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# The Pricing of Small Business Loans

John K. Ford

A major difficulty in determining the appropriate risk premium for lending to small businesses is the lack of market value information. This paper develops a mean-variance model that uses available failure rate data to establish a benchmark risk premium for lending to firms in specific industries. This model incorporates the benefits of diversifying across firms and industries. This paper also presents evidence that a random walk model provides the best forecast of future failure rates.

## I. INTRODUCTION

The traditional approach to equity valuation is to conduct a fundamental analysis of the economic characteristics of an individual firm and its industry. The modern approach adds the extra dimension of considering the effect of a stock on the risk and return of the portfolio. The professional and academic investment literature reflects the consensus that including portfolio considerations provides valuable insights into the correct pricing of a risky security.

Unfortunately, the important field of credit analysis still relies solely on the fundamental approach to loan evaluation. The usual procedure is to gather credit information, review financial statements, project cash flows and inspect possible collateral. Some banks also apply discriminant analysis to financial ratios in order to improve their ability to distinguish between acceptable and unacceptable borrowers (Eisenbeis & Avery, 1972; Fulmer, 1984; Johnson & Grace, 1990; Maniktala, 1991).

The usual loan pricing models in the banking literature include factors such as the cost of funds, origination costs, and compensating balances (Brick, 1984; Cramer & Sterk, 1982; Ferrari, 1992; Johnson & Grace, 1990). Some models do recommend using the bond rating as a measure of risk, but

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do not indicate how to translate the rating into an explicit risk premium (Maniktala, 1991). The lack of a systematic procedure for including uncertainty yields wide variations in the pricing of risk among banks (Edminster, 1984; Slater, 1986; Snyder, 1988; Wyman, 1991).

The purpose of this paper is to suggest a portfolio approach to including risk in commercial loan pricing. The idea to apply portfolio theory to the asset decisions of financial institutions is certainly not a new concept. (Hart, Oliver, & Jaffe, 1974; Koehn & Santomero, 1980; Pyle, 1971). Neither is the notion that the risk of an individual loan depends on portfolio considerations (Ford & Stanley, 1988; Paroush, 1992; Stanley & Ford, 1986). However, this previous work has not produced a practical way to incorporate risk in the pricing of commercial loans. An attempt by the American Bankers Association to study commercial loan pricing failed because banks were not willing to invest the time and money to develop the necessary data base (Makeever, 1984).

The relative availability of data may account for the difference in the development of portfolio applications for equities and commercial loans. For large firms with publicly traded securities market values provide a basis for gauging risk. In most cases bank customers are small firms and their securities are not traded. This paper avoids the problem by using accessible failure rate data to develop a benchmark interest rate for each category of commercial lending. Table 1 shows the average industry failure rates for recent years. This table includes all the business categories with information available from the Business Failure Record of Dun & Bradstreet Inc. for the years from 1972 through 1992. The Dun & Bradstreet definition of business failure includes those firms that ceased operations following assignment or bankruptcy; ceased with loss to creditors after execution, foreclosure or attachment; voluntarily withdrew leaving unpaid obligations; were involved in receivership, reorganization or arrangement; or voluntarily compromised with creditors. In other words, the business failure statistics indicate the proportion of firms that caused severe problems and losses for creditors.

# **Risk Measures**

The objective of this paper is to derive a benchmark risk premium for extending credit to a specific industry. The basic approach is to calculate the mean and variance of the failure rate of the loan portfolio assuming equal investment in a random draw of (n) firms from a population with a failure rate (F). The failure rate (f) of a portfolio chosen in this fashion depends on both the population failure rate (F) and the luck of the draw. According to

Industry	Average	Standard Deviation	Auto-correlation of Failure Rates	Auto-correlation of First Differences
Food & Kindred Products	0.57%	0.36%	84%	13%
Textile Mill Products	0.81	0.28	35	-29
Apparel & Other Textile Products	0.84	0.31	50	-2
Lumber & Wood Products	0.65	0.43	84	28
Furniture & Fixtures	1.17	0.51	70	-4
Paper & Allied Products	0.48	0.26	74	-17
Printing & Publishing	0.57	0.26	79	42
Chemicals & Allied Products	0.57	0.33	73	7
Leather & Leather Products	0.88	0.40	69	10
Stone, Clay & Glass Products	0.49	0.30	80	15
Machinery, except Electrical	0.60	0.38	84	38
Electrical & Electronic Equipment	0.88	0.38	80	29
Transportation Equipment	1.09	0.48	70	6
<b>Building Materials &amp; Garden Supplies</b>	0.51	0.33	83	36
General Merchandise Stores	0.47	0.25	74	-11
Food Stores	0.33	0.30	82	12
Automotive Dealers & Service Stations	0.34	0.25	80	2
Apparel & Accessory Stores	0.87	0.51	85	15
Furniture & Home Furnishing Stores	0.73	0.31	78	2
Eating & Drinking Places	0.53	0.49	80	-2
Drug & Proprietary Stores	0.23	0.09	69	-1
Sporting Goods	0.73	0.29	73	5
Jewelry Stores	0.39	0.31	77	-11
Hobby, Toy & Game Shops	0.44	0.34	71	-32
Camera & Photographic Supply Stores	0.76	0.43	72	7
Gift, Novelty & Souvenir Shops	0.50	0.23	72	-10

Table 1Annual Failure Rates by Category of Firm 1972–1992

Source: Dun & Bradstreet, Inc., The Business Failure Record, from 1972 to 1992.

statistical theory the expectation of the failure rate [E(f)] is equal to the population failure rate (F) (Wonnacutt & Wonnacott, 1977).

$$E(f) = F. \tag{1}$$

The variance of the failure rate  $[\sigma^2(f)]$  is approximately equal to (F/n) (Wonnacott & Wonnacott, 1977).

$$\sigma^{2}(f) = (F)(1-F)/n = (F-F^{2})/n \approx F/n.$$
(2)

Two reasons permit ignoring the  $(-F^2)$  term and using the simplifying assumption that the conditional variance equals (F/n). First, this assumption is conservative because it overstates the variance by  $(F^2)$ . Second, the actual values of (F) are on the order of one percent so  $(F^2)$  is quite small.

Equation (2) captures the risk of selecting an inordinate number of failed firms. The information in Table 1 suggests yet another source of uncertainty in the loan portfolio: the substantial variability in industry failure rates from year to year. The second column of Table 1 shows that the variability of the failure rate is highest for furniture and apparel retailers (0.51%) and lowest for drug stores (0.09%). In general, these standard deviations are quite large relative to the average values of the failure rates.

The derivation of equations (1) and (2) makes no allowance for variations in the population failure rate. In this particular case the population failure rate (F) is uncertain and equations (1) and (2) must be treated as conditional values. The expected failure rate [E(f|F)] of the portfolio given (F) is equal to the population failure rate (F). The variance  $[\sigma^2(f|F)]$  of the portfolio failure rate given (F) is approximately equal to the population failure rate (F) divided by the number of loans (n).

$$E(f \mid F) = F \tag{3}$$

$$\sigma^2(f|F) \approx F/n. \tag{4}$$

Hansen, Hurwitz, and Madow (1953) developed the use of conditional expectations in statistical sampling theory. One of their results is that the expected value of a random variable is equal to the expected value of the conditional expectation. In this case the expected value of the failure rate [E(f)] is equal to the expected population failure rate [E(F)].

$$E(f) = E[E(f|F)] = E(F) .$$
(5)

The variance of a random variable is equal to the variance of the conditional expected value plus the expected value of the conditional variance.

$$\sigma^{2}(f) = \sigma^{2}[E(f|F)] + E[\sigma^{2}(f|F)].$$
(6)

Substituting from equations (3) and (4) yields the following expression.

$$\sigma^{2}(f) = \sigma^{2}[F] + [E(F)/n].$$
(7)

Equation (7) shows the two components of risk: variations in the overall failure rate of the industry  $[\sigma^2(F)]$  and variations in the selection of individual firms [E(F)/n].

### **Portfolio Considerations**

The objective of this paper is to develop a loan pricing model that includes only the nondiversifiable portion of risk. The second component of risk in equation (7) can be eliminated by diversifying the loan portfolio over a large number of customers. The following analysis assumes that the bank has removed this component of risk [E(F)/n] and addresses the problem of reducing the first component of risk  $[\sigma^2(F)]$  by diversifying over a number of industrial categories. The importance of a particular loan category (i)depends on its relative weight  $(w_i)$  in the portfolio. The effect of a category on the risk of the portfolio  $[\sigma(f_p)]$  depends on its covariance with the loss rates of the other categories.

$$\sigma^2(f_p) = \sum \sum w_i w_j \operatorname{cov}(F_i, F_j).$$
(8)

The major difficulty in portfolio analysis is the computational burden of estimating the covariances of the loss rates of a large number of categories. The common practice to reduce the number of calculations is to use linear regression to relate all categories to an index (I). In the following regression equation, the index (I) and the error term (e) are the two sources of variation in the loss rate.

$$F_i = a_i + b_i I + e_i. \tag{9}$$

The coefficient  $(b_i)$  is a measure of the covariance of the loss rate and the index. This relationship can also be shown in terms of the correlation  $[\rho(F_{i},I)]$  between the loss rate and the index.

$$b_{i} = cov(F_{i},I) / \sigma^{2}(I)$$
  
=  $(\rho_{i,I}) \sigma(F_{i})\sigma(I) / \sigma^{2}(I)$   
=  $(\rho_{i,I}) \sigma(F_{i})\sigma(I).$  (10)

A major assumption is that the covariance of the error terms  $(e_i, e_j)$  is zero. In other words, the loss rates of the various loan categories are related only through their common association with the index. As shown below, the covariance of categories can then be expressed in terms of the regression coefficients (Francis & Archer, 1971).

$$cov(F_{i},F_{j}) = b_{i}b_{j}\,\sigma^{2}(I).$$
(11)

Substituting the definition of the covariance in equation (11) into equation (8) yields this result.

$$\sigma^{2}(f_{p}) = \sum \sum w_{i}w_{j} \ b_{i}b_{j} \ \sigma^{2}(I)$$
  
$$\sigma^{2}(f_{p}) = \sigma^{2}(I) \sum \sum w_{i}w_{j} \ b_{i}b_{j}.$$
 (12)

Taking the square root of both sides of this equation produces this expression.

$$\sigma(f_p) = \sigma(i) \sum w_i b_i. \tag{13}$$

Substituting the definition of the  $(b_i)$  coefficient from equation (10) yields this result.

$$\sigma(f_p) = \sigma(I) \sum w_i \rho(F_i, I) \sigma(F_i) / \sigma(I)$$
  
$$\sigma(f_p) = \sum w_i \rho(F_i, I) \sigma(F_i).$$
 (14)

The preceding analysis separates the total risk of a loan category  $[\sigma(F_i)]$  into two components. The first component  $[\rho(F_i,I) \sigma(F_i)]$  is perfectly correlated with the index and is not diversifiable. The second component  $[(1 - \rho(F_i,I)) \sigma(F_i)]$  is not correlated with the index and is diversifiable. Equation (14) shows that the total risk of the loan portfolio is the weighted sum of the nondiversifiable risk of the constituent loan categories. The essential insight of portfolio theory is that diversification among loan categories that are not perfectly correlated can reduce uncertainty. The lower the correlation of a category with the others, the greater the diversification benefit of the category.

# **A Loan Pricing Model**

Since a bank can always elect to invest funds in riskless Treasury bills, the riskless opportunity cost of commercial lending  $(R^*)$  is the short-term rate on government securities plus an allowance for higher administrative costs. The interest rate on commercial loans must also compensate for the risk of lending. One approach to the problem is to use the Chebyshev inequality to limit the probability of a loss (Roy, 1952). As shown below, the probability (P) that the rate of return  $(R_p)$  for the loan portfolio will be less than  $(R^*)$  depends on the expected value  $[E(R_p)]$  and standard deviation  $[\sigma(R_p)]$  of the return.

$$P(R_{p} \le R^{*}) \le \sigma^{2}(R_{p}) / [E(R_{p}) - R^{*})]^{2}.$$
(15)

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In equation form the bank imposes the additional condition that the probability of an inferior return be equal to or less than a specified confidence level  $(P^*)$ .

$$P(R_{\rho} \le R^*) \le P^*. \tag{16}$$

The right side of equation (15) gives the maximum probability of an inferior return. If this maximum probability is to be equal to or less than  $(P^*)$ , then the right side of equation (16) has to be equal to or less than  $(P^*)$ .

$$\sigma^2(R_p) / [E(R_p) - R^*]^2 \le P^*.$$
(17)

Rearranging equation (17) yields equation (18).

$$E(R_p) \ge R^* + (P^*)^{-1/2} \sigma(R_p).$$
(18)

The net return  $(R_p)$  of the loan portfolio is the overall portfolio interest rate  $(IR_p)$  less the loss rate  $(f_p)$  of the portfolio.

$$R_p = IR_p - f_p. \tag{19}$$

The expected value  $[E(R_p)]$  of  $(R_p)$  and its standard deviation  $[\sigma(R_p)]$  are shown below.

$$E(R_p) = IR_p - E(f_p) \tag{20}$$

$$\sigma(R_{p}) = \sigma(f_{p}). \tag{21}$$

Substituting equations (20) and (21) into equation (18) yields the following result.

$$IR_{p} \ge R^{*} + E(f_{p}) + (P^{*})^{-1/2} \sigma(f_{p}).$$
<sup>(22)</sup>

Equation (22) indicates the portfolio interest rate should be at least the sum of the opportunity cost  $(R^*)$ , the expected loss rate  $[E(f_p)]$  and an allowance for uncertainty  $[(P^*)^{-1/2} \sigma(f_p)]$ .

At the strategic or policy level of the bank the important issue is the effect of a particular loan category on the return and risk of the portfolio. The overall interest rate for the portfolio  $[IR_p]$  is just the weighted average of the rates of the individual categories  $[IR_i]$ . The expected loss rate of a portfolio  $[E(f_p)]$  is simply the weighted average of the expected loss rates of the individual categories  $[E(F_i)]$ .

$$IR_{p} = \sum w_{i} IR_{i}$$
<sup>(23)</sup>

$$E(f_p) = \sum w_i E(F_i).$$
<sup>(24)</sup>

The risk of the portfolio, however, depends on just the nondiversifiable portion of risk.

$$\sigma(f_p) = \sum w_i \, \rho(F_i, I) \, \sigma(F_i). \tag{14}$$

Substituting equations (23), (24), and (14) into equation (22) shows the effect of each loan category on the overall portfolio.

$$\sum w_i \, IR_i \ge R^* + \sum w_i \, E(F_i) + (P^*)^{-1/2} \sum w_i \, \rho(F_i, I) \, \sigma(F_i). \tag{25}$$

Equation (26) shows the appropriate price for each loan category consistent with its impact on the portfolio. This pricing equation contains only the nondiversifiable component of risk.

$$IR_i \ge R^* + E(F_i) + (P^*)^{-1/2} \rho(F_i, I) \sigma(F_i).$$
(26)

For example, assume the industry loss rate has an expected value of 1.2 percent, a standard deviation of 0.5 percent and a correlation with the index of 80 percent. In addition, suppose the bank wants the probability of an inferior return to be 10 percent or less. As shown below, the interest rate for the industry should be at least the opportunity cost plus a premium of 2.46 percent.

$$\begin{split} IR_i &\geq R^* + 1.2\% + (10\%)^{-1/2} \ (80\%) (0.5\%) \\ IR_i &\geq R^* + 1.2\% + (3.16) (0.4\%) \\ IR_i &\geq R^* + 1.2\% + 1.26\%. \end{split}$$

#### **Applications of the Model**

Using this loan pricing model requires forming expectations about the loan losses for each particular line of business. Bankers can rely on experience to form subjective opinions or develop causal models that relate loss rates to economic variables such as corporate profits and growth rates. However the apparent advantage of the latter approach is usually an illusion simply shifting the problem to predicting the surrogate variables. Fortunately, a study of the historical record shows that a simple time-series model offers an efficient way to develop the necessary information.

A time-series analysis indicates that industry failure rates display a high degree of autocorrelation. Table 1 shows the autocorrelation coefficients for a lag of one year. All of these coefficients are significantly different from zero for a five percent confidence level. This result suggests there is merit in using the history of failure rates to predict future values. This paper finds that the random walk model offers a good representation of the historical pattern of failure rates for these 26 business categories. In general, failure rates show no affinity for a mean value and change randomly from year to year. With an increase just as likely as a decrease, the best forecast of the future rate is the current rate. This behavior is consistent with a business environment in which chance events generate changes in economic circumstances and failure rates each year.

This relationship is evident in the autocorrelation coefficients of the first differences of the failure rates as shown in the last column of Table 1. None of these coefficients is significantly different from zero for a five percent confidence level. This study does not rule out the possibility that there are better ways to use past failure rates to predict the future. Bankers should continue to search for a method to predict business failures in the same way that technical analysts ought to look for methods to predict stock prices. In both cases the enormous value of a reliable forecast justifies the small effort involved in trying a new approach.

The following equation assumes a random walk and shows the loss rate  $(F_{i,t})$  in period (t) as the sum of the previous loss rate  $(F_{i,t-1})$  and a random term  $(u_{i,t})$ .

$$F_{i,t} = F_{i,t-1} + u_{i,t}.$$
 (27)

If the random element has an expected value of zero and a standard deviation  $[\sigma(u_i)]$ , then the following equations describe the expected value and standard deviation of the loss rate.

$$E(F_{i,t}) = F_{i,t-1} \tag{28}$$

$$\sigma(F_i) = \sigma(u_i). \tag{29}$$

The following equation expresses the nondiversifiable portion of total risk.

$$[\rho(F_i,I) \sigma(F_i)] = \rho(u_i,I) \sigma(u_i). \tag{30}$$

Substituting equations (28) and (30) into equation (26) yields the following pricing relationship.

$$IR_{i,t} \ge R^* + F_{i,t-1} + (p^*)^{-1/2} \rho(u_i, I) \sigma(u_i).$$
(31)

The index (*I*) used in this paper is the change in the loss rate of the portfolio of all loan categories with each category receiving equal weight. The first column of Table 2 shows the standard deviation of the annual change in the failure rate for each category while the second column shows its correlation with the index. The third column reports the 1992 failure rate and the last column indicates the appropriate risk premium  $[(p^*)^{-1/2}\rho(u_i,I)\sigma(u_i)]$  for 1993 using a 10 percent confidence level.

i ne kisk Premium							
Industry	Standard Deviation of First Differences	Correlation with Failure Index	1992 Failure Rate	1993 Risk Premium			
Food & Kindred Products	0.16%	89%	1.19%	1.64%			
Textile Mill Products	0.26	42	1.31	1.65			
Apparel & Other Textile Products	0.27	85	1.79	2.51			
Lumber & Wood Products	0.17	80	1.28	1.71			
Furniture & Fixtures	0.32	58	2.21	2.79			
Paper & Allied Products	0.18	53	1.35	1.65			
Printing & Publishing	0.13	71	1.04	1.33			
Chemicals & Allied Products	0.20	76	1.25	1.73			
Leather & Leather Products	0.28	55	1.17	1.65			
Stone, Clay & Glass Products	0.14	76	1.1	1.43			
Machinery, except Electrical	0.18	66	1.05	1.42			
Electrical & Electronic Equipment	0.20	81	1.46	1.97			
Transportation Equipment	0.31	76	1.96	2.70			
Building Materials & Garden Supplies	0.13	72	0.92	1.21			
General Merchandise Stores	0.15	58	0.78	1.05			
Food Stores	0.16	70	0.62	0.97			
Automotive Dealers & Service Stations	0.12	78	0.77	1.06			
Apparel & Accessory Stores	0.24	82	1.48	2.10			
Furniture & Home Furnishing Stores	0.17	90	1.26	1.74			
Eating & Drinking Places	0.27	76	1.01	1.65			
Drug & Proprietary Stores	0.06	88	0.45	0.62			
Sporting Goods	0.19	69	0.74	1.15			
Jewelry Stores	0.19	88	0.78	1.31			
Hobby, Toy & Game Shops	0.25	70	0.72	1.27			
Camera & Photographic Supply Stores	0.28	77	1.42	2.10			
Gift, Novelty & Souvenir Shops	0.16	67	0.76	1.10			

Table 2 The Risk Premium

An implicit assumption in the derivation of this model is that banks are able to adjust interest rates on both new and existing loans to reflect changes in economic circumstances and failure rates. This assumption is consistent with the variable rate pricing employed by banks on most commercial loans. Extending this model to a multi-period context for fixed rate loans will be the subject of future research.

# **II. CONCLUSION**

There is every reason to be concerned about the effects of business loan losses on the profitability and financial condition of commercial banks. According to *Dun & Bradstreet*, the annual failure rate of U.S. firms more than doubled from an average of 0.4 percent in the 1970's to 0.9 percent in the 1980's. With an average return on assets of only about one percent, a 0.5 percent increase in business loan losses is a significant problem for banks.

The goal in commercial lending is not to avoid risk altogether but to ensure that loan prices adequately compensate for loan losses. This paper advocates that banks use a portfolio approach to determine the appropriate risk premium for commercial lending. This system for pricing risk does not eliminate the need to conduct a thorough credit analysis of potential borrowers on a case by case basis. The important point is that the data cited in this paper indicate the failure rates for firms that met prevailing credit standards. If a firm were unacceptable it would be denied credit and would not be included in the failure statistics. This paper complements traditional credit analysis by suggesting a practical approach to establishing a benchmark rate of interest for firms deemed acceptable. The actual interest rate can be tailored to reflect the particular circumstances of each firm. For instance, an implicit assumption in this analysis is that a loan to a failed firm is a total loss. A smaller risk premium is appropriate if a firm offers solid collateral that reduces the risk of loss.

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