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ON THE ROLE OF SUBJECTIVE UNCERTAINTY IN THE BUSINESS CYCLE

Thomas E. Chamberlain, Ph.D.*

August 31, 2000

“Modeling the labor market is a challenge and a puzzle for all classes of business cycle theories. A model of business cycles should be able to explain one of the most robust facts of economic fluctuations: that consumption and labor input both track the business cycle. ..Reproducing this comovement, however, has proved a challenge for business cycle theory.” (Basu and Taylor, 1999, p. 57.)

SUMMARY

While the present new approach to economics opposes the ordinal-utility school promoted by Pareto and Hicks early in the twentieth century, it bears some similarity to modern Real Business Cycle theory with its necessarily cardinal utility. A difference that cuts across all of economics, however, is how economic behavior of the individual human being is approached: Contrary to modern economic theory, production and consumption are modeled together (in the same model) as distinct or separate human activities (following Gossen [1854]). This epistemologically sound approach is denied by neoclassical theory, due to the keystone “direct commodity utility” assumption initially posited by Walras [1874]. By adopting human activity instant-utility as fundamental (Occam’s razor), the undivided or complete intratemporal time constraint on individual behavior is achieved yielding an exactness that helps advance the science. In the present paper, the bearing of social psychology on the business cycle is addressed. It is shown in this preliminary study how a crisis-in-confidence can effect an immediate contraction of labor. Just prior to the crisis all agents are unified in their exuberant over-confidence that a cataclysm threatening to cease all economic activity will not occur. With the arrival of the unexpected crisis-in-confidence, the society experiences an immediate shift-to-pessimism. As a consequence, all agents at all levels of the (implicitly) modern institutionalized economy suddenly decrease their indirect (capital-production/maintenance) labor, in accordance with postulated decreasing returns on labor. In preparing the model, a novel exponential formulation of intertemporal uncertainty allows functional combination with (assumed) exponential subjective discounting. Short-term activity-regimen periodicity is achieved by postulated acute myopia due to rapid or steep subjective discounting of intertemporal utility. The representative agent owns and controls a share of the aggregate capital thereby allowing semi-autonomy. Following the initial crisis-shock to the economy, capital-intensity and business activity decline to a diminished stationary state, where increased labor is necessary to supply the (postulated) fixed demand for consumables.

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I. INTRODUCTION¹

In its fundamentals the present new approach to the business cycle is a far-reaching departure from mainstream economics (as codified by Hausman [1992]). It has, however, a more compatible relationship with Real Business Cycle (RBC) theory (see Hartley, J. E., et. al. [Eds.] [1998]). It is interesting in this regard that the point of departure of the present theory from both standard and RBC economics is in utility theory, specifically in the formulation of consumption utility. Simply stated, in the present approach (instant) utility is directly identified with consumptive *activity* whereas in standard theory utility is directly identified with consumptive *commodities*. This difference could be judged cosmetic, inasmuch as a simple transformation (with qualifications) relates commodity utility to activity instant utility. But the difference is crucial regarding the character or tenor of economic theory.

It is not that standard theory proscribes consumptive instant utility—i.e., rejects the identification of instant utility (pleasure) with the consumptive activity. Such treatment has long been evident in the literature (e.g., Ramsey [1927],² and Strotz [1956]). What is *methodologically proscribed* is the conjunction of labor and consumption as mutually exclusive activities. This union cannot be accommodated in neoclassical theory. In particular, it would call into question the great pillars of neoclassical theory—including the equilibrium postulate, rational greed assumption, and exogenous preferences. To allow the non-overlapping conjunction of consumptive instant utility and labor instant utility in an economic model would expose the shortcomings of neoclassical economics. The fall of direct consumable utility would knock down other dominos—first ordinal utility, next exogenous preferences, then unbounded rationality, and so on. To introduce another metaphor, direct consumable utility is the *keystone* of the great edifice of neoclassical economics. Negate it and the edifice is fatally weakened. But replace this keystone with consumptive *instant utility* and the neoclassical edifice may be transformed into a stronger and more useful structure.

Returning to RBC theory, it is, as noted above, not far removed from the path of the present methodology. RBC theory would be on the present path were direct consumable utility set

aside in favor of the direct identification of instant utility with consumptive activity. Then consumption, labor, and leisure could all coherently reside on the same methodological page. (See Georgescu's introduction to Gossen's book, p. lxxxix.)

It is interesting that this step would not necessarily depart from neoclassical theory. The modeler could, in specifying the constraint tying consumptive activity to consumed amount, simply write that the consumable-amount produced and consumed is exclusively dependent on the activity duration. This, in conjunction with consumption instant utility, is functionally equivalent to directly identifying utility with consumables in the first place—subject to the condition that leisure utility is zero. If leisure utility is not zero the marginal increase of consumptive duration partially eclipses leisure duration with its finite instant utility, thereby leaving a net marginal utility of consumption—i.e., a specific marginal utility of the consumable that is smaller than it would be under direct assignment. This epistemologically sound accounting of utility in economic theory is one of the justifications for the exclusive identification of (instant) utility with human activity.

Another justification for the exclusive identification of utility with human activity is that it promotes economics as an exact science. This could be understood to be the crucial reason. When utility is directly identified with consumables the (overall) time constraint on individual behavior is denied as a rigorous analytic tool. In particular, the entire duration of the individual's intratemporal period (e.g., 24 hour day) cannot be mathematically "filled" with his or her activity. Gaps must remain—i.e., consumptive *durations* have not been defined. Accordingly, one can no more derive compelling economic solutions and theorems on the basis of incomplete activity-time than can, say, Laplace transforms be processed on the basis of integrands that are indeterminate in time. The power of *exactness*, so important in mathematics and physics, is untapped in standard economics.

The present approach to modeling economic behavior extends beyond RBC theory in other ways—e.g., the transformation of aggregate intertemporal utility into real-time instant utility as the basis for deciding between candidate expectational plans; the introduction of human-activity worldlines as the basis for accommodating subjective uncertainty; and the extinction of intertemporal (worldline) uncertainty as concomitant with learning. These aspects have been presented in earlier articles (e.g., [1997] and [1998a]), and will be briefly addressed in the following section on basic theory.

After the section on basic theory, the paper proceeds to business cycle theory. While the present approach can accommodate the exogenous-shock hypothesis of RBC theory it also accommodates an endogenous driver—in particular, the crisis in confidence. In the present article, the approach is used to provide insight into the effect of uncertainty on capital and labor in the business cycle. In particular, it will be shown how the rigorous modeling of uncertainty, human activity, and utility yields individual and collective behavior that reflects the real-world business cycle. In performing the analysis, the point of departure is Lange's 1936 article "The Place of Interest in the Theory of Production." In Lange's paper the Clark/Wicksell real interest rate is accommodated in a narrow sector of a near-subsistence economy, wherein capital (axes) is produced and used to make new axes and a finished good (wood). The model is modified in the present work to reflect the modern industrialized economy, albeit one subjected to a number of simplifying assumptions. Most prominent among these assumptions is postulated activity-regimen periodicity in the individual's near-term planning. This is accomplished by his or her steep discounting of intertemporal experience relative to the rates of change of the physical parameters of the economic system—a discounting that accommodates, through special assumptions, expected uncertainty in return-on-investment. The economy is accordingly modeled as though in a quasi "equilibrium" so far as each individual is concerned, while strong transients proceed in response to the crisis.

In order to mathematically formulate the above-described effect of a crisis in confidence, and also help in its visualization, a particular class of economic systems is considered wherein all agents are united in their expectation of some threatening disaster or cataclysm that would effectively extinguish economic life. This could be an impending natural event (e.g., a great volcanism, or meteor impact) or it could have a social basis (nuclear holocaust). While the event does not occur, an unexpected (surprising) increased belief that it could results in a reduced level of capital production (reduced indirect labor). In this regard, the agents decide that the "sacrifice" (investment) of discounted marginal utility (value) in capital production will not be fully compensated by the valued return-on-investment (overall optimal planning is not achieved). In particular, the value of the resulting consumption would not fully compensate for the sacrifice (intratemporal-efficiency failure). Nor, alternatively (at the differential margin), would the value of

the labor reduction (indirect and direct) fully compensate for the sacrifice (intertemporal-efficiency failure). The economy accordingly declines to a diminished stationary state. By postulating, first, that there is no economic after-life following the cataclysm, and second, that the crisis in confidence arrives as a surprise, two contingent scenarios are avoided or set aside, thereby greatly simplifying the model while retaining the expository character.

II. BASIC THEORY³

Economic theory, and social science in general, can be understood to transcend the natural sciences in the sense of having two rather than one time dimensions. One dimension—real time—is of course fundamental to both of the great branches of science: Whether the evolution of a supernova is considered or a child walking across a room, in either case the material dynamics must conform to the laws of physical reality. But the behavior of a child—or any conscious individual—in response to environmental conditions differs from the reaction of non-conscious material entities in that a purposeful scenario is prepared or created in imaginary, or intertemporal, time—the second dimension—on how the future should evolve. Purposeful intent (subject to the expected constraints)—or the *expectational plan*⁴—is applied to the material world, this world including the individual's own (physical and biopsychological) material; and with the advance of real time the world sensibly accommodates, or tends to conform (subject to the real-time *virtual* constraints), to the child's plan. Inevitably, and frequently, an unexpected (i.e., surprising) experience occurs, and the new information necessitates or fosters a new expectational plan. Once again the child follows the plan until interrupted by further surprise, and so on.

It is clear that economics has not satisfactorily come to grips with time in either dimension (see, for example, Currie and Steedman [1990]). In this regard it may be judged noteworthy that the direct measurability of instant utility—i.e., *pleasure*—has remained unrecognized in economics for over thirty years. What makes this remarkable is the potentially great importance it holds for the discipline: If instant utility is measurable (and it is: see Rolls [1975]), then satisfaction—i.e., utility, in the language of economic theory—is empirically cardinal, inasmuch as utility is the time-integral of instant utility. And if utility is empirically cardinal, then the emerging behavioral science based on Gossen's insights [1854]—see, Georgescu-Roegen's references to

Becker's and Linder's work in his introduction to Gossen's book [Ibid., c-cv and cxxvi-cxxvii]—is significantly advanced. However, mainstream (neoclassical) economic theory of the twentieth century has been formulated on the presumption of *ordinal* utility (Hausman [1992]). Given the tendency to preserve dominant paradigms (Kuhn [1962]), it could perhaps be expected that the measurability of instant utility would go unrecognized.

Not only is instant utility measurable, but other aspects of economic behavior that extend deeper than standard theory have an established empirical basis. For example, expectation has been measured in neuropsychological tests on monkeys (Snyder [1997]). And tests on brain-damaged patients and "normals" have indicated that emotion and feeling are contributors to practical thought (Damasio [1994]; Bechara, et. al. [1997])—thereby adding to existing doubt on the rationality postulate of standard theory. In view of the evidence, it could be concluded that standard economics does not correctly represent behavioral reality—and, as has been argued elsewhere (Chamberlain [1998 June] and [1998 August]), this condition is significantly responsible for the chronic schism within the social sciences, due to modal economists taking preferences to be exogenous while modal sociologists understand preferences to be endogenous. (See also Baron and Hannan [1994].)

The emotive equation, based on measurable instant utility, coherently accommodates advances in neuropsychology where standard economic theory, based on ordinal utility, cannot succeed. In the essential or canonical form, this equation may be written:

$$(1a) \quad E_k^i = \sum_{w=1, \infty} [f_{kw}^i \int_0^\infty \lambda_{kw}^i(.,.,.,., t) P_{kw}^i(.,.,.,., t) dt]$$

where the terms are defined in the list of symbols.⁵ The individual in his or her purposeful activity—planned "without a sense of unrealism" (Shackle [(1958) 1967, p. 41-42])—seeks to maximize E_k^i , subject to the expected constraints

$$(1b) \quad \Phi_{kw}^{ic} = 0, \quad c(w) = 1, \infty.$$

In initiating the operative expectational plan, the individual selects the candidate plan that provides the maximum anticipatory-pleasure (or *anticipatory value*,⁶ as an alternative expression).⁷

Adjuncts to the emotive equation and corresponding expected constraints are the "virtual constraints,"

$$(1c) \quad \Psi_{kw}^{im} = 0, \quad m(w) = 1, \infty.$$

These statements comprise the real-world material, psychological, emotional, social, etc. constraints on behavior. With the progress of real-time, surprise is ultimately experienced when some aspect of the expectational plan is negated by a virtual constraint (or, alternatively, a creative thought), and on each such occasion the new information initiates a new or revised expectational plan.⁸

The derivation of the emotive equation was provided in the June 1998 (AAAS) paper by the writer, and its substantive discussion in relation to standard economic theory was given in the 1997 (WEAI) paper.

In a brief overview, it may first be noted that the emotive equation formulates the emotional response—specifically, *anticipatory pleasure* E_k^i —experienced by individual i in anticipation of expectational plan k . In this expression, the integrand consists of the expected differential utility Pdt in product with the emotive mapping function λ , the latter transforming (mapping) intertemporal utility into real-time anticipatory pleasure (instant utility) experienced by the individual.⁹ Performing the indicated integration, applying the expectational occurrence probability f , and summing over all worldlines, yields the net (positive or negative) instant utility experienced in anticipation of the expectational plan.

The emotive equation differs from standard economics in a number of significant ways. Perhaps the most prominent of these departures is the maximization of cardinal plan-value E as a determining factor in the individual's purposeful activity.¹⁰ E is categorically cardinal in the present theory, in contrast with mainstream theory's ambivalence between cardinal versus ordinal utility.¹¹ Plan value is realized, as has been noted, by the emotive mapping of total intertemporal utility into a real-time datum. A new direction such as this in a science must have empirical substantiation, and the justification rests with studies performed on brain-damaged patients and normals by Damasio and others (see, e.g., Damasio [1994]).

Another departure from the standard approach is to dismiss as unsound the direct identification of utility with consumables. Alternatively, *instant utility* is exclusively identified with the process-of-knowing (P-O-N) attending all mental and physical activity—including, of course, consumption. This identification is effected by multiplying the P-O-N instant utility P by the differential time dt . Plan value is ultimately received by consumables—as a subset of all entities, including productive tools and other persons—by imputation.

While there are significant departures of the present approach from standard theory, there are also important agreements. One concerns equimarginality of the objective parameter—i.e., intertemporal utility in standard theory versus plan value in the present theory. In either case, constrained perturbations at the margins must sum to zero in purposeful planning. This, of course, is a fundamental principle of mathematical behavioral theory. However, to briefly reiterate, while mainstream theory identifies utility directly with consumables, in the present approach all commodities—indeed, all imagined entities—receive anticipatory value by imputation, with utility entering the mathematics through identification of instant utility exclusively with the P-O-N. This is not a cosmetic difference that might be casually ignored in theory, basic and applied, but one of fundamental importance. As observed by Keynes over a half-century ago:

Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when the storm is long past the ocean is flat again. (Keynes [1971, 65])

Standard (core) theory is basically timeless.¹² The present theory accommodates time as an explicit parameter in its canonical formulation, and can substantively represent the profoundly temporal character of real economic life. In this regard, it treats “equilibrium”¹³—or, more correctly, *periodicity*—as a special condition that can be modeled for both the short term and long term.¹⁴ It is expectational (subjective) discounting of expected instant utility that permits economic systems in transition to be advantageously modeled as periodic, as demonstrated herein.¹⁵ Also demonstrated is the condition of long-term periodicity, wherein the stationary state can persist for an indefinite time.

Other points of consistency, albeit with degrees of qualification, include: emotive discounting of expected instant utility,¹⁶ subjective (expectational) uncertainty,¹⁷ capital function,¹⁸ indifference curves,¹⁹ and price theory.²⁰

III. UNCERTAINTY AND THE BUSINESS CYCLE: CONCEPTUAL DEVELOPMENT

By taking empirically meaningful *instant utility* as the essential parameter, rather than nonessential consumable utility, time is realistically and coherently admitted into economic theory. In this regard, as has been noted, it is not real-time but imaginary-time that is fundamental. However, behavior must, of course, conform to the laws of physics, and in this sense real-time still “governs” our activity. But the individual, to a significant degree, purposefully creates or defines the emergent future. This process occurs in imaginary thought, performed (as noted earlier) “without a sense of unrealism.” And it is in the substantive mathematical formulation of expectational plans in imaginary (i.e., intertemporal) time that uncertainty (and all other salient parameters) are constructively represented.

Uncertainty, or the more mathematically meaningful *occurrence probability*, has been shown to have an important effect on capital function. (See the WEAI paper, 1997.) In particular, should there be a decrease in the expectational probability that a given enterprise by an individual (in isolation) will continue in its present profitably, the real interest rate imparted to the enterprise by the individual—i.e., its normalized net marginal return on investment in a given time-span—will increase. This must occur in order to balance cause and effect marginal value, in the spirit of expected utility theory.²¹ And this character of the individual’s planning is similarly true for the community, and for the individual in command of substantial human and material capital.

In response to the reduction of expected occurrence-probability the community adjusts its activity regimens, and capital “on-line,” to restore balance. These adjustments include decreasing the planned duration (hours) of periodically recurring capital-producing labor (e.g., day-to-day) in accord with the required increase in marginal productivity. A decline in the capital on-line (intensity) follows. In this way, a shift in expectational uncertainty can effect an increase or decrease of economic activity. It is this effect of expectational occurrence-probability, or

subjective uncertainty—i.e., to increase or decrease economic activity—that is the primary focus of the present work.

In the following section, a verbal and graphical description of capital/labor function under uncertainty is first provided. Then, later in the paper, the corresponding mathematical formulation is addressed. Having prepared the conceptual and mathematical foundation, the effect of a crisis in confidence on economic function—i.e., on the quantities of labor and capital in productive activity—is assessed.

A. Model Definition

In seeking insight into the causes of the business cycle, with particular interest given to the effect of uncertainty, a number of assumptions will be applied. One assumption—*periodicity*—similar to the bedrock “equilibrium” (i.e., timeless²²) condition of standard economics, is prominent in the development to follow, but essentially different. Invariant macroeconomics may prevail in either case. But a crucial difference in the present approach is the explicit modeling of postulated-periodic human activity, and its effect (also periodic) on the economic world. At a deeper level, steep or severe subjective discounting retains periodic regimens in planning even though people recognize transients in their economic circumstances. This “deeper level” is modeled in the present work, as steep discounting yields stationary-state or “equilibrium” planning while strong economic transients are in progress. .²³

Periodicity requires a number of supportive or ancillary assumptions—e.g., that the expected interactive plans of all agents are compatible and satisfy the virtual constraints (with no innovations), thereby permitting the real-time economy to follow plan (similar to *Rational Expectations*). This, in turn, requires that all have identical (exponential) discounting of expected instant-utility. Additional simplifying assumptions not required by periodicity, such as invariant instant-(dis)utility of labor, will be introduced.

It is, of course, advantageous (and appropriate, i.e. advances should relate to the state of the science) in developing new theory to start with existing theory. For this purpose the work of Lange [1936] is particularly well suited. Lange postulated a communal (or, alternatively, Crusoe)

economy wherein labor in conjunction with capital (axes) is employed to reproduce both (depleted) capital (as a circular factor) and a consumable (wood), with two separate functions representing these distinct productive processes. His formulation was greatly constrained in certain respects in comparison with the present study, in part due to the neglect of subjectivity and the attending analytic opportunities. In particular:

- Capital (equipment): Lange postulated hand axes as the form of capital, whereas diverse forms of capital are (implicitly) accommodated herein (representative of the modern economic system).
- Consumable: Wood was the sole output versus diverse kinds herein (including food), albeit only one kind of consumable per form of capital, as in Lange's work.
- Production Functions: As in Lange's treatment, two production-functions are employed.²⁴ However, in the present study the problem definition and analysis are (moderately) simplified. In particular, the capital on hand is used first in the reproduction of itself in a given intratemporal period and then in the production of a finished good,²⁵ whereas Lange divided capital (axes) into two parts, one for ax reproduction and the second for finished-good production. Additionally, in the present paper newly-produced capital is immediately available for use, versus Lange's one-year delay. Furthermore, Lange's capital abruptly disappears after a year's use, while in the present work capital wears out over extended time (during use).
- Activity Regimen: Only (direct and indirect) labor is represented in Lange's work, while consumptive activity is included in the present study. (In this regard, the present expectational approach determines, in principle, the durations of all activities, whereas the capability must, of course, be absent in Lange's mechanistic formulation.)
- Time Constraint: Lange postulated that the sum of indirect and direct labor equals an invariant constant, whereas in the present work, the end-to-end summation of all activity—labor, consumption and rest—is subject to the time constraint. (However, as will be seen in the mathematical development, rest instant-utility is set to zero to enable decoupling of multiple labor-activities in the intratemporal regimen.)

Institutions are (implicitly) accommodated in the present study. As a related matter, money is represented (also implicitly) as a neutral "numeraire," as in Lange's formulation.

The upper half of Figure 1 provides an overview of Lange's model of capital and labor for the Crusoe economy. Real time (i.e., clock time) is represented on the x-axis, and "on-line" capital (i.e., when producing in conjunction with labor) is on the y-axis. The dashed and dotted line traces the activity of the individual as he or she performs the periodic regimen. It is seen that a dashed line first spans the ax (capital) production interval, after which the individual switches to

one or more undefined activities, as represented by the dotted line. Productive activity then resumes with the cutting of wood (consumable), again making use of axes (but not the same axes used in the first productive activity). Undefined activity again occurs followed by the return to ax production, thus completing one cycle of an infinite sequence of identical cycles. As has been noted, Crusoe waits a year before using each new ax, with the ax suddenly wearing out after the lapse of a second year.

Moving now to discussing the present model of labor/capital function shown at the bottom of Figure 1, a significant difference may be immediately noted. In particular, coincident real and imaginary (expectational) time is represented on the x-axis, versus real-time in Lange's model.²⁶ This extension allows the accommodation of diverse activity in a determinate manner—i.e., one that permits, in principle, the determination of all activity durations.²⁷ In this regard, the substantive formulation of expectation, including discounting, allows a nonzero interest rate in the stationary-state economy (contrary to some reasoning in mainstream economics).

It is seen in the figure, in a regimen similar to that portrayed by Lange, that the individual first applies labor to the capital to produce new capital of the same particular kind (thereby restoring depleted or worn-out capital). After engaging in other activity, the capital (old plus new) is used to produce the corresponding consumable. Following another suspension, the individual consumes the commodity and soon returns to producing capital, thereby completing the cycle. As will be seen, this cycle is rigorously periodic in the near-term—and it will be periodic in the long-term as well, except for the possibly extended interval following the crisis in confidence.

In an assumption indispensable to the analysis, each individual is now understood to be *mathematically* independent of all other agents of the economy, although profoundly cooperative in collective enterprise. Furthermore, each productive/consumptive component of his or her overall activity is postulated independent of the remaining components. It is required in this regard that each component generate an income commensurate with its product.²⁸ This, of course, means, in addition to other conditions, that all are rigorously cooperative in their intertemporal activities.

Having described the present model in relation to Lange's, the next step is to address expectational theory in greater detail. Figure 2 shows expectational (intertemporal) time preference (I) and instant-utility (III) in their connection to the activity regimen (II). Note that in

this figure, and throughout the analysis to follow, the datum for expectational discounting is taken to be the margin of the initial capital-producing session. (The margins are identified by circles.) It is seen that labor instant utility is assumed constant as an advantageous approximation.²⁹ A short time after the end of capital production, consumable production is entered. Instant utility at the margin is now fractionally diminished to a point just above the original (negative) labor instant utility (see [*] in the figure). Here the tendency to discount expected instant utility is recognized, and assigned an exponentially declining character (I). The next activity is consumption with expectedly positive instant utility, also discounted. Indirect labor is again encountered, and it is seen that marginal instant utility is discounted even more steeply. And so on.

Figure 2 begins to exhibit the power of the activity-based instant utility approach. In particular, it is seen that activity, instant utility, discounting, and capital are coherently interrelated. Basically timeless standard theory cannot do this (i.e., in a dimensionally consistent manner).³⁰ Further validation of the instant-utility approach is provided in Figure 3, where perturbation analysis shows how expectational planning determines the activity-regimen.

In Figure 3 the objective is to show how the periodic solution is determined. (A revision of this figure in the next section (B) will be used to show the effect on economic activity of a society-wide shift-to-pessimism.) In demonstrating how the individual determines the solution, it is first recognized that capital tends to diminish or degrade over time—i.e., of a given amount of capital that is produced at a point in time, a quantity thereof will revert to a non-capital (non-useful) state in each subsequent time unit.³¹ (This differs from Lange's approach, wherein it was assumed that each new ax abruptly disappears after its use in the second year.) Just as capital persists (albeit declining), so does the infinitesimal perturbation thereof, due in our problem to a small increase of indirect labor at the time-zero datum. And as the perturbation persists it of course induces corresponding marginal responses of labor (reductions) and consumption (increases) over intertemporal time.

The capital increment is assumed to diminish with time in a manner shown at the top of the figure.³² In particular, the increment diminishes during direct labor, increases during indirect labor, and is constant in between, following an exponential decline in the long run. The (small) perturbation linearizes the activity response (to first order), yielding an important advantage—i.e.,

the induced changes in marginal activity throughout the intertemporal interval are functionally tied to the original marginal perturbation of indirect labor. To reiterate this point, the indirect labor perturbation at [1] can be rigorously connected to the direct labor, indirect labor, and consumptive (marginal) responses throughout expected time. This character of the response is crucial. In particular, for any cause and effect relationship in a given expectational plan, the activity-marginal-values of the cause and effect actions must sum to zero.³³ Essentially germane to this character, and indispensable in the present analysis, is the mathematical option of suppressing the consumptive response to the perturbation in favor of an increased direct labor response. (This, of course, is at the heart of the Wicksell real interest rate.)³⁴

To see how the activity-marginal-values sum to zero when the consumptive response is suppressed, refer again to point [1] in Figure 3A. At [1] the activity-marginal-value has a negative magnitude. At point [3], and all subsequent consumptive margins (e.g., [3] and [6]), a zero response is postulated. This shifts the direct-labor responses at the margins to their maximum possible (negative) magnitudes (decreases—at (2), (5), etc.). Note that the marginal utilities are discounted—relative to the undiscounted “hats.” Summing these (discounted) contributions (over infinite time), and adding the indirect-labor contributions at [4], [7], etc., gives a result that must, in the spirit of (Gossenian) utility theory, exactly cancel the (negative) activity-marginal-value at [1].

It is seen how marginal value may be balanced in a manner similar to standard utility theory. A significant difference, however, is the coherent and rigorous way in which labor-utility, discounting, and capital are modeled in this activity-based approach. The approach will now be used to investigate the effect on business activity (i.e., employment and capital-intensity) of a universally experienced shift-to-pessimism.

B. Confidence Crisis: Effect on Labor and Capital

It appears advantageous to draw a parallel with the science of fluid mechanics. To satisfactorily visualize or conceptualize aerodynamics, simplifying assumptions are helpful (when admissible)—such as thermally and calorically perfect gases, incompressibility, and uniform free-stream conditions. Analogous assumptions are necessary in economics to advance conceptual, and

mathematical, investigation. A number of assumptions have been made to this point in the present work—e.g., periodicity, and an invariant instant utility of labor. To this list are added several more. Including in these is the postulate that the expected time constant of capital increase or decrease is much greater than the intertemporal time constant that may be identified with expectational discounting—i.e., before a significant change is expected in capital intensity following a shift-to-pessimism, or an imposed perturbation, expected instant utility has already been discounted to near zero. Additionally, a relatively small elasticity of consumption is assumed, to in effect “freeze” the immediate direct-labor response to the crisis. It will be seen in the study that a sudden shift-to-pessimism throughout society produces an immediate reduction of indirect labor. This is followed by a diminution of capital intensity over extended time with an associated gradual increase in direct labor.

Lange’s 1936 work continues as the foundation or point-of-reference for the discussion. In particular, his Figure 1 will be used to good advantage in showing how the uncertainty shift leads to labor reduction (and increased idle time). First it is necessary to model the postulated shift-to-pessimism. To aid the discussion, Figure 3 is repeated as Figure 4, with the original discount function and the more steeply declining “crisis” discount function (both exponential) shown at the top. At the bottom of the figure, the labor marginal-utilities subsequent to the perturbation at [1] are seen to be more deeply discounted, in accordance with the “crisis” discount factor. Clearly their summation no longer balances or cancels the (negative) marginal utility at [1]. An adjustment of the activity regimen to optimize the individual’s purposeful activity is accordingly necessary.

It is here that we postulate an almost vanishingly small consumption-elasticity as a simplification, where the small elasticity is effected by a steeply descending instant utility at and near the margin of the consumption activity. Also assumed, to reiterate an important condition, is a capital transient with an order greater time constant than the expectational-discount time constant. As a consequence of these (expectational) assumptions, direct labor is not immediately affected or changed by the shift-to-pessimism. This must, of course, be a consequence inasmuch as: (1) the consumable amount produced by labor is unchanged (i.e., the consumed amount is unchanged); and (2) the marginal productivity of direct labor is unchanged (because the capital

on-line is substantially invariant). Indirect labor therefore becomes the focus of attention in assessing the effect of a shift-to-pessimism on economic behavior.

Continuing to work with Figure 4, it can now be seen that the marginal productivity of indirect labor must increase in order to balance the more deeply discounted marginal utilities of direct and indirect labor over intertemporal time. Decreasing returns on indirect-labor investment mean, of course, that indirect-labor duration must decrease. This may be seen more clearly by reference to Lange's Figure 1.

Figure 1 of Lange's 1936 work, as (substantially) modified, is provided in Figure 5, where the modifications show the indirect labor reduction resulting from the shift-to-pessimism. In addressing the figure it is recalled that all individuals are, in a sense, quasi-independent. In this regard, each individual, although interactive and cooperative, has a self-specific share of the total capital that is worked in parallel. This means that the individual may be treated as if in *mathematical* isolation, although, of course, he or she remains profoundly integrated in the (large-scale) economic system. It also means that each individual's expectational planning and real-time behavior is in accord with that of all agents in the economy.

In Figure 5, the x-axis represents both indirect and direct labor, where indirect labor is measured to the right from O and direct labor is measured to the left from P. As defined by Lange, the sum of indirect and direct labor equals O-P, postulated invariant in his study. Only marginal productivity in the production of the consumable is represented, on the y-axes, by way of the lines I-I' and D-D'.

The principal way in which the present study departs from Lange's is by accommodating expectation. With this *complementary* capability it is possible to investigate not only how each individual's activity regimen is determined, but also how the regimen is changed when the economic prognosis is changed. In the present model scenario, the expectational shift-to-pessimism requires an increased indirect labor marginal productivity. With reference to Figure 5 it is seen that a reduction of indirect-labor duration must be the response. In particular, the required greater magnitude of marginal productivity along line I-I' is obtained by the contraction of interval OA to OA'. This is also depicted at the top of the figure, in its relation to the constrained direct labor. Here it is seen that indirect labor decreases faster than direct labor can increase, due to the latter's inelasticity (i.e., consumption is substantially inelastic so direct labor must be

similarly inelastic, for effectively invariant capital on-line). Productive hours accordingly become idle hours.

The question now concerns the system's behavior over extended intertemporal time. All agents expect capital to gradually decline, asymptotically approaching its new stable level. As capital declines, direct labor must increase to maintain the near inelastic consumption. (It is recognized here that, all else equal, labor productivity declines when on-line capital declines.) Indirect labor will also increase in order to balance direct labor and indirect labor marginal value. Attending the reduction of capital intensity is a commensurate reduction of consumption, in accordance with residual consumptive elasticity.

As an additional perspective, reflection on the considered model suggests that long-run direct and indirect labor in the depressed economy may be greater than that in the more capital intensive economy. More to the point, inelastic consumption requires that direct labor increase as capital intensity drops (in accordance with the reduced productivity of labor). It could accordingly be concluded that labor is countercyclic in the long-term business cycle. (This has an intuitively agreeable tenor, in the sense that increased capital per individual should reduce the labor to meet needs—and conversely.)

This long-term countercyclic variation of labor stands in opposition to its short-term procyclic variation. One is accordingly led to the conclusion that labor will lag the (sinusoidal) uncertainty bias depending on its period—zero degrees for short periodic intervals and 180 degrees for long periodic intervals (both periods relative to the material or physical time constant of the economic system).

IV. UNCERTAINTY AND THE BUSINESS CYCLE: MATHEMATICAL DEVELOPMENT

The mathematical formulation is intended to reflect the conceptual treatment given above. Following the reformulation of the emotive equation in terms of discrete activities, and definition of the differential solution process, attention will turn to the capital/labor model and the effect of a shift-to-pessimism on business activity.

A. Formulation and Solution

The starting point is the emotive equation with the corresponding expectational constraints, i.e. (1a) and (1b). One such (operative) equation exists for each individual, and all equations are, in varying degrees, coupled to each other across the large-scale economic system by the interrelated constraints. However, rigorous near-term³⁵ periodicity of the entire economy is postulated, and this greatly simplifies the analysis. In this regard, all individuals are postulated to identically and steeply discount instant utility (to within a constant factor) across intertemporal time. This allows all to function as (quasi) independent although profoundly united in their cooperative, capital-assisted productive efforts.³⁶ As a consequence of this condition, and others, it will be seen that each emotive equation with its constraints may be addressed as mathematically independent.

In the rigorously periodic economy, a number of ancillary conditions must be satisfied. Here we are not invoking the postulate in a semi-rigorous sense: the ancillary conditions must exactly conform to the periodic requirement. As a particular, each individual must be comprehensively and rigorously knowledgeable of the expectational plan of every other individual with whom he or she interacts. It is recognized in this regard that interlocking expectational plans provide overall compatibility. Should near-term compatibility not be the case, aperiodic activity would occur as agents adjust their near-term regimens. As a related matter, it is not necessary that the future be determinate, as in standard (equilibrium) theory: The economy moves into an uncertain future.

It is recognized, then, that the near-term activity regimens and economy at all levels of the community (or nation) remain periodic with the progress of real time.^{37,38} As an additional consideration, liquidity preference cannot be accommodated in the present model inasmuch as the psychological need is absent. In the first place, the impending cataclysm is of such expected severity that sustenance is not an issue. Secondly, the crisis in confidence arrives as a surprise and there is no vision to initiate and guide preparation. And finally, steep discounting suppresses liquidity preference—even when suffering for lack of preparation is foreseen. Relaxation of the first two conditions would promote liquidity preference, but the question arises (not answered here) as to whether activity-regimen periodicity is preserved.

The mathematical form of the discount function is also significantly predetermined (in effect) by periodicity. In this regard it is greatly advantageous, and possibly mandatory³⁹ (recognizing Strotz's work [1956]), that the expectational discounting of expected instant utility be assigned an exponential character. On this basis the relative discounting of expected experience over the intertemporal period is unaffected by the advance of real time.

Closely related is the consideration—previously noted—that exponential discounting of expected instant utility must be identical (to within a constant factor) for all participants of the economic system. Should this not be the case, transients defeating the periodic character would result from participants either saving or borrowing, depending on the relationship of their self-specific discounting to the (systemic) interest rate.

With the recognition that autonomic (emotive) discounting of expected instant utility is exponential and, further, that this discounting of future experience has the same exponential argument for all,⁴⁰ λ_{kw}^i in (1a) may be written:

$$(2) \quad \lambda_{kw}^i(.,.,.,., t) = K_{kw}^i \exp(-\alpha^i t) \\ = K^i \exp(-\alpha t)$$

where

$$K_{kw}^i = K^i = \text{constant}; \\ \alpha^i = \alpha = \text{inverse time-constant of emotive discounting.}$$

Note that the dependence of the individual's autonomic discounting on plan k and worldline w has been dropped. (The operational expectational plan has been selected, and worldline dependence is suppressed as a simplification.)

As is the case with the emotive mapping (discount) function just considered, periodicity requires that the expected longevity of the economy have an exponentially diminishing character—i.e., the expected chance that the economy will survive declines exponentially with (imaginary) time. Towards the corresponding formulation, it may first be observed that people do not have an exact vision of how their economic lives will evolve. They might be optimistic, while recognizing that fortune could be suddenly reversed in some indefinite manner. This frame of mind, including the exponential character, may be mathematically formulated by postulating that in each imaginary (i.e., intertemporal) unit of time a (small) fraction of the still “viable” worldlines

represents the expectational probability of an immediate cessation of economic activity. In the circumstance that the economy-ending disaster does not occur with the advance of time, the corresponding worldlines simply vanish, leaving the remaining worldlines to continue into the intertemporal interval.⁴¹ Accordingly,

$$d[\sum f_w^i]/dt = -\kappa^i [\sum f_w^i]_t$$

where $[\sum f_w^i]_t$ is the expected probability that the economy survives to time t and κ^i is the expected fraction of the worldlines that “peels off” or disappears per unit time. Then

$$(3) \quad [\sum f_w^i]_t = [\sum f_w^i]_0 \exp(-\kappa^i t)$$

where

$$[\sum f_w^i]_0 = \sum_{w=1, \infty} f_w^i = 1.0$$

represents the unit-probability (certainty) that the economy exists at the datum $t=0$.

A change to (3) is introduced before completing the reformulation of the emotive equation. As noted earlier, it is not uncommon for people to adopt an unrealistic expectational occurrence-probability, i.e. to be optimistic, or pessimistic, beyond what is warranted by experience and fundamental considerations. This is, of course, an important cause of imbalance in the individual’s finances—and in the economy as a whole, when such unrealism is concerted (as will be addressed). In the present application, we can represent an unrealistic expectational probability by substituting $\kappa^i = \beta^i - \gamma^i$ in (3), resulting in

$$(4) \quad [\sum f_w^i]_t = [\sum f_w^i]_0 \exp(-[\beta^i - \gamma^i] t)$$

where

$$\begin{aligned} \beta^i &= \text{experiential or rational discount coefficient;} \\ \gamma^i &= \text{irrational deviation.} \end{aligned}$$

As will be discussed later in the paper, a positive magnitude of γ^i , when in significant concert with all others, induces or promotes excessive economic expansion, and the opposite occurs for a

negative γ^i . On the other hand, β^i represents the prudent assessment of future prospects, based on fundamental considerations and long-term experience.

While it is certainly true in the real economy that everyone will not be of the same mind regarding future prospects, history suggests a significant tendency in this regard. The observation is taken to its limit in order to permit an analytic formulation in this expository treatment. More to the point, it is postulated, as a necessary condition for periodicity, that all participants have the same rational and deviant coefficients—i.e., $\beta^i \Rightarrow \beta$ and $\gamma^i \Rightarrow \gamma$. Equation (4) accordingly becomes:

$$(5) \quad [\sum f_w]_t = [\sum f_w^i]_0 \exp(-[\beta - \gamma] t).$$

It may be noted here that γ is a measure, for the stationary-state (or “equilibrium”) condition, of the degree of imbalance of the economic system.

At this point, continuous P-O-N instant utility evident in (1a) is resolved into its labor, consumption, and rest components. Attending this step is the advantageous assumption that the instant utility of all labor that a given individual expects and performs has a fixed (negative) magnitude (invariant, in real and imaginary time). This postulate, of course, sets aside the complication of Gossen’s law of declining instant utility. In this regard, it may be noted that since typically little or nothing is ingested as a normal part of labor, the rapid decrease of marginal instant utility characteristic of some consumptive activity—e.g., consumption of food and water—may be understood to be absent. Furthermore, decreasing marginal instant utility of labor is not necessary for the determinate solution. Since the postulate greatly facilitates the visualization and mathematical treatment of capital/labor function while retaining a plausible representation of labor instant utility, it will be employed in the analysis.

Turning now to consumptive activity, the usual decreasing instant utility of positive magnitude is assumed. Exact recurrence-of-wants is postulated for all consumptive activity—i.e., when any given consumptive activity is resumed after an interruption, the initial intensity is restored. It is not required that the arbitrarily diverse array of consumptive activities have the same instant-utility functions.

Rest instant utility is set to zero, as an assumption that serves the present expository treatment in a crucial manner: It frees productive and consumptive activity from the intratemporal time constraint. This means that during the economic transients following the crisis in confidence, production and consumption activity durations *in aggregate* can expand, or contract, without being affected by eclipsed rest utility—i.e., independent production/consumption relationships can be formulated. (In effect, the assumption suspends the intratemporal time constraint.)

Collecting the several foregoing adjustments to the emotive equation allows the following reformulation of (1a):

$$\begin{aligned}
 (6) \quad E^i &= [\sum f_w^i]_0 K^i \left\{ \int_0^{A_{i,1}} \exp(-[\alpha + \beta - \gamma]\tau') p_1^i(\tau') d\tau' \right. \\
 &\quad + \int_0^{A_{i,2}} \exp(-[\alpha + \beta - \gamma] (A_{i,1} + \tau')) p_2^i(\tau') d\tau' \\
 &\quad + \int_0^{A_{i,3}} \exp(-[\alpha + \beta - \gamma] (A_{i,1} + A_{i,2} + \tau')) p_3^i(\tau') d\tau' \\
 &\quad \left. + \dots \right\} \\
 &= [\sum f_w^i]_0 K^i \left\{ \sum_{n=1, \infty} \int_0^{A_{i,n}} \exp(-[\alpha + \beta - \gamma] (\sum_{m=1, n-1} A_{i,m} + \tau')) p_n^i(\tau') d\tau' \right\}
 \end{aligned}$$

where

A_n^i = the duration of activity n ($A_n^i = A_{i,n}$ in the integral limit).
 $p_n^i(\tau')$ = process-of-knowing instant utility for activity n :
 $p_1^i(\tau')$ = negative constant (assumed) for all labor by individual i ;
 $p_n^i(\tau') > 0$, and $\partial p_n^i(\tau')/\partial \tau' < 0$ for all consumptive activity;
 $p_n^i(\tau') = 0$ for rest.

In his or her purposeful activity, the plan value (i.e., anticipatory instant-utility) is maximized, subject of course to the expectational constraints. The plan will be optimal when $dE^i = 0$. Equation (6) accordingly becomes:

$$\begin{aligned}
 (7) \quad \delta E^i &= \exp(-[\alpha + \beta - \gamma] A_1^i) p_1^i(A_1^i) \delta A_1^i \\
 &\quad + \exp(-[\alpha + \beta - \gamma] (A_1^i + A_2^i)) p_2^i(A_1^i + A_2^i) \delta A_2^i \\
 &\quad + \exp(-[\alpha + \beta - \gamma] (A_1^i + A_2^i + A_3^i)) p_3^i(A_1^i + A_2^i + A_3^i) \delta A_3^i
 \end{aligned}$$

$$\begin{aligned}
 & + \dots = 0 \\
 & = \sum_{n=1, \infty} \exp(-[\alpha + \beta - \gamma] (\sum_{m=1, n} A_m^i)) p_n^i(\sum_{m=1, n} A_m^i) \delta A_n^i = 0
 \end{aligned}$$

where p_n^i continues to conform to the earlier prescriptions. The constraints (1b) of course complement the differential expression. (Whereas δA , etc., is used in place of dA in the treatment to aid the intuition, it is understood that the limit $\delta(\bullet) \rightarrow 0$ applies.)

While the periodicity assumption limits economic detail, a considerable scope remains. In particular, the great diversity of economic behavior at the macro-level of the modern economy largely continues in its implicit representation in the emotive equation and constraints. Additionally, at the other extreme, the behavior of the economic system at the “nanoeconomic” level—i.e., in the activity regimens of individuals—is represented. The result is a mathematical formulation that retains an exact character, one that permits analytically derived conclusions. This formulation will now be used to investigate the effect of uncertainty on the labor/capital relationship, and the level of business activity.

B. Effect of Uncertainty on the Business Cycle

Capital serves, in part, as the overreaching and connective agency, of all economic systems, of whatever character. It unites agents in cooperative endeavors. And capital strings are understood to be extensive or perpetuated, elements of capital under (mathematically autonomous) individual-specific control in the considered economic system. In the idealized form, any given capital string consists of a quantity of a particular kind of capital of capital that is maintained and utilized across intertemporal time. Its use is by definition periodic, on a daily basis in the present formulation, but possibly having longer intervals between use, such as monthly or seasonally.

In the rigorous periodicity that is assumed in the present work, the individual produces or maintains a particular kind of capital early in the interval, uses the capital to produce a corresponding consumable, and consumes the product. Rest, also on a periodic basis, is assumed (although rest instant-utility is identically zero, as earlier noted). This regimen proceeds for a given capital string, one of an arbitrary number in the individual’s overall economic life.

As a consequence of the assumptions, each capital string is independent of the remaining capital strings in the individual's control, or in the control of other agents in the economic system. Of course, as noted above, the activities of individuals are interdependent due, in large part, to productivity that is coupled by "overreaching" or "encompassing" labor-assisting capital. This is a property of real economic systems that greatly complicates substantive analysis. However, quasi-independent behavior, despite "interconnectedness," can be formulated. It is this specialized system that is addressed below.

(i) ***Uncertainty as a Determinant of Economic Behavior.*** The first step is to organize or collect the activity differentials in (7) on the basis of capital strings. Accordingly, in a treatment conforming to Figure 2:

$$\begin{aligned}
 (8) \quad \delta E^i = & \sum_{I=1, N} \left\langle \begin{aligned} & \text{(a)} \\ & \langle (IU_{L1}^i \delta L_1^i) \exp(-[\alpha + \beta - \gamma] [T_{L1}^i=0.0]) \\ & + (IU_{L2}^i \delta L_2^i) \exp(-[\alpha + \beta - \gamma] T_{L2}^i) + (IU_{C(C^i)}^i \delta C^i) \exp(-[\alpha + \beta - \gamma] T_C^i) \rangle_a \\ & + \langle (IU_{L1}^i \delta L_1^i) \exp(-[\alpha + \beta - \gamma] T_{L1}^i) \\ & + (IU_{L2}^i \delta L_2^i) \exp(-[\alpha + \beta - \gamma] T_{L2}^i) + (IU_{C(C^i)}^i \delta C^i) \exp(-[\alpha + \beta - \gamma] T_C^i) \rangle_b \\ & \cdot \\ & \cdot \end{aligned} \right\rangle_I = 0
 \end{aligned}$$

In this expression, each set of large brackets $\langle \dots \rangle_I$ encompasses a single capital string, where the capital string consists of an (infinite) sequence of terms. These terms are themselves organized into term-sets designated by the brackets $\langle \dots \rangle_a$, $\langle \dots \rangle_b$, etc., with each consisting of an indirect-labor component (a), a direct-labor component (b), and a consumption component (c). It is seen that the activity-specific terms have been assigned symbols representing their meaning. Accordingly, the activity perturbations are δL_1^i , δL_2^i , and δC^i for indirect labor, direct labor, and consumption, respectively. The corresponding marginal instant utilities are IU_{L1}^i , IU_{L2}^i , and $IU_{C(C^i)}^i$, where the functional dependence of $IU_{C(C^i)}^i$ recognizes, contrary to IU_{L1}^i and IU_{L2}^i , that instant utility varies

(declines) with increasing activity-duration. The remaining new-parameter names consist of $\langle\langle T_{L2}^i \rangle_a \rangle_1$ for ΣA_m^i , etc., where the datum is the margin of the first activity in (7). Note that while each capital string will occur sequentially through time in the order indicated, it is understood that the activities of the capital string may be interspersed—i.e., the individual may perform on one string of capital, then another, return to the first later in the day, and so on.

Reformulating (7) into (8) does not, of course, affect or change the overall character of the expression—in particular, the entire ensemble of differential terms must still sum to zero. However, by organizing the terms according to the postulated “quasi-autonomous” capital strings an important simplification has been achieved: the activity-marginal-value terms of each capital string must sum to zero independently of all remaining capital strings of the economic system.⁴² We may accordingly investigate the single capital-string in seeking insight into economic behavior at the individual and large-scale levels.

It is appropriate at this juncture to define in mathematical terms how discounted marginal-utility is expectationally balanced. Imagine, with reference to Figure 2 (C), that activity-marginal-value is balanced only up to the point “(+)” in time. From (8), it is required that

$$\begin{aligned}
 (9) \quad & \text{(a)} \\
 & (IU_{L1}^i \delta L_1^i) \exp(-[\alpha + \beta - \gamma] [T_{L1}^i=0.0]) \\
 & \text{(b)} \quad \text{(c)} \\
 & + (IU_{L2}^i \delta L_2^i) \exp(-[\alpha + \beta - \gamma] T_{L2}^i) + (IU_C^i(C^i) \delta C^i) \exp(-[\alpha + \beta - \gamma] T_C^i) = 0
 \end{aligned}$$

(Intratemporal efficiency). The consumable-constraint for this capital-string element may be written:

$$(10) \quad b^i \delta C^i = [\langle mp_{L1} \rangle^i \delta L_1^i] \langle mp^{cap-L2} \rangle^i + \langle mp_{L2} \rangle^i \delta L_2^i$$

where

- b = consumable-consumption rate per unit time of consumption;
- mp_{L1} = marginal productivity of indirect labor;
- mp^{cap-L2} = marginal productivity of capital with respect to direct labor;
- mp_{L2} = marginal productivity of direct labor.

At the solution point, making use of (10), each of the three term-pairs—(a)-(b), (a)-(c), and (b)-(c)—equals zero, this being an essential mathematical property. For example, with δC^i suppressed, substituting (10) into (9) gives

$$\langle [mp_{L1}][mp^{cap-L2}] / [mp_{L2}] \rangle^i \exp(-[\alpha + \beta - \gamma] T_{L2}^i) = 1$$

where simplifications and cancellations have been introduced. It is seen how the marginal productivities of labor and capital must be (expectationally) adjusted (in overall effect) to accommodate the expectational discounting of direct labor instant utility, in accordance with the individual's purposeful planning.

Returning now to balancing activity-marginal-value across intertemporal time, moving (+) to the right in Figure 2 (III) invalidates the expectational balancing represented in (9). This may be shown by first recognizing that the capital perturbation, $\delta Q^{cap} = mp_{L1} \delta L_1^i$ in (10), persists over extended time. And so the benefit in increased consumption $b^i \delta C^i$ (or, alternatively, decreased direct-labor δL_2) is repeatedly received—in principle, albeit diminishing, to time infinity (in the present formulation). More to the point, terms (b) and (c) in (9) can no longer, by themselves, balance the negative activity-marginal-value of term (a) because this would ignore the substantial contributions to the total or aggregate activity-marginal-value of subsequent labor/consumption cycles: A more comprehensive treatment than the foregoing is appropriate.

For the present purposes this treatment may be omitted. Suffice it to note that (9) is logically extended to account for the expected persistence of the capital perturbation. The result is a balancing of activity-marginal-value throughout intertemporal time, a balancing that accounts for the discounting of expected instant utility. In the following development, we investigate the effect on labor, capital, and business activity of a concerted, society-wide shift in expectational uncertainty.

(ii) *Effect of a Shift-to-Pessimism on Employment, Capital-Intensity, and Business Activity.* The immediate objective is to isolate the single capital string in (8), and represent the effect of the capital perturbation on δL_1^i and δL_2^i throughout the intertemporal interval—where,

as above, δC^i is set to zero so that labor-activity is both the investment and return exclusively. For the single capital string, (8) becomes (converting a,b,c... to 0,1,2,...):

$$\begin{aligned}
 (11) \quad & \langle \Delta L_1^i \rangle_0 + \delta L_2^i \exp(-[\alpha + \beta - \gamma] T_{L2}^i) \rangle_0 \\
 & + \langle \delta L_1^i \exp(-[\alpha + \beta - \gamma] T_{L1}^i) + \delta L_2^i \exp(-[\alpha + \beta - \gamma] T_{L2}^i) \rangle_1 \\
 & + \langle \delta L_1^i \exp(-[\alpha + \beta - \gamma] T_{L1}^i) + \delta L_2^i \exp(-[\alpha + \beta - \gamma] T_{L2}^i) \rangle_2 \\
 & \vdots \\
 & = 0
 \end{aligned}$$

where, in addition to suppressing the consumption terms, the identical labor instant utilities have been canceled. Note that the “generating” indirect-labor perturbation is assigned the symbol ΔL_1^i rather than δL_1^i . This is done to set apart, or identify, the perturbation source, and also to recognize that $\langle \Delta L_1^i \rangle_0$ is substantially larger than the induced $\langle \delta L_1^i \rangle_n$ and $\langle \delta L_2^i \rangle_n$ (on a relative basis, of course, since all perturbations are infinitesimal).

In addressing $\langle \delta L_1^i \rangle_n$, it is first recalled that the capital perturbation at the end of the initial indirect-labor session (point [1] in Figure 3A) is subsequently diminished only during direct labor. Its amplitude is accordingly constant to the start of consumable production, decreases during the productive activity, and then remains constant to the start of the following indirect labor session, thereafter experiencing a recovery, as seen in Figure 3B. The recovery, however, is incomplete—i.e., it falls short of the original incremental magnitude—as (indirect) labor-duration is marginally reduced in accordance with the necessary decay of the capital increment. (In this regard, the “surviving” or residual capital-increment at the end of a direct-labor session would be restored to its initial magnitude to first order in the subsequent indirect labor session—were the indirect labor to remain unchanged.)

Recognizing that the decay has an exponential tendency, as is typical of small perturbations about a stable solution, and, further, that it is commensurate with the initial perturbation magnitude, allows the following formulation,

$$(12) \quad \langle \delta Q_{LI}^{cap} \rangle_n^i = \langle \Delta L_1 \rangle_0^i \langle mp_{LI} \rangle^i \exp(-\langle \Lambda^{-1} \rangle^i \langle T_{LI} \rangle_n^i)$$

where $\langle \Lambda \rangle^i$ is the time constant of the decay and $\langle T_{L1} \rangle_n^i$ is intertemporal time at the margin of the n th indirect-labor session, measured from the initial ($n = 0$) indirect-labor margin ($\langle T_{L1} \rangle_0^i = 0$). As just noted, indirect labor during the perturbation decay is marginally smaller than its unchanging, periodic magnitude—this being the only way that the capital increment can be reduced back to zero. In this regard, it is not the magnitude of $\langle \delta Q^{\text{cap}}_{L1} \rangle_n^i$ that determines $\langle \delta L_1 \rangle_n^i$, but the change $\langle \delta[\delta Q^{\text{cap}}_{L1}] \rangle_n^i$ from the margin of the previous indirect labor session. In particular,

$$\begin{aligned} \langle \delta[\delta Q^{\text{cap}}_{L1}] \rangle_n^i &= -\langle \Delta L_1 \rangle_0^i \langle mp_{L1} \rangle^i \left\{ \exp(-\langle \Lambda^{-1} \rangle^i \langle T_{L1} \rangle_{n-1}^i) - \exp(-\langle \Lambda^{-1} \rangle^i \langle T_{L1} \rangle_n^i) \right\} \\ &= -\langle \Delta L_1 \rangle_0^i \langle mp_{L1} \rangle^i \left\{ \exp(-\langle \Lambda^{-1} \rangle^i [n-1] \underline{P}) - \exp(-\langle \Lambda^{-1} \rangle^i n \underline{P}) \right\} \end{aligned}$$

making use of the identity $T_{L1}^i = \langle T_{L1} \rangle_0^i + n \underline{P}$ with $\langle T_{L1} \rangle_0^i = 0$, where \underline{P} is the periodic interval. The corresponding incremental reduction of indirect labor is given by

$$\begin{aligned} (13) \quad \langle \delta L_1 \rangle_n^i &= \langle \delta[\delta Q^{\text{cap}}_{L1}] \rangle_n^i / \langle mp_{L1} \rangle^i \\ &= -\langle \Delta L_1 \rangle_0^i \left\{ \exp(-\langle \Lambda^{-1} \rangle^i [n-1] \underline{P}) - \exp(-\langle \Lambda^{-1} \rangle^i n \underline{P}) \right\} \end{aligned}$$

Regarding the incremental reduction of direct labor it is recalled that the quantity of capital at the beginning of the direct-labor session enters the production function—i.e., explicit account is not taken of capital wear during the work session. The resulting direct-labor increment is accordingly

$$\begin{aligned} (14) \quad \langle \delta L_2 \rangle_n^i &= -\langle \delta Q^{\text{cap}}_{L2} \rangle_n^i \langle [mp^{\text{cap-L2}}] / [mp_{L2}] \rangle^i \\ &= -\langle \delta Q^{\text{cap}}_{L1} \rangle_n^i \langle [mp^{\text{cap-L2}}] / [mp_{L2}] \rangle^i \\ &= -\left\{ \langle \Delta L_1 \rangle_0^i \langle [mp_{L1}] [mp^{\text{cap-L2}}] / [mp_{L2}] \rangle^i \exp(-\langle \Lambda^{-1} \rangle^i n \underline{P}) \right\} \end{aligned}$$

where $\langle T_{L1} \rangle^i = \langle T_{L1} \rangle_0^i + n \underline{P}$ with $\langle T_{L1} \rangle_0^i = 0$ is again used. This, to reiterate, is the second mode of return on the original indirect-labor investment $\langle \Delta L_1 \rangle_0^i$, the first being for (subsequent) indirect labor $\langle \delta L_1 \rangle_n^i$.⁴³ Each term is, of course, one of an infinite number, as the capital increment

(expectedly) encounters repeated labor-activity in its (diminishing) projection across intertemporal time.

Substituting (13) and (14) into (11), again making use of the periodicity conditions (i.e., $T_{L1}^i = n \underline{P}$ and $T_{L2}^i = \langle T_{L2} \rangle_0^i + n \underline{P}$), and simplifying gives:

(15)

Labor Cycle 0a:

1

Labor Cycle 0b:

$$- \langle [mp_{L1}] [mp^{cap-L2}] / [mp_{L2}] \rangle^i \exp(-[\alpha + \beta - \gamma] \langle T_{L2}^i \rangle_0)$$

Labor Cycle 1a:

$$- \{ 1 - \exp(-\langle \Lambda^{-1} \rangle^i \underline{P}) \} \exp(-[\alpha + \beta - \gamma] \underline{P})$$

Labor Cycle 1b:

$$- \langle [mp_{L1}] [mp^{cap-L2}] / [mp_{L2}] \rangle^i \exp(-\langle \Lambda^{-1} \rangle^i \underline{P}) \exp(-[\alpha + \beta - \gamma] (\langle T_{L2}^i \rangle_0 + \underline{P}))$$

Labor Cycle 2a:

$$- \{ \exp(-\langle \Lambda^{-1} \rangle^i \underline{P}) - \exp(-\langle \Lambda^{-1} \rangle^i 2\underline{P}) \} \exp(-[\alpha + \beta - \gamma] 2\underline{P})$$

Labor Cycle 2b:

$$- \langle [mp_{L1}] [mp^{cap-L2}] / [mp_{L2}] \rangle^i \exp(-\langle \Lambda^{-1} \rangle^i 2\underline{P}) \exp(-[\alpha + \beta - \gamma] (\langle T_{L2}^i \rangle_0 + 2\underline{P}))$$

·
·

= 0.

In this expression, to provide a brief overview, part “a” of Labor Cycle 0 represents the applied (indirect) labor perturbation (at expected time $\langle T_{L1}^i \rangle_0 = 0$) and part “b” represents the initial direct-labor expected response at time $\langle T_{L2}^i \rangle_0$. During Labor Cycle 1, the first indirect-labor response expectedly occurs “1a” along with the second direct-labor response “1b”. And so on. The equation represents, for any given capital string of the economic system, the balancing of increased activity-marginal-value of labor across intertemporal time, against the decreased activity-marginal-value due to the indirect labor perturbation at the datum $\langle T_{L1}^i \rangle_0 = 0$.

The mathematical development, with the objective of showing how a general shift-to-pessimism affects business activity, has not yet reached its final form. In this regard, to properly employ (15) for this purpose it is necessary that the periodic character be preserved. This requires that the time constant of the capital structure (in response to the “crisis in confidence”) be much greater than the intertemporal time constant identified with the expectational discounting: In

effect, all agents ignore capital diminution as occurring too far in the future, as compared to their near-term interests. The capital-response time constant may be understood to be approximated by Λ , the time constant of the capital-perturbation decay. Then the requirement may be stated: $\Lambda^{-1} \ll (\alpha + \beta - \gamma)$. This requirement has the effect of not only removing Λ throughout (15), but also suppressing indirect labor responses to the original perturbation. (That is, as $\langle \Lambda^{-1} \rangle^i$ becomes negligible vis-à-vis $(\alpha + \beta - \gamma)$, $\{ \exp(-\langle \Lambda^{-1} \rangle^i [n-1] \underline{P}) - \exp(-\langle \Lambda^{-1} \rangle^i n \underline{P}) \} \rightarrow 0$.) Equation (15) accordingly becomes:

$$(16) \quad \langle [mp_{L1}][mp^{\text{cap-L2}}]/[mp_{L2}] \rangle^i \exp(-[\alpha + \beta - \gamma] \langle T_{L2}^i \rangle_0) \times \\ \times [1 + \sum_{n=1, \infty} \exp(-[\alpha + \beta - \gamma] n \underline{P})] = 1.$$

One final specialization is now introduced. It is assumed that consumptive activity is substantially inelastic compared to labor—this being accomplished by postulating a nearly “vertical” consumptive instant utility at the margin. The result is the analytical benefit of, in effect, “freezing” the direct-labor duration in the initial response to the crisis. In particular, the invariant consumptive duration fixes the amount of consumed—and hence produced—consumable, and this, in conjunction with the (temporarily) invariant marginal productivity of capital with respect to direct labor fixes the direct-labor duration. Indirect labor is then left as the sole *immediate* response to the confidence shift. For this case (16) becomes:

$$(17) \quad \langle mp_{L1} \rangle^i = [\langle [mp^{\text{cap-L2}}]/[mp_{L2}] \rangle^i]^{-1} \times \\ \times \exp([\alpha + \beta - \gamma] \langle T_{L2}^i \rangle_0) / [1 + \sum_{n=1, \infty} \exp(-[\alpha + \beta - \gamma] n \underline{P})].$$

where it is seen that the marginal productivity of indirect labor $\langle mp_{L1} \rangle^i$ is shown in its initial (immediate) functional dependence on the biased-discount parameter γ .

It is appropriate to recognize at this point that the crisis in confidence, or shift-to-pessimism, sets the economic system off in a new (unexpected) direction—i.e., along an (infinite)

set of (postulated coincident) worldlines whereon concerted, society-wide business activity is moving to a contracted level. Contrary to the “old” economy, which adhered to a periodic character in all respects (up to the crisis) along the original worldlines, this “new” economy is periodic only within the beginning of the intertemporal interval where activity utility hasn’t been discounted to near zero—i.e., in the much longer time-scale, transient responses of capital and activity to the crisis are aperiodic. Eventually the economy will reacquire a comprehensive long-term periodicity (in the absence of further changes in outlook), albeit at the depressed level. To investigate how these processes unfold, we now shift the deviant discount coefficient γ from a positive (excessively favorable) magnitude to a negative (excessively unfavorable) magnitude, and address (17) to gain insight into the system response.

From (17), it is first seen that a shift of the biased-discount parameter γ from a positive magnitude to a negative—i.e., the individual, and society in general, abandon the unrealistically rosy prognosis of the economic future for one of an opposite character—has the necessary consequence of increasing the marginal productivity of indirect labor (i.e., to maintain intertemporal efficiency). In reaching this interim conclusion, it is recognized that the marginal productivities (of capital $\langle mp^{\text{cap-L2}} \rangle^i$ and direct-labor $\langle mp_{L2} \rangle^i$)—along with (fixed) α , β , and \underline{P} —are invariant during the sudden shift from a positive to negative γ . The required increase of indirect-labor marginal productivity, in turn, reduces the indirect-labor duration (Figure 5).⁴⁴ In the real world, of course, the reduced employment will not be uniformly distributed as in the present simplified analysis. However, it may be understood that the direction of the overall employment response has been obtained.

To elaborate on the foregoing analysis, the sudden drop of employment in the production of capital is necessary because of the newly pessimistic state of the collective mind, one that recognizes an increased probability that the economy will cease. All accordingly judge, in optimizing their intertemporal activity, that present investments (debits of activity-marginal-value) under the operative plan will likely not be fully compensated by later returns (corresponding intertemporal credits). The return on the marginal investment must accordingly increase, and this

requires a decrease in capital-producing labor (for decreasing returns on labor). Eventually, however, as alluded to earlier, total employment in the long run at the diminished capital intensity could exceed pre-crisis levels, as people, laboring with reduced capital assistance, work longer hours to meet their needs.

Regarding capital, it follows that reduced productive-labor shifts the advantage to dissipation, with the resulting decline of capital intensity to a new level. Significant changes in the price structure will, of course, attend the decline,⁴⁵ particularly in (durable) capital which is more sensitive to intertemporal discounting. Because of the reduced level of capital intensity there will be a correspondingly diminished domestic product, even if labor returns to or exceeds the pre-crisis level. Cyclic business activity can be expected as a concerted and unrealistically favorable prognosis returns to again promote excessive investment and production, leading once again to a crisis in confidence with the resulting decline in industry and trade.

V. CONCLUSION

The effect of uncertainty on the business cycle has been addressed by first representing the Clark/Wicksell real interest rate in the differential form of the canonical emotive equation. In this treatment it is recognized that increased indirect-labor may be deemed the investment, rather than sacrificed finished good, with the return-on-investment being a reduction of subsequent labor in the production of capital and consumables. Simplifying assumptions are introduced to yield an exclusive dependence of indirect (capital-producing) labor on a sudden society-wide shift-to-pessimism. Most prominent among the assumptions, and one imposing an overriding influence, is near-term activity-regimen periodicity for all agents—i.e., despite profound transients due to the crisis, all plan their near-term activities as though the stationary-state (“equilibrium”) condition exists, due to their steep intertemporal discounting. A second prominent assumption is the relative inelasticity of consumption—yielding a collateral inelasticity of direct labor at the initiation of the economic downturn. A third assumption (related to the first assumption) required the time constant of capital-intensity change to be much greater than that of the exponential discounting of intertemporal instant utility. It has been illustrated how a sudden downward shift of public

confidence concerning future economic performance, from one of excessive optimism to the reverse, can immediately reduce indirect labor, followed by a gradual decline of capital intensity and consumption along with a related increase in labor.

As a concluding note, a hallmark of the mature mathematical science is a canonical formulation that receives wide acceptance. Clearly, economics has yet to reach this high plateau. (See the articles by Elster, Bowles, Hodgson, and Rabin in the 1st quarter 1998 issue of the *Journal of Economic Literature*.) What is necessary is that the discipline break free of its mechanistic tenor dating from the nineteenth century, and recognize that understanding and explanation of economic behavior will come from the substantive modeling of cognitive function. In the present work, additional progress along this line is offered. Contrary to standard theory which has been “challenged” in modeling the procyclic variation of labor and consumption with the business cycle (Zarnowitz [1999]), the present approach, accounting for uncertainty in individual expectational planning, has shown some progress. This approach is predicated on a simple, yet far-reaching, change in utility theory: Instead of directly identifying utility with consumables in the problem definition, *instant utility* is exclusively identified with the process-of-knowing attending thought and action. After all, instant-utility is clearly more essential than utility, and it is empirically measurable.

NOTES

¹ Significant refinements have been introduced since the paper’s original presentation at the 4th Biennial Pacific Rim Allied Organizations Conference in Sydney, Australia, on 12 January 2000.

² Ramsey, however, employed *community* instant utility—i.e., productive and consumptive activity were superposed with both filling time to infinity, rather than resolve activity into discrete components. While this basis facilitates macroeconomics where essential details are sometimes lost in order to minimize complexity, it is of course alien to individual behavior.

³ This section is largely obtained from the IAEC paper.

⁴ The present theory is consistent with emerging (empirical) behavioral science, in that once a plan has been adopted it guides the individual’s activity as though free will has been suspended. (For an overview see Nichols and Newsome [1999, p. C38].)

⁵ Terms without indices, e.g. time *t*, have implicit indices in accordance with their context in the expression, as an adopted convention.

⁶ The concept of value lies at the heart of the present methodology. Value is anticipatory pleasure (with the dimension [PLEASURE], or [INSTANT-UTILITY]). Frequent reference to value will be made in the text—e.g., to *plan value*, and *activity-marginal-value*.

⁷ Of course, the consideration of identified alternative expectational plans could itself be judged part of an already initiated operative plan.

⁸ Here we maintain total mathematical rigor. In this regard, one can easily think of surprising experiences that seemingly have no effect on the operative plan. But all conscious experience is neurologically imprinted in the brain, and hence exerts some influence on cognitive function, however slight this influence may be.

⁹ It may be noted here that a prime reason for giving the dimension [TIME⁻¹] to λ in the emotive equation was to avoid indeterminacy in the decision process (i.e., in the selection of the operative expectational plan). In particular, were λ to be dimensionless, as in Strotz's treatment, decisions, all else equal, would inappropriately depend on anticipatory utility (in the present methodology)—i.e., on how long the individual deliberated. In other words, decisions between plans would be time-extensive cognitive events, contrary to "point-in-time" experience—which, of course, is not to say that such point-in-time events are independent of deliberation. (As an aside, the individual's "decisions" play an essential role in the operational plan—in that these uncertain intertemporal events guide his or her experience. And planned decisions—conditional or not—have, in a sense, already been made. Recent biopsychological research is tending to substantiate this conclusion—See fn 2, and Bechara, et. al. [1997].)

¹⁰ In this regard, principle, morality, ethics, habit, etc., enter the individual's expectational planning in varied ways, including candidate plan selection and expected constraints.

¹¹ When commodities are directly assigned utility, as frequently occurs in economic theory, ordinality is the logical conclusion (as discerned by Edgeworth [1881], Pareto [(1906) 1971], Hicks [(1939) 1946], Samuelson [1947], et. al.; See also Hausman [1992]).

¹² "Core" theory refers to basic methodology that incorporates *equilibrium* of the Walras genre [1874-1877]; see Hausman [1992]). Regarding time, for many years mathematical economists have sought a satisfactorily accommodation of this crucial parameter in theory. But the direct identification of utility with consumables (initiated by Walras) has sidetracked the effort.

¹³ "Equilibrium" is in quotes to distinguish its special meaning herein relative to (static) neoclassical equilibrium: In particular, the economy may be transient (due to an internal or external shock) from the macro-perspective, yet stationary-state in the individual's planning. This apparent contradiction is due to the individual's steep subjective discounting relative to the characteristic time of the macroeconomic parameters. Here the individual may recognize that the economy is evolving over the long-term, but he or she concentrates exclusively on the short-term.

¹⁴ Expectational (or subjective) uncertainty can be accommodated in the periodic economy, under given restrictive conditions. As a particular, while the individual remains uncertain about future economic events and conditions, Nature (or the modeler) orchestrates uncertainty extinction—including the individual's uncertainty about his or her own future actions—such that a prescribed scenario plays out. This capability is exploited in the present article.

¹⁵ As has been noted, and will be later developed, the expectational discounting of intertemporal instant utility is postulated to proceed much "faster" than the response of capital to an economic shock—i.e., the discounting time constant is much smaller. Expected transients in activity regimens and capital intensity are therefore negligible in the individual's planning. (In this regard, the individual could foresee a great personal injury, or even death, at a future time. But sufficiently steep discounting would suppress any effect of this expectation on the planning of near-term activity.)

¹⁶ The emotive discount function (or emotive mapping function) is identical to Strotz's [1956] "discount function," except for *dimension*. In particular, the emotive discount function has the dimension [TIME⁻¹] while the Strotz discount function is dimensionless. The salient effect is that while plan intertemporal-utility is maximized as a purely intellectual exercise in Strotz's study, in the present theory decisions are substantively emotive. In this regard, the product of the emotive discount function [TIME⁻¹] with intertemporal utility [PLEASURExTIME] yields real time anticipatory instant-utility (or plan-value [PLEASURE]).

¹⁷ A prominent distinction, however, is the identification of uncertainty (or its mathematically-meaningful, *occurrence-probability*) with action schemes (worldlines) in the present theory, versus the inappropriate incorporation of probability in the intratemporal utility functions.

¹⁸ The present methodology renders prominent models of capital function expectationally determinate—i.e., crusionia plant (see Knight [1944]), costless conversion (see Solow [1965]), and two production function (see Lange [1936]).

¹⁹ For individuals as price takers, commodity indifference curves can be constructed for the individual's fixed expectational plan. However, utility (or, more fundamentally, anticipatory pleasure) is imputed to all commodities (productive and consumptive) in the present theory, rather than preemptively assigned to consumables alone as in equilibrium theory. This departure may significantly affect the scope of indifference curves in economic theory.

²⁰ The standard plots or graphs (Marshallian: see Rima [1991, 326]) of commodity supply and demand versus price may be determined—where the intersection point of the two curves gives the price. The present approach differs from standard neoclassical theory by determining value, on which prices depend, as part of the solution.

²¹ A difference, however, is that the marginal disutility of the initiating action is recognized in the present methodology.

²² This not to say that time is never introduced in economic theory, but rather that it is always done on an ad hoc or part-way basis. The work of Strotz, for example, which is a significant antecedent to the present methodology, did identify instant utility with consumption, but he did not extend the scope to include the instant utility of labor and leisure. The work of Becker [1965] and DeSerpa [1971], among others, similarly did not take substantive and comprehensive account of instant utility. (See Georgescu-Roegen's introduction to Gossen [(1854) 1983].)

²³ To elaborate on this crucial point, periodicity is expected only over the intertemporal interval wherein instant utility has not been discounted to (sensibly) zero. Beyond this interval significant non-periodicity can occur, and the individual may objectively recognize this. But (as has been noted) such “long-term” expectation will not affect “short-term” planning. This distinction is prominent in the development to follow.

²⁴ In Lange's formulation, production functions yield a product-flow (i.e., dimensions [GOOD/TIME]) at each point in time, while in the present approach production functions yield aggregate or accumulated product (dimension [GOOD]) corresponding to duration of the work-session.

²⁵ Examples of this function include blacksmithing, wherein the tools are used to produce both replacement tools and finished goods, and road building, where roads are used in their own production/maintenance and in commerce.

²⁶ Lange's agents could, of course, be understood to plan their activities. But the treatment is mechanistic, and “real time” may be assigned to the x-axis.

²⁷ Of course it should not be understood that the mind manages an exact or precise prediction of future experience. First, uncertainty does not permit a unique timeline (or worldline) of how the future may evolve. Second, the human mind lacks the infinite power to model any scenario in complete, rigorous detail, particularly in concert with all other participants in the economic system. (Uncertainty is used to great advantage in this regard. See the IAES paper, [1999a].) And third, inexorable surprise will frequently negate plans. Nevertheless, the postulate of “coincident” real and imaginary time can be useful in the investigation of behavior, as demonstrated herein.

²⁸ This condition may be relaxed, but it would introduce unnecessary complication into the present study.

²⁹ Here it may be noted that in economics we are not interested so much in the behavior of the individual, as we are in the behavior of ensembles of great numbers of people. This is analytically beneficial, inasmuch as the definition of sufficiently stable instant-utility functions for the individual must be considered somewhat of a challenge. For very large numbers of individuals, however, it is conceivable that the magnitude of labor instant utility relative to leisure, and consumption, may be ascertained, and, further, that the relations are stable, or of some functional dependence. In this quality, utility theory is similar to heat transfer of mechanical engineering: In either discipline functional forms defined on the basis of theoretical and empirical insight are fit to empirical data, and subsequently employed in analysis.

³⁰ The problem in standard theory, to reiterate, is that the specific utility of consumption has the dimensions “utility per unit consumable”—i.e., [PLEASURExTIME/GOOD]—whereas the specific utility of labor has the dimensions “utility per unit time”—i.e., [PLEASURExTIME/TIME] = [PLEASURE] = [INSTANT-UTILITY]. This obvious incoherence in economic theory could be resolved by adopting the principle that *measurable* instant utility is fundamental.

³¹ This, of course, is an abstraction from real-world economics, to render the problem analytic.

³² One of the mathematical characteristics of labor/capital function (for infinitely divisible capital) is that the capital perturbation would infinitely-persist (in the periodic formulation, to first order) were the indirect-labor to remain unchanged. In this regard, just as the indirect-labor 100% restores the normal (physical) capital depletion in the periodic formulation, so would the capital-perturbation be 100% restored. It is the indirect-labor shortfall (marginal reduction) that determines the capital-perturbation decay with time. (Lange did not acknowledge this effect in his analysis.)

³³ Value is understood to be essentially identified with the psychosomatic response attending the expectation of thought and action. Hence the reference to the balancing of activity-marginal-value in the discussion.

³⁴ Alternatively, the (infinitesimal) direct-labor response could be postulated zero, thereby yielding maximum consumptive responses throughout intertemporal time.

³⁵ Prior to the crisis-in-confidence, and after stationary-state is again achieved at the depressed level of economic activity, long-term periodicity will exist. In either case, however, steep myopic discounting is assumed.

³⁶ As an aside, steep discounting should aid the study of spatially and demographically non-uniform economics. Innovation diffusion and distribution are examples that may be amenable to analysis.

³⁷ The reader may observe that immortality is yet another “derived” conclusion of the periodicity postulate. Here it may be noted that extreme consequences of an advantageous assumption (to promote analysis) are not uncommon in the physical sciences. As an example, in fluid mechanics the incompressible gas is sometimes assumed, with the mathematical consequence of an infinite speed of sound.

³⁸ Capital is, of course, ubiquitous, existing not just in business and commerce but throughout the social system, including government, as integral parts of the lives of each individual. (The simple task of investing time in a shopping list—an elementary form of capital—to realize saved time in the market is an example. One can, alternatively, invest in a mental shopping list, so capital need not be tangible in the normal sense.)

³⁹ In this regard, preliminary assessment suggests that experience could cause the individual to adjust expectational plans such that “myopic surprise” does not occur.

⁴⁰ That is, α is the same, but not necessarily K (which becomes moot upon differential maximization of E).

⁴¹ As has been noted in the introduction, hypothetical economy-ending disasters include a great volcanism, large meteor impact, and global nuclear war. The postulate offers an intuitive basis for both conceptualizing and formulating the model.

⁴² In the integrated economy, each individual will not typically consume his own finished product. Here it is assumed that trade provides commodities that will be consumed commensurately with his or her production.

⁴³ It may be noted here that the Clark/Wicksell expression for the real interest rate does not recognize the indirect-labor reduction subsequent to (and resulting from) the perturbation. This omission is in addition to neglect of the repeated returns-on-investment through intertemporal time as the (persisting) capital perturbation is employed in recurring (intratemporal) productive sessions.

⁴⁴ Of course, the opposite can occur. In particular, an increase in confidence, realistic or otherwise, may produce increased employment.

⁴⁵ This in addition to the immediate change in the price structure due to the sudden shift-to-pessimism.

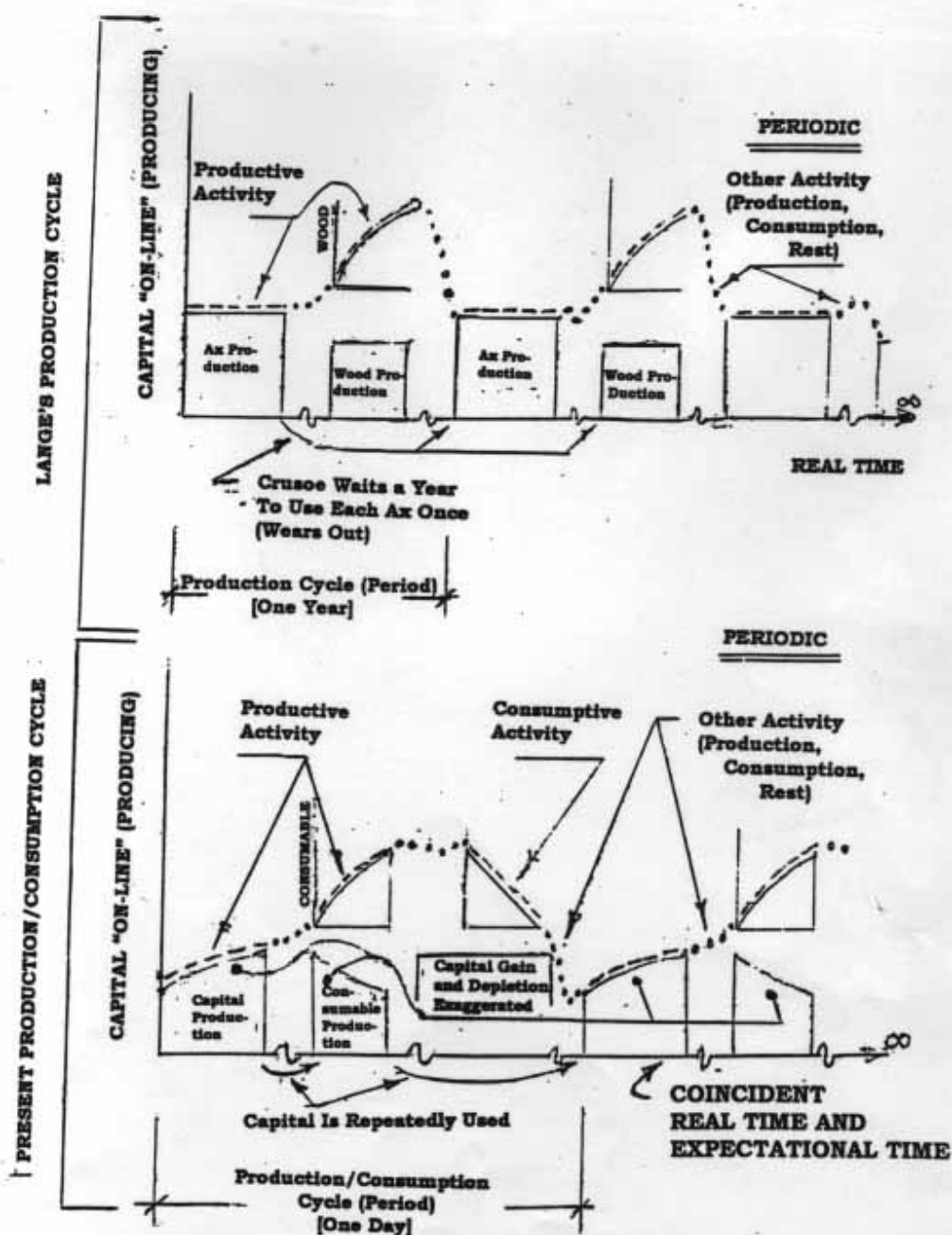


FIGURE 1. COMPARISON OF LANGE'S PRODUCTION CYCLE WITH THE PRESENT PRODUCTION/CONSUMPTION CYCLE. (SINGLE INDIVIDUAL IN FUNCTIONAL ISOLATION.)

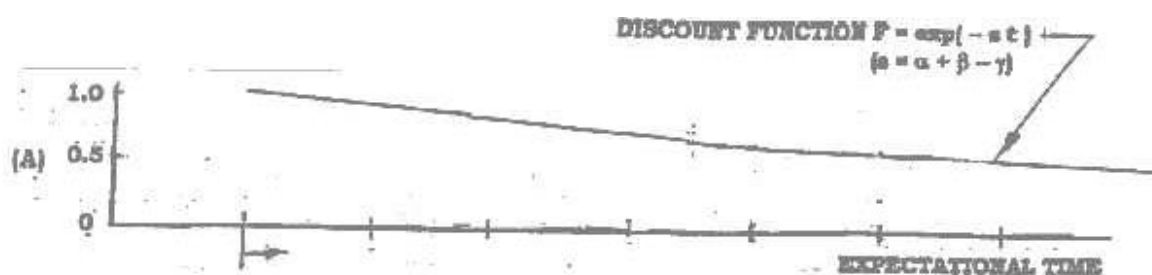


FIGURE 2B. DISCOUNT FUNCTION FOR EXPECTED INSTANT UTILITY

α = emotive discounting
 β = rational assessment of risk/uncertainty
 γ = irrational exuberance or pessimism

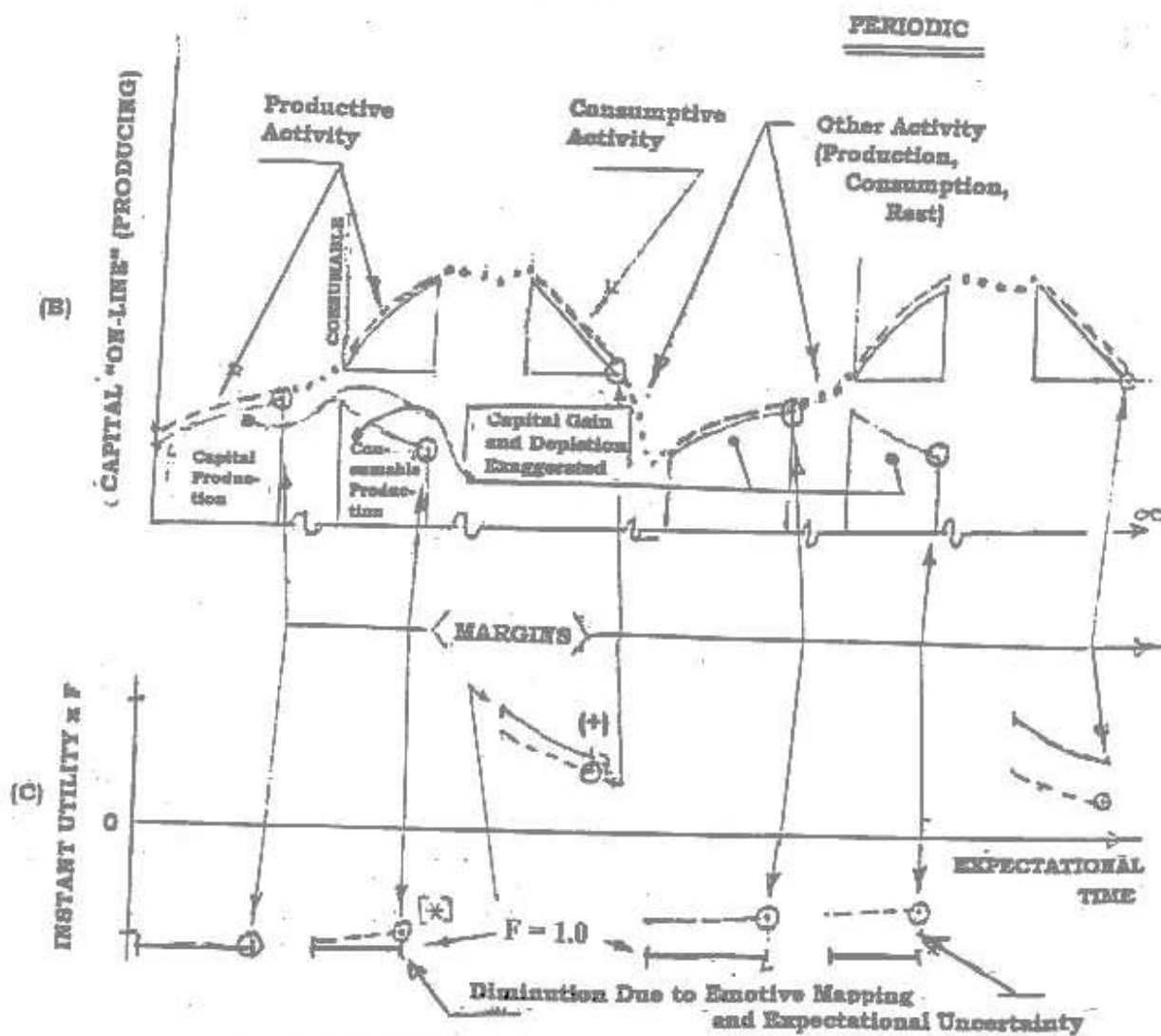
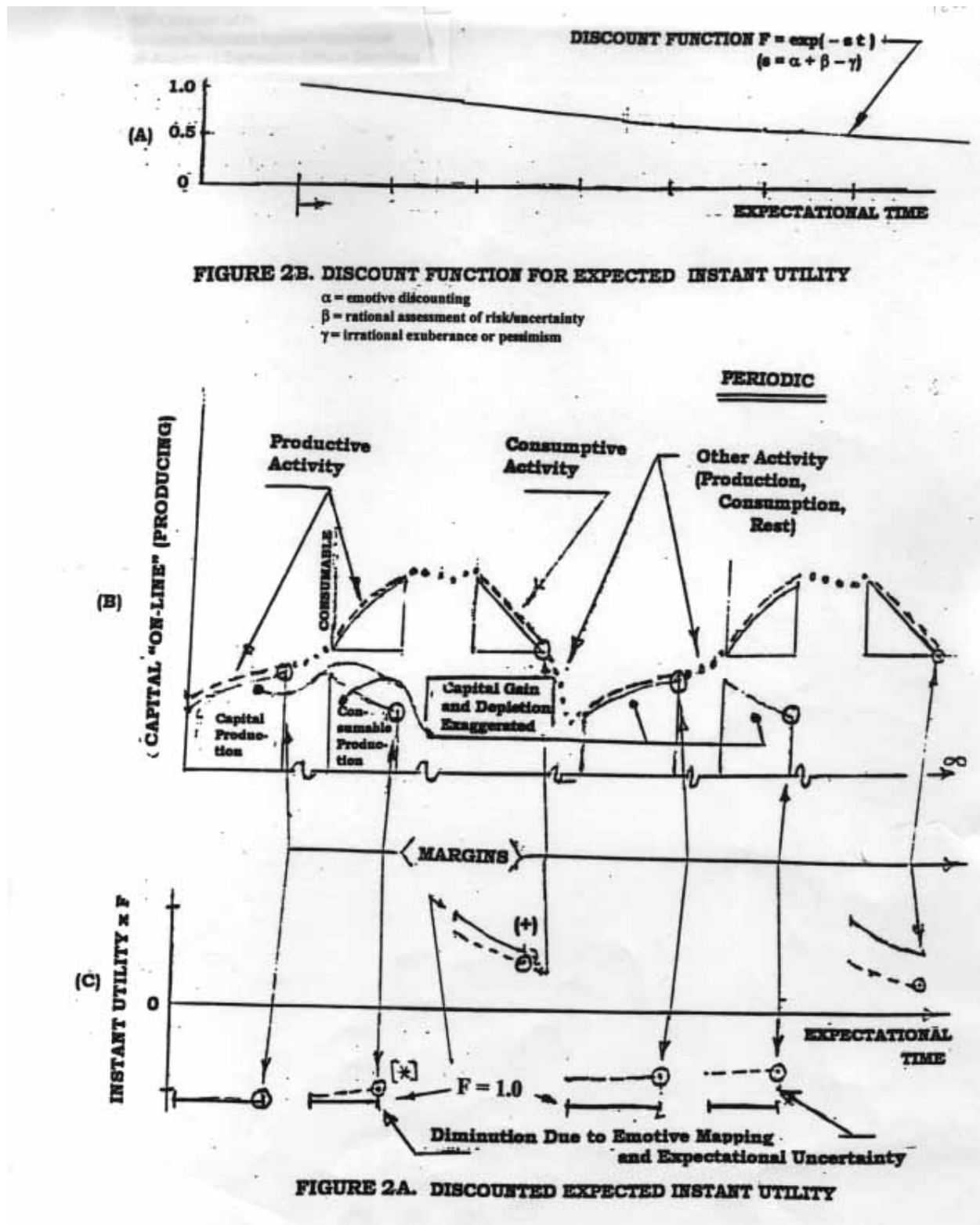


FIGURE 2A. DISCOUNTED EXPECTED INSTANT UTILITY



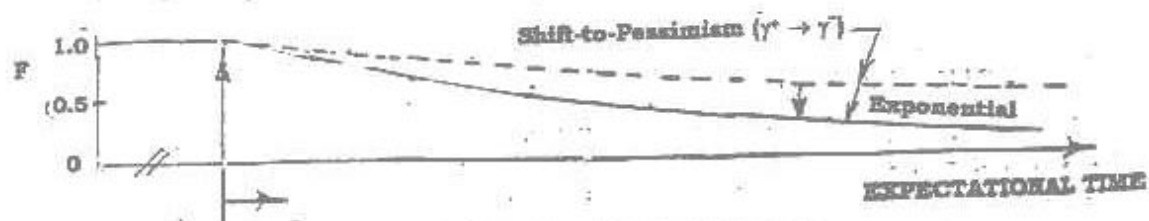


FIGURE 4B. DISCOUNT FUNCTION FOR EXPECTED
INSTANT UTILITY—ADJUSTED.

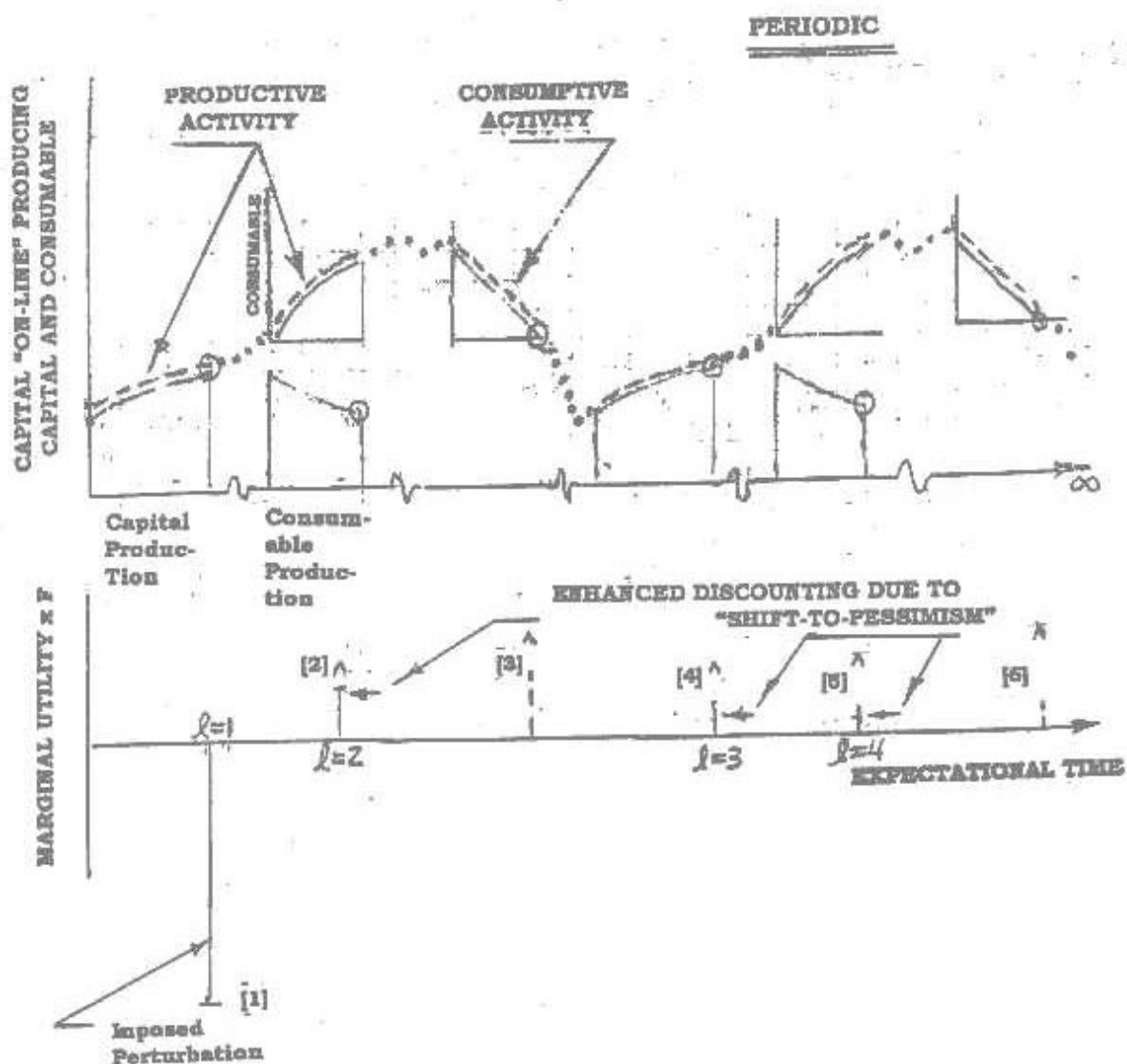
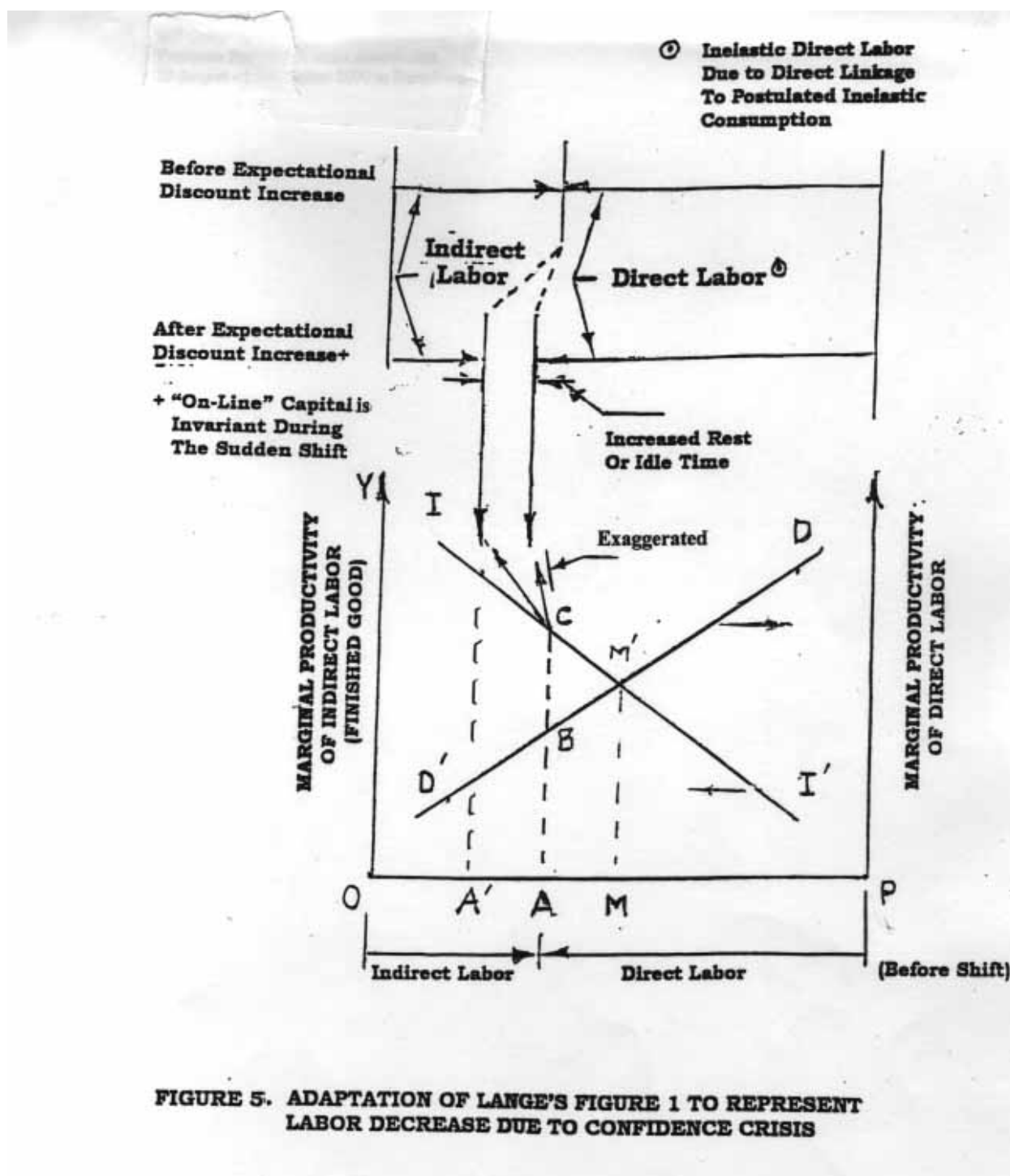


FIGURE 4A. EFFECT OF A SHIFT-TO-PESSIMISM ON
DISCOUNTED MARGINAL UTILITY.



NOMENCLATURE

A_n^i	Duration of activity n (dimension [TIME]). (Also expressed $A_{i,n}$.)
b	Expected rate of consumption (dimensions [GOOD/TIME]).
C	Expected duration of consumption (dimension [TIME]).
δC^i	Expected marginal differential of consumptive activity (dimension [TIME]).
E_k^i	Real-time instant utility (plan value) experienced by individual i in anticipation of expectational plan k (dimension [PLEASURE] = [INSTANT UTILITY]).
f	Expectational (subjective) occurrence-probability of a given worldline.
IU_C^i	Instant utility of consumption (dimension [PLEASURE]).
IU_{L1}^i	Instant utility of indirect labor in the production of capital, invariant (dimension [PLEASURE]).
IU_{L2}^i	Instant utility of direct labor in the production of a consumable, invariant (dimension [PLEASURE]).
K	Emotive mapping/discount coefficient (dimension [TIME ⁻¹]).
L_1	Expected duration of (indirect) labor in the production of capital (dimension [TIME]).
δL_1^i	Expected marginal differential of indirect labor (dimension [TIME]).
L_2	Expected daily duration of (direct) labor in the production of a consumable (dimension [TIME]).
δL_2^i	Expected marginal differential of direct labor (dimension [TIME]).
mp_{L1}	Marginal productivity of indirect labor (dimensions [GOOD/TIME]).
mp^{cap-L1}	Marginal productivity of capital with respect to indirect labor (dimensionless). (Not used.)
mp^{cap-L2}	Marginal productivity of capital with respect to direct labor (dimensionless).
mp_{L2}	Marginal productivity of direct labor (dimensions [GOOD/TIME]).
P	Expected process-of-knowing instant utility: datum is start of plan (dimension [PLEASURE]).
\underline{P}	Periodic interval of intertemporal activity (dimension [TIME]). (The economy is “quasi-periodic” when the characteristic time of emotive and uncertainty discounting is much shorter than the time constant of capital-response to a perturbation or shock to the economy.)
p_n^i	Expected process-of-knowing instant utility in the performance of activity n: datum is the expected start of activity (dimension [PLEASURE]).
Q^{cap}	Quantity of capital (dimension [GOOD]).
t	Expectational (intertemporal) time, from start plan initiation (dimension [TIME]).
$\langle T_C \rangle_n^i$	Expectational (intertemporal) time at the margin of the nth consumptive activity (dimension [TIME]).
$\langle T_{L1} \rangle_n^i$	Expectational (intertemporal) time at the margin of the nth indirect labor activity (dimension [TIME]).
$\langle T_{L2} \rangle_n^i$	Expectational (intertemporal) time at the margin of the nth direct labor activity (dimension [TIME]).

Greek Symbols:

α	Inverse time constant of autonomic (emotive) discounting of expected instant utility (dimension [TIME ⁻¹]).
β	Inverse time constant of the expectational occurrence probability of an economic cessation: rational assessment (dimension [TIME ⁻¹]).

γ	Inverse time constant of the expectational occurrence probability of an economic cessation: irrational assessment (dimension [TIME ⁻¹]).
Λ	Time constant of capital-intensity perturbation (dimension [TIME]).
λ	Expectational (emotive) mapping function (and expected instant utility discount coefficient). (Dimension [TIME ⁻¹].)
κ	Expectational occurrence probability coefficient (dimensionless).
Φ	Expectational constraint function.
Ψ	Virtual constraint function.
τ'	Intertemporal time, measured from the start of an activity (dimension [TIME]).

Superscripts:

c	Expectational constraint (along worldline-year wy of expectational plan k).
cap	Capital.
i	Individual i.
m	Virtual constraint (along worldline-year wy of expectational plan k).
T	Time.

Subscripts:

k	Expectational plan.
L1, L2, C	Indirect labor, direct labor, consumption.
w	Worldline.

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