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Mitigating Carbon Leakage: Combining Output-Based Rebating with a Consumption Tax

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Mitigating Carbon Leakage: Combining Output-Based Rebating with a Consumption Tax

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Abstract

Unilateral climate policy induces carbon leakage through the relocation of emission-intensive and trade-exposed industries to regions with no or more lenient emission regulation. Both analytical and numerical studies suggest that emission pricing combined with border carbon adjustments may be a second-best instrument, and more cost-effective than output-based rebating, in which case domestic output is indirectly subsidized. No countries have so far imposed border carbon adjustments, while variants of output-based rebating have been implemented. In this paper we demonstrate that it is welfare improving for a region who has already implemented emission pricing along with output-based rebating for emission-intensive and trade-exposed goods to also introduce a consumption tax on these goods. Moreover, we show that combining output-based rebating with a consumption tax can be equivalent with border carbon adjustments.

JEL-Code: D610, H200, Q540.

Keywords: carbon leakage, output-based rebating, border carbon adjustments, consumption tax.

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1. Introduction

In response to the threat of climate change, many countries consider or have introduced unilateral climate policies. However, greenhouse gases are global pollutants and unilateral action leads to carbon leakage, such as relocation of emission-intensive and trade-exposed (EITE) activities to countries with no or more lenient climate regulations. Constraints on emissions raise production costs for industries such as steel, cement, and chemical products, reducing their competitiveness in the world market, and hence trigger more production and emissions in unregulated regions.

To mitigate counterproductive leakage, countries have either exempted EITE industries from the regulation, or searched for supplemental anti-leakage measures. As a prime example, EITE industries in the EU, which are regulated under an emissions trading system (ETS), have received large amounts of free allowances. Currently, allowances are mainly allocated in proportion to installations’ (past) production.¹ Free allowances have also been introduced in other ETSs such as in New Zealand, South Korea and California, and in the regional ETSs in China (World Bank, 2014). Free allowance allocation conditional on output can be interpreted as output-based rebating (OBR) of emission tax payments (e.g., Bernard et al., 2007; Böhringer et al., 2014a).

Another potential anti-leakage measure that figures prominently in the economic literature is border carbon adjustments (BCA) with carbon tariffs on imports and rebates on exports of EITE goods. Most studies on carbon leakage suggest that BCA outperform OBR with respect to leakage reduction and cost efficiency of reducing global emissions (Monjon and Quirion, 2011a; Fischer and Fox, 2012; Böhringer et al., 2014a). BCA are however politically contentious, and experts differ in their views about whether or not it is compatible with WTO rules (see e.g. Horn and Mavroidis, 2011, Tamiotti, 2011, and Böhringer et al., 2012b).² One signal for its critical political feasibility is that – so far – border measures have only been proposed but not implemented.³

One main difference between OBR and BCA is that whereas the latter dampens foreign supply of EITE goods to the regulated country, the former stimulates domestic production. The reason is that OBR acts as an implicit production subsidy (Böhringer and Lange, 2005). As a consequence, production and consumption of EITE goods will be too high under OBR, compared to the first-best

¹ As pointed out by Meunier et al. (2014), the allocation mechanism in the EU ETS may be better characterized by capacity-based allocation, as new (and expansion of existing) installations receive allowances in proportion to their installed capacity.
² In 2010, the Indian Environment Minister threatened to “bring a WTO challenge against any ‘carbon taxes’ that rich countries impose on Indian imports” (ICTSD, 2010). There is also a fear that BCA could trigger a trade war (Holmes et al., 2011). On the other hand, Nordhaus (2015) argues that trade penalties can induce countries to join a “Climate Club” (see also Helm and Schmidt, 2015).
³ For example, border measures have been included in the American Clean Energy and Security Act of 2009 that passed the U.S. Congress but not the Senate (see https://www.congress.gov/bill/111th-congress/house-bill/2454; Fischer and Fox, 2011). Border measures have also been put forward by the EU Commission (2009) as a possible future alternative to free allowance allocation.
outcome (except in the case of market power, cf. Gersbach and Requate, 2004). In other words, the incentives to switch from buying emission-intensive to less emission-intensive products are weakened under OBR. As demonstrated by Böhringer et al. (2014a), whereas border adjustments automatically become inactive as the coalition of regulating countries embrace the whole world, OBR continues to induce excess costs as it stimulates too much output of the EITE goods. Similarly, whereas BCA for non-exposed goods have little or no impacts, OBR triggers too much production.

In this paper we first show analytically that it is welfare-improving for a country that have already implemented a carbon tax (or emissions trading system) along with OBR to EITE goods, to also impose a consumption tax on the same EITE goods. By consumption tax, we refer to product-specific taxes on all purchases of these goods, i.e., not only on final consumption but also on intermediate use in production. The intuition behind the welfare-improving effect of such a consumption tax is that OBR stimulates too much output of EITE goods. Note that in a closed economy OBR and the consumption tax cancel each other out, just as BCA become inactive in a closed economy. We also find that even in the case without any rebating, it is welfare-improving to implement a consumption tax on EITE goods as it reduces foreign production (and hence emissions) of such goods.

Second, we show that combining OBR with a consumption tax may in fact be equivalent with BCA. The equivalence requires that the consumption tax for an EITE good is equal to the OBR rate, which in turn must equal the carbon tariff and the export rebate. Thus, our finding suggests that the contentious issue of border adjustments can be ameliorated by coupling OBR with a consumption tax on all products that are entitled to OBR – the outcome will be the same. To our best knowledge, this equivalence result has not been shown in any previous studies.

We substantiate our analytical findings with complementary numerical results based on a stylized computable general equilibrium (CGE) model with two regions and four goods, where the goods can be either consumed or used as input into production. The numerical results are in accordance with our analytical findings. Moreover, the simulations demonstrate that the advantage of a consumption tax is particularly big if the EITE good is only slightly trade-exposed. Then the leakage exposure is limited, and thus the distortive effects of stimulating output are becoming more critical. By combining OBR with a consumption tax, the distortive effect of OBR can be controlled for. Such a hedging strategy becomes particularly policy-relevant if there is uncertainty about leakage exposure for individual

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4 All instruments are applied in monetary value per unit of the EITE good. For instance, with 100% rebating, i.e., all emission payments from the EITE-industry are rebated back to the industry in proportion to firms’ output, the equivalence requires that the carbon tariff is based on domestic emission intensities, and that there is 100% export rebating. Böhringer et al. (2012b, 2015) examine different tariff regimes, concluding that differentiated tariffs are better than non-differentiated tariffs. However, differentiated tariffs require more administrative efforts, and are more likely to conflict with GATT rules (see Ismer and Neuhoff, 2007, and Monjon and Quirion, 2011b).

5 Note that as long as benchmarks are already determined for the OBR rates (such as the benchmarks currently used in the EU ETS), there are no extra administrative costs in determining the consumption taxes.
sectors. The actual practice in EU climate policy sheds some light on the issue at stake. In the EU ETS, sectors that are “exposed to a significant risk of carbon leakage” receive a high share of free allowances. A majority of industry sectors have been put into this group. In contrast, Sato et al. (2015) find that “vulnerable sectors account for small shares of emission”, and Martin et al. (2014) conclude that the current allocation results in “substantial overcompensation for given carbon leakage risk”. Note that supplementing OBR with a consumption tax does not only provide a hedge against uncertainty on data statistics grounds but also with respect to lobbying activities.

There is a large body of literature on carbon leakage triggered by unilateral emission regulation. The seminal paper by Markusen (1975) derives the first-best combination of a domestic production tax and a tariff on imported goods (in his model, emissions are functions of production only), where the optimal tariff depends on both leakage and terms-of-trade effects. In a similar vein, Hoel (1996) determines an optimal combination of an emission tax and a carbon tariff (or export subsidy), where he also includes the indirect emission effects of the tariff (see also Copeland, 1996, for an early analytical contribution).

There exist many numerical modeling studies of carbon leakage, the bulk of them using multi-region and multi-sector CGE models of the world economy. For policy-relevant parameters on key dimensions (such as the stringency of emission regulation or the choice of abating regions), most studies conclude that the leakage rate of a unilateral carbon tax (or ETS) is in the range of 5-30%, i.e., a reduction of 100 units of CO₂ in the regulating country leads to an increase of 5-30 units of CO₂ in non-regulating countries (see, e.g., the review by Zhang, 2012, and the special issue edited by Böhringer et al., 2012a). There are, however, a few outliers with negative leakage (Elliott and Fullerton, 2014) or leakage rates above 100% (Babiker, 2005), adopting less conventional assumptions on international factor mobility or market power. Studies that calculate leakage from single EITE industries often find somewhat higher leakage rates (e.g., Ponssard and Walker, 2008, and Fischer and Fox, 2012).

Leakage mainly occurs through two intertwined channels. In this paper we focus on leakage through the market for EITE goods, often referred to as the competitiveness channel. The second channel is the so-called fossil-fuel channel: Reduced demand for fossil fuels in climate policy regions depresses international fuel prices, stimulating fuel consumption and thus emissions in other regions (Böhringer et al, 2014c). The policy debate focuses on leakage through the competitiveness channel, mirroring concerns of regulated EITE industries on adverse competitiveness effects. The policy focus goes also along with broader scope of policy options – such as BCA or OBR – to mitigate leakage through EITE markets rather than leakage through fossil fuel markets.

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6 http://ec.europa.eu/clima/policies/ets/cap/leakage/index_en.htm
Our paper also relates to a strand of literature that examines consumption taxes in environmental regulation, either alone or in combination with other instruments. In particular, Holland (2012) shows that adding a consumption tax to an emission intensity standard can improve efficiency as standards trigger inefficiently high consumption. Tradable intensity targets can be re-interpreted as a combination of an emission price and OBR – in this respect, Holland’s finding is comparable with our result on the efficiency gains through supplemental consumption taxes. However, Holland’s model includes only one good, with domestic and foreign goods being perfect substitutes, whereas we use a model with three goods and consider both perfect and imperfect substitutes. Eichner and Pethig (2015a) examine consumption-based taxes as an alternative to emission (production-based) taxes in a two-period two-country analytical general equilibrium model with a finite stock of fossil fuels. They find that consumption-based taxes may reduce first-period emissions in both countries, and thereby reduce the cost of unilateral climate policy. In follow-up work, Eichner and Pethig (2015b) show that a combination of production- and consumption-based taxes outperform production-based taxation standalone. There is also a large literature on consumption taxes based on carbon footprint or life cycle emissions (see e.g. McAusland and Najjar, 2015).

The remainder of this paper is organized as follows. In Section 2 we lay out our theoretical model and analyze the optimal consumption tax in a situation where an emission tax combined with OBR is already in place; we then demonstrate the equivalence between BCA on the one hand and the combination of OBR and consumption tax on the other hand. In Section 3, we develop a stylized computable general equilibrium model calibrated to empirical data for the world economy and substantiate our analytical results with numerical simulations. Section 4 concludes.

2. Analytical model

Consider a model with two regions, \( j = \{1, 2\} \), and three goods \( x, y \) and \( z \). Good \( x \) is carbon-free and tradable, good \( y \) is carbon-intensive and tradable, while good \( z \) is carbon-intensive and non-tradable. Same goods produced in different regions are assumed to be homogenous,\(^7\) with no trade cost (for the two tradable goods). It is reasonable to interpret \( y \) as emission-intensive and trade-exposed (EITE) sectors where output-based rebating is considered (e.g., chemicals, metal and other mineral production), and \( z \) as sectors where leakage is of less concern (e.g. electricity production and transport). The market prices (excluding taxes) of goods \( x, y \) and \( z \) in region \( j \) are denoted \( w^j, p^j \) and \( ω^j \), respectively.

\(^7\) In the Appendix we show that our results generalize to heterogeneous \( y \) goods (see Corollary 3).
The representative consumer’s utility from consumption in region $j$ is given by $u'(x^j, y^j, z^j)$, where $x^j$, $y^j$, and $z^j$ are consumption of the three goods. As usual, we assume that the utility function is twice differentiable, increasing and strictly concave; i.e., we have $u''_x > 0, u''_y > 0, u''_z > 0$ and the Hessian matrix is negative definite.

Production of good $x$ in region $j$ is $x^j = x^j_1 + x^j_2$, where $x^j$ denotes goods produced in region $j$ and sold in region $i$. We use similar notation for good $y$. The market equilibrium conditions are then:

$$
\begin{align*}
x^1 + x^2 &= x^j_1 + x^j_2 \\
y^1 + y^2 &= y^j_1 + y^j_2 \\
z^j &= z^j
\end{align*}
$$

That is, total production of each good must equal total consumption.

Cost of producing good $x$, $y$ and $z$ in region $j$ is given by $c^x(x^j), c^y(y^j, e^j)$ and $c^z(z^j, e^j)$, respectively, with $e^j$ and $e^j$ denoting emissions. We assume that cost is increasing in production for all goods, and that cost of producing $y$ and $z$ is decreasing in emissions (below those associated with no emission regulation). More precisely, $c^y_0, c^y_1, c^y_2 > 0$ and $c^z_0, c^z_1, c^z_2 \leq 0$, with strict inequality when emissions are regulated. Further, cost is assumed to be twice differentiable and strictly convex. Last, all derivatives are assumed to be finite.

2.1 Output-based rebating and consumption tax

Consider the following available policy variables in region 1: $t^1$ is an emissions tax, $s^1$ is an output subsidy to production of good $y$, and $v^1$ is a consumption tax on buying good $y$. With no uncertainty, output-based rebating (OBR) is equivalent with an output subsidy, where the subsidy is linked to the emission tax. In particular, if the tax revenues are fully redistributed back to the producers, the implicit subsidy of OBR is $s^1 = t^1 e^j / y^j$, a case we will refer to as 100% OBR.\(^8\) We assume no climate policy in region 2, i.e., $t^2 = s^2 = v^2 = 0$.

Competitive producers in region $j$ maximize profits:

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\(^8\) Most studies of OBR in the literature consider 100% rebating. In the EU ETS, the most leakage-exposed industries, accounting for more than half of total emissions from installations that receive free allowances, have around 100% rebating on average. Note that this does not mean that the allowances they receive cover all their needs, as $e^j$ in the expression above denotes regulated emissions, which typically are lower than baseline emissions.
This gives the following first order conditions for interior solution:

\[ w^j = w^2 = c_s^{1} = c^{s2} \]
\[ p^1 + s^1 = p^2 + s^1 = c_y^{1} \quad ; \quad p^1 = p^2 = c_y^{s2} \]
\[ \omega^j = c_e^{s1} = -t^1 \quad ; \quad c_e^{s2} = c_e^{s2} = 0 \]

Note that an interior solution requires that there is one global price for each of the tradable goods \( x \) and \( y \), as both goods are homogenous with no trade cost (this is not the case with heterogeneous goods, see Corollary 3 and the proof in Appendix A). The domestic emissions tax \( t^j \) induces higher cost of producing good \( y \) in region 1, which implies higher output and emissions in region 2 through the international market for good \( y \). The motivation for the subsidy \( s^j \) (or OBR) is to target this leakage, by driving a wedge between marginal production cost in region 1 and the market price on good \( y \), and hence stimulate domestic output of this good. The net effect of \( t^j \) and \( s^j \) on \( y^1 \) is ambiguous.

The representative consumer in region \( j \) maximizes utility, given consumer prices and a budget restriction. After constructing the Lagrangian function and then differentiating, we get the following first order conditions:

\[ u^j_{x} = w^j, u^j_{y} = p^j + v^j, u^j_{z} = \omega^j. \]\n
We assume that the regions have a budget constraint, so that import expenditures must equal export revenues in both regions. Net export for region \( j \) is equal to production minus consumption in that region, i.e., \( x^j - x^j \) and \( y^j - y^j \). Using \( p^1 = p^2 \equiv p \) and \( w^1 = w^2 \equiv w \) from the first order conditions in (2), we have:

\[ p\left(y^j - y^j\right) + w\left(x^j - x^j\right) = 0. \]

### 2.2 The optimal consumption tax under OBR

We now want to derive the optimal consumption tax on good \( y \) in region 1, given that the region has already implemented an emission tax \( (t^j) \) on goods \( y \) and \( z \), combined with OBR \( (s^j) \) to good \( y \).

Welfare in region 1 is given by:

\[ W^1 = u'(x^1, y^1, z^1) - c^{x1}(x^1) - c^{y1}(y^1, e^1) - c^{z1}(z^1, e^1) - \tau(e^1 + e^2 + e^1 + e^2), \]
where \( \tau \) is the fixed shadow cost of emissions, i.e., the Pigouvian tax. We assume that emissions abroad are valued by the same shadow cost as emissions at home. This is a reasonable assumption for greenhouse gas emissions, with spatially independent emissions damage. We then have the following result:

**Lemma 1.** Let welfare in region 1 be given by equation (5), and assume that the emissions tax is set equal to the Pigouvian tax, i.e., \( \tau^1 = \tau \). Then the welfare maximizing consumption tax \( v^1 \) on good \( y \) is given by:

\[
(6) \quad v^1 = \left( \frac{\partial y^1}{\partial y^1} \right)^{-1} \left[ s^1 \frac{\partial y^1}{\partial y^1} + \tau \left( \frac{\partial e^2 \partial y^2}{\partial y^2} \frac{\partial y^2}{\partial y^2} + \frac{\partial e^2 \partial z^2}{\partial z^2} \frac{\partial y^2}{\partial y^2} \right) - \frac{\partial p}{\partial y^1} \left( y^1 - y^1 \right) - \frac{\partial w}{\partial y^1} \left( x^1 - x^1 \right) \right].
\]

**Proof.** See Appendix A.

The first factor \((a)\) in (6) is negative, as a higher consumption tax on good \( y \) in region 1 reduces consumption of this good in that region (see Appendix A). Hence, the sign of \( v^1 \) is the opposite of the sign of the bracket.

The first term inside the bracket \((b)\) is negative, as reduced demand for good \( y \) in region 1 reduces the market price of \( y \) and hence output of good \( y \) in both regions. This term reflects that the OBR-subsidy, which reduces leakage through depressing foreign production, has a negative by-effect as it leads to too much consumption (marginal production cost in region 1 exceeds the consumer price in both regions). The optimal consumption tax corrects for this.

The two next terms capture emission effects in the foreign region. Term \((c)\) is negative by the same reasoning as for term \((b)\), and the fact that emissions are increasing in output. The sign on term \((d)\) is a priori ambiguous and depends on the cross derivatives of the utility function in region 2, in particular whether \( z \) is a complement or a substitute to good \( y \). As the consumption tax reduces the price of \( y \), consumption of this good in region 2 increases. This will tend to reduce the consumption of other goods, and hence production of the non-tradable \( z \) good, in region 2 unless \( y \) and \( z \) are complements (in consumption). Moreover, because \( z \) is typically dominated by electricity generation and transport, and electricity is an important input into production of many EITE-goods, reduced output of \( y \) in Region 2 will also tend to decrease consumption (and thus production) of \( z \). For these two reasons, we find it likely that the sign of \( \frac{\partial e^2}{\partial y^1} / \frac{\partial z^2}{\partial y^1} \) is negative. In any case, it is very likely that this second-order effect is dominated by the first-order effect \((c)\). We will henceforth make the following assumption:

\[
(7) \quad \frac{\partial e^2}{\partial y^1} \frac{\partial y^2}{\partial y^1} + \frac{\partial e^2}{\partial z^2} \frac{\partial z^2}{\partial y^1} < 0,
\]
which of course is always true if \( \frac{\partial z^2}{\partial v^1} < 0 \). \(^9\)

The last term (e) is terms-of-trade effects. Whereas the price of good y (p) decreases, the price of good x (w) will increase due to increased demand. If region 1 is initially a net importer (exporter) of good y and net exporter (importer) of good x, both last terms are negative (positive). Note that the balance of payments constraint (4) requires that if region 1 imports good y, it must export good x (and vice versa). Hence, we have shown the following result:

**Proposition 1.** Consider a region that combines a Pigouvian tax on emissions with a subsidy to production of an emission-intensive, tradable good y, and considers a consumption tax on good y. Then we have the following:

- The optimal consumption tax on good y is unambiguously positive if the region is not a net exporter of good y.

- If the region is a net exporter of good y, then the optimal consumption tax on good y is positive if and only if the disadvantageous terms-of-trade effects are dominated by the beneficial effects from reducing excessive production of good y and emissions abroad.

**Proof.** The proposition follows from equations (4), (6) and (7).

So far, we have assumed that region 1’s policy objective when setting the consumption tax is to maximize welfare in region 1. It is also interesting to consider the case where region 1 is concerned about effects on global welfare, including the cost of emissions as before. \(^10\) Global welfare is:

\[
W^G = \sum_{j=1,2} \left[ u^j(x^j, y^j, z^j) - c^{yj}(x^j) - c^{yj}(y^j, e^j) - c^{yj}(z^j, e^j) - \tau(e^j + e^j) \right].
\]  

The consumption tax \( v^{1**} \) that maximizes global welfare (8) is given by (see Appendix A):

\[
v^{1**} = \left( \frac{\partial y^1}{\partial v^1} \right)^{-1} \left[ s^1 \frac{\partial y^1}{\partial v^1} + \tau \left( \frac{\partial c^{yj}}{\partial y^1} \frac{\partial^2 y^1}{\partial v^1} + \frac{\partial c^{yj}}{\partial z^1} \frac{\partial^2 z^1}{\partial v^1} \right) \right] > 0.
\]

We observe that equation (9) is equal to equation (6) when terms-of-trade effects are zero. Thus, we have the following result:

\(^9\) In the simulations in Section 3, the sign of \( \frac{\partial z^2}{\partial v^1} \) is consistently negative.

\(^10\) For example, in Böhringer et al. (2014a), a coalition of countries concerned about leakage chooses the policy that maximizes global welfare. Böhringer et al. (2014b) decomposes leakage and terms-of-trade motives of differential emission pricing, as such pricing can be used as a “beggar-thy-neighbor policy” to exploit terms of trade.
**Proposition 2.** Consider a region that combines a Pigouvian tax on emissions with a subsidy to production of an emission-intensive, tradable good \(y\). If the regulator in this region maximizes global welfare, then the optimal consumption tax on good \(y\) in this region is unambiguously positive.

**Proof.** The proposition follows from equations (7) and (9).

There are some interesting cases worth elaborating on. To simplify the discussion, we here disregard the terms-of-trade effect, i.e., assume that it is equal to zero. First, the optimal consumption tax on good \(y\) obviously increases in the OBR subsidy \(s^1\). However, we also observe that the tax is unambiguously positive also without OBR (i.e., \(s^1 = 0\)). The reason is that reduced domestic demand for good \(y\) reduces imports of \(y\), and hence reduces environmental damages from emissions abroad (emissions at home are already accounted for by the emissions tax). Thus, in the case where region 1 has implemented (only) a Pigouvian tax, the region should also tax consumption of emission-intensive, tradable goods. We state this finding in the following corollary:

**Corollary 1.** Consider a region that has implemented a Pigouvian tax on emissions. Then the optimal consumption tax on an emission-intensive, tradable good \(y\) is unambiguously positive if the region is not a net exporter of good \(y\), or if the regulator in region 1 maximizes global welfare.

**Proof.** The corollary follows from the discussion above.

Next, consider the case where region 1 is much larger than region 2. In this case, changes in domestic production and consumption are much larger than changes in foreign production and consumption. We see from equation (6) that this yields:

\[ v^1 \approx s^1 \]

The reason is that OBR is typically implemented to mitigate emission leakage and loss in competitiveness induced by environmental regulation. However, the effects of this policy are not only to shift market shares towards the domestic firm, but also to stimulate excessive global use of this good. Therefore, the regulator wants the consumption tax to reduce the demand for good \(y\). In this special case, when impacts in region 2 are negligible compared to in region 1, the optimal consumption tax completely offsets the distortion to the economy caused by the OBR subsidy.\(^{12}\) The economic reasoning behind this is simple: carbon leakage is not an important issue when the domestic region is much larger than the foreign region. Hence, it is not a good idea to introduce OBR in the first place, and the optimal consumption tax negates the effects of OBR. The same result holds if the size of

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\(^{11}\) A similar result is found by Eichner and Pethig (2015b), who demonstrate that a combination of production-based (i.e., emission) and consumption-based taxes is less expensive than a production-based tax alone.

\(^{12}\) The first order conditions (2) and (3) imply \(u^i_y - c^i_y = v^i - s^i\), which becomes \(u^i_y \approx c^i_y\) if \(v^i \approx s^i\).
region 2 is more comparable with region 1, but both production and consumption in region 2 are insensitive to the consumption tax in region 1. In our model with homogenous goods, this would be the case if, e.g., large transport cost were introduced or if both the marginal cost function and marginal utility for good y in region 2 are very steep. In a model with heterogeneous goods (see the numerical analysis in Section 3), the (Arminston) substitution elasticities between domestic and foreign goods are important for how sensitive foreign consumption and production are to the consumption tax.

For many sectors, it may be difficult to determine how leakage exposed the sector really is and, correspondingly, whether or not it should be included in an OBR regime. The above results suggest that a policy which combines OBR with a consumption tax is more robust with respect to uncertainties about sector leakage than OBR alone. The reason is that, because the consumption tax offsets the distortive effects of the output subsidy, the risk of including too many sectors in an OBR-regime is reduced when the consumption tax is added.

Another interesting special case is 100% OBR, i.e., \( s^t = t^i y^t \) (see above). Given a Pigouvian emissions tax \( t^i = \tau \), this implies \( \tau = s^i y^t / e^t \). Assume, for the sake of argument, that the average emissions intensity of good y in region 1 is equal to the marginal emissions intensity in region 2, i.e., \( e^t / y^t = \partial e^2 / \partial y^2 \). Equation (6) then becomes:

\[
\nu^t = \left( \frac{\partial y^1}{\partial v^1} \right)^{-1} \left[ \frac{\partial \nu^1}{\partial v^1} + \frac{\partial \nu^2}{\partial v^1} + \frac{y^1 \partial e^2 / \partial v^2}{e^1 \partial e^2 / \partial v^2} \right] s^1 .
\]

Since consumption in region 2 increases in the consumption tax in region 1, the sum of the two first terms inside the bracket is lower in absolute value than \( \partial y^1 / \partial v^1 \). Thus, unless \( \partial z^2 / \partial v^1 \) is too large and negative, we have \( \nu^t < s^t \). That is, contrary to the first special case, the regulator does not wish to completely offset the OBR subsidy, because the tax also stimulates consumption in region 2. For instance, if the tax only shifts consumption from region 1 to region 2, with no net effects on production, the tax has no impact on emissions and the optimal consumption tax is zero. More generally, the more the consumption tax is able to reduce overall production rather than shifting consumption abroad, the higher should the tax be.

Regarding emissions intensities in the two regions, we note that it may be more reasonable to assume \( e^t / y^t < \partial e^2 / \partial y^2 \), mainly because the emissions tax will reduce the emission intensity in region 1, but also because marginal emission intensities will tend to exceed average intensities.\(^{13}\) Hence, the optimal

\(^{13}\) At least this is most likely the case in the short run, when marginal production cost exceeds average production cost. Higher unit cost will typically be associated with higher energy intensities and thus emission intensities. In the long run, increased production may be associated with new installations with more efficient technologies, in which case marginal emission intensities could be lower than the average.
size of the consumption tax is not necessarily lower than \( s^1 \), and could exceed the subsidy if the emissions intensity in region 2 is significantly higher than in region 1 and the consumption tax affects global production more than consumption in region 2.

### 2.4 Equivalence between OBR and consumption tax, and border carbon adjustment

In this subsection we show that the combination of OBR and consumption tax on good \( y \) is equivalent to a certain specification of border carbon adjustments (BCA) on good \( y \) (assuming that the same emissions tax is in place). Let \( \pi^1 \) denote the carbon tariff on imports of good \( y \) to region 1, and let \( \gamma^1 \) denote the export rebate to exports of good \( y \) from region 1. We still assume no climate policy in region 2, so that \( \gamma^2 = \pi^2 = t^2 = 0 \).

A carbon tariff is an import tariff on the embodied carbon in the imported good, proportional to the emission price in the importing region. Ideally, the tariff should reflect the emission intensity of the exporting firm, giving this firm incentive to reduce emissions (see Böhringer et al., 2015). However, such a system may be difficult and costly to implement, and hence analysis of carbon tariffs usually assume that the tariff is determined based on some average emission intensity. This average can either be the average emission intensity in the exporting region (which could be differentiated across regions if there were more than one export region), or the average emission intensity in the importing region. Both these variants are examined in the literature (see, e.g., Böhringer et al., 2012b; Kuik and Hofkes, 2010; and Mattoo et al., 2009). As mentioned in the introduction, non-differentiated tariffs are more likely to be compatible with the GATT rules, and this is what we consider here. When the tariff is based on the emission intensity in the import region, \( \pi^1 = t^1 e^i / y^i \). Export rebates under BCA proposals are usually set equal to \( \gamma^i = t^i e^i / y^i \), so the export rebate and the carbon tariff are equal in this case. Moreover, we notice that \( \gamma^i = \pi^i = s^i \) in the case we have referred to as 100% OBR.

The maximization problems for producers of goods \( x \) and \( z \) under BCA are equal to the OBR case. Hence, their first order conditions are as given in equation (2). Producers of good \( y \) in region \( j \) maximize profits:

\[
\max_{\gamma^j, \pi^j, e^j} \left[ (p^1 - \pi^i) y^{ij} + (p^2 + \gamma^j) y^{2j} - c^y(y^j, e^j) - t^i e^i \right],
\]

where \( i \neq j \). This gives the following first order conditions for interior solution:

\[
\begin{align*}
    p^1 &= p^2 + \gamma^1 = c^{y1}_y; \quad p^1 - \pi^1 = p^2 = c^{y2}_y, \\
    -c^{y1}_{e^i} &= t^1; \quad c^{y2}_{e^i} = 0
\end{align*}
\]

(10)
For producers in region 1, the net price home and abroad are \( p^1 \) and \( p^2 + \gamma^1 \), respectively, while for producers in region 2, the net price home and abroad are \( p^2 \) and \( p^1 - \pi^1 \), respectively. An interior solution requires equal net prices on exports and domestic sales, implying \( p^1 = p^2 + \gamma^1 \) and \( p^2 = p^1 - \pi^1 \). That is, the price in region 1 must equal the price in region 2 plus a “markup” equal to \( \gamma^1 = \pi^1 \). Notice that if we had specified the carbon tariff differently, so that \( \gamma^1 \neq \pi^1 \), we would not have an interior solution in this model with homogenous goods.\(^{14}\)

The consumer utility maximization problem is similar as under OBR and a consumption tax, but with \( v^j = 0 \) in (3). The budget constraint under BCA is still given by equation (4), where \( p \) denotes the international price of good \( y \) and also the price in region 2 (\( p \equiv p^2 \)). The first order conditions for good \( y \) in (2), (3) and (10) may then be rewritten as in Table 1.

### Table 1. First order conditions for good \( y \) under the two types of regulations

<table>
<thead>
<tr>
<th></th>
<th>OBR+Tax</th>
<th>BCA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Production</strong></td>
<td>( p + s^1 = c^1_y ); ( p = c^2_y )</td>
<td>( p + \gamma^1 = c^1_y ); ( p = c^2_y )</td>
</tr>
<tr>
<td><strong>Abatement</strong></td>
<td>( -c^1_z = t^1 ); ( c^2_z = 0 )</td>
<td>( -c^1_z = t^1 ); ( c^2_z = 0 )</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td>( u^1_y = p + v^1 ); ( u^2_y = p )</td>
<td>( u^1_y = p + \pi^1 ); ( u^2_y = p )</td>
</tr>
</tbody>
</table>

In addition, equilibrium requires the market equilibrium condition (1) and the budget constraint (4) to hold under both types of regulation. It is also straightforward to see that net government revenues are the same in the two cases.

We then have the following result:

**Proposition 3.** The two types of regulation i) emissions tax with OBR and consumption tax, and ii) emissions tax with BCA as specified above, induce equal production, consumption and emissions in both regions if \( v^1 = s^1 = \pi^1 = \gamma^1 \).

**Proof.** According to Table 1, all first order conditions for good \( y \) are equal. Moreover, first order conditions (2) and (3) for the goods \( x \) and \( z \) are equal, too. Market equilibrium conditions and budget constraints for all goods are given by equations (1) and (4), respectively, in both cases. The second

\(^{14}\) In a model with heterogeneous goods, interior solution is feasible also when the carbon tariff deviates from the export rebate. However, equivalence still requires that these are identical, see (the proof of) Corollary 3 and the numerical analysis in Section 3.
order conditions put identical constraints on the cost and utility functions under both types of regulations. The proposition follows.

Proposition 3 implies that under certain conditions, combining output-based rebating with a consumption tax has the same effect as full border carbon adjustments. As BCA is regarded as more contentious, though more effective than OBR, combining OBR with a consumption tax can be a viable policy alternative to implementing BCA.

In the discussion leading up to Proposition 3, we assumed that the carbon tariff is determined based on the emission intensity in region 1. However, it is straightforward to see that the proposition also may hold for different levels of carbon tariffs, given that the export rebate is equal to the tariff. Then by adjusting the OBR rate and the consumption tax accordingly, the equivalence still holds. The only requirement is that \( v^1 = s^1 = \gamma^1 = \pi^1 \). Thus, if the regulator in region 1 would like to impose a higher carbon tariff (and export rebate) than the one following from the domestic emission intensity, e.g., because emission intensities abroad are higher than at home, the same result can be achieved by imposing a combination of OBR and consumption tax. We state this generalization as a separate corollary:

**Corollary 2.** The two types of regulation i) emissions tax with OBR and consumption tax, and ii) emissions tax with BCA, are equivalent for any level of carbon tariff as long as \( v^1 = s^1 = \pi^1 = \gamma^1 \).

**Proof.** The proof follows from the proof of Proposition 3.

Whereas the motivation for OBR and BCA typically is to mitigate carbon leakage through the international product markets, the assumption that the good \( y \) is homogeneous and independent of region of origin is unrealistic for many emission-intensive and trade-exposed goods. We therefore state the following corollary:

**Corollary 3.** Consider the case where \( y \) goods produced in different regions are imperfect substitutes. Then we have the following:

- The optimal consumption tax on good \( y \) is positive unless the terms-of-trade effects are sufficiently negative.

- The two types of regulations i) emissions tax with OBR and consumption tax, and ii) emissions tax with BCA as specified above, induce equal production, consumption and emissions in both regions if \( v^1 = s^1 = \pi^1 = \gamma^1 \).

**Proof.** See Appendix A.
Finally, in the real world there are several EITE goods that are exposed to leakage, and these will typically have different carbon tariffs in a BCA system. It is straightforward to show that the equivalence result still holds in this case, too, as long as the output-based rebating is good specific, i.e., emission payments from the production of one specific good is rebated back to producers of this specific good. Thus, we have the following corollary:

**Corollary 4.** Consider the case with several EITE goods $y_j$. Then the two types of regulation i) emissions tax with OBR-rates $s_j$ and consumption taxes $v_j$ for good $j$, and ii) emissions tax with BCA specified by $\pi_j$ and $\gamma_j$, are equivalent as long as $v_j = s_j = \pi_j = \gamma_j$.

**Proof.** The proof is straightforward and is available from the authors upon request.

### 3. Stylized Numerical Analysis

We transfer our theoretical analysis to numerical simulations with a stylized computable general equilibrium (CGE) model in order to gain insights into the potential magnitude of economic effects based on empirical data and accommodating more functional complexity. Below we first summarize the main characteristics of the numerical model in a non-technical manner (see Appendix B for an algebraic model summary). We then discuss the parameterization of the model based on empirical data. Finally, we describe the specification of illustrative policy scenarios and interpret the simulation results.

#### 3.1 Non-technical model summary

We consider two regions (R1 and R2) with four production sectors: carbon-free and tradable production ($NC_T$), carbon-intensive and tradable production ($C_T$), carbon-intensive and non-tradable production ($C_NT$), and (fossil) energy production ($FE$). Sectors $NC_T$, $C_T$, and $C_NT$ correspond to the goods $x$, $y$ and $z$ in the analytical model, respectively. In the numerical model, these goods can be used both as intermediate inputs into production and in final consumption. Emissions are modelled as proportional to energy use instead of as a direct input. To keep in line with the analytical model, energy can not be used in final consumption, and can neither be traded between regions. Thus, we implicitly suppress the energy channel for carbon leakage, as we want to focus on the competitiveness channel examined in the theoretical analysis.

Primary factors of production include labor, capital, and specific energy resources. In the central case model settings, labor and capital are intersectorally mobile within a region but immobile between regions. The energy resource is specific to the energy production sector.

Producers combine primary factors and intermediate inputs at least cost subject to technological constraints. Production of non-energy goods is captured by three-level constant elasticity of
substitution (CES) cost functions describing the price-dependent use of capital, labor, energy and other intermediate inputs. At the top level, non-energy intermediate inputs trade off with a composite of energy, capital and labor, subject to a constant elasticity of substitution. At the second level, a CES function describes the substitution possibilities between energy and a value-added composite of labor and capital. At the third level, capital and labor enter the CES value-added composite. In the production of energy, all inputs except for the specific energy resource are combined in fixed proportions. This Leontief composite trades off with the energy resource at a constant elasticity of substitution.

Final consumption demand in each region is determined by the representative agent who maximizes welfare subject to a budget constraint. Total income of the representative household consists of factor income and net revenues from emission regulation. Consumption demand of the representative agent is given as a CES composite of final consumption goods.

Figures 1-3 sketch the nesting of functional forms in production and consumption together with the default elasticities underlying our central case simulations.

**Figure 1: Nesting in non-energy production**

![Diagram of nesting in non-energy production](image)

Elasticities: $\sigma_{KLE,M} = 0.25$; $\sigma_{KLE} = 0.5$; $\sigma_M = 0$; $\sigma_{KL} = 1$

**Figure 2: Nesting in energy production**

![Diagram of nesting in energy production](image)

Elasticities: $\sigma_R = 0.9$; $\sigma_{NR} = 0$
Figure 3: Nesting in final consumption

Elasticities: $\sigma_C = 0.5$

Since emissions are linked in fixed proportions to the use of energy, emission reductions in response to emission pricing will take place by energy savings. The latter can take place either through substitution of energy through other non-energy inputs or through scale reduction of production and final demand activities.

Only the two goods $C_T$ and $NC_T$ can be traded bilaterally (with no transport cost). A balance of payment constraint incorporates the base-year trade deficit or surplus for each region. The stylized model can reflect two alternative trade paradigms – either trade in homogeneous goods or trade in heterogeneous goods. In case of heterogeneous goods, we follow Armington’s differentiated goods approach, where domestic and foreign goods are distinguished by origin (Armington, 1969). All goods used on the domestic market in intermediate and final demand correspond to a CES composite that combines the domestically produced good and the imported good from the other region. The size of the (Armington) substitution elasticities determine how close substitutes goods produced in different regions are. In case of homogeneous trade, only net trade flows matter such that there is no crosshauling. The simulations presented below are based on the heterogeneous goods setting, and we only briefly refer to the homogeneous goods setting.

3.2 Data and parametrization

As to parameterization, we adopt the standard calibration procedure in applied general equilibrium analysis in which a balanced base-year dataset determines the free parameters of the functional forms (i.e., cost and expenditure functions) such that the economic flows represented in the data are consistent with the optimizing behavior of the economic agents.

To have the stylized numerical analysis closely related with our theoretical exposition, we restructure an empirical dataset in line with the fundamental settings of the theoretical part. Our dataset is based on the most recent GTAP9 data for the world economy (base-year 2011) with 57 sectors and 140

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15 While the focus of our stylized numerical simulations is to substantiate the theoretical analysis, the more specific assumptions of the theoretical part can be readily relaxed in the numerical setting. As a matter of fact, we have performed comprehensive sensitivity analysis on base-year data, trade specifications, elasticity values, factor mobility assumptions, etc. to test the robustness of our theoretical analysis.
regions. We first allocate all 57 GTAP sectors into the four sectors in our model as displayed in Table 2. Then we construct a social accounting matrix (SAM) for the global economy based on the GTAP data (King, 1985). Since the $NC_T$ good is assumed to be carbon-free, we set (fossil) energy use in this sector equal to zero.\(^\text{16}\)

**Table 2. Allocation of GTAP sectors (GTAP acronyms are provided in the brackets)**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE</td>
<td>Coal (COA), Crude oil (CRU), Natural gas (GAS, GDT)</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Refined oil (OIL), Ferrous metals (I_S), Metals (NFM), Mineral products (NMM), Chemical, rubber, plastic products (CRP), Machinery and equipment (OME), Paper products, publishing (PPP)</td>
</tr>
<tr>
<td>$C_{NT}$</td>
<td>Electricity (ELE), All transport sectors (ATP, OTP, WTP)</td>
</tr>
<tr>
<td>$NC_T$</td>
<td>All remaining goods and services</td>
</tr>
</tbody>
</table>

Next, we divide the world into two identical regions to follow the symmetry assumption in the theoretical analysis.\(^\text{17}\) Thus, each entry in the SAM for region $j$ is half of the corresponding entry in the global SAM. As there is no trade in the global SAM, we have to make an assumption about initial trade volumes between the two regions. For each of the two goods $C_T$ and $NC_T$ we simply assume that 50\% of the trade observed in 2011 (according to the GTAP data) takes place between regions 1 and 2. As mentioned before, we assume no trade for $C_{NT}$ and $FE$.

The derived SAM for each region is displayed in Table 3. The entries constitute value flows with negative values being inputs (demands) and positive values being output or endowments (supplies). Since the base-year data is given in value terms, we have to choose units for goods and factors to separate price and quantity observations. A commonly used convenient convention is to choose units for both goods and factors to have a price of unity in the base-year such that values readily transfer into quantities.\(^\text{18}\) In general, data consistency of a social accounting matrix requires that the sums of each of the rows and columns equal zero. Whereas market equilibrium conditions (including trade balance) are associated with the rows, the columns capture the zero-profit condition for production sectors as well as the income balance for the aggregate household sector.

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\(^{16}\) In the original GTAP9 dataset, this sector just accounts for 3-4\% of total fossil energy use.

\(^{17}\) This implies that there are no terms-of-trade effects by the policies (at the margin).

\(^{18}\) We abstract from explicit tax wedges and use gross-of-tax values throughout to suppress initial tax distortions which are absent in our analytical model.
Table 3. Base-year data for stylized model simulations. Social accounting matrix (in bn USD)
for each region*

<table>
<thead>
<tr>
<th></th>
<th>$C_{T}$</th>
<th>$C_{NT}$</th>
<th>$NC_{T}$</th>
<th>$FE$</th>
<th>$X$</th>
<th>$M$</th>
<th>$FD$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{T}$</td>
<td>4521</td>
<td>-659.5</td>
<td>-3281</td>
<td>-40.5</td>
<td>-565</td>
<td>565</td>
<td>-540</td>
<td></td>
</tr>
<tr>
<td>$C_{NT}$</td>
<td>486</td>
<td>3136.5</td>
<td>-1495.5</td>
<td>-51</td>
<td></td>
<td></td>
<td>-1104</td>
<td></td>
</tr>
<tr>
<td>$NC_{T}$</td>
<td>-1189</td>
<td>-816.5</td>
<td>26189</td>
<td>-221</td>
<td>-1440</td>
<td>1440</td>
<td>-23962.5</td>
<td></td>
</tr>
<tr>
<td>$FE$</td>
<td>-994.5</td>
<td>-203.5</td>
<td>1198</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$LAB$</td>
<td>-957</td>
<td>-733.5</td>
<td>-13001.5</td>
<td>-127</td>
<td></td>
<td></td>
<td>14819</td>
<td></td>
</tr>
<tr>
<td>$CAP$</td>
<td>-894.5</td>
<td>-723.5</td>
<td>-8411</td>
<td>-462</td>
<td></td>
<td></td>
<td>10491</td>
<td></td>
</tr>
<tr>
<td>$RES$</td>
<td>-296.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>296.5</td>
</tr>
<tr>
<td>$INC_{EXP}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25606.5</td>
<td>-25606.5</td>
<td></td>
</tr>
<tr>
<td>$BOP$</td>
<td>2005</td>
<td>-2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $X$ denotes exports, $M$ imports, $FD$ final demand, $C$ consumption, $LAB$ labour, $CAP$ capital, $RES$ energy resource, $INC_{EXP}$ income-expenditure constraint, $BOP$ balance-of-payment constraint

Elasticity values are provided in Figures 1-3 along with the nesting structure in production and consumption. As to the heterogeneous goods variant, the default Armington elasticity is set to 2. However, we vary this elasticity in our simulations.

### 3.3 Scenarios

Our reference scenario (REF) for unilateral climate policy is a situation where a single country (or country coalition) – here: region R1 – undertakes uniform emission pricing to achieve an exogenous domestic emission reduction target. In our central case simulations, we set the unilateral emission reduction target at 20 percent of the base-year emissions. We use the stylized numerical model to quantify how the REF outcome changes if the region adopts in addition either full border carbon adjustments (BCA), or output based rebating combined with a consumption tax ($OBR+Tax$). In both cases, the additional policies are directed only towards the $C_{T}$ good, i.e., the emission-intensive and trade-exposed commodity. In the BCA case, the carbon tariff and the export rebate are determined based on the domestic emission intensity (see Section 2). In the $OBR+Tax$ case, we assume full rebating and consider different levels of the consumption tax, which is levied on both final consumption and intermediate use of the $C_{T}$ good. We indicate the different levels of the consumption tax as a fraction $x$ of the OBR rate where we increase $x$ subsequently in steps of 20 percentage points from 0% to 200%. Obviously, $OBR+Tax$ includes output-based rebating stand-alone.

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19 Uniform emission pricing to achieve some emission reduction target can either be implemented through an emission tax which is set at a sufficient level or equivalently through a cap-and-trade system.
as a special case when we set the consumption tax to zero ($x=0\%$). As demonstrated in our theoretical analysis (see Proposition 3 and Corollary 3), $OBR+Tax$ is equivalent to $BCA$ when the consumption tax is set equal ($x=100\%$) to the implicit output subsidy under output-based rebating (given our specification of $BCA$).

Table 4. Policy scenarios for region R1

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$REF$</td>
<td>Emission price only</td>
</tr>
<tr>
<td>$OBR+Tax$</td>
<td>Full output-based rebating + $x%$ consumption tax for the $C_T$ good</td>
</tr>
<tr>
<td>$BCA$</td>
<td>Full border carbon adjustments</td>
</tr>
</tbody>
</table>

Considering that the climate is a global public good, a coherent cross-comparison of results requires that we keep global emissions constant unless we can value the damage from emissions. Here, we do not attempt to trade off the abatement cost with the benefit from avoided climate change but restrain ourselves to a cost-effectiveness analysis. Therefore, we require the abating region R1 to adjust its unilateral emissions reduction effort to meet a given global emission cap. The cap is taken as the global emission level which emerges from scenario $REF$. If additional policy measures such as $OBR+Tax$ turn out to reduce leakage compared to $REF$, then the effective unilateral emission reduction requirement will be lower than the $REF$ target. Technically, the global emissions constraint requires an endogenous scaling of the unilateral emission target to compensate leakage toward the exogenous global emissions constraint. The shadow price on the emission cap corresponds to the endogenous emission tax under price regulation.

A key parameter regarding the magnitude of emission leakage through the competitiveness channel is the Armington elasticity for emission-intensive and trade-exposed goods ($C_T$ in our case). The choice of the Armington elasticity determines the ease of substitution between the domestically produced good and its foreign counterpart. The higher this elasticity, the more pronounced leakage ceteris paribus becomes. To investigate the robustness of our findings, we provide simulation results for alternative choices of the Armington elasticity (for $C_T$) ranging from a lower end value of 1 to an upper end value of 8.

3.4 Results

In our results discussion, we first check if the equivalence result between $BCA$ and $OBR+Tax$ holds (when $x=100\%$). We then investigate changes in leakage rates, welfare, and production output as the key indicators of policy interest. The leakage rate is defined as the ratio of the emission change in the non-abating region over the emission reduction in the abating region. Welfare effects are defined as
Hicksian equivalent variation (HEV) in income as a percentage of the pre-policy equilibrium levels – the so-called business-as-usual (BAU).

We find that the numerical results are in accordance with the equivalence results in Proposition 3 and Corollary 3. That is, given emissions pricing (tax or quotas), the combination of output-based rebating and a consumption tax equal to the OBR rate \((OBR+\text{Tax} \text{ with } x=100\%)\) gives exactly the same outcome as border carbon adjustment (BCA) as specified above. This equivalence result is robust independent of whether we assume homogenous or heterogeneous goods, and whatever assumptions we make about elasticities, emission target etc.

Next we turn to the leakage mitigation which is a central policy motivation for supplementing unilateral emission pricing with either OBR or BCA. Previous studies have suggested that BCA is more effective in leakage mitigation than OBR. Figure 4 shows how the combination of OBR and consumption tax affects leakage across alternative choices of the \(C_{T}\) Armington elasticity as we increase the consumption tax from 0% to 200% of the OBR-rate (note that \(x=0\%\) and \(x=100\%\) are replaced with respectively OBR and BCA in all the figures). As expected, leakage rates go up towards higher Armington elasticities. Figure 4 clearly shows that introducing OBR reduces the leakage rate significantly, and more so the higher is the Armington elasticity. The leakage rates in fact become negative in all four cases.\(^{20}\) Next, we notice that introducing and increasing the consumption tax decreases the leakage rate further: when the consumption tax rate is set equivalent to the OBR-rate (BCA) the leakage rate drops by another 1-2 percentage points compared to OBR. We also can see that leakage is further reduced when the consumption tax is increased beyond 100%.

Figure 4. Leakage rates under different policy scenarios and Armington elasticities (in %)

\(^{20}\) Note that we deliberately suppress the fuel-fuel-price channel in our analysis, which makes leakage rates rather low to start with in the REF scenario (relative to most numerical analysis in the literature), and explains why anti-leakage measures have a strong potential to drive leakage rates even negative.
In Figure 5 we show how the policies affect economic welfare in region R1. Since we assume that global emissions are the same across all policy scenarios, we do not have to value emission changes. We first notice that the welfare effects of OBR are positive if the Armington elasticity is sufficiently high, i.e., if domestic and foreign goods are sufficiently close substitutes. In this case, the implicit subsidy given by OBR reduces the inefficient relocation of production from region R1 to region R2. On the other hand, if the substitution possibilities between domestic and foreign goods are more limited (here: Armington elasticities for C_T at values of 2 and 1), then the distortionary negative effect of subsidising this good becomes relatively more important and dominates the former positive effect. In this case the welfare cost for region R1 increases rather than decreases when shifting from emission pricing only (REF) to emission pricing combined with OBR.

Figure 5. Welfare effects (HEV) for region R1 under different policy scenarios and Armington elasticities (% change from BAU)

Figure 5 furthermore shows that it is welfare improving for region R1 to implement a consumption tax when output-based rebating is already in place, which is in line with Proposition 1 of our analytical section. Even with high Armington elasticities, the welfare gains of the consumption tax are noticeable peaking at tax rates around 120% of the OBR-rate.

The numerical results provide evidence that OBR may serve as a decent second-best policy for goods that are much exposed to foreign competition, but not so for goods that are less exposed. Moreover, supplementing OBR with a consumption tax is beneficial whether or not the good is much exposed to foreign competition. Thus, when output-based rebating is applied to a certain group of goods, the policy-relevant conclusion from our analysis is to also introduce a corresponding tax on all purchases of the same goods.
Figure 5 shows that a consumption tax is beneficial for region R1, but what about global welfare? According to Proposition 2, introducing such a tax should also be beneficial from a global perspective. Figure 6 shows the global welfare cost of the different policies. Here we notice that $OBR$ increases global welfare cost across alternative values for Armington elasticities, with cost increasing towards lower Armington elasticities. Next, we see that introducing a consumption tax in addition to $OBR$ reduces global welfare cost in all four cases, which is in accordance with Proposition 2. The lowest welfare cost is actually obtained when the consumption tax is around 100% of the $OBR$-rate, i.e., the $BCA$-equivalent rate. This holds irrespective of the choice of Armington elasticities.

**Figure 6. Global welfare effects (HEV) under different policy scenarios and Armington elasticities (% change from BAU)**

Whereas the consumption tax is advantageous for region R1 and also for the two regions jointly, region R2 is worse off by the consumption tax. Welfare in region R2 is monotonically falling in the consumption tax in all the simulations carried out.

Finally, we consider how the policies affect production output in the two regions. Output effects are shown in Figure 7 for the case with an Armington elasticity equal to 2. As expected, the emission price ($REF$) reduces output of the two carbon-intensive goods $C_T$ and $C_{NT}$ in region R1, and increases output of the good $C_T$ in the other region R2. When $OBR$ is introduced, the effects on output of the good $C_T$ are turned around, as region R1 (R2) increases (decreases) its output compared to the BAU level (see the negative leakage rates in Figure 4). When the consumption tax is introduced on the good $C_T$ in region R1, we observe that output of this good is reduced in both regions. This effect is strengthened as the consumption tax is increased.

---

21 Global welfare accounting is based on a utilitarian (Benthamite) perspective on efficiency where welfare changes of individual regions are treated as perfect substitutes.
We see from Figure 7 that also the $C_{NT}$ output in region R1 increases notably when $OBR$ is implemented for the good $C_T$. The explanation behind is that the two emission-intensive goods are used quite a lot as intermediate inputs into each other’s production, see Table 3 (relative to the $NC_T$ good). Thus, when the $C_T$ production is stimulated by the $OBR$ policy, this indirectly stimulates $C_{NT}$ production, too. Output of the carbon-free good $NC_T$ in region R1 declines with the implementation of the emission price as well as with the introduction of $OBR$ and the increase of the consumption tax from zero. This is partly due to reduced income in region R1, and partly because production of this good uses the two carbon-intensive goods as inputs.

**Figure 7. Production output in regions R1 and R2 under different policy scenarios (% change from BAU)**

![Graph showing production output in regions R1 and R2 under different policy scenarios.](image)

### 4. Concluding remarks

In the absence of global cooperation to mitigate global warming, many countries consider or have introduced unilateral climate policies. This causes carbon leakage associated with the relocation of emission-intensive and trade-exposed (EITE) industries. Economic theory and numerical studies suggest that border carbon adjustments, in addition to emission pricing, can be used as a second-best instrument to improve the economic efficiency of unilateral climate policy. However, as carbon tariffs and export rebates are politically contentious to implement, policy makers have typically chosen other instruments such as variants of output-based rebating to EITE industries.

One prime example for such supplemental anti-leakage policy is the EU Emission Trading System (EU ETS) where emission allowances are allocated to EITE industries conditional on output. Martin et al. (2014) find that there has been substantial overallocation of allowances in the EU ETS for the given carbon leakage risk, and they propose a more efficient allocation scheme. However, as the optimal
scheme relies on data that are not publicly observable, they also propose a more “feasible” allocation scheme based on easily observable characteristics of firms such as employment and historic CO$_2$ emissions.

Our paper suggests an alternative strategy, namely to combine output-based rebating to production of EITE goods with a consumption tax on all use of the same EITE goods. We have shown analytically that it is welfare improving for a region to introduce such a consumption tax if output-based rebating is already in place. The theoretical result is confirmed when using a stylized numerical general equilibrium model calibrated to data for the world economy, highlighting also that the welfare gains from additional consumption taxes can be substantial. As tradable emissions allowances and emissions taxes are equivalent when there is no uncertainty (Weitzman, 1974), these results are also valid for emissions trading combined with output-based allocation. The administrative cost of adding such a consumption tax is likely to be moderate as the tax level could be set in proportion to the benchmarks already determined through the allocation mechanisms.

Moreover, we have shown that a certain combination of output-based rebating and a consumption tax is equivalent to full border carbon adjustments as long as the carbon tariffs (and the export rebate) are based on average domestic emission intensities. Thus, whereas border carbon adjustments may be politically contentious to introduce due to WTO considerations, the same outcome can in fact be achieved by supplementing output-based rebating with a corresponding consumption tax.

Last but not least, combining output-based rebates with a consumption tax makes the policy more robust with respect to uncertainties about leakage exposure, because the distortive effects of including too many sectors with limited carbon leakage risk in the allocation regime are moderated. To conclude: supplementing free allocation with a corresponding consumption tax could be a more feasible and cost-effective strategy than tightening the number of industries that receive free allowances.

**References**


Appendix A: Proofs and derivations

Proof of Lemma 1:

Differentiating welfare (5) with respect to the consumption tax we get:

\[
\frac{\partial W^l}{\partial \tau} = u'_y(\cdot) \frac{\partial x^l}{\partial \tau} + u'_z(\cdot) \frac{\partial y^l}{\partial \tau} + u'_v(\cdot) \frac{\partial z^l}{\partial \tau} - c'_y(\cdot) \frac{\partial x^l}{\partial \tau} - c'_z(\cdot) \frac{\partial y^l}{\partial \tau} - c'_v(\cdot) \frac{\partial z^l}{\partial \tau} \\
-(c'_e(\cdot) + \tau) \frac{\partial e^l}{\partial \tau} - (c'_e(\cdot) + \tau) \frac{\partial e^l}{\partial \tau} - \tau \left( \frac{\partial e^l}{\partial \tau} + \frac{\partial e^l}{\partial \tau} \right)
\]

By using the first order conditions (2) and (3) we can simplify this equation:

\[
\frac{\partial W^l}{\partial \tau} = w \left( \frac{\partial x^l}{\partial \tau} - \frac{\partial x^l}{\partial \tau} \right) + \left( p + v^l \right) \frac{\partial y^l}{\partial \tau} - \left( p + s^l \right) \frac{\partial y^l}{\partial \tau} + \omega \left( \frac{\partial z^l}{\partial \tau} - \frac{\partial z^l}{\partial \tau} \right) \\
-(\tau - t^l) \frac{\partial e^l}{\partial \tau} - (\tau - t^l) \frac{\partial e^l}{\partial \tau} - \tau \left( \frac{\partial e^l}{\partial \tau} + \frac{\partial e^l}{\partial \tau} \right)
\]

In addition, from (4), we must have:

\[
\frac{\partial p}{\partial \tau} (y^l - y^l) + \frac{\partial w}{\partial \tau} (x^l - x^l) + w \left( \frac{\partial x^l}{\partial \tau} - \frac{\partial x^l}{\partial \tau} \right) = 0 ; \quad \frac{\partial \tau}{\partial \tau} = \frac{\partial z^l}{\partial \tau}
\]

We assume that the emissions tax is set equal to the Pigouvian tax, i.e., \( t^l = \tau \). Using equation (12) we can then further simplify equation (11):

\[
\frac{\partial W^l}{\partial \tau} = \frac{\partial p}{\partial \tau} (y^l - y^l) + \frac{\partial w}{\partial \tau} (x^l - x^l) + \left( p + v^l \right) \frac{\partial y^l}{\partial \tau} - \left( p + s^l \right) \frac{\partial y^l}{\partial \tau} - \tau \left( \frac{\partial e^l}{\partial \tau} + \frac{\partial e^l}{\partial \tau} \right)
\]

where we also used the fact that emissions in region 2 are only affected via production changes of good \( y \) and \( z \) in region 2. For a given \( s^l \) (from the OBR regulation), we can solve for the optimal \( v^l \) by setting \( \frac{\partial W^l}{\partial \tau} = 0 \). This gives equation (6).

Sign of first factor in equation (6):

To see that the first factor of (6) is negative, note that equations (2) and (3) imply \( u'_y - c'_y = v^l - s^l \).

Because the second order derivatives are non-zero and finite, an increase in \( v^l \) entails that \( u'_y \) increases
and \( c_{ij}^{y} \) decreases when \( s^1 \) is constant. This implies \( \partial y^i / \partial v^j < 0 \) and \( \partial y^j / \partial v^i < 0 \), because \( u_{y^i}^j < 0 \) and \( c_{ij}^{y} > 0 \).

**Derivation of equation (9):**

We differentiate equation (8) and follow the steps explained in the proof of Lemma 1. This gives:

\[
\frac{\partial W^G}{\partial v^j} = \sum_{j=1}^{J} \left[ u_{x^j}^j + u_{y^j}^j + u_{z^j}^j + u_{\tilde{y}^j}^j - c_{x^j}^{y} \frac{\partial y^j}{\partial v^j} - c_{y^j}^{y} \frac{\partial y^j}{\partial v^j} - c_{z^j}^{y} \frac{\partial y^j}{\partial v^j} - \left( c_{x^j}^{y} + \tau \right) \frac{\partial y^j}{\partial v^j} - \left( c_{y^j}^{y} + \tau \right) \frac{\partial y^j}{\partial v^j} \right]
\]

\[
= \sum_{j=1}^{J} \left[ u_{y^j}^j \left( p + v^j \right) \frac{\partial y^j}{\partial v^j} + \left( p + v^j \right) \frac{\partial y^j}{\partial v^j} \left( y^j - y \right) + \left( p + v^j \right) \frac{\partial y^j}{\partial v^j} \left( x^j - x \right) \right] - \tau \left( \frac{\partial e^2}{\partial v^j} + \frac{\partial e^2}{\partial v^j} \right)
\]

We also used \( c_{ij}^{y} = c_{ij}^{y} = s^2 = 0 \). Setting \( \partial W^G / \partial v^G = 0 \) gives equation (9).

**Proof of Corollary 3:**

We now extend the model to heterogeneous \( y \) goods, i.e., \( y \) goods produced in different regions are imperfect substitutes. The representative consumer’s utility from consumption in region \( j \) is given by

\[
u \left( x^j, y^j, z^j \right).
\]

We assume that the Hessian matrix associated with the consumers’ utility maximization problem is negative definite and that \( u_{y^j}^j = \partial u^j / \partial x^j, u_{y^j}^j, u_{x^j}^j, u_{x^j}^j > 0 \). The market equilibrium conditions and the first order conditions w.r.t. goods \( x \) and \( z \) are not affected by the extension to heterogeneous \( y \) goods (i.e., they remain as in equations (1), (2), (3) and (10)). We therefore omit good \( x \) and \( z \) from the analysis below (except in budget constraints).

The market equilibrium condition for each \( y \) good is:
and we have \( y^i = y^{ij} + y^{2j} \). We now show that the first order conditions w.r.t. \( y^i \) are equal across the regimes. The profit maximization problem for \( y^i \) under OBR is:

\[
\max_{y^{ij}, y^{2j}, e^j} \left[ (p^{ij} + s^j)y^{ij} + (p^{2j} + s^j)y^{2j} - c^{y^i}(y^i, e^i) - t^i e^i \right].
\]

The associated first order conditions imply:

\[
\begin{align*}
  p^{11} + s^1 &= p^{21} + s^1 = c^{y^1}_y \\
  p^{12} &= p^{22} = c^{y^2}_y \\
  -e^i_y &= t^i ; \quad c^{e^2}_y = 0
\end{align*}
\]

In the BCA case, competitive producers of \( y^i \) in region \( j \) maximize profits:

\[
\max_{y^{ij}, y^{2j}, x^j, e^j} \left[ (p^{ij} - \pi^j)y^{ij} + (p^{2j} + \gamma^j)y^{2j} - c^{y^i}(y^i, e^i) - t^i e^i \right] \quad i, j \in \{1, 2\} \quad (i \neq j),
\]

This gives the following first order conditions for interior solution:

\[
\begin{align*}
  p^{11} &= p^{21} + \gamma^j = c^{y^1}_y \\
  p^{12} - \pi^j &= p^{22} = c^{y^2}_y \\
  -e^i_y &= t^i ; \quad -c^{e^2}_y = 0
\end{align*}
\]

Finally, the representative consumer in region \( j \) maximizes welfare:

\[
\max_{x^j, y^j, z^j} \left[ u^j(x^j, y^j, z^j) - \left( w^j x^j + \left( p^{11} + v^1 \right)y^{1j} + \left( p^{12} + v^2 \right)y^{2j} - w^j z^j \right) \right].
\]

The associated first order conditions for the \( y^i \) goods are:

\[
(16) \quad u^i_{y^i} = p^y + v^j \quad i = \{1, 2\},
\]

which is valid under OBR and BCA (\( v^j = 0 \) under BCA).

The budget constraint required for import expenditures to equal export revenue in region \( j \) is:

\[
(17) \quad p^y y^i - p^y y^i + w(x^j - x^j) = 0 \quad i, j \in \{1, 2\} \quad (i \neq j),
\]

under OBR and BCA.

Following the steps in the derivation of equation (6), we find the optimal consumer tax with heterogeneous \( y^i \) goods:

\[
v^r = \left( \frac{\partial y^{11}}{\partial v^j} + \frac{\partial y^{12}}{\partial v^j} \right)^{-1} \left( s \frac{\partial y^1}{\partial v^j} + r \left( \frac{\partial e^2}{\partial v^j} \frac{\partial y^2}{\partial v^j} + \frac{\partial e^2}{\partial v^j} \frac{\partial z^2}{\partial v^j} \right) - \frac{\partial p^{21}}{\partial v^j} y^{21} + \frac{\partial p^{12}}{\partial v^j} y^{12} + \frac{\partial w}{\partial v^j} \left( x^1 - x^j \right) \right).
\]
The interpretation is similar to that of equation (6). The first part of Corollary 3 follows.

We now turn to the second part of Corollary 3. Table A1 summarizes and compares the first order conditions for the y goods:

**Table A1. First order conditions under the two regimes with heterogeneous y goods.**

<table>
<thead>
<tr>
<th></th>
<th>OBR + tax</th>
<th>BCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production y1</td>
<td>( p^{11} + s^1 = p^{21} + s^1 = c^y_1 )</td>
<td>( p^{11} = p^{21} + \gamma^1 = c^y_1 )</td>
</tr>
<tr>
<td>Production y2</td>
<td>( p^{12} = p^{22} = c^y_2 )</td>
<td>( p^{12} - \pi^1 = p^{22} = c^y_2 )</td>
</tr>
<tr>
<td>Abatement</td>
<td>(-c^y_1 = t^1 ); ( c^y_2 = 0 )</td>
<td>(-c^y_1 = t^1 ); ( c^y_2 = 0 )</td>
</tr>
<tr>
<td>Consumption y11</td>
<td>( u_{y11} = p^{11} + v^1 = p^{21} + v^1 )</td>
<td>( u_{y11} = p^{11} = p^{21} + \gamma^1 )</td>
</tr>
<tr>
<td>Consumption y12</td>
<td>( u_{y12} = p^{12} + v^1 = p^{22} + v^1 )</td>
<td>( u_{y12} = p^{12} = p^{22} + \pi^1 )</td>
</tr>
</tbody>
</table>

Note that equal consumption across regimes in region 1 implies equal consumption in region 2, given equal production levels and the market equilibrium condition (13). Table A1 shows that the first order conditions are equal across the regimes if \( v^1 = s^1 = \gamma^1 = \pi^1 \). This proves the last part of Corollary 3.
Appendix B: Algebraic summary of the numerical CGE model

Our stylized multi-sector multi-region computable general equilibrium model is formulated as a system of nonlinear inequalities. The inequalities correspond to the two classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero profit) conditions for producers with constant returns to scale; and (ii) market clearance for all goods and factors. The former class determines activity levels, and the latter determines price levels. In equilibrium, each variable is linked to one inequality condition: an activity level to an exhaustion of product constraint and a commodity price to a market clearance condition. In our algebraic exposition, the notation $\Pi_{gr}$ used to denote the unit profit function (calculated as the difference between unit revenue and unit cost) for production with constant returns to scale of sector $g$ in region $r$, where $z$ is the name assigned to the associated production activity. Differentiating the unit profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotelling’s lemma), which appear subsequently in the market clearance conditions. We use $g$ as an index for all sectors/commodities except primary fossil energy and index $r$ (aliased with $s$) to denote region.

Tables B1–B6 explain the notations for variables and parameters employed within our algebraic exposition. Numerically, the model is implemented in GAMS (Brooke et al., 1996) and solved using PATH (Dirkse and Ferris, 1995).

Table B.1: Indices and sets

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>Set of all commodities {NC_T, C_T, C_NT, FE}</td>
</tr>
<tr>
<td>$EG$</td>
<td>Subset of primary energy goods {FE}</td>
</tr>
<tr>
<td>$R$</td>
<td>Set of regions {R1, R2}</td>
</tr>
<tr>
<td>$g$ \textit{(alias} $i$)</td>
<td>Index for sectors and commodities</td>
</tr>
<tr>
<td>$r$ \textit{(alias} $s$)</td>
<td>Index for regions</td>
</tr>
</tbody>
</table>

Table B.2: Activity variables

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{gr}$</td>
<td>Production of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$M_{gr}$</td>
<td>Material composite for commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$KL_{gr}$</td>
<td>Value-added composite for commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$A_{gr}$</td>
<td>Armington aggregate of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$IM_{gr}$</td>
<td>Import aggregate of commodity $g$ in region $r$</td>
</tr>
</tbody>
</table>

Consumption composite in region $r$

**Table B.3: Price variables**

| $p_{gr}$  | Price of commodity $g$ in region $r$ |
| $p_{rM}$  | Price of material composite for commodity $g$ in region $r$ |
| $p_{rKL}$ | Price of value-added composite for commodity $g$ in region $r$ |
| $p_{rA}$  | Price of Armington aggregate of commodity $g$ in region $r$ |
| $p_{rIM}$ | Price of aggregate imports of commodity $g$ in region $r$ |
| $p_{rc}$  | Price of consumption composite in region $r$ |
| $w_r$     | Price of labor (wage rate) in region $r$ |
| $v_r$     | Price of capital services (rental rate) in region $r$ |
| $q_r$     | Rent for primary energy resource in region $r$ |
| $p_{rCO2}$| Price of carbon emissions in region $r$ |

**Table B.4: Cost shares**

| $\theta_{gr}^{M}$ | Cost share of material composite in production of commodity $g$ in region $r$ |
| $\theta_{gr}^{FE}$ | Cost share of primary energy in capital-labor-energy composite input to production of commodity $g$ in region $r$ |
| $\theta_{gr}^{MN}$ | Cost share of input $i$ in material composite of commodity $g$ in region $r$ |
| $\theta_{gr}^{K}$ | Cost share of capital within the value-added of commodity $g$ in region $r$ |
| $\theta_{gr}^{O}$ | Cost share of primary energy resource in primary energy production in region $r$ |
| $\theta_{FE,j}^{LN}$ | Cost share of labor in non-resource composite of primary energy production in region $r$ |
| $\theta_{FE,j}^{KN}$ | Cost share of capital in non-resource input to primary energy production in region $r$ |
| $\theta_{FE,j}^{N,FE,j}$ | Cost share of good $g$ in non-resource input to primary energy production in region $r$ |
| $\theta_{gr}^{A}$ | Cost share of domestic input $g$ in the Armington composite of commodity $g$ in region $r$ |
| $\theta_{gr}^{IM}$ | Cost share of commodity $g$ from region $s$ in import composite of region $r$ |
| $\theta_{gr}^{C}$ | Cost share of commodity $g$ in consumption composite of region $r$ |
Table B.5: Elasticities of substitution

<table>
<thead>
<tr>
<th>符号</th>
<th>意义</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{KLEM}^{gr}$</td>
<td>Substitution between the material composite and the energy-value-added aggregate in production of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$\sigma_{KLE}^{gr}$</td>
<td>Substitution between primary fossil energy and the value-added nest in production of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$\sigma_{M}^{gr}$</td>
<td>Substitution between material inputs within the material composite in production of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$\sigma_{KL}^{gr}$</td>
<td>Substitution between the capital and labor within the value-added composite in production of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$\sigma_{Q}^{gr}$</td>
<td>Substitution between natural resource input and the composite of other inputs in primary energy production in region $r$</td>
</tr>
<tr>
<td>$\sigma_{L}^{gr}$</td>
<td>Substitution between import composite and domestic input to Armington production of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$\sigma_{IM}^{gr}$</td>
<td>Substitution between imports from different regions within the import composite of commodity $g$ in region $r$</td>
</tr>
<tr>
<td>$\sigma_{C}^{gr}$</td>
<td>Substitution between commodity inputs to composite consumption in region $r$</td>
</tr>
</tbody>
</table>

Table B.6: Endowments

<table>
<thead>
<tr>
<th>符号</th>
<th>意义</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_r$</td>
<td>Aggregate labor endowment in region $r$</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Capital endowment in region $r$</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>Resource endowment of primary fossil energy in region $r$</td>
</tr>
<tr>
<td>$CO_2_r$</td>
<td>Endowment with CO$_2$ emission allowances in region $r$</td>
</tr>
<tr>
<td>$\theta_{FE,r}$</td>
<td>CO$_2$ emissions coefficient for primary fossil energy in region $r$</td>
</tr>
</tbody>
</table>

Zero profit conditions

- Production of goods except fossil primary energy ($g \notin EG$):

  $$\Pi_{gr}^{y} = p_{gr} - \left[ \theta_{gr}^{M} p_{gr}^{1-\sigma_{gr}^{KLEM}} + (1-\theta_{gr}^{M}) \left( \theta_{gr}^{FE} p_{FE,r} + a_{FE,r}^{CO_2} p_{r}^{1-\sigma_{gr}^{FE}} \right) \right] + (1-\theta_{gr}^{FE}) p_{gr}^{KL} \left[ \frac{1}{1-\sigma_{gr}^{KL}} \right] \leq 0$$

- Sector-specific material composite ($g \notin EG$):
\[ \Pi^M_{gy} = p^M_{gy} - \left[ \sum_{i \in EG} \theta_{gy}^{MN} p_{gy}^{A(1-\sigma_{gy})} \right]^{1/(1-\sigma_{gy})} \leq 0 \]

- Sector-specific value-added aggregate \((g \notin EG)\):

\[ \Pi^{KL}_{gy} = p^{KL}_{gy} - \left[ \theta_{gy}^{K} v_{gy}^{(1-\sigma_{gy})} + \left(1 - \theta_{gy}^{K}\right) w_{gy}^{(1-\sigma_{gy})} \right]^{1/(1-\sigma_{gy})} \leq 0 \]

- Production of primary fossil fuel:

\[ \Pi^Y_{FE,r} = p^Y_{FE,r} - \left[ \theta_{FE,r}^{Y} d_{FE,r}^{(1-\sigma_{FE,r})} + \left(1 - \theta_{FE,r}^{Y}\right) p_{FE,r}^{A(1-\sigma_{FE,r})} \right]^{1/(1-\sigma_{FE,r})} \leq 0 \]

- Armington aggregate \((g \notin EG)\):

\[ \Pi^A_{gy} = p^A_{gy} - \left[ \theta_{gy}^{A} p_{gy}^{(1-\sigma_{gy})} + \left(1 - \theta_{gy}^{A}\right) p_{gy}^{IM(1-\sigma_{gy})} \right]^{1/(1-\sigma_{gy})} \leq 0 \]

- Import composite \((g \notin EG)\):

\[ \Pi^{IM}_{gy} = p^{IM}_{gy} - \left[ \sum_{i \in EG} \theta_{gy}^{IM} p_{gy}^{A(1-\sigma_{gy})} \right]^{1/(1-\sigma_{gy})} \leq 0 \]

- Consumption composite:

\[ \Pi^C_r = p^C_r - \left[ \sum_{g \in EG} \theta_{gy}^{C} p_{gy}^{A(1-\sigma_{gy})} \right]^{1/(1-\sigma_{gy})} \leq 0 \]

**Market clearance conditions**

- Labor:

\[ L_r \geq Y_{FE,r} \frac{\partial \Pi^Y_{FE,r}}{\partial w_r} + \sum_{g \in EG} Y_{KL}^{KL} \frac{\partial \Pi^{KL}_{gy}}{\partial w_r} \]

- Capital:

\[ K_r \geq Y_{FE,r} \frac{\partial \Pi^Y_{FE,r}}{\partial v_r} + \sum_{g \in EG} Y_{KL}^{KL} \frac{\partial \Pi^{KL}_{gy}}{\partial v_r} \]

- Primary fossil energy resource:
\[ \bar{q}_r \geq Y_{FE_r}^Y \frac{\partial \Pi_{FE_r}^Y}{\partial q_r} \]

- Material composite \((g \not\in EG)\):

\[ M_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial p_M} \]

- Value-added \((g \not\in EG)\):

\[ KL_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial p_{KL}} \]

- Armington aggregate \((g \not\in EG)\):

\[ A_{gr} \geq C_r \frac{\partial \Pi_{gr}^C}{\partial p_{gr}^A} + Y_{FE,r} \frac{\partial \Pi_{FE,r}^Y}{\partial p_{gr}^A} + \sum_{i \in EG} M_{gr} \frac{\partial \Pi_{gr}^M}{\partial p_{gr}^A} \]

- Import composite \((g \not\in EG)\):

\[ IM_{gr} \geq A_{gr} \frac{\partial \Pi_{gr}^A}{\partial p_{IM}^A} \]

- Goods except primary energy \((g \not\in EG)\):

\[ Y_{gr} \geq A_{gr} \frac{\partial \Pi_{gr}^A}{\partial p_{gr}} + \sum_{i \in EG} \frac{\partial \Pi_{gr}^{IM}}{\partial p_{gr}} \]

- Primary energy:

\[ Y_{FE,r} \geq \sum_{g \in EG} Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial \left(p_{FE,r} + a_{FE,r}^CO p_r^{CO2} \right)} \]

- Private consumption \((g = C)\):

\[ p^C_r C_r \geq w_r \bar{L}_r + v_r \bar{K}_r + q_r \bar{Q}_r + p_r^{CO} \overline{CO2}_r \]

- Carbon emissions:

\[ \overline{CO2}_r \geq a_{FE,r}^{CO} Y_{FE,r} \]