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Housing policies for Asia: A theoretical analysis by use of a demand and supply model

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ADB Working Paper Series

**Housing Policies for Asia:
A Theoretical Analysis by Use of a
Demand and Supply Model**

Naoyuki Yoshino,
Matthias Helble, and
Toshiaki Aizawa

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Abstract

The main objective of this paper is to give an overview of the most commonly used housing policies and to illustrate their economic impact. To facilitate the analysis, we first introduce a simple two-period housing demand model for owner-occupied houses and rental houses. We then add a standard stock-flow housing supply model. Using this modelling framework, we explain the qualitative effects of various housing policies on supply and demand. In the last section of the paper, we provide a quantitative estimation of the impact of each policy and assess its effectiveness using a simple analysis of cost effectiveness. We hope that the model's versatility makes it a simple tool for policymakers to better understand the economic consequences of various housing policies.

JEL Classification: R21, R28, R31, R38

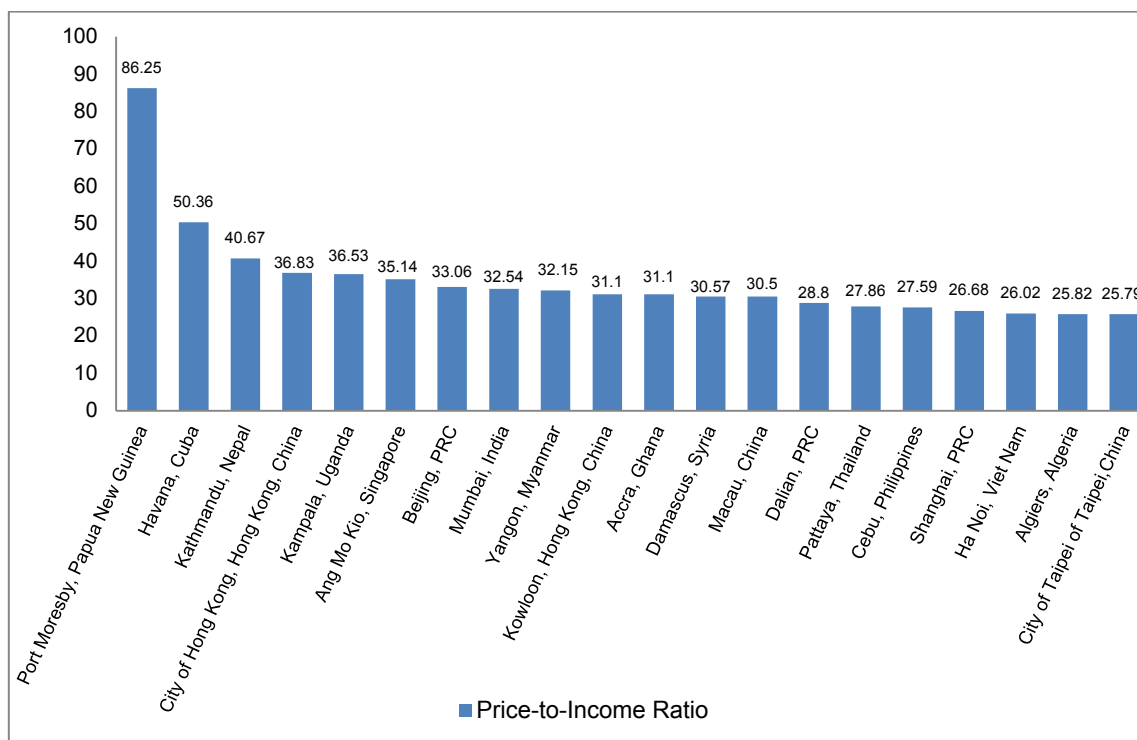
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1. INTRODUCTION

Access to affordable and adequate housing has become a key concern for many low- and middle-income households around the world. Rapid urbanization combined with population growth has led to a surge in housing prices in many urban areas, especially in developing countries. As a consequence, housing has become less affordable for many middle- and low-income households. A recent survey showed that the problem of affordability is particularly severe in Asia and the Pacific. Among the top 20 cities in the world ranked according to the price-to-income ratio, 16 are located in Asia and the Pacific (Figure 1).

Figure 1: Price-to-Income Ratio in 20 Most Expensive Cities in the World



PRC = People's Republic of China.

Source: Numbeo Property Prices Index 2015.

If prices in the housing market are getting beyond the reach of many dwellers, it can have several severe consequences. First, households are forced to live in dwellings that are too small or in bad condition. Second, households are forced to allocate a large share of their income for housing, neglecting other needs or taking substantial financial risks. Third, some low-income households are unable to pay for regular housing and end up in illegal dwellings or slums, which often lack basic services such as fresh water or sanitation.

To avoid these negative consequences, policymakers are attempting to intervene in the housing market using different policies. The outcome of these housing policies is not always well understood, even though the problem of providing enough affordable and adequate housing for the population is not new.¹ The first public housing project started in 1890 in the Old Nichol in London, a notorious slum at that time. Throughout the 20th century, governments around the world intervened in the housing markets and

¹ Public housing was already known in the Roman Empire.

attempted to provide adequate housing for their citizens. Government interventions were particularly bold after natural disasters or wars, when large parts of the population were lacking shelter. Governments then often massively built public houses for low income-groups.

Today, the main challenge in housing policies in Asia stems from rapid urbanization. According to the United Nations (UN) (2014), in Asia only 48% of the population is currently living in urban areas, which is substantially lower than in all other regions of the world (North America 82%, Latin America and the Caribbean 80%, as well as Europe 73%), except for Africa (40%). Given the relatively low share of urbanized people in Asia, the UN expects that in Asia urbanization will rapidly increase and have reached 64% (3.313 billion) by 2050. UN–HABITAT (2011) estimates that this growth means that in Asia every day urban areas will need to accommodate 120,000 new residents, which equals a daily demand of around 20,000 housing units. It is a well-known fact from urban economics, that the supply of housing is very inelastic (see for example, Ozanne and Struyk 1978). As a rule of thumb, urban economists assume that new construction in a given year is only 2% to 3% of the total housing stock (O’Sullivan 1996). Given the strong increase in demand for housing and the small elasticity of supply, it is no surprise that the affordability of housing has become a central concern in many economies in Asia and the Pacific. Many governments throughout the region have started to intervene in the housing market, but in most places access to affordable and adequate housing remains elusive.

The objective of this paper is to develop a simple theoretical framework that allows us to better understand and assess various housing policies. The proposed model allows for the illustration of supply- as well as demand-side policies. Moreover, it covers both the case of home ownership and that of renters. Our main intention is to provide a simple tool for policymakers to enable them to better understand the implications of various policies and compare them. It is meant to serve as a toolkit to better manage housing markets and facilitate progress towards increasing affordable housing in Asia and the Pacific.

The paper is structured as follows: Section 2 reviews the related economic literature. Section 3 gives an overview of the housing policies we discuss in this paper. Section 4 introduces theoretical frameworks for housing and Section 5 applies them and analyzes the policies. Section 6 provides quantitative estimations of the impact of each policy. Section 7 draws a conclusion and provides a further discussion.

2. RELATED LITERATURE

The purpose of this paper is to study the qualitative and quantitative effects of housing policies. A large number of textbooks in urban economics have already illustrated the qualitative effects of housing policies (e.g., Brueckner 2011; O’Sullivan 1996; Harvey 2000; McDonald 1997). Typically, the authors use simple pedagogical diagrams to analyze different housing policies. However, the diagrams used for illustration are rarely based on derivation from utility maximization. Instead, the authors draw on the large body of economic literature on the topic of demand for housing.

In this literature, the demand for housing is typically derived from the households’ utility maximization. As Megbolugbe et al. (1991) pointed out, housing as a commodity can be distinguished by some principal features such as its durability, its heterogeneity, and its spatial immobility. Particularly, the durability of housing makes it different from other consumption goods. Given these features, numerous attempts and modifications have been made to better describe housing demand. According to Rothenberg et al. (1991),

housing demand analysis can be categorized into the following four types: (1) demand for housing services or housing units, (2) demand for housing attributes such as the distance to the central business district and amenities in the neighborhood, (3) tenure choice, and (4) spatial allocation of households. Each of these categories requires very different modelling and estimation strategies (Zabel 2004).

Research on housing services or units studies the demand for housing, assuming housing units to be homogenous (e.g., Ghavari 1986; Malpezzi et al. 1985; Kau and Keenan 1980). Research on the demand for housing attributes often develops a single period model and adopts a hedonic approach for estimation (e.g., Rosen 1974; Bajic 1984; Cheshire and Sheppard 1998). One of the disadvantages of the single period model lies in the difficulties of incorporating the durable aspect of housing. In contrast, research on the tenure choice tends to treat housing consumption as a discrete choice (rent/own) in multiperiod models, but the discreteness makes it difficult to analyze a policy effect quantitatively. For example, a recent study by Attanasio et al. (2012) studied the tenure choice and the change in the home ownership rate by modelling the demand for housing over the life cycle, treating housing consumption in a discontinuous fashion. Research on spatial allocation studies the choice as to where people dwell and discusses segregation of residence (e.g., McFadden 1978; Rapaport 1997; McDonald 1997).

The research objective of this paper falls into the first category, as it considers housing demand as a continuous quantity. However, in contrast to most papers in this category, we develop a two-period model that focuses on the finance of housing. Our paper mainly draws from two earlier contributions to the literature on housing demand: First, we suggest a utility function similar to Zabel (2004). Zabel (2004) developed a single period model, assuming that individual utility depends on housing and non-housing composite consumption and individual demographic characteristics. The model gives a very simple and intuitive presentation of housing demand, but it is only a one-period model and is unable, therefore, to illustrate important cases, such as the effect of mortgage on the household's budget constraint.

Second, we model housing finance similar to Gahvari (1986). Gahvari (1986) adopted a multiperiod model and implicitly incorporated housing finance in the budget constraint. Optimal housing consumption in each period is derived in a way that an individual agent maximizes his/her utility in each period. The model developed by Gahvari (1986) is based on the idea of the consumer choice model and elegantly proposes a substitution relationship between housing and all-encompassing consumption goods. However, the model ignores the aspect of durability of housing. In the model, the individual agent is allowed to choose the optimal housing consumption level in each period without being affected by the choice made in the previous period. In our model, we explicitly incorporate the aspect of durability following earlier contributions to the literature, for example by Fallis (1983).²

In summary, this paper adopts the modelling framework proposed by Zabel (2004) and extends it to a two-period decision-making model incorporating housing finance aspects similar to Gahvari (1986). The continuity of the housing units and the durability of housing are explicitly assumed in our model for owner-occupied housing.

We add to the existing literature in two main ways: First, we introduce a new theoretical model that is able to accommodate most of the commonly used housing policies. It

² Fallis (1983) introduced a dynamic model for the demand for general durable goods. In the same paper, the author presented a static single period model to explain housing demand and tenure choice.

thereby constitutes a formidable tool for assessing and comparing different housing policies. And second, we go beyond a simple graphical illustration of the model, and provide numerical simulation results.

3. OVERVIEW OF HOUSING POLICIES

Various types of housing policies have been implemented throughout the world since World War II. In this paper, we suggest categorizing all policies into four dimensions (see Table 1): demand- and supply-side policies as well as policies to promote home ownership and policies to support renters.

Table 1: Overview of Housing Policies along Four Dimensions

	Owner-occupied Housing Market	Rental Housing Market
Demand Side	<ul style="list-style-type: none"> • Cash benefits for housing • Housing subsidies • Mortgage interest rate reduction • Mortgage interest deduction from income tax • Upgrading quality standards • Property tax on housing purchases • Loan-to-value (LTV) and debt-to-income (DTI) ratio regulations • Restriction of new purchases 	<ul style="list-style-type: none"> • Fixed amount cash subsidies • Rental subsidies • Rent certificates • Housing vouchers • Slum prevention • Rent controls
Supply Side	<ul style="list-style-type: none"> • Public housing • Subsidies to suppliers • Upgrading quality standards 	<ul style="list-style-type: none"> • Public housing • Subsidy to suppliers • Slum upgrading

Source: Authors.

Demand-side policies encompass all those policies directly targeted at demand. The only exception is the policy of upgrading quality standards. This policy directly affects the demand- and supply-side. On the demand-side, we have first listed those policies that increase demand for housing and then those that are implemented to curb demand. (The order of the policies in Table 1 is made according to the order of their introduction later in the paper.)

Supply-side policies were implemented by many governments after World War II. The destruction caused by the war and the rapid growth in population made it necessary to quickly address the shortage of housing. In other emerging countries, rapid economic growth also caused shortages in housing and triggered supply-side interventions by the government. When the economy is in a more mature state, demand-side policies typically become the preferred policy instrument.

A good example of this shift from supply- to demand-side policies is the Republic of Korea. The Republic of Korea first adopted a supply-side policy called Two Million New Housing Construction Project to deal with a severe housing shortage in 1989. After

overcoming the housing shortage and price hikes, housing policy in the Republic of Korea shifted toward demand-side approaches, such as an interest rate deregulation, the introduction of a reverse mortgage loan, and a cash subsidy. Singapore provides another example of this shift. Under Prime Minister Lee Kuan Yew (1959–1990), most housing policies were supply-side-oriented with an objective to increase home ownership rates. In the 1990s, the government shifted to demand-side subsidies, which were considered more cost-effective than supply-side subsidies. However, supply-side policies are still playing vital roles in many economies in Asia and the Pacific. For example, in India supply-side policies are considered to be as important as demand-side policies for providing affordable housing.

4. A THEORETICAL FRAMEWORK OF HOUSING POLICIES

We use the consumer choice model based on standard tools of microeconomics. We assume there are only two types of goods: housing (H) and other consumption goods (C). The household allocates its budget to the two goods. Our two-period housing demand model for owner-occupied housing allows us to analyze the interplay between housing demand and supply. In the second part of this section, we also look into the rental house market, applying a similar two-period model by slightly changing some assumptions. These two demand models (homeowners and renters) are used to analyze the effects of major housing policies. An advantage of setting up a theoretical model is that it enables us to evaluate the policy effects graphically as well as numerically. In the next section, we will show the numerical effects of several policies that are based on our two-period housing demand model.

As for the supply of housing, we employ the idea of the stock and flow model (Brueckner 2011; Pirounakis 2013; DiPasquale and Wheaton 1996). The stock is independent of the price, but the flow is dependent on it. The stock is inelastic to the price and the stock level changes gradually via the flow market. As the stock and flow model is not derived from the profit maximization of a representative producer, we focus only on the qualitative effect in analyzing supply-side policies.

4.1 Demand for Housing

4.1.1 Two-period Owner-occupied House Model

We assume a representative household that lives only for two periods and seeks to maximize its utility:

$$u(C_1, H_1) + \beta u(C_2, H_2), \quad \beta \in (0,1) \quad (1)$$

where

$$u(C_t, H_t) = \frac{C_t^{1-\theta}}{1-\theta} + b \frac{H_t^{1-\omega}}{1-\omega}, \quad \theta \neq 1, \omega \neq 1 \quad (2)$$

$$u(C_t, H_t) = \ln(C_t) + b \ln(H_t), \quad \theta = \omega = 1 \quad (3)$$

$$H_2 = (1 - \delta)H_1 \quad (4)$$

C_t and H_t represent the quantity of consumption goods consumed and housing units owned by the household in period t . Every housing unit is assumed to be homogenous and of the same quality. Differences, for example in location, tranquility, and range of amenities in neighborhoods, are not considered in the model. In other words, we treat all units equally and differences in characteristics are assumed not to affect demand.

Houses as durable goods affect utility in both periods, but they are subject to depreciation due to wear and tear. The dilapidation is assumed to be at the rate of δ . b shows the weight for housing in the utility function and β is the discount factor for future utility.

We assume that the household buys a new house with the aid of a loan, L , in period 1 and pays off the loan and its interest in period 2. The interest rate of the loan is r . Y_1 and Y_2 represent the household's income in each period. G_1 and G_2 stand for the cash subsidies from the government in period 1 and 2. t_h and t_y stand for the property tax rate and income tax rate, respectively. The budget constraint of the households can thus be written as:

$$C_1 + (1 + t_h)P_h H_1 = (1 - t_y)Y_1 + L + G_1 \quad (5)$$

$$C_2 + (1 + r)L = (1 - t_y)Y_2 + G_2 \quad (6)$$

$$Y_2 = (1 + g)Y_1 \quad (7)$$

where P_h is the price of a housing unit and the price of consumption goods is set to unity as numeraire. g denotes the economic growth rate, which is assumed to be determined exogenously in the model. From (5) to (7) we obtain the following inter-temporal budget constraint:

$$C_1 + \frac{C_2}{1+r} + (1 + t_h)P_h H_1 = (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \quad (8)$$

When we assume $\theta = \omega = 1$, the optimal H_1^* , H_2^* , C_1^* , C_2^* are the bundles that maximize $\ln(C_1) + b \ln(H_1) + \beta\{\ln(C_2) + b \ln((1 - \delta)H_1)\}$

$$(9)$$

subject to the inter-temporal budget constraint.

The optimal levels of consumption and housing units are expressed as functions of income and the housing price (the derivations can be found in the Appendix):

$$H_1^* = \frac{b}{(1+t_h)(1+b)P_h} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (10)$$

$$H_2^* = (1 - \delta)H_1^* \quad (11)$$

$$C_1^* = \frac{1}{(1+\beta)(1+b)} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (12)$$

$$C_2^* = \frac{\beta(1+r)}{(1+\beta)(1+b)} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (13)$$

The optimal level of housing loan, L^* , and Debt-to-Income (DTI) ratio and Loan-to-Value (LTV) ratio can be expressed as follows:

$$L^* = C_1^* + (1 + t_h)P_h H_1^* - (1 - t_y)Y_1 - G_1 \quad (14)$$

$$DTI = \frac{L^*}{Y_1} \quad (15)$$

$$LTV = \frac{L^*}{P_h H_1^*} \quad (16)$$

4.1.2 Two-period Rental Housing Model

Similar to the owner-occupied house model, we assume that a representative household that lives only for two periods seeks to maximize the following utility function:

$$u(C_1, R_1) + \beta u(C_2, R_2), \quad \beta \in (0,1) \quad (17)$$

where

$$u(C_t, R_t) = \frac{C_t^{1-\theta}}{1-\theta} + b \frac{R_t^{1-\omega}}{1-\omega}, \quad \theta \neq 1, \omega \neq 1 \quad (18)$$

$$u(C_t, R_t) = \ln(C_t) + b \ln(R_t), \quad \theta = \omega = 1 \quad (19)$$

C_t represents the quantity of consumption products consumed and R_t denotes the housing units rented by the household in period t .

The household can choose the quantity of housing units in each period. It can substitute its consumption inter-temporally through savings, for which the interest rate is r .

$$C_1 + P_r R_1 + S = (1 - t_y)Y_1 + G_1 \quad (20)$$

$$C_2 + P_r R_2 = (1 - t_y)Y_2 + (1 + r)S + G_2 \quad (21)$$

$$Y_2 = (1 + g)Y_1 \quad (22)$$

where P_r , t_y , and g denote the price for rented house per unit, the income tax rate, and the exogenous economic growth rate. G_t is a subsidy from the government in period t .

From (20) to (22), the inter-temporal budget constraint for rental houses takes the form:

$$C_1 + \frac{C_2}{1+r} + P_r R_1 + \frac{P_r R_2}{1+r} = (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \quad (23)$$

The optimal level of rental housing units and the consumption goods, R_1^* , R_2^* , C_1^* , C_2^* , can be obtained by maximizing the whole life utility subject to the inter-temporal budget constraint.

When we assume that $\theta = \omega = 1$, the optimal levels of rental housing units and consumption goods become:

$$R_1^* = \frac{b}{(1+\beta)(b+1)P_r} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (24)$$

$$R_2^* = \frac{b\beta(1+r)}{(1+\beta)(b+1)P_r} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (25)$$

$$C_1^* = \frac{1}{(1+\beta)(b+1)} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (26)$$

$$C_2^* = \frac{\beta(1+r)}{(1+\beta)(b+1)} \left\{ (1 - t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (27)$$

4.2 Supply of Housing

4.2.1 A Stock-flow Housing Model

Following Pirounakis (2013) and DiPasquale and Wheaton (1996), we assume that the supply of housing can be decomposed into a stock side and a flow side—the housing

stock from the previous period and the new constructions. In the stock-flow model, the current period stock level, H_t , is the last period's housing stock, H_{t-1} , plus the current period's new construction, ΔH_t , minus the last period's stock, which needs to be demolished, δH_{t-1} . We assume that the new constructions depend on the current price for housing and that it is also affected by exogenous conditions, such as policy changes, v_t .

$$H_t^s = H_{t-1}^s + \Delta H_t^s - \delta H_{t-1}^s \tag{28}$$

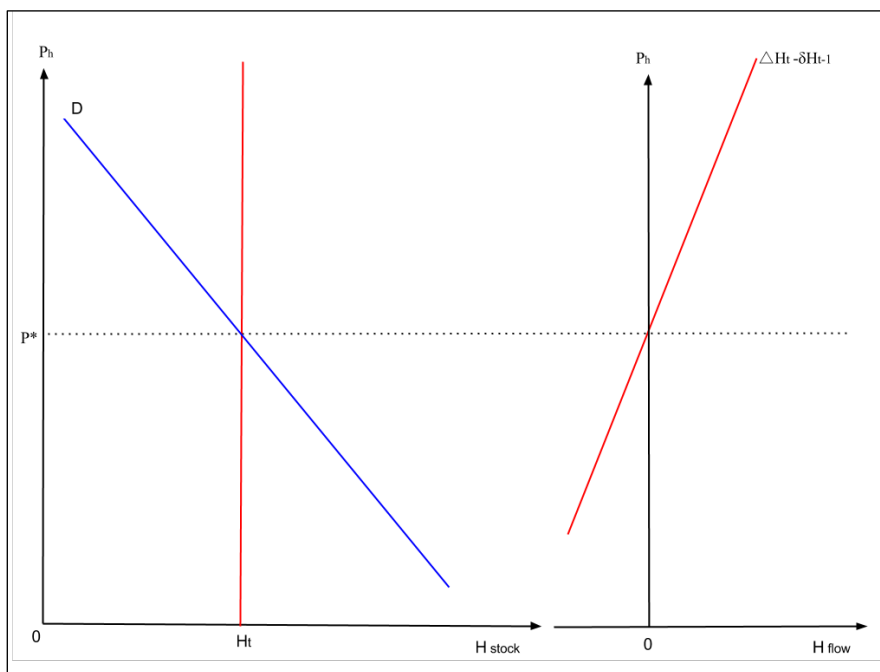
$$\Delta H_t^s = aP_{h_t} + b + v_t, \quad a > 0 \tag{29}$$

From equation (28) and (29), it is apparent that

$$H_t^s = aP_{h_t} + (1 - \delta)H_{t-1}^s + b + v_t \tag{30}$$

Figure 2 shows the stock and flow market of housing. The existing stock of housing is illustrated in the left panel and the right panel shows whether and how much the housing stock changes given a certain price. In every period new housing units are constructed, but at the same time decrepit houses are demolished. As long as construction exceeds demolitions, the stock increases over time. If the opposite is true, the stock decreases. The equilibrium price is determined at the point where demand intersects with the housing stock.

Figure 2: Stock-flow Supply Model



Source: Authors.

The stock-flow model tells us that, in the short run, the housing price adjusts quickly to equalize demand to the existing units. As well as the price, the housing stock is adjusted to help the price level go back to the original equilibrium level. However, the adjustment of the housing stock occurs only slowly over time and often with substantive lags. When the stock does not change, $H_t^s = H_{t-1}^s$, the stock is said to be in a steady state. Under the steady state equilibrium price, P_h^* , new construction and demolition offset each other, $\Delta H_t^s = \delta H_{t-1}^s$. If for some reason the price is higher than the steady state equilibrium price, then the new constructions outnumber the units depreciated and the stock grows gradually. If the price is below P_h^* , then the flow of housing units becomes negative and the housing stock decreases continuously. The supply of

housing in the steady state can be derived simply by substituting $H_t^{S*} = H_{t-1}^{S*}$ into equation (30).

$$H_t^{S*} = aP_h^* + (1 - \delta)H_t^{S*} + b + v_t \tag{31}$$

Solving for H_t^* , we obtain the steady state of housing supply.

$$H_t^{S*} = \frac{aP_h^* + b + v_t}{\delta} \tag{32}$$

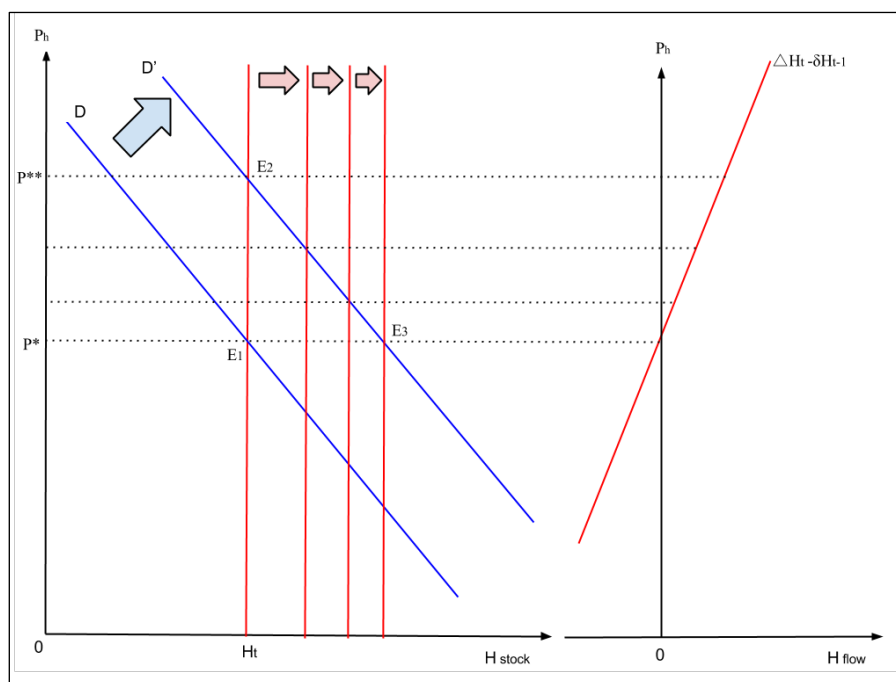
where P_h^* is a steady state equilibrium price.

4.2.2 Demand Shock and Supply Shift

Ignoring large exogenous shocks (such as natural disasters), the supply of housing can be assumed to be fixed in the short run for any price level, which translates into a vertical short-run supply curve, inelastic to price changes. On the other hand, in the intermediate run and long run, the supply responds to a price change via changes in the flow. The higher the price becomes, the more units are constructed and the stock of housing increases.

Suppose the demand for housing goes up because of an increase in the population—the increase shifts the demand curve upward and pushes up the price in the short run to P_h^{**} (Figure 3). Then the higher price encourages new construction and therefore the supply also gradually goes up because newly constructed housing outnumbers abandoned housing. The short-run equilibrium price P_h^{**} and the newly constructed units become smaller and smaller in every period. The shift continues until the short-run equilibrium price reaches the original steady state equilibrium level, P_h^* . We need to stress that under normal circumstances the supply of housing adjusts only gradually, as the building of new houses (or the destruction of old ones) cannot be done immediately. However, exogenous shocks—such as wars, natural disasters, or large policy interventions—might directly and suddenly affect the supply.

Figure 3: The Effect of Demand Shift on Supply



Source: Authors.

5. APPLICATIONS TO HOUSING POLICIES

This section analyzes the qualitative effect of each policy in Table 1. We pay attention firstly to the owner-occupied house market and then move to policies for the rental house market. We make use of indifference curves and a budget constraint curve for the analysis. The representative household chooses the point which gives the highest utility of all the feasible points. An indifference curve is a contour line realizing the same utility, so any point on the same indifference curve gives the same level of utility. Indifference curves located in the north-west give higher levels of utility, but the household can only choose the point in the feasible area, which is the south-west area divided by the budget constraint line.

5.1 Owner-occupied Housing Market

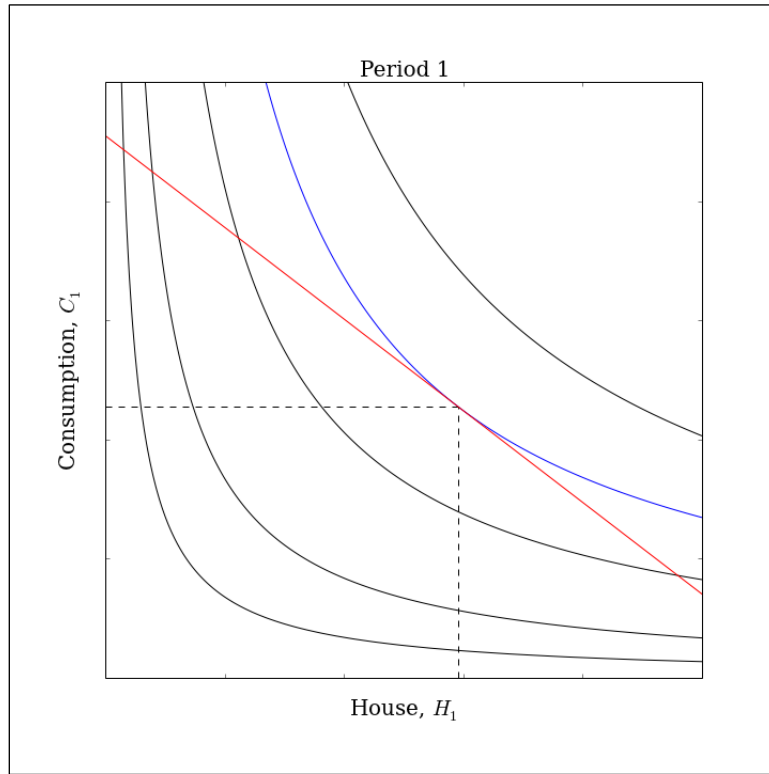
We are going to provide the effect of the following seven demand-side policies:

- i. cash benefits for housing;
- ii. housing subsidies;
- iii. mortgage interest rate reduction;
- iv. mortgage interest deduction from income tax;
- v. quality improvement;
- vi. property tax on the purchase of housing;
- vii. loan-to-value (LTV) ratio regulation;
- viii. debt-to-income (DTI) ratio regulation; and
- ix. restrictions of new purchases.

Figures 4 and 5 show the optimal level of consumption goods and housing units before implementing any policy. The optimal levels are determined at the tangency point between the budget constraint line and the indifference curve. Since the representative household rationally substitutes its consumption inter-temporally and decides its optimal expenditure on consumption goods and housing intra-temporally with perfect foresight in each period, the household allocates its budget such that its allocation will maximize its utility.

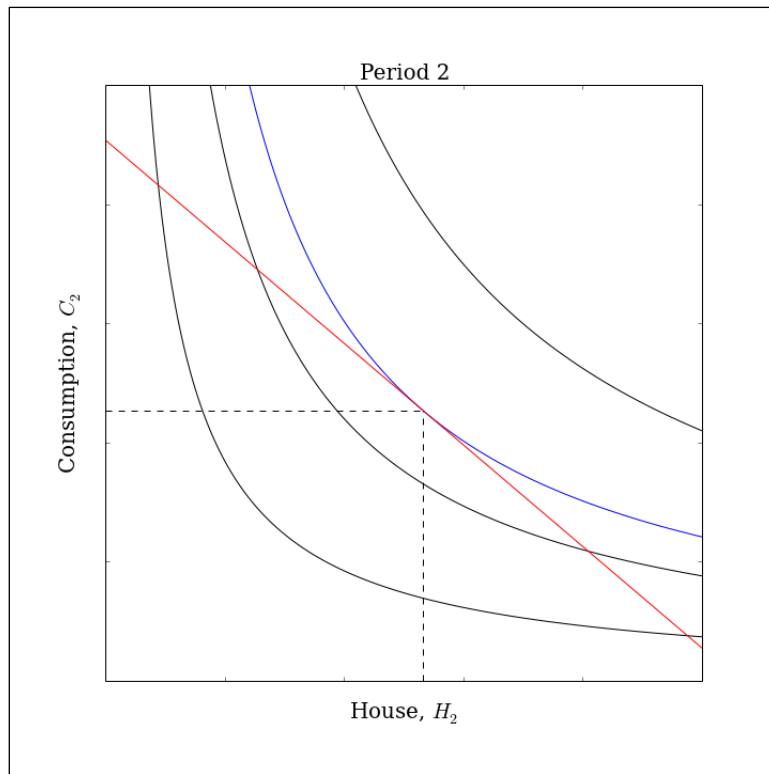
In the two-period model, the housing consumption level in period 2 is automatically determined based on the level of the housing consumption chosen in period 1. In this sense, the household cannot choose in period 2 how many units of housing it consumes. However, the optimal housing units in each period are simultaneously determined by the households with perfect information and foresight, which allows us to draw the budget constraint line in the second period diagram, because the choice in period 2 is not independent of the choice in period 1.

Figure 4: The Optimal Bundle in Period 1



Source: Authors.

Figure 5: The Optimal Bundle in Period 2



Source: Authors.

Most of the housing policies affect the budget constraint line of the household and therefore change the demand for consumption goods and housing units. Each policy affects a value of exogenous variables such as the interest rate, the subsidy, and the tax rate. The budget constraints in period 1 and period 2 are:

$$C_1 = \frac{1}{1+\beta} (1 + t_h) P_h H_1 + \frac{1}{1+\beta} \left\{ (1 - t_y) Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\} \quad (33)$$

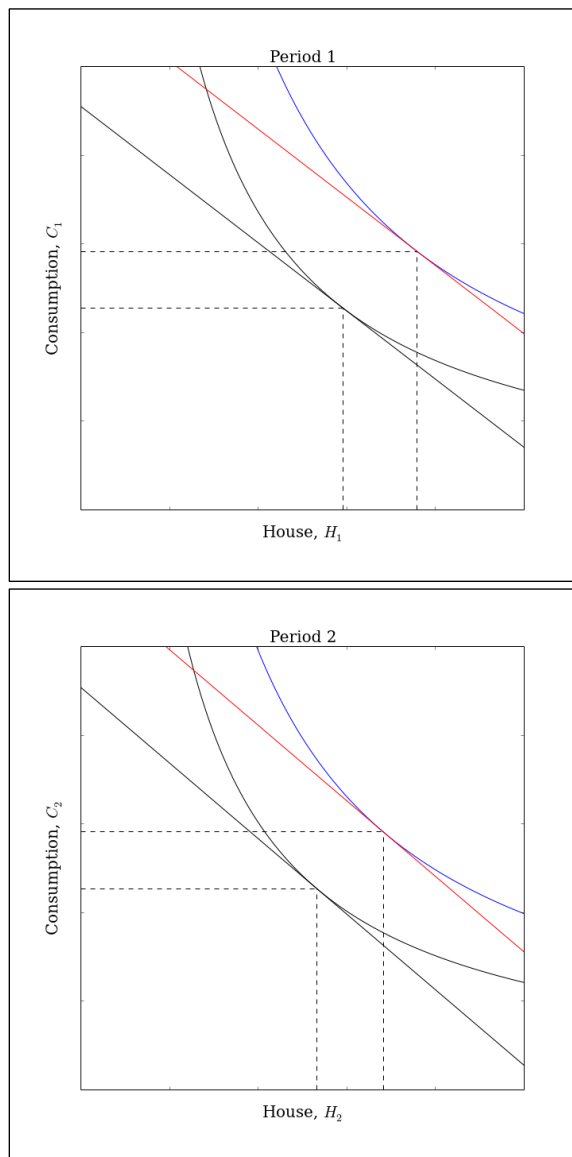
$$C_2 = \frac{\beta}{(1+\beta)(1-\delta)} (1 + t_h) (1 + r) P_h H_2 + \frac{\beta}{1+\beta} \left\{ (1 - t_y) (1 + r) Y_1 + (1 - t_y) (1 + g) Y_1 + (1 + r) G_1 + G_2 \right\} \quad (34)$$

5.1.1 Cash Benefits for Housing

We now assume that the government gives a grant to those who do not yet own a house, but wish to buy one. In some countries these are either low-income households or young households. The housing grant can take the form of a cash benefit and can either be used to purchase housing or for any other purpose. The different effects of cash benefits versus in-kind benefits, such as a housing grant, were briefly explained by Brueckner (2011).

A subsidy in the form of a cash benefit shifts up the budget constraint, both in period 1 and period 2 (Figure 6). As a result, both the optimal consumption level and the housing consumption level become higher, leading to higher utility levels in both periods.

Figure 6: Housing Subsidy as Cash Benefit



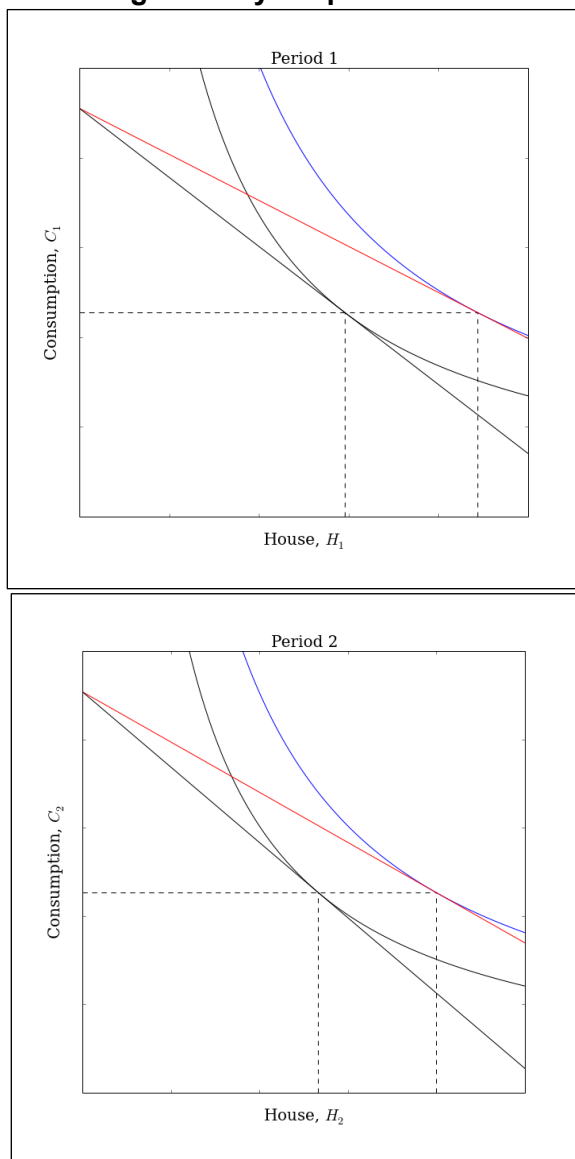
Source: Authors.

5.1.2 Housing Subsidy

We would now like to introduce another type of subsidy. Suppose the amount of the subsidy depends on the price of housing the household is going to buy. Housing financial aid, which is sometimes called housing subsidy, is an example of this type of subsidy. The more expensive the house the household will buy, the larger the amount of subsidy that will be paid. Due to the subsidy, housing is now cheaper to purchase for the household. The perceived cheaper cost for housing means that the budget constraint line rotates outward (Figure 7). The distance between the original budget constraint line and the new budget constraint line is subsidized by the government. Now housing becomes cheaper and thus the households will increase their quantity consumed and obtain higher utility. The relative price change, however, does not affect the consumption of other goods in this model, because of our parameter setting. There are two effects at work: the decrease in the housing price compels the households to consume less in terms of consumption goods, because consumption goods became relatively more expensive (substitution effect). At the same time, the cheaper cost for

housing makes the households better off and increases their consumption of other goods (income effect). In our model, the substitution effect and the income effect exactly offset each other when we assume $\theta = \omega = 1$. As a result, the households purchase the same level of consumption goods before and after the introduction of the subsidy.

Figure 7: Housing Subsidy Proportional to Housing Price



Source: Authors.

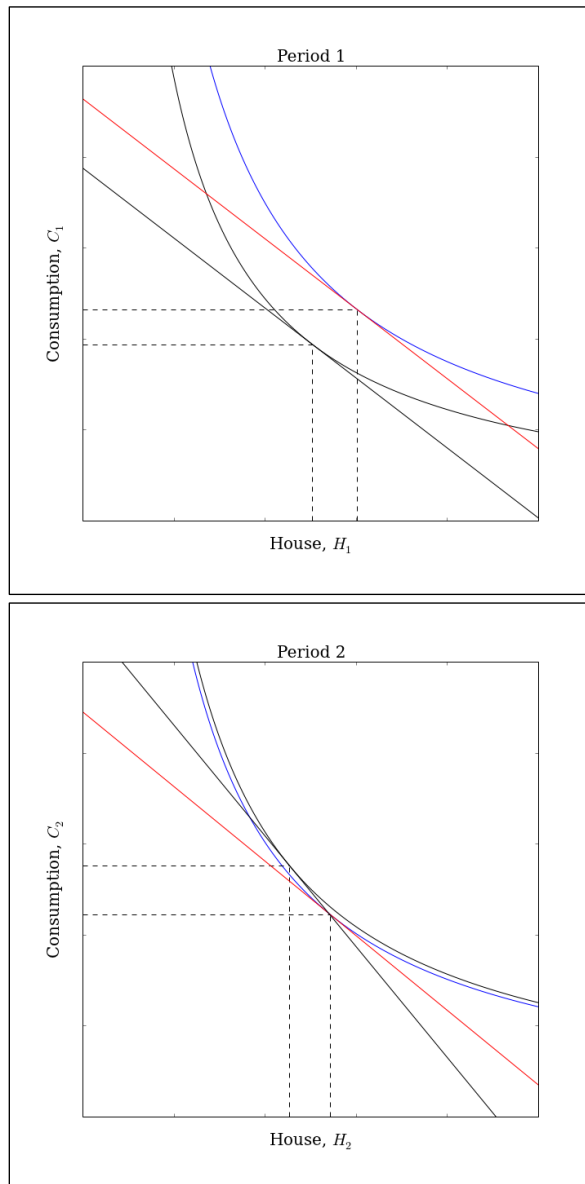
It is known that housing subsidy is generally less efficient than cash benefits, even when the amount of both subsidies is equal. This is because the subsidy in the form of cash benefits, which is equivalent to a lump-sum cash transfer, gives the household more options than the housing subsidy. This welfare difference is not due to our parameter settings, but due to the distortion of the price system caused by the housing subsidy. As standard neo-classical microeconomics argues, the intervention in the competitive price system causes a deadweight loss. In this case, the deadweight loss shows up as a welfare loss of the household, that is to say, lower utility. On the contrary, as a lump-sum subsidy, such as cash benefits, does not intervene in the price system, and an efficient allocation can be achieved.

Housing subsidy policies have been adopted in many countries. India, for example, started to provide subsidies extensively to make housing more affordable in the 1950s and 1960s. Initially, it had worked well, as long as affordability had been a key issue. However, it turned out to be very costly and the rate of new construction lagged behind the increased demand. In Singapore, housing grants were introduced as part of housing policies under Prime Minister Goh (1990–2004). Capitalization of housing subsidies facilitated households' movement up the housing "ladder," but housing policies under Prime Minister Goh accentuated the house price bubble that preceded the Asian financial crisis.

5.1.3 Mortgage Interest Rate Reduction

The economic implications of a reduction in the mortgage interest rate are slightly more complicated to analyze, compared with the previous policies. A reduction in the mortgage interest rate is achieved by introducing a subsidy to cover the difference between market interest rates for mortgages and the targeted level of mortgage interest rates. The lower interest rate shifts up the first period budget constraint in a parallel fashion, which pushes up consumption of both goods and housing units (Figure 8). In the second period, the budget constraint line pivots around the original optimal point in counterclockwise direction. Although consumption of housing units in the second period becomes higher than the units that could be consumed but for the policy, the optimal consumption level in period 2 becomes lower than the level before the reduction in the mortgage interest rate.

Figure 8: Mortgage Interest Rate Reduction



Source: Authors.

A long-term, low interest rate finance policy was introduced in Japan in the 1950s to promote the construction of housing. A publicly sponsored agency, the Government Housing Loan Corporation, offered favorable interest rates to potential homeowners. This policy enhanced competition with private banks and helped to successfully eliminate the shortage of housing stocks in the post-war era and thus raised the living standards of many Japanese people. The Government of the United Kingdom started in 2013 to provide mortgage guarantees, which is another way of lowering mortgage interest rates. The program increased demand for housing, however, as the supply did not react accordingly and the prices of new houses boomed as a consequence, lowering housing affordability.

Instead of an intervention by the government in the market for housing finance, another option is to increase competition in the housing finance market with the objective of lowering financing costs for households. For example, the Republic of Korea started in

1999 to liberalize the housing finance market, which helped to increase access to housing finance and eventually raise the home ownership rate.

5.1.4 Mortgage Interest Deduction from Income Tax

Mortgage interest deduction allows households to deduct the interest payments on mortgages from their taxable income. Our model can be extended to incorporate the mortgage interest deduction program as follows:

The household's disposable income without the mortgage interest deduction system is:

$$(\text{Income}) - (\text{Tax}) = Y_2 - Y_2 t_y = (1 - t_y) Y_2 \quad (35)$$

Once the program is introduced, the interest payment rL can be deducted from the household's income and its disposable income then becomes:

$$(\text{Income}) - (\text{Tax}) = Y_2 - (Y_2 - rL) t_y = (1 - t_y) Y_2 + r t_y L \quad (36)$$

The new inter-temporal budget constraint is:

$$C_1 + \frac{C_2}{1+r-rt_y} + (1+t_h)P_h H_1 = (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \quad (37)$$

The representative household again chooses its optimal consumption level and optimal numbers of housing units subject to the inter-temporal budget constraint. The optimal levels of housing units and consumption goods then become:

$$H_1^* = \frac{b}{(1+t_h)(1+b)P_h} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\} \quad (38)$$

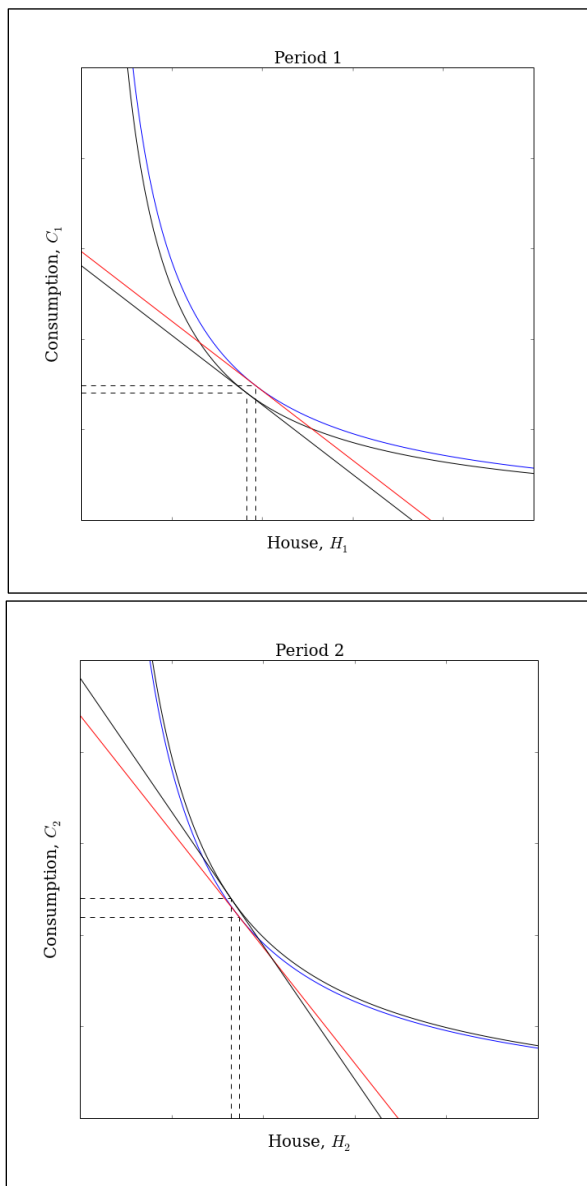
$$H_2^* = (1-\delta)H_1^* = \frac{(1-\delta)b}{(1+t_h)(1+b)P_h} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\} \quad (39)$$

$$C_1^* = \frac{1}{(1+\beta)(1+b)} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\} \quad (40)$$

$$C_2^* = \frac{\beta(1+r-rt_y)}{(1+\beta)(1+b)} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\} \quad (41)$$

The mortgage interest deduction from income tax shifts up the budget constraint in period 1 and the household increases both its consumption of goods and of housing units, resulting in a higher level of utility (Figure 9). However, in the second period, its optimal consumption level decreases, and its utility also becomes lower compared with its utility before the introduction of the policy. The overall qualitative effect of mortgage interest deduction is the same as that of mortgage interest rate reduction.

Figure 9: Mortgage Interest Rate Deduction from Income Tax



Source: Authors.

The United States provides the most prominent example of mortgage interest deduction. In the United States, a mortgage interest deduction (MID) program was started in 1986 to facilitate access to home ownership, especially for low- and middle-income households. The MID allows homeowners to deduct interest payment on mortgage balances up to \$1.1 million and home equity loans up to \$100,000. Ironically, however, the benefits of MID went disproportionately to higher-income taxpayers.³ Hilber and Turner (2014) found that MID has worked less well for low- and middle-income households than for high-income households, as most of the latter would own a house with or without MID. The home ownership rate did not increase as a result of MID and, overall, the policy has turned out to be ineffective and expensive to maintain for the government.

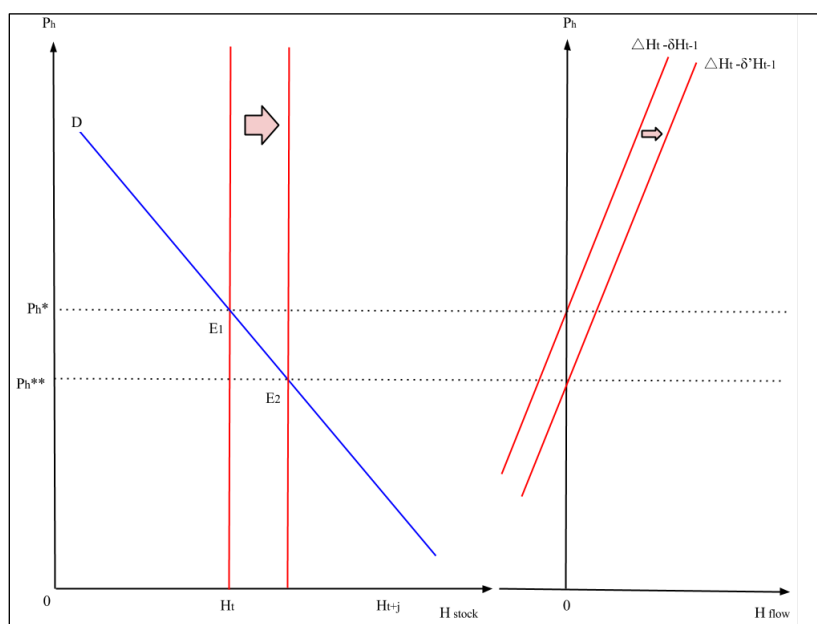
³ Tax Payers for Common Sense (2013).

5.1.5 Quality Improvement (reduction in depreciation rate)

We assume that quality improvements such as enhancement of earthquake resilience or energy efficiency lowers the pace at which houses become decrepit, translating into a decrease of δ in the two-period demand model. Quality improvements do not affect the optimal bundle in period 1 because δ only affects the housing stock level in period 2. However, thanks to the lower depreciation rate, the second-period housing stock (inherited from the first period) increases, which results in a higher utility level.

Quality investments also affect the supply of housing. Fewer demolitions, thanks to quality improvements, shift the flow of housing curve to the right (Figure 10). The reason for the shift is that, although new constructions are independent of the quality improvement, the remaining housing stock from the previous period increases. The rightward shift of the flow curve lowers the steady state equilibrium price from P_h^* to P_h^{**} . The housing stock starts to grow gradually because the construction industry produces more housing even at the original price, which is now higher than P_h^{**} . As new houses are constructed, the housing stock curve moves to the right. The movement of the housing stock continues until the price drops to the new steady state price, reaching the new equilibrium, E_2 .

Figure 10: Effects of Quality Improvement on Housing Flow and Housing Stock



Source: Authors.

Japan has been promoting the improvement of housing quality in recent years. After quantitative housing needs had been met in the 1970s, the emphasis shifted to factors relating to the quality of housing, including residential environments and housing performance (The Building Center of Japan 2014). The Basic Act for Housing enacted in 2006 aims to promote safe, secure, and high-quality housing and to develop housing safety nets for people with difficulties securing a house. The law attempts to achieve enhanced residential living standards for the Japanese today and in the future. Enhancement of earthquake resilience and energy efficiency, and promotion of elderly accessible houses are high priorities. So far, it has managed to enhance housing quality and living conditions, according to recent studies.⁴

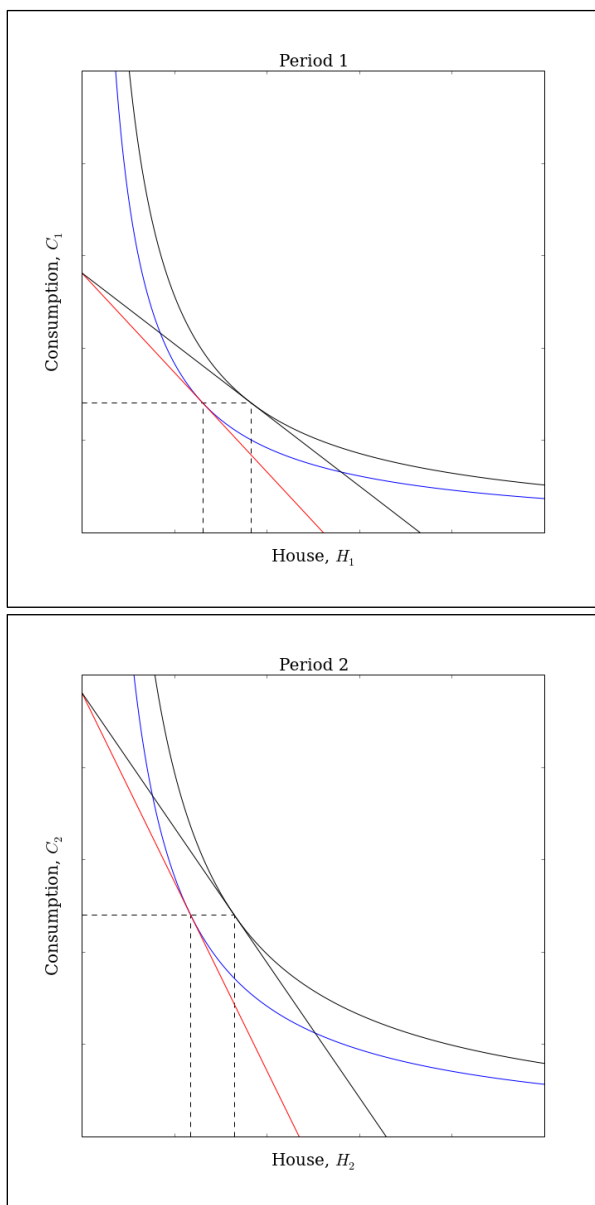
⁴ A *Quick Look at Housing in Japan May 2014*, edited by the Building Center of Japan, reported that the percentage of households living in dwellings below the minimum housing standard has shrunk to

5.1.6 Property Tax on the Purchase of Housing

The introduction of a property tax on the purchase of housing has the effect of increasing the price of housing. The introduction of the tax thus rotates the budget constraint inward and lowers housing consumption in both periods (Figure 11). The change in the price for a housing unit, however, does not affect the choice of how many consumption goods are consumed due to our parameter setting. The substitution effect and the income effect exactly offset each other again.

The tax introduction cools down demand for housing and the increase in the housing price makes the household worse off.

Figure 11: Introduction of a Property Tax



Source: Authors.

approximately 5%, while the number of households living in dwellings exceeding the targeted housing standard is above 50%.

A property tax is typically used to curb speculation in the housing market. For example, in 2003, Hong Kong, China began to introduce several stamp duties as part of its 10-year Long Term Housing Strategy—special stamp duty (SSD), buyer stamp duty (BSD), and ad valorem stamp duty (ADC). The level of the SSD duty rates depends on the holding period of properties. The BSD is imposed on residential properties acquired by any person except a Hong Kong, China permanent resident. These stamp duties did indeed manage to lower demand from non-local buyers and speculators. However, it did not completely stop the housing price from further increasing.⁵

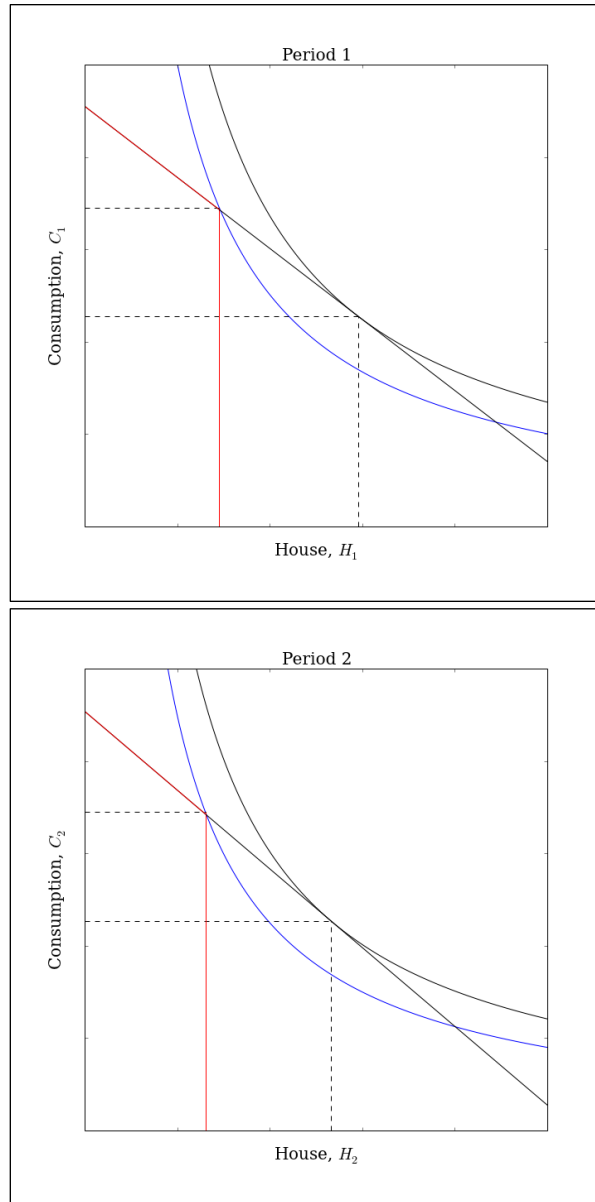
Similarly, the People's Republic of China (PRC) introduced a property tax as a pilot program in Shanghai and Chongqing in 2011. The main objective was to curb speculation on luxurious housing and to limit new home buyers. Another objective was to give incentives to rent out more apartments, thereby reducing the number of vacant apartments. However, the program has not yet been extended nationwide, as it has faced strong opposition from local governments, property developers, as well as the public, who saw their investment opportunities constrained. Another objective was to stimulate economic growth—real-estate construction has continuously made an important contribution to economic growth in the PRC.

5.1.7 Introduction of LTV and DTI Ratio Regulations

Under the LTV and DTI ratio regulations, households are not allowed to borrow money above a certain level. In our model, this type of restrictions bends the budget constraint because the amount of housing units that can be purchased are strictly restricted through the limited availability of mortgages (Figure 12). If the regulation is binding, households become worse off because they cannot choose the optimal level of housing units that would be feasible before the introduction of the regulation. If the constraint is not binding, they can still choose the optimal bundle and have the same level of utility as before (Figure 13).

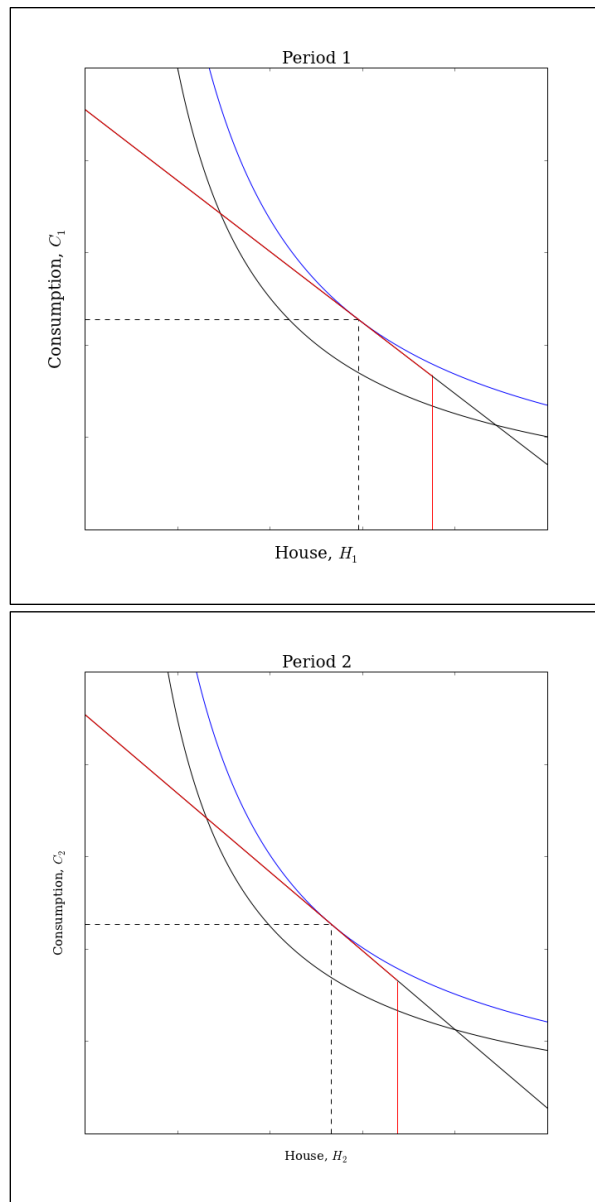
⁵ Under the 10-year Long Term Housing Strategy, 470,000 new housing units are to be built to improve the affordability of housing.

Figure 12: Introduction of Loan-to-Value and Debt-to-Income Ratio Regulations (when regulations are binding)



Source: Authors.

Figure 13: Introduction of Loan-to-Value and Debt-to-Income Ratio Regulations (when regulations are not binding)



Source: Authors.

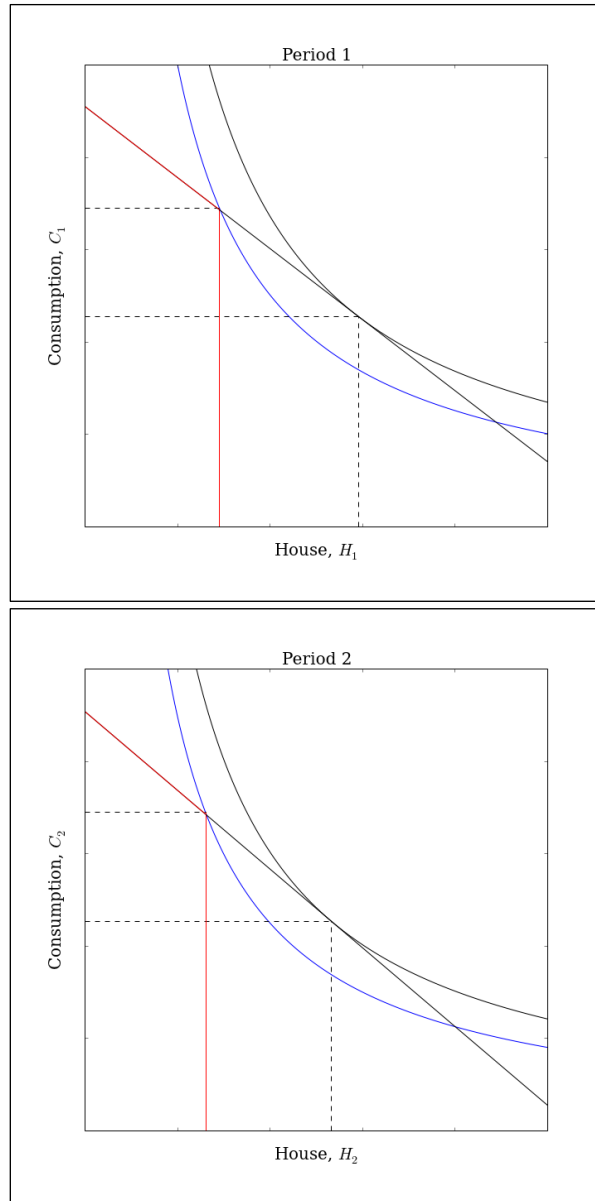
Examples of the use of LTV and DTI ratio regulations are ample. In 2003, the Republic of Korea adjusted the LTV and DTI ratio ceilings to suppress demand for housing loans as the housing market was overheating and posing a systematic risk. In 2004, Singapore reduced the LTV ratio and introduced the DTI ratio to lower investment demand under Prime Minister Hsien Loong. In both countries, LTV and DTI regulations curbed speculative demand for housing and helped to prevent further price hikes.

5.1.8 Restriction of Housing Purchase

Another housing policy that has recently become popular is placing restrictions on additional housing purchases. The objective is to prohibit a household from buying additional homes to lower housing demand. In our model we assume that households are allowed to buy housing units only up to a certain level (Figure 14). The economic

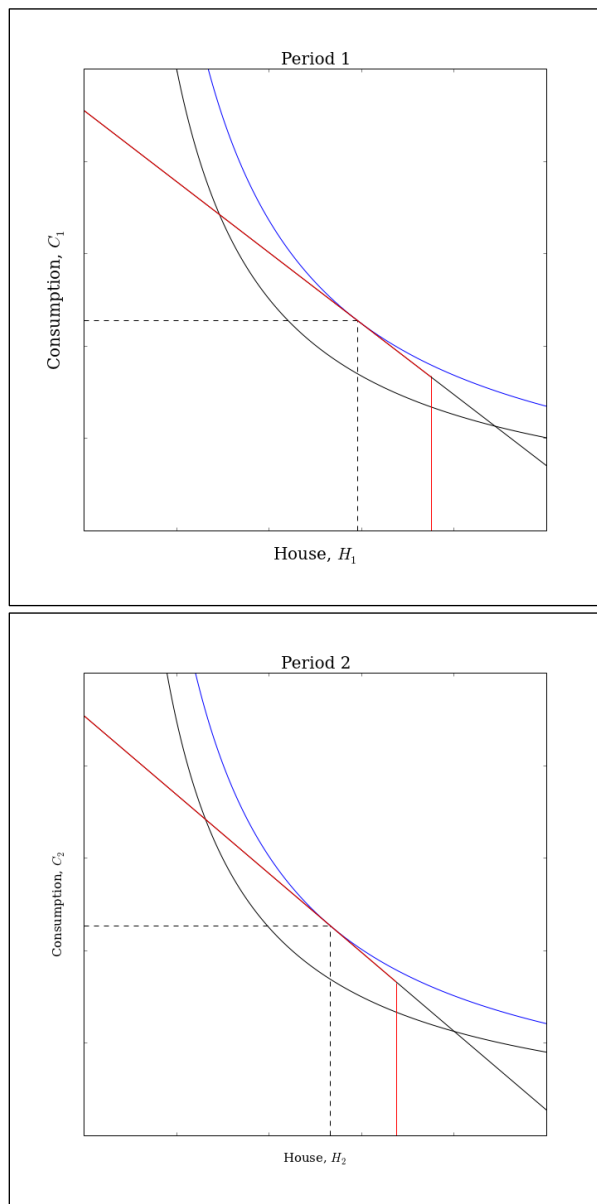
effect is similar to the LTV and DTI ratio regulation. Households that are facing such a restriction spend more on their other consumption instead of housing purchase, but their utility becomes smaller compared with a situation without any restrictions. If the regulation does not influence their optimal decision, they can still choose the same amount of housing units and consumption goods as before (Figure 15).

Figure 14: Restricted Purchases with Binding Regulation



Source: Authors.

Figure 15: Restricted Purchase with Non-binding Regulation



Source: Authors.

Switzerland provides a recent example of this policy. The county started to ban the construction of new second homes in areas with a high share of second homes to limit the sprawling of empty second homes in tourist areas. As a consequence, the demand for second homes in the affected municipalities dropped drastically. The lower housing prices made owner-occupied primary homes more affordable for young local residents. However, local owner-occupiers and owners of second homes in the targeted areas were made worse off, as their housing prices fell. Moreover, no reduction was found in overall sprawling as demand for second homes just shifted from areas constrained by the ban to unconstrained areas.

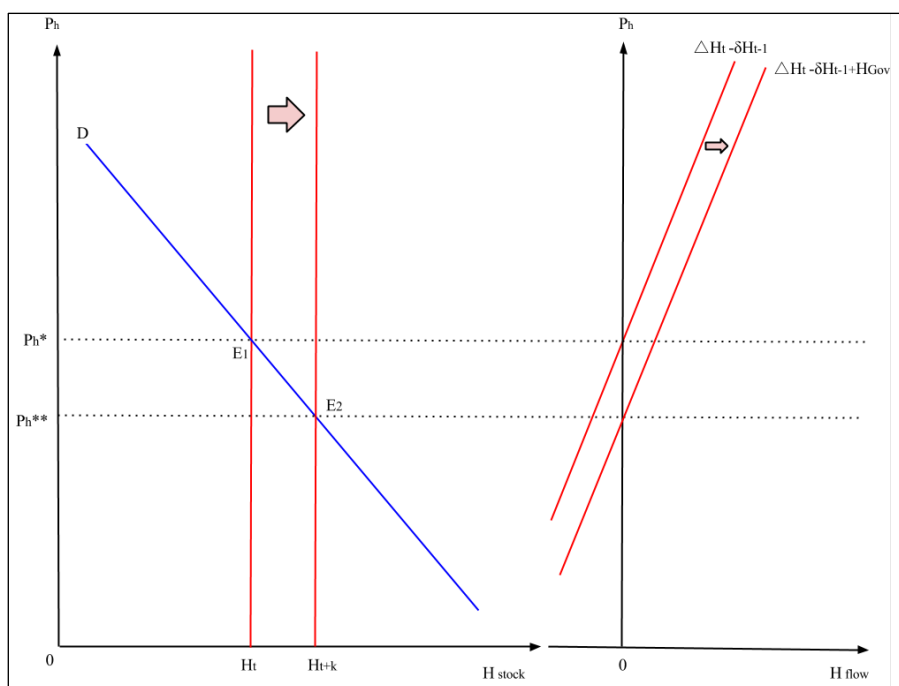
Another example is found in the PRC. In 2010, the PRC started to restrict purchases in 45 cities to limit ownership of more than two properties and to avoid potential housing bubbles. Non-local residents or foreigners were no longer allowed to buy a second house and local residents needed to have at least two-year intervals before buying a second house. Furthermore, those wishing to buy a second or third house with a loan

were required to have a higher down payment. The restriction policy showed positive effects in terms of preventing housing prices from rising further or eventually bringing down housing prices substantially. However, many local governments felt that the price drop went too far and the unsold housing inventory quickly increased. In 2014, many local governments decided to lift and loosen the restrictions on purchasing housing. By the beginning of 2015, 42 cities had abolished all restrictions.

5.1.9 Government Provision of Housing and Subsidy to Suppliers

Another option to promote the provision of housing is through the construction of houses by the government. Under the assumption that the government provides a certain number of new housing units independent of the current housing price level, construction outside the private market shifts the flow curve rightward in a parallel fashion, which lowers the steady state equilibrium price to P_h^{**} (Figure 16). The additional construction by the government increases the stock of housing in every period and the equilibrium changes along the demand curve. Once the short-run equilibrium price reaches the new steady state equilibrium price, the supply becomes stable at E_2 . Using the same logic and diagram, we can analyze the case where the government gives a fixed amount of subsidy to private construction companies.

Figure 16: Government Provision of Housing

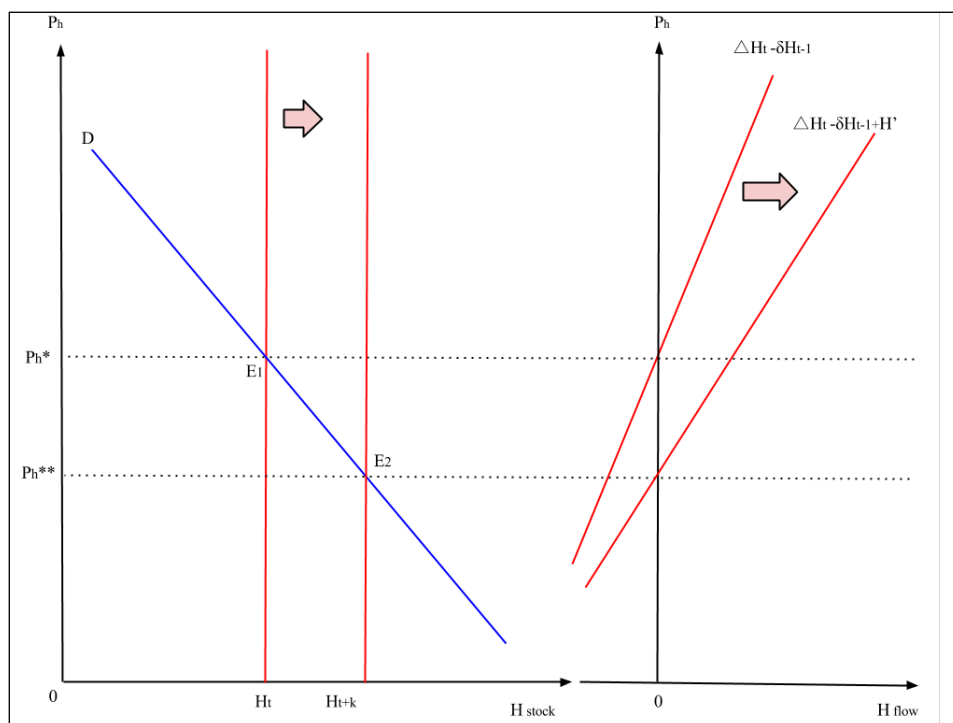


Source: Authors.

The lower price makes housing more affordable, but public housing construction has a few disadvantages compared with demand-side policies. Weicher (1979) introduced several studies showing that the cost of new public housing is higher than that of new private housing. Moreover, when there is a plentiful supply of pre-owned housing, new housing construction is much more costly than demand-side policies. Even the least expensive construction is usually more expensive than the reuse of secondhand housing. According to Harvey (2000), “Often governments have been so preoccupied with new building programmes that present stock has been neglected by being allowed to remain unoccupied or to fall into disrepair.” (p.301) And O’Sullivan (1996) showed that public housing produces a relatively small increase in recipient welfare per dollar.

When the amount of subsidy to the private constructing industry depends on the units the private sector supplies, the flow curve shows a different movement. The more housing the private sector supplies, the more subsidy is paid. As a result, the flow curve pivots, as Figure 17 demonstrates. The steady state equilibrium price goes down and the steady state equilibrium changes from E_1 to E_2 .

Figure 17: Subsidy to Housing Suppliers



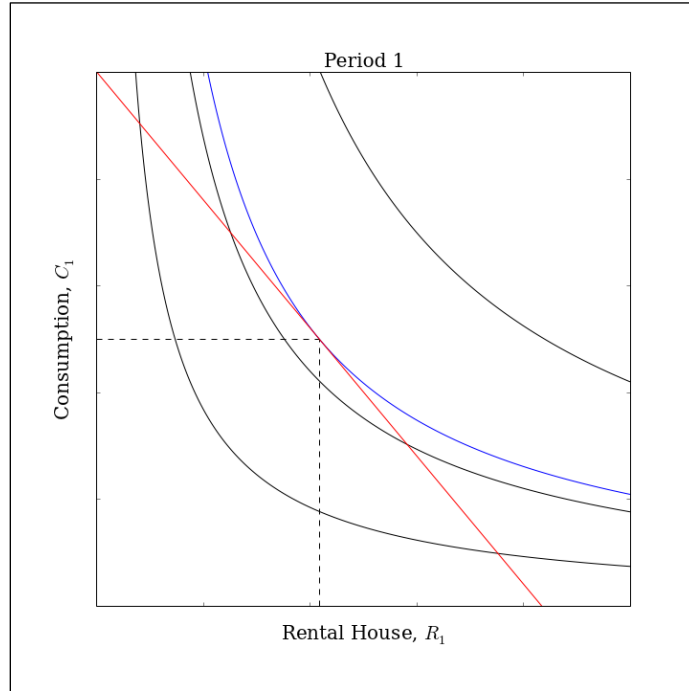
Source: Authors.

In 1989, the Republic of Korea started the Two Million New Housing Construction Project to overcome its housing shortage and housing price hike. As one of the main measures, the Government of the Republic of Korea increased the supply of developable land through public sector developers. Despite the fact that the massive supply of land resulted in a lack of diversity and overstretched the capacity of the construction industry, the measure helped to boost housing construction, eliminate the housing shortage, and stabilize housing prices.

5.2 Rental Housing Market

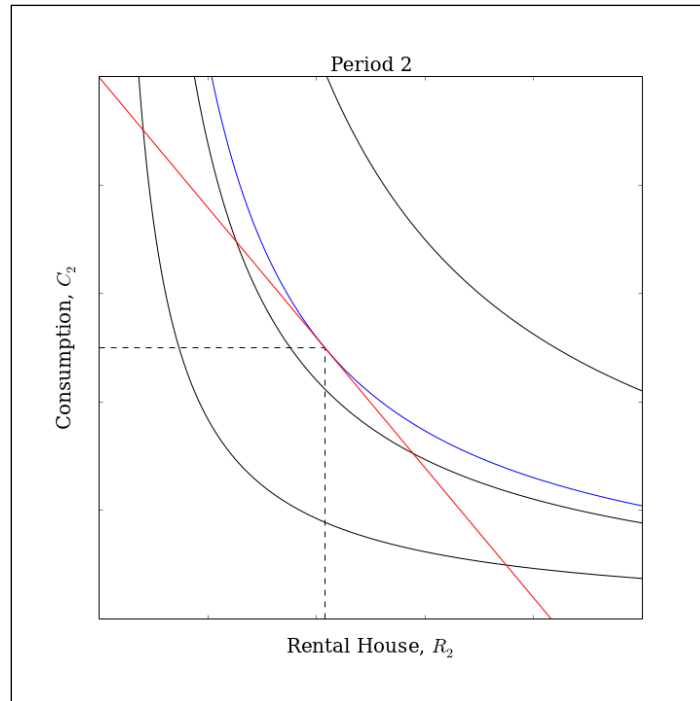
As shown in Figure 18 and Figure 19, households choose the optimal bundle of (C_t, R_t) such that they maximize their utility in each period. Next we analyze different housing policies for rental houses.

Figure 18: The Optimal Bundle of Tenants in Period 1



Source: Authors.

Figure 19: The Optimal Bundle of Tenants in Period 2



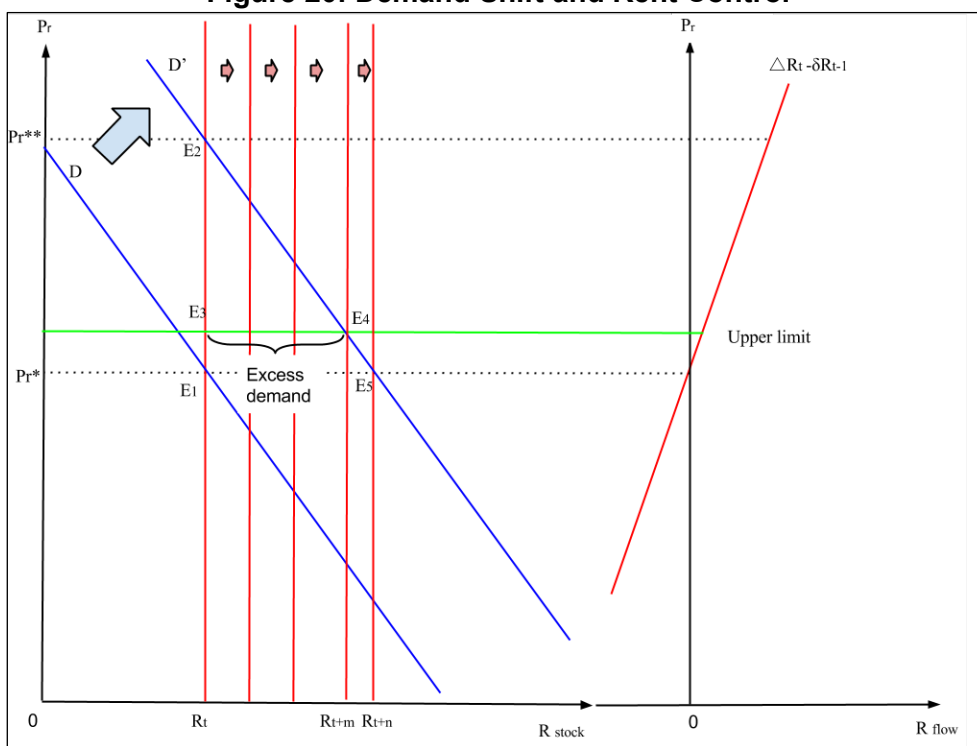
Source: Authors.

5.2.1 Rent Control

First, we assume that the housing market starts in equilibrium, E_1 (Figure 20). Let us suppose that the demand curve shifts rightward due to the increase in the population, it pushes up the equilibrium price to P_r^{**} , and the equilibrium changes from E_1 to E_2 . Suppose the government, however, sets an upper limit of rent, which is below the

market equilibrium price. This rent control translates into an excess demand in the constrained equilibrium under the rent control, E_3 . When the upper limit of rent is set below the short-run equilibrium price, P_r^{**} , the supply curve shifts very slowly and it takes even longer for the stock to grow. This is because the constrained price level is not attractive enough to encourage producers to supply housing promptly. The excess demand is cleared only after the housing stock reaches R_{t+m} ($m > 0$), where the equilibrium becomes E_4 . The growth of housing stock still continues because the upper limit of rent is still higher than the steady state equilibrium price. When the stock finally reaches R_{t+n} ($n > m > 0$), the market equilibrium price returns to the original steady state equilibrium price at E_5 and the supply curve shift stops. If rent were controlled exactly at the original steady state equilibrium level, P_r^* , the supply would not move and excess demand could not be eliminated through the market adjustment process because the housing stock does not change in the face of the controlled rent. In short, rent control reduces the incentive to construct new housing and causes excess demand. The controlled lower rent prolongs the process of the shortage elimination.

Figure 20: Demand Shift and Rent Control



Source: Authors.

In addition to the slow adjustment process, the excess demand due to rent control is also problematic from a viewpoint of efficient allocation. Those who are willing to pay most cannot necessarily find their dwellings. Households that only need to pay a low controlled rent are reluctant to move out, which makes it difficult for households that have high willingness to pay to access proper housing. Only the first generation of renters can become “winners” under the rent control policy. Landlords are obviously worse off and subsequent-generation tenants are also worse off because they will face higher search cost. In this sense, the rent control tends to allocate housing inequitably. The misallocation benefits go only to households enjoying the low controlled rent and this market distortion negatively affects social welfare. That is to say, the social surplus cannot be maximized under rent control. The lower controlled rent also discourages

landlords from maintaining dwellings. As a result, their reduced profit may be recouped by their cutting maintenance cost, resulting in lower durability and quality of housing.

To control the decrease in supply because of the rent control, several adjustments are usually made to rent control. The first way to diminish the supply decrease is to exempt new housing from rent control. If new housing is not affected by rent control, a potential price increase stimulates the supply of new housing, offsetting the loss in housing stock as a result of rent control. However, if builders and landlords suspect that the rent of new housing might come under rent control in the future, they may become reluctant to supply new housing. The second option is to subsidize new construction to diminish the decrease in the housing supply caused by rent control. The third option is to adjust the controlled rent with regularity. For example, allowing for a periodic increase in controlled rent can mitigate the negative effect of rent control. However, O'Sullivan (1996) pointed out that "in most rent-control cities that use rent adjustment, controlled rents rise more slowly than the cost of building and maintaining rental housing." (p.440) The fourth way is to control rents only of existing tenants. Whenever a new tenant moves into the dwelling, the rent can be freely adjusted. This moderate form of rent control slackens some negative aspects, but is still highly ineffective. Most importantly, the incentive of the owners to invest into maintenance is low. Tenants are also still very reluctant to move even though their needs in terms of space and location change drastically. For example, retired couples might decide to remain in their apartment in the business district, as they would have to pay substantially higher rents when moving to a new apartment with a new lease, whereas young couples wishing to be closer to work see themselves obliged to commute into the city center.

The major alternatives to rent control are income redistribution by the national government through use of a land tax. Although rent control itself has a function of income redistribution, it is not always efficient because the "winners" of rent controls can be both low-income and high-income households. Whoever can continue to dwell in a house for which rent is controlled are beneficiaries of the policy. Redistribution by use of a land tax has an advantage over rent control in that the land tax does not affect the supply of land or the supply of housing because the land supply is totally inelastic to price.

Rent control can be seen in many cities. Rent controls in New York City after World War II are a well-known example. Rent control was implemented in Switzerland in 1936 to stabilize rents and combat urban sprawl. Until today, rents of current tenants are tied to mortgage interest rates. The policy was reasonably successful in stabilizing rents, but it led to significant excess demand. Vacancy rates in the bigger cities, such as Basel, Geneva, and Zurich are notoriously low, typically less than 1%. The other two major drawbacks are the low level of maintenance as well as a mismatch in the allocation of apartments.

5.2.2 Rent Certificates

Rent certificates are typically distributed to low-income groups, and are in effect a direct subsidy. They provide financial assistance for eligible households residing in standard quality housing. The basic idea is that the eligible household is not allowed to spend more than the fair market rent, which is determined as the prevailing rent in a competitive rental housing market for a standard low-income dwelling. If the household wanted to rent a house for which rent is higher than the fair market rent, it would not be able to get a rent certificate and would have to pay the entire rent out of pocket.

Let us assume that an eligible household spends 30% of its income on housing and that it receives a rent certificate that covers the rest of the actual rent:

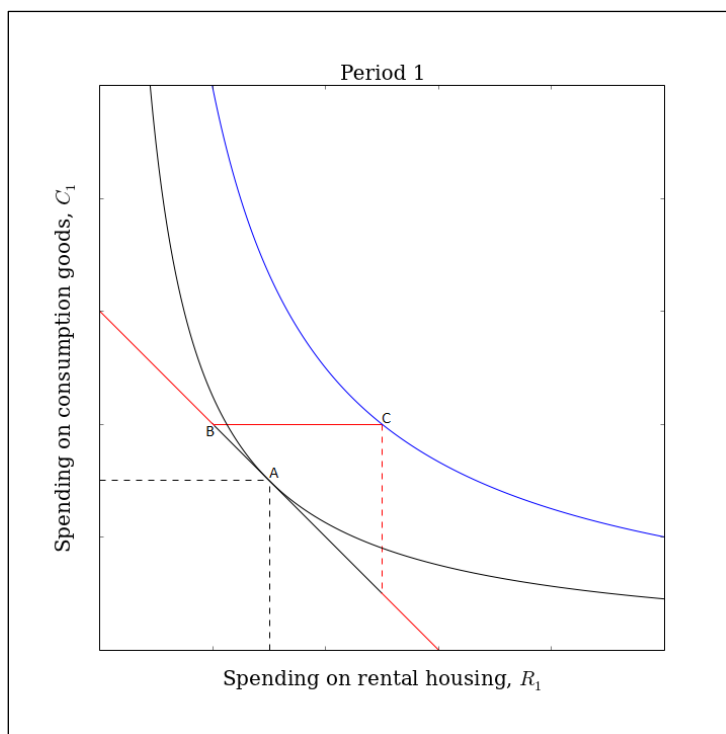
$$\text{Actual rent} = \text{Rent certificate} + 0.3 \times \text{Income}$$

The government covers the difference between the 30% of the household's income and the actual rent, in case the actual rent exceeds 30% of the household's income. Thanks to the rent certificate, the household has more income available to spend on consumption goods. The budget constraint line becomes horizontal in the area where the actual rent is below the fair market rent.

Figure 21 shows the effects of a rent certificate program on housing consumption. To make the discussion simpler and more intuitive, we use a slightly different diagram compared with the previous section. Figures 21 and 22 now show on the horizontal axis the spending on housing, instead of the amounts of housing units consumed in the previous graphs.⁶ Having a graph on spending helps to better visualize rent certificates and housing vouchers as both of them are relative to spending on housing.

In Figure 21, the optimal bundle of housing and consumption is A, under the initial budget constraint. Once the rent certificate program is implemented, the budget constraint bends at point B. As the household could not get any subsidy if it wished to live in a dwelling with rent higher than the fair market rent, the new budget constraint has a discontinuity at point C on the fair market rent. Under the new discontinuous budget constraint, the household chooses point C, which gives the highest utility of all feasible points. The distance between B and C is subsidized by the government.

Figure 21: Rent Certificate Program



Source: Authors.

⁶ The vertical axis measures spending on other goods. Given that we assume that these goods are the numeraire for prices, the scale does not change.

5.2.3 Housing Voucher

Similar to rent certificates, housing vouchers also are in effect a direct subsidy to the eligible household. However, housing vouchers are different from rent certificates because housing vouchers can be used for any types of dwellings as long as the dwellings meet certain minimum standards. There is as such no upper limit on how much the household can spend on rent.

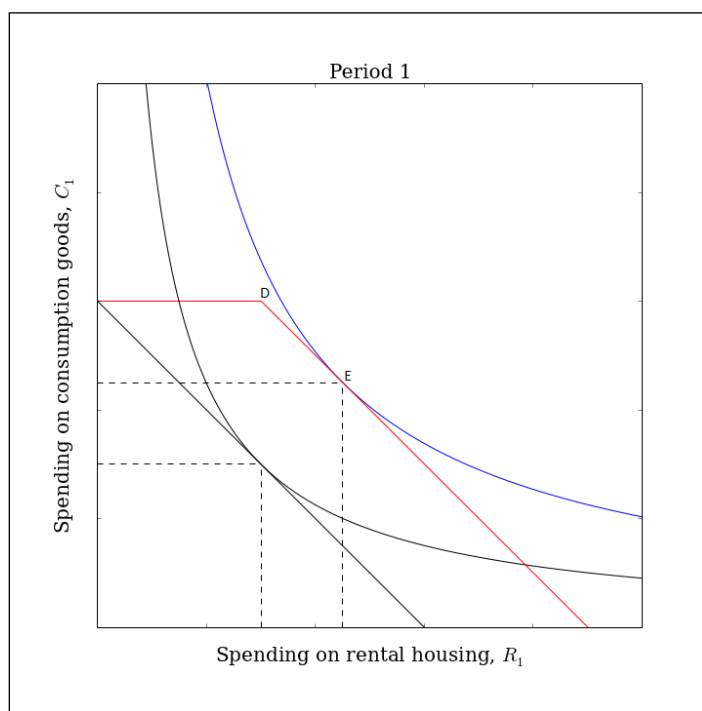
Similar to the certificates, let us assume that the household has to spend 30% of its income on rent. The face value of the voucher is thus the difference between the fair market rent and 30% of the household's income. The amount of the housing voucher is based on the fair market rent and thus independent of the actual rent. In other words, the amount of subsidy is fixed, no matter what type of housing they choose to rent. As a consequence, if the household rents a dwelling that costs more than the fair market rent, it will still receive the same housing voucher, but will have to spend more than 30% of the household's income to pay the rent.

$$\text{Housing voucher} = \text{Fair market rent} - 0.3 \times \text{Income}$$

In contrast to the rent certificate policy, under the housing voucher program the household's maximum expenditure on housing is not limited. The recipients of the housing vouchers are even allowed to live in luxurious dwellings for which rent is higher than the fair market rent.

Figure 22 illustrates the effects of housing vouchers on household utility. To make our argument consistent, we employ the same diagram settings as in Figure 21. Thanks to the housing voucher program, the budget constraint line shifts upward. However, the household is not allowed to spend more on consumption goods than would be feasible without the housing voucher program, because the housing vouchers cannot be used for consumption goods. This requirement produces a kink in the budget constraint line at point D. Under the new budget constraint, the household chooses point E, which gives the highest utility of all the feasible points.

Figure 22: Housing Voucher Program



Source: Authors.

Housing vouchers have several advantages compared with rent certificates. First, rent certificates do not provide incentives to households to look for low-rent housing. As the subsidy fully covers the difference between the actual rent and 30% of households' income, households choose the most expensive rental housing as long as the rent does not exceed the fair market rent. In contrast, under a housing voucher program, the face value of the housing voucher is fixed wherever households decide to live. Households thus efficiently choose their optimal rental housing and pay a lower rent than before.

Additionally, under certain assumptions, we can show that the housing voucher program gives households higher utility compared with rent certificates, even if the subsidy paid to the eligible households is the same. This is because the voucher program as a lump-sum cash transfer gives the households more options to choose their optimal consumption bundle compared with the rent certificate program (details are explained in Section 5.1.2.).

However, it does not always mean that housing vouchers are a superior policy instrument compared with rent certificates. If the objective of housing policy is to increase housing consumption rather than increase utility, rent certificates could be more effective.

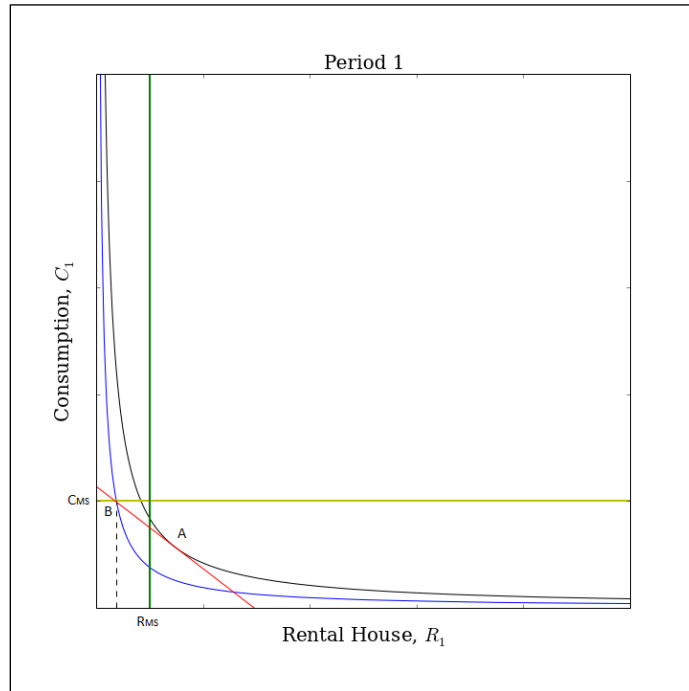
In the Republic of Korea, a housing voucher program was launched in 2015 aiming to ensure the minimum housing standard will be met. The amount of monthly cash subsidy is determined based on household income, rent, family size, and location of residence. The effectiveness of the program still needs to be assessed.

5.2.4 Slum Prevention and Rehabilitation

Poor households typically find it difficult to access adequate housing. The main reason is that their income is too low to be able to afford the rent for a dwelling of a minimum standard. Most of their income is spent on expenditure for basic survival.

In our model, we can easily introduce such minimum standards for survival as well as minimum standards for housing. In Figure 23, C_{MS} stands for the minimum consumption for survival and R_{MS} denotes the rent of the smallest possible dwelling of a minimum housing standard. We assume that every households needs to keep its consumption at or above C_{MS} to survive. If the income is too small to access minimum standard housing while having to maintain the minimal level of consumption, the household's only choice is to sacrifice part of its housing consumption for survival. The housing units consumed then fall below R_{MS} , which means the household is forced to live in substandard quality housing. These are often slums, where dwellings tend to be overcrowded and lack adequate ventilation, light, or sanitation. Moreover, access to safe drinking water is often limited and security of tenure tends to be lacking. Figure 23 illustrates the case of a household's income being too low to be able to afford living in a standard quality house and is forced to live in a substandard dwelling.

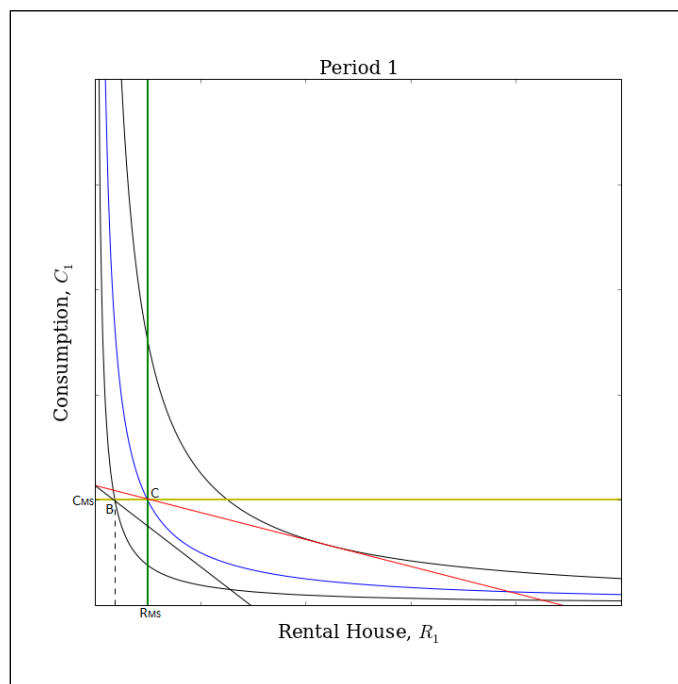
Figure 23: Substandard Quality Housing



Source: Authors.

The objective of slum prevention policies is typically to create an affordable housing stock by increasing the supply of housing of a minimum housing standard. The increase of the housing supply lowers the price for housing and it allows poor households to access dwellings of a minimum standard (Figure 24). It is hoped that once a household's income becomes larger, it will be able to choose the optimal bundle with a consumption level above the minimum line.

Figure 24: Slum Prevention and Rehabilitation



Source: Authors.

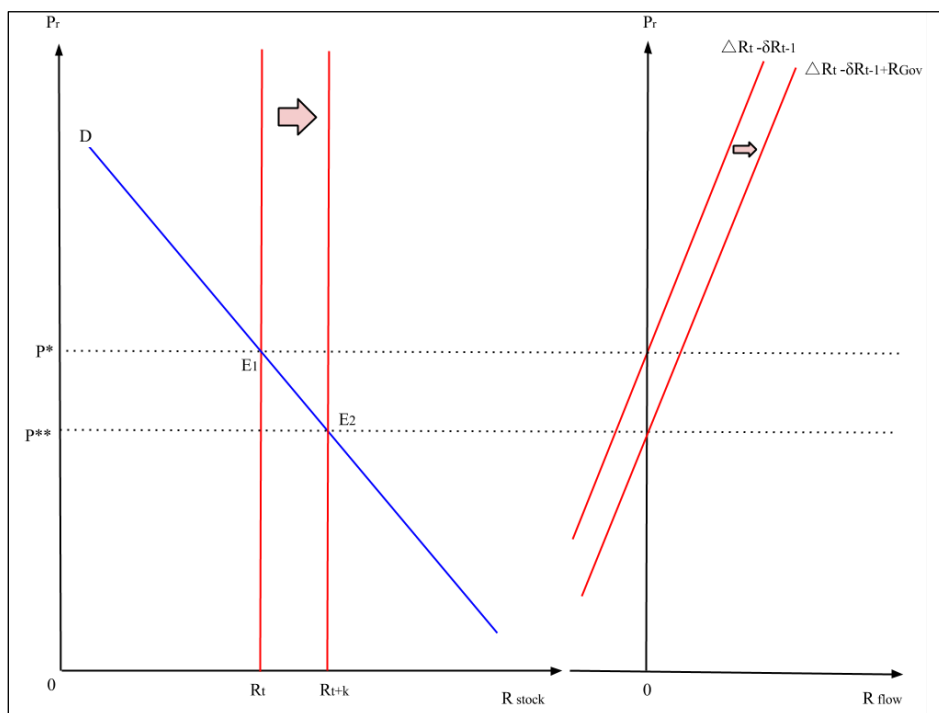
In 2013, the Indian government started a large slum prevention program, which attempts to give slum dwellers access to appropriate housing, while at the same time tackling the process by which slums are created. The main policy instrument is to provide affected people with shelter or housing free of cost. The program will run until 2022 and has the ultimate goal of making India slum free by that time.

5.2.5 Government Provision of Rental Houses

Another option for the government to intervene in the rental market is to directly invest in construction of public rental housing. Especially after World War II, governments of countries affected by the war decided to provide public rental houses to ensure their citizens found adequate shelter.

Suppose in our model that the government promotes the construction of rental houses to address the shortage of housing. The construction shifts the flow curve rightward, which lowers the steady state equilibrium price (Figure 25). The additional construction by the government increases the stock of housing in every period and the equilibrium changes along the demand curve. Once the short run equilibrium price has reached the new steady state equilibrium price, the supply becomes stable.

Figure 25: Government Construction of Public Rental Housing



Source: Authors.

Since we employ the same stock-flow supply model for rental houses as for owner-occupied housing, the supply-side policy applied to the rental housing market shows the same effects as we discussed in the previous section.

Public rental housing has been provided in various countries. Japan, for example, enacted the Publicly-Operated Housing Act in 1951 and supplied publicly-operated low-rent housing for low-income people through local government units. The act enabled the central government to provide subsidies to local governments to increase the housing supply. In 1995, the Japan Housing Corporation, which was the predecessor of the Urban Renaissance Agency established in 2004, started the collective construction of rental housing for low- to middle-income households mainly living in

major cities to overcome a housing shortage due to an influx of people into urban areas. By 1973, the total number of houses exceeded the total number of households in all metropolitan areas and prefectures. Japan had reached its goal of one house per household, ending two decades of postwar housing shortages.⁷

6. COMPARING HOUSING POLICIES

The graphical analysis in the previous section has given us valuable insights into how housing, the consumption of other goods, and household utility change after the introduction of different housing policies. Tables 2 and 3 summarize the qualitative results of all demand-side policies discussed in the previous section. The objective of this section is to introduce a quantitative analysis of the housing policies described above. For simplicity, we focus on demand-side policies and ignore their effects on the supply-side.

Table 2: Policy Effects on the Demand and Utility (Owner-occupied houses)

Owner-occupied Houses	Period 1			Period 2		
Policy	H_1	C_1	U_1	H_2	C_2	U_2
Cash subsidy	↑	↑	↑	↑	↑	↑
Housing subsidy	↑	←	↑	↑	←	↑
Mortgage interest rate reduction	↑	↑	↑	↑	↓	↓
Mortgage interest deduction from income tax	↑	↑	↑	↑	↓	↓
Quality improvements	←	←	←	↑	←	↑
Introduction of property tax on housing purchase	↓	←	↓	↓	←	↓
LTV, DTI ratio regulations	↓	↑	↓	↓	↑	↓
Restrictions on purchases	↓	↑	↓	↓	↑	↓

LTV = loan-to-value; DTI = debt-to-income.

Source: Authors.

Table 3: Policy Effects on Demand and Utility (Rental houses)

Rental Houses	Period 1			Period 2		
Policy	R_1	C_1	U_1	R_2	C_2	U_2
Cash Subsidy	↑	↑	↑	↑	↑	↑
Rent aid	↑	←	↑	↑	←	↑
Rent certificates	↑	↑	↑	↑	↑	↑
Housing vouchers	↑	↑	↑	↑	↑	↑

Source: Authors.

⁷ "A Quick look at Housing In Japan" edited by The Building Center of Japan, May 2004

6.1 Numerical Examples of Housing Policies

The first step in our simulation is to choose all parameters that are exogenously given and not subject to change in our simulations. For our two-period housing model we assume the following values for these variables:

- Discount factor for future utility: $\beta = 0.95$
- Coefficient of relative risk aversion: $\theta = 1.0, \omega = 1.0$
- Weight for house in utility: $b = 1.0$
- Income in the first period: $Y_1 = 10$
- Economic growth rate: $g = 0.02$
- Income tax rate: $t_y = 0.10$
- Price for house per housing unit: $P_h = 1.5$
- Rent per housing unit: $P_r = 0.75$

We will study the effect of housing policies on demand and utility when changing the value of the following policy variables:

- Subsidy in period 1 and in period 2: $G_1 = 0, G_2 = 0$
- Interest rate: $r = 0.05$
- Depreciation rate of house: $\delta = 0.10$
- Property tax rate: $t_h = 0.00$

The housing policies and variables we will simulate are shown in Table 4.

Table 4: Housing Policy and its Policy Variables

Policy	Policy Variable	
Cash subsidy for potential homeowners	G_1	↑
Housing subsidy	P_h	↓
Mortgage interest rate reduction	r	↓
Mortgage interest deduction form income tax	$rt_y L^*$	↑
Quality improvement	δ	↓
Property tax on the purchase of housing	t_h	↑
Cash subsidy for tenants	G_1, G_2	↑
Rent aid	P_r	↓

Source: Authors.

Our main interest lies in the percentage change of the housing units demanded (ΔH) and the change in utility (ΔU). In addition to the changes in housing demand and utility, we are interested in the change of the housing price in the short run. The short-run price level is defined as the price level immediately after the implementation of the new policy and it thus reflects the initial shock. In the medium to long run, our model predicts that supply will respond and adjust the housing stock according to the new demand. The housing price will eventually return to the steady state equilibrium level, as long as the policy does not alter the supply-side or change the steady state housing price. In other words, in our model the supply reacts in the medium to long run in a way that excess demand or excess supply is cleared, whereas in the short run the excess demand or the excess supply is cleared by the change in the price, not in the supply change. In this section, we will analyze the policy effect on the short-run housing price.

6.1.1 Cash Subsidy

In Table 5, the first column G_1 denotes the possible cash subsidies by the government (from 0 to 5). The upper part of the table shows the absolute demand for housing units, consumption goods, mortgage loans, and utility for the cases of 0 to 5 units of cash subsidies. The lower part of the table lists the percentage changes from the benchmark.

Suppose that the government grants a fixed amount of subsidy in period 1 to every household that wishes to buy a new house. In our numerical example, one unit of subsidy is equivalent to a 10% increase in households' income in the first period. As a consequence, the demand for housing units and the demand for consumption goods both increase at the same rate of 5.64%. Looking at the change in utility, we see that the subsidy improved the welfare of households by 3.33%. (Every additional unit of subsidy increases the total utility in a concave way, that is, the marginal increase in utility by the one-unit additional subsidy becomes smaller as the absolute number of subsidy becomes larger.) Furthermore, the subsidy program reduces the need of the households to seek a loan, because the subsidy mitigates the necessity for the households to rely on a mortgage for the purchase of new housing. A subsidy of one unit lowers mortgage demand by 5.51%.

In the second period, demand for housing units and consumption goods remains larger compared with the situation without intervention. The increase is equal to that observed in the first period. The utility in the second period also increases, and its response is even bigger compared with the first period. As the cash subsidy increases demand for housing, in the short run we observe an increase in the housing price. One unit of additional subsidy pushes up the housing price by 5.64% in the short run, thus exactly offsetting the demand increase. (Remember that we assume a vertical housing supply in the short run.)

Table 5: Numerical Simulation of Cash Subsidy

G_1	H_1	C_1	U_1	L	$P_{h \text{ short run}}$	H_2	C_2	U_2	$H_1+\beta H_2$	$C_1+\beta C_2$	$U_1+\beta U_2$
0	5.91	4.55	3.29	4.42	1.50	5.32	4.54	3.18	10.97	8.86	6.32
1	6.25	4.81	3.40	4.18	1.58	5.62	4.79	3.29	11.59	9.36	6.53
2	6.58	5.06	3.51	3.93	1.67	5.92	5.05	3.40	12.21	9.86	6.73
3	6.91	5.32	3.60	3.69	1.75	6.22	5.31	3.50	12.83	10.36	6.93
4	7.25	5.58	3.70	3.45	1.84	6.52	5.56	3.59	13.44	10.86	7.11
5	7.58	5.83	3.79	3.20	1.92	6.82	5.82	3.68	14.06	11.36	7.29
G_1	H_1	C_1	U_1	L	$P_{h \text{ short run}}$	H_2	C_2	U_2	$H_1+\beta H_2$	$C_1+\beta C_2$	$U_1+\beta U_2$
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1	5.64%	5.64%	3.33%	-5.51%	5.64%	5.64%	5.64%	3.44%	5.64%	5.64%	3.38%
2	11.27%	11.27%	6.49%	-11.02%	11.27%	11.27%	11.27%	6.71%	11.27%	11.27%	6.59%
3	16.91%	16.91%	9.49%	-16.53%	16.91%	16.91%	16.91%	9.81%	16.91%	16.91%	9.64%
4	22.54%	22.54%	12.35%	-22.04%	22.54%	22.54%	22.54%	12.77%	22.54%	22.54%	12.55%
5	28.18%	28.18%	15.08%	-27.55%	28.18%	28.18%	28.18%	15.59%	28.18%	28.18%	15.33%

Source: Authors' calculations.

6.1.2 Housing Subsidy

As discussed above, the government may decide to give a subsidy proportional to the housing cost and which can only be used to buy housing. The simulation results for different levels of housing subsidies are summarized in Table 6. Similar to Table 5, the upper part of Table 6 shows the absolute changes and the lower part the relative changes.

We observe that, when more generous rates of subsidy are offered, demand for housing grows more than proportionally. In other words, housing demand responds more strongly as the percentage point change of the subsidy rate increases. For example, when the subsidy rate is 2% (a 2 percentage point change from the status quo), housing demand increases by 2.04% compared with the status quo. When the subsidy rate changes from 2% to 4% (the same 2 percentage point change), housing demand shows a 2.13 percentage point increase. Given our model assumptions, the housing subsidy does not affect the consumption of other goods. The reason is that the substitution effect exactly offsets the income effect. Finally, the utility exhibits a convex increase in response to any additional housing subsidy.

In the second period, the demand for housing changes by the same amount as in period 1. However, the response of the utility to the housing subsidy in period 2 is larger compared with period 1. When the subsidy rate is 6%, for example, the utility increase in period 1 is 1.88% and the increase in period 2 is 1.94%. The increase in lifetime utility (last column) is 1.91%, which is between the percentage change in the first period and the second period. As the housing subsidy increases the demand for housing, we observe a corresponding increase in the housing price in the short run.

Table 6: Housing Subsidy Numerical Simulation

subsidy rate	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ +βH ₂	C ₁ +βC ₂	U ₁ +βU ₂
0%	5.91	4.55	3.29	4.42	1.50	5.32	4.54	3.18	10.97	8.86	6.32
2%	6.03	4.55	3.31	4.42	1.53	5.43	4.54	3.20	11.19	8.86	6.36
4%	6.16	4.55	3.33	4.42	1.56	5.54	4.54	3.23	11.43	8.86	6.40
6%	6.29	4.55	3.35	4.42	1.60	5.66	4.54	3.25	11.67	8.86	6.44
8%	6.43	4.55	3.38	4.42	1.63	5.79	4.54	3.27	11.93	8.86	6.48
10%	6.57	4.55	3.40	4.42	1.67	5.91	4.54	3.29	12.19	8.86	6.52
subsidy rate	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ +βH ₂	C ₁ +βC ₂	U ₁ +βU ₂
0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	2.04%	0.00%	0.61%	0.00%	2.04%	2.04%	0.00%	0.63%	2.04%	0.00%	0.62%
4%	4.17%	0.00%	1.24%	0.00%	4.17%	4.17%	0.00%	1.28%	4.17%	0.00%	1.26%
6%	6.38%	0.00%	1.88%	0.00%	6.38%	6.38%	0.00%	1.94%	6.38%	0.00%	1.91%
8%	8.70%	0.00%	2.53%	0.00%	8.70%	8.70%	0.00%	2.62%	8.70%	0.00%	2.57%
10%	11.11%	0.00%	3.20%	0.00%	11.11%	11.11%	0.00%	3.31%	11.11%	0.00%	3.25%

Source: Authors' calculations.

6.1.3 Mortgage Interest Rate Reduction

Table 7 indicates that when the mortgage interest rate is lowered, demand for housing units and other consumption goods in period 1 increases. When the interest rate falls from 5% to 4% (a 1 percentage point decrease from the status quo), the housing units and other goods demanded increase by 0.47%, resulting in a 0.29% utility increase. The 1 percentage point reduction in the mortgage interest rate leads to 1.44% greater use of mortgages. In our parameter settings, the increase in demand for housing units and consumption goods exhibits a linear relation to the percentage point change in the mortgage interest rate.

Table 7: Numerical Simulation of Mortgage Interest Rate

r	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ +βH ₂	C ₁ +βC ₂	U ₁ +βU ₂
5%	5.91	4.55	3.29	4.42	1.50	5.32	4.54	3.18	10.97	8.86	6.32
4%	5.94	4.57	3.30	4.48	1.51	5.35	4.52	3.18	11.02	8.86	6.33
3%	5.97	4.59	3.31	4.55	1.51	5.37	4.49	3.18	11.08	8.86	6.34
2%	6.00	4.62	3.32	4.62	1.52	5.40	4.47	3.18	11.13	8.86	6.35
1%	6.03	4.64	3.33	4.68	1.53	5.43	4.45	3.18	11.19	8.87	6.36
r	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ +βH ₂	C ₁ +βC ₂	U ₁ +βU ₂
5%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4%	0.47%	0.47%	0.29%	1.44%	0.47%	0.47%	-0.48%	0.00%	0.47%	0.01%	0.15%
3%	0.96%	0.96%	0.58%	2.90%	0.96%	0.96%	-0.97%	-0.01%	0.96%	0.02%	0.30%
2%	1.45%	1.45%	0.87%	4.40%	1.45%	1.45%	-1.45%	-0.01%	1.45%	0.04%	0.45%
1%	1.95%	1.95%	1.17%	5.92%	1.95%	1.95%	-1.93%	-0.01%	1.95%	0.06%	0.61%

Source: Authors' calculations.

While housing consumption in period 2 increases at the same rate as in period 1, the change in the demand for consumption goods is negative. In case of a 1 percentage point decrease in the interest rate from 5%, housing demand increases by 0.47%, whereas consumption of other goods falls by 0.48%. Why? The change in the utility in the second period is close to zero, irrespective of the decrease in mortgage interest rates. However, the lifetime utility becomes larger compared with the status quo. For example, a 4 percentage point decrease in the interest rate leads to a 0.61% increase in lifetime utility. As observed above, the demand increase pushes up the housing price in the short run at the same rate.

6.1.4 Mortgage Interest Rate Deduction from Income Tax

When mortgage interest deduction from income tax is introduced, income in the second period increases by $rt_h L^*$ (Table 8). The mortgage interest deduction increases demand for housing and consumption goods equally, by 0.24%, in the first period. It increases utility by 0.14% and mortgage use by 0.72%. In period 2, however, demand for consumption goods falls by 0.24%. In contrast, demand for housing increases by 0.24% compared with the status quo.⁸ Overall, demand for housing increases by 0.24%, whereas demand for consumption goods stays the same. Lifetime utility increases by 0.07%. We estimate a 0.24% increase in the housing price after the introduction of the deduction system.

Table 8: Effect of Mortgage Interest Deduction from Income Tax

	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ +βH ₂	C ₁ +βC ₂	U ₁ +βU ₂
status quo	5.91	4.55	3.29	4.42	1.50	5.32	4.54	3.18	10.97	8.86	6.32
MID	5.93	4.56	3.30	4.45	1.50	5.34	4.53	3.18	11.00	8.86	6.32
	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ +βH ₂	C ₁ +βC ₂	U ₁ +βU ₂
status quo	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
MID	0.24%	0.24%	0.14%	0.72%	0.24%	0.24%	-0.24%	0.00%	0.24%	0.00%	0.07%

Source: Authors' calculations.

6.1.5 Quality Improvement

In our model, quality improvements translate into a lower depreciation rate. In the following numerical example, we will only look at the demand-side effects of a lower

⁸ As a result, utility in the second period marginally falls, from 3.18451 to 3.18445.

depreciation rate and ignore possible supply-side effects. As Table 9 shows, quality improvements do not affect demand or utility in the first period, but it increases housing consumption in period 2, resulting in higher utility in period 2. A 2% decrease in the depreciation rate leads to a 2.22% increase in the housing demand in period 2. Utility responds to a fall in the depreciation rate in a concave way, i.e., the marginal increase in utility becomes smaller as the depreciation rate decreases.

When the depreciation rate is 2 percentage point lower, overall demand for housing units increases by 1.02%. Lifetime utility also shows a concave relation with the depreciation rate reduction. When the depreciation rate changes from 10% to 4% (a 6 percentage point decrease from the status quo), lifetime utility increases by 0.97%. In the short run, the housing price does not change.⁹

Table 9: Numerical Simulation of Quality Improvement

δ	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ + β H ₂	C ₁ + β C ₂	U ₁ + β U ₂
10%	5.91	4.55	3.29	4.42	1.50	5.32	4.54	3.18	10.97	8.86	6.32
8%	5.91	4.55	3.29	4.42	1.50	5.44	4.54	3.21	11.08	8.86	6.34
6%	5.91	4.55	3.29	4.42	1.50	5.56	4.54	3.23	11.20	8.86	6.36
4%	5.91	4.55	3.29	4.42	1.50	5.68	4.54	3.25	11.31	8.86	6.38
2%	5.91	4.55	3.29	4.42	1.50	5.80	4.54	3.27	11.42	8.86	6.40
δ	H ₁	C ₁	U ₁	L	P _{h short run}	H ₂	C ₂	U ₂	H ₁ + β H ₂	C ₁ + β C ₂	U ₁ + β U ₂
10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
8%	0.00%	0.00%	0.00%	0.00%	0.00%	2.22%	0.00%	0.69%	1.02%	0.00%	0.33%
6%	0.00%	0.00%	0.00%	0.00%	0.00%	4.44%	0.00%	1.37%	2.05%	0.00%	0.65%
4%	0.00%	0.00%	0.00%	0.00%	0.00%	6.67%	0.00%	2.03%	3.07%	0.00%	0.97%
2%	0.00%	0.00%	0.00%	0.00%	0.00%	8.89%	0.00%	2.67%	4.10%	0.00%	1.28%

Source: Authors' calculations.

6.1.6 Property Tax on the Purchase of Housing

The introduction of a property tax lowers demand for housing units (Table 10). However, it does not affect demand for consumption goods, because the substitution effect and the income effect exactly offset each other. The decrease in demand for housing is less than proportional to the change in the rate of the property tax. In other words, the marginal decrease in housing demand becomes smaller as the tax rate becomes higher. Similarly, utility is decreasing, but its marginal change becomes smaller as the interest rate of the tax becomes higher. For example, if the rate of property tax changes from 0% to 2% (a 2 percentage point change from the status quo), housing demand and utility decrease by 1.96% and 0.60%, respectively. However, when it changes from 2% to 4% (the same 2 percentage point change), they decrease by 1.89% and by 0.59%, respectively.

In the second period, demand for housing units decreases at the same rate as in the first period in response to the increase in the tax rate. The tax does not affect consumption goods for the same reason as mentioned above. The percentage point loss of utility in period 2 is bigger than that in period 1. Therefore, the change rate of lifetime utility lies between the change rate in the first period and that in the second period. The percentage point change of overall demand for housing units shows the same value as in period 1 and in period 2. Property tax lowers the short-run housing price at the same rate as the fall in housing demand.

⁹ However, the housing price will decrease in the long run due to the supply-side effect of the policy, as discussed in Section 5.1.6.

Table 10: Numerical Simulation of Property Tax Interest

ty	H1	C1	U1	L	Ph short run	H2	C2	U2	H1+βH2	C1+βC2	U1+βU2
0%	5.91	4.55	3.29	4.42	1.50	5.32	4.54	3.18	10.97	8.86	6.32
2%	5.80	4.55	3.27	4.42	1.47	5.22	4.54	3.16	10.76	8.86	6.28
4%	5.69	4.55	3.25	4.42	1.44	5.12	4.54	3.15	10.55	8.86	6.24
6%	5.58	4.55	3.23	4.42	1.42	5.02	4.54	3.13	10.35	8.86	6.20
8%	5.48	4.55	3.22	4.42	1.39	4.93	4.54	3.11	10.16	8.86	6.17
10%	5.38	4.55	3.20	4.42	1.36	4.84	4.54	3.09	9.97	8.86	6.13
ty	H1	C1	U1	L	Ph short run	H2	C2	U2	H1+βH2	C1+βC2	U1+βU2
0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	-1.96%	0.00%	-0.60%	0.00%	-1.96%	-1.96%	0.00%	-0.62%	-1.96%	0.00%	-0.61%
4%	-3.85%	0.00%	-1.19%	0.00%	-3.85%	-3.85%	0.00%	-1.23%	-3.85%	0.00%	-1.21%
6%	-5.66%	0.00%	-1.77%	0.00%	-5.66%	-5.66%	0.00%	-1.83%	-5.66%	0.00%	-1.80%
8%	-7.41%	0.00%	-2.34%	0.00%	-7.41%	-7.41%	0.00%	-2.42%	-7.41%	0.00%	-2.38%
10%	-9.09%	0.00%	-2.89%	0.00%	-9.09%	-9.09%	0.00%	-2.99%	-9.09%	0.00%	-2.94%

Source: Authors' calculations.

6.1.7 Subsidy for Tenants

A subsidy increases demand for rental houses and for consumption goods equally by 5.50% (Table 11). Because of the diminishing marginal utility, the marginal response of utility to the additional subsidy becomes smaller and smaller as the amount of subsidy increases. The second period shows almost the same percentage increase, but the change in utility is slightly bigger than that in the first period. In both periods, the housing price surges in the short run to equalize the increased housing demand. Lifetime utility follows the same concave trend and its increase is between that in the first period and that in the second period.

Table 11: Numerical Simulation of Cash Subsidy for Rental Houses

G1	G2	R1	C1	U1	Pr 1 short run	R2	C2	U2	Pr 2 short run	R1+βR2	C1+βC2	U1+βU2
0	0	6.07	4.55	3.32	0.75	6.05	4.54	3.31	0.75	11.81	8.86	6.46
0.5	0.5	6.40	4.80	3.42	0.79	6.38	4.79	3.42	0.79	12.46	9.35	6.67
1	1	6.73	5.05	3.53	0.83	6.72	5.04	3.52	0.83	13.11	9.84	6.87
1.5	1.5	7.07	5.30	3.62	0.87	7.05	5.29	3.62	0.87	13.76	10.32	7.06
2	2	7.40	5.55	3.72	0.92	7.38	5.54	3.71	0.92	14.41	10.81	7.24
2.5	2.5	7.73	5.80	3.80	0.96	7.72	5.79	3.80	0.96	15.06	11.30	7.41
G1	G2	R1	C1	U1	Pr 1 short run	R2	C2	U2	Pr 2 short run	R1+βR2	C1+βC2	U1+βU2
0	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.5	0.5	5.50%	5.50%	3.23%	5.50%	5.50%	5.50%	3.23%	5.50%	5.50%	5.50%	3.23%
1	1	11.00%	11.00%	6.29%	11.00%	11.00%	11.00%	6.30%	11.00%	11.00%	11.00%	6.30%
1.5	1.5	16.51%	16.51%	9.21%	16.51%	16.51%	16.51%	9.22%	16.51%	16.51%	16.51%	9.22%
2	2	22.01%	22.01%	11.99%	22.01%	22.01%	22.01%	12.01%	22.01%	22.01%	22.01%	12.00%
2.5	2.5	27.51%	27.51%	14.65%	27.51%	27.51%	27.51%	14.67%	27.51%	27.51%	27.51%	14.66%

Source: Authors' calculations.

6.1.8 Rental Subsidy

In case the household receives a rental subsidy in both periods, the household increases its housing consumption in both periods (Table 12). In contrast to housing demand, demand for consumption goods stays the same in both periods, because the substitution effect exactly offsets the income effect under the assumption, $\theta = \omega = 1$. The utility also increases in response to the introduction of the rental subsidy. Since both periods show the same increase, lifetime demand and utility follow the same

convex response. As in the other cases, the short-run rental price in both periods increases in response to the housing demand increase.

Table 12: Numerical Simulation of Rent Aid

subsidy rate	R1	C1	U1	Pr 1 short run	R2	C2	U2	Pr 2 short run	R1+βR2	C1+βC2	U1+βU2
0%	6.07	4.55	3.32	0.75	6.05	4.54	3.31	0.75	12.12	9.09	6.46
2%	6.19	4.55	3.34	0.77	6.17	4.54	3.33	0.77	12.36	9.09	6.50
4%	6.32	4.55	3.36	0.78	6.30	4.54	3.35	0.78	12.62	9.09	6.54
6%	6.45	4.55	3.38	0.80	6.44	4.54	3.37	0.80	12.89	9.09	6.59
8%	6.59	4.55	3.40	0.82	6.58	4.54	3.40	0.82	13.17	9.09	6.63
10%	6.74	4.55	3.42	0.83	6.72	4.54	3.42	0.83	13.46	9.09	6.67
subsidy rate	R1	C1	U1	Pr 1 short run	R2	C2	U2	Pr 2 short run	R1+βR2	C1+βC2	U1+βU2
0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	2.04%	0.00%	0.61%	2.04%	2.04%	0.00%	0.61%	2.04%	2.04%	0.00%	0.61%
4%	4.17%	0.00%	1.23%	4.17%	4.17%	0.00%	1.23%	4.17%	4.17%	0.00%	1.23%
6%	6.38%	0.00%	1.87%	6.38%	6.38%	0.00%	1.87%	6.38%	6.38%	0.00%	1.87%
8%	8.70%	0.00%	2.51%	8.70%	8.70%	0.00%	2.52%	8.70%	8.70%	0.00%	2.52%
10%	11.11%	0.00%	3.18%	11.11%	11.11%	0.00%	3.18%	11.11%	11.11%	0.00%	3.18%

Source: Authors' calculations.

6.2 Comparison of Housing Policies

Suppose that the government decides to intervene in the housing market by providing a subsidy. One of the most important questions is about the effectiveness of its intervention. In other words, what is the increase in households' welfare as a result of a particular policy intervention? As our main interest lies in those policies intended to make housing more accessible for low-income households, we focus solely on policies for stimulating housing demand (and not on policies that suppress it).

We make a comparison based on an analysis of cost effectiveness. The costs of a policy are defined as the discounted government expenditure or the discounted revenue loss of the government over the two periods. Table 13 shows the details of how the policy costs are calculated. For instance, the policy cost of a lower mortgage interest rate can be interpreted in two ways. First, the lower interest rate lessens the revenue of the government and the lost revenue is measured as policy cost. The second interpretation is that the government compensates the private financial sectors for its losses caused by offering a lower mortgage interest rate compared with the market interest rate.

Table 13: Policy Cost for Housing Demand Policies

Policy	Policy Cost (discounted)
Cash subsidy for potential homeowners	G_1
Housing subsidy	$(subsidy\ rate)P_hH_1$
Mortgage interest rate reduction	βrL^*
Mortgage interest deduction form income tax	$\beta r t_y L^*$
Cash subsidy for tenants	$G_1 + \beta G_2$
Rent aid	$(subsidy\ rate)P_rR_1 + \beta(subsidy\ rate)P_rR_2$

Source: Authors.

The change in discounted lifetime utility for each household measures the benefit of each housing policy. In Table 14, we compare the effectiveness of four housing policies

that have the same cost (0.02215).¹⁰ The second last column of Table 14 shows the absolute change in utility after the introduction of the new policy. The final column lists the percentage change in utility compared with the status quo. Comparing the effectiveness on the basis of the size of the increase in utility, Table 14 indicates that the mortgage interest reduction policy yields the highest return in terms of welfare increase. The mortgage interest deduction has the second highest policy effectiveness. As we discussed in section 5.1.2, housing subsidy is less efficient than cash subsidy when the amount of the subsidy necessary for them is the same. This theoretical prediction is confirmed in our numerical simulation. A housing subsidy thus becomes the least effective policy in our table.

Table 14: Comparison of the Effectiveness of the Demand Policy for Homeowners

	Policy	Policy variable	Change		Policy Cost	Δ Utility	Δ Utility(%)
			From	To			
Owner occupied house	Cash subsidy for potential homeowners	G_1	0	0.02115	0.02115	0.004646	0.07354%
	Housing subsidy	Subsidy rate	0%	0.23783%	0.02115	0.004643	0.07350%
	Mortgage interest rate reduction	r	5%	4.46204%	0.02115	0.005004	0.07921%
	Mortgage interest deduction from income tax	$rtyL^*$	0	0.02115	0.02115	0.004649	0.07360%

Source: Authors.

Similar to the government interventions in the owner-occupied housing market, we can compare interventions in the rental housing market. Table 15 shows the effectiveness of two basic policy interventions in the rental housing market—cash subsidies for tenants and rent aid. In contrast to the case of owner-occupied housing, we assume that the government grants a subsidy in both periods. The policy cost is thus discounted by β . Our simulations indicate that a cash subsidy is preferable to rent aid. This result is consistent with what our microeconomic model predicted in Section 5.2.3.

Table 15: Comparison of the Effectiveness of the Demand Policy for Tenants

	Policy	Policy variable	Change		Policy Cost	Δ Utility	Δ Utility(%)
			From	To			
Rental house	Cash subsidy for tenants	G_1 and G_2	0	0.51781	1.00973	0.216116	3.34299%
	Rent aid	Subsidy rate	0%	10%	1.00973	0.205453	3.17805%

Source: Authors.

7. CONCLUSION AND DISCUSSION

Together with food and clothing, housing can be considered as one of three basic material needs of every person. To respond to these needs, governments around the world have made various efforts to facilitate access to housing for their citizens, especially for lower-income groups.

The objective of this paper was to give an overview of some of the most commonly applied housing policies and to illustrate their impact on households' welfare. To facilitate the analysis, we first set up a simple two-period housing demand model for owner-occupied houses and rental houses. We then introduced a standard stock-flow housing supply model. Given this modelling framework, we explained the qualitative

¹⁰ To compare the effectiveness, the ratio between benefits and costs are often used as a criterion for comparison. However, since our utility function is non-linear, the marginal benefit per additional one-unit cost is not independent of the level of the cost. We therefore need to hold the policy costs constant to be able to properly compare the effectiveness of each policy.

effects of various housing policies on supply and demand. In the last section of the paper, we provided an estimation of the quantitative impact of each policy. We hope that the model's versatility makes it a simple tool for policymakers to better understand the economic consequences of various housing policies.

The theoretical model presented in this paper can be extended in several directions. One possibility is to explicitly model the externalities that housing exhibits to society. For example, well-maintained houses not only help to increase the health of the dwellers, but also exhibit a positive externality to the neighborhood. Another option would be to include moving costs in our model. It is well documented in the literature (e.g., O'Sullivan 1996) that moving costs may prevent households from benefitting from better housing opportunities. Yet another interesting extension could be to model the link between the markets for homeowners and renters. The implementation of a policy in one of the two markets certainly affects the other one. Finally, like for food and clothing, housing conditions need to meet certain standards to be adequate and deliver the expected benefits. In our paper, we assumed that all housing units meet the same standards. However, in many developing countries, large numbers of dwellings are below the minimum standard. A more nuanced analytical approach to the question of housing standards could be a worthwhile undertaking.

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APPENDIX

Derivation of Equations (10)–(13)

Setting the following Lagrangian function,

$$L = u(C_1, H_1) + \beta u(C_2, H_2) + \lambda \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_1}{1+r} - C_1 - \frac{C_2}{1+r} - (1+t_h)P_h H_1 \right\} \quad (42)$$

or

$$L = \ln(C_1) + b \ln(H_1) + \beta \{ \ln(C_2) + b \ln[(1-\delta)H_1] \} + \lambda \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_1}{1+r} - C_1 - \frac{C_2}{1+r} - (1+t_h)P_h H_1 \right\} \quad (43)$$

Taking first order conditions with respect to C_1 , C_2 , H_1 , and λ , we obtain

$$\frac{1}{C_1} = \lambda \quad (44)$$

$$\frac{\beta}{C_2} = \lambda \frac{1}{1+r} \quad (45)$$

$$\frac{b}{H_1} + \frac{\beta b}{H_1} = \lambda(1+t_h)P_h \quad (46)$$

$$(1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_1}{1+r} = C_1 + \frac{C_2}{1+r} + (1+t_h)P_h H_1 \quad (47)$$

Substituting (44) into (45) and (46), we get

$$\beta(1+r)C_1 = C_2 \quad (48)$$

$$b(1+\beta)C_1 = (1+t_h)P_h H_1 \quad (49)$$

(47) to (49) and $H_2 = (1-\delta)H_1$ give

$$H_1^* = \frac{b}{(1+t_h)(1+b)P_h} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\}$$

$$H_2^* = (1-\delta)H_1^* = \frac{(1-\delta)b}{(1+t_h)(1+b)P_h} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\}$$

$$C_1^* = \frac{1}{(1+\beta)(1+b)} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\}$$

$$C_2^* = \frac{\beta(1+r-rt_y)}{(1+\beta)(1+b)} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r-rt_y} + G_1 + \frac{G_2}{1+r-rt_y} \right\}$$

Derivation of Equations (24)–(27)

Setting the following Lagrangian function,

$$L = u(C_1, R_1) + \beta u(C_2, R_2) + \lambda \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_1}{1+r} - C_1 - \frac{C_2}{1+r} - P_r R_1 - \frac{P_r R_2}{1+r} \right\} \quad (50)$$

or

$$L = \ln(C_1) + b \ln(R_1) + \beta \{ \ln(C_2) + b \ln[R_2] \} + \lambda \left\{ (1+t_y)Y_1 + \frac{(1+t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_1}{1+r} - C_1 - \frac{C_2}{1+r} - P_r R_1 - \frac{P_r R_2}{1+r} \right\} \quad (51)$$

Taking first order conditions with respect to C_1, C_2, R_1, R_2 , and λ we obtain

$$\frac{1}{C_1} = \lambda \quad (52)$$

$$\frac{\beta}{C_2} = \lambda \frac{1}{1+r} \quad (53)$$

$$\frac{b}{R_1} = \lambda P_r \quad (54)$$

$$\frac{\beta b}{R_2} = \lambda \frac{P_r}{1+r} \quad (55)$$

$$(1+t_y)Y_1 + \frac{(1+t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_1}{1+r} = C_1 + \frac{C_2}{1+r} + P_r H_1 + \frac{P_r R_2}{1+r} \quad (56)$$

Substituting (52) into (53), (54) and (55), we get

$$\beta(1+r)C_1 = C_2 \quad (57)$$

$$bC_1 = P_r R_1 \quad (58)$$

$$\beta b(1+r)C_1 P_r R_2 \quad (59)$$

(56) to (59) give

$$R_1^* = \frac{b}{(1+\beta)(b+1)P_r} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\}$$

$$R_2^* = \frac{b\beta(1+r)}{(1+\beta)(b+1)P_r} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\}$$

$$C_1^* = \frac{1}{(1+\beta)(b+1)} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\}$$

$$C_2^* = \frac{\beta(1+r)}{(1+\beta)(b+1)} \left\{ (1-t_y)Y_1 + \frac{(1-t_y)(1+g)Y_1}{1+r} + G_1 + \frac{G_2}{1+r} \right\}$$