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## A Model Of Commuting Distances: Some Preliminary Insights From A Spatial Model Of Job Search

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# A Model of Commuting Distances: Some Preliminary Insights from a Spatial Model of Job Search

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## Abstract

A spatial model of job search is developed. From this model some results that are important in the analysis of commuting distances and labour market interaction are highlighted. The discussion focuses on labour market interactions, the spatial weights matrix and functional form in empirical work.

**Keywords:** Job search, spatial model, commuting distances, labour market interaction, spatial weights matrix

**JEL Classification:** R1, R23

## 1. Introduction

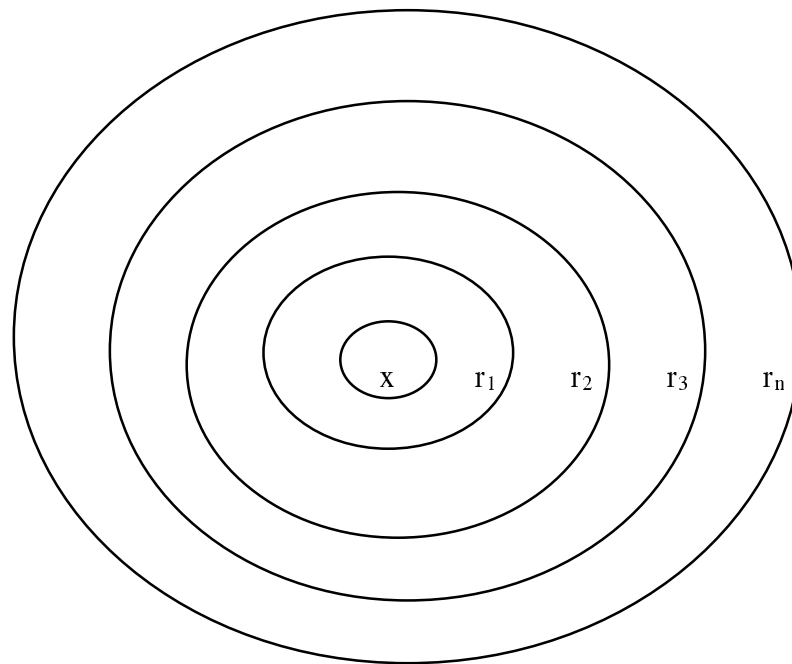
Fundamental changes in the organisation of production and the growth towards a service and information based economy have strong implications for commuting distances and the configuration of labour market areas (Van der Lann, 1998; Mabogunje, 1980). These concepts are increasingly seen as important in small area empirical models of spatial development where the challenge is to understand intrametropolitan population and employment movements at a very fine level of spatial detail. Well known examples of these models include that of Carlino and Mills (1987) who use a two equation simultaneous system to provide an empirical structure for reconciling the employment/population direction of causality issue in county level development processes. Boarnet (1994) provides an extension of the Carlino/Mills model by explicitly recognising that city labour markets are larger than the municipality of residence of the labour force. Recognising that location within a region matters, Boarnet adjusts for the mismatch between locations of employment and locations of residences by constructing labour market potential variables that capture interactions that are a function of distances between zones and the numbers of employees and residents in each zone.

This paper gives a spatial orientation to the well established economic theory of job search (McKenna, 1990) in order to derive some insights into labour market and commuting behaviour. The results from the model provide some useful preliminary background for the modelling of empirically observed commuting distances and labour market interactions between areas.

## 2. Spatial Job Search and Commuting Distances

The spatial model is one that describes jobs as distributed evenly over space away from the searcher. The wages of all jobs,  $W$ , is the same but the distance between home and work differs between the various job offers. Thus, the searcher evaluates jobs over some geographical range  $r_n$  on the basis of the wage minus commuting costs. It is assumed that commuting costs are proportional to distance  $r$ , with  $t$  the costs per kilometre. Thus, since jobs are distributed over space, the wage minus commuting costs ( $W - tr$ ) varies. There is also an opportunity cost,  $C$ , associated with each unit of search activity. Using the concept

of the reservation wage, defined as the minimal acceptable wage, the searcher can evaluate each opportunity in turn (sequentially) and compare the wage offer ( $W \setminus tr$ ) with their reservation wage. When the worker uncovers an offer which at least matches their reservation wage they will stop searching and accept that job. The spatial model is illustrated in Figure 1.



**Figure 1. Spatial Job Market**

In the aspatial version of this model the problem is defined as finding the optimal reservation wage  $W^*$  which will maximise the net return from job search activity. In this spatial version this optimal reservation wage is measured as  $W^* = W \setminus tr^*$  so that information about the optimal reservation wage which maximises the net return from job search activity will also yield information about a bound on the commuting distance  $r^*$ .

In this sequential decision rule search model (Smith, 1994) there are only two possible outcomes: finding a wage offer where  $W \geq (W \geq tr^*)$  or obtaining a wage offer such that  $W < (W \geq tr^*)$ . Thus:

$$\boxed{\hspace{10em}} \tag{1}$$

Given that the individual is exploring each location in turn, the expected return from searching a single site is

$$\boxed{\hspace{10em}} \tag{2}$$

which McKenna (1990, 44 – 45) interprets as "The expected return to searching once using a reservation wage,  $W^*$ , is the mean (expected) wage of all wages no less than  $W^*$ . ... multiplied by the probability of finding such a wage, plus the expected return to searching once more multiplied by the probability that an unacceptable wage is found, less the cost of the first search." Solving (2) for  $R(W^*)$  we obtain the following expression:

$$\boxed{\hspace{10em}} \tag{3}$$

where:

$$\boxed{\hspace{5em}}$$

and

$$\boxed{\hspace{10em}}$$

The optimal reservation wage  $W^*$  in the spatial version of the search model is (4), see appendix A

$$\boxed{\hspace{10em}} \tag{4}$$

This equation (4) is then maximised over  $r^*$  to reveal the “reservation” commuting distance associated with this optimal reservation wage. This concept of a reservation commuting distance can also be found in the model of Rouwendal (1998). The general solution used to generate this reservation distance, equation 6a in Appendix A, exhibits two types of behaviour depending on the extent of the geographical area over which the job searcher chooses to search. If a small area is searched, a reservation distance does not exist, see Figure 2a, but when the area is large there is a well defined reservation commuting distance, see Figure 2b. Some implications of these different results are discussed more fully in the next two sections of the paper.

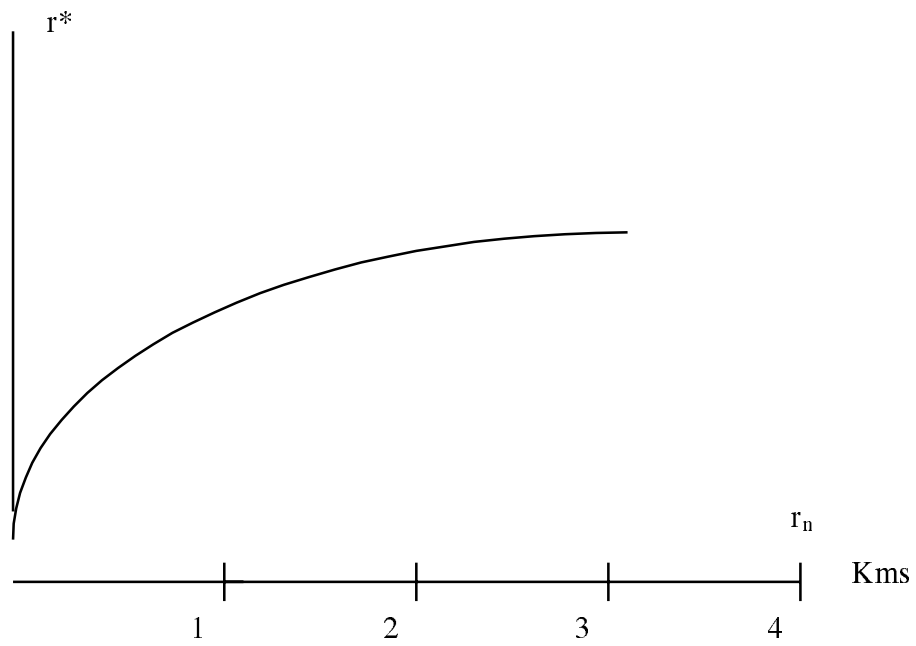


Figure 2a. A Small Geographical Range  $r_n$  and Commuting Distance  $r^*$

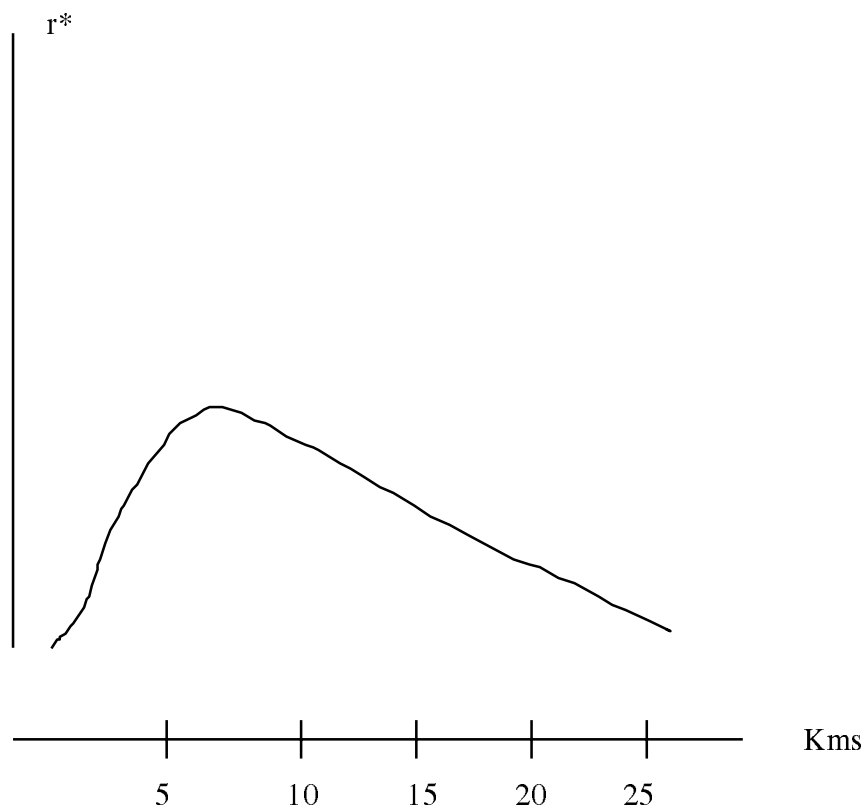


Figure 2b. A Large Geographical Range and Commuting Distance  $R^*$

**Figure 2. Geographical Range and Commuting Distance**



### 3. Labour Market Interaction and the Spatial Weights Matrix

The trend towards an information oriented economy, particularly reflected in the presence of business services, suggests new patterns of commuting across spatial zones. This interaction connects locations and defines new commuting sheds and labour market areas. This labour market interaction between zones is expected to decrease as the distances involved increase. In the empirical work cited earlier, this feature is captured by a spatial weights matrix  $W = \{w_{ij}\}$  with  $w_{ij} \downarrow 0$  as interactions fall. In some applications the row or column sums of  $W$  are scaled to one. Two examples that are commonly used (Haining, 1990) are:

- (a)  $w_{ij} = d_{ij}^{-\gamma}$  ( $\gamma \geq 0$ ) where  $d_{ij}$  is the distance between  $i$  and  $j$  and  $\gamma$  is a constant
- (b)  $w_{ij} = \exp(-\beta d_{ij}^\gamma)$  where  $\exp$  denotes the exponential function

The  $W$  matrix used in Boarnet (1994), for example, has as its elements  $d_{ij}^{-\alpha}$  where again the parameter  $\alpha$  describes how the labour market interactions decrease with distance. Since the strength of this labour market interaction is based on commuting patterns across US municipalities, the value of  $\alpha$  ( $= 0.67$ ) was estimated by Boarnet from US census commuting data. In contrast, in a recent extension of the Boarnet model to Denmark, Kristensen and Henry (1997) set  $w_{ij} = 1$  if municipalities  $i$  and  $j$  are within the same commuting shed of each other and  $w_{ij} = 0$  otherwise. This decision is consistent with Bartels (1979) who suggests that simple binary weights are as adequate as more complex schemes in most instances. An earlier and detailed discussion on the choice of  $W$  can be found in Cliff and Ord (1973) and in Haggett et al. (1977).

The spatial model outlined here strongly supports the exponential form,  $w_{ij} = \exp(-\beta d_{ij}^\gamma)$ . This result is illustrated with the aid of Figure 3. Figure 3a describes the relationship between  $r_n$  and  $r^*$ , as defined by equation 6a in the Appendix. Figure 3b illustrates how labour interaction declines with distance. The measure of interaction is expressed as the ratio of  $r^*$  to  $r_n$  as shown in 3a. As suggested earlier, there is a small geographical range over which labour interactions are fully maximised (these are areas that are, in Kristensen's and Henry's terms, within the same commuting shed of each other) but beyond that range interaction declines exponentially. It is true, of course, that

the particular characterisation that is shown in Figure 3b reflects a given set of values for  $C$  and  $t$  in the underlying model. An increase in  $t$ , for example, will reduce the range over which interaction is maximised and will also strengthen the degree of distance decay present. The general shape of the result in 3b is, however, unaffected by changes in parameter values.

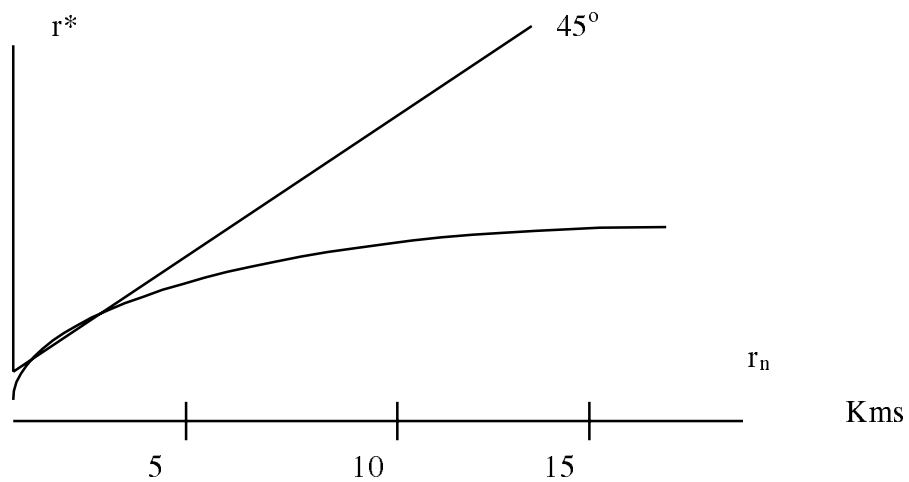


Figure 3a. Geographical Range and Commuting Distance

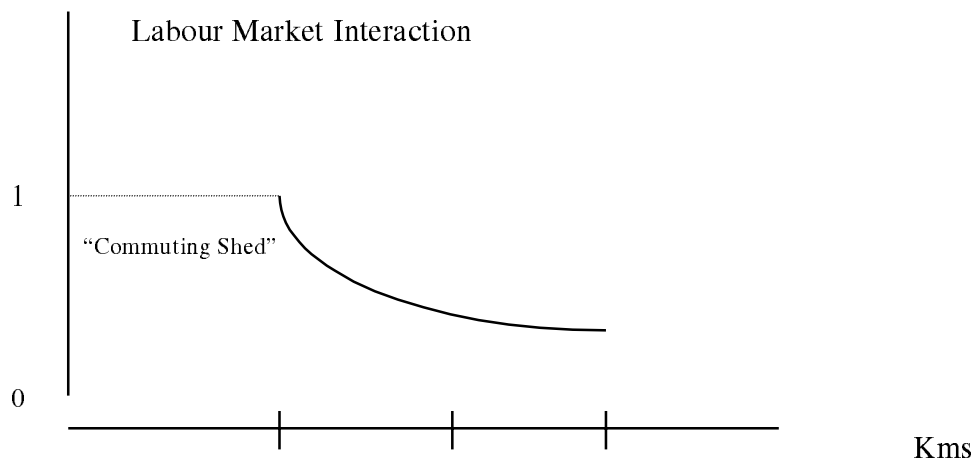


Figure 3b. Labour Market Interaction and Distance

Figure 3. Geographical Range, Commuting Distances and Labour Market Interactions

#### 4. Commuting Distances: An Empirical Test

The reservation commuting distance defines the extent of the commuting that will take place from a given location  $i$ . What we see from the model is that this distance increases as a result of households increasing their search area but does so at a decreasing rate, see Figure 3a. We can briefly compare this relationship with some commuting data. The data is from the Census of Population Ireland 1996. The empirical relationship is average commuting distances and spatial job search area.

In rural areas jobs and people are unevenly distributed and this reality must be accommodated in devising any measure of potential search area. Clearly the number of jobs in all other parts of a region will have an influence on the search area used by an individual located at  $i$ . The increasing likelihood is that it will be the set of urban locations that matters in defining spatial job search. Houston (1998) discusses a measure of spatial job accessibility which is adopted here, albeit in a simplified, and more generalised, form. The assumption made is that a higher level of accessibility will encourage workers to engage in more extensive search. The measure used is:

$$A_i = \frac{P_i}{\sum_o P_o d_{io}}$$

where  $A_i$  = spatial accessibility of a worker living at location  $i$ ;  $P_i$  = population of nearby (within a distance of 8kms.) urban place(s);  $P_o$  = population in each other urban place; and  $d_{io}$  = straight-line distance of each other place from location  $i$ .

Average commuting data computed for households in 15 randomly chosen rural locations (District Electoral Divisions), along with spatial accessibility measures, are reported in Table 4. The least-squares fit for both the model and empirical relationship are shown in Figure 4. The analysis supports the idea that spatial interactions increase in complexity as spatial accessibility increases.

Table 4. Average Commuting Distance and Accessibility Measures for 15 rural areas

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<b>Location</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>
<b>Distance</b>	2.5	5.7	3.2	3.3	2.8	5.3	4.6	6.1	6.0	6.2	4.2	3.5	4.8	4.4	5.3
<b>Accessill bility</b>	1.8	15.6	0.6	0.6	4.0	4.0	0.2	21.3	4.0	21.3	5.0	0.2	2.4	1.9	15.6

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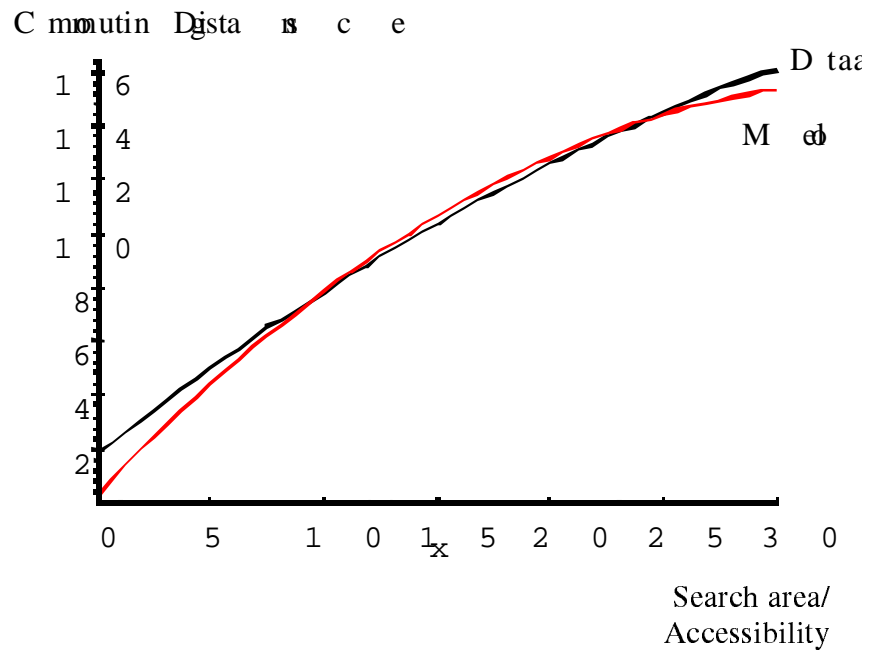


Figure 4. Least Squares Fit

## 5. Conclusions

There is a renewed research interest in local labour markets and in how they work. A serious problem encountered in empirical models of local labour markets and employment and population change concerns model specification. One particular set of issues is how the spatial weight matrix that is used in these models to construct labour market potentials and define labour market areas is itself constructed and how sensitive the empirical results are to different constructions. A spatial model of search has been outlined here as a basic theoretical framework that appears to be useful in thinking about some of the modelling questions involved. Despite its initially simple form, the model is quite fruitful in terms of the insights it offers into labour market interactions. In addition, this model of search suggests that the appropriate function in any exercise that seeks to relate commuting distance, or the distribution of commuting distances, with the spatial distribution of employment opportunities around the home location is a logarithmic form. In fact, this specification can be found in some of the recent empirical literature on commuting distances (Rouwental and Rietveld, 1994; Immergluck, 1998).

APPENDIX A

Figure 1 describes the spatial model. Table 1A summarises the probabilities faced by the searcher located at  $x$ , see Figure 1.

**Table 1A: Wage Probabilities**

Wage	Number of Possibilities	Probability of Wage ( $W/r_i$ )
$W \leq r_1$	$\Pi r_1^2$	$r_1^2/r_n^2$
$W \leq r_2$	$\Pi r_2^2 \square \Pi r_1^2$	$(r_2^2 \square r_1^2)/r_n^2$
.	.	.
.	.	.
.	.	.
$W \leq r^*$	$\Pi r^{*2} \square \Pi r_{n01}^2$	$(R^{*2} \square r_{n01}^2)/r_n^2$
.	.	.
.	.	.
.	.	.
$W \leq r_n$	$\Pi r_n^2 \square \Pi r_{n01}^2$	$(r_n^2 \square r_{n01}^2)/r_n^2$

[ ]

The calculation of [ ] is the sum of the probabilities associated with all wage offers equal to or greater than  $W^*$ .

= [ ] ... [ ]

= [ ] (1a)

[ ]

=

$$\boxed{\phantom{\text{Equation}}}$$

Taking a particular Example;  $\boxed{\phantom{\text{Equation}}}$

$$\boxed{\phantom{\text{Equation}}}$$

(2a)

which simplifies to:

$$\boxed{\phantom{\text{Equation}}}$$

(3a)

Using summation of series results (3a) simplifies further to:

$$\boxed{\phantom{\text{Equation}}}$$

(4a)

and generalised yields

$$\boxed{\phantom{\text{Equation}}}$$

(5a)

Thus

$$\boxed{\phantom{\text{Equation}}}$$

maximising  $R(W^*)$  w.r.t.  $r^*$  and checking 2nd order conditions (Char, et al. 1992) gives

$$\boxed{\phantom{\text{Equation}}}$$

(6a)

where

$$\boxed{\phantom{\text{Equation}}}$$

## References

Bartels, C.P.A. 1979. Operational Statistical Methods for Analysing Spatial Data. In Bartels C.P.A., Ketallapper, R.A. (eds.), *Exploratory and Explanatory Statistical Analysis of Spatial Data*. Martinus Nijhoff, Boston.

Boarnet, M.G. 1994. An Empirical Model of Intrametropolitan Population and Employment Growth. *Papers in Regional Science: The Journal of the RSAI* 73: 135 – 152

Carlino, G.A., Mills, E.S. 1987. The determinants of county growth. *Journal of Regional Science* 27: 39 – 54

Char, B.W., Geddes, K.O., Gonnet, G.H., Leong, B.L., Monaghan, M.B., Watt, S.M. 1992. *First Leaves: A Tutorial Introduction to Maple V*. Springer-Verlag, New York

Cliff, A.D., Ord, J.K. 1973. *Spatial Autocorrelation*. Pion, London

Haggett, P., Cliff, A.D., Frey, A. 1997. *Locational Analysis in Human Geography*. Arnold, London

Haining, R. 1990. *Spatial Data Analysis in the Social and Environmental Science*. Cambridge University Press, Cambridge

Immergluck, D. 1998. Neighbourhood Economic Development and Local Working: The Effect of Nearby Jobs on where Residents Work. *Economic Geography* 74: 170–187

Kristensen, K., Henry, M.S. 1997. Peripheral Population and Employment Change In: Denmark's Functional Economic Regions: Does the Hinterland have a Stake in Urban? Paper presented at the 48th European Association of Agricultural Economists Seminar *Rural Restructuring within Developed Economies*, Dijon France, March

Mabogunje, A.L. 1984. *The Development Process*. Hutchinson, London



McKenna, C.J. 1990. The Theory of Search in Labour Markets. In Sapsford, D., Tzannatos Z. (eds.), *Current issues in Labour Economics*. Macmillan, London

Smith, S.W. 1994. *Labour Economics*. Routledge, London

Rouwendal, J. 1998. Search Theory, Spatial Labour Markets, and Commuting. *Journal of Urban Economics* 43: 1-22.

Rouwendal, J., Rietveld, P. 1994. Changes in Commuting Distances of Dutch Households. *Urban Studies* 31: 1545 - 1557

Van der Lann, L. 1998. Changing Urban Systems: An Empirical Analysis of Two Spatial Levels. *Regional Studies* 32: 235-247