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A search model of optimal pricing and production

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ABSTRACT

A Search Model of Optimal Pricing and Production

This paper presents an overview of the application of the mathematical theory of 'high-low' search to firms' pricing and production decisions. We show how this methodology can be used to determine an optimal sequence of price-quantity decisions by a firm through time.

We suppose that the firm chooses a sequence of prices and quantities supplied over time not only with a view to earning current profit (given the current information about the demand curve) but also in order to acquire information about the demand curve by observing its inventory stocks as a result of these price and quantity decisions.

We compare and contrast the high-low model with the conventional microeconomic model of pricing and production. We show how the firm uses its pricing and production decisions to partition the uncertainty interval it faces and thereby influence the value of the information which it receives.

JEL classification: 022, 210, 227

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NON-TECHNICAL SUMMARY

In traditional microeconomic theory, the firm is assumed to maximize its profits, given a known demand curve for its product and a known total cost curve. In practice, firms have little information on their demand curves and whatever information they do have is usually gleaned from selling their products at varying prices and observing the resulting inventories.

This paper aims to capture the idea that the firm learns about its demand curve through its pricing, production and inventory holding decisions. With this in mind, we outline a new methodology, based on the mathematical theory of high-low search (developed by Alpern, Baston and Bostock) for determining an optimal sequence of price-quantity decisions through time. In previous work we have applied this methodology to models in which a firm faces a fixed price and an unknown quantity of demand (Discussion Paper No. 223). By contrast, this paper deals with the more general problem of formulating joint pricing and production decisions on the basis of what is currently known about the demand curve, and learning about the demand curve from the outcomes of these decisions.

We suppose that the firm chooses a sequence of prices and quantities over time with the purpose of earning current profit (given the current information about the demand curve) and acquiring information about the demand curve by observing its inventories. The novel feature of our analysis is that a firm's pricing and production decisions may be made with a view to the information it thereby gains about its demand curve.

In the conventional microeconomic model, in which the firm's demand curve is known, the firm has no incentive to hold inventories, provided that the demand function and cost function remain unchanged through time. When the demand function is unknown, however, we show that inventory holdings can arise as the result of optimal pricing and production decisions.

We assume that the firm's initial information about its demand is only that price and quantity demanded lie in known intervals and that the demand function does not vary over time. At the beginning of each period, the firm makes a decision about its price and the quantity put up for sale; we call this decision its 'guess', since the firm is trying to find the price-quantity combination that maximizes the present value of profits under its unknown demand function. When making the guess, the firm has an information set consisting of all previous guesses and all previous inventory stocks. Once a new guess has been made, the firm observes its new inventory stock and from this observation it makes inferences about its demand function. In particular, if inventories are zero it infers that demand at the current price must be at least as large as the quantity put up for sale. If inventories

are positive then the firm learns the exact level of demand at the current price.

In our framework the firm chooses a price-quantity strategy which is a sequence of rules for determining its guess, based on the current information available. This general model illuminates a trade-off which faces the firm: on the one hand, it may expect more current profit by reducing production and thereby reducing its chances of holding inventories; on the other, it can expect more future profit by increasing production, increasing its chances of holding inventories and thereby increasing its chances of gaining more information about its demand curve. In this sense inventories are held on account of the information they convey.

1. Introduction

In traditional microeconomic theory, the firm is assumed to maximize its profit, given a known product demand curve (relating quantity demanded to the price) and a known total cost curve (relating the total cost of production to the quantity produced). Of course, economists have long recognized that in Practice the managers of firms have little information on the demand curves they face and whatever information they do have is usually gleaned from selling their products at varying prices and observing the resulting inventories. The traditional theory has made little headway in accounting for this pervasive learning phenomenon.

In the microeconomic literature, the most common way of analyzing the behaviour of the firm under imperfect information about demand is simply to include an additive or multiplicative error term in the demand and to assume the distribution of the error term is known. However, we are told little about how the firm discovers this distribution. This paper aims to capture the idea that the firm learns about its demand curve through its pricing, production and inventory holding decisions. With this in mind, we outline a new methodology, based on the mathematical theory of high-low search (developed by Baston and Bostock (1985) and Alpern (1985)) for determining an optimal sequence of price-quantity decisions through time. See also Gal

(1974).

This methodology has already been applied to models in which a firm faces a fixed price and an unknown quantity demanded by Alpern and Snower (1986) and Reyniers (1986). The solution to these models is discussed in Section 3. By contrast, this paper deals with the more general problem of formulating joint pricing and production decisions. In particular, it is concerned with the twin activities of

- (a) making pricing and production decisions on the basis of what is currently known about the demand curve, and
- (b) learning about the demand curve on the basis of outcomes of the pricing and production decisions.

We suppose that the firm chooses a sequence of price-quantity combinations through time with the purpose of earning current profit (given the current information about the demand curve) and acquiring information about the demand curve by observing its inventory stocks.

The formulation of a general model for this purpose, as shown in Section 2, involves a number of important conceptual issues which have not been addressed in the microeconomic literature thus far. Indeed, our economic analysis of this general model leads to the formulation of a new class of mathematical search models. These models are outlined in Section 4.

2. The General Model

As noted, the novel feature of our analysis is that a firm's pricing and production decisions may be made (at least in part) with a view to the information it thereby gains about its demand curve. To bring this feature into sharp relief, we begin with the conventional, elementary microeconomic theory on the behaviour of the firm when the demand curve is known. We then inquire how this theory needs to be modified when the firm does not know its

demand curve, but can reveal it gradually through its pricing and production decisions.

In the conventional theory, the firm maximizes its profit (π) subject to the known demand curve $q=f(p)$, where q is the quantity demanded at the price p . The profit is expressed as the difference between revenue ($p \cdot q$) and cost ($c(q)$). In short, the standard problem is

$$(1) \quad \text{Max } \pi = p \cdot q - c(q),$$

p

where $q=f(p)$. For convenience of presentation, we assume that the demand and cost functions are such that Problem 1 has a unique, economically meaningful solution, with the optimal price-quantity combination given by $H_f \equiv (p_f^*, q_f^*)$ and the optimum profit is π_f^* . (The subscript "f" draws attention to the fact, significant below, that the optimal decisions depend on the demand function, $f(p)$.)

In this formulation, the firm has no incentive to hold inventories, provided that the demand function ($f(p)$) and the cost function ($c(q)$) remain unchanged through time and $c' \geq 0$. Thus, it is not necessary to distinguish among quantities demanded, produced, and sold.

When the demand function is unknown, however, inventory holdings can arise, even (as shown below) under optimal pricing and production decisions. In order to compare the firm's decision making under unknown demand with that under known demand, let us rewrite the known-demand problem (1) as an intertemporal maximization of the present value (PV) of a stream of profits. Let the inventory stock at time t be I_t . We assume that $I_0 = 0$. The time discount factor is α ; the inventory holding cost is $h(I_t)$ (where $h(0) = 0$ and $h > 0$ for $I_t > 0$); δ is the inventory depreciation factor; q_t is the quantity produced; d_t is the quantity demanded; z_t is the quantity sold; and x_t is the stock of goods put up for sale at time t . Then Problem (1)

becomes

$$(2) \quad \text{Max}_{p_t, x_t} PV = \sum_{t=1}^{\infty} \alpha^{t-1} \cdot [p_t \cdot z_t - c(q_t) - h(I_t)]$$

subject to $d_t = f(p_t)$

$$z_t = \min(d_t, x_t)$$

$$I_t = \max [(x_t - d_t), 0] \quad \text{for } t=1, 2, \dots,$$

where the choice of x_t implies a value of q_t , since the quantity offered for sale is equal to the quantity produced plus the inventory stock carried forward from the previous period: $x_t \equiv \delta \cdot I_{t-1} + q_t$.

Since the firm does not hold inventories at the optimum ($I_t^* = 0, \forall t$), the quantities demanded, produced, and sold are all equal ($d_t = q_t = x_t = k, \forall t$, where k is a non-negative constant). The firm's behaviour may be characterized in terms of a sequence of decisions about price and the quantity put up for sale at each point in time: (g_1, g_2, \dots) , where $g_t = (p_t, x_t)$. The optimal sequence of these decisions is (H_f, H_f, \dots) , where H_f is the solution of Problem (1). The corresponding optimal present value is $PV_f^* = (1/(1-\alpha)) \cdot \pi_f^*$.

We now consider the case in which the demand function is unknown. We examine how the analysis above (Problem (2)) must be extended to capture the idea that the firm may learn about its demand curve through its sequence of decisions (g_1, g_2, \dots) . To this end, we assume that the firm's initial information about its demand is only that p and d belong to the known intervals $P = [\underline{p}, \bar{p}]$ and $D = [\underline{d}, \bar{d}]$, respectively (where $\underline{p} < \bar{p}$ and $\underline{d} < \bar{d}$), and that the demand function f is time invariant and taken from a known set F (say, the set of all decreasing, continuous functions $f: P \rightarrow D$).

At the beginning of each period t , the firm makes a decision about its

price (p_t) and its quantity put up for sale (x_t); we call this decision the "guess" $g_t = (p_t, x_t)$ about H_f , since the firm is trying to find the price-quantity combination which maximizes its present value under its actual (but unknown) demand function. Each guess g is taken from the set $G = P \times D$, called the "price-quantity rectangle" (or "search region").

When making the guess g_t , the firm has an information set consisting all previous guesses and all previous inventory stocks:

$(g_1, g_2, \dots, g_{t-1}; I_0, I_1, I_2, \dots, I_{t-1})$. (We still assume that the initial inventory stock is zero.) Once the guess g_t has been made, the firm observes its new inventory stock I_t and, from this observation, it makes inferences about its demand function. In particular,

(a) if $I_t = 0$ (i.e. a stock-out), then it infers that $f(p_t) \geq x_t$ (i.e. its demand at the price p_t must be at least as large as the quantity put up for sale); and

(b) if $I_t > 0$ (i.e. a positive inventory stock), then it infers the exact level of demand at the price p_t : $d_t = f(p_t) = x_t - I_t$.

These inferences are illustrated in Figure 1. Here we have drawn the actual, but unknown, demand function $d = f(p)$. The firm has made five guesses (denoted by the points g_1, \dots, g_5) and has observed that $I_1, I_4, I_5 = 0$ and $I_2, I_3 > 0$. Consequently, the firm infers that the demand function must pass through the points g'_2 and g'_3 (where $g'_i = [p_i, f(p_i)]$) and through the vertical lines L_1, L_4 , and L_5 (where $L_i = \{(p_i, s) : x_i \leq s \leq \bar{d}\}$).

Of course, if the demand function f is known to be decreasing, then the firm can infer that f passes through the same points and the shorter vertical lines given in Figure 2.

As noted, the maximum attainable present value is $PV_f^* = \sum_{t=1}^{\infty} \alpha^{t-1} \cdot \pi_f^*$ (of Problem (2)). In general, the firm does not achieve this present value,

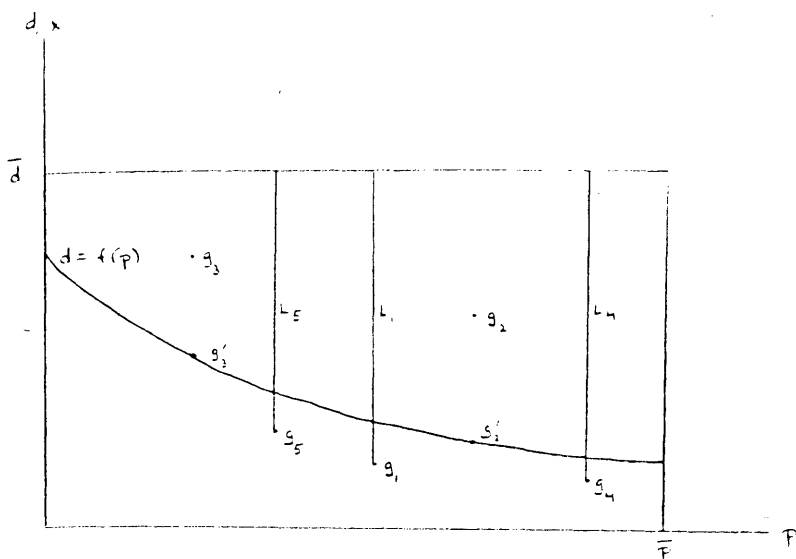


Figure 1

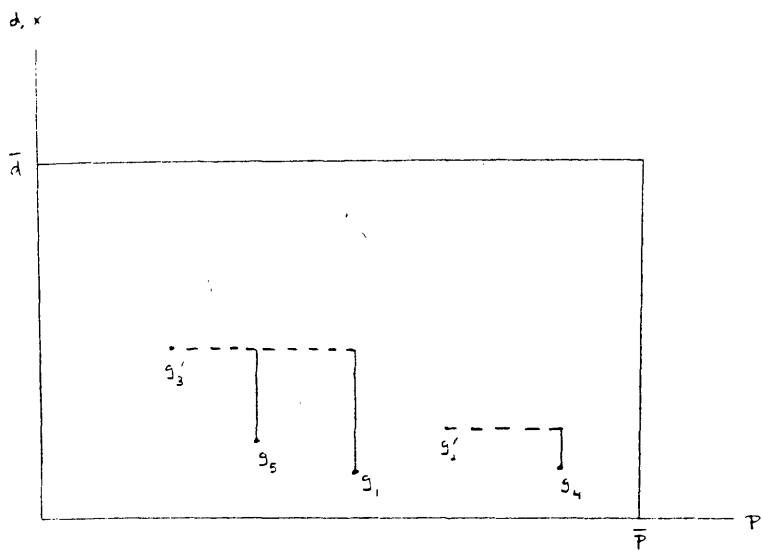


Figure 2

since it has imperfect information about its demand function. The opportunity cost associated with its sequence of guesses $g = (g_1, g_2, \dots)$ is

$$(3) \quad R(g, f) = \sum_{t=1}^{\infty} \alpha^{t-1} \cdot [\pi_f^* - (p_t \cdot z_t - c(q_t)) + h(I_t)] ,$$

where z_t , q_t , and I_t may all be expressed in terms of $g_t = (p_t, x_t)$ (as shown in Problem 2).

A price-quantity strategy $S = (S_1, S_2, \dots)$ is a sequence of rules S_t , $t=1, 2, \dots$, for determining the guess $g_t = (p_t, x_t)$ based on information available when g_t is chosen. Since all previous guesses and inventories are known at that time, we have $g_t(S, f) = S_t(g_1, g_2, \dots, g_{t-1}; I_1, I_2, \dots, I_{t-1})$ is the t 'th guess assuming a strategy S and an actual demand function f , with $g(S, f) = (g_1(S, f), g_2(S, f), \dots)$. If the firm uses strategy S and the demand curve is f , then the opportunity cost is given by $\hat{R}(S, f) = R[g(S, f), f]$. If \mathcal{S} denotes the set of all strategies, then we seek a minimax strategy S^* and a minimax value V^* defined by

$$(4) \quad \max_{f \in F} \hat{R}(S^*, f) = \min_{S \in \mathcal{S}} \max_{f \in F} \hat{R}(S, f) = V^* .$$

We may interpret V^* as being the optimal worst-case cost to the firm of not knowing the actual demand curve.

In this general model, the guesses g will in general never reach the theoretical equilibrium point H_f , although it is possible that the guesses g_t will converge to it. However, for an important class of demand function sets F , the equilibrium can be attained in finite time: Suppose that every demand curve f in the set F has the property that it is completely determined by any n distinct valuations $f(p_1), f(p_2), \dots, f(p_n)$. We say that such a set F has "complexity n ". For example, the set of n 'th degree polynomials has complexity $n+1$.

If f has complexity n , then any n periods with overproduction at distinct

prices will determine f and hence the equilibrium point H_f . This observation gives an upper bound for V^* by consideration of a strategy with $g_t = (p_t, \bar{d})$ for distinct p_t , $t=1, \dots, n$. Moreover, we conjecture that for all f with finite complexity, there is an optimal strategy S^* for which an a priori estimate can be made on the number of guesses needed to find H .

3. Models of Given Price and Unknown Demand

While the general unknown demand curve model described in the previous section is too complex for an analytical solution, two models which may be interpreted as special cases of the one above have been solved. We briefly describe these models and their solutions here.

The model of Alpern and Snower (1986) assumes that the product has a fixed unit price p_0 and that the only unknown is the amount d demanded at this price. The demand d is known to belong to a given interval $D = [\underline{d}, \bar{d}]$. We assume linear production and inventory holding costs of c and h per unit, respectively.

The firm's sequence $g = \{g_t, t=1, 2, \dots\}$ of guesses can now be identified with the amounts x_t offered for sale, since p_t is always p_0 . We view x_t as a guess about the unknown level d of demand. (If demand d is known, then of course the optimal sequence of guesses $g_t = x_t$ is (d, d, \dots) .) Let N be the first time period in which the guess g_t is greater than the demand d : $N = \min \{i: g_i \geq d\}$. At the end of the period N , the demand can be inferred by observing the inventory stock I_N and computing $d = g_N - I_N$. Thereafter demand will be met exactly (i.e. all g_t with $t > N$ will be set at $g_t = d$).

The opportunity costs R of the sequence of guesses (g_1, g_2, \dots) is given by

$$(5) \quad R(g, d) = \sum_{g_t \leq d} \alpha^{t-1} \cdot (p_0 - c - h) \cdot (d - g_t) \\ + \sum_{g_t > d} \alpha^{t-1} \cdot [h + c \cdot (1-\alpha)] \cdot (g_t - d)$$

where $(p - c - h)$ is the unit cost of underproduction and $[h + c \cdot (1-\alpha)]$ is the unit cost of overproduction. A pure search strategy $S = (S_1, S_2, \dots)$ is now quite simple to describe. Each S_i is a number belonging to $[d, \bar{d}]$ and $\underline{d} \leq S_1 \leq S_2 \leq \dots \leq \bar{d}$. The strategy S together with the actual value d of demand determine the guesses g_t as follows. For $t \leq N$, $g_t = S_t$; and for $t \geq N$, $g_t = d$. Using recursive methods adapted from the mathematical theory of high-low search as formulated in Baston and Bostock (1985) and Alpern (1985), a minimax strategy $S^* = (S_1^*, S_2^*, \dots)$ was calculated.

As an example, we give the strategy S^* corresponding to a demand interval of $[d, \bar{d}] = [20, 31]$, unit production costs of $c=8$ and holding costs of $h=2$. We assume each unit sells for the fixed price of $p = 12$ and the discounting factor is $\alpha = 1/2$. The unique minimax strategy is then $S = (23, 29, 31, 31, \dots)$. The maximum opportunity cost corresponding to S^* is 18 and occurs when d is 20, 23, or 31.

This problem can be viewed as a special case of the general problem of Section 2. The set F of demand curves is given by $F = \{f_d: \underline{d} \leq d \leq \bar{d}\}$ where $f_d(p) = d$ for $p \leq p_0$ and $f_d(p) = 0$ for $p > p_0$. This family has complexity 1, i.e. a single guess above the demand curve determines the curve completely.

An interesting variation on this fixed price model has been considered by Reyniers (1986). Reyniers allows a time lag of one period so that only I_{t-2} (and not I_{t-1}) is known when g_t is chosen. This time lag may be interpreted as either a production lag (where g_t must be chosen at time $t-2$) or a delay in

information feedback on inventory stocks from retailers (where I_{t-1} is not known until the end of period t). Reyniers develops recursive formulae which show that with time lag, the minimax strategy S is not monotone, but rather alternating. In particular,

$$S_1 < S_2 < \dots < S_{2n+1} < S_{2n+3} \dots \text{ and } S_{2n+1} + S_{2n+2} = 2 \cdot S_{2n-1}.$$

4. The General Nature of the Search Problems

We now investigate in greater detail the information structure of the models of Sections 2 and 3. In Section 3, a guess g (offering an amount g for sale) can be viewed as an "experiment" which will determine whether the actual "state of nature" d belongs to the set $[g, \bar{d}]$ (stock-out) or to one of the singleton sets $\{b\}$ where $\underline{d} \leq b < g$. These sets form a "partition" of the set $D = [\underline{d}, \bar{d}]$ of states of nature. This means (definition of partition) that every state of nature belongs to exactly one of these sets. We denote this partition of $[\underline{d}, \bar{d}]$ into one interval and a continuum of singleton sets as \mathcal{P}_g . The firm knows in advance that if g is the chosen guess, it will learn which element of the partition \mathcal{P}_g (i.e. which subset) contains g . Similarly, in the more complicated model of Section 2, the guess $g = (p, x)$ partitions the set of demand curves F into $\{f: f(p) \geq x\}$ and the sets $\{f: f(p)=d\}$, $\underline{d} \leq d < x$. The search for the actual demand (in Section 3) or the actual demand curve (in Section 2) is carried out by successive partitioning of the states of nature.

This general search problem may be described as follows:

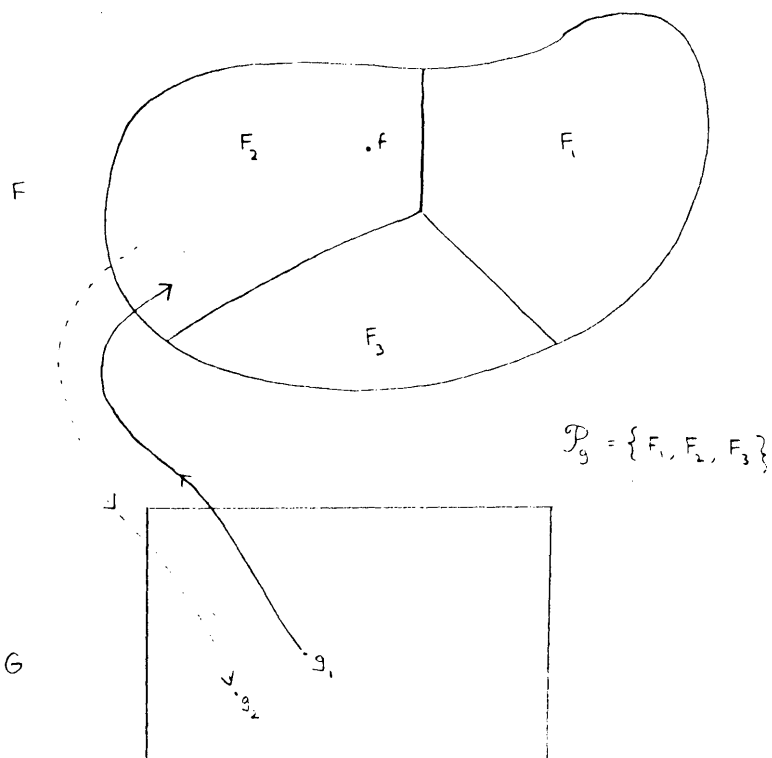
Simple sequential search problem: An unknown point d is hidden in a search set D . A discounting factor $0 < \alpha \leq 1$ is given. A searcher seeks to find d by making successive guesses $g_t \in D$, $t=1, 2, \dots$, trying to minimize the sum $R = \sum_{t=1}^{\infty} \alpha^{t-1} \cdot k_d(g_t)$ where k_d ($d \in D$) is a family of

non-negative functions depending on d , satisfying $k_d(d) = 0$. Corresponding to every possible guess g in D , there is a known partition \mathcal{P}_g of D . If the searcher makes the guess g_t , he is told the element of \mathcal{P}_{g_t} which contains d before he picks g_{t+1} . Which strategy for picking the g_t 's guarantees the lowest value of R ?

This "simple sequential search problem" subsumes the model of Section 3, where ρ_g has already been described, and $k_d(g)$ is the function $(p_0 - c - h) \cdot (d - g)$ if $d \geq g$ and $[h + c \cdot (1 - \alpha)] \cdot (g - d)$ if $g > d$.

The general model of Section 2, however, has a more complicated search structure. There is also a hidden point $H_f \in G = P \times D$ which the searcher tries to approximate. However, each guess $g \in G$ gives not information about the equilibrium point $H_f \in G$ but rather information about the demand function f which lies in another space F . Information about H_f (and hence where to make the next guess) must be inferred from knowledge of the way H_f depends on f , which we call a map $\phi: F \rightarrow G$, where $\phi(f) = H_f$. The compound nature of the search problem is captured in the following abstraction, which has not been studied in the mathematical search literature, but deserves to be:

Compound sequential search problem: There is a primary search region G and a secondary search region F with a given map $\phi: F \rightarrow G$, as illustrated in Figure 3. To each point g in G there is a corresponding partition \mathcal{P}_g of F . To each f in F there is a non-negative function π_f on G with maximum value $\pi_f^* = \pi_f(\phi(f))$. A point f is hidden in F . The searcher makes a sequence of guesses $g_t \in G$, $t=1, 2, \dots$. After each guess g_t he is told the element of \mathcal{P}_{g_t} containing f . He seeks to minimize the opportunity cost function $R = \sum_{t=1}^{\infty} \alpha^{t-1} \cdot [\pi_f^* - \pi_f(g_t)]$ in the worst case



After guess g_1 , the searcher learns that f is in F_2 and uses this information to pick g_2

Figure 3

with respect to f . Thus the "compound sequential search problem" may be summarized by the question, What is the strategy of picking the guesses g_t so as to minimize the maximum (over f) of R ?

5. Final Remark

This paper has set out a methodology for formulating the firm's pricing and production decisions when these decisions are not only made on the basis of given demand information, but are also meant to reveal further demand information. In particular, the firm infers the position of its demand curve by observing its inventory stock. In this setup, which is quite common in practice, the firm faces a tradeoff between two desirable objectives: on the one hand, it may expect more current profit by reducing production and thereby reducing its chances of holding inventories; on the other, it can expect more future profit by increasing production, increasing its chances of holding inventories and thereby increasing its chances of gaining more information about its demand curve. It is in this sense that inventories are held on account of the information they convey.

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