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COMPETITIVE FEDERALISM: A BARGAIN-THEORETIC GENERAL EQUILIBRIUM APPROACH

by

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Abstract: Both conventional computable general equilibrium (CGE) models and game-theoretic models have been widely used in examining federal systems. There has been little attempt, however, at linking such models. This paper describes initial work on a project aimed at developing a political-economy CGE framework to examine fiscal federalism in Australia. Within this model we examine the incentive system that motivates the governments at various levels and evaluate their optimal policy choices in the equilibrium of the political economy.

Embedded in the overall political-economy model are two linked sub-models: a conventional multiregional CGE model of the economic sphere of the federation, and a game-theoretic (GT) model of the political sphere. Taxes and expenditures of each government are exogenous to the CGE model, but endogenous to the GT model. Policy makers decisions as to optimal fiscal policies are determined in the GT model in the light of outcomes from the economic sphere. In other words, policy choices are made to achieve simultaneous equilibrium in both of the two sub-models.

This paper describes our initial modelling steps. We develop a non-numerical political-economy model. The general equilibrium (GE) component of the model has been designed to enable it to be transformed easily into a CGE model. The GE component of the model covers two regions, each of which contains a single industry producing a private and a public good, a representative household and a regional government. A federal government is added subsequently to the model. The GT component assumes that each regional government chooses its fiscal variables so as to maximize an objective function appropriate to its jurisdiction, with the fiscal variables of the other jurisdiction treated as parameters of the problem. Thus we have a two-player competitive game with a government's pay off function represented by their objective function. The reaction functions for each government are the first-order conditions arising from the government maximizing its pay off.

1. Introduction¹

This paper describes the first steps in developing a modelling framework capable of handling fiscal federal questions within the context of competitive federalism. Computable general equilibrium (CGE) models form one good approach for examining such issues. There are many examples of studies using regional and multiregional CGE models to look at fiscal federalism issues. See for example, Jones and Whalley (1989), Dixon, Madden and Peter (1993), Madden (1993), Morgan, Mutti and Rickman (1996) and Nechyba (1997).

However, conventional CGE models contain little, if any, optimizing-behaviour theory relating to economic decision-making by governments. This imposes a clear limitation on such models for analysing competitive federalism.

An alternative modelling method would be to follow that developed by game-theorists who have analysed decision-making by governments in a political federation in terms of a non-cooperative, strategic-form game. Examples of this approach can be found in Mintz and Tulkens (1986), Wildasin (1988), Hoyt (1993) and Laussel and Le Breton (1998).

A way forward in combining the above two approaches was shown by Pant (1997) who analysed tariff determination by means of a "mini" one-region CGE model on to which a relationship serving to endogenise tariff decision-making by the government, had been grafted. We intend to develop a similar idea for analysing fiscal federalism. In this paper we take the first step by describing and analysing a simple two-region GE model in which maximising behaviour is extended to regional governments. The model put forward here is of an analytical type, but will be calibrated as a CGE model in due course. We call this a regional political-economy GE (PEGE) model. We proceed in two stages. In the first we set out a PEGE model for a single region. This allows us to establish notation and develop results and intuition which carry over in large part to the two-region model, the development of which constitutes the second stage of the work we report.

Most economies with regional governments also have a national or federal government. In our examination of an economy with two regions we begin by considering a model with only regional governments. We derive and explore the nature

of the solution for this model both with and without government optimisation. We then introduce a rudimentary federal government and consider two cases; in the first the federal government carries out a lump-sum transfer of resources from one regional government to another and in the second it imposes lump-sum income taxes on households and uses this revenue to make transfers to regional governments. We compare the solution with and without the federal government transfers and conclude that optimising regional governments change their own tax rates to offset the effects on their citizens of the federal government action. But this offsetting action is only partial since the regional governments have access only to distorting payroll taxes so that any attempt to offset lump-sum transfers or lump-sum income taxes generates changes in the other endogenous variables of the system such as employment, consumption and government expenditure. These “secondary” effects imply that the federal government will be able to influence the optimum which each regional government can achieve for its own region.

The structure of the paper is as follows. We begin our account by presenting, in section 2, a one-region GE model which we use as our starting point and go on in section 3 to describe its conversion to a one-region PEGE model by adding optimising government behaviour. In sections 4 and 5 we undertake the corresponding discussion for the two-region GE and PEGE models which were constructed from the one-region versions. In section 6 the model is extended to incorporate a federal government. Conclusions are presented in the final section.

2. The One-region CCGE Model

We begin with a one-region GE model in which there are households, firms and a regional government. The firms produce a single good using labour. Households supply the labour which firms require. Labour is in fixed supply and we assume that the wage adjusts to clear the labour market. The output of the good is sold to the households and, after costless transformation into a second good, to the regional government. The regional government finances its purchases of the second good by imposing a payroll tax on the firms and distributes its purchases, free of charge, to the households.

Both households and firms are optimizers - the representative household chooses its purchases of the good so as to maximize utility subject to an income constraint, with

the product-price and income taken as parameters, while the representative firm chooses its purchases of labour services so as to maximize profits subject to a production function constraint, with the product-price and the wage-rate taken as parameters. Each house hold has an equal share in the firms in its region and the firms distribute all profits to households.

Consider the representative household first. It maximises utility subject to a budget constraint. Utility depends on the consumption of the private good, C , and the government-provided good, G . We assume that the utility function is additively separable in its two arguments so that it may be written as:

$$(1) \quad I = U(C) + V(G).$$

We assume that U and V have the following properties:

$$(2) \quad \begin{aligned} &U' > 0, \quad V' > 0 \\ &U'' < 0, \quad V'' < 0 \\ &\lim_{C \rightarrow 0} U' = \infty \\ &\lim_{G \rightarrow 0} V' = \infty \end{aligned}$$

Utility is maximised subject to a budget constraint which constrains consumption to equal income which, in turn, consists of wage income and profit income:

$$(3) \quad PC = M = \pi + WL,$$

where P denotes the price of the consumption good, M denotes income, π denotes profits, W the wage rate and L labour supply. The household takes both W and π (and therefore M) as given. There is therefore only one feasible solution to the household's problem:

$$(4) \quad C = M/P.$$

Consider now the representative firm's problem. The firm maximises profit, π , defined by:

$$(5) \quad \pi = P(C + G) - WL(1 + T),$$

where G is the amount of the firm's output supplied to the government and T is the payroll tax rate. We assume that production takes place only if profits are positive. Note that we have assumed that the firm sells its output to the government and the private sector at the same price. Since the firm transforms output from C to G costlessly, any difference between the price charged to the government and the price

charged to private consumers would be inconsistent with profit maximisation. The firm is assumed to produce output with a single factor, labour, according to the production function:

$$(6) \quad O = L^\alpha, \quad 0 < \alpha < 1$$

where O is real output:

$$(7) \quad O = C + G.$$

A necessary and sufficient condition for profit maximisation is the standard marginal-productivity condition:

$$(8) \quad \alpha PL^\alpha = W(1 + T).$$

This condition determines employment (labour demand) for given P , W and T . Output supplied is then determined via the production function (6).

Equilibrium in the labour market requires equality between demand for labour (or employment, L) and the fixed supply of labour, \bar{L} :

$$(9) \quad L = \bar{L}.$$

The final component of the model relates to the government. It is assumed to satisfy the budget constraint:

$$(10) \quad PG = WLT$$

where the left-hand side measures the value of government expenditure and the right-hand side revenue. The government budget constraint implies that the government cannot treat both T and G as instruments. We assume that it treats T as its policy instrument and adjusts G to satisfy (10). G is therefore treated as endogenous and T as exogenous in our GE model.

Note that the consumption function, (4), and the definitions of household income and profits, (3) and (5), together imply that $PG = WLT$ which is the government budget constraint, (10). Hence, one of the equations of the model is redundant and we eliminate the government budget constraint.

We are therefore left with seven equations, (3) – (9) which can be reduced to the following three by substitution:

$$(11) \quad PG = W\bar{L}T$$

$$(12) \quad aP\bar{L}^{a-1} = W(1 + T)$$

$$(13) \quad C = \bar{L}^a - G$$

The variables are now C, G, W, \bar{L}, T . We treat C, W and G as endogenous and P, \bar{L} and T as exogenous. Further, we choose units so that $P = 1$, thus treating output as the numeraire.

From (12) with $P = 1$ we can obtain the solution for W in terms of \bar{L} and T as:

$$(14) \quad W = \frac{a\bar{L}^{a-1}}{1+T}$$

Substituting (14) and $P = 1$ in (11) we get the solution for G as:

$$(15) \quad G = \left(\frac{a\bar{L}^{a-1}}{1+T} \right) \bar{L} T = (a\bar{L}^a) \left(\frac{T}{1+T} \right)$$

Finally, from (6), (7) and (15) we get the solution for C in terms of \bar{L} and T as:

$$(16) \quad C = \bar{L}^a - (a\bar{L}^a) \left(\frac{T}{1+T} \right)$$

Thus all three endogenous variables are affected by both T and \bar{L} . An increase in \bar{L} will clearly increase output and will, in turn, increase both C and G . It will depress the wage since, with declining marginal product of labour, a larger labour force will be employed by profit-maximising firms only if the gross wage rate falls. A rise in T will also depress the wage (by approximately the same proportion as the increase in the tax rate). It will leave the level of output unchanged and therefore affect only the distribution of output between C and G . A higher tax rate results in a rise in G at the expense of C .²

3. The One-region PEGE Model

The model developed in section 2 assumes that the government's policy instrument, T , is exogenous to the model. While it is quite conventional to derive private behaviour from maximising assumptions while assuming government behaviour to be exogenous, we have argued above that this is inconsistent and we now extend the assumption of maximisation to government behaviour and in so doing move from the one-region GE model to the corresponding one-region PEGE model.

The first step is to decide on the government's objective. The literature in regional economics identifies various objectives of regional government policy. Common examples with counterparts in this model are employment and real output. However, in the present model these are effectively fixed by the exogenous labour supply

and cannot therefore be influenced by government action, making them unsuitable as objectives of government policy. An alternative is consumption, which appears suitable since it is subject to government influence and it is a source of utility. However, it is not the only source of utility – the government also influences household welfare via government-provided goods. Thus we take the government's objective function as the maximised value of the household's utility, i.e. equation (1) with C and G evaluated at their utility-maximising levels.

The constraints facing the government are assumed to be the structure of the economy as captured by the model set out in the previous section. Hence, we replace C and G by their solutions derived in section 2. Thus there is some asymmetry in the way in which households, firms and government are treated: the government knows the way in which households and firms will react to changes in T and G but we assume that firms and households take T and G as given and ignore the government's own maximising behaviour.

There are two variables which may be used for government instrument, viz., T and G . However, as pointed out in the previous section, the government is subject to a budget constraint which precludes it from using both independently. We assume arbitrarily but for reasons of convenience that the government uses T as its instrument and that it allows G to vary to satisfy its budget constraint, equation (10).

The government's objective is, therefore to choose T to maximise:

$$(1) \quad I = U(C) + V(G)$$

subject to the equations of the GE model. Substituting for C and G using the solutions from the GE model, reduces the problem to an unconstrained one of choosing T to maximise:

$$(17) \quad I = U \left[\bar{L}^a - aL^a \frac{T}{(1+T)} \right] + V \left[a\bar{L}^a \frac{T}{(1+T)} \right]$$

The first-order condition may be rewritten to give:

$$(18) \quad aL^a \frac{d}{dT} \left(\frac{T}{1+T} \right) \{V' - U'\} = 0$$

Since $aL^a \frac{d}{dT} \left(\frac{T}{1+T} \right) > 0$, (18) implies that at the optimum:

$$(19) \quad V' = U'$$

The optimal T will be positive and (under reasonable conditions) less than unity.

To show that it will be positive we argue as follows. With a positive labour supply, output will be positive so that, given the properties of the utility function, (19) can be satisfied only if both C and G are positive so that both U' and V' are finite. With G positive the government's budget constraint ensures that T will be positive.

We argue that T will be less than unity as long as the labour share in output is greater than 50%. From the definition of profits, equation (5), the product-market clearing condition, (7), and the production function, (6), our assumption that in equilibrium profits must be positive implies that:

$$T < \frac{\bar{L}^a}{WL} - 1$$

It follows that the optimal T will be less than unity so long as $\frac{\bar{L}^a}{WL}$ is less than 2 at the optimum, i.e. so long as the optimal labour-share exceeds 0.5.³

4. The Two-Region GE Model

We turn now to the two-region models which correspond with the one-region models presented in sections 2 and 3, respectively. In this section we develop the two-region counterpart of the GE model presented in section 2. In the next section we convert this into a PEGE model thus obtaining the two-region counterpart of the one-region PECGE model of section 3.

4.1 The Model

At the outset we need to make assumptions about the nature of inter-regional relations. We assume that each regional economy is identical to the one-region one we set out in section 2 except that we allow the possibility of inter-regional migration so that regional labour supplies are not fixed. Instead, we assume that the national labour force is fixed and that households migrate (costlessly) between regions in response to inter-regional wage differentials. In equilibrium, therefore, the wage is equalised across regions.

It may be argued that our modelling of households is inconsistent – households are assumed to choose their consumption to maximise a utility function dependent on both C and G but make their location decision based only on W – in effect on C alone. A

theoretically preferable alternative would be to assume that households choose their location to maximise the same utility function as is used to motivate their consumption choice. In that case the equilibrium condition for inter-regional migration would be

$$U(C_1) + V(G_1) = U(C_2) + V(G_2)$$

where subscripts refer to regions. While preferable theoretically, this would greatly complicate the analysis. Not only does it introduce four endogenous variables; it also makes the equilibrium dependent on the form of the utility function. At this stage we use the simpler assumption of wage inter-regional wage equality in the interests of tractability.

We assume that households reside in the region in which they work and they receive government goods provided by the regional government in that area. We therefore abstract from inter-regional spillovers in the provision of government goods. We further assume that the ownership of firms is not inter-regionally transferable so that firms are owned by households living in the region in which the firm is located and that households receive profit distributions only from firms in the region in which they live.

Under these assumptions the two regional economies are replicas of the single-region economy of section 2 except for inter-regional labour flows. Hence equations (11), (12) and (13) apply to both regions (with the fixed labour supply assumption relaxed and $P = 1$):

$$(20) \quad G_i = W_i L_i T_i \quad i = 1, 2$$

$$(21) \quad a L_i^{a-1} = W_i (1 + T_i) \quad i = 1, 2$$

$$(22) \quad C_i = L_i^a - G_i \quad i = 1, 2$$

To these equations we add:

$$(23) \quad L_1 + L_2 = \bar{L}, \text{ and}$$

$$(24) \quad W_1 = W_2$$

where \bar{L} is national employment.

Relationships (20) – (24) constitute our two-region GE model. This is a set of eight relationships in eleven variables. The eleven variables are: \bar{L} , G_i ($i=1,2$), W_i ($i=1,2$), L_i ($i=1,2$), C_i ($i=1,2$) and T_i ($i=1,2$). We take \bar{L} and T_i ($i=1,2$) as exogenous leaving G_i ($i=1,2$), W_i ($i=1,2$), C_i ($i=1,2$) and L_i ($i=1,2$), as the eight endogenous variables.

4.2 The Solutions

The solutions given by the model for L_i ($i=1,2$) can be obtained from (21), (23) and (24):

$$(25) \quad L_1 = \frac{\bar{L}}{1 + \left(\frac{1+T_1}{1+T_2} \right)^B}.$$

The solution for L_2 can be obtained from (25) by using $L_2 = \bar{L} - L_1$:

$$(26) \quad L_2 = \frac{\bar{L}}{1 + \left(\frac{1+T_1}{1+T_2} \right)^B}$$

Having obtained the solution for L_1 , the solution for W_1 (in terms of L_1) can be obtained from (21):

$$(27) \quad W_1 = \frac{aL_1^{a-1}}{1+T_1}$$

From (24) it follows that the resulting expression will also be the solution for W_2 .

To obtain the solution for G_1 (again, in terms of L_1) we use (20) and (21) to get:

$$(28) \quad G_1 = \frac{aT_1}{(1+T_1)} L_1^a$$

Similarly the solution for G_2 (in terms of L_2) is:

$$(29) \quad G_2 = \frac{aT_2}{(1+T_2)} L_2^a$$

Finally, we obtain the solutions for C_1 and C_2 (in terms of L_1 and L_2 respectively). From (20), (22) and (27) we get:

$$(30) \quad C_1 = L_1^a - T_1 W_1 L_1 = L_1^a - T_1 \frac{aL_1^{a-1}}{1+T_1} L_1 = \left\{ \frac{1+(1-a)T_1}{1+T_1} \right\} L_1^a$$

Arguing along the same lines we get the solution for C_2 :

$$(31) \quad C_2 = \left\{ \frac{1+(1-a)T_2}{1+T_2} \right\} L_2^a$$

4.3 The Multipliers

While we are not interested in the GE model *per se*, we derive several multipliers at this stage of the analysis since they will be useful in the analysis of the two-region

PEGE model to be developed in the next section. Besides, they can be used to throw some light on the implications of moving from the single-region to the two-region model. Multipliers can be derived for each of the endogenous variables with respect to each of the exogenous variables but, given the nature of our interests, we restrict the derivation to multipliers for the region-1 variables with respect to T_1 . Similar results can be derived for the second region.

Consider L_1 first. The multiplier for L_1 with respect to T_1 can be derived by taking the partial derivative of (25) with respect to T_1 to get:

$$(32) \quad \frac{\partial L_1}{\partial T_1} = - \frac{\partial \left\{ \bar{L} \left[1 + \left(\frac{1+T_1}{1+T_2} \right) \right]^\beta \right\}}{\partial T_1} = \bar{L} \beta \left[1 + \left(\frac{1+T_1}{1+T_2} \right) \right]^{\beta-1} \left(\frac{1}{1+T_2} \right) < 0,$$

where the negative sign follows immediately from the fact that $\beta = 1/(\alpha-1)$ and the restriction that $0 < \alpha < 1$ so that $\beta < 0$. Since output is monotonically related to employment, a rise in T_1 reduces not only employment in the region but also output.

The multiplier for W_1 with respect to T_1 follows from the following expression for W_1 which follows from the equilibrium condition that $W_1 = W_2$:

$$W_1 = W_2 = \frac{a}{1+T_2} \bullet L_2^{a-1} = \frac{a}{1+T_2} (\bar{L} - L_1)^a$$

so that

$$(33) \quad \frac{\partial W_1}{\partial T_1} = \left\{ - \frac{a(a-1)}{(1+T_2)} \right\} \left\{ (\bar{L} - L_1)^{a-2} \right\} \left\{ \frac{\partial L_1}{\partial T_1} \right\} < 0$$

where the sign of the multiplier again follows from the restrictions on α (and the sign of the multiplier for L_1). Hence a tax rise in region 1 depresses wages in both regions.

The effect on G_1 of a change in the tax rate in region 1 follows from the differentiation of (28) with respect to T_1 to give:

$$(34) \quad \frac{\partial G_1}{\partial T_1} = \frac{a}{(1+T_1)^2} L_1^a + \frac{a^2 T_1}{(1+T_1)} L_1^{a-1} \frac{\partial L_1}{\partial T_1}$$

We have already established that $\partial L_1 / \partial T_1$ is < 0 . From this it follow that the second term in (34) is negative (assuming once again that T_1 and T_2 are positive). The first term, however, is positive. Consequently, unlike the multipliers of L_1 and W_1 with respect to T_1 , the sign of the multiplier of G_1 with respect to T_1 is indeterminate.

Finally consider the effects of a tax change on consumption expenditure. From (20), (21) and (22) we have:

$$C_1 = \frac{(1 + (1 - a)T_1)}{1 + T_1} L_1^a$$

so that

$$(35) \quad \frac{\partial C_1}{\partial T_1} = -\frac{aL_1^a}{(1 + T_1)^2} + \left(\frac{1 + (1 - a)T_1}{1 + T_1} \right) aL_1^{a-1} \frac{\partial L_1}{\partial T_1} < 0$$

where the negative sign follows from the restriction that $0 < \alpha < 1$ and the sign of $\partial L_1 / \partial T_1$.

Recall that in the 1-region GE model, a rise in the payroll tax rate leaves output unaffected (since it is determined by the fixed supply of labour) but reduces the wage and redistributes output from consumption to the government good. In the present region, in contrast, output is also affected. This is because a rise in region 1's tax rate "initially" depresses the wage in region 1, causing labour to migrate to region 2 in search of higher wages. This reduces employment and output in region 1 and increases employment and output in region 2. The effect in region 1, therefore, is both to reduce output and to redistribute output from C to G. These effects ensure an unambiguous effect on C_1 but produce an ambiguous effect on G_1 as shown by the multipliers in equations (34) and (35).

The effects of the tax rise on wages and employment are illustrated in Figures 1 and 2. Figure 1 shows the initial equilibrium. The length of the horizontal axis, O_1O_2 represents the fixed national labour supply. Wages are measured up the vertical axes – W_1 along the left-hand axis and W_2 along the right-hand axis. The two curves are the marginal product curves adjusted for the presence of the payroll tax. In equilibrium the tax-adjusted MPL must be equal to the wage in each region and the immigration condition is that wages are equalised across the two regions. Hence equilibrium is represented by the wage $W_1^* = W_2^*$ where national employment is distributed to the two regions as O_1E and EO_2 respectively.

In Figure 2 we show the effects of an increase in the tax rate in region 1 from T_1 to T_1' which shifts its MPL curve down to $MPL/(1+T_1')$. The result is a reduction in the wage rate from W_1^* to $W_1^{*'}$ which causes migration of labour to region 2 so that employment in region 1 falls to O_1E' and employment in region 2 increases by the same

amount. The fall in the wage is smaller than it would be in the absence of migration in which case the wage in region 1 would have fallen to W_1^{**} . Thus, in the two-region model, there are spillover effects on region 2 of a tax rise in region 1 and, while region 2 “gains” in terms of increased employment (and population), it “loses” in terms of a lower wage.

5. The Two-Region PEGE Model

We now extend the model of the previous section to include optimisation on the part of the two regional governments and so move to the two-region PEGE model. We assume, as in the one-region case, that each regional government chooses its own payroll tax rate to maximise the welfare of its own citizens. We also assume that in solving its maximisation problem each government is constrained by the set of relationships constituting the GE model determining consumption and government expenditure in its own region. Each government is assumed to take the tax rate in the other region as given so that the resulting equilibrium will be a Nash equilibrium.

5.1 The Optimizing Relationships

Our discussion of the relationships defining the regional governments’ optimal tax-rates will be conducted throughout in terms of region 1. A parallel discussion holds for region 2.

The government of region 1 chooses T_1 to maximise:

$$(36) \quad I_1 = U(C_1) + V(G_1)$$

subject to the solutions for C_1 and G_1 derived from the GE model in section 4, equations (28) and (30) with L_1 replaced by the expression in (25) and T_2 and \bar{L} treated as parameters. The first-order condition for this problem is:

$$(37) \quad \frac{\partial I_1}{\partial T_1} = U' \frac{\partial C_1}{\partial T_1} + V' \frac{\partial G_1}{\partial T_1} = 0$$

The terms $\partial C_1 / \partial T_1$ and $\partial G_1 / \partial T_1$ are simply the relevant multipliers derived from the GE model in the previous section. On substituting these expressions, equations (34) and (35), the first-order condition for the government’s problem can be written as:

$$(38) \quad U' - V' = \frac{a \left\{ U' \frac{C_1}{L_1} + V' T_1 W_1 \right\}}{W_1 L_1 / (1 + T_1)} \bullet \frac{\partial L_1}{\partial T_1}$$

The coefficient of $\partial L_1 / \partial T_1$ on the right-hand side of (38) is positive and we have seen in section 4 that $\partial L_1 / \partial T_1$ itself is negative so that the right-hand side of the condition for the optimal value of T_1 is negative. Hence at the optimum $U' < V'$.

The requirement that $U' < V'$ at the optimum is in contrast to the single-region case where the government's maximising condition is that U' and V' are equal.⁴ In the case without inter-regional migration T determines only the division of a given output between C and G . Hence, an increase in tax increases G and decreases C by the same amount so that at the optimum the welfare benefit of the increase in G (V') must be exactly offset by the welfare foregone from lost consumption (U').

Once inter-regional migration is permitted there is an additional effect of a tax change. Now a change in tax affects not just the distribution of given output between C and G but affects the level of output itself. In particular, an increase in T_1 results not only in a shift of output from C_1 to G_1 but also in a reduction in output in region 1. Hence the cost in terms of consumption foregone of a given increase in G is greater than it is in the no-migration case so that at the welfare optimum the welfare effects are balanced only if $U' < V'$.

In the single-region case we argued that the optimal tax rate is positive (since G is positive at the optimum) and, under reasonable conditions, less than unity. The same argument may be applied to the present model so that we may conclude that the optimal payroll taxes rates fall between 0 and 1.

The complete two-region PEGE model consists of 10 equations: two optimality conditions of the form of (38) and the solution equations for L_i , G_i , C_i and W_i given by equations (25)-(31). The system has 10 endogenous variables (T_i , G_i , C_i , L_i and W_i , $i=1,2$) with a single exogenous variable, \bar{L} .

Before turning, in section 6, to an examination of the reaction of optimising regional governments to the policy actions of a federal government, we briefly explore the effects within the present model of a shock to national labour supply, \bar{L} .

We concentrate on the effects on employment in region 1. From the solution for L_1 , equation (25), it follows that

$$(39) \quad dL_1 = \frac{\partial L_1}{\partial \bar{L}} d\bar{L} + \frac{\partial L_1}{\partial T_1} dT_1 + \frac{\partial L_1}{\partial T_2} dT_2$$

where the partial derivatives are simply the multipliers for L_1 with respect to \bar{L} , T_1 and T_2 . It follows from inspection of equation (25) that the first of these, $\partial L_1 / \partial \bar{L}$, is positive, from our analysis in section 4 that the second is negative and, again, from equation (25) that the third is positive. That is, a rise in national labour force increases employment in both regions in the absence of regional government action and a tax rise in one region results in the movement of employment from that region to the other. Hence the first term in (39) will tend to increase employment in region 1 but the effects of the others on dL_1 depend on the regional governments' reactions.

The government in region 1 observes the "immediate" effect of an increase in \bar{L} being an increase in employment in its own region which is accompanied by an increase in output and in consumption. To maintain its optimality condition it will need to increase G_1 to balance the effect on household welfare of the increase in C_1 . This does not necessarily require an increase in the payroll tax rate since the tax base will have risen with the increase in output.⁵ Hence, the equilibrium effects on the regional tax rates is indeterminate and will depend on the precise forms of U and V . A similar argument may be applied to region 2.

6. The Two-Region PEGE Model with a Federal Government

The PEGE model developed in the two preceding sections has two optimizing regional governments but no federal government. We now introduce a federal government which uses its authority to modify the equilibrium generated by the regional governments' optimizing strategy. We consider two possibilities.

The first is that the federal government takes from one regional government some of the output which it has purchased for distribution to households in its region and gives it to the other regional government. The second possibility is that the federal government imposes a lump-sum tax on households in both regions and uses the combined proceeds to purchase outputs of the transformed from the two regional governments. The output so purchased it then distributes, on a lump-sum basis, directly to households in each of the two regions.

6.1 Lump-sum Inter-governmental Transfers

Denote the output transferred to the government of region 1 by the federal government by TR_1 and the output transferred to the government of region 2 by TR_2 . They satisfy:

$$(40) \quad TR_1 + TR_2 = 0$$

We add this relationship to the relationships of the GE model and treat TR_i ($i=1,2$) as exogenous.

The question we now consider is: How will a federal-government intervention of the type now under discussion change the equilibrium generated by the two-region PEGE model of sections 5 and 6 and, in particular, how are the regional governments likely to react?

We focus on the two regional government optimising conditions of the form of (38). We begin by noting that we need to distinguish between the amount of output purchased by regional government i and that distributed to the households in region i . We continue to use the notation G_i to refer to government purchases so that the amount consumed by citizens of region i is now $G_i + TR_i$. With this interpretation of G_i none of the solution expressions for L_i , W_i , C_i and G_i given by equations (25)-(31) is affected by the introduction of the federal governments transfers. Hence the private sector will respond to the federal government re-distribution only if the regional governments change their tax rates. Whether they do will be governed by their optimising conditions.

Consider case of region 1. Equation (38) may be written as:

$$(41) \quad \frac{V'}{U'} = \frac{\frac{aL_1^a}{(1+T_1)^2} - a \frac{C_1}{L_1} \frac{\partial L_1}{\partial T_1}}{\frac{aL_1^a}{(1+T_1)^2} + aT_1 W \frac{\partial L_1}{\partial T_1}}$$

The optimising T_1 must satisfy this condition both before and after a federal-government intervention of the type described. Suppose that T_1^* is the tax rate which satisfies the condition before the lump-sum transfer. It will no longer satisfy (41) after the transfer since the argument of V' is now $(G_1 + TR_1)$ and if we assume that the transfer is from region 2 to region 1 so that $TR_1 > 0$, we find that after the transfer V'/U' will be less than the right-hand side of (41) at the original taxes rates. Hence, the optimality condition for region 1's government is violated at unchanged tax rates.

To restore optimality it will need change its tax rate so as to increase C_1 or reduce G_1 or both. Both of these will be achieved by a reduction in T_1 . The opposite is true for region 2 since TR_2 will be negative. Hence the government in region 2 will need to increase the payroll tax rate to restore optimality.

We can conclude, therefore, that the reactions of the regional government to the federal government re-distribution will move in the direction of offsetting the effects of the transfer. Optimising regional governments will therefore undo (at least part of) the actions of the federal government. But, they will not be able to completely undo the federal government's action since that would require a fall in G_1 equal to TR_1 . But the tax change necessary to achieve the fall in G_1 will also increase C_1 , reducing U' , so that not all of the adjustment can be in G_1 , some of the adjustment necessarily being in C_1 . This reflects the fact that the federal government has available a lump-sum transfer while the regional governments have only distorting payroll tax instruments.

The above argument contains two omissions which should be noted by way of qualification. In the first place, it ignores the fact that if $TR_1 > 0$, then $TR_2 < 0$ and T_2 will need to rise; the rise in T_2 will work against the fall in T_1 as regards region 1's $\frac{V'}{U'}$ ratio. Likewise, no account is taken of the fact that the fall in T_1 will work against the rise in T_2 as regards region 2's $\frac{V'}{U'}$.

Secondly, no account is taken of the fact that both the fall in T_1 and the rise in T_2 will have effects on the right-hand side of the equalities set out in (41) as well as on the left-hand side.

6.2 *Lump-sum Income Taxes and Transfers*

We turn now to the second type of federal government intervention distinguished at the outset. This is where the federal government uses its authority to impose a lump-sum income tax on households in each of the two regions. It then uses the proceeds of the tax to purchase output of the transformed good from the regional governments. Finally it distributes this output directly to regional households on a lump-sum basis.

Denote the lump-sum income tax imposed on households in region i by F_i ($i=1,2$) and the lump-sum transfers of the transformed good to households in region i , by GF_i ($i=1,2$). These four variables are linked by the federal government's budget constraint:

$$(42) \quad F_1 + F_2 = GF_1 + GF_2$$

We continue to denote the output purchased by the regional government by G_i so that the regional government budget constraint remains as before:

$$G_i = T_i W_i L_i$$

As in the lump-sum-transfer case, the introduction of GF_i does not affect the solutions for C_i , G_i , W_i and L_i at given payroll tax rates. The only change is that the amount of government good consumed by residents of region i is now $(G_i + GF_i)$.

The introduction of the lump-sum income tax does, however, change the consumption of the private good since it reduces the amount of income households have to spend. Private consumption expenditure is now

$$C_i = O_i - G_i - F_i = L_i^\alpha - T_i W_i L_i - F_i$$

As in the previous case, condition (41) must hold both before and after the federal government intervention.⁶ Suppose, again, that T_1^* is the tax rate which satisfies the condition before the federal government's action. Will it continue to be optimal after the federal government policy? The answer is "No" for two reasons. The first is similar to that given in the simpler case of a transfer – the new argument of V' is now $G_1 + GF_1$ so that V' is now "too low" (assuming that GF_1 is positive). The second reason is that the argument of U' is now the original C_1 less F_1 so that U' is "too high" (assuming that F_1 is positive). Both of these changes require a rise in T_1 to restore optimality for region 1.

Thus in the tax and transfer case, the federal government not only provides goods to the citizens of region 1 which skews the distribution of output towards the government good (requiring an offsetting action by the regional government) but it also raises taxes on the citizens of region 1 which further skews the allocation of output towards the government good, requiring a further shrinking of the regional government to maintain optimality for the citizens of region 1. The federal government essentially does what the regional government also does – transform taxes into the government good – and to maintain a welfare maximum the regional government reduces its operations in response to the federal government's attempt to redistribute output from private consumption to government consumption and from one region to another. As in the simple transfer case, the offsetting action of the regional governments will not be perfect because of the differences in the nature of the instruments available at the two levels of government.

Exactly the same argument holds for region 2. Thus, in the case of the intervention now under discussion both regional governments will need to reduce their labour tax if they are to remain in an optimal situation. This is in contrast to the case of federal intervention analysed in section 6.1 where one regional tax needs to fall and the other to rise.

It will be recalled that the argument developed in section 6.1 for the lump-sum-transfer case of federal intervention was subject to two qualifications which were noted. Similar qualifications apply here. Once again our argument ignores the effect of the fall in T_2 (T_1) on region 1's (region 2's) situation, though here the effect will be supportive rather than offsetting. Likewise no account is taken in the argument that both the fall in T_1 and the fall in T_2 will have effects on the right-hand side of the two relationships set out in (41), as well as on the left-hand side.

8. Conclusions

This paper set out to build a small regional general equilibrium (GE) model and extend it to include optimising behaviour on the part of regional governments. The motivation for this research was the observation that standard models assume optimising behaviour on the part of private agents (firms and households) but assume government behaviour to be exogenous. In this paper we assumed, instead, that regional governments choose their policy instruments so as to maximise the utility of the representative household.

We started by describing a single-region model in which the government raises tax revenue from a payroll tax which it uses to purchase output from the firms and which it provides free of charge to households. We derived the condition for optimal government policy and found that it involves the equality of the marginal utility of the private good and that of the government-provided good. This follows from the property of the model that changes in the tax rate simply redistribute a given output from private consumption to government consumption.

When the model was extended to two regions, the optimality condition was adjusted to take account of the effects of inter-regional migration. In the two-region model a change in tax not only shifts output from the private to the government sector but also affects the total amount of output produced as workers migrate in response to inter-regional wage differentials.

The final section of the paper introduced a federal government which attempts to change the distribution of resources between the regions by lump-sum tax and transfer mechanisms. We found that the optimising regional governments operate to frustrate the redistributive aims of the federal government but they are only partially successful in doing so since their taxes have allocational consequences. Hence, there is still a redistributive role for a federal government even though the possibilities are more limited and there may be unintentional consequences when they face optimising regional governments.

One of our proposed extensions of the work described here is to replace this rudimentary treatment of the federal government with optimising behaviour. In this connection the work of Boadway and Keen (1997) will be taken as a starting point.

NOTES

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² The multiplier for G with respect to T is $dG/dT = \alpha L^\alpha (1/(1+T^2)) > 0$. Since output is unaffected by a rise in T , C must fall.

³ To understand why a production-parameter constrains the feasible tax-rate, consider a case where the equilibrium labour-share is 0.3. In that case a 200% tax on wages is consistent with positive profits – wages will account for 30% of the value of output, taxes for 60% of the value of output and profits will still be positive at 10% of the value of output. However, once the labour share rises to 0.5 a tax-rate of over 100% would result in negative profits, the possibility of which has been ruled out.

⁴ Note that if labour is not inter-regionally mobile (i.e., $\partial L_1/\partial T_1 = 0$) condition (38) reduces to $U' = V'$.

⁵ The tax base is $W_1 L_1 = [\alpha/(1+T_1)] O_1$ which is clearly increasing in output, O_1 . However, since tax revenue is $T_1 [\alpha/(1+T_1)] O_1$ the increase in tax does not absorb all the increase in the value of output.

⁶ Note that C_1 also appears on the right-hand side of (41). However, in the derivation of (41) this was introduced when $\alpha C_1/L_1$ was substituted for $[(1+T_1)(1-\alpha)/(1+T_1)] \alpha L_1^{\alpha-1}$ which is not affected by the federal intervention.

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