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A General Equilibrium Analysis of the Waste-economic System with Material Recycling - A CGE-modeling Approach -

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1. Introduction

The authors (Miyata, 1995 and 1997) have earlier proposed a waste-economic accounting matrix, which is an extended social accounting matrix that expresses interaction between economic activities and waste generation/ abatement. We have then constructed static and intertemporal *CGE* (computable general equilibrium) models regarding the estimated waste-economic accounting matrix as a benchmark dataset. These studies have examined the economic effects of charging household waste and promotion of recycling via comparative static and dynamic analyses.

In these studies, however, material balance and circulation, which are considered to be one of the most important issues in the environmental economics, are dealt with only in a very simplified way. Promotion of material recycling, i.e. reuse and reproduction of materials, aims not only to save physical and energy inputs, but also aims to realize a zero-emission oriented society in which unused materials generated by socio-economic activities are designated to be ultimately eliminated.

The concept of zero-emission oriented society was originally proposed by Gunter Pauli (Capra and Pauli, 1995), former Vice President of the University of United Nations, and the studies on this field has received much attention recently. Current studies on this topic have, however, shown an initial step in which investigations are being made on what materials are inputted and generated in various production processes (Suzuki, 1999). Apart from the natural sciences and engineering related to material circulation, it has been a relatively unexplored field in environmental economics how the realization of a zero-emission oriented society would transform or change our present society. Taking these backgrounds into account, this article aims to present a *CGE* model incorporating material transformation/circulation for the 1994 economy of Aichi Prefecture in Japan as a study region.

2. Economic-Material Balance Accounting Matrix

In our previous studies, a waste-economic accounting matrix was estimated, and then a *CGE* model was constructed incorporating the accounting matrix as a benchmark dataset. Instead of the waste-economic accounting matrix, in this study, we propose a concept of economic-material balance (E-M) accounting matrix extending our previous approach. The structure of a new accounting matrix is illustrated in Table 1.

In the E-M matrix, wastes generated by industries and households are further classified into different types of wastes. A part of materials discharged by economic activities are transformed into reusable materials which are in turn reinputted in industries. Materials that can no longer be used by any industries are denoted as final unused materials. Explanation for other sectors listed in columns and rows, and transactions between sectors are skipped because they are based on the standard definition in social accounting study.

The estimated E-M matrix is shown in Table 2 in aggregated form. The original E-M matrix in this study consists of 34 industries, 34 internal material transformation sectors, one ex-

ternal material transformation sector, the government, households, capital, labor, capital account, and the external sector. In Table 2, 34 industries and 34 internal material transformation sectors are aggregated into two sectors.

From this table, one can observe that industries in Aichi prefecture totally yielded 67 trillion yen of output in 1994 with 17.19 million tons of industrial waste. Households obtained 26.63 trillion yen of income spending 15.1623 trillion yen on consumption expenditures, and at the same time generated 2.76 million tons of waste. Industrial waste was internally treated at the cost of 38.37 billion yen, while the external material transformation sector treated a part of industrial waste and entire household waste at the cost of 16.1 billion yen.

3. The Structure of the Model

A static model is constructed in the present study based on assumptions mentioned below. This model differs from our previous model in introduction of material transformation/circulation. Therefore this section emphasizes explanation of industries, material transformation sectors, prices, and market equilibrium conditions, which are significantly modified by the introduction of material transformation. Figure 1 illustrates the hierarchical structure of the model.

Figure 1. Hierarchical Structure of the E-M CGE Model

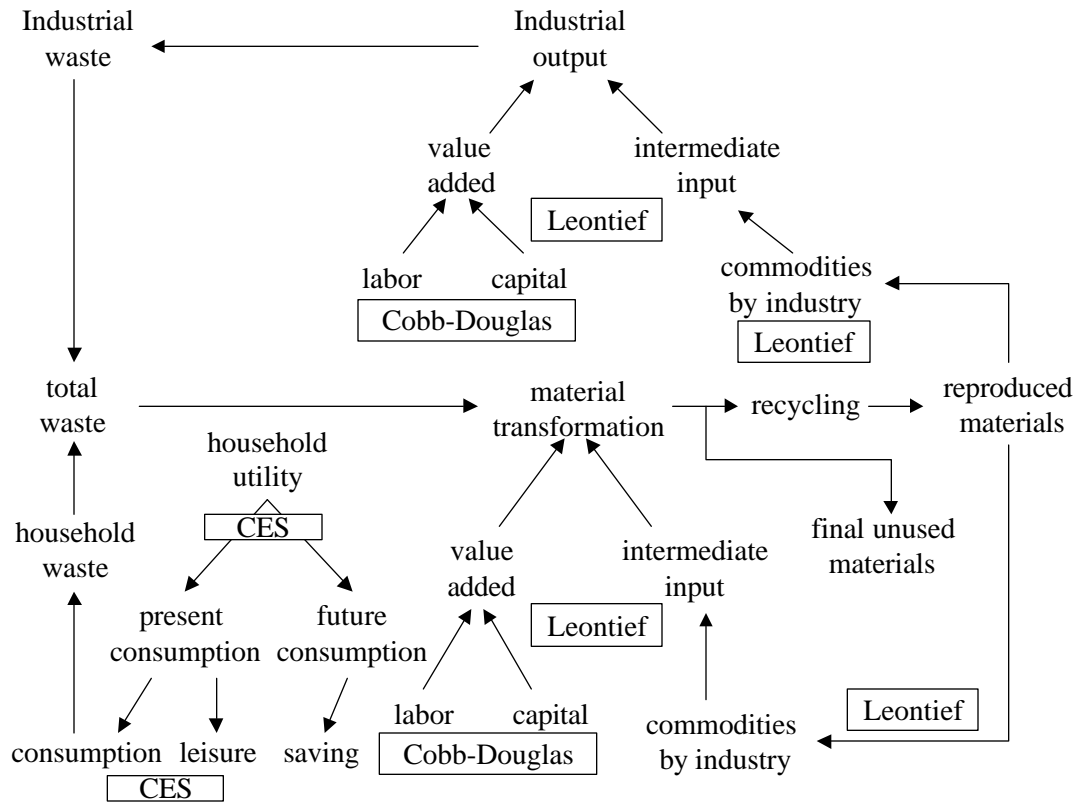


Table 1 Structure of the Economic-Material Balance Accounting Matrix

		production activities	material transformation sectors		institution		production factors		capital accumulation	external sector	total	
		34 industries	internal	external	government	households	capital	labor	capital accumulation	exports	total commodity demand	
production activities	34 industries	intermediate input	intermediate input	intermediate input	government consumption	household consumption	•	•	capital accumulation	•	total commodity demand	
material transformation sectors	internal	material transformation services	•	•	•	•	•	•	•	•	total demand for internal material transformation	
	external	material transformation services	•	•	current transfers	•	•	•	•	•	total demand for external material transformation	
institution	government	net indirect tax	net indirect tax	net indirect tax	•	direct taxes	•	•	•	current transfers	government income	
	households	•	•	•	current transfers	•	capital income	labor income	•	current transfers	household income	
production factors	capital	operating surplus	operating surplus	operating surplus	•	•	•	•	•	capital income inflows	total capital income	
	labor	compensation of employees	compensation of employees	compensation of employees	•	•	•	•	•	labor income inflows	total labor income	
capital finance		capital depreciation	capital depreciation	capital depreciation	savings	savings	•	•	•	capital transfers	total capital finance	
external sector		imports	•	•	current transfers	current transfers	capital income outflows	labor income outflows	current surplus	•	receipts of external sector	
total		commodity supply	supply of material transformation	supply of material transformation	disposes of income	disposes of income	disposes of income	disposes of income	total capital accumulation	total expenditures	receipts-expenditures	
waste generation / volume of reuse		waste generation	volume of reproduced materials	volume of reproduced materials	•	waste generation	•	•	•	•	final unused materials	
cinder organic sludge inorganic sludge general waste oil waste solvent solid waste oil sludge waste acid waste alkali waste plastic waste tyre shredder dust waste paper waste wood waste fiber animal and vegetable residue waste rubber waste metal waste glass and porcelain slag concrete waste asphalt other construction waste soot and dust animal waste other industrial waste		volume of waste generation by industry	volume of reproduced materials	volume of reproduced materials	•	waste generation by households	mixed garbage	•	•	•	•	volume of final unused materials by type
							inflammable garbage					
							non-inflammable garbage					
							recyclable garbage					
							large size garbage					
							direct haulage garbage					
							self disposal garbage					
							group collection garbage					
							other garbage					

Table 2 Estimated Waste-Economic Accounting Matrix of 1994 Aichi's Economy

		production activities	material transformation sectors		institution		production factors		capital accumulation	external sector	total
		34 industries	internal	external	government	households	capital	labor			
production activities	34 industries	35,877,673	14,327	29,550	1,962,981	15,162,290	0	0	733,735	25,483	85,868,531
material transformation sectors	internal	38,307	0	0	0	0	0	0	0	0	38,307
	external	96,280	0	0	57,085	7,643	0	0	0	0	161,008
institution	government	2,294,188	0	3,152	0	6,118,189	0	0	0	1,324,915	9,740,444
	households	0	0	0	2,149,822	0	8,008,547	16,197,355	0	274,696	26,630,420
production factors	capital	7,542,630	4,327	13,483	0	0	0	0	0	448,107	8,008,547
	labor	16,677,003	13,127	103,499	0	0	0	0	0	253,257	17,046,886
capital finance		4,473,611	6,526	11,324	1,753,243	5,342,298	0	0	0	232,738	11,819,740
external sector		18,868,839	0	0	3,817,313	0	0	849,531	4,482,005	0	28,017,688
total		85,868,531	38,307	161,008	9,740,444	26,630,420	8,008,547	17,046,886	11,819,740	28,017,688	187,331,571
waste generation / treatment		17,194	- 10,527	- 9,424	0	2,757	0	0	0	0	0

Note: Figures in transaction of economic sectors are in million yen, while waste generation/treatment are in thousand tons.

3.1 Assumptions of the model

Main assumptions applied in the model are summarized in the following. Although there are many other conditions/specifications are assumed in the model by nature of the *CGE*-modeling, rest of the assumptions are described with explanation of model specifications in order.

1994 Aichi's economy is examined. Economic sectors in the model are; households, 34 industries, the government, the external sector, 34 internal material transformation sectors, and one external material transformation sector. Internal and external material transformation sectors are abbreviated to *IT* and *ET* sectors hereafter.

IT sectors are associated with the respective industries. They treat and dispose of the wastes that are generated in production process, and then yield recyclable/reusable materials. *ET* sector consists of public waste abatement activities and private waste abatement firms, and it disposes of a part of industrial waste that is not treated by industries, and treats household waste. A part of waste treated by *ET* sector is recycled for intermediate inputs in industries as well.

Virgin commodities and recycled materials are assumed to be homogenous in quality. Recycled goods are used only for intermediate inputs but not for final uses. Recycled materials are uniformly contained in intermediate inputs.

Markets considered in the model are 34 commodity, labor, and capital markets.

These markets are assumed to be in long-run equilibrium in 1994.

3.2 Industries

Industries employ intermediate goods/services, labor, and capital, and then produce commodities/ services generating industrial waste. A part of industrial waste is treated by the respective *IT* sectors, while the rest of it is disposed by *ET* sector. Industries pay costs of reutilization of waste. The technology of industries is assumed to be *Leontief* type for intermediate inputs, and *Cobb-Douglas* type for labor and capital inputs. Constant returns to scale are supposed. Due to linear homogeneity in the production functions, profit maximization in industries' behavior becomes cost minimization.

Table 3 Classification of Industries

Industry abbreviati on	Industry abbreviati on
1. agriculture and forestry	18. metal product
2. fishery	19. general machinery
3. mining	20. electric machinery
4. food	21. transportation equipment
5. beverage and feed	22. precision machinery
6. fabric and textile	23. other manufacturing
7. wooden product	24. construction
8. furniture	25. electric power
9. pulp and paper	26. gas, steam, and hot water supply
10. chemical	27. water supply and sewage disposal
11. petroleum refinery and coal	28. trade
12. plastic	29. financial service, insurance, and real estate
13. rubber	30. land transport
14. leather	31. other transport
15. ceramic, stone, and clay	32. communication and broadcasting
16. iron and steel	33. car repairing
17. non-ferrous metal	34. other services

3.3 Waste generation

Wastes are discharged by both industries and households. The volume of industrial waste generated by each industry is assumed to be proportional to its output, while household waste is supposed to be proportional to the amount of composite consumption. These are expressed in the following specifications in which the types of waste are shown in Table 1.

$$WG_{kj} = RWG_{kj} \cdot X_j \quad (k = 1, \dots, 26, j = 1, \dots, 34) \quad (1)$$

$$WGH_l = RWH_l \cdot CC \quad (l = 1, \dots, 9) \quad (2)$$

where

WG_{kj} waste k generated by industry j

RWG_{kj} marginal generation of waste k by industry j

X_j output of industry j

WGH_l household waste of type l

RWH_l marginal household waste discharge l

CC household composite consumption

3.4 Material transformation sectors

Material transformation sectors, like industries, employ intermediate commodities/services, capital, and labor to treat waste, and then supply reusable materials for industries.

The technology in material transformation sectors is *Leontief* type for intermediate inputs,

and *Cobb-Douglas* type for capital and labor inputs. Constant returns to scale are also assumed. The behavior of material transformation sectors is supposed to minimize operating costs under the given volume of waste discharged in production process of industries.

3.5 Behavior of industries and internal material transformation sectors

Before proceeding to explanations of behavior of industries mentioned in subsections 3.2 to 3.4, we explain a specification of recycled materials. Since recycled materials are assumed to be homogeneous as virgin materials in quality, and contained uniformly in intermediate goods, the amount of recycled good i contained in intermediate input x_{1ij} is calculated as ;

$$x_{R1ij} = \frac{x_{1ij}}{\sum_{j=1}^{34} x_{1ij} + \sum_{j=1}^{35} x_{2ij}} x_{Ri} \quad (3)$$

where

x_{R1ij} : recycled material contained in industry j 's intermediate input of commodity i

x_{1ij} : industry j 's intermediate input of commodity i

x_{2ij} : material transformation sector j 's intermediate input of commodity i

x_{Ri} : recycled material as commodity i

Since costs of x_{R1ij} , $p_i x_{R1ij}$, are paid by industry j to *IT* and *ET* sectors, $p_i x_{R1ij}$ is not counted as costs of intermediate inputs. Therefore denoting intermediate input of virgin goods by x_{V1ij} , the behavior of industries and *IT* sectors are expressed as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{i=1}^{34} p_i x_{V1ij} + (1 + tp_{1j})(w \cdot L_{1j} + r \cdot K_{1j}) + \sum_{i=1}^{34} p_i x_{V2ij} \\ & + (1 + tp_{2j})(w \cdot L_{2j} + r \cdot K_{2j}) + q_{35} WT_{2j} \end{aligned} \quad (j = 1, \dots, 34) \quad (4)$$

with respect to x_{V1ij} , L_{1j} , x_{V2ij} , L_{2j} , K_{2j} , and WT_{2j}

subject to

$$X_j = \text{Min} \left\{ \frac{1}{a_{10j}} f_{1j}(L_{1j}, K_{1j}), \frac{x_{V11j} + x_{R11j}}{a_{11j}}, \dots, \frac{x_{V134j} + x_{R134j}}{a_{134j}} \right\} \quad (5)$$

$$WG_{kj} = RWG_{kj} X_j \quad (k = 1, \dots, 26, j = 1, \dots, 34) \quad (6)$$

$$WT_j \equiv RWT_j \sum_{k=1}^{26} WG_{kj} \quad (7)$$

$$WT_j = \text{Min} \left\{ \frac{1}{a_{20j}} f_{2j}(L_{2j}, K_{2j}), \frac{x_{V21j} + x_{R21j}}{a_{21j}}, \dots, \frac{x_{V234j} + x_{R234j}}{a_{234j}} \right\} \quad (8)$$

$$WT_{2j} \equiv RWT_{2j} \sum_{k=1}^{26} WG_{kj} \quad (9)$$

$$f_{1j}(L_{1j}, K_{1j}) \equiv A_{1j} L_{1j}^{a_{1j}} K_{1j}^{(1-a_{1j})} \quad (10)$$

$$f_{2j}(L_{2j}, K_{2j}) \equiv A_{2j} L_{2j}^{a_{2j}} K_{2j}^{(1-a_{2j})} \quad (11)$$

where

p_i : price of product of industry i

x_{V1ij} : industry j 's intermediate input of virgin good i

x_{R1ij} : industry j 's intermediate input of recycled good i

tp_{1j} : net indirect tax rate imposed on industry j

w : wage rate

r : capital return rate

L_{1j} : labor input in industry j

K_{1j} : labor input in industry j
 WT_j : waste treated by IT sector j
 q_{35} : price of material transformation service by ET sector
 WT_{2j} : waste of industry j treated by ET sector
 a_{10j} : value added rate in industry j
 a_{1ij} : intermediate input coefficient in industry j
 RWT_j : rate of internal transformation in industry j
 RWT_{2j} : rate of external transformation in industry j
 A_{1j}, \hat{a}_{1j} : technological parameters in industry j
 x_{V2ij} : IT sector j 's intermediate input of virgin good i
 x_{R2ij} : IT sector j 's intermediate input of recycled good i
 tp_{2j} : net indirect tax rate on IT sector j
 L_{2j} : labor input in IT sector j
 K_{2j} : capital input in IT sector j
 a_{20j} : value added rate in IT sector j
 a_{2ij} : intermediate input coefficient in IT sector j
 A_{2j}, \hat{a}_{2j} : technological parameters in IT sector j

Solving the above-mentioned optimization problem (4)–(11), conditional demands for intermediate, capital, and labor inputs are obtained under the given X_j .

$$x_{1ij} = a_{1ij}X_j \quad (x_{1ij} \equiv x_{V1ij} + x_{R1ij}) \quad (12)$$

$$LD_{1j} = \left[\frac{(1-a_{1j})r}{a_{1j}w} \right]^{a_{1j}} \frac{a_{0j}X_j}{A_{1j}} \quad (13)$$

$$KD_{1j} = \left[\frac{a_{1j}w}{(1-a_{1j})r} \right]^{(1-a_{1j})} \frac{a_{0j}X_j}{A_{1j}} \quad (14)$$

$$x_{2ij} = a_{2ij}WT_j \quad (x_{2ij} \equiv x_{V2ij} + x_{R2ij}) \quad (15)$$

$$LD_{2j} = \left[\frac{(1-a_{2j})r}{a_{2j}w} \right]^{a_{2j}} \frac{a_{20j}WT_j}{A_{2j}} \quad (16)$$

$$KD_{2j} = \left[\frac{a_{2j}w}{(1-a_{2j})r} \right]^{(1-a_{2j})} \frac{a_{20j}X_j}{A_{2j}} \quad (17)$$

where

LD_{1j} : labor demand of industry j
 LD_{2j} : labor demand of IT sector j
 KD_{1j} : capital demand of industry j
 KD_{2j} : capital demand of IT sector j

Free entry assumption yields the following zero profit condition for industry j in equilibrium.

$$\begin{aligned}
profit = p_j X_j - \sum_{i=1}^{34} p_i x_{V1ij} - (1+tp_{1j}) [w \cdot LD_{1j} + r \cdot KD_{1j}] - \sum_{i=1}^{34} p_i x_{V2ij} \\
- (1+tp_{2j}) [w \cdot LD_{2j} + r \cdot KD_{2j}] - q_{35} WT_{2j} = 0
\end{aligned} \quad (18)$$

Moreover, the shadow price of IT sector j 's service, q_j , can be derived by the Lagrange multiplier associated with the constraint condition (8).

$$q_j = \sum_{i=1}^{34} p_i (a_{2ij} - x_{R2ij} / WT_j) + w \cdot ld_{2j} + r \cdot kd_{2j} \quad (19)$$

$$ld_{2j} \quad LD_{2j} / WT_j \quad kd_{2j} \quad KD_{2j} / WT_j$$

3.6 External material transformation sector

ET sector actually disposes of the wastes generated by both industries and households. Therefore it is assumed that the entire household waste and a part of industrial waste that is not abated by *IT* sectors are treated by *ET* sector. *ET* sector is further supposed to behave like a private firm, leading to cost minimization under the given quantity of household and industrial wastes to be disposed of. Thus we have;

$$\text{Min} \sum_{i=1}^{34} p_i x_{V2i35} + (1 + tp_{235})(w \cdot L_{235} + r \cdot K_{235}) \quad (20)$$

with respect to $x_{V2i35}, L_{235}, K_{235}$

subject to

$$WT_{35} = \sum_{j=1}^{34} RWT_{2j} \sum_{k=1}^{26} WG_{kj} + \sum_{l=1}^9 WGH_l \quad (21)$$

$$WT_{35} = \text{Min} \left\{ \frac{1}{a_{2035}} f_{235}(L_{235}, K_{235}), \frac{x_{V2135} + x_{R2135}}{a_{2135}}, \dots, \frac{x_{V23435} + x_{R23435}}{a_{23435}} \right\} \quad (22)$$

$$f_{235}(L_{235}, K_{235}) \equiv A_{235} \cdot L_{235}^{a_{235}} \cdot K_{235}^{(1-a_{235})} \quad (23)$$

where

x_{V2i35} : *ET* sector's intermediate input of commodity i

tp_{235} : net indirect tax rate on *ET* sector

L_{235} : labor input in *ET* sector

K_{235} : capital input in *ET* sector

WT_{35} : waste treated by *ET* sector

RWT_{2j} : rate of waste treated by *ET* sector in industry j

a_{2035} : value added rate in *ET* sector

a_{2i35} : intermediate input coefficient in *ET* sector

A_{235}, \hat{a}_{235} : technological parameters in *ET* sector

Conditional demands of *ET* sector for intermediate goods, labor, and capital are, therefore, obtained as follows;

$$x_{2i35} = a_{2i35} WT_{35} \quad (x_{2i35} \equiv x_{V2i35} + x_{R2i35}) \quad (24)$$

$$LD_{2j} = \left[\frac{(1 - a_{2j})}{a_{2j} w} \right]^{a_{2j}} \frac{a_{20j} WT_j}{A_{2j}} \quad (j = 1, \dots, 35) \quad (25)$$

$$KD_{2j} = \left[\frac{a_{2j} w}{(1 - a_{2j}) r} \right]^{(1-a_{2j})} \frac{a_{20j} WT_j}{A_{2j}} \quad (j = 1, \dots, 35) \quad (26)$$

Zero profit condition also holds in equilibrium for *ET* sector.

$$\text{profit} = q_{35} WT_{35} - \sum_{i=1}^{34} p_i x_{V2i35} - (1 + tp_{235}) [w \cdot LD_{235} + r \cdot KD_{235}] = 0 \quad (27)$$

3.7 Households

Households in Aichi prefecture are assumed to be identical, therefore, households are assumed to share an aggregate *CES* utility function of future good, i.e. saving, and of present commodity which is a composite of present consumption and leisure time.

The present good is then divided into composite consumption good and leisure time maximizing a sub-utility function. Finally, the composite consumption good is divided into commodities produced by 34 industries maximizing a sub-sub-utility function.

Household full income is defined as the total of full wage income that is obtained if a household supplied its entire labor endowment, post depreciation capital income, current transfers from the government, labor and property incomes from the external sector, and other current transfers. A part of wage and capital incomes of households are transferred to the ex-

ternal sector. Households expend the post-tax-and-current-transfers income on present and future commodities.

Let us now explain the household behavior by starting with the future goods. Future goods can be regarded as composite investment goods. Investment is made using produced goods. Assuming the technology employed in investment as *Leontief* type, then we have;

$$I = \text{Min} \{I_1/b_1, \dots, I_{34}/b_{34}\} \quad (28)$$

where

I_i : commodities used in investment I

b_i : parameters in *Leontief* technology combining I and I_i . ($b_i > 0, \sum_{i=1}^{34} b_i = 1$)

Assuming that investment I is made minimizing investment costs $\sum_{i=1}^{34} p_i I_i$ commodity demand for industry i associated with investment I is expressed as $I_i = b_i I$. Denoting the price of investment composite by p_I , $p_I I = \sum_{i=1}^{34} p_i I_i$ is realized. Therefore we obtain the price of capital good as $p_I = \sum_{i=1}^{34} b_i p_i$. p_I can also be regarded as the price of saving, p_s .

Now the net capital returns associated with one unit of investment is expressed as $(1-ty)(1-k_o)(1-k_r)r$ the ratio of expected net capital returns to price of saving good p_s , that is, the expected net return rate on savings r_s is written as follows;

$$r_s = (1-ty)(1-k_o)(1-k_r)r / p_s \quad (29)$$

where

ty : direct tax rate on household income

k_o : rate of property income transfers to the external sector

k_r : capital depreciation rate

\ddot{a} : ratio of capital stock measured in physical unit to that in service unit

Suppose that the expected net returns finances household future consumption expenditures, and the price of future consumption good H is identified as the price of present consumption good p with myopic expectation, then the following equation holds.

$$p \cdot H = (1-ty)(1-k_o)(1-k_r)r \cdot S \quad (30)$$

This yields $[p_s p / (1-ty)(1-k_o)(1-k_r)r] H = p \cdot S$ and let the price of future consumption good p_H be

$$p_H = p_s p / (1-ty)(1-k_o)(1-k_r)r \quad (31)$$

then $p_s S = p_H H$ holds.

Applying these specifications of future good and its price, household utility maximization is now described as follows;

$$\text{Max}_{G,H} u(G,H) \equiv \{ \mathbf{a}^{1/v_1} G^{(v_1-1)/v_1} + (1-\mathbf{a})^{1/v_1} H^{(v_1-1)/v_1} \}^{v_1/(v_1-1)} \quad (32)$$

subject to

$$p_G \cdot G + p_H \cdot H = (1-ty)FI - TrHO \quad (33)$$

$$FI \equiv (1-l_o)w \cdot E + LI + (1-k_o)(1-k_r)r \cdot KS + KI + TrGH + TrOH \quad (34)$$

where

\mathbf{a} : share parameter

v_1 : elasticity of substitution between present and future goods

G : present consumption good

H : future consumption good

p_G : price of present consumption good

p_H : price of future consumption good

FI : household full income

$TrHO$: current transfers from households to the external sector

l_o : transfer rate of labor income to the external sector

E : initial labor endowment of households (= 2.045 times the initial household labor supply, which is

estimated from the data on working time and leisure time in Aichi prefecture.)

LI : labor income transfers from the external sector to households (exogenous variable)

KS : initial household capital endowment

KI : property income transfers from the external sector to households (exogenous variable)

$TrGH$: current transfers from the government to households

$TrOH$: current transfers from the external sector to households

Solving this utility maximization problem, the present and the future consumption demand functions, and the saving function are obtained.

$$G = \frac{\mathbf{a}[(1-ty)FI - TrHO]}{p_G^{v_1} \cdot \mathbf{D}} \quad (35)$$

$$H = \frac{(1-\mathbf{a})[(1-ty)FI - TrHO]}{p_H^{v_1} \cdot \mathbf{D}} \quad (36)$$

$$SH = p_H H / p_s \quad (37)$$

$$\mathbf{D} \equiv \mathbf{a} p_G^{1-v_1} + (1-\mathbf{a}) p_H^{1-v_1} \quad (38)$$

Let us now turn to derivation of composite consumption and leisure time from the present consumption composite (35). The present consumption G is an aggregate of composite consumption and leisure time, and G is derived from the following sub-utility maximization problem.

$$\underset{C,F}{\text{Max}} G \equiv \{ \mathbf{b}^{1/v_2} C^{(v_2-1)/v_2} + (1-\mathbf{b})^{1/v_2} F^{(v_2-1)/v_2} \}^{v_2/(v_2-1)} \quad (39)$$

subject to

$$p \cdot C + (1-ty)(1-l_o)w \cdot F = (1-ty)FI - TrHO - SH \quad (40)$$

where

$\hat{\mathbf{a}}$: share parameter

v_2 : elasticity of substitution between composite consumption and leisure time

C : composite consumption

F : leisure time

p : price of composite consumption good

Solving this sub-optimization problem, composite consumption, leisure time, and labor supply are obtained.

$$C = \frac{\mathbf{b}[(1-ty)FI - TrHO - SH]}{p^{v_2} \cdot \mathbf{\Omega}} \quad (41)$$

$$F = \frac{(1-\mathbf{b})[(1-ty)FI - TrHO]}{[(1-ty)(1-l_o)w]^{v_2} \cdot \mathbf{\Omega}} \quad (42)$$

$$LS = E - F \quad (43)$$

$$\mathbf{W} = \mathbf{b} p^{(1-v_2)} + (1-\mathbf{b})[(1-ty)(1-l_o)w]^{(1-v_2)} \quad (44)$$

where

LS : household labor supply

Substituting the composite consumption and leisure time into equation (39), the present consumption demand can be obtained as an indirect sub-utility function yielding the following price index of present consumption good.

$$p_G = \{ \mathbf{b} p^{1-v_2} + (1-\mathbf{b})[(1-ty)(1-l_o)w]^{1-v_2} \}^{1/(v_2-1)} \quad (45)$$

Moreover consumption demands for industries' products are derived from maximizing a *Cobb-Douglas* sub-sub-utility function given the income and leisure time.

$$\text{Max} \prod_{j=1}^{34} C_j^{g_j} \quad (\sum_{j=1}^{34} g_j = 1) \quad (46)$$

subject to

$$\sum_{j=1}^{34} p_j \cdot C_j = (1-ty)Y - TrHO - SH \quad (47)$$

where

C_j : household consumption demand for commodities produced by industry j

p_j : price of commodity j

Y : household income $(= (1-l)w \cdot LS + LI + (1-k) \cdot r \cdot KS + KI + TrGH + TrOH)$

Then we have consumption demands for industries, and the price index of composite consumption good as follows;

$$C_j = \frac{r_j}{p_j} [(1-ty)Y - TrHO - SH] \quad (j = 1, \dots, 34) \quad (48)$$

$$p = \prod_{j=1}^{34} \left(\frac{p_j}{g_j} \right)^{g_j} \quad (49)$$

3.8 The government

The government obtains its revenues from direct and net indirect taxes, and current transfers from the external sector. Then it expends the revenues on government consumption expenditures, current transfers to households, expenditures to *ET* sector, and current transfers to the external sector. The balance between receipts and payments are saved. It is assumed that nominal government consumption expenditures, current transfers to households and to the external sector are proportional to government income. These specifications are expressed as the following balance of payments.

$$\begin{aligned} \sum_{i=1}^{34} p_i \cdot CG_i + TrGH + WTC + TrGO + SG = ty \cdot Y + \sum_{i=1}^{34} tp_{1i} (w \cdot LD_{1i} + r \cdot KD_{1i}) \\ + \sum_{i=1}^{34} tp_{2i} (w \cdot LD_{2i} + r \cdot KD_{2i}) + TrOG \end{aligned} \quad (50)$$

where

CG_i : government consumption expenditures on commodity i

WTC : government expenditures on the external transformation activity

$TrGO$: current transfers from the government to the external sector

SG : government savings

$TrOG$: current transfers from the external sector to the government

3.9 The external sector

The external sector receives its income from Aichi's imports, current transfers from the

government, labor and property income transfers. Then it expends its income on Aichi's exports, current transfers to households and to the government, labor and property income transfers to Aichi prefecture. The balance between income and expenditures is saved. Here it is also assumed that nominal Aichi's exports are fixed, and real imports are proportional to Aichi's domestic demand. These specification are illustrated as the following balance of payments.

$$\sum_{i=1}^{34} p_i \cdot EX_i + TrOH + TrOG + KI + LI + SG = \sum_{i=1}^{34} p_i \cdot EM_i + TrHO + TrGO + KIO + LIO \quad (51)$$

where

EX_i : Aichi's export of commodity i

EM_i : Aichi's import of commodity i

SO : savings of the external sector (= - prefectural current surplus)

LIO : labor income transfers to the external sector (= $l \quad w \quad LS$)

KIO : property income transfers to the external sector (= $k \quad r \quad kS$)

3.10 Balance of Investment and savings

Savings of the households, the government, and the external sector, and capital depreciation determine the total investment.

$$\sum_{i=1}^{34} p_i \cdot I_i = SH + SG + SO + \sum_{i=1}^{34} DR_{1i} + \sum_{i=1}^{35} DR_{2i} \quad (52)$$

where

DR_{1i} : capital depreciation in industry i

DR_{2i} : capital depreciation in material transformation sector i

3.11 Commodity prices

From the zero profit condition for industries and ET sector, the following cost composition can be derived.

$$p_j X_j = \sum_{i=1}^{34} p_i (x_{1ij} - x_{R1ij}) + (1 + tp_{1j}) [w \cdot LD_{1j} + r \cdot KD_{1j}] + \sum_{i=1}^{34} p_i (x_{2ij} - x_{R2ij}) - (1 + tp_{2j}) [w \cdot LD_{2j} + r \cdot KD_{2j}] - q_{35} WT_{2j} \quad (53)$$

$$q_{35} WT_{35} = \sum_{i=1}^{34} p_i (x_{2ij} - x_{R2ij}) + (1 + tp_{235}) (w \cdot LD_{235} + r \cdot KD_{235}) \quad (54)$$

By a simple calculation from equations (53) and (54), commodity prices can be expressed by the following equation given the wage and capital return rates.

$$P = [I - B_1'(Q) - DWT \cdot DWG \cdot B_2'(Q) - DWT_2 \cdot DWG \cdot B_3'(Q)]^{-1} \cdot [(1 + tp_{1j})(w \cdot ld_{1j} + r \cdot kd_{1j}) + (1 + tp_{2j})(w \cdot ld_{2j} + r \cdot kd_{2j}) + (1 + tp_{235})(w \cdot ld_{235} + r \cdot kd_{235})] \quad (55)$$

where

P : vector of commodity prices

$B_1'(Q)$: transposed matrix of input coefficients in which recycled goods are subtracted
($a_{1ji} - x_{R1ji}/X_i$)

Q : vector of generalized prices including commodity prices, prices of material transformation services,

wage rate, and capital return rate

DWT : diagonal matrix whose diagonal elements are RWT_j .

DWG : diagonal matrix whose diagonal elements are $\sum_{i=1}^{26} RWG_{kj}$.

$B_2'(Q)$: transposed matrix of input coefficients in which recycled goods are subtracted
 $(a_{2ji} - x_{R2ji} / WT_i)$

DWT_2 : diagonal matrix whose diagonal elements are RWT_{2j} .

$B_3'(Q)$: transposed matrix of input coefficients in which recycled goods are subtracted.
 $(a_{2j35} - x_{R2j35} / WT_{35})$

[] : column vector whose j -th element is shown in the parentheses

ld_{1j} LD_{1j} / X_j kd_{1j} KD_{1j} / X_j , ld_{2j} LD_{2j} / WT_j kd_{2j} KD_{2j} / WT_j
 ld_{235} LD_{235} / WT_{235} kd_{235} KD_{235} / WT_{235}

Prices of material transformation services are denoted by using commodity prices as follows;

$$q_j = \sum_{i=1}^{34} p_i b_{2ij}(Q) + (1 + ty_{2j})(w \cdot ld_{2j} + r \cdot kd_{2j}) \quad (j = 1, \dots, 35) \quad (56)$$

where

$b_{2ij}(Q)$: input coefficient matrix in material transformation sectors being subtracted by recycled goods.

4. Market Equilibrium Condition

In this study, introduction of material transformation sectors also influences the commodity market equilibrium condition. Therefore we describe the treatment of material circulation in commodity market.

Industry j generates waste WG_{kj} in producing X_j . Then RWT_j WG_{kj} of it is internally treated at a rate of RWT_j , supplying recycled goods x_{Rij} . Assuming that this process is implemented by a liner technology, the process such as production waste generation material recycling can be written in matrix representation as follows;

$$Z_1 = {}_1 RWG \ DWT \ X \quad (57)$$

where

Z_1 : vector of materials recycled by IT sectors

${}_1$: matrix of transformation of waste into recycled goods by IT sectors

RWG : matrix of waste generation coefficients, RWG_{kj} 's

X : vector of outputs of industries

Similar to IT sectors, supply of recycled goods from industrial and household wastes transformed by ET sector can be denoted, respectively, as follows;

$$Z_2 = {}_2 RWG \ DWT_2 \ X \quad (58)$$

$$Z_3 = {}_3 RWH \ CC \quad (59)$$

where

Z_2 : vector of recycled goods transformed from industrial waste by ET sector

${}_2$: matrix of transformation of waste into recycled goods by ET sector

Z_3 : vector of recycled goods transformed from household waste by ET sector

${}_3$: matrix of transformation of household waste into recycled goods

RWH : column vector of household waste generation coefficients, RWH_l 's

Regarding recycling of waste as supply of commodities or negative input in industries, the market equilibrium conditions in the present model can be summarized as follows;

commodity market

$$\begin{aligned} X + EM = & A_1 X + A_2 \cdot DWG \cdot DWT \cdot X + A_3 \cdot DWG \cdot DWT_2 \cdot X \\ & + a_3 \sum_{l=1}^9 RWH_l \cdot DWH \cdot CC + C + CG + I + EX \\ & - \Theta_1 \cdot RWG \cdot DWT \cdot X - \Theta_2 \cdot RWG \cdot DWT_2 \cdot X - \Theta_3 \cdot RWH \cdot CC \end{aligned} \quad (60)$$

internal material transformation

$$WT_j = \sum_{k=1}^{26} RWG_{kj} \cdot RWT_i \cdot X_j \quad (j=1,2,\dots,34) \quad (61)$$

external material transformation

$$WT_{35} = \sum_{k=1}^{26} \sum_{j=1}^{34} RWG_{kj} \cdot RWT_{2j} \cdot X_j + \sum_{l=1}^9 RWH_l \cdot CC \quad (62)$$

labor market

$$LS(p(w, r), w) = \sum_{j=1}^{34} LD_{1j}(w, r) + \sum_{j=1}^{35} LD_{2j}(w, r) \quad (63)$$

capital market

$$KS = \sum_{j=1}^{34} KD_{1j}(w, r) + \sum_{j=1}^{35} KD_{2j}(w, r) \quad (64)$$

where

EM : vector of domestic and international imports

A_1 : input coefficient matrix in industries

A_2 : input coefficient matrix in IT sectors

A_3 : matrix whose column vectors are input coefficient vector in ET sector

a_3 : vector of intermediate input coefficient in ET sector

C : vector of household consumption

CG : vector of government consumption

I : vector of investment

EX : vector of domestic and international exports

In this study, equilibrium commodity prices, wage and capital return rates are computed by applying the condition on material recycling (3), commodity price equation (55), and equation of material transformation service prices (56). Numerical computation is implemented by using Newton-Raphson method letting labor be the numeraire ($w=1$).

5. Parameter Setting

For numerical computation, it is necessary to estimate parameters in functions specified in the model. Since technological parameters in production and material transformation functions are specified as *Leontief-Cobb-Douglas* type, they can easily be estimated by applying the economic-material balance accounting matrix as a benchmark dataset in the usual CGE-modeling framework (Shoven and Whalley, 1992). The detailed results of parameter estimation is beyond the scope of this paper, therefore, they are skipped here.

For the parameters in the utility function, estimation of them is made in a standard way and results are shown in Table 3 though the description of the estimation method is skipped as well.

Marginal waste generations by type, RWG_{kj} and RWH_l , are obtained by dividing the volumes of waste generation, WG_{kj} and WGH_l , by industrial output X_j and household composite consumption CC , respectively.

The waste-commodity transformation matrix α_i is estimated by applying the data on waste recycling published by Aichi prefecture. The results are presented in Tables 4 and 5. In these tables, for example, figures such as 0.36 and 4.1 express that 0.36% of organic sludge is re-used as mining products

Table 3 Parameters in Utility Function

Commodities	share parameter
present goods	0.77767
future goods	0.22233
composite consumption	0.58506
leisure	0.41494
elasticity of substitution between present and future goods	1.11909
elasticity of substitution between composite consumption and leisure	1.07054

Table 4 Rate of Industrial Waste Recycling
(in %)

product		3	5	6	7	9	10	12	13	15	16	17	
waste		mining products	beverages and feeds	fabrics and textiles	wooden products	pulp and paper	chemicals	plastics	rubber	ceramic, stone, and clay	iron and steel	non-ferrous metals	total
1	cinder	13.60	0.30										13.90
	sludge												
2	organic sludge	0.36	4.10										4.46
3	inorganic sludge	15.10	3.00										18.10
	waste oil												0.00
4	general waste oil						31.80						31.80
5	waste solvent						56.58						56.58
6	solid waste						83.82						83.82
7	oil sludge						5.79						5.79
8	waste acid						22.00						22.00
9	waste alkali						6.00						6.00
	waste plastics												
10	waste plastic							23.51					23.51
11	waste tyre								75.94				75.94
12	shredder dust												0.00
13	waste paper					71.00							71.00
14	waste wood				24.00								24.00
15	waste fiber			29.00									29.00
16	animal and vegetable residue		56.00										56.00
17	waste rubber							17.00					17.00
18	waste metal									83.90	14.00		97.90
19	waste glass and porcelain									51.00			51.00
20	slag	77.50									10.90	2.40	90.80
	construction waste												
21	concrete	83.69											83.69
22	waste asphalt	98.00											98.00
23	other construction waste												
24	soot and dust						63.00						63.00
25	animal waste		90.00										90.00
26	other industrial waste												

Table 5 Rate of Household Waste Recycling
(in %)

		3	5	6	7	9	10	12	13	15	16	17	
waste	product	mining products	beverages and feeds	fabrics and textiles	wooden products	pulp and paper	chemicals	plastics	rubber	ceramic,stone, and clay	iron and steel	non-ferrous metals	total
	1	mixed garbage			4.666		28.500		11.740	0.220	3.460	1.090	1.870
2	inflammable garbage			5.716		35.000							40.716
3	non-inflammable garbage							63.200	1.230	18.400	5.880	10.060	98.770
4	recyclable garbage			9.090		55.580		22.850		6.700	2.130	3.640	99.990
5	large size garbage												0.000
6	direct haulage garbage			4.676		28.500		11.740	0.220	3.460	1.090	1.870	51.556
7	other garbage												0.000
8	self disposal garbage												0.000
9	group collection garbage			9.130		55.580		22.850		6.700	2.130	3.640	100.030

Table 6 Price of Product Reproduced from Waste
(in million yen / ton)

product		3	5	6	7	9	10	12	13	15	16	17
waste		mining products	beverages and feeds	fabrics and textiles	wooden products	pulp and paper	chemicals	plastics	rubber	ceramic,stone ,and clay	iron and steel	non- ferrous metals
1	cinder sludge	0.0016	0.0204									
2	organic sludge	0.0016	0.0204									
3	inorganic sludge	0.0016	0.0204									
	waste oil											
4	general waste oil						0.0518					
5	waste solvent						0.0518					
6	solid waste						0.0518					
7	oil sludge						0.0518					
8	waste acid						0.0110					
9	waste alkali						0.0130					
	waste plastics											
10	waste plastic							0.0783				
11	waste tyre								0.2197			
12	shredder dust											
13	waste paper					0.0827						
14	waste wood				0.0005							
15	waste fiber			0.4294								
16	animal and vegetable residue		0.0204									
17	waste rubber							0.1163				
18	waste metal									0.0099	0.1887	
19	waste glass and porcelain								0.0038			
20	slag	0.0015									0.0192	0.0007
	construction waste											
21	concrete	0.0237										
22	waste asphalt	0.0237										
23	other construction waste											
24	soot and dust											
25	animal waste		0.0204									
26	other industrial waste											

(garden soil), and 4.1% as beverage and feed products (fertilizer), respectively.

In addition, data of economic sectors are applied in monetary term, while in material transformation sector in physical term. Therefore a transformation matrix which links physical unit to monetary unit is introduced as depicted in Table 6. These data are estimated from the physical table associated with the I-O table of the Japanese economy.

6. Simulation Analysis

6.1 Simulation cases

Applying the model described above, in this section we proceed to numerical experiments focusing on the effects of material recycling on the economy of the study region. Two cases are examined in this paper. The first one is business as usual case (Case 1), while another case assumes a promotion of material recycling in intermediate inputs in industries.

6.2 Simulation results

Comparison of results in the two cases is illustrated in Figure 2 to Figure 9. In these figures, results of Case 2 are presented in variational ratio to results of Case 1. To what follows, we summarize the simulation results. Variables expressed in monetary term are measured at the prices in Case 1.

(1) Outputs of industries

As presented in Figure 2, changes in outputs of industries significantly differ in reusable / reproducible commodities and non-reusable products. Large decreases are -92.1 % in mining products, -38.1% in non-ferrous metal products, -6.1 % in beverages and feeds, and -2 % in pulp and papers. These products are reused / reproduced at high recycling rates, respectively. Therefore reductions in these industries result from decreases in demands for virgin commodities of them due to promotion of recycling of waste.

On the other hand, output growth of assembly industries is enhanced by promotion of recycling, yielding a very small expansion of total industrial output, 0.04 %.

(2) Industrial waste

Looking at wastes by industry in Figure 3, their changes show a very similar manner to those in industrial outputs since industrial waste discharges are assumed to be proportional to outputs. However, as marginal industrial waste generations differ across industries, the total waste discharge including household waste slightly falls. Moreover, household waste increases by 0.6 % due to a growth of consumption.

(3) Outputs of material transformation activities

Observing material transformation sectors in monetary term in Figure 4, *IT* sectors behave in similar manner to industrial outputs. However, due to an increase in output of *ET* sector mainly derived from the expansion in household consumption, the total output of material transformation sectors shows an increase of 0.2 %.

(4) Prices

Prices of recycled goods reflect costs of transforming waste into reusable materials. The data applied in this paper result in lower prices of recycled goods than those of virgin goods. Therefore as the price equation (55) depicts, a promotion of recycling lowers prices of virgin commodities (see Figure 5). Specifically, the price of non-ferrous metal product shows a significant fall of -8.6 % because non-ferrous metal industry purchases a lot of recycled products as intermediate goods.

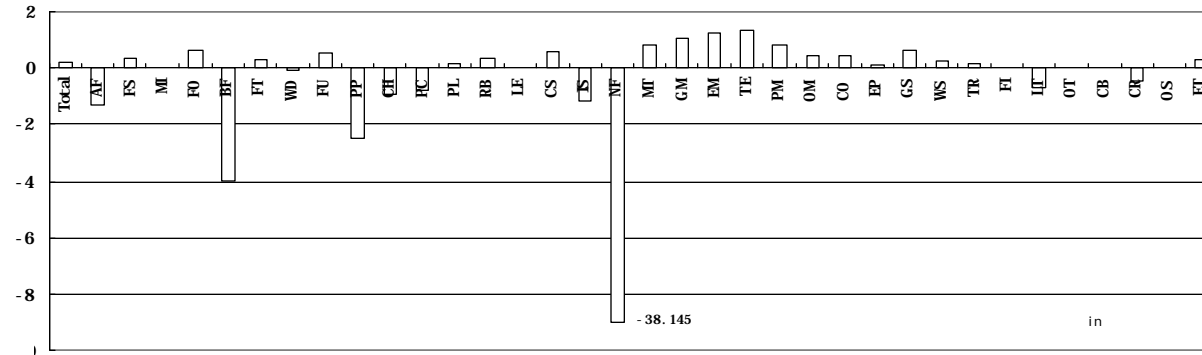
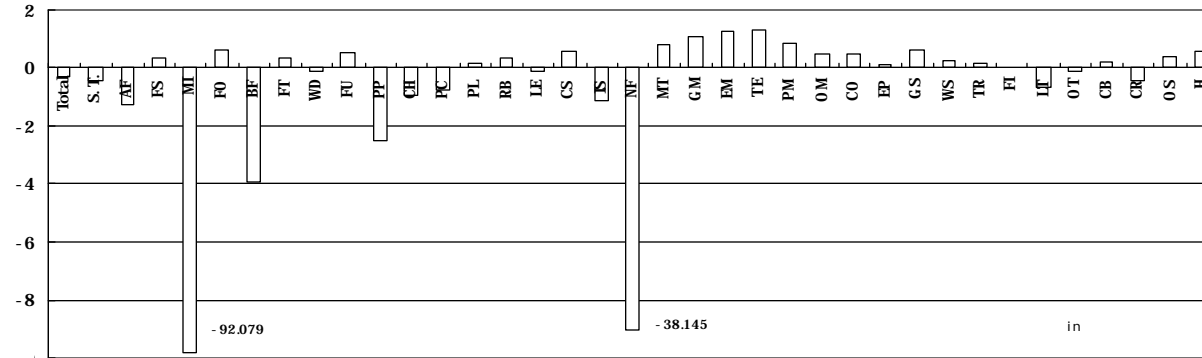
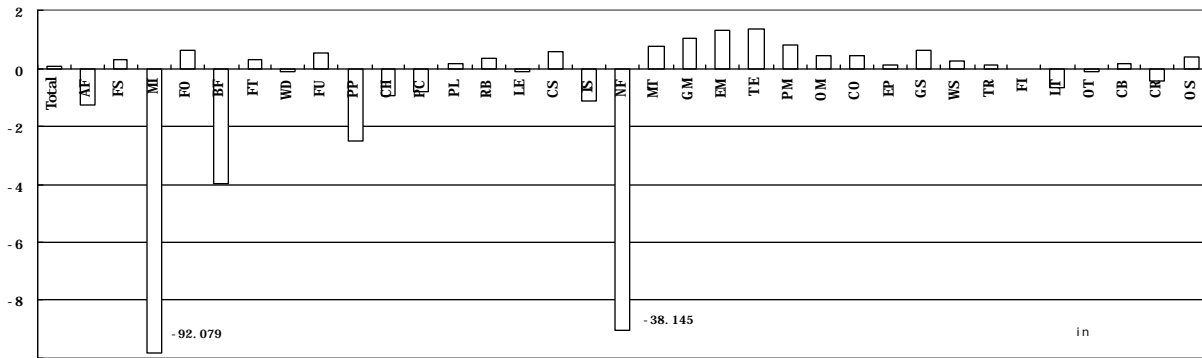
Moreover, prices of material transformation services fall as well due to decreases in prices of intermediate inputs though the falls are not as large as those of virgin commodities.

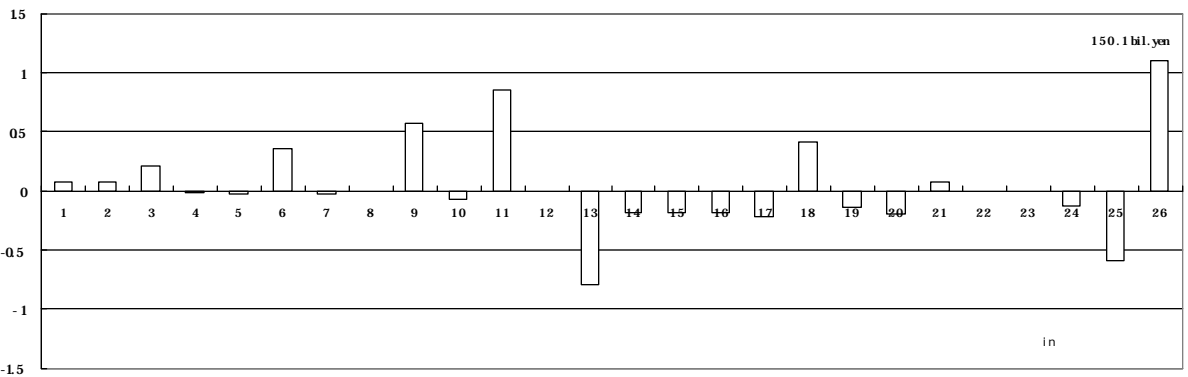
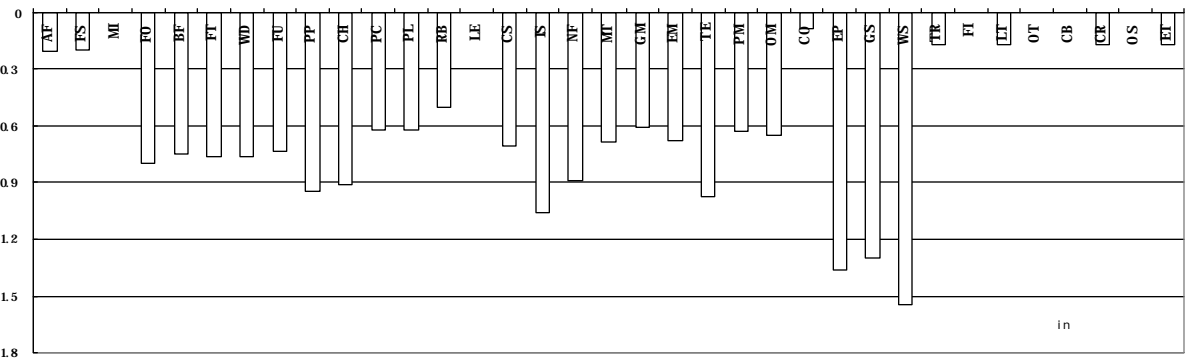
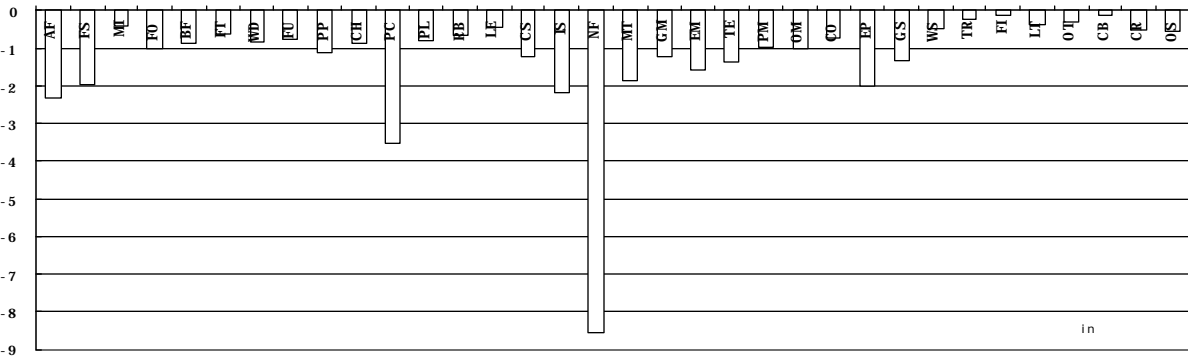
(5) Other variables

Behaviors of other variables are summarized in Figure 7. The prefectural gross product shows a slight decrease because total industrial gross products decreases despite an increase in that of material transformation sectors. Due to a small change in the gross product, household income almost stay at a constant level. However, commodity prices fall yielding an increase in real income with a raise in real household consumption expenditures by 0.6 % .

The capital return rate falls by -0.1 %, but the price index of composite consumption good decreases by -0.6 % showing an increase in the real capital return rate. This results in an increase in household savings (= future consumption). A fall in the price index of composite consumption good is regarded as an increase in the real wage, leading to an increase in labor supply (= a decrease in leisure demand).

In summary, despite a negative effect on household utility due to a reduction in leisure time, increases in present and future consumptions raises household utility leading to a welfare improvement as the equivalent variation shows 150 billion yen.





tors,

4. prefectural gross product, 5. gross product of industries, 6. gross product of material transformation sectors,
7. household full income, 8. household income, 9. household composite consumption, 10. leisure time,
11. household savings, 12. direct taxes, 13. net indirect taxes, 14. government income, 15. government consumption,
16. government current transfers to households, 17. government current transfers to the external sector,
18. government current transfers to the external material transformation sector, 19. government savings, 20. investment,
21. labor demand, 22. capital demand, 23. wage rate, 24. capital return rate, 25. price index of composite consumption,
26. equivalent variation.

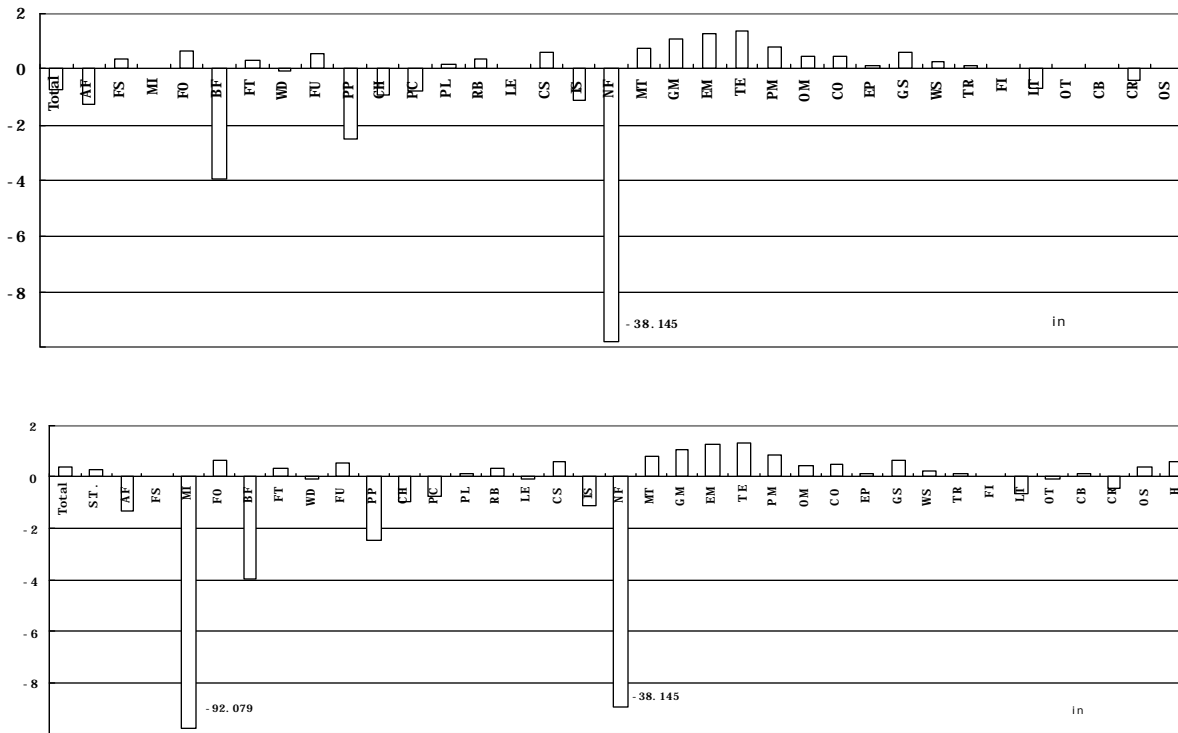


Figure 9 Changes in Wastes Treated by External Material Transformation Sector
 Note: S.T.: subtotal of volumes of industrial wastes treated by ET sector, H : households

7. Concluding Remarks

This study has extended a computable general equilibrium model of waste-economic system previously constructed by the authors to the analysis of internalizing material transformation and recycling activities. The simulation results obtained in the study suggest that promotion of recycling reduces the need for industries to produce raw materials, yielding expansion of the outputs of assembling, processing, and tertiary industries.

Though the operational levels of *IT* sectors associated with raw material producing industries are reduced due to a decrease in production of virgin raw materials, those in assembling and processing industries, and in *ET* sector show a growth.

As we have mentioned in the earlier part, the zero-emission oriented society has recently been referred to very often. This study gives an idea of the type of environmentally related business in a zero-emission oriented society, and nature of impact on the economic structure of our society.

However, in current studies on material circulation, in particular, crucial lack in the availability of data on costs of material transformation or recycling affects the accuracy of our study as well. Improvement of data precision would be the most significant issue in this field.

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References

Capra, F. and Pauli, G. ed. 1995. *Steering Business Toward Sustainability*, United Nations University

- Press, Tokyo.
- Miyata, Y. 1995. A General Equilibrium Analysis of the Waste-Economic System - A CGE-modeling Approach -, *Infrastructure Planning Review*, 12 : 259-270.
- Miyata, Y. 1997. An Intertemporal General Equilibrium Analysis of the Waste-Economic System, *Infrastructure Planning Review*, 14 : 421-432.
- Shoven, J.B. and Whalley, J. 1992. *Applying General Equilibrium*, Cambridge University Press.
- Suzuki, M. ed. 1999, *Constructing Material Circulation Processes Toward a Zero-Emission Society*, Report of the Scientific-Grant-in-Aid of the Ministry of Education, No. 292, the Ministry of Education, the Government of Japan, Tokyo (in Japanese)

Appendix Structure of the Model

(1) Households

Households in Aichi prefecture are assumed to be identical, therefore, aggregate household is considered in the model. Households share an aggregate CES utility function of present commodities which are composite of present consumption and leisure, and future goods, i.e. savings.

Then the present good is divided into composite consumption good and leisure maximizing sub-utility function. Finally, composite consumption good is divided into commodities produced by 34 industries maximizing sub-sub-utility function.

Household full income is defined as the total of full wage income that is obtained if a household supplied its entire labor endowment, post depreciation capital income, current transfers from the government, labor and property incomes from the external sector, and other current transfers. A part of wage and capital incomes of households are transferred to the external sector. Households expense the post-tax-and-current transfers income on present and future commodities.

Let us now explain the household behavior by starting with explanation of the future good. Future goods can be regarded as composite investment goods. Investment is made using produced goods. Then assume the technology employed in investment as *Leontief* type. Thus we have;

$$I = \text{Min} \{I_1/b_1, \dots, I_{34}/b_{34}\} \quad (40)$$

where

I_i : produced goods used in investment I

b_i : parameters in *Leontief* technology combining I and I_i . ($b_i \geq 0$, $\sum_{i=1}^{34} b_i = 1$)

Assuming that investment I is made minimizing investments costs $\sum_{i=1}^{34} p_i I_i$ commodity demands for industries associated with investment I is expressed as $I_i = b_i I$. Denoting the price of investment composite by p_I , $p_I I = \sum_{i=1}^{34} p_i I_i$ is realized. Therefore we obtain the price of capital good as $p_I = \sum_{i=1}^{34} b_i p_i$. p_I can also be regarded as the price of saving p_s .

Now the net capital return associated with one unit of investment is expressed as $(1-ty)(1-k_o)(1-k_r)r$ expected return rate to price of saving good p_s , that is, expected net return rate on savings r_s is written as follows;

$$r_s = (1-ty)(1-k_o)(1-k_r)r / p_s \quad (41)$$

where

ty : direct tax rate on household income

k_o : transfer rate of property income to the external sector

k_r : capital depreciation rate

\dot{a} : ratio of capital stock measured in physical unit to that in service unit

Suppose that the expected net returns finances household future consumption expenditures, and the price of future consumption good H is the price of present consumption good p , the following equation holds.

$$p \cdot H = (1-ty)(1-k_o)(1-k_r)r \cdot S \quad (42)$$

This yields $[p_s p / (1-ty)(1-k_o)(1-k_r)r] H = p \cdot S$ and let the price of future consumption good p_H be

$$p_H = p_s p / (1-ty)(1-k_o)(1-k_r)r \quad (43)$$

then $p_s S = p_H H$ holds.

Applying these specifications of future good and its price, household utility maximization is now described as follows;

$$\underset{G,H}{Max} u(G, H) \equiv \{ \mathbf{a}^{1/v_1} G^{(v_1-1)/v_1} + (1-\mathbf{a})^{1/v_1} H^{(v_1-1)/v_1} \}^{v_1/(v_1-1)} \quad (44)$$

subject to

$$p_G \cdot G + p_H \cdot H = (1-ty)FI - TrHO \quad (45)$$

$$FI \equiv (1-l_o)w \cdot E + LI + (1-k_o)(1-k_r)r \cdot KS + KI + TrGH + TrOH \quad (46)$$

where

\dot{a} : share parameter

v_1 : elasticity of substitution between present and future goods

G : present consumption good

H : future consumption good

p_G : price of present consumption good

p_H : price of future consumption good

FI : household full income

$TrHO$: current transfers from households to the external sector

l_o : transfer rate of labor income to the external sector

E : initial labor endowment of households (= 2.045 times of the initial household labor supply, which is based on actual figures of working time and leisure time in Aichi prefecture.)

LI : labor income transfers from the external sector (exogenous variable)

KS : initial household capital endowment

KI : property income transfers from the external sector (exogenous variable)

$TrGH$: current transfers from the government to households

$TrOH$: current transfers from the external sector to households

Solving this utility maximization problem, present and future consumption demand functions, and saving function are obtained.

$$G = \frac{\mathbf{a}[(1-ty)FI - TrHO]}{p_G^{v_1} \cdot \mathbf{D}} \quad (47)$$

$$H = \frac{(1-\mathbf{a})[(1-ty)FI - TrHO]}{p_H^{v_1} \cdot \mathbf{D}} \quad (48)$$

$$SH = p_H H / p_s \quad (49)$$

$$\mathbf{D} \equiv \mathbf{a} p_G^{1-v_1} + (1-\mathbf{a}) p_H^{1-v_1} \quad (50)$$

Let us now turn to derivation of composite consumption and leisure demands from the present consumption (47). The present consumption G is a composite of composite consumption and leisure time, and G is derived from the following sub-utility maximization problem.

$$\underset{C,F}{\text{Max}} G \equiv \{ \mathbf{b}^{1/v_2} C^{(v_2-1)/v_2} + (1-\mathbf{b})^{1/v_2} F^{(v_2-1)/v_2} \}^{v_2/(v_2-1)} \quad (51)$$

subject to

$$p \cdot C + (1-ty)(1-l_o)w \cdot F = (1-ty)FI - TrHO - SH \quad (52)$$

where

$\hat{\mathbf{a}}$: share parameter

v_2 : elasticity of substitution between composite consumption and leisure time

C : composite consumption

F : leisure time

p : price of composite consumption good

Solving this sub-optimization problem, composite consumption and leisure demands and labor supply are obtained.

$$C = \frac{\mathbf{b}[(1-ty)FI - TrHO - SH]}{p^{v_2} \cdot \mathbf{W}} \quad (53)$$

$$F = \frac{(1-\mathbf{b})[(1-ty)FI - TrHO]}{[(1-ty)(1-l_o)w]^{v_2} \cdot \mathbf{W}} \quad (54)$$

$$LS = E - F \quad (55)$$

$$\mathbf{W} = \mathbf{b} p^{(1-v_2)} + (1-\mathbf{b})[(1-ty)(1-l_o)w]^{(1-v_2)} \quad (56)$$

where

LS : household labor supply

Substituting the composite consumption and leisure demands into equation (51), present consumption demand can be obtained as an indirect sub-utility function yielding the following price of present consumption good.

$$p_G = \{ \mathbf{b} p^{1-v_2} + (1-\mathbf{b})[(1-ty)(1-l_o)w]^{1-v_2} \}^{1/(v_2-1)} \quad (57)$$

Moreover consumption demand for industries' products is derived from maximizing a

Cobb-Douglas sub-sub-utility function given the income and leisure time.

$$\text{Max } \prod_{j=1}^{34} C_j^{g_j} \quad \left(\sum_{j=1}^{34} g_j = 1 \right) \quad (58)$$

subject to

$$\sum_{j=1}^{34} p_j \cdot C_j = (1-ty)Y - TrHO - SH \quad (59)$$

where

C_j : household consumption demand for commodities produced by industry j

p_j : price of commodity j

Y : household income $(=(1-l)w \cdot LS + LI + (1-k)(1-k)r \cdot KS + KI + TrGH + TrOH)$

Then we have consumption demands for industries, and the price index of composite consumption good as follows;

$$C_j = \frac{r_j}{p_j} [(1-ty)Y - TrHO - SH] \quad (j = 1, \dots, 34) \quad (60)$$

$$p = \prod_{j=1}^{34} \left(\frac{p_j}{g_j} \right)^{g_j} \quad (61)$$

(2) The government

The government obtains its revenues from direct and net indirect taxes, and current transfers from the external sector. Then it expends the revenues on government consumption expenditures, current transfers to households, expenditures to the external material transformation sector, and current transfers to the external sector. Balance between receipts and payments are saved. It is assumed that nominal government consumption expenditures, current transfers to households and the external sector are proportional to government income. These specifications are expressed as the following balance of payments.

$$\begin{aligned} \sum_{i=1}^{34} p_i \cdot CG_i + TrGH + WTC + TrGO + SG = ty \cdot Y + \sum_{i=1}^{34} t p_{1i} (w \cdot LD_{1i} + r \cdot KD_{1i}) \\ + \sum_{i=1}^{34} t p_{2i} (w \cdot LD_{2i} + r \cdot KD_{2i}) + TrOG \end{aligned} \quad (62)$$

where

CG_i : government consumption expenditures on commodity i

WTC : government expenditures on the external transformation activity

$TrGO$: current transfers from the government to the external sector

SG : government savings

$TrOG$: current transfers from the external sector to the government

(3) The external sector

The external sector receives its income from Aichi's imports, current transfers from the government, labor and property income transfers. Then it expends its income on Aichi's exports, current transfers to households and the government, labor and property income transfers to Aichi prefecture. Balance between income and expenditures is saved. Here It is also assumed that nominal Aichi's exports are fixed, and real imports are proportional to Aichi's domestic demand. These specification are illustrated as the following balance of payments.

$$\begin{aligned} \sum_{i=1}^{34} p_i \cdot EX_i + TrOH + TrOG + KI + LI + SG = \sum_{i=1}^{34} p_i \cdot EM_i + TrHO + TrGO \\ + KIO + LIO \end{aligned} \quad (63)$$

where

EX_i : Aichi's export of commodity i

EM_i : Aichi's import of commodity i

SO : savings of the external sector (= - prefectural current surplus)

LIO : labor income transfers to the external sector (= $l - w - LS$)

KIO : property income transfers to the external sector (= $k - r - kS$)

(4) Balance of Investment and savings

Savings of households, the government, and the external sector, and capital depreciations determine total investment.

$$\sum_{i=1}^{34} p_i \cdot I_i = SH + SG + SO + \sum_{i=1}^{34} DR_{1i} + \sum_{i=1}^{35} DR_{2i} \quad (64)$$

where

DR_{1i} : capital depreciation in industry i

DR_{2i} : capital depreciation in material transformation activity i