Buettner, Thiess

Conference Paper

Local Capital Income Taxation and Competition for Capital: The Choice of the Tax Rate


Provided in Cooperation with:
European Regional Science Association (ERSA)


This Version is available at:
http://hdl.handle.net/10419/114283

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Local Capital Income Taxation and Competition for Capital: The Choice of the Tax Rate

Thiess Buettner, ZEW, Mannheim, Germany

Mai 1999

Abstract:

This paper discusses how local tax rates of a capital tax are set when communities compete for capital as a mobile factor. In a theoretical model communities provide public inputs financed by a tax on capital income in order to maximize their objective function. It is shown that communities will respond to each others taxing decisions irrespective of the actual weights in the objective functions. However, in the tax equilibrium differences in tax rates are not eliminated if communities differ in size or in the councils’ preferences. These propositions are then related to the empirical distribution and development of the collection rates of the business tax in West Germany’s districts. The results indicate that collection rates are set in response to the fiscal decisions of local neighbors. Yet, competition does not eliminate all tax differences between locations. In particular, tax rates are positively related to communities’ population size even when controlling for density. This conforms with the hypotheses that large jurisdictions are less concerned with a tax policy aimed at attracting mobile capital. In addition federally mandated local welfare expenses are established as a determinant of local tax differences raising concerns about distortions induced by the German federal system.

Keywords: fiscal competition, capital income taxation, public inputs, local taxation, empirical study, spatial econometrics

---

1An earlier version of this paper circulates under the title “Local Business Taxation and Competition for Capital: The Choice of the Tax Rate”
2ZEW, Mannheim, email: buettner@zew.de

I would like to thank H. Buscher, B. Fitzenberger, K. Göggelmann, and P. Winkler for helpful comments and J. Beck for able research assistance.
1 Introduction

The last decade has provided us with ample literature on the consequences of economic integration for fiscal policy. In particular, the nature and consequences of fiscal competition and especially of tax competition were discussed intensively. Whereas most theoretical contributions were dealing with normative issues such as the question whether competition between governments is efficient (see Wellisch, 1998, for an overview), a small but growing number of recent papers deals with the positive issue, how taxes are determined and whether in fact competition can be identified in the actual taxation or expenditure decisions of public authorities (for an overview, see Devereux, 1995, and Schulze/Ur sprung, 1999). Yet, national tax systems show such a huge degree of complexity that competition effects are hard to identify by means of international comparison. But, most federations allow for some fiscal autonomy at the local level and thus offer a rich experience with provision of public goods and taxation in the presence of mobility. Furthermore, the autonomy at the local level is often regulated, such that local differences in taxation are restricted to differences in a few parameters.

There already exist studies showing that local fiscal policy in the U.S. is involved in local tax competition or mimicking of neighbors’ tax burdens (Ladd, 1992, see also Brett and Pinske, 1997). There is also evidence for spatial effects in expenditure decisions (Case et al., 1993, and Seitz, 1994) including development incentives (McHone, 1987). Inman (1989) provides evidence that the proximity of opportunity locations is related with reduced tax rates. Furthermore, there is evidence that voters and thus politicians compare policies at neighboring locations (see Besley and Case, 1995. See also the analysis by Ashworth and Heyndels, 1997, using data for Belgium). This suggests that at the local level interjurisdictional competition in fact matters, and the present paper will provide empirical evidence for the case of Germany.

Despite evidence of tax competition observed local tax rates display marked differences between locations. In particular, large cities tend to set relatively high tax rates in the US (e.g., Hoyt, 1992). Besides more conventional explanations such as urban characteristics affecting demand or costs of public goods, Epple and Zelenitz (1981) and, more recently, Hoyt (1992) argued that the market power of larger jurisdictions might explain the urban tax premium. This hypothesis is of particular importance as it sheds doubts on the efficiency of mobile capital to constrain the taxing power of governments. Therefore, the analysis in this paper takes into account asymmetries...
between jurisdictions and also presents empirical evidence on the long-run distribution of tax rates.

In contrast to Bucovetsky (1991) and Wilson (1991) the theoretical model presented in this paper discusses capital income taxation between asymmetric jurisdictions when the tax revenue is used to provide productive government expenditures. This is of importance because especially local business taxation may contain elements of benefit taxation, which has strong implications for location. In difference to Seitz (1994) which compares the tax rate on mobile capital and the provision of public inputs as instruments of interjurisdictional competition, we focus on the local tax rate. Since the consequences of tax competition are dependent on the behavioral assumption of public authorities (Hange / Wellisch, 1998), the analysis allows for different government objectives including income maximization of residents and revenue maximization, similar to Edwards / Keen (1996). After deriving central empirical implications, evidence is provided by an empirical analysis of the collection rates of the local business tax (Gewerbesteuer) in Germany. This case is of particular interest since it is the most important element of decentral taxing autonomy in Germany and the collection rates show considerable cross-sectional variation. Focusing on the within-distribution and using spatial econometric techniques it is shown that the taxing decisions of neighbors are interdependent, which conforms with positively sloped response functions in the tax space. Yet, when focusing on the between-distribution it turns out that competition does not eliminate all tax differentials between districts. Instead, tax rates are found to be positively related to the population size of the communities and the share of local welfare recipients.

The paper is organized as follows. The next section derives central empirical implications from a model of local setting of the rates of capital income taxation. The empirical investigation starts with an overview of the extent and evolution of local tax differences in Germany before the differences are analyzed with respect to their determinants. Finally, the results are summarized and a short conclusion is given.

2 A Model of Local Tax Setting

In order to gain insights into the question how communities choose their rates of business taxation, this section presents a theoretical model. Local business taxation is analyzed in a context where public spending has an impact on productivity (cf. Seitz, 1994). Furthermore, the model employs a generalized objective function similar to Edwards and Keen (1996) which
allows for maximization of revenue as well as for maximization of residents’ utility. The following section shows the basic setup of the model. Then a single community’s choice of the tax rate is explained analytically, before the existence of the interregional tax equilibrium as well as its properties are discussed.

2.1 The Setup

Suppose the local council at location \( i \) seeks to maximize a simple objective function \( V_i (E_i, U_i) \). Both, the utility of local residents \( U_i \) and the total level of spending on infrastructure \( E_i \) enter as arguments. The utility of residents is assumed to be a function of the income of local factors \( Y_i^L \). This is justified by the stronger interest of owners of local, immobile factors in influencing local policies (see Wellisch, 1998). Thus, local councils aim at maximizing \( V_i (E_i, U (Y_i^L)) \). As argued by Edwards and Keen (1996) this specification encompasses alternative assumptions on the policy makers’ preferences.\(^1\) In case that the community’s council maximizes the residents’ utility the objective function degenerates to \( V_i (U (Y_i^L)) \). In the more general case where the level of spending \( E_i \) and the utility of residents \( U (Y_i^L) \) both have significant positive weights in the objective function, the council is biased towards public spending. In the extreme case where income of residents plays no role at all (\( V_i = V_i (E_i) \)) the council maximizes its budget. When discussing tax competition Edwards and Keen (1996) as well as Wellisch (1998) employ this function but concentrate on public goods, whereas the current discussion focuses on public inputs.

Output at location \( i \) is described by the following function:

\[
Y_i = A (E_i, K_i) F (K_i, L_i) \quad \frac{\partial A}{\partial E_i} > 0, \quad \frac{\partial A}{\partial K_i} < 0
\]

\[
= A (E_i, L_i k_i) L_i f (k_i) ,
\]

where \( F \) is a neoclassical production function with labor \( L_i \) and capital \( K_i \) as inputs and the output price is set to unity. The second equation is simply the expression in intensity form, where \( f \) is the labor productivity and \( k_i \) is the capital intensity. \( A (E_i, K_i) \) is a shift–term capturing the location specific total factor–productivity. It is formulated as a function of local public

\(^1\)In difference to Edwards and Keen (1996) no distinction is made between “wasteful” expenditures and other local public expenditures.
expenditures $E_i$ and the total usage of those public inputs captured by the amount of the mobile factor, capital, installed locally. The positive impact of public expenditures is formulated analogous to the treatment of external scale economies as used for instance by Helpman (1984) and Henderson (1985). Following Matsumoto (1998) this specification may be referred to as a factor augmenting public input. The ceteris-paribus effect of the total stock of capital on total factor productivity is assumed to be zero or negative in order to allow for some rivalry in the usage of public inputs. Only if expenditures are purely public inputs the impact is zero. The other extreme is the case where only the intensity of expenditures relative to the amount of installed capital has an effect on productivity. Then, the goods locally supplied are rival such as private goods. Though, as was emphasized recently by Sinn (1997) an ideal public sector will focus on the case of inputs with only some degree of rivalry.

With maximization of profits and parametric treatment of wages and public expenditures the local labor income $Y_i^L$ can be derived as:

$$ Y_i^L = \left[ f \left( k_i \right) - f' \left( k_i \right) k_i \right] A \left( E_i, k_i L_i \right) L_i, $$

where $f' \left( k_i \right)$ denotes the derivative of $f \left( k_i \right)$.

The level of public spending is determined from the government budget constraint. In an atemporal context without debt and when neglecting grants spending equals income from taxation:

$$ E_i = t_i Y_i^C, $$

where $t_i$ denotes the tax rate on capital income and $Y_i^C \equiv Y - Y_i^L$ is the pretax return to capital:

$$ Y_i^C = A \left( E_i, k_i L_i \right) f' \left( k_i \right) k_i L_i. $$

From equations (1) and (2) it can be seen that higher taxes have a positive impact on the income of local labor because higher taxes increase public spending and thus increase labor productivity. Taxes on capital income and the level of local spending will also affect the interregional allocation of mobile capital and thus affect the local capital intensity $k_i$. Thus, we need to determine the equilibrium of the interregional capital allocation. It is characterized by an equalization of the after-tax return to capital across regions. For simplicity, let us assume that there is only one other region, indexed by $j$. Then, the following condition will hold in an equilibrium with two communities:

$$ \left( 1 - t_i \right) A \left( E_i, k_i L_i \right) f' \left( k_i \right) \overset{!}{=} \left( 1 - t_j \right) A \left( E_j, k_j L_j \right) f' \left( k_j \right), $$

For the pretax return to capital.
where the left (right) hand side gives the after-tax rate of return to capital in region $i$ ($j$). As the focus is on the distribution of capital between the two locations total capital can be held fixed. When the total supply of labor is set equal to unity, the following equation shows the relation between the regional capital intensities.

$$ k = L_i k_i + L_j k_j, \quad L_i + L_j = 1. \tag{5} $$

### 2.2 The Choice of the Tax Rate

Given the tax rates $(t_i, t_j)$ and the interregional allocation of labor $(L_i, L_j)$ the five equations (1)-(5) and the three equations obtained by replacing $i$ with $j$ in equations (1)-(3) determine the capital intensities $(k_i, k_j)$, the levels of expenditures $(E_i, E_j)$, and the labor and capital income at the two locations $(Y_i^L, Y_j^L, Y_i^C, Y_j^C)$. Thus, when the council of the local community shows Nash-behavior and treats the tax rate of the other community as given, it is facing an optimization problem of the following kind:

$$ \max_{t_i} V_i \left( E_i \left( Y_i^C( t_i, t_j) \right), \ U \left( Y_i^L( t_i, t_j) \right) \right), $$

where the first-order condition is:

$$ \left( \frac{\partial V_i}{\partial E_i} \right) \frac{\partial E_i}{\partial t_i} = - \left( \frac{\partial V_i}{\partial U} \frac{\partial Y_i^L}{\partial t_i} \right) - \left( \frac{\partial V_i}{\partial E_i} \frac{\partial E_i}{\partial Y_i^C} \right) \frac{\partial Y_i^C}{\partial t_i}. \tag{6} $$

The left hand side represents the council’s gain from higher tax revenues due to a higher tax rate. The right hand side gives the loss from the income effects of a higher tax rate: the first is the direct income effect on the household’s utility, the second is the income effect on the budget. In the optimum gain and loss just match, implying that with the tax rate at its optimum either one or both income effects $(\frac{\partial Y_i^L}{\partial t_i}, \frac{\partial Y_i^C}{\partial t_i})$ are negative or the optimum is a corner solution with $t_i = 1$.

To simplify matters, a log-linear production function is assumed. Then, the two income effects are equal and the first-order condition becomes:

$$ - \frac{\partial \log Y_i^L}{\partial \log t_i} = \nu_i \tag{7} $$

$$ \nu_i \equiv \frac{\epsilon_i^{V,E}}{\epsilon_i^{V,U} \epsilon_i^{V,Y} + \epsilon_i^{V,E}}, \quad 0 \leq \nu_i \leq 1, $$
where $\epsilon_i^{V,E}$, $\epsilon_i^{V,U}$ are the elasticities of the council's objective function with respect to spending and residents' utility, and $\epsilon_i^{U,Y}$ is the elasticity of residents' utility with respect to income. Equation (7) states that the negative of the elasticity of the local factor's income is equated to a certain parameter $\nu_i$, which is capturing the preferences in the local council. $\nu_i$ is larger the smaller the weight of the income of the local factor. If the local council does not at all care for the local factor income, $\nu_i$ approaches unity. Then, the optimum tax maximizes the tax revenue. On the other hand, if the council is only interested in raising the income of the local factor, $\nu_i$ is zero, and local income is maximized.

Graphically, an interior solution is depicted in Figure 1. With public spending enhancing factor productivity, local income ($Y_i^L$) first increases with the level of taxation and then decreases. This gives the solid curve, representing the constraint to the optimization problem. The dashed curve depicts an indifference curve between the tax rate and local income, as obtained from inserting equation (2) into the objective function. In the figure the level of utility is chosen such that this curve tangents the solid line. If $\nu_i$, the elasticity of the indifference curve, equals zero, the indifference curve is flat,
whereas with higher values of $\nu_i$ the tangent point lies to the right of the maximum of the solid line.\(^2\)

As it is difficult to present an explicit solution for the optimum tax rate we need to establish the existence of a solution. For simplicity, in the following a log-linear formulation of the total factor productivity term is assumed:

$$A (E_i, k_i L_i) = E_i^\beta (k_i L_i)^{-\gamma}, \quad 0 \leq \gamma \leq \beta. \quad (8)$$

$\beta$ determines the productivity impact of public spending and $\gamma$ determines the degree of rivalry in the use of the public inputs. Equation (8) allows to isolate public expenditures from equations (2) and (3), yielding:

$$E_i = \left[ t_i f'(k_i) (k_i L_i)^{1-\gamma} \right]^{\left(\frac{1}{1-\gamma}\right)}. \quad (9)$$

Given the log-linearity of the production function the elasticity of local income with respect to the tax rate is a linear function of the perceived elasticity of local capital supply with respect to the local tax rate:

$$\frac{\partial \log Y_i}{\partial \log t_i} = \frac{\beta}{1-\beta} + \frac{\alpha - \gamma}{1-\beta} \left( \frac{\partial \log k_i}{\partial \log t_i} \right), \quad \alpha > \gamma, \quad (10)$$

where $\alpha$ is the capital elasticity of the production function and thus the pre-tax share of capital. It is reasonable to require that $\alpha$ is larger than $\gamma$, since only then a higher capital intensity translates into higher labor productivity and there is an incentive for the local council to attract capital even when it cares only for the residents’ utility. When considering the first-order condition for the optimal tax rate (7) and the definition of the income elasticity (10) it turns out that since $\nu_i$ is assumed constant an equilibrium will exist, if the elasticity of the local capital intensity to the local tax rate $\left( \frac{\partial \log k_i}{\partial \log t_i} \right)$ is non-negative for low tax rates, but approaches minus infinity for high tax rates. As this is true for $(1 - (\alpha - \gamma) > \beta)$ the following lemma holds:

**Lemma:** If the productivity of public spending is restricted $(1 - (\alpha - \gamma) > \beta)$, there is a unique tax rate which fulfills the council’s optimality condition (7).

**Proof:**

In order to proof this lemma, we need to inspect the tax elasticity of capital supply:

$$\frac{\partial \log k_i}{\partial \log t_i} = \frac{\frac{\beta}{1-\beta} - \frac{t_i}{1-t_i}}{\frac{1-(\alpha-\gamma)-\beta}{1-\beta} \left( 1 + \frac{k_i}{k_j L_j} \right)}, \quad (11)$$

\(^2\)Note that the value of $\nu_i$ is restricted to the interval $[0, 1]$, thereby preventing tangency points in the nonconvex area of the constraint.
Since \( \left( \frac{t_i}{1-t_i} \right) \) increases from zero to infinity as \( t_i \) grows from zero to unity, the numerator is positive for low tax rates. At \( t_i = \beta \) it becomes zero, for higher tax rates \( (t_i > \beta) \) it is negative, and when the tax rate approaches unity \( (t_i = 1) \) the numerator approaches minus infinity. If the productivity of public spending is not too large \((1 - (\alpha - \gamma) > \beta)\), the denominator is positive. With a given stock of capital in the other region, the ratio of capital intensities in the denominator is decreasing when the tax rate rises and approaches zero for \( t_i = 1 \). Consequently the tax elasticity of capital supply increases from zero to minus infinity as \( t_i \) increases from zero to unity. Thus, a solution of the first-order condition exists, where \( t_i > \beta \). Inspection of equation (11) also shows that the elasticity is strictly increasing in absolute size with \( t_i \) since \( \frac{t_i}{1-t_i} \) increases and the ratio of capital installed at location \( i \) relative to \( j \) decreases. Therefore, the elasticity of capital supply with respect to the tax rate changes monotonically with the tax rate for \( t_i > \beta \), and the solution is unique. 

\textit{End of Proof.}

If the productivity of spending is large \((1 - (\alpha - \gamma) < \beta)\) the denominator of the elasticity of capital supply is negative. Then, above a certain tax rate, tax increases attract more and more capital such that a tax rate of unity becomes a global maximum. In order to rule out this perverse case, the productivity of spending needs to be restricted \(1 - (\alpha - \gamma) > \beta\). Then, the diminishing returns caused by holding the local factor constant outweigh the returns from public spending, and the interregional allocation of the local factor predetermines the locational equilibrium. In order to obtain a determinate locational equilibrium a similar condition needs to hold in the context of agglomeration economies, see Henderson (1985) and Buettner (1999a).

An equation for the optimum tax rate can be found by inserting equations (10) and (11) into the optimum condition (7):

\[
t_i = \frac{\beta}{1-\beta} + \left( \nu_i + \frac{\beta}{1-\beta} \right) \frac{1-|\alpha-\gamma|-\beta}{\alpha-\gamma} \left( 1 + \frac{k_i L_i}{k_j L_j} \right) \bigg/ \left( 1 + \frac{\beta}{1-\beta} + \left( \nu_i + \frac{\beta}{1-\beta} \right) \frac{1-|\alpha-\gamma|-\beta}{\alpha-\gamma} \left( 1 + \frac{k_i L_i}{k_j L_j} \right) \right),
\]

Note, that the expression employs the ratio of the local stocks of capital at the right hand side which are endogenous to local taxation.

From equation (4) the ratio of capital at the two locations can easily be
solved in the log-linear setting:

\[
\frac{k_i L_i}{k_j L_j} = \left( \frac{1 - t_i}{1 - t_j} \right)^{1-\beta} \left( \frac{t_i}{t_j} \right)^{\beta} \left( \frac{L_i}{L_j} \right)^{1-\alpha} \frac{1}{1-\alpha-\gamma-\beta}. \tag{13}
\]

Accordingly, the ratio of capital installed at the two locations increases with the ratio of labor supply and is also determined by the tax rates.

### 2.3 Equilibrium in Tax Competition

In the previous section it was shown that with a log-linear production function and with constraints on the productivity of public spending there is a unique solution to the council’s optimization problem. Under the Nash assumption about the other region’s tax rate the two equations (12) and (13) determine region \(i\)’s tax rate given the tax rate set by region \(j\), i.e. they determine a response function. Actually, the response function is positively sloped, such that the following proposition holds:

**Proposition 1 (Tax Competition):** If the tax rate on capital earnings is reduced (increased) elsewhere, the local community will also set a lower (higher) tax rate.

**Proof:**

Total differentiation of equation (12) gives:

\[
\dot{t}_i = \mu_i \left\{ \dot{k}_i - \dot{k}_j + \dot{L}_i - \dot{L}_j \right\}, \tag{14}
\]

where

\[
\mu_i = \left[ \frac{\left(1 - t_i\right) \left(\nu_i + \frac{\beta}{1-\beta}\right) \left(\frac{1-(\alpha-\gamma)-\beta}{\alpha-\gamma}\right) \left(\frac{k_i L_i}{k_j L_j}\right)}{\left(\nu_i + \frac{\beta}{1-\beta}\right) \left(\frac{1-(\alpha-\gamma)-\beta}{\alpha-\gamma}\right) \left(1 + \frac{k_i L_i}{k_j L_j}\right)} \right].
\]
The hat denotes relative changes. By total differentiation of equation (13) the relative change in the capital ratio can be derived:

\[
\{ \hat{k}_i - \hat{k}_j + \hat{L}_i - \hat{L}_j \} = \left( \frac{1 - \alpha}{1 - (\alpha - \gamma) - \beta} \right) \{ \hat{L}_i - \hat{L}_j \} - \left( \frac{1 - \beta}{1 - (\alpha - \gamma) - \beta} \right) \left[ \frac{t_i}{1 - t_i} - \frac{\beta}{1 - \beta} \right] \hat{t}_i + \left( \frac{1 - \beta}{1 - (\alpha - \gamma) - \beta} \right) \left[ \frac{t_j}{1 - t_j} - \frac{\beta}{1 - \beta} \right] \hat{t}_j
\]

(15)

Because the tax rate in both regions is set above \( \beta \) the two terms in squared brackets on the right hand side are positive. Therefore, a rising tax rate in region \( j \) implies a relative increase in region \( i \)’s capital. Thus, the other community’s fiscal policy exerts an external effect on the considered community (cf. Wildasin, 1994) with the consequence that the tax rates of the communities are interdependent. When holding constant the regional labor allocation \( (\hat{L}_i - \hat{L}_j = 0) \) the gradient of the response function can be found from equations (14) and (15):

\[
\frac{dt_i}{dt_j} = \frac{\mu_i \left( \frac{1 - \beta}{1 - (\alpha - \gamma) - \beta} \right) \left[ \frac{t_j}{1 - t_j} - \frac{\beta}{1 - \beta} \right] t_i}{1 + \mu_i \left( \frac{1 - \beta}{1 - (\alpha - \gamma) - \beta} \right) \left[ \frac{t_i}{1 - t_i} - \frac{\beta}{1 - \beta} \right] t_i},
\]

(16)

which is positive for \( t_i, t_j > \beta \).

*End of Proof.*

Although it was already shown that for each location there is a unique best response to the tax rate of the other location, a Nash-equilibrium in tax competition might not exist. As it is defined by a pair of tax rates \( t_i^*, t_j^* \) which are the best responses to each other, we need to ensure that the two response functions intersect for tax rates \( t_i, t_j > \beta \). A characteristic numerical solution is depicted in Figure 2. Here, the two response lines intersect once in the interior of the interval \([\beta, 1]\) at point P. Additionally, the responses coincide for tax rates equal to unity at point Q. The latter equilibrium is unstable, as small deviations from a tax rate of unity might be followed by an iterative process of tax responses until the interior equilibrium is reached.\(^3\)

\(^3\)A more general way to rule out an equilibrium with \( t_i = t_j = 1 \) is to remove the simple assumption of completely inelastic capital supply at the national level. As long as capital is supplied with less than infinite elasticity at the national level, the general properties of the model remain unchanged, as there is still an effect of the local market share on the perceived elasticity of capital supply. However, the equilibrium with \( t_i, t_j = 1 \) would imply a lower pay off in terms of both councils’ utility.
Analytically, the existence of an interior equilibrium can simply be shown for the symmetric case, where the councils at the two locations have the same preferences $\nu_i = \nu_j$ and the locations are of equal size $L_i = L_j$. Then, according to equations (12) and (13) both optimum taxes coincide. Furthermore, the symmetric equilibrium is stable, since from equation (16) the gradient of the response function is less than unity at $t_j = t_i$. If there are differences either in the endowment with the immobile factor ($L_i \neq L_j$) or in the councils’ preferences ($\nu_i \neq \nu_j$) the response functions are shifted.

The impact of the size of the communities on the position of the response function reflects the fact that the elasticity of capital supply depends on the size of the local community relative to the country (cf. Bucovetsky, 1991, and Wilson, 1991). Thus, the larger the share of capital installed locally, the weaker are adverse income effects and thus the higher the tax rate is set. This leads to the following proposition (cf. Hoyt, 1992, Bucovetsky, 1991, and Wilson, 1991):
Proposition 2 (Size of Communities): If local communities differ in size, large communities will set higher tax rates on the earnings of mobile capital.

Proof:
Solving the equations (14), (15), and their counterparts for country \( j \) for the relative tax change at location \( i \) yields:

\[
\hat{t}_i = \frac{\hat{\mu}_i \left( \frac{1-\alpha}{1-(\alpha-\gamma)(\alpha-\beta)} \right)}{1 + \hat{\mu}_i \left( \frac{1-\beta}{1-(\alpha-\gamma)(\alpha-\beta)} \right) \left[ \frac{\hat{t}_i}{1-\hat{t}_i} - \frac{\beta}{1-\beta} \right]} \left\{ \hat{L}_i - \hat{L}_j \right\}, \tag{17}
\]

where

\[
\hat{\mu}_i = \mu_i \left[ 1 - \frac{\mu_j \left( \frac{1-\beta}{1-(\alpha-\gamma)(\alpha-\beta)} \right) \left[ \frac{t_j}{1-t_j} - \frac{\beta}{1-\beta} \right]}{1 + \mu_j \left( \frac{1-\beta}{1-(\alpha-\gamma)(\alpha-\beta)} \right) \left[ \frac{t_j}{1-t_j} - \frac{\beta}{1-\beta} \right]} \right].
\]

As \( \hat{\mu}_i \) is positive for \( t_j > \beta \), also \( \hat{t}_i \) is positive for \( \hat{L}_i > \hat{L}_j \).

End of Proof.

A second deviation from the symmetric case arises from differences in the councils’ preferences. If, for instance, council \( i \) puts less emphasis on residents income, \( \nu_i \) is higher than \( \nu_j \). Then, it can be seen from equation (12) that the optimum tax rate at location \( i \) will be higher \( (t_i > t_j) \), which leads to the following proposition.

Proposition 3 (Preferences of Communities): If local communities differ in preferences, communities putting less weight on residents’ income will set higher tax rates on the earnings of mobile capital.

Proof:
Holding constant the tax rate at the other region \( (\hat{t}_j = 0) \) and the labor allocation it follows from total differentiation of equations (12) and (13):

\[
\hat{t}_i = \left( \nu_i \left( \frac{1-(\alpha-\gamma)-\beta}{1-\beta} \right) \left[ \frac{t_i}{1-t_i} - \frac{\beta}{1-\beta} \right] \left( \frac{1-t_i}{t_i} \right)^2 \right) \hat{\nu}_i, \tag{18}
\]

which is positive for \( t_i, t_j > \beta \), indicating that the response function of the community shifts upwards with an increase in \( \nu_i \).

End of Proof.

It was already pointed out that the specification of total factor-productivity bears some resemblance to the case of agglomeration economies. In fact,
by assuming constant returns to scale in the factor inputs the productivity effect of public inputs introduces a non-convexity which strongly alters the properties of the interregional factor allocation (cf. Richter, 1994). The consequence is that, as long as public inputs display some degree of nonrivalry \((\beta - \gamma > 0)\), the value of output at the aggregate level is not maximized. However, for an inefficiency due to market-size effects on the capital markets or to differences in preferences the non-convexity issue does not matter: even with complete rivalry \((\beta = \gamma)\) or no productivity effects at all \((\beta = \gamma = 0)\) deviations from strong symmetry in size and preferences cause a situation where reallocation of the mobile factor would increase total output.

The theoretical model outlined above makes use of strong simplifying assumptions. First, the locational equilibrium assumes a determinate spatial pattern of production and thus neglects many important difficulties of the spatial economy, such as multiple equilibria, as emphasized by Krugman (1991), or strong productivity effects of public inputs (see Martin and Rogers, 1995). Also, capital is assumed to be immediately relocatable (see Koch and Schulze, 1998, for a discussion). Beside these more technical points it is important to note that the analysis assumes that the local supply of local public expenditures is fully determined by the tax rate, omitting aspects of intercommunity benefit spillovers and grants from higher level governments. And, finally, there is only one good and thus one production function, which leaves no room for Tiebout-type explanations of business tax differences, where heterogeneous producers separate themselves into spatial clubs with similar public input demands. Leaving the analysis of more complex cases for future research, the present paper poses the more decent question, whether the implications already drawn from the simple model hold empirically.

3 Empirical Evidence

The most important element of subnational taxing autonomy in Germany is the business tax (Gewerbesteuer). Besides locally varying collection rates the terms and conditions of the business tax are the same for all communities. The collection rates set by the communities define the factor by which base tax rates of about 5% on profits and 0.2% on the value of capital are increased in order to compute the local tax.\(^4\) Due to difficulties of obtaining data on fiscal variables at community level, the following empirical analysis

\(^4\)The implied tax rate on profits is lower, as tax payments are deductible. Furthermore, there are tax exemptions.
focuses on the local business tax rates at district level. The investigation uses the complete set of collection rates in the 327 districts in West Germany in the years 1980–1996. The majority of districts consists of several local communities. In case of districts with several communities, the reported collection rates are weighted averages of the communities’ collection rates.\(^5\)

Figure 3 shows the distribution of the collection rates. The lines in the figure show various quantiles of the distribution. The solid line depicts the median of the collection rates across West Germany’s districts. From the 25 % and the 75 % quantiles it can be seen that throughout the years, a quarter of

\(^5\)The weights are the communities’ shares of the tax bases. For the details see series 10.1 (“Finanzen und Steuern – Realsteuervergleich”) of the German federal statistical office (Statistisches Bundesamt).
districts contains communities which tax firms with an at least 22 % higher rate than those at the median district. On the other hand, a quarter of districts tax them with at least 13 % lower rates. According to Figure 3 all location measures show a positive trend. The distribution of tax rates seems to be quite stable. For instance, it is evident from Figure 3 that there is always a wedge between the median of tax rates at urban and rural districts of about 60 percentage points. Of course, one would like to know whether the theory provides any testable explanations for this urban vs. rural district differential. Yet, it is useful to postpone further analysis of this issue until having dealt with another important element in the theoretical discussion, namely spatial effects in tax-setting.

3.1 Spatial Effects in Local Tax Rates

According to Proposition 1 competition effects matter in the determination of business tax collection rates. In order to test for those effects the correlation between the collection rate in a considered district and its competing jurisdictions should be analysed. In the above theoretical discussion only the case of two communities was modelled. This would suggest to determine simply the correlation between the two district’s collection rate. But with multiple regions we may impose a structure on the districts determining which are more likely to engage in an interdistrict tax competition. Assuming spatial transaction costs in a broad sense, including also information costs, fiscal competition will be particularly strong with communities in the neighborhood, whereas more distant locations constitute a less relevant location option for residents and investors. Additionally, the perceived political costs or benefits are higher for tax differentials with the local neighborhood (see Ashworth / Heyndels, 1997, and Besley and Case, 1995). Therefore, we may distinguish a general competition between the district and all other districts and a local competition between geographic neighbors. This suggest, to use the collection rate in the local neighborhood and at the national level as determinants of local collection rates.\(^6\) Due to the differences in the number of communities involved the simple pooling of the data of districts is inadequate. More generally, since only a very limited set of local characteristics can be entered in the regression given the data limitations, we should expect local fixed effects to be important and focus on the within-distribution. As prior analysis has revealed the presence of autoregression pointing to

\(^6\)In his study of tax policy of cities Seitz (1994) also uses the similarity of communities’ sectoral employment composition in order to identify competitors.
sluggish adjustment of collection rates to changes in the local conditions, an error-correction framework is adequate where adjustment and long-run solution are explicitly distinguished.

Consider the results presented in Table 1, where tax rate changes are regressed on previous as well as neighbors' tax rate changes and levels. As some of the districts in the aftermath of unification were exposed to the neighborhood of East German districts, a dummy variable for districts close to the intra-German border in the post-unification period (1992-1996) is added. (Various other specifications including the border dummy for 1991 and for the period (1991-1996) did not show better fit.) The population size is suppressed, as it is subsumed to average regional differences which are captured by the regional fixed effects. The average collection rate for the nation as a whole is suppressed, since the regression employs time dummies in order to capture common national effects, as for instance fiscal consolidation after unification.

In column (1) the results from a regression including fixed regional and time effects are presented. According to the Wald statistic at the bottom of the table the regional fixed effects are highly significant. The current as well as the lagged change in the neighbors' tax rate shows significant effects indicating that an increase in the collection rate in the neighborhood of a community allows one to predict an increase in the considered district as well. As this is in line with Proposition 2, it is an indication of tax competition between neighboring communities. Also, the lagged levels of the own as well as the neighbors' collection rates are significant.

It was emphasized by Cliff and Ord (1973) and Anselin (1988) that the introduction of a spatial lag introduces a simultaneity bias and in order to estimate the simultaneous spatial model maximum-likelihood (ML) estimation is appropriate under standard assumptions. The estimates from the application of a ML procedure are reported in column (2). Although the size of the coefficients of the neighbors' tax changes is much lower, the ef-

---

7The present panel data setting is nonstandard, since the fixed effects lead to the incidental parameter problem as the number of parameters rises with the cross-section dimension. In case of OLS, a conditional likelihood based on the transformed variables exists, which gives consistent estimates (cf. Chamberlin, 1980). In the spatial model ML estimation is also consistent if the coefficient of spatial correlation is close to zero, since the difference between the likelihood of the model with a spatial lag and the likelihood of the linear regression model is the value of the determinant $|I - \rho W|$, which is unity if $\rho = 0$ (cf. Anselin, 1988) ($I$ is an identity matrix, $W$ is a spatial weight matrix). In the present estimation the unconditional ML estimator is applied, but the variance-covariance matrix is corrected for the degrees of freedom lost in the fixed effects.
Table 1: Collection Rate Changes 1981-1996

<table>
<thead>
<tr>
<th>Observations</th>
<th>4905</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. variable</td>
<td>Collection Rate Change</td>
</tr>
<tr>
<td>Method</td>
<td>OLS</td>
</tr>
<tr>
<td>Hetsced. Rob. S.E.</td>
<td>yes</td>
</tr>
<tr>
<td>Specific Trends</td>
<td>(1)</td>
</tr>
<tr>
<td>Regressors</td>
<td></td>
</tr>
<tr>
<td>Own tax rate</td>
<td>-.126 ***</td>
</tr>
<tr>
<td>Change, lagged</td>
<td>(.052)</td>
</tr>
<tr>
<td>Neighb.' tax rate change</td>
<td>.180 ***</td>
</tr>
<tr>
<td>Neighb.' tax rate change, lagged</td>
<td>(.045)</td>
</tr>
<tr>
<td>Own tax rate, last year</td>
<td>-.335 ***</td>
</tr>
<tr>
<td>Neighb.' tax rate, last year</td>
<td>(.031)</td>
</tr>
<tr>
<td>Post unification dummy</td>
<td>.123 ***</td>
</tr>
<tr>
<td>Wald statistics regeff (P-val.)</td>
<td>.004 ***</td>
</tr>
<tr>
<td>spec.trd. (P-val.)</td>
<td></td>
</tr>
<tr>
<td>bias (P-val.)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. Column (1) shows OLS estimates, where standard errors are heteroscedasticity robust following White (1980). Columns (2) to (5) report results from Maximum Likelihood estimation of the simultaneous spatial model. Columns (2) and (4) report analytic standard errors, whereas columns (3) and (5) report standard errors from a spatial block bootstrap estimator based on 5,000 resamples. Significant coefficients are marked with one, two, or three stars for levels of 10%, 5%, and 1%. All estimations include fixed regional and time effects. Columns (4) and (5) employ district-type specific time effects.
ffects of current and lagged changes in the neighbors’ tax rates are supported. Similarly the obtained long-run relationship shows a significant coefficient of spatial correlation.

Yet, the maximum-likelihood estimation may overstate the significance in particular since heteroscedasticity is not taken into account. But for the given dimension of the spatial model, incorporation of heteroscedasticity into the ML estimation is simply unfeasible. In order to at least robustify inference, therefore, a heuristically block bootstrap approach is applied to the regression. Instead of drawing single observations in order to obtain resamples this approach consists of drawing presumably dependent blocks of observation jointly which retains the dependency between observations. As in Buettner (1999b) due to the large dimension of the estimation there is no room for flexible block design and the blocks consist of the considered districts and its neighbors in all years. Since ML estimation is no longer consistent in the presence of heteroscedasticity at the bottom of column (3) in Table 1 a Wald statistic is displayed testing for joint differences between the bootstrap estimator and the ML estimator. However, no significance is found. As shown in column (3) the resulting standard errors are about twice as large as the ML estimates. Accordingly, the short-run dynamics show only weak spatial effects, but the level relation in taxes is still highly significant. The estimation supports the following long-run relationship in the tax rates:

\[ t_i = 0.286 \bar{t}_i + a_i, \]  \hspace{1cm} (19)

where \( \bar{t}_i \) is an average of the tax rates of \( i \)'s neighboring communities. If equation (19) identifies the response function, the estimate is consistent with a stable tax competition equilibrium, because the coefficient of other communities’ taxes is less than unity (see above).

However, the estimation has assumed away differences in the evolution of districts’ collection rates over time except those arising from differences in the neighborhood. This may cause problems, since especially cities have increased their tax rates during the sixteen years considered, as is documented by the statistics in Table 2. Without being able to explicitly employ potential determinants of differences in the evolution of tax rates, we should therefore allow for district-type specific evolutions in the analysis of collection rate changes. Columns (4) and (5) report the ML estimates from an estimation with district-type specific time effects with original and robust standard errors, respectively. According to a Wald test the district-type specific time effects are highly significant. It turns out that the results are quite

\footnote{See Fitzenberger, 1997, for a treatment of the time-series case.}
Table 2: Tax Trends among District Types

<table>
<thead>
<tr>
<th>District Type</th>
<th>Median Level 1996</th>
<th>Median Level 1980</th>
<th>Median Change abs.</th>
<th>Median Change rel.</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>440.0</td>
<td>360.0</td>
<td>65.0</td>
<td>17.0 %</td>
</tr>
<tr>
<td>Highly Dense</td>
<td>372.5</td>
<td>310.0</td>
<td>42.0</td>
<td>13.5 %</td>
</tr>
<tr>
<td>Dense</td>
<td>339.0</td>
<td>317.0</td>
<td>20.0</td>
<td>5.7 %</td>
</tr>
<tr>
<td>Rural</td>
<td>323.5</td>
<td>310.0</td>
<td>10.5</td>
<td>3.3 %</td>
</tr>
</tbody>
</table>

Median collection rates among the 327 districts in 1996 and 1980 as well as median absolute and relative changes between 1980 and 1996. District types according to the classification of the BBR. Source: own computations.

similar. Even with bootstrapped standard errors spatial effects in the lagged tax rate levels are significant.

The empirical analysis of spatial effects in the evolution of tax rates thus confirm the theoretical hypothesis that local tax setting in the German business tax gives rise to an interdistrict tax competition.

3.2 Differentials in Local Tax Rates

Whereas the previous section has established the existence of tax competition effects, long-run differentials in local tax rates were removed by the fixed regional effects. In order to identify factors behind tax rate in this section differentials the fixed effects from the previous section are regressed on local characteristics.

The theoretical discussion has provided us with at least two causes for those differentials. In view of Proposition 2, we should expect that collection rates are higher in larger jurisdictions. The size effect should therefore lead to higher collection rates taxes at more populous districts and at districts with a lower number of communities.

Proposition 3 suggests that differences in councils’ preferences will also cause tax differentials. Yet, it will be difficult to measure those differences. As the demand for local public expenditures will be affected by the age structure of the population the share of children and the share of old citizens could indicate those differences. Given the German institutional setting, especially the local share of welfare recipients among the population should be important,
as communities are federally mandated to pay social assistance, which constitutes an important part of local expenditures. One might think of further district conditions which determine to what degree tax policy is aimed at increasing the economic performance of residents by means of tax policy rather than at increasing tax revenues. A possibly important indicator would be the local rate of unemployment, although there might exist a problem with simultaneity.

The theoretical model considers districts as simple points in space and the implied differences in population density are neglected. But, the considered districts show large differences in density, which affect local governments’ tax policy in a variety of ways. Higher density reflects advantages from agglomeration such as urbanization economies which translate into a higher taxing power analogous to the simple size effect. Yet, higher density also induces crowding externalities which increase the local cost of production but may also lead to a higher demand for public goods. Although it will be difficult to distinguish these implications for tax policy, it is important to take density into account in order to check whether the pure size effect is not a spurious finding. We therefore use not only density but also the price for developed vacant land, the travel time to the next agglomeration and to the next international airport in order to control for density effects.

Besides differences in density the identification of population size and preference effects is hindered by several other determinants of location. Some of them can explicitly be considered in the analysis: as regional policy in Germany is aimed at increasing the after tax rate of return in selected areas, a dummy is included indicating whether a specific district contains a specific development area (Schwerpunktort). Furthermore, local differences in the supply price of electric power are controlled for by employing average power prices at district level.

Column (1) of Table 3 presents the results from regressing the fixed effects according to the estimation presented in column (3) of Table 1 on the explanatory variables. The size of population as well as the number of communities are highly significant indicating that the lon–run tax rate is high where population per single community is large. As several density related variables are employed this significance is not simply due to density. From the preference variables especially the share of welfare recipients is significant, indicating that higher federally mandated social assistance payments lead to higher tax rates.

Since the fixed effects are not observed directly but estimated one may
Table 3: Long-Run Tax Rate Differentials

<table>
<thead>
<tr>
<th>observations</th>
<th>327</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>dep. variable</strong></td>
<td>Regional Fixed Effects</td>
</tr>
<tr>
<td><strong>method</strong></td>
<td>OLS</td>
</tr>
<tr>
<td><strong>exp. variables</strong></td>
<td>(1)</td>
</tr>
<tr>
<td>constant</td>
<td>44.8 ***</td>
</tr>
<tr>
<td>log av. Population</td>
<td>13.4 ***</td>
</tr>
<tr>
<td>log no. of Communities</td>
<td>-11.1 ***</td>
</tr>
<tr>
<td>log av. Density</td>
<td>6.05 ***</td>
</tr>
<tr>
<td>Price Vacant Dev. Land</td>
<td>.005</td>
</tr>
<tr>
<td>Travel Time to Agglomeration</td>
<td>-.007</td>
</tr>
<tr>
<td>Travel Time to Airport</td>
<td>.008</td>
</tr>
<tr>
<td>share of Recreation Area</td>
<td>.085 ***</td>
</tr>
<tr>
<td>share of Welfare Recipients</td>
<td>.153 ***</td>
</tr>
<tr>
<td>share of Children</td>
<td>1.05 **</td>
</tr>
<tr>
<td>share of Citizens Age &gt; 65</td>
<td>.843 **</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>-.315</td>
</tr>
<tr>
<td>dummy Development Area</td>
<td>1.93 *</td>
</tr>
<tr>
<td>Power Price</td>
<td>-.866 *</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.726</td>
</tr>
<tr>
<td>MSD</td>
<td>120.7</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. Results in columns (1) and (3) obtained from OLS regressions. Columns (2) and (4) report results from minimum-distance estimation based on the bootstrap estimate of the variance-covariance matrix. Significant coefficients are marked with one, two or three stars for levels of 10%, 5%, and 1%.
increase the efficiency of the estimation by applying a minimum-distance estimator (MDE) (see Greene, 1993) which makes use of the (estimated) variance-covariance matrix. The MDE minimizes:

\[
\left(\hat{f} - S\gamma\right)' \left(V\hat{C}M(f)\right)^{-1} \left(\hat{f} - S\gamma\right),
\]

where \(\hat{f}\) denotes the vector of fixed effects, \(V\hat{C}M(f)\) their variance-covariance estimator, \(S\) a matrix of local characteristics, and \(\gamma\) a vector of parameters. The results of the minimum-distance estimation are presented in column (2). Only the population size effect and the share of welfare recipients remain significant. Nevertheless, according to the mean squared distance (MSD) displayed at the bottom of the table the restrictions imposed on the fixed effects by the estimated linear relationship cannot be rejected.

Yet, as the analysis of the tax rate evolution has found relevant differences between district types, we would like to know, whether the results still hold for the regional fixed effects as estimated after inclusion of district-type specific time effects (cf. columns (4) and (5) of Table 1). The corresponding estimates are presented in columns (3) and (4) of Table 3. Again, we find significant effects of population size and the share of welfare recipients. In addition, there is also a significantly higher tax rate at regional development areas indicating that regional development policy tends to offset higher tax rates.

Therefore, we can conclude that, indeed, the size of communities in terms of population is an important determinant behind the local tax rate differences, even when taking into account several characteristics of location including density. Given the German institutional setting it is also important to note that the population size effect is not driven by the social assistance payments. The significance of these payments, however, raises concerns that the joint presence of federally mandated spending and local taxing autonomy may cause distortions the spatial allocation of productive activities.

4 Summary

In the theoretical discussion in accordance to the literature, three basic propositions were derived from a simple model of local tax-setting. Tax rates set by local communities will rise with the population size as well as with the council’s bias in favor of public expenditures, and tax rates will be positively related to the neighbors’ tax rates.
In the empirical part these propositions were then confronted with the distribution of collection rates of the business tax across West Germany’s districts. The results confirm the existence of tax competition, in the sense that collection rates are set in response to the fiscal decisions of local neighbors: tax rates were found to be positively related to the tax rates in the neighborhood. Yet tax competition does not eliminate all differences in the local tax rates. The analysis of the long-run distribution of tax rates has revealed a robust positive relation between tax rates and population size but not with density in the German case. This conforms with the theoretical hypothesis that larger communities are less concerned with a tax policy aimed at increasing the local supply of mobile factors. A further interesting result is the significant positive relationship between the share of welfare recipients and the local tax rates, indicating that federally mandated local expenditures affect local tax policy. This raises concerns about the distribution of responsibilities in the German federal system.

An important qualification of the analysis is the underlying assumption that local fiscal differences are sufficiently described by the tax rate. Although this assumption certainly is adequate for an empirical study mainly based on a large set of tax rate observations, further research should take into account subsidies, intercommunity benefit spillovers, and grants, which partly intend to affect the bundle of inputs supplied.

5 References


Buettner, T. 1999a. *Agglomeration, growth, and adjustment.* A theoretical and empirical study of regional labor markets in Germany. Heidelberg et al..


6 Sources and Definitions of Data:

**Local Collection Rates**: Collection of rates of the business tax among 327 districts from 1980 until 1996 are published in series 10.1 of the Statistisches Bundesamt (German federal statistical office). In districts with several communities the collection rate is an average weighted by the communities’ share of the tax base.

**No. of Communities**: Taken from the official registry of communities in Germany (Amtliches Gemeindeverzeichnis).

**District area**: Total area in squared kilometers taken from Eurostat database Regio referring to the district definitions in 1980.

**Share of Recreation Area**: Referring to 1988 taken from the INKAR CD-ROM of the BBR (federal office for regional planning).

**Travel Time Agglomeration**: Travel time to the next density point (Verdichtungsraumkern) in minutes, source: BBR (federal office for regional planning).

**Travel Time Airport**: Travel time (by car) to the next international airport in minutes, source: BBR (federal office for regional planning).

**Share of Citizens with Age > 65:** Number of citizens with age > 65 relative to total population. Average of 1983, 1989, and 1995, source: Laufende Raumbeobachtung of the BBR (federal office for regional planning), own computations.

**Share of Children:** Number of children (age < 15) relative to total population in 1989, source: Laufende Raumbeobachtung of the BBR (federal office for regional planning).

**Unemployment Rate:** Average rate of unemployment in the 327 districts in the years 1986 until 1995, source: Institut of Employment Research (IAB) of the federal ministry of labor (BMA), own computations.


**Power Price:** Average price of electricity in DM per 100 kWh calculated at standardized demand values of four hypothetical firms. Average of 1982, 1985, 1991, and 1994 adjusted for price changes by the energy price index for former West Germany, source: Council of Experts on Economic Development (SVR), Laufende Raumbeobachtung of the BBR (federal office for regional planning), own computations.

**Development Area:** Dummy variable determining whether one of the communities is a development area (Schwerpunktort) according to the regional development act (Schwerpunktaufgabe Verbesserung der Regionalen Wirtschaftsstruktur). Average of 1983 and 1990, source: Laufende Raumbeobachtung of the BBR (federal office for regional planning), own computations.