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Business concentration through the eyes of the HHI

George Djolov¹

Abstract

This paper examines the understanding of business concentration through the Herfindahl-Hirschman Index (HHI), by showing that this index is conceptually a model according to which this concentration is the consequence of a renewal process. This process is prompted by firms engaging in different types of economic activity as the means by which to vie for market share. The resultant rivalry produces departures between the market shares of firms. These departures ultimately transmit into differing concentration levels attributable only to the economic activity with which firms vie. As a consequence, while the HHI is commonly interpreted to be a screening indicator of market structure, it is in fact first and foremost a screening indicator of market conduct, which incidentally doubles-up as an indicator of market structure. As part of this, the paper shows that while the HHI cannot identify the exact economic conduct that produces the corresponding business concentration of the observed market structure, it does reveal that whatever this conduct is, it is always subordinated to some type of regenerative or revitalising process.

Keywords: Business concentration, Market conduct, Market structure, Renewal processes

JEL Classification: C15, C46, L11

1. Introduction

There is an established thinking in the economics field that while statistical theory is helpful in the measurement of business concentration, for instance through the likes of the Herfindahl-Hirschman Index (HHI), it is not helpful with understanding what is the driving force behind this concentration. Nutter (1968, p. 219) was among the first to spell this out, giving the following detailed explanation:

“Observed density distributions of firm size are almost always unimodal and skewed upwards: firms are clustered about a relatively limited range of sizes with a longer taper toward the larger sizes than toward the smaller ones. It is reasonable

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to suppose that this characteristic shape results at least in part from a ... growth process, and some economists have therefore tried ... to approximate observed size distributions by lognormal, Yule, Pareto, and similar distributions.... Even if good fits could always be obtained ... a collection of curves would mean little without some theoretical framework for interpreting them. Unfortunately, no systematic theory of industrial structure has yet emerged from studies of this type to command ... agreement among ... economists. In the absence of such a theory, measures of industrial concentration are generally confined to descriptive indexes not amenable to formal statistical analysis.”

Almost thirty years later, Davies (1996, pp. 105-106) has supported Nutter’s conclusion, observing that while statistical theory has proven successful in accounting for the characteristically positive skewness of firm size distributions, it offers no understanding of the forces at work that determine business concentration. Through an illustration of the HHI, in light of it being the most practically relevant measure of business concentration, the present paper seeks to demonstrate that such thinking is mostly a matter of belief. This exposition will firstly proceed with the innate statistical mechanics of the HHI in light of new evidence showing that the Chi-square distribution is the approximate sampling distribution of the HHI (Djolov, 2013, pp. 213-214). From this evidence, it will become apparent that in and of itself the HHI is a model in the sense of being a conceptual portrayal of economic activity, according to which a renewal process drives this activity. By extension the same process drives business concentration. To make this easier to absorb, the model-based nature of the HHI will be empirically confirmed through a simulation. Zickar (2006, pp. 427-428) advises that there are two main advantages to doing this type of analysis. One, simulation permits for a firm control over the compilation and analysis of estimates by making their generation a controlled event. This is not always achievable or feasible with primary or secondary data, which in one or another way are always subjected to the vagaries of field collections. In turn, this makes the artificial data from simulation preferable to work with. Two, as simulation mimics the outcome of a real situation, this becomes especially valuable when it is otherwise impractical or impossible to collect primary or secondary data in the volume that is needed to confirm a theoretical supposition. Both of these advantages matter currently because the artificial generation of the HHI estimates becomes controllable and hence a relatively secure event for confirming the model-based nature of the HHI. From here-on, the economic framework, which connects to the statistical framework of the HHI will be considered, to explain why the HHI is intertwined with a renewal process in economic activity. It will become clear that the renewal process on which the HHI is predicated, firstly operates on the implicit assumption of Max Weber’s characterisation that economic activity is purposefully done with the intention to alter prices and competition itself (Weber, 1947, p. 145, p. 167), and secondly it also rests on the implicit assumption of Milton Friedman’s characterisation that the modus operandi of economic action is essentially to create proprietary knowledge, obtain direct and indirect government assistance, or secure government sanctioned private collusion (Friedman,

2002, pp. 26-27, pp. 128-130). For illustrative purposes, a practical illustration of this explanation will be given using the Australian retail banking industry as an example. This example will assist with showing how the renewal process functions practically in affecting business concentration through the economic activity of firms. In the end, a conclusion will be drawn, to recapture the main arguments presented.

Over the course of the discussion, there are a number of operative assumptions being made, namely:

- a. In accordance with Gini (Gini, 1947, p. 24; Gini, 1965, pp. 94-95), the Gini index is treated as a measure of relative variability, and likewise the same extends to the HHI given this latter index is a variant of the Gini index.
- b. To promote easier understanding as well as any other independent verification of results, the statistical analysis is confined to common, well-understood statistical methods (Andrews, 2007, p. 342).
- c. The terms business concentration, market concentration, and industry concentration are synonymous with each other, and hence used interchangeably. Likewise the same is done with the terms market and industry.

2. HHI's Statistical Mechanics

The starting point in considering the statistical mechanics of the HHI is a recent finding that shows that the HHI is a variant of the Gini index whose approximate sampling distribution is the Chi-square distribution (Djolov, 2013, p. 211). According to the first aspect of this finding, the HHI is given by the sum of 1 and the squared Gini index (G) of firms' market shares, divided by the number of firms (n):

$$HHI = \frac{G^2 + 1}{n} = \frac{G^2}{n} + \frac{1}{n} \quad (1)$$

Expression (1) readily permits to establish what degrees of freedom are inherent in the HHI. To remind, degrees of freedom (df), are the number of independent variations or the so called unrestricted chances for variation in the measurement being made (Sharma, 2010, p. 261). These chances of variation apply as much to the moments describing the measurements – as for instance their location, scale, skewness and kurtosis – as they do to the constituent elements of the construct or instrument producing them (Gini, 1965, p. 105). In the present case, the HHI is the instrument for the measurement of business concentration and the sample estimates it gives rise to are the measurements of business concentration themselves. Applying the foregoing definition of degrees of freedom to the HHI as per expression (1) shows that the HHI has two independent sources of variation. In short, as a construct comprising two constituent elements of variation, the HHI has two degrees of freedom. The one source of variation – $1/n$ – describes whether firms' market shares are uniform, i.e. unvarying, while the second source of variation – G^2/n –

describes the similarity of firms' market shares relative to uniformity. In this context firms' market shares are identical if the relative variation between them is the same. Then the HHI will tend towards its known lower limit of 0. On the other hand, firms' market shares are not identical if the relative variation between them is not the same. The extreme of this is achieved whenever there is only 1 firm, and this is attained when absolutely speaking there is no relative variability. Then the HHI will tend towards its known upper limit of 1. Knowledge of the limits of the HHI exposes that the scale of its inherent distribution as measured by the range, which is the difference between the upper and lower limits, is 1. From here the location of its inherent distribution can be also determined.

This leads us to the second aspect of the above-referred to recent finding, namely that the approximate sampling distribution of the HHI is the Chi-square distribution. It is approximate because the exact probability distribution of the sample HHI is unknown except for knowing that whatever this distribution is, it is unimodal and positively skewed. The cited works of Nutter (1968, p. 219) and Davies (1996, pp. 105-106) have already suggested as much by their referral to positive skewness being something characteristic of business concentration. The fact that we are being confronted with an approximation for the sampling distribution of the HHI as opposed to something exact, should not lessen its significance. As Kleinke (1979, p. 163) notes, approximations in general serve three purposes:

- a. They provide computational simplicity, which does not necessarily come at the expense of accuracy, and as such they practically offer computational solutions that are not arduous to implement;
- b. They enable the formulation of an estimator that can be subjected to statistical analysis, as much as an estimator attained from an exact solution;
- c. Lastly, and perhaps most important of all, they give insight into the nature of a statistic that may otherwise be impossible to attain through an exact solution.

This last point essentially captures the benefits from having an approximation with a known distribution for the HHI. It is known that for the Chi-square distribution, the ratio of its scale to location is the square root of double the inverse of its degrees of freedom (Evans et al., 2000, p. 53; Krishnamoorthy, 2006, p. 156). Or:

$$\frac{scale}{location} = \sqrt{2 \cdot df^{-1}} \quad (2)$$

To remind, the scale of a distribution refers to its spread, and its location to its typical value. Since in the present case we have two degrees of freedom ($df = 2$) and a unit value for scale ($scale = 1$), it follows that expression (2) reduces to:

$$\frac{1}{\sqrt{2 \cdot 2^{-1}}} = location \Rightarrow location = 1 \Rightarrow location = scale \quad (3)$$

Thus, as expression (3) highlights, the location and scale of the inherent HHI distribution are equal. The meaning of this is that the typical value of the HHI, which varies from case to case, is given by its spread. Stated more simply, the HHI's typical value is the relative value it assumes within the limits of its continuum. This is understandable since, as can be seen from expression (1), by including the Gini index in its formulation, the HHI is also a measure of relative variability. Like the Gini index it too captures the extent to which this variability is bunched together in few observations. That is, the HHI is a measure of concentration. By focusing on this concentration in the context of firms' market shares, it acts as an index of business concentration.

It will be of interest to draw attention that for Kendall and Stuart (1977, pp. 48-49) the distinction between concentration and relative variability was merely one of semantic idiom. The two are the same, just going by a different name. Respectively, it should come as no surprise that the sample HHI follows the variance distribution, i.e. the Chi-distribution. By implication, given that as a concept the HHI has two degrees of freedom, it follows that a likely candidate for its inherent distribution is the Chi-square distribution with two degrees of freedom. This well-known distribution is also called the Rayleigh distribution (Hirano, 2006, pp. 6987-6988). From expression (3), it is clear that the HHI adheres to the shape of this distribution without any modifications to its location and scale as per the equality between the latter two. Thus, as a concept, the HHI should be considered to be subordinated to the basic, i.e. unmodified Rayleigh distribution. The ensuing section is concerned with confirming this by means of simulation.

3. HHI's Inherent Distribution

The four main methods to generate artificial data for the purpose of producing simulated estimates are bootstrapping, jack-knifing, balanced repeated replications, and cross-validation (Diaconis and Efron, 1983, pp. 107-108). On review of the practices of these methods, Efron and Tibshirani (1991, pp. 390-391) advised that there is no right or wrong decision as to which method to use for simulation. The important thing firstly is to choose a simulation method in the same way as choosing any other statistical technique, namely taking the simplest that would be applicable to one's situation since this will ease the difficult task of interpreting the final results, and secondly to implement the selected method as accurately as possible. Acting on this advice, in the present case bootstrapping is used. As Press et al. (2007, p. 810) explain the basic idea behind the bootstrap is to take a representative sample, and to carry on repeatedly sampling this with replacement – analogously to reshuffling a card deck – until more data is created from which to estimate a statistic. Essentially by reshuffling the representative sample, an estimate of the statistic is derived from each reshuffle until in the limit the generated number of estimates is collectively able to show the statistic's distribution.

In the present case, the bootstrap is done with the Leontitsis and Pagge algorithm (Leontitsis and Pagge, 2007, pp. 337-338). To recall, to execute an algorithm is to carry out a mechanical procedure for achieving a specified result much like using a calculator to

perform some mathematical operation of interest (Copeland, 1996, p. 335, p. 337). In the present instance, the specified result is the confirmation that the likely candidate for the inherent distribution of the HHI is the Rayleigh distribution. The Leontitsis and Page (LP) bootstrap algorithm is promising in this regard because it has 95% accuracy in detecting the likely distribution of a statistic. The algorithm attains this level of accuracy by relying on two established facts:

- a. A representative sample for the accurate estimation of a statistic can be created with as few as 20 measurements (Lehr, 1992, pp. 1099-1100; Van Belle, 2008, pp. 30-34).
- b. In the estimation of a statistic, a 5% lack-of-precision is attainable with a minimum number of 800 measurements, if any error in measurement is limited to at most 1 in 3 measurements (Van Belle, 2008, p. 34).

Based on the aforementioned, the LP bootstrap algorithm involves the following three steps:

- a. Create a representative sample consisting of a randomly selected seed of 20 observations. This sample must resemble the situation being studied.
- b. Sample this seed repeatedly at random with replacement for 1000 times, to create 1000 data sets each of 20 observations.
- c. From the 1000 data sets, compute 1000 estimates of the statistic under study, and either analyse or plot these to get the statistic's distribution.

Analogously, running the LP algorithm on the HHI also involves three steps:

- a. Creating a representative sample consisting of a random selection of 20 market shares, which exhibit positive skewness in their distribution. In the present instance the Pearson skewness coefficient of this distribution is a positive 41%, signalling moderate positive skewness. As described by Nutter (1968, p. 219) and Davies (1996, pp. 105-106) at the beginning, positive skewness is the anticipated resemblance for the distribution of firm market shares.
- b. Re-sampling randomly with replacement 1000 times the random seed of 20 market shares to create 1000 replicated data sets each of 20 observations.
- c. Computing the HHI for each data set from expression (1), to create 1000 estimates of the HHI, from which in turn its inherent distribution is determined.

Table 1 gives the percentiles of the distribution of the HHI estimates from running the LP algorithm. It is known that the percentile values of such generated distribution provide estimates of the lower and upper confidence limits of a statistic at a designated confidence level (Wood, 2004, pp. 180-181). This method of confidence interval derivation

is commonly referred to as the percentile method, because it takes the percentile values of a statistic's bootstrap distribution, to be the lower and upper limit estimates of its confidence interval at the adopted level of significance, regardless of the statistic's underlying distribution (Efron, 1988, p. 296).

The 5% significance level is a common lack-of-precision yardstick for a confirmatory test, and the same convention is followed in the present instance since there is no reason to suppose that an alternative significance level will do better.

Table 1: Percentiles of simulated HHI values

Percentiles (%)	HHI values (%)
100.0 (maximum)	6.95
99.5	6.87
97.5	6.75
90.0	6.59
75.0 (3 rd quartile)	6.44
50.0 (median)	6.24
25.0 (1 st quartile)	6.02
10.0	5.82
2.5	5.54
0.5	5.26
0.0 (minimum)	5.01

Note: Calculations are done from generated values of the HHI by the LP bootstrap algorithm.

Thus, at the 95% confidence level, or alternatively at the 5% significance level, the 95% confidence interval for a statistic is given by the values of the 2.5th and the 97.5th percentile of its bootstrap-derived distribution, since these are the corresponding percentiles for this confidence or significance level. Inferring from Table 1, the resultant 95% confidence interval for the HHI irrespective of its distribution is:

$$\Lambda_{HHI}^{95\%} = (5.54\%, 6.75\%) \tag{4}$$

However, if conceptually the HHI is inherently Rayleigh distributed, then at the 5 significance level, its critical Rayleigh value, i.e. the tabular Chi-square value with 2 degrees of freedom, is 5.99%. As can be seen from expression (4), this value falls within the limits of the 95% confidence interval for the HHI, thereby indicating that a null hypothesis

in favour of the Rayleigh distribution cannot be rejected. In short, we have just gained an empirical confirmation that as a *concept* the HHI is inherently Rayleigh distributed. As reminded in the 2008 edition of the Oxford Dictionary of Statistics, as well as by the prior work of Samuels (1974, pp. 73-74, p. 83), the Rayleigh distribution is the distribution of the distance between the points in the data emanating as the outcome of a spatial renewal process. Thus, while the HHI is an index of business concentration, it is in fact a model of business concentration predicated on a renewal process that ends up separating firms apart, essentially by creating differences in their market shares. The implication of this finding is now considered in the following section.

4. Implications of HHI's Inherent Distribution

So far we have found that conceptually the HHI is inherently subordinated to the Rayleigh distribution, which directly implies that as a concept the HHI is a model of business concentration, which evolves according to a renewal process. This is because the Rayleigh distribution is established by such a process, and by default the same extends to the HHI.

After a detailed review of renewal processes, Mitov and Zazanis (2008, p. 1220) reached the conclusion that:

“Alternating renewal processes arise in a natural way in many situations ... where working (busy) periods ... interchange with idle periods.”

As observed by Mitov and Zazanis (2008, p. 1220), the classical statistical explanation for this alternation in the human sciences is the existence of some regenerative or revitalising action, which brings new and more vigorous life to prevailing conduct before leading to two types of renewal events. One being the moment of bringing-in an introduction, the other being the moment when this introduction is accommodated or made operative. The busy periods of human activity transpire when these events occur, whereas the periods of idle human activity arise when these events do not occur. The regenerative or revitalising action is itself initiated by a preferential attachment to a desired method of operation (Albert et al., 2000, p. 379, p. 381; Rybski et al., 2009, p. 12641, p. 12645). Moreover, as Albert et al. (1999, p. 130) observe, this initiation is dependent on an intelligent agent who can interpret the consequences of any regeneration and/or revitalisation, in order to become informed as to the relevant course of action to take in reaching the desired method of operation.

The foregoing depiction of how a renewal process should be interpreted in the human sciences has a number of immediate parallels to the economics field. For instance, as Max Weber (1947, p. 167) spells out, economic activity is any activity a firm does with the known opportunity for exchanging products for the purpose of orienting prices and competition. It is a small step from here to appreciate that this orientation cannot happen without informed actions designed to bring something into existence such as a product introduction or its respective adoption for usage. In turn, behind such actions is a preferred

way, i.e. a method by which the firm wants to do this. After an extensive review, Milton Friedman (2002, pp. 26-27, pp. 128-130) outlined the existence of three such methods:

- a. Creation of proprietary knowledge, which is the technical and operational information a firm exercises to produce products;
- b. Securing direct and indirect government assistance; and
- c. Government vetted private collusion.

There is nothing special in the methods referred to by Friedman. Almost one-hundred years earlier Sir Giffen (1904, p. 188) had already identified the same, and more recently Baumol (2002, p. 15, p. 67) finds that the first method dominates firm operations whenever pursuit of the last two methods is absent or limited in scope to them. What is different about the methods is the nature of the renewal process they operate by.

As a method of economic activity, the act of creating proprietary knowledge revolves around creating knowledge that is exclusively held by the firm, through a range of ownership instruments by which it vies for market share (Reekie, 1989, p. 105-106). Examples of these instruments include trade-marks, branding, royalties, copyrights, patents, as well as trading and production licenses. Firms, whose market position rests on the ownership of such instruments, already automatically participate in a renewal process of supplying the market they operate in. As Pretnar (2003, p. 901) explains the reason for this is that the outcome of creating proprietary knowledge is a different product innovation or imitation, with the effect that the stock of knowledge available for use in further production is more knowledge come the next round of renewed economic activity. This is what makes the creation of proprietary knowledge a gradual cumulative process. As Pretnar (2003, pp. 894-897) clarifies, the coupling of a firm's market position with the creation of proprietary knowledge, means that this knowledge is exclusive to the firm which holds it, in the sense that no other firm has access to the same market position with the same knowledge. Since this type of vying for market share is not foreclosed to any other firm, then economic activity by such competing means turns into a renewal process exemplified by ongoing product innovations and imitations. This process comes to a halt if the firm loses the ownership of its proprietary knowledge by any free-riding means (Pretnar, 2003, p. 895, p. 897). In that case the firm is driven away from creating any such knowledge precisely because it cannot establish ownership for it. If this was to be the case for all firms, then none will want to create proprietary knowledge, and so by extension the renewal process fostered by this knowledge will be terminated. Such market situations are exemplified by retarded or ceased product innovations and imitations. Thus, the principal function of economic activity predicated on the act of creating proprietary knowledge, is to establish a renewal process, where each firm's pricing and competing decisions limit or prevent other firms from taking ownership of its proprietary knowledge (Pretnar, 2003, p. 897). Firms who can better interpret the consequences of these decisions relative to others also become well informed as to the course of action that will either give them an advantage,

or a lead in the market. In response, differences in market shares emerge so that those firms in the lead gain more market share than those firms in the following. This will see a market concentrating, and likewise the HHI will bear this out by measuring what the concentration is. Effectively, what the HHI will be measuring, is whether the observed differences in market shares arising from such a renewal process are relatively different from the same process producing a situation of no difference. If there are such differences in market shares, then business concentration is present and vice versa.

Without doubt, the lineage from proprietary knowledge to competitive economic activity is direct. To picture this, we can easily think of instances where such knowledge intimates the existence of a renewal process, which is incubated from a new idea, a better new product, more innovative pricing strategies, a new technique of production, as well as more innovative ways of firm structuring and restructuring. Kirzner (1997, p. 49) suggests that sometimes the by-products of the renewal process might be easier to see than the introductions themselves. For example, if the new product is a technological breakthrough that induces the production of more output at lower costs than ever before, economies of scale are an obvious spin-off. The same goes for any newly patented production technique that gives a firm superior costs of production and distribution that no other rival can share in. Then the resultant absolute cost advantage becomes an evident result. So too is the product variety created, if the better new product a firm introduces is distinctive from anything else available on offer.

By contrast, it is often hard to recognise that the remaining two methods of economic activity – government assistance and government vetted private collusion – also function through a renewal process. Tollison (1982, p. 575) and Tullock (1993, p. 22) have pointed out that this is a renewal process where the action is on firms capturing artificially contrived benefits from government-bestowed privileges. This is done through the introduction of legislative instruments in the case of government assistance, or politically sanctioned gentlemen agreements in the case of private collusion. Some common examples include: subsidies, tariffs, quantitative or supply restrictions, licensures, secured profit margins or guaranteed rates of return, stipulated prices, prescribed trading practices, cartelisation and any other collective monopoly fixture. Lal (2012, pp. 498-500) finds that while the underlying nature of these legal and political instruments has remained intact, over time their proliferation has increased, in the sense that the variety or variations they come in has grown. As George Stigler (1988, pp. 210-214) explained, each such introduction rests on the action of the benefitting firm or firms enlisting the support of a State organ, to confer the sought after benefit or privilege. It is known that such privilege gives the benefitting firm an entrenched market position as opposed to the case of having to vie for this position by means of creating proprietary knowledge (Buchanan, 1993, pp. 29-30). This in and of itself is the reason why the firm or firms to benefit will act to enlist the express or implied support of the State, provided they can correctly identify if the State is in the business of giving this support (Lal, 2012, p. 499). But the effect of such activity, if successful, is to make the market closed from rivalry or weakened by its suppression. For the benefitting firm or firms market dominance is the gain, reflected in their relatively higher market share, as

distinct from that of the firms without the privilege. Consequently business concentration also arises, but its sources come from renewable actions by firms wanting to legally or politically shut-out rivalry in their favour. The resultant market concentration will be captured by the HHI, which will now measure if the observed differences in market shares arising from this type of renewal process, are relatively different from the same process producing a situation of no difference. Here too, if such differences in market shares exist, then business concentration is present and vice versa. What is different from the situation of firms creating proprietary knowledge is the cause of the concentration, which now is to be traced to the enlistment of government assistance or political approval.

Due to the principal of neutrality, as a statistical index, the HHI is silent on identifying what the cause of the observed business concentration is, except firstly to indicate that such concentration exists, and secondly that as per the type of economic activity pursued, there are three possible reasons for it. The index cannot explain which reason holds. This is incumbent on economic theory to explain. It is also clear that compared to economic textbook orthodoxy, the primary diagnostic function of the HHI is different from that normally assigned to it, as an index of business concentration that is suggestive of prevailing market structure. For example, Ertl and McCarrell (2002, p. 9) and Besanko et al. (2013, pp. 172-173) advise that if the HHI falls between 0% and 20% it signals the presence of perfect competition, if it is between 21% to 40% it signals the presence of monopolistic competition, if it is between 41% to 70% it signals the presence of oligopoly, and lastly if it is between 71% to 100% it signals the presence of monopoly as the prevailing market structure.

In the context of the above-considered discussion, it is apparent that as the initiator of different types of renewal processes, it is perpetually reinventing economic activity that is responsible for anyone of the aforementioned market structures. Ultimately, as the economic activities responsible for these processes unfold, the result is differently-bred business concentration. Not unexpectedly, the economic activity responsible for the renewal process becomes conflated with the resultant market structure it brings. A naïve deduction from here would to infer that the HHI is just an index of business concentration that is predictive of market structure. It is much more than this. Firstly, in terms of its primary diagnostic function, the HHI is an index of business concentration to the extent of signalling that this concentration is due to market conduct. This is in the sense that such conduct is attributable to different types of economic activity as already seen. Secondly, as a spin-off of this, it happens to incidentally be able to suggest what the market structure from this conduct is. This however is a secondary diagnostic function of the HHI, because it does not draw a lineage to the explanatory reasons for the observed, i.e. estimated business concentration. By contrast, the primary diagnostic function of the HHI does.

In the next section the above diagnostic functions of the HHI are demonstrated. For illustrative purposes this is done with an example of the Australian retail banking industry.

5. Illustration

It was shown in the previous section that the HHI has two diagnostic functions.

According to the first and primary function, it is an index of business concentration that is indicative of the presence of this concentration as a result of: renewable economic activity in the creation of proprietary knowledge, the securing of government assistance, or the partaking in private collusion. According to its secondary function the HHI is an index of business concentration indicative of prevailing market structure that predicatively reveals what market conditions exist, and how they compare to a situation of monopoly should any other market structure be encountered. Table 2 captures what conventional economic theory holds about the market conditions firms face in different market structures associated with particular HHI values, in addition to the expected outcomes of these structures at these values, relative to monopoly.

As the contents of Table 2 show, while the secondary diagnostic function of the HHI gives it attractive screening abilities as to the anticipated consequences associated with a particular business concentration level, this function departs from the primary diagnostic function of the HHI in two ways. Firstly, it makes the index silent on the fact that what is responsible for the observed business concentration level, and the signal this sends about the likely market structure and its outcomes, is some kind of pervasive economic activity. Secondly, that the identified structure and its outcomes are traceable to three kinds of economic activity as elaborated already. Consequently, the primary diagnostic function of the HHI does not seek to replace the secondary one. It adds to it, by highlighting that whatever the detected business concentration, this concentration is explainable by the economic activity it is incubated from. Thus knowing what the observed market structure and its outcomes are as suggested by the HHI, is just as important as knowing what produced it. Here the HHI suggestively signals that we should look for market conduct that is responsible for proprietary knowledge, government involvement, and collusion. While the HHI will numerically give an estimate of the business concentration belonging to each of these situations, it will not show the type of conduct that created it. The reason for this is the same as that for any other data. As Forrester (1980, p. 558) explains:

“Missing from numerical data is direct evidence of the structure and policies that created the data. The numerical data do not reveal the cause-to-effect direction between variables. ... That still leaves unanswered the question of internal causality.”

The same also holds for the values of the HHI. As per their primary diagnostic function, these values will only inform whether there is business concentration due to distinct kinds of perpetual economic activity, but will not answer which of them caused the observed concentration. There is another, statistical way, to understand this. It stems from the inclusion of the Gini index in the HHI. As known, the Gini index measures the extent to which a distribution of values deviates from a uniform distribution (Gini, 1947, p. 24). Specifically, the Gini index identifies this deviation, by identifying whether the changes in the values from their mean produce any displacement in their rankings, as compared to a situation of no displacement in their ranks. By analogy, the meaning of this for the HHI is that, it detects business concentration according to whether changes in firms' market shares

Table 2: Market conditions and outcomes of prevailing market structures

	Perfect Competition	Monopolistic competition	Oligopoly	Monopoly
HHI Range, %	$0 \leq \text{HHI} \leq 20$	$20 < \text{HHI} \leq 40$	$40 < \text{HHI} \leq 70$	$70 < \text{HHI} \leq 100$
Market conditions faced by firms	Free entry	Free entry	Entry is restricted	Entry is restricted
	Complete product information	Complete product information	Incomplete product information	Incomplete product information
	No private collusion	No private collusion	Private collusion is possible	No private collusion
	Product variety is missing	Product variety exists	Product variety is possible	Product variety is limited or missing
	Equal access to production technologies	Equal access to production technologies	Equal access to production technologies is possible	Unequal access to production technologies
	Capital and labour are immediately mobile	Capital and labour are immediately mobile	Capital and labour are not immediately mobile	Capital and labour are not immediately mobile
	No market power	Market power is immediately contestable	Immediate contestability of market power is possible	Market power is not immediately contestable
Market outcomes with monopoly as baseline	Industry output is 100% higher than in monopoly	Industry output is 90% to 95% higher than in monopoly	Industry output is 60% to 75% higher than in monopoly	Baseline
	Price-cost margin at industry level is 99% lower than in monopoly	Price-cost margin at industry level is 90% to 95% lower than in monopoly	Price-cost margin at industry level is 60% to 75% lower than in monopoly	Baseline
	Industry profit is 99% lower than in monopoly	Industry profit is 82% to 90% lower than in monopoly	Industry profit is 36% to 56% lower than in monopoly	Baseline
	Consumer surplus is 300% higher than in monopoly	Consumer surplus is 263% to 280% higher than in monopoly	Consumer surplus is 156% to 206% higher than in monopoly	Baseline

Sources: Andreosso and Jacobson (2005, pp. 104-114), Besanko et al. (2013, pp. 172-173), Cabral (2000, pp. 89-95), Carlton and Perloff (1994, p. 243), and Ertl and McCarrell (2002, p. 9).

from the mean share displace their rankings, as compared to a situation of no changes in their ranks. As such, the HHI does not answer what type of economic activity produces this displacement (the value of which captures the observed business concentration). Clearly then, failure to incorporate any such activities in accounting for any estimated HHI value, removes the context or background that could explain why the HHI estimate is revealing of a particular market structure as opposed to any other. In this sense, the job of the HHI is not only to point out what business concentration is expected out of a particular market structure, but also to alert that there are specific economic activities that could explain it. This point will now become clearer with the illustrative example of business concentration in the Australian retail banking industry.

In 2011, the Australian Trade Commission published the findings of its review into the Australian banking market, in its report *Australia's Banking Industry*. One of the reviewed banking sectors was the retail banking industry, which the Commission examined by a snapshot, as at 2010, of the banks registered at that time to grant loans. Table 3 gives the market shares of this sample of firms as extracted from the Commission's report (2011, p. 14). The market shares are derived as the proportion of the loaned amount by each bank relative to the total amount loaned by all banks at that time. Based on the figures in Table 3, the Commission (2011, p. 5) concluded that:

“Australia's retail banking sector is relatively concentrated....While the major... banks have dominant market shares across most consumer ... lines, there is ... increasing competition from ... lenders ... and ... specialist finance companies.”

The Commission did not clarify how this conclusion was arrived at, except to present it as a statement of fact. Certainly, looking at the numbers alone will not reveal according to which standard is Australia's retail banking industry relatively concentrated, or why it is undergoing increased competition at the same time as being concentrated. This of course can be seen by performing an HHI analysis on the figures in Table 3.

Table 3: Australian retail banking industry

Firms (Banks)	Market share in 2010, %	Rank, from worst to best
Investec Bank	0.010	1
Rabobank	0.020	2
Beirut Hellenic Bank	0.031	3
Bank of Cyprus	0.032	4
Bank of China	0.033	5
Arab Bank Australia	0.036	6
Members Equity Bank	0.370	7
AMP Bank	0.530	8

Macquarie Bank	0.612	9
HSBC Bank	0.670	10
Citigroup	1.190	11
Bank of Queensland	1.626	12
Bendigo and Adelaide Bank	1.993	13
Suncorp-Metway	2.497	14
ING Bank	3.390	15
Bank of Western Australia	3.690	16
Australia and New Zealand Banking Group	15.730	17
National Australia Bank	16.020	18
Commonwealth Bank of Australia	24.610	19
Westpac	26.910	20
Total	100.000	

Source: Australian Trade Commission (2011, p 14).

The first step in the HHI analysis is to estimate the HHI for the industry. By solving for the Gini index in expression (1), Djolov (2013, p. 212) finds a robust estimator of the HHI given by:

$$HHI = \frac{4 \left(\sum_{i=1}^n ix_i \right)^2 - 4(n+1) \sum_{i=1}^n ix_i + (n+1)^2 + n^2}{n^3} \quad (5)$$

For reminder, a robust estimator is any estimator, which maintains its accuracy in the estimation of a statistic whenever the encountered data conditions change (Morgenthaler, 2007, p. 272, pp. 277-278).

The second step in the HHI analysis is to estimate the corresponding confidence intervals for the point estimate of the HHI. Here, Djolov (2013, pp. 215-216) shows that because McKay's approximation for the sample coefficient of variation extends to the sample Gini index, McKay's confidence intervals for the sample coefficient of variation are equally applicable to the Gini index. Consequentially, the first or original McKay confidence interval, with respect to the HHI is given by:

$$\Lambda_1^{HHI} = \left(\frac{\sqrt{nHHI-1}}{\sqrt{\left((nHHI-1) \cdot \left(\frac{\chi_u^2}{n} - 1 \right) \right) + \frac{\chi_u^2}{n-1}}}, \frac{\sqrt{nHHI-1}}{\sqrt{\left((nHHI-1) \cdot \left(\frac{\chi_l^2}{n} - 1 \right) \right) + \frac{\chi_l^2}{n-1}}} \right) \quad (6)$$

While, the second or modified McKay confidence interval with respect to the HHI is given by:

$$\Lambda_2^{HHI} = \left(\frac{\sqrt{nHHI-1}}{\sqrt{\left| (nHHI-1) \cdot \left(\frac{2+\chi_u^2}{n} - 1 \right) \right| + \frac{\chi_u^2}{n-1}}}, \frac{\sqrt{nHHI-1}}{\sqrt{\left| (nHHI-1) \cdot \left(\frac{2+\chi_l^2}{n} - 1 \right) \right| + \frac{\chi_l^2}{n-1}}} \right) \quad (7)$$

In both expressions, the lower and upper critical Chi-square values (χ^2) are denoted by “l” and “u” respectively. They may either be obtained from a Chi-square distribution table with n-1 degrees of freedom, or in the event of large samples for which tabular Chi-square values are not available, from the familiar normal-based Wilson-Hilferty approximation.

Given that expressions (6) and (7) are effectively confidence intervals for the Gini index expressed in terms of the HHI, if they are applied to data containing only the number of observations and the HHI, then they give the lower and upper estimates for the Gini index. In turn, to get the lower and upper confidence limits for the HHI, these estimates will have to be run through expression (1).

The calculation results of the first and second step of the HHI analysis with the figures from Table 3 are presented in Table 4. For reminder, lack of precision in estimation is derived from the width of each interval, and the bias from estimation is obtained as half the width of each interval. A 95% confidence level is selected merely for illustrative purposes. Any other level from among the conventionally available ones, such as the 90% or the 99% confidence level, is just as usable depending on the particular application involved.

Table 4: HHI analysis of Australian retail banking industry

HHI statistics	Estimates
Point estimate	7.7%
95% HHI confidence limits from original McKay confidence interval	6.3% , 8.5% Bias = 1.1% Lack of precision = 2.2%
95% HHI confidence limits from modified McKay confidence interval	6.3% , 8.8% Bias = 1.3% Lack of precision = 2.5%

Note: Calculations are firstly done with expressions (5), (6), and (7), and the solutions from the latter two expressions are re-entered into expression (1) to get the HHI confidence limits.

Of the two intervals, the HHI confidence limits from the original McKay confidence interval are the ones carried forward in the third step of the HHI analysis, since this interval has comparatively smaller loss of precision and smaller bias to the same counterparts from the modified McKay confidence interval.

In the third step of the HHI analysis, the computed HHI estimate of 7.7% and its lower confidence limit of 6.3% and upper confidence limit of 8.5%, as attained at the 95% confidence level, are matched to the market conditions and outcomes outlined in Table 2. This is done to determine the likely market conditions and/or outcomes that prevail in the Australian retail banking industry. It is the interpretation of the numbers at this step that gives an insight into the Commission's view on the retail banking industry. Examined according to the HHI point estimate, the view by the Commission that the retail banking industry is relatively concentrated, is unsupported since its value of 7.7% indicates that the industry operates under a perfectly competitive market structure. Similarly, the estimates of the HHI's lower and upper confidence limits, of 6.3% and 8.5% respectively, corroborate this to be a persistent market structure for the industry, since none of them overlap with the anticipated HHI range of other market structures. By the same token, the foregoing HHI numbers support the Commission's view that the industry is open to increasing competition. In this regard, the secondary diagnostic function of the HHI makes it possible to describe from the industry's market shares in Table 3, what its observed business concentration suggests about its likely market conditions and/or outcomes. But, it is also clear that performing the HHI analysis up to the information disclosure the secondary function gives, is unsatisfactory. This is due to this information being silent on identifying the economic activities that can account for the observed business concentration level, or the consequences it is associated with. For instance, in the current instance, we are left to grapple with a picture of the retail banking industry that is not entirely conforming to expectations – it is competitively unconcentrated as opposed to competitively concentrated. *This is why an HHI analysis should not leave out its primary diagnostic function.* Essentially this function of the index runs concurrently with its secondary counterpart, by requiring that the market or trade environment underpinning the economic activities leading to the observed HHI must be considered inseparably from its estimates.

In the present example, the Australian Trade Commission (2011, pp. 13-19, pp. 44-46) advises that the trading environment of Australia's retail banking industry has five main attributes:

- a. The firms with the four largest market shares as reported in Table 3 are legally prohibited from merging, which in effect bars them from collectively monopolising the industry;
- b. Firms are legally compelled to fully disclose to consumers the contents of any loanable offer, which in effect encourages a market nexus with complete information;

- c. By regulation, government is excluded from market participation by government intervention or political participation, which in effect means a market functioning without government assistance or government induced private collusion, such that the competing means available to firms are limited to the creation of proprietary knowledge;
- d. By regulation, the market is open for anyone to enter, in the sense that any firm can become a licensed supplier of capital provided it accepts it has to operate according to the foregoing requirements, which in effect means that consumers have more competing offers from different firms to choose from, irrespective of whether this leads to product variety or more of the same being offered.
- e. By regulation, firms are given free contracting abilities to trade and maintain their property rights in the products they offer, which in effect means that no firm loses ownership in its proprietary knowledge to any other firm that may want to free-ride on it.

Lachmann (1992, p. 25, pp. 28-29, p. 31) defined an open market in one of two mutually consistent ways. On the one hand, it is a market composed of economic activity including only the creation of proprietary knowledge. Alternatively, it is a market composed of economic activity excluding any type of government assistance and government-vetted private collusion. The juxtaposition between either of these definitions and the above-described trading environment of Australia's retail banking industry, readily reveals that this industry operates as an open market since firms only have the ability to acquire market presence through the power of the property they create, as per the products – in the form of the varied loans – they offer to consumers.

Extracting from Newman's work (2000, pp. 412-413), an open market may be seen as the result of firms purposefully designing or evolving the best possible conduct for contesting each other, which takes us full circle to renewal processes as discussed earlier on. Imagine then a market where a competing firm – in this case a bank – wants to grow product choice as much as possible. The principal activity of the firm is producing competing offers – in this case loans. Such an offer starts in the market with a moderate frequency of demand that can replace large numbers of other existing or prospective offers if left unchecked. So the job of the other competing firms is to counteract this competing offer in order to prevent its spread from eating away their actual or potential share of the market. The best way to minimise such possible damage that the competing offer may inflict, is to separate the market into equally sized segments if the competing offer is provided uniformly, i.e. in the same way to everyone in the market. However, if there is more of the offering to some consumers than to others, the possible damage is minimized by dividing the market into segments whose sizes in terms of the numbers of consumers vary in inverse proportion to the rate at which the competing offer is supplied to the different consumers. This inverse proportionality is readily reconcilable with the already communicated finding, that business concentration depicted by the HHI is inherently distributed according to the Rayleigh distribution, where the probability of the occurrence of an event is inversely proportional to

its different possibilities (Evans et al., 2000, pp. 167-168). In turn, if the event in question is the act of countering a replacement offer, the probability of this act – which will work to alter market concentration – by prospectively dividing a market into segments, will be inversely proportional to the possibility of the replacement occurring (the latter being captured by the rate at which the replacement enters a segment). Consequentially, the size of a market segment will increase proportionately to the decrease in the replacement offer's rate of entry in the segment, because the market share prevalence of the counter offer in the segment will be proportionately higher. The reverse will hold in the opposite situation. Either way, each firm's conduct becomes highly sensitive to that of competing rivals, by forcing them to design or evolve activities that have ingenuity, specifically through the perpetuation of proprietary knowledge and/or independent actions that are limited in their prospects for collusion or government aid. Individually or together, each of these actions are robust, as they enable the firm to gain or maintain market share, while also being fragile in that the firm could reduce or lose its market share if its rivals perform these actions better. Thus, as known, open markets are in a continual state of firm rivalry, which works against them becoming permanently concentrated. This explains the current finding of the retail banking industry being competitively unconcentrated, as expected from the perfectly competitive market structure signalled by its HHI, while at the same time also helping to explain why the Australian Trade Commission finds the industry to be competitive. The current explanation also suggests that the Commission's concurrent view that the industry is relatively concentrated actually presumes a concentration level at which firm rivalry is unstoppable. In such a context, it is possible to conclude, as the Commission has, that in the presence of relative concentration, competition prevails. But to clarify, the concentration is only relative to the extent that competition in the industry is unaffected by its magnitude. Hence the Commission's finding of a relatively concentrated industry with increasing competition.

The foregoing concludes the illustration on the diagnostic functions of the HHI as set out to demonstrate. The next section summarises the work thus far.

6. Conclusion

This paper began by recalling that according to established thinking in the economics field, the HHI or more generally statistical measures of business concentration are merely descriptive indices of such concentration incapable of formal statistical analysis and disconnected from the economic theory on the formation of industrial structure. At least in the case of the HHI, the present work finds that these suppositions are of questionable substance. For instance, Schumpeter's (1961, p. 66; 1976, pp. 84-85) seminal works have demonstrated that the formation of industrial structure begins and ends with a renewal process, which is made real by the three types of economic activity considered here, namely the creation of proprietary knowledge, the levitation to government assistance, and the explicit or implicit sanctioning by government of private collusion. By finding that conceptually the HHI is inherently Rayleigh distributed, it emerges that the underlying

behaviour of this index is likewise governed by a renewal process, given that this distribution is only established by such a process. It was shown that in the context of human activity in commerce, the statistical explanation for such process rests on regenerative or revitalising actions in economic activity. This gives the basis to differentiate between two types of diagnostic functions embedded in the HHI. These are identified as the primary and secondary diagnostic function. The primary diagnostic function of the HHI is contained in the ability of the index to measure business concentration primarily as an outcome of market conduct. This is because each index value is an estimate of business concentration fostered by any of the aforementioned three possible methods of economic activity. A by-product of the HHI's primary function is its secondary diagnostic function, which is contained in the ability of the index to anyhow give an estimate of business concentration, irrespective of the economic activity responsible for it or the resultant market structure it points to. Consequentially, the HHI should be seen as a screening indicator of market conduct, which incidentally doubles-up as an indicator of market structure.

The HHI's diagnostic functions become obvious only after enlisting the support of statistical theory. This is done by calling attention to the fact that the Chi-square distribution is the approximate sampling distribution of the HHI. Then this fact is used to show that as a theoretical construct, the HHI is inherently distributed as a special case of the Chi-square distribution – called the Rayleigh distribution – that only comes about by a renewal process. In the present instance, the renewal process involves economic activities thereby reinforcing that the HHI is a measure of business concentration. This same statistical theory also shows that the index is perfectly capable of formal statistical analysis that is subordinated to the familiar Chi-square distribution. Such an analysis highlights that the HHI is primarily an index of suggestive market conduct diagnosis, and secondarily an index of suggestive market structure diagnosis. This is in line with the correspondingly different but interwoven diagnostic functions of the index.

The above casts doubt on any premise that as an index of business concentration the HHI will offer no understanding of the forces at work that determine this concentration. As the current work demonstrates, this premise is only true to the extent that while the HHI cannot identify the exact economic activity that accounts for the observed business concentration level, or the consequences it is associated with, it does suggest that the explanation for it must be sought for in the renewal process governing the three distinct methods of economic activity. These are the activities that a firm chooses to compete for market share. In the illustrated example of the Australian retail banking industry, the renewal process was tied to economic activity resting on the creation of proprietary knowledge, as the mechanism that gives each firm the technical and operational abilities to produce competing offers, whether they are counteroffers or replacements. But depending on the particular case involved, the encountered economic activity will be different and so might be the features of its renewal process. Then the HHI estimates will point to business concentration in terms of the suggestive market conduct that is explainable or identifiable with the method of economic activity prevailing in the case. It is clear however that whatever the case, a formal HHI statistical analysis of it, offers a systematic economic theory for interpreting

its business concentration. By contrast, and questionably so, the current uses of the HHI assume otherwise (Andreosso and Jacobson, 2005, pp. 98-99).

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