

where \(\tau^N\) is the tax on the wage bill. The technology faced by the formal sector is given by

\[ Y_t^F = (K_t^F)^\alpha (\Gamma_t^F h_t^F)^{1-\alpha} \]  \hspace{1cm} (9)

where \(\Gamma_t^F\) is a labor-augmenting productivity process. We allow for a different capital income share \(\alpha\) than in the informal sector. The static problem of the firm is to maximize (8) subject to (9).

3.3 Government

The government runs a balanced budget in every period:

\[ \tau^N W_t h_t^F + (W_t h_t^F + r_t K_t^F) \tau^Y = G_t \]  \hspace{1cm} (10)
where government spending $G_t$ equals total tax revenue. We assume public expenditure is entirely in formal goods.

### 3.4 Interest Rates

The interest rate on the debt issued in world capital markets is equal to the inverse of the price of the debt, which we assume to be equal to a constant interest rate and an interest premium. We assume that

$$
1/q_t = R + \tilde{\psi}\left(\tilde{D}_{t+1}/\Gamma_t^F\right)
$$

where $\tilde{\psi}\left(\tilde{D}_{t+1}/\Gamma_t^F\right)$ is a function representing an aggregate-debt elastic premium stemming from deviations from a long-run level of debt, and $R$ is the long-run interest rate that the small open economy faces in world capital markets. Following Schmitt-Grohé and Uribe (2003) we define $\tilde{\psi}(\cdot)$ as follows

$$
\tilde{\psi}\left(\tilde{D}_{t+1}/\Gamma_t^F\right) = \psi \left[\exp\left(\tilde{D}_{t+1}/\Gamma_t^F - d\right) - 1\right]
$$

with $\psi > 0$, and $d$ being the long-run (detrended) steady state level of debt. This premium will have no effect on the short-run fluctuations of the model because we will calibrate $\psi$ to be small following the literature. Last, note that, in equilibrium, aggregate debt $\tilde{D}_{t+1}$ and consumer’s debt $D_{t+1}$ coincide.

### 3.5 Market Clearing

The equations that describe market clearing in the goods market for both types of goods are:

$$
Y_t^F = C_t^F + I_t^F + G_t + D_t - q_t D_{t+1}
$$

$$
Y_t^I = C_t^I + I_t^I.
$$
Hence we define the trade balance share as
\[
ny_t = \frac{D_t - q_t D_{t+1}}{Y_t^F} = \frac{Y_t^F - C_t^F - I_t^F - G_t}{Y_t^F}.
\]

We also define total aggregate output as \( Y_t^A = Y_t^F + p_t Y_t^I \).

### 3.6 Productivity Processes

We assume a process for the growth factor of productivity in the formal sector
\[
\frac{\Gamma_t^F}{\Gamma_{t-1}^F} = g_t^F
\]
where \( g_t^F \) is assumed to follow an AR(1) process
\[
\ln \left( \frac{g_{t+1}^F}{\mu} \right) = \rho_g \ln \left( \frac{g_t^F}{\mu} \right) + \varepsilon_{t+1}^g
\]
with \( 0 < \rho_g < 1 \) and variance of the shock \( \sigma_g^2 > 0 \). We call \( \varepsilon_{t+1}^g \) a growth shock. Parameter \( \mu \) is the long-run growth factor of labor-augmenting productivity.

We assume that the growth shock in the formal sector relates to the informal sector as follows
\[
\frac{\Gamma_t^I}{\Gamma_{t-1}^I} = g_t^I
\]
\[
g_t^I = \left( g_{t-1}^I \right)^{1-\omega} \left( g_t^F \right)^\omega.
\]

Hence, we assume that growth shocks in the formal sector are passed through to the informal sector with an elasticity of \( \omega \), with \( 0 < \omega < 1 \). We do so in order to make the shocks to both markets imperfectly correlated, thereby generating incentives for labor to reallocate across sectors in equilibrium. This will allow us later on to match the countercyclicality of informal labor. More fundamentally, as argued above, it attempts to capture, albeit in reduced form, institutional and other types of barriers that prevent business cycle drivers from transmitting uniformly across formal and informal markets.
We can express the levels of labor-augmenting productivity in both sectors as a product of the growth shocks:

\[ \Gamma_t^F = \Gamma_0^F \prod_{j=1}^{t} g_j^F \]

\[ \Gamma_t^I = \Gamma_0^I \prod_{j=1}^{t} g_j^I. \]

We assume that the initial difference between \( \Gamma_0^F \) and \( \Gamma_0^I \) is pinned down by a parameter \( \gamma \) via \( \Gamma_0^I = \gamma \Gamma_0^F \). Parameter \( \gamma \), with \( 0 < \gamma < 1 \), governs the productivity gap between the two sectors in the steady state. In the next section we calibrate \( \gamma \) based on the productivity differentials across formal and informal sectors in Mexico. As shown in the Appendix, under a plausible calibration \( \Gamma_t^I < \Gamma_t^F \) for all \( t \). Last, we assume that in the long run

\[ \frac{\Gamma_t^F}{\Gamma_{t-1}^F} = \frac{\Gamma_t^I}{\Gamma_{t-1}^I} = \mu \]

which allows us to compute a balanced growth path equilibrium. Hence we are assuming that in the balanced growth path the informal sector grows at the same speed, although at a lower level, as the formal sector.\(^{16}\)

### 3.7 Competitive Equilibrium

Given initial conditions \( K_0^F, K_0^I, \) and \( D_0 \), and exogenous state-contingent sequences of growth shocks \( g_t^F \) in the formal sector, an equilibrium is a set of state-contingent allocations

\[ \{C_t^A, C_t^F, C_t^I, h_t^A, h_t^F, h_t^I, D_{t+1}, I_t^F, I_t^I, Y_t^A, Y_t^F, Y_t^I, G_t, K_{t+1}^F, K_{t+1}^I\} \]

and prices

\[ \{W_t, r_t, p_t\} \]

such that, given the laws of motion of shocks:

1. The allocations solve the consumer’s problem given prices and the laws of motion

\(^{16}\)In the Appendix we present data that shows that the share of informal establishments in economic censuses has remained stable.
for the capital stocks.

2. The allocations solve the formal firm’s problem given prices.
3. The government satisfies its budget constraint each period.
4. Markets clear for capital, labor and goods in the formal and informal sectors.

3.8 Solution of the Model

When solving the model, given that it exhibits a balanced growth path, we first detrend the system of equations. To find the stationary solution we apply a first-order Taylor approximation to the set of detrended equilibrium conditions. The log-linearized system is then solved using perturbation methods following Schmitt-Grohé and Uribe (2003). The Appendix presents the list of nonstationary and stationary equations as well as further technical details of the solution.

4 Results

This section presents the main results of our work. It is divided into three subsections. We first describe our strategy for calibrating the model to Mexican data. Then we present the model’s performance when accounting for the Mexican business cycle. Finally, we provide a careful analysis of the model’s main internal mechanism.

4.1 On the Calibration

A summary of the calibration of the model is reported in Table 3. The unit of time is assumed to be a quarter. Some parameters have a standard calibration. We set $\sigma$, which determines the intertemporal elasticity of substitution, $1/\sigma$, equal to 2. Parameter $\kappa$, determining the wage elasticity of labor supply under GHH preferences, is set to $\kappa = 1.6$, following Aguiar and Gopinath (2004).$^{17}$

We set $\delta^F = \delta^I = \delta$. The choice of an identical depreciation rate for the two types of capital is clearly arbitrary. However, the lack of data on capital or investment in the informal sector

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$^{17}$We refer to the working paper version of Aguiar and Gopinath (2007). We do this because only in this version were GHH preferences used.
sector forces us to make this choice. We set $\delta$ equal to the value used in Aguiar and Gopinath (2007), which is 5 percent. We calibrate $\mu$ to 1.006, the average growth factor of the Mexican economy in our dataset. The constant gross interest rate $R$ paid by the economy in world capital markets is calibrated with data on country interest rates for Mexico from Uribe and Yue (2006). Specifically, we set $R$ equal to 1.0145. With these last two parameters and the stationary Euler equation for debt evaluated at the steady state, $1 = R\beta\mu^{-\sigma}$, we get a value for the discount factor $\beta$ equal to 0.9976. We set the interest rate premium parameter $\psi$ to a small value as in Schmitt-Grohé and Uribe (2003), $\psi = 0.00001$.

[Table 3. Benchmark Model Calibration]

The absence of data on informal consumption prevents us from directly calibrating $e$, which governs the elasticity of substitution between formal and informal goods. We follow Restrepo-Echavarría (2011), who chooses a value of $e = 0.875$, implying an elasticity of 8. She argues that formal and informal goods are close substitutes and provides an illustration by describing the kind of goods sold in some well-known informal markets of large metropolitan areas in Latin America with attributes similar to those found in formal markets. Another argument in her work for using such a high elasticity of substitution is a comparison to the household production literature. Restrepo-Echavarría (2011) reports that previous research in that literature has used an elasticity of 5 between market and non-market activities. Arguably, there is a higher substitution between formal and informal goods relative to that between market and non-market activities. We will nonetheless consider robustness checks where we set this elasticity to arbitrarily higher and lower values.

We calibrate $\tau^N$ and $\tau^Y$ following the methodology in Mendoza et al. (1994). They measure the income tax rate as the ratio of aggregate individual income tax revenue to pre-tax household income. Pre-tax household income is defined as the sum of wages and salaries, property and entrepreneurial income, and the operating surplus of private unincorporated enterprises. We obtain annual data for these variables from 2003 to 2008 and calibrate $\tau^Y$ as the ratio of aggregate individual income tax revenue to the sum of wages and salaries,
and household income from capital.\footnote{Data on income tax come from the OECD, as in Mendoza et al. (1994). The data on factor payments come from INEGI’s website. Payments to labor are wages and compensations of the entire economy (“sueldos y salarios, economía total” in Spanish). Payments to capital is operational gross surplus of households (“excedente bruto de operacion, hogares” in Spanish).} Unlike Mendoza et al. (1994), we exclude the operating surplus of private unincorporated enterprises from the tax base because a part of this income comes from informality, and is hardly taxed. This follows from INEGI (2006), which reports that measured informal GDP in Mexico comes from the operating surplus of private unincorporated enterprises, or mixed income (“ingreso mixto”, in Spanish).\footnote{INEGI lists activities which are not included in the official measurement of the informal sector. These activities include, for example, drug trafficking.}

The average value of $\tau^Y$ is 0.07223. To calibrate $\tau^N$ we follow a similar strategy. Using INEGI’s website as our main source, in particular the section on national accounts, we put together data on tax collection regarding the payments that firms make as social contributions, and on tax base (wage income). We find an average value of $\tau^N = 0.1142$.

Regarding capital income shares, in the formal sector we set it equal to $\alpha = 0.35$, following García-Verdu (2005), who finds that the share in Mexico has a value similar to that in the United States. We set the capital share in the informal sector following Restrepo-Echavarría (2011) who uses $\alpha_I = 0.2$, citing evidence that informal production is less capital intensive.

When calibrating $\gamma$ we use the production functions of the two sectors. In the steady state the ratio of formal to informal total factor productivity levels is:

\[
\frac{TFP_F}{TFP_I} = \mu^{-\alpha + \alpha_I} \gamma^{1-\alpha_I},
\]

and, using the firm level measures of total factor productivity in Mexico from Busso et al. (2012), we set $\frac{TFP_F}{TFP_I} = 2.1901$. Therefore the calibrated value of $\gamma$ is 0.38. More details on how we perform this calculation as well as how the steady state is solved appear in our Appendix.\footnote{In the Appendix we also show that the dynamics of the model, summarized by the predicted second moments, do not change as we vary the TFP ratio across sectors. In those experiments we keep other parameters fixed such as $\omega$ and $\sigma_p$ whose calibration we explain below.}

The parameter $a$, determining the importance of formal goods in the consumption aggregator, is pinned down numerically when we solve the system of equations that define
the non-stochastic steady state. To solve this system we add two steady state ratios. One is the share of informal employment, \( h^I/(h^I + h^F) \). We set this equal to 0.3514, which is the average across the proxies of informality \( h^I_1, h^I_2 \) and \( h^I_4 \) in the first sample (see Section 2). To be consistent with our model, we exclude \( h^I_3 \) as it pertains to a kind of informality that we are not capturing in our model, that is one where informal labor can be hired by formal firms and an informal wage is determined. The other share that we manually fix is the debt to (formal) income ratio which we fix equal to 10 percent, the same value used by Aguiar and Gopinath (2007).

The remaining three parameters \( \{\omega, \rho_g, \sigma_g\} \) govern the dynamics around the steady state of the model and are therefore key for its business cycle properties. We calibrate the persistence of the shock to the value in Aguiar and Gopinath (2004), \( \rho_g = 0.72 \), who also embed a growth shock into an otherwise standard small open economy real business cycle model and estimate it using Mexican data. We calibrate \( \sigma_g \) and \( \omega \) so that the model’s implied volatility of formal output, \( \sigma (Y^F) \), and correlation of informal employment with formal output, \( \rho (h^I, Y^F) \), match their data counterparts, \( \sigma (Y^D) \) and \( \rho (h^I, Y^D) \), respectively.\(^{21}\)

We are hence implicitly taking a stance in making a one-to-one mapping between Mexican aggregate output, \( Y^D \), and the model’s level of formal production, \( Y^F \). While this is likely an extreme assumption, we are guided by the evidence on the poor measurement of informal activities in EMEs (see Schneider and Enste, 2000, and the discussion in the Appendix).

We will also present moments using aggregate output, \( Y^A \). More importantly, in one of the robustness cases considered later we relax the assumption of a one-to-one mapping between data and the model’s allocations to the formal sector. In the next section we discuss the calibrated values for \( \sigma_g \) and \( \omega \).

\(^{21}\)In principle, an alternative way to pin down \( \omega \) would be to estimate it from direct measures of productivity in formal and informal activities. In practice, this is impossible to achieve as it would require data for the capital stock and output in the informal sector. To the best of our knowledge there are annual data only for few years on informal production in Mexico, and no data on capital stocks. This makes it impossible to construct even raw series on TFP measures of the informal sector which one can use to estimate \( \omega \).
4.2 On the Model’s Performance

We now turn to the model’s performance in terms of replicating some of the business cycle properties of the Mexican data. We first explore performance in terms of its unconditional business cycle moments. Then we assess the model’s ability to reproduce the dynamics during large economic contractions. But before doing this we discuss the calibration results of parameters $\omega$ and $\sigma_g$ that are crucial for the model’s business cycle.

The results of the calibration of $\omega$ and $\sigma_g$ in our benchmark model are presented in the first column of the upper panel in Table 4. We calibrate $\sigma_g$ to be 0.2986 percent. To assess how small or large this volatility needs to be to match the empirical volatility of output, we construct an alternative SOE model without an informal sector and compute the required volatility of the growth shock to match the empirical target. In this alternative model $h^I = Y^I = 0$ (see the Appendix for further details on the solution and calibration of this model). The results of this experiment are reported in the last column of Table 4 and show that the required volatility of the shock is calibrated to be 0.3244 percent. Hence a first quantitative result is that of an amplification of shocks, in the sense that the inclusion of informal employment in an otherwise standard SOE model amplifies the effects of growth shocks. Or, to put it differently, one requires smaller shocks to reproduce the same volatility in output.

| Table 4. Business Cycle Moments: Data and Benchmark Model |

In terms of $\omega$, we calibrate it to be 0.64. Thus, seen through the lens of our model, the strong countercyclicality of informal labor implies a relatively low pass-through level of shocks across sectors, as roughly 36 percent of a shock in the formal sector is not contemporaneously propagated into the informal sector. Hence a second quantitative result is that the amplification of shocks in the model is linked to an imperfect propagation of shocks across sectors. This will be further studied in the next subsection, where we will provide more intuition about the model’s mechanism.

We turn now to the model’s performance in terms of its ability to account for other second moments that we do not match by construction in our calibration. The results are
reported in the lower panel of Table 4. We focus on the moments that are central to our investigation: labor, output and consumption. In the case of labor we report statistics for aggregate employment as well as formal and informal subcomponents, given that we have empirical proxies for all three. In the case of output and consumption we report the moments for aggregate and formal allocations. We will look at a wider set of moments in a further section on robustness. Finally, for comparison purposes, we also report in the last column the moments derived from the simplified model without informal employment. In this case, by construction, formal and aggregate variables are identical.

Overall, the model is capable of simultaneously matching the volatility of output and the countercyclicality of informal labor while performing relatively well in the other second moments considered. In particular, looking closely at the model’s performance in terms of output, a third quantitative result of interest is that total output is actually less volatile relative to formal output. Thus the amplification result is mostly channeled through formal output. This also highlights that measurement matters to the extent that the performance of the model changes if one looks at formal or aggregate variables. This issue will further be studied when we analyze the model’s mechanism.

A fourth result of interest is that, in line with what is observed empirically, the model is capable of generating high procyclicality of formal labor, \( \rho \left( h^F, Y^F \right) = 0.84 \), stronger than that of aggregate labor, \( \rho \left( h^A, Y^F \right) = 0.66 \). However, the model understates the volatility of both formal and informal labor. On the other hand, it manages to capture the fact that both formal and informal labor are more volatile than total labor.

A fifth result documented in Table 4 is related to the ratio of volatilities between formal consumption and output, \( \sigma \left( C^F \right) / \sigma \left( Y^F \right) \). The benchmark model generates a ratio of 1.18, which is considerably closer to the empirical counterpart of 1.31 relative to the model without informal labor, where this ratio is slightly below one, 0.98. Two comments are in order about this result. First, measurement is again crucial. Indeed, the results show that if one were to measure the ratio in terms of aggregate consumption and output, fully incorporating informality, then it would fall below one, \( \sigma \left( C^A \right) / \sigma \left( Y^A \right) = 0.96 \), hence consumption would not be more volatile than output. This result echoes a point stressed by Restrepo-Echavarria (2011), who argued that, if aggregate data in emerging economies correctly accounted for
the informal activity, then consumption would not be more volatile than output. Second, the relatively lower volatility of consumption in the simplified model without informality is a result of having no capital adjustment costs, as more resources are devoted to investment and less to consumption following a productivity shock.\textsuperscript{22}

The sixth and final result that we want to highlight from Table 4 is that the benchmark model is capable of reproducing the strong procyclicality of consumption and also the persistence of output, regardless of whether one measures formal or total aggregates. Relative to the former, the model implies that $\rho(Y^F, C^F) = 0.97$, and $\rho(Y^A, C^A) = 1.00$ which are in line with the value in the data, $\rho(Y^D, C^D) = 0.92$. In terms of the latter, the model generates a first order serial correlation of formal and aggregate output equal to $\rho(Y^F) = 0.94$, and $\rho(Y^A) = 0.96$, which are relatively higher but still close to their data counterpart, $\rho(Y^D) = 0.81$.

As documented in the empirical section, a distinctive property of informal labor in Mexico over the business cycle is that it is both countercyclical and lags the cycle. We now turn to assessing the model’s ability to reproduce this stylized fact. To be concrete, we compute the model’s unconditional cross correlation between leads and lags of informal employment and formal output, $\rho(Y^F_t, h^{I}_{t+j})$ for $j = -4, -3, \ldots, 4$. Recall that the model is calibrated so that the contemporaneous correlation between informal labor and (formal) output matches the one we observe in the data, $\rho(Y^F_t, h^{I}_t) = \rho(Y^D_t, h^{I,D}_t)$. But the calibration is silent about the leads and lags of this correlation. It is thus important to inquire if the model also matches the distinctive lagging property of this correlation. The results of this experiment are reported in Figure 3. A seventh quantitative result derived from this plot is that the model matches quite well the dynamic correlations between output and informal employment. It reproduces the lagging property of the latter, and its skewed U-shape is within the 95 percent confidence interval. We take this as validating evidence for the structure of the model.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{cross_correlation.png}
\caption{Cross Correlation between Output and Informal Employment: Mexican Data}
\end{figure}

\textsuperscript{22}This reconciles the results of the model without informal employment with other works where growth shocks alone can account for consumption volatility that is higher than that of output but which feature capital adjustment costs (Aguiar and Gopinath, 2007; Chang and Fernández, 2013).
A final dimension in which we assess the model’s performance is by conditioning the analysis to large economic recessions. Our dataset includes the two largest economic contractions that Mexico has undergone in its modern history (after the Great Depression) in the years 1994-1995, with the Tequila Crisis, and more recently in the Great Recession of 2008-2009. Our assessment is based on a simulation of the benchmark model where the growth shocks are drawn such that (formal) output dynamics exactly match the data for these two episodes. With that specific sequence of shocks we calculate the predicted time series of informal employment. There is obviously no guarantee that matching output perfectly implies a perfect match of informal employment in the data. Figure 4 displays HP-filtered output, together with three (filtered) time series of informal employment. The figure also displays informal employment as predicted by the model.

Overall, the model does a fair job at capturing the dynamics during crisis-type events. This is more so in the case of the Tequila Crisis, where the predictions of the model are very close to the data. There is an increase in informal employment during 1995. The comovement between the different empirical measures of informal employment and the predicted one is very strong. In the case of the Great Recession the model predicts a rise in informal employment in line with the data, but the increase occurs before we observe it empirically.

4.3 On the Mechanism

The previous subsection documented the good performance of the model in terms of conditional and unconditional moments. As argued above, although not exhaustively, key to

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23Shocks are such that the model matches the entire sequence of observed formal output.

24The drop in TFP in both simulations is also considerably close to the one observed in Mexican Solow residuals (results not shown but available upon request). Another salient feature is that the model’s implied fall in TFP is relatively lower than that in output. In the Tequila Crisis simulation, for instance, the fall in output, from peak to trough is about 12 percentage points, while that in TFP is about half of that. Similar magnitudes are obtained when using direct measures of the Mexican Solow residual.
this good performance is the conjunction between three different but related elements: the amplification role of informal employment, the imperfect propagation of shocks and the measurement of aggregate variables in the economy. This section sheds more light on the way in which these elements are linked and give rise to the mechanics of the model. The main message is that introducing informal employment into a standard SOE model amplifies the effects of productivity shocks. This comes from assuming that shocks to the formal sector do not fully propagate into the informal sector, i.e., their pass-through is incomplete. Additionally, the more informal economic activity is poorly measured in national accounts, the more variability will formal economic activity display.

The first instrument with which we inspect the mechanism of the model is via its impulse response functions (IRF). Figure 5 presents the IRF following a productivity shock for output (left column) and employment (right column), both formal and informal. It plots deviations of these variables relative to their steady state levels. The two upper panels consider the responses under the benchmark calibration of $\omega = 0.64$. The middle and lower panels display the same IRF but, ceteris paribus, considering arbitrarily higher and lower values of this parameter, respectively, relative to their benchmark calibration: a high value, $\omega^H = 0.9$; and a low value, $\omega^L = 0.2$.

[Figure 5. Impulse Response Functions: Benchmark Case]

Under the benchmark calibration for $\omega$, a productivity shock contemporaneously raises both formal and informal production levels. In the next period, however, the dynamics of the two follow paths with opposite directions. This can be traced back to what happens in formal and informal employment. As depicted by the upper right quadrant in Figure 5, following the shock both types of employment increase, but in the next few periods their paths take opposite directions. While formal employment further increases away from its steady state level, informal employment actually contracts. Because a shock in the formal sector will not be transmitted as strongly to the informal sector, there are changes in relative productivities across sectors. Hence, in equilibrium, there is a reallocation of labor into the formal sector and away from the informal sector. This explains the different paths of formal
and informal economic activity and is the heart of the model’s mechanism for delivering countercyclical informal employment.

Central to this mechanism is the level of $\omega$, the parameter that governs the extent to which productivity shocks in the formal sector are propagated into the informal sector, i.e., their pass-through level. A comparison between the upper panels in Figure 5 and the ones below illustrates this. The less shocks are propagated (the lower is $\omega$), the stronger will be the opposite movements in employment across the two sectors and, hence, the stronger will be the countercyclical properties of informal employment.

An important corollary of the comparison across plots in Figure 5 is that the response of formal economic output is further amplified as the propagation of shocks across sector decreases. This implies that the volatility of formal and informal production may be also affected by the mechanism that governs the propagation of shocks in the model. To dig deeper into this, Figure 6 reports the second moments that were reported in Table 4 across various levels of $\omega$, while keeping the size of the primitive shocks unchanged. The upper panel of this figure reports the volatility of formal output together with the relative volatilities of consumption and labor. The figure confirms that, as the propagation of shocks is smaller, the volatility of formal economic activity is further amplified, and so are the volatilities of formal and informal labor due to the reallocation of employment across sectors. Despite this increase in the variability of formal and informal employment, aggregate employment’s relative volatility, $\sigma (h^A) / \sigma (Y^F)$, actually decreases as shocks are less correlated across sectors. This is explained by the fact that the covariance between the two types of labor further decreases. The lower panel in Figure 6 illustrates this by showing how the correlation between (in)formal labor and formal output (decreases) increases as $\omega$ is closer to 0.

[Figure 6. Model’s Implied Second Moments at Various Levels of Pass-Through]

A final element of the mechanism behind the model’s performance that we want to study is the role of imperfect measurement of informal economic activity. Figure 7 again reports the IRF of labor and output to a productivity shock but complements them in several dimensions. First we also include the IRF of consumption, and in all three cases the total
allocations into their formal and informal components. Finally, we include the IRF derived from the model without an informal sector. There are two important results derived from Figure 7. The amplification of shocks in our benchmark model is extended to aggregate output ($Y^A$) as well given that the response is more pronounced relative to the case without an informal economy ($Y^N$). However, the opposite occurs for labor and consumption. Hence, to the extent that informal economic activity is poorly captured in national accounts, the relative volatility of the main macro variables, particularly consumption, will be higher.

[Figure 7. Impulse Response Functions: Benchmark Case and Model without Informal Labor]

5 Robustness

This section evaluates the robustness of the results of the benchmark model when four of the initial assumptions are relaxed. We first assess the performance of the model when the elasticity of substitution between formal and informal goods changes. In this case we also extend the number of second moments studied. Second, we allow for more sources of uncertainty by introducing stationary perturbations to total factor productivity, in addition to existing growth shocks. In this case we assume that the pass-through parameter is identical in both shocks. In the third and fourth extensions we introduce capital adjustment costs and allow for the pass-through to be shock-specific in the model with growth and stationary shocks. Finally, we relax the assumption made when calibrating the model of a one-to-one mapping between Mexican aggregate data and the model’s allocations in the formal sector. Instead, we assume that a fraction of informal output is counted in the model’s measure of output and other macroeconomic variables. Overall, the main results are robust to these modifications to the original benchmark case.
5.1 Elasticity of Substitution between Formal and Informal Goods

We compute the model using different values for the elasticity of substitution between formal and informal consumption goods. We find that a higher elasticity of substitution yields more reallocation of labor across sectors and higher output volatility. This experiment is relevant to the extent that there is uncertainty about the elasticity between formal and informal goods. The results appear in Table 5. Note that we have incorporated into the analysis four new moments: the relative volatility and cyclicality of investment and trade balance share. Also, for simplicity, all measures of output refer to formal income.

The column in Table 5 labeled "High Elasticity – benchmark calibration" shows the consequences of arbitrarily doubling the elasticity of substitution from the benchmark value of 8 to 16; for comparison, the previous column shows the results of the benchmark model. Given our calibration, this requires a virtually null change in $a$, from 0.6831 to 0.6849. We initially keep constant $\omega$ and $\sigma_g$. The first observation is that the predicted volatility of output $\sigma(Y^F)$ becomes higher. As mentioned earlier, the higher substitution across types of goods increases the volatility of output. Other predicted moments do not change much. An exception is the correlation between output and informal employment: it becomes more negative, almost doubling the one in the data. The following column, labeled “High Elasticity – new calibration” reflects what happens if we recalibrate $\omega$ to match the correlation between output and informal employment and $\sigma_g$ to match the volatility of output. Because of the higher volatility in output, matching our targets implies a slight decrease in the required volatility of the growth shock and a relatively higher $\omega$. Once we match our targets, there is little difference compared to the benchmark results.

[Table 5. Business Cycle Moments: Extension 1]

If we consider the opposite case and assume that the elasticity is lower, equal to 2, we see in the column titled “Low Elasticity – benchmark calibration” that the model predicts a much lower volatility of output. The predicted correlation between output and informal employment goes up, taking positive values. Once we recalibrate $\omega$ and $\sigma_g$ to match our calibration targets, as we do in the last column of the table, results become very similar

29
to the benchmark values. Overall, once calibration targets are chosen and matched, the model’s predicted statistics do not change much and are similar to those in the data. More importantly, these experiments show that the presence of an informal sector and the degree of substitution between goods affect the volatility of output.

A caveat highlighted in Table 5 is that the model underperforms in terms of investment dynamics. It overstates investment volatility and generates counterfactually low cyclical for it. As a consequence the trade balance share is not only much more volatile than in the data, but its countercyclicality is small. Further below we explore the extent to which this can be amended by the presence of capital adjustment costs.

We also look at predictions regarding dynamic correlations between output and informal employment. Figure 8 shows that changing parameter $e$, and therefore the elasticity of substitution, does not affect the success of the model replicating these correlations. The shape of the predicted correlation function, either with a low $e$, or with a high $e$, continues to be very close to the one in the data.

[Figure 8. Cross Correlation between Income and Informal Employment: Extensions]

5.2 Transient Shocks

We now extend our benchmark model to include stationary productivity shocks. The motivation for doing this is twofold. First, we first want to assess if the mechanism that we highlighted above depends on the structure of the productivity shocks. We assume a structure of shocks as in Aguiar and Gopinath (2007). We want to know if our results are robust to the inclusion of stationary shocks. We find that the answer is positive. Second, we also want to know if growth shocks are preponderant compared to transient shocks, as found by Aguiar and Gopinath (2007) when trying to account for business cycles in emerging economies. It is of interest to assess the extent to which their result continues to hold when informal labor is explicitly taken into account. Our findings show that it is robust.
Formally, we introduce transient productivity shocks by rewriting the production functions in the formal and informal sectors, (9) and (5), respectively, as follows:

\[ Y_t^F = a_t^F \left( K_t^F \right)^\alpha \left( \Gamma_t^F h_t^F \right)^{1-\alpha} \]

\[ Y_t^I = a_t^I \left( K_t^I \right)^\alpha_i \left( \Gamma_t^I h_t^I \right)^{1-\alpha_i} \]

where \( a_t^F \) is assumed to follow an AR(1) process

\[
\ln(a_{t+1}^F/a^F) = \rho_a (\ln a_t^F/a^F) + \varepsilon_{t+1},
\]

\( a^F \) is the steady state level; \( 0 < \rho_a < 1 \) and the variance of the shock is \( \sigma_a^2 > 0 \).

We assume that the relationship between \( a_t^F \) and \( a_t^I \) is similar to the one between \( g_t^F \) and \( g_t^I \) in our benchmark model. Concretely, the process for the informal transitory technology process is a function of its previous value and of the current transitory value of the process in the formal sector:

\[ a_t^I = (a_{t-1}^I)^{1-\omega} \left( \gamma a_t^F \right)^\omega \]

where \( \omega \) and \( \gamma \) play the same role as in our benchmark case. Hence we are assuming that the degree of pass-through of shocks, governed by \( \omega \), is identical between the two shocks. We will relax this assumption below. Also, \( \gamma \) governs the productivity gap between the two transient shocks in steady state. To see this note that, without uncertainty, \( a^I = \gamma a^F \). As in our benchmark model, we calibrate \( \gamma \) based upon the productivity differentials across formal and informal sectors in Mexico (details in the Appendix).

An important question that arises in this new setting is how to assign values to the parameters that govern the persistence, variance and pass-through of the two driving forces: \( \rho_a, \rho_g, \sigma_a, \sigma_g, \omega \). We cannot in principle apply the same calibration technique as in our benchmark model by matching only two moments, as this could be achieved with at least two different combinations of shocks.

We choose to calibrate the persistence of both shocks to the values estimated by Aguiar and Gopinath (2004), \( \rho_a = 0.94, \rho_g = 0.72 \). In addition, we calibrate \( \omega, \sigma_a \) and \( \sigma_g \) by minimizing the distance of a set of second moments with respect to the data. Formally, we
solve
\[
\min_{\{\omega, \sigma_a, \sigma_g\}} \left[ \frac{M(\omega, \sigma_a, \sigma_g) - M^d}{M^d} \right]' \Gamma \left[ \frac{M(\omega, \sigma_a, \sigma_g) - M^d}{M^d} \right]
\]
(12)

where \(M^d\) denotes the set of moments in the data that we choose, \(M(\omega, \sigma_a, \sigma_g)\) is its model counterpart, and \(\Gamma\) is a weighting matrix associated with that set. We set \(M^d\) to be a 22x1 vector containing the 14 moments in Table 6 plus the 8 cross-correlations between formal output and informal labor, \(\rho(Y_t^F, h_{t+j}^I)\), \(j = -4, \ldots, 4\), and \(\Gamma = I_{22}\). In the Appendix we document additional sensitivity results when alternative specifications for \(M^d\) and \(\Gamma\) are chosen. The calibrated values of \(\omega, \sigma_a\) and \(\sigma_g\) appear in Table 6 in the column labeled “Extension 2: Transient Shocks.”

[Table 6. Business Cycle Moments: Extensions 2 to 5]

In terms of calibrated values, we see that \(\omega\) remains virtually unchanged, compared to the benchmark. Parameter \(\sigma_g\) also has a very similar value. The calibration procedure yields a low value for \(\sigma_a\). In fact the ratio between volatilities, \(\sigma_a/\sigma_g = 0.03\), is an order of magnitude lower than the one reported in Aguiar and Gopinath (2004), which is 0.38. We use it to compute the random walk component (RWC) of productivity.\(^{26}\) It is 6.14, higher than the value of 5.33 in their work, indicating a pervasive importance of growth shocks.

Regarding predicted moments, note that in this experiment we do not match the correlation between output and informal employment by construction, as in the benchmark. The calibration procedure is trying to match simultaneously all moments that we are interested in. However, the predicted value (-0.37) is very similar to the one in the data (-0.45). Most of the moments that were not included in the calibration procedure have values similar to those in the benchmark experiment. Therefore, extending the model to include stationary shocks does not affect our results.

As in the previous experiment, after making this modification to the model, it continues to predict well the dynamic correlations between output and informal employment. Figure

\(^{25}\)We solved this minimization problem by postulating a grid for each of the three parameters and computing the minimum element among all possible elements of the grid, while checking that the minimum was not satisfied at the boundaries of the grid.

\(^{26}\)Following Aguiar and Gopinath (2007), the RWC is defined as
\[
\left[ \alpha^2 \sigma_g^2 / (1 - \rho_g)^2 \right] \Big/ \left\{ [2/(1 + \rho_a)] \sigma_a^2 + \left[ \alpha^2 \sigma_g^2 / (1 - \rho_g^2) \right] \right\}.
\]
8 shows that using this version of the model does not affect its success replicating these correlations. The predicted correlations are labelled "transient" and are similar to those in the data.

5.3 Capital Adjustment Costs

So far we have found that investment is relatively less procyclical and more volatile than in the data. In this modification we extend the model with transient and growth shocks by introducing a cost for adjusting the stock of capital in each of the two sectors. We follow the literature on business cycles in emerging economies (e.g., Aguiar and Gopinath, 2007; García-Cicco et al., 2010; Chang and Fernández, 2013) and assume the presence of convex costs from deviations in the growth of capital with respect to its balanced-growth path factor. We assume that the laws of motion for the capital stock in the formal and informal sectors are, respectively

\[
K_{t+1}^F = I_t^F + K_t^F (1 - \delta) - \frac{\phi}{2} \left( \frac{K_{t+1}^F}{K_t^F} - \mu \right)^2 K_t^F
\]

\[
K_{t+1}^I = I_t^I + K_t^I (1 - \delta) - \frac{\phi}{2} \left( \frac{K_{t+1}^I}{K_t^I} - \mu \right)^2 K_t^I
\]

where the cost function parameter \(\phi\) governs the degree to which these costs are present. We continue to assume that the depreciation rate is the same across sectors and that the adjustment cost parameter is the same. We make this choice for the same reasons that we choose identical depreciation rates: even though we know that the informal sector uses capital to produce, there is no data on investment by this sector that would allow us to calibrate a parameter for each capital. When assessing the robustness of our benchmark results to this extension, we included \(\phi\) in the set of parameters considered when solving (12).

The column labeled “Extension 3: Capital Adjustment Costs” in Table 6 presents the results. In this case \(M^d\) includes all 22 moments and \(\Gamma\) is the identity matrix. The calibrated value for \(\phi\) is 0.40. We find a reduction in investment volatility and an increase in

\[27\text{The Appendix contains the details of how the equilibrium conditions change under this modification.}\]
its procyclicality. This in turn contributes to closing the gap between model and data in regard to the dynamics of the trade balance. The correlation between output and informal employment becomes less negative but remains at a low value of $-0.24$, whereas the benchmark model was calibrated to replicate the one observed in the data, $-0.45$. Other predicted moments take values similar to those in the benchmark experiment.

This modification predicts a smoother dynamic correlation compared to previous experiments. Figure 8 shows a smoother correlation function. The predictions are labeled CAC (for capital adjustment costs). At the same time, the correlation’s shape continues to be very similar to that in the data, perhaps even more than in previous experiments.

5.4 Asymmetric Pass-Through

We now allow for a more general transmission of shocks from the formal to the informal sector in the model with two shocks and capital adjustment costs. In particular, we assume that the degree of propagation of shocks from the formal to the informal sector can be different across shocks. Formally, the processes for the transitory and growth technology shocks in the informal sector are

\[ a^I_t = (a^I_{t-1})^{1-\omega_a} (\gamma a^F_t)^{\omega_a} . \]
\[ g^I_t = (g^I_{t-1})^{1-\omega_g} (g^F_t)^{\omega_g} . \]

where $\omega_a$ and $\omega_g$ are between 0 and 1. We calibrate both parameters by including them in the set of parameters considered when solving (12).

The results of this experiment are reported in Table 6, under “Extension 4: Asymmetric pass-through.” The most interesting outcome is that the calibrated value for $\omega_a$ is 0.05, considerably lower than the value for $\omega_g$, 0.70. This reiterates previous results in the sense that, through the lens of our model, TFP shocks are imperfectly transmitted from one sector to the other. Simultaneously, most results are similar to the benchmark ones. The predicted correlation between informal employment and output is slightly less negative than in the previous extensions.

In terms of the dynamic correlations between output and informal employment, Figure 8 shows that this extension also replicates the lagging property of informal employment in
the data. In this case, the predicted pattern is smoother than the data, but both series are similar.

5.5 Imperfect Measurement of the Informal Sector

In this last extension, we relax the assumption that no dynamics of the informal economy are included in the measurement of the Mexican macro statistics that we use to calibrate the model. Instead, we assume that fraction \( \theta \) of the informal economy is registered. This parameter will be calibrated in such a way that, in the steady state, the registered informal economy as a share of the total economy is equal to what the Mexican statistical agency reports.

To implement this, we create four new \textit{measurement} equations that define output, consumption, investment, and the trade balance including a constant and equal fraction \( \theta \) of informal activity:

\[
Y_{t}^{IM} = Y_{t}^{F} + \theta p_{t}Y_{t}^{I}
\]
\[
C_{t}^{IM} = C_{t}^{F} + \theta p_{t}C_{t}^{I}
\]
\[
I_{t}^{IM} = I_{t}^{F} + \theta p_{t}I_{t}^{I}
\]
\[
x_{t}^{IM} = Y_{t}^{IM} - C_{t}^{IM} - I_{t}^{IM} - G_{t}
\]

where superscript "IM" stands for imperfectly measured. For example, \( Y_{t}^{IM} \) stands for the model’s counterpart of output assuming that all formal activity was included in national accounts but, due to the presence of imperfect measurement, only a fraction \( \theta \) of informal activity was included. Note that in our previous calibration strategies we have been implicitly assuming \( \theta = 0 \).

The steady state in this new extension is the same as the one found in the benchmark case. We use it to calibrate \( \theta \) as follows. Once \( Y^{F}, p, \) and \( Y^{I} \) are pinned down, we calibrate \( \theta \) so that:

\[
\frac{\theta p Y^{I}}{Y^{F} + \theta p Y^{I}} = \Omega
\]

where \( \Omega \) is the share of the total economy that is both informal and registered by the Mexican
statistical agency. We set \( \Omega = 0.124 \) to be consistent with the evidence that the share of informal value added in total value added was on average 12.4 percent between 1998 and 2003 in Mexico, according to INEGI (2006). This yields a calibrated value of \( \theta \) equal to 0.39.

We replicate the experiment in Extension 3 (the one with capital adjustment costs and two shocks) using the new four variables in the model when computing the model-based moments and comparing those to Mexican data. The results of this experiment are reported in Extension 5 in Table 6. Overall, the results are robust in terms of calibrated values and predicted moments. The predicted correlation between output and informal employment does become less negative, but it continues to replicate countercyclical informal employment. We also find that the model produces dynamic correlations in a pattern similar to that in the data. This can be seen in Figure 8, in which the prediction is labelled “Imp. Meas.” (for imperfect measurement of the informal sector).

6 Concluding Remarks

Despite the fact that informal labor markets are a distinctive characteristic of EMEs, they have received little attention from macroeconomists interested in business cycles. We argued that this is an important omission in the literature and have presented empirical and theoretical evidence to support our case. On the empirical side we built relatively long quarterly time series for various proxies of informal employment in Mexico, and documented how they are all strongly countercyclical. This in turn accounts for why aggregate employment is less procyclical and volatile relative to Canada, a developed economy.

From a theoretical perspective, we also support our case by documenting the non-trivial effects of introducing an informal labor market into an otherwise standard business cycle model of a small open economy. We introduce informality by assuming that workers optimally choose between a (taxable) formal wage or informal (tax-free) compensation from self-employment, and we calibrate the productivity and tax levels of both activities from Mexican firm-level and fiscal data, respectively. We also assume that growth shocks are imperfectly transmitted from the formal to the informal sector. We show that an imperfect transmission of shocks across sectors leads to a powerful amplification mechanism. The
model can additionally capture key stylized facts of the labor market such as the moderate procyclicality and volatility of total employment, although the predicted volatility is much smaller than in the data. Finally, it also highlights that proper measurement of informal economic activity matters to the extent that the performance of the model changes if one looks at formal or aggregate variables. The more informal economic activity is poorly measured in national accounts, the more variability will aggregate economic activity display.

We have only scratched the surface of the role of the informal sector in emerging market business cycles, and our work can be extended in many ways. The theoretical framework with which we model informal labor in a dynamic general equilibrium set-up is deliberately simple in order to gain intuition about the channels through which informal employment is related to the business cycle. But the cost of doing so is that we abstract from interesting issues. For instance, it would be worth exploring more linkages between formal and informal sectors, as the two have remained mostly isolated in our theoretical framework. Yet there may be important empirical linkages between the two (beyond a high degree of labor mobility). Another potentially interesting area of future research would be to dig deeper into the imperfect transmission of shocks across sectors that we find when taking our model to the data. In our model this is a reduced-form device to capture barriers that prevent shocks from spreading uniformly across labor markets. It is interesting to further explore the sources of these barriers.

References

[1]


Figures and Tables

Figure 1. Cross Correlation of Output and Informal Employment in Mexico


Note: The figure shows the cross-correlation between cyclical Mexican output in period $t$ and the cyclical component for each of the five definitions of informal labor in $t+j$, where $j$ is denoted in the horizontal axis. The data are quarterly, and the cyclical component was computed using a Hodrick-Prescott filter with a smoothing parameter of 1600. $h_1^I, h_2^I, h_3^I, h_4^I$ and $h_5^I$ refer, respectively, to employment in establishments with 1 to 5 employees; employment not covered by labor legislation benefits; employment from wage earners who do not receive benefits provided by labor legislation; self-employment; and employment in economic units not distinguished from households.
Figure 2. Rolling Correlations between Output and Formal/Informal Employment in Mexico

Note: The two lower (upper) panels show rolling window correlations between the cyclical component of output and self-employment $h^I_4$ (and non-self-employed workers ($h^F_4$)). Each correlation was computed using 21 quarters, from $t-10$ to $t+10$. The dotted lines represent 95% interval confidence bounds. Panels on the left refer to the first sample, while the two on the right refer to the second sample. Shaded areas represent recessions according to INEGI.
Figure 3. Cross Correlation between Output and Informal Employment: Mexican Data and Model

Note: This figure shows in the blue/squared line the model-based cross-correlation between cyclical (HP-filtered) informal employment in $t+j$, where $j$ is denoted in the horizontal axis, and output in $t$. The plot also shows in the solid line the empirical cross-correlation using Mexican data on output cycle and cyclical self-employment ($h_4^t$), and its 95% confidence interval (CI), from the first sample, 1987.Q1-2003.Q2 (see Figure 1, upper panel).
Figure 4. Large Recessions: Mexican Data and Model

Tequila Crisis: 1994 - 1995

The Great Recession: 2008 - 2009

Note: The upper/lower panels plot the (HP-filtered) cyclical components of output and all available definitions of informal employment in the two largest contractions in our dataset: the Tequila Crisis of 1994-1995 and the Great Recession of 2008-2009. Each panel also plots the simulated path of the model’s implied informal employment. Growth shocks are drawn such that (formal) output dynamics exactly match the data. See footnote in Figure 1 for definitions of each proxy for informality.
Figure 5. Impulse Response Functions: Benchmark Case

Note: The figure displays the impulse response functions (IRF) of the (HP-filtered) variables in the model. The units are percentage deviations from steady state. We use the calibration in Table 3 in the upper “Benchmark” case where $\omega$ is 0.64. In the middle and lower panels we set $\omega$ equal to 0.9 and 0.2, respectively. The IRF are generated following a one standard deviation shock.
Figure 6. Model’s Implied Second Moments at Various Levels of Pass-Through

Panel A. Standard Deviation

Panel B. Correlation with Cyclical Formal Income

**Note:** Panels A and B reproduce, respectively, the model-based relative standard deviation and cross-correlation with income across various values of ω, the parameter that governs the level of pass-through of TFP shocks from the formal to the informal sector in the model. Other parameters were calibrated according to Table 3.
Figure 7. Impulse Response Functions: Benchmark Case and Model without Informal Labor

Note: The figure displays the impulse response functions of the benchmark model’s (HP-filtered) output (y), labor (h) and consumption (c) in the upper, middle and lower panels, respectively. The units are percentage deviations from steady state. The IRF are generated following a one standard deviation shock. Superscripts F, I and A, refer to the formal, informal, and aggregate allocations in the benchmark model. Superscript N refers to the model without an informal sector.
Figure 8. Cross-Correlation between Income and Informal Labor: Extensions

Note: This figure shows the model-based cross-correlation between cyclical (HP-Filtered) informal employment in $t+j$, where $j$ is denoted in the horizontal axis and output in $t$, in each of the five extensions considered (“Ext.”). Extension 1 considers, separately, high and low elasticity of substitution between formal and informal goods. Extension 2 adds stationary productivity shocks. Extension 3 adds capital adjustment costs. Extension 4 allows for $\omega$ to be different across the two shocks. Extension 5 allows for a fraction of informal activity to be included in the macro aggregates. The plot also shows in the solid line the empirical cross-correlation using Mexican data on the output cycle and cyclical self-employment ($h_4^t$), and its 95% confidence interval (CI), from the first sample, 1987.Q1-2003.Q2 (see Figure 1, upper panel).
Table 1. Business Cycles: Mexico and Canada

<table>
<thead>
<tr>
<th>Moment</th>
<th>Canada</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>1.63 (.08)</td>
<td>2.32 (.20)</td>
</tr>
<tr>
<td>$\sigma(C) / \sigma(Y)$</td>
<td>0.76 (.04)</td>
<td>1.31 (.11)</td>
</tr>
<tr>
<td>$\sigma(I) / \sigma(Y)$</td>
<td>2.68 (.13)</td>
<td>3.79 (.18)</td>
</tr>
<tr>
<td>$\sigma(NX/Y) / \sigma(Y)$</td>
<td>0.53 (.04)</td>
<td>0.80 (.08)</td>
</tr>
<tr>
<td>$\rho(Y)$</td>
<td>0.93 (.04)</td>
<td>0.81 (0.10)</td>
</tr>
<tr>
<td>$\rho(C,Y)$</td>
<td>0.89 (.03)</td>
<td>0.92 (.01)</td>
</tr>
<tr>
<td>$\rho(I,Y)$</td>
<td>0.76 (.06)</td>
<td>0.95 (.01)</td>
</tr>
<tr>
<td>$\rho(NX/Y,Y)$</td>
<td>-0.26 (.13)</td>
<td>-0.80 (.05)</td>
</tr>
<tr>
<td>$\sigma(h) / \sigma(Y)$</td>
<td>0.74 (.03)</td>
<td>0.42 (.04)</td>
</tr>
<tr>
<td>$\rho(h,Y)$</td>
<td>0.88 (.03)</td>
<td>0.54 (.09)</td>
</tr>
</tbody>
</table>

Note: $\sigma(x)$ denotes the standard deviation of the cyclical component of $x$. $\rho(x,y)$ denotes the correlation between the cyclical components of $x$ and $y$. Variables $Y, C, I, NX, h$ stand for quarterly data on output, consumption, investment, net exports and total employment, respectively. Standard deviations are reported in percentages. All the variables were HP-filtered using a smoothing parameter of 1600. All variables in logarithms except for the ratio $NX/Y$. Standard errors are reported in parentheses. Moments and their standard errors were computed using GMM. Data from Canada cover from 1981.Q1 to 2002.Q1; Mexican data cover from 1987.Q1 to 2003.Q2.
Table 2. Second Moments of Formal and Informal Employment in Mexico

| Variable | $\sigma_i$ | $\sigma_i/\sigma_Y$ | $\rho(i,Y)$ | $\rho(h_{i,t},i)$ | $\rho(i,Y|\text{exp})$ | $\rho(i,Y|\text{cont})$ | $\rho_{\text{INEGI}}(i,Y|\text{exp})$ | $\rho_{\text{INEGI}}(i,Y|\text{cont})$ |
|----------|----------|----------------|-----------|----------------|----------------|----------------|----------------|----------------|
| $Y$      | 2.32     | 2.32/100       | 0.54      | 0.05           | 0.56           | 0.84           | 0.35           | 0.44           |
| $h$      | 0.98     | 0.98/200       | 0.54      | 0.05           | 0.56           | 0.84           | 0.35           | 0.44           |
| $h_1^F$  | 1.81     | 1.81/200       | 0.84      | -0.44          | 0.88           | 0.77           | 0.90           | 0.80           |
| $h_2^F$  | 2.14     | 2.14/200       | 0.84      | -0.53          | 0.87           | 0.72           | 0.92           | 0.81           |
| $h_3^F$  | 2.48     | 2.48/200       | 0.84      | -0.23          | 0.66           | 0.46           | 0.76           | 0.61           |
| $h_4^F$  | 1.34     | 1.34/200       | 0.74      | -0.38          | 0.78           | 0.63           | 0.81           | 0.63           |
| $h_5^F$  | 1.72     | 1.72/200       | 0.74      | -0.46          | 0.74           | -0.52          | -0.42          | -0.51          |
| $h_1'$   | 1.94     | 1.94/200       | 0.74      | -0.46          | 0.74           | -0.46          | -0.43          | -0.50          |
| $h_2'$   | 1.72     | 1.72/200       | 0.72      | -0.46          | 0.72           | -0.46          | -0.43          | -0.50          |
| $h_3'$   | 5.33     | 5.33/200       | 0.37      | -0.50          | 0.37           | -0.50          | -0.18          | -0.28          |
| $h_4'$   | 2.57     | 2.57/200       | 1.00      | -0.48          | 1.00           | -0.48          | -0.47          | -0.43          |
| $h_5'$   | 3.03     | 3.03/200       | 0.58      | 0.56           | 0.56           | 0.58           | 0.57           | 0.37           |

**First Sample:** 1987Q1 - 2003Q2

**Second Sample:** 2000Q2 - 2010Q4

Note: $\sigma_i$ refers to the standard deviation (in percentage) of the cyclical component of variable $i$. $\sigma_i/\sigma_Y$ refers to the ratio of volatility between the cyclical component of variable $i$ to the volatility of the cyclical component of output. $\rho(x,z)$ refers to the correlation between the cyclical components of variables $x$ and $z$. $\rho(i,Y|\text{exp})$ is the correlation between the cyclical components of $i$ and output, calculated for the periods between troughs and the peaks of the HP-cycle. Similarly, $\rho(i,Y|\text{cont})$ refers to the correlation coefficient calculated for the periods between peaks and troughs of the HP-cycle. $\rho_{\text{INEGI}}(i,Y|\text{exp})$ is similar to $\rho(x,Y|\text{exp})$, except that it takes the expansion definition from the official business cycle dates in Mexico. $\rho_{\text{INEGI}}(i,Y|\text{cont})$ keeps the same relation with $\rho(x,z|\text{cont})$. $Y$, $h$ refer to output and aggregate employment, respectively. See note in Figure 1 for definitions of each of the five informality measures, $h_1^F$, $h_2^F$, $h_3^F$, $h_4^F$ and $h_5^F$. Also $h_1'$, $h_2'$, $h_3'$, $h_4'$ and $h_5'$ refer to the residual when each of the five series on informal employment are subtracted from total employment. All variables were log HP-filtered using a smoothing parameter of 1600. Standard errors are shown in parentheses.
### Table 3. Benchmark Model Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>Sets elasticity of substitution $1/(1-e)$ between goods</td>
<td>0.875</td>
<td>Restrepo-Echavarría (2011)</td>
</tr>
<tr>
<td>$\tau^N$</td>
<td>Tax on wage bill</td>
<td>0.1142</td>
<td>Methodology by Mendoza et al. (1994) based on Mexican data</td>
</tr>
<tr>
<td>$\tau^Y$</td>
<td>Tax on income</td>
<td>0.0722</td>
<td>Methodology by Mendoza et al. (1994) based on Mexican data</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Formal capital share</td>
<td>0.35</td>
<td>García-Verdú (2005)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Sets intertemporal elasticity of substitution $1/\sigma$</td>
<td>2</td>
<td>Aguiar and Gopinath (2007)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.05</td>
<td>Aguiar and Gopinath (2007)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Wage elasticity of labor supply</td>
<td>1.6</td>
<td>Aguiar and Gopinath (2004)</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Interest rate debt elasticity</td>
<td>0.00001</td>
<td>Schmitt-Grohé and Uribe (2003)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Long run productivity growth factor</td>
<td>1.006</td>
<td>Data, Average GDP growth</td>
</tr>
<tr>
<td>$R$</td>
<td>External Interest Rate</td>
<td>1.0145</td>
<td>Data by Uribe and Yue (2006)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.9976</td>
<td>Satisfies steady state condition: $l=R\beta\mu^\omega$</td>
</tr>
<tr>
<td>$\alpha_I$</td>
<td>Informal capital share</td>
<td>0.20</td>
<td>Restrepo-Echavarría (2011)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Productivity gap between formal and informal technology</td>
<td>0.3749</td>
<td>Using micro data from Busso et al. (2012)</td>
</tr>
<tr>
<td>$a$</td>
<td>Share of formal goods in aggregate consumption</td>
<td>0.6831</td>
<td>Pinned down with a share of informal labor $h^f$ of 0.3514</td>
</tr>
<tr>
<td>$d$</td>
<td>Steady state debt to (formal) income</td>
<td>0.10</td>
<td>Aguiar and Gopinath (2007)</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence of growth shock</td>
<td>0.72</td>
<td>Aguiar and Gopinath (2004)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Pass-through of shocks from formal to informal sectors</td>
<td>0.64</td>
<td>Match correlation of informal labor with (formal) output</td>
</tr>
<tr>
<td>$100\sigma_g$</td>
<td>Standard deviation of growth shock</td>
<td>0.2986</td>
<td>Match (formal) output volatility</td>
</tr>
</tbody>
</table>

*Note:* The period is a quarter in our calibration.
Table 4. Business Cycle Moments: Data and Benchmark Model

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Model</th>
<th>Model with no Informal Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Selected Calibrated Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100σₙ</td>
<td>0.2986</td>
<td>0.3244</td>
</tr>
<tr>
<td>ω</td>
<td>0.64</td>
<td>n.d.</td>
</tr>
<tr>
<td><strong>Mexican Data</strong></td>
<td><strong>Models</strong></td>
<td></td>
</tr>
<tr>
<td>σ(Y_D)</td>
<td>2.32</td>
<td>2.32</td>
</tr>
<tr>
<td>σ(Y_I)</td>
<td>2.09</td>
<td>2.32</td>
</tr>
<tr>
<td>ρ(Y_D, h_D)</td>
<td>-0.45</td>
<td>-0.45</td>
</tr>
<tr>
<td>ρ(Y_I, h_I)</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
<td>ρ(Y_D, h_I)</td>
<td>0.66</td>
<td>0.77</td>
</tr>
<tr>
<td>σ(h_D)/σ(h_D)</td>
<td>0.97</td>
<td>0.43</td>
</tr>
<tr>
<td>σ(h_I)/σ(h_I)</td>
<td>0.39</td>
<td>0.65</td>
</tr>
<tr>
<td>σ(C_D)/σ(Y_D)</td>
<td>0.75</td>
<td>0.60</td>
</tr>
<tr>
<td>σ(C_I)/σ(Y_I)</td>
<td>3.04</td>
<td>2.73</td>
</tr>
<tr>
<td>σ(C_D)/σ(C_D)</td>
<td>1.99</td>
<td>1.99</td>
</tr>
<tr>
<td>σ(C_I)/σ(C_I)</td>
<td>1.31</td>
<td>1.18</td>
</tr>
<tr>
<td>ρ(Y_D, C_D)</td>
<td>0.92</td>
<td>0.96</td>
</tr>
<tr>
<td>ρ(Y_I, C_D)</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>ρ(Y_I, C_I)</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>ρ(Y_D)</td>
<td>0.81</td>
<td>0.94</td>
</tr>
<tr>
<td>ρ(Y_I)</td>
<td>0.96</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Note: σ(x) denotes the standard deviation of the cyclical component of x. ρ(x, y) denotes the correlation between the cyclical components of x and y. Variables Y, C, h stand for output, consumption, and employment, respectively. Superscript “D” denotes moments computed using Mexican data that cover from 1987.Q1 to 2003.Q2. Superscripts “F”, “I” and “A” denote moments computed using the model-based formal, informal and aggregate allocations. All variables were log HP-filtered using a smoothing parameter of 1600. “n.d.” means that a particular moment is not defined in the model without informal labor.
Table 5. Business Cycle Moments: Extension 1

<table>
<thead>
<tr>
<th>Second Moments</th>
<th>Models</th>
<th>Mexican Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark Model</th>
<th>High Elasticity – benchmark calibration of $\omega, \sigma_g$</th>
<th>High Elasticity – new calibration of $\omega, \sigma_g$</th>
<th>Low Elasticity – benchmark calibration of $\omega, \sigma_g$</th>
<th>Low Elasticity – new calibration of $\omega, \sigma_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100\sigma_g$</td>
<td>0.2986</td>
<td>0.2986</td>
<td>0.2868</td>
<td>0.2986</td>
<td>0.3670</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.64</td>
<td>0.64</td>
<td>0.81</td>
<td>0.64</td>
<td>0.24</td>
</tr>
<tr>
<td>$1/(1-e)$</td>
<td>8</td>
<td>16</td>
<td>16</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$a$</td>
<td>0.6831</td>
<td>0.6849</td>
<td>0.6849</td>
<td>0.6725</td>
<td>0.6725</td>
</tr>
</tbody>
</table>

Note: $\sigma(x)$ denotes the standard deviation of the cyclical component of $x$. $\rho(x, y)$ denotes the correlation between the cyclical components of $x$ and $y$. Variables $Y, C, I, TBY, h$ stand for output, consumption, investment, trade balance share and employment, respectively. Superscript “D” denotes moments computed using Mexican data that cover from 1987.Q1 to 2003.Q2. Superscripts “F”, “I” and “A” denote moments computed using the model-based formal, informal and aggregate allocations. All the variables were log HP-filtered using a smoothing parameter of 1600. “Benchmark Calibration” refers to the calibration in Table 3. “New Calibration” refers to the cases where $\omega$ and $\sigma_g$ are set to match the standard deviation of cyclical output and the cyclical correlation of informal employment.
Table 6. Business Cycle Moments: Extensions 2 to 5

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Model</th>
<th>Extension 2: Transient Shocks</th>
<th>Extension 3: Capital Adjustment Costs</th>
<th>Extension 4: Asymmetric pass-through</th>
<th>Extension 5: Imperfect Measurement*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calibrated Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.64</td>
<td>0.60</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>( \omega_a )</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \omega_g )</td>
<td>0.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varphi )</td>
<td>0</td>
<td>0</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>100( \sigma_a )</td>
<td>0.30</td>
<td>0.31</td>
<td>0.32</td>
<td>0.32</td>
<td>0.45</td>
</tr>
<tr>
<td>100( \sigma_g )</td>
<td>6.14</td>
<td>5.62</td>
<td>3.70</td>
<td>5.59</td>
<td>5.59</td>
</tr>
<tr>
<td></td>
<td>Second Moments</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexican Data</td>
<td>Models</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma(Y^d) )</td>
<td>2.32</td>
<td>2.32</td>
<td>2.32</td>
<td>1.80</td>
<td>2.34</td>
</tr>
<tr>
<td>( \sigma(Y^f) )</td>
<td>0.92</td>
<td>0.94</td>
<td>0.94</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>( \rho(Y^d, Y^f) )</td>
<td>-0.82</td>
<td>-0.14</td>
<td>-0.17</td>
<td>-0.68</td>
<td>-0.61</td>
</tr>
<tr>
<td>( \sigma(h^d)/\sigma(Y^d) )</td>
<td>0.42</td>
<td>0.18</td>
<td>0.18</td>
<td>0.21</td>
<td>0.28</td>
</tr>
<tr>
<td>( \rho(Y^d, h^d) )</td>
<td>0.54</td>
<td>0.66</td>
<td>0.65</td>
<td>0.06</td>
<td>0.71</td>
</tr>
<tr>
<td>( \sigma(h^d)/\sigma(Y^f) )</td>
<td>0.56</td>
<td>0.31</td>
<td>0.30</td>
<td>0.20</td>
<td>0.55</td>
</tr>
<tr>
<td>( \sigma(h^d)/\sigma(Y^f) )</td>
<td>1.07</td>
<td>0.28</td>
<td>0.28</td>
<td>0.44</td>
<td>0.30</td>
</tr>
<tr>
<td>( \rho(Y^d, h^f) )</td>
<td>0.74</td>
<td>0.84</td>
<td>0.81</td>
<td>0.30</td>
<td>0.65</td>
</tr>
<tr>
<td>( \rho(Y^d, h^f) )</td>
<td>-0.45</td>
<td>-0.45</td>
<td>-0.37</td>
<td>-0.24</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

Notes: See note in Table 5 for definitions. “RWC” refers to the random walk component. See text for details. (*) In Extension 5, model-based moments of output, consumption, investment and trade balance are computed using total aggregates that include a fraction \( \theta \) of informal economic activity. See text for details.