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Political economics of external sovereign defaults

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No. 508

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Political Economics of External Sovereign Defaults
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Political Economics of External Sovereign Defaults*

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Abstract

We develop a dynamic recursive model where political and economic decisions interact, to study how excessive debt-GDP ratios affect political sustainability of prudent fiscal policies. Rent seeking groups make political decisions— to cooperate (or not)—on the allocation of fiscal budgets (including rents) and issuance of sovereign debt. A classic commons problem triggers collective fiscal impatience and excessive debt issuing, leading to a vicious circle of high borrowing costs and sovereign default. We analytically characterize debt-GDP thresholds that foster cooperation among rent seeking groups and avoid default. Our analysis and application helps in understanding the politico-economic sustainability of sovereign rescues, emphasizing the need for fiscal targets and possible debt haircuts. We provide a calibrated example that quantifies the threshold debt-GDP ratio at 137%, remarkably close to the target set for private sector involvement in the case of Greece.

Keywords: sovereign debt, rent seeking, world interest rates, international lending, incentive compatibility, tragedy of the commons, EU crisis, Grexit, Graccident

JEL classification: H63, F34, F36, G01, E44, E43, D72
1. Introduction

The Maastricht treaty has been explicit about two fiscal requirements in order to justify participation in the Eurozone: (i) that the fiscal deficit-GDP ratio never exceeds 3%, and (ii) that the fiscal debt-GDP ratio never exceeds 60%. Here we investigate whether such fiscal rules go beyond narrow-minded economic accounting. Specifically, we examine whether quotas on fiscal debt-GDP ratios guarantee the political feasibility of fiscal prudence once a country is already member of a monetary union.

As Figure 1 indicates, corruption and fiscal profligacy correlate strongly across Eurozone countries, and corruption is particularly acute in the EU periphery.1 In order to explain the strong correlation illustrated by Figure 1, we suggest that, (a) the interplay between politics and corruption is central to explaining the EU core/periphery fiscal imbalances, and (b) causality may go the other way around, too: fiscal imbalances may reinforce channels through which corrupt politics lead to excessive fiscal debt, i.e., corruption may lead to more corruption on a way towards sovereign default.

The channel we explore is whether outstanding debt-GDP ratios affect practices of well-organized groups within partisan politics that seek fiscal rents. In particular, we investigate whether debt-GDP ratios provide incentives to rent-seeking groups to cooperate (or not) in order to comply with fiscal-prudence practices. Our emphasis on such cooperation decisions is corroborated by excerpts of IMF country reports (see Appendix A), which refer to Eurozone countries that either received rescue packages or faced excessively high 10-year government bond spreads during the sovereign crisis. IMF monitoring experts explicitly state the need for coalition governments or for partisan cooperation in order to implement

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1 The correlation coefficient between fiscal surplus/deficit-GDP ratios and the corruption perception index is 73%. Grechyna (2012) reports similar correlation results to this depicted by Figure 1, referring to OECD countries. Figure 1 partly reflects the sovereign “debt shocks” in EU countries, presented by Mendoza et al. (2014, Figure 1).
programs of controlled fiscal spending.\footnote{One goal of our analysis is to understand more about policy prescription for sovereign-crisis problems that involve politics more heavily than usual. One example is the severe sovereign crisis of Greece (reaching peaks in years 2010, 2012, and 2015, outlined by Sinn, 2015). As a currency-union member, Greece cannot have any partial benefits from currency devaluation. This feature gives a stronger role to politics, requiring from politicians of different parties to collaborate on strong reforms and on internal devaluation policies in order to avoid disorderly default (the so-called “Graccident”) and sudden exit from the Euro zone (the so-called “Grexit”, discussed extensively by Sinn, 2015).}

1.1 The cooperation dilemma faced by rent-seeking groups

The payoff matrix of the cooperation game in Table 1 shows why debt-GDP ratios may affect incentives for cooperation on prudent policies. In Table 1, the cooperation strategy is denoted by “C” and the noncooperation strategy by “NC”. If $V_i^C < V_i^{NC}$, $i \in \{1, 2\}$, then there are three Nash equilibria, $(NC, NC)$, $(C, NC)$ and $(NC, C)$, i.e., if noncooperation is more rewarding for both rent seekers, then noncooperation is a sure outcome. If, instead, $V_i^C > V_i^{NC}$, $i \in \{1, 2\}$, then there are two Nash equilibria, namely, $(C, C)$ and $(NC, NC)$, i.e., if cooperation is more rewarding for both rent seekers, then cooperation becomes a possible outcome. Debt-GDP ratios influence cooperation decisions because cooperation on fiscal prudence involves a cost of servicing the outstanding debt. Thus, when debt-GDP levels are high, it may be more profitable for each rent-seeking group to refuse cooperation.

<table>
<thead>
<tr>
<th>Rent seeker 2</th>
<th>(C)</th>
<th>(NC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent seeker 1</td>
<td>(V_1^C, V_2^C)</td>
<td>(V_1^{NC}, V_2^{NC})</td>
</tr>
<tr>
<td>(V_1^{NC}, V_2^{NC})</td>
<td>(V_1^{NC}, V_2^{NC})</td>
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</tbody>
</table>
1.2 Mechanism

Our model seeks to understand which economic fundamentals shape the values of $V^C_i$ and $V^{NC}_i$, $i \in \{1, 2\}$, in games of the form given by Table 1. Specifically, we want to uncover the determinants of threshold debt-GDP ratios that encourage political cooperation on fiscal prudence. To this end, we introduce the mechanism explained by the game of Table 1 in a dynamic environment with the following features: (i) rent-seeking groups jointly influence debt dynamics, government spending, and taxes through (non)cooperation decisions, and, (ii) sovereign bond rates are determined in international markets where foreign creditors buy government debt. These features capture both the willingness as well as the ability to repay debt.

The ability to repay is compromised by the emergence of a commons problem as explained in Persson and Tabellini (2000, pp. 163-164). Without permanent cooperation among rent-seeking groups there is excessive debt issuing, a type of endogenous fiscal impatience: noncooperating rent-seeking groups exploit additional resources earlier, to avoid being crowded out by extra rents of other groups. This impatience causes a mismatch between creditors and a government: the rate of time preference of creditors is lower than that of the borrowing government. This mismatch leads to high interest rates and immediate sovereign default, with groups permanently extracting rents out of balanced fiscal budgets thereafter.

As critical role is played by an endogenous threshold of the initial outstanding debt-GDP ratio. Above that debt-GDP-ratio threshold, servicing the debt becomes too costly, and it leads to dominant noncooperation incentives among rent-seeking groups, because the payoff of noncooperation becomes bigger. This noncooperation triggers fiscal impatience and leads to prohibitively high interest rates which lead to default. In this way this critical debt-
GDP-ratio threshold aligns the ability of the economy to repay the debt with the strategic
ingleness of rent-seeking groups to repay the debt. To offer an analysis that can identify
the determinants of such a critical debt-GDP-ratio threshold is one of the key goals of our
study.

1.3 The need for a recursive corrupt-politics formulation that
bridges two literatures

Our analysis builds an indispensable analytical tool for introducing microfoundations to the
politics of sovereign defaults. This tool is a recursive mapping from economic fundamentals to
corrupt-politics choices, and then back from corrupt-politics choices to economic fundamen-
tals, specifying initial conditions that determine alternative politicoeconomic regimes (cutoff
debt-GDP ratios that may encourage default through a political channel). This recursive
mapping falls in the broad class of the self-selected dynamic politicoeconomic mechanisms
suggested by Lagunoff (2009, pp. 576-579).\(^3\)

Our recursive mapping bridges two literatures. The one literature, which is more devel-
oped in the context of studying sovereign debt, has been pioneered by Cole and Kehoe (1995)
and Cole et al. (1995) and it links sovereign-default decisions with economic fundamentals
and international-market mechanisms.\(^4\) The other literature is related to the political econ-

\(^{3}\) An earlier, stricter concept, that was a predecessor of Lagunoff’s (2009) stable political rules is the
“structure-induced” political equilibrium suggested by Denzau and Mackay (1981) in the context of a spatial
political model. In his literature review, Lagunoff (2009, pp. 579-581) mentions previous several applica-
tions of his concept, which have a common advantage: putting microfoundations on the dynamics of
political choices, reforms, or institutional changes. Three notable examples of studies employing a recursive
politicoeconomic equilibrium are Krusell and Rios-Rull (1996, 1999), and Krusell et al. (1997). Haag and
Lagunoff (2007) study the possibility of cooperation in games with the possibility of free riding in a recursive
setting, which is an analysis related to our cooperation mechanism.

\(^{4}\) See also Cole and Kehoe (2000), and recent related work by Mendoza and Yue (2012) on the link between
sovereign defaults and business cycles. Other related literature includes Eaton and Gersovitz (1981), who
study government borrowing in a dynamic setting, and show that lenders –taking into account the cost and
benefit of default by the government– impose debt ceilings on governments. While Cole and Kehoe (2000)
study debt crises that arise from a loss of confidence on governments’ ability to roll over fiscal debt, Conesa
or not, gambling on the possibility of recovery of fiscal revenues. Arellano (2008) extends Eaton and Gersovitz
omy of rent-seeking (special-interest) groups, pioneered by Schattschneider (1935), Tullock (1959), Olson (1965), Weingast, Shepsle, and Johnsen (1981), Becker (1983, 1985), and Taylor (1987). Our setup extends the formulation suggested by Persson (1998), who studies political competition among rent-seeking groups which consume within-group club goods.\(^5\) One discrepancy between the two literatures is that the first literature, of sovereign defaults, uses formal dynamic-programming tools which pave the way to formal applications with data,\(^6\) whereas the second literature relies on stylized static or two-period models.

In order to resolve this discrepancy, we build a model falling in Lagunoff’s (2009, p. 577) specific class of politicoeconomic games which are a collection of, (i) economic primitives, (ii) political rules, and (iii) initial conditions. The political rules we study are Markovian decisions of rent-seeking groups to cooperate (or not) on fiscal prudence, while the key initial condition that matters in our analysis is the initial debt-GDP ratio. Our analysis is able to shed light on issues related to the recent sovereign crisis in the Eurozone, specifically on the desirability and politically feasibility of bailouts, which is the core application in this paper.\(^7\)

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\(^5\) Roberts (1999) studies politico-economic mechanisms of clubs that share such within-club public goods and their formation. Our analysis is more tied to the Persson (1998) model in which the size of these clubs is exogenously predetermined.

\(^6\) One notable example is the Mendoza and Yue (2012) business-cycle analysis of bond spreads and international-trade statistics in models with sovereign defaults.

\(^7\) Our model focuses on a setup in which there is a common currency between a domestic economy and foreign
1.4 Applications

A calibration exercise indicates that, if there are only two rent-seeking groups, then, above a cutoff debt-GDP ratio of about 137%, rent-seeking groups prefer to not cooperate, to default, and to resort to fiscal autarky forever. Extracting noncooperative rents from balanced budgets becomes preferable beyond 137%, since noncooperative rents under fiscal autarky will be higher compared to shared cooperative rents minus the servicing cost of a high debt.

Insights on the determinants of such debt-GDP-ratio cutoff levels help in understanding the design of bailout rescue packages. A binding commitment for a debt haircut tries to exclude an equilibrium in which rent-seeking groups would want to swing to noncooperation even for one period. Securing that debt-GDP ratios stay below such cutoff levels may contribute to the politicoeconomic sustainability of debt. We also find that international agreements (among foreign governments or by the IMF) to roll over fiscal debt using lower pre-agreed interest rates, increase the debt-GDP-ratio cutoffs that support cooperation. So, lower interest rates foster political cooperation among rent-seeking groups, making rescue packages politically feasible even at high outstanding debt-GDP ratios. High interest rates make the servicing burden of new debt socially unsustainable as it implies higher taxes and/or lower public consumption, reducing welfare. Our model’s mechanics are compatible with these features, which perhaps explain the stated rationale behind bailouts: the need to make the servicing costs of debt socially and politically bearable.

Our analysis offers potential for further applications and modeling extensions: therecur-

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8 In our Online Appendix we provide evidence on observations motivating us to suggest that corruption and rent-seeking, as endemic problems in Eurozone periphery countries, play a central role as both causes and effects within the vicious circle of the Eurozone sovereign crisis.

9 Indeed, one feature of bailout plans in the Eurozone is the tool of lowering interest rates. Out of all the bailaout plans, we believe, the Greek experience is the best example of all the model's mechanics at work.
sive structure of our model allows for the introduction of shocks, different forms of capital and capital markets, and recursive numerical-solution approaches. Numerical approximations with such model extensions may help toward developing sovereign-default-risk assessment indicators. These indicators can arguably be a valuable core input for public institutions (IMF, World Bank, Eurogroup) and private institutions that interact in sovereign debt markets.

2. Model

2.1 The domestic economy

2.1.1 Production

The domestic economy is populated by a large number of identical infinitely-lived agents of total mass equal to 1. A single composite consumable good is produced under perfect competition, using labor as its only input through the linear technology,

\[ y_t = z_t \cdot l_t , \]

in which \( y \) is units of output, \( l \) is labor hours, and \( z \) is productivity. Assume that there is no uncertainty and that productivity at time 0 is \( z_0 > 0 \), growing exogenously at rate \( \gamma \), i.e.,

\[ z_t = (1 + \gamma)^t z_0 . \]

2.1.2 Non rent-seeking households

A representative non rent-seeking household (one among a large number of such households) draws utility from private consumption, \( c \), leisure, \( 1 - l \) (a household’s time endowment per period is equal to 1), and also from the consumption of a public good, \( G \), maximizing the life-time utility function

\[ \sum_{t=0}^{\infty} \beta^t [\ln (c_t) + \theta_l \ln (1 - l_t) + \theta_G \ln (G_t)] , \]
in which $\beta \in (0, 1)$ is the utility discount factor, while $\theta_l, \theta_G > 0$ are the weights on leisure and public consumption, $G$, in the utility function. Public consumption is financed via both income taxes and fiscal debt. Yet, for simplicity, we assume that agents in this economy cannot hold any government bonds, so fiscal debt is external in all periods. Finally, we assume that agents cannot have access to domestic government bonds in the future, and that there is no storage technology. Under these assumptions, the budget constraint of an individual households is,

$$c_t = (1 - \tau_t) z_t l_t .$$

(4)

The representative non-rent-seeking household maximizes its lifetime utility given by (3), subject to equation (4), by choosing the optimal stream of consumption and labor supply, $\{(c_t, l_t)\}_{t=0}^{\infty}$, subject to any given stream of tax rates and public-good quantities, $\{(G_t, \tau_t)\}_{t=0}^{\infty}$. Since the solution to this problem is based on intra-temporal conditions only, we obtain a simple formula, namely,

$$l_t = \frac{1}{1 + \theta_l} = L , \quad t = 0, 1, \ldots ,$$

(5)

with $L$ being both the individual and the aggregate labor supply. That labor supply does not respond to changes in marginal tax rates is due to using logarithmic utility. Under logarithmic utility the income and substitution effects of taxation on leisure cancel each other out.

2.1.3 Rent-seeking groups and rent-seeking households

We introduce $N$ rent-seeking groups in the domestic economy that may be heterogeneous in size. Total population in the economy has normalized size 1, and the population mass of each rent-seeking group is $\mu_j, j \in \{1, ..., N\}$, with $\sum_{j=1}^{N} \mu_j \leq 1$. These groups have the power to expropriate resources from the fiscal budget. In each period $t \in \{0, 1, \ldots\}$, a rent-
seeking group \( j \in \{1, \ldots, N\} \) manages to extract a total rent of size \( C_{j,t}^R \). Changing slightly the formulation of Persson (1998), \( C_{j,t}^R \) is a composite club good subject to rivalness (public good within but with congestion). Examples of components of \( C_{j,t}^R \) are civil-servant jobs for which devoted group members can put less effort at work, tax evasion for which the group supports a network of non-transparency which is exclusive for group members, preferential legal treatment, privileges regarding the management of real estate, fiscal over invoicing, or wasteful public infrastructure related to private benefits, etc. These goods, \( C_{j,t}^R \), are equally available to every member of rent-seeking group \( j \) (every member of the group is the same), but with each member taking advantage from a smaller club size.\(^{10}\) In each rent-seeking group there is a large number of individuals, with each individual being unable to influence the group’s aggregate actions.\(^{11}\) Denoting by \( C_{j,t}^R \) the individual member’s consumption of the club good \( C_{j,t}^R \), the utility function of an individual rent seeker belonging to group \( j \) is,\(^{12}\)

\[
\sum_{t=0}^{\infty} \beta^t \left[ \ln(c_{j,t}) + \theta_l \ln(1 - l_{j,t}) + \theta_G \ln(G_t) + \theta_R \ln(C_{j,t}^R) \right],
\]

with \( \theta_R > 0 \), and her economic problem is maximizing (6) subject to the budget constraint

\[
c_{j,t} = (1 - \tau_t) z_t L_{j,t}.
\]

Optimal choices for a rent seeker are given by,

\[
l_{j,t} = l_t = \frac{1}{1 + \theta_l} = L, \quad t = 0, 1, \ldots.
\]

Since labor supply is identical across rent seekers and non rent seekers, private consumption is also the same across rent seekers and non rent seekers, namely,

\[
c_{j,t} = c_t = (1 - \tau_t) z_t L.
\]

\(^{10}\)On club goods see Mueller (2003, Chapter 9), and especially Sandler and Tschirhart (1997), and Roberts (1999, Section 6), in which club goods with congestion are studied.

\(^{11}\)Nevertheless, club strategies are fully compatible with individual-member incentives. We assume that even if rent-seeking groups have to lobby, this is a costless collective action: it requires no individual effort or any other sacrifice.

\(^{12}\)For the formulation of the utility function see, for example, Sandler and Tschirhart (1997, eq. 1, p. 339).
2.1.4 Aggregate production and fiscal budget

Combining $L$ with (1) and (2) gives the competitive-equilibrium GDP level,

$$Y_t = (1 + \gamma)^t z_0 L .$$

(10)

For simplicity, we assume that the domestic government issues only one-period zero coupon bonds. So, in every period there is a need for full debt rollover to the next period.\(^1\) The government’s budget constraint is,

$$\frac{B_{t+1}}{1 + r_{t+1}} = B_t + G_t + \sum_{j=1}^{N} \omega_j C_{j,t}^R - \tau_t Y_t ,$$

(11)

in which the weight $\omega_j = N \mu_j / \sum_{i=1}^{N} \mu_i$, $B_{t+1}$ is the value of newly issued bonds in period $t$ that mature in period $t + 1$, evaluated in terms of the consumable good in period $t + 1$, and $r_{t+1}$ is the interest rate which reflects the intrinsic return of a bond maturing in period $t + 1$. Assuming that the one-period zero-coupon bond delivers one unit of the consumable good at maturity, $B_t$ reflects the quantity of bonds maturing in period $t$. The weights $\omega_j$ in equation (11) play the role of an efficiency factor, transforming and mapping each dollar extracted by the fiscal budget into goods enjoyed by each member of group $j$.\(^2\) Specifically, given that $C_{j,t}^R$ is an individual member’s consumption of the total rents extracted by group $j$, $\bar{C}_{j,t}$, the relationship between $\bar{C}_{j,t}$ and $C_{j,t}^R$ is given by,

$$\bar{C}_{j,t} = \omega_j C_{j,t}^R , \text{ for all } j \in \{1, ..., N\}, t \in \{0, 1, ...\} .$$

The smaller the size of group $j$, the smaller the weight $\omega_j$, which means more special goods $C_j^R$ for each member of $j$. If all groups have the same size ($\mu_i = \mu_j = \mu$ for all $i, j \in \{1, ..., N\}$,

\(^1\)This assumption of issuing exclusively one-year zero-coupon bonds rules out concerns about strategic supply of bonds with different maturity. The short maturity time of bonds does not affect our qualitative results.

\(^2\)For the formulation of weights $\omega_j$ in the fiscal budget constraint (11), see, for example, Sandler and Tschirhart (1997, eq. 5, p. 341), which is based on the more general formulation of McGuire (1974), adapted for a continuum of agents within the group, and assuming a constant within-group congestion cost.
the symmetric-equilibrium case), then $\omega_j = 1$ for all $j \in \{1, \ldots, N\}$. So, by convention, the price per unit of $C_{j,t}^R$ equals the consumer-basket price.\footnote{We follow this convention as it is not straightforward to impose a market price on such special-interest club goods. Nevertheless, our equilibrium which emphasizes politicoeconomic inefficiencies, uncovers the effects and costs of providing such rents.} In the case of heterogeneity in group size, weights $\omega_j$ affect the rent-seeking-strategy incentives that each group member promotes, by taking into account that in larger groups there is a smaller portion of goods enjoyed per group member.

2.1.5 Impact of tax rates on GDP performance versus impact of tax rates on welfare

The absence of any marginal tax rates in equation (10) demonstrates that our logarithmic-utility setup neutralizes the impact of taxes on GDP performance and rules out dynamic Laffer curves. While taxes do not affect GDP performance, they directly reduce consumption and utility (see equation (4)). So, taxes have a profound impact on welfare. Also, despite that taxes do not have the classic distortionary effects on GDP performance, our analysis does not rule out considerations about an economy’s ability to repay fiscal debt. As it will be clear later, international interest rates at which a country borrows externally, influence its ability to repay fiscal debt in the future. It is an analytical advantage that our model clearly distinguishes the impact of interest-rate pressure on the ability to repay from other factors affecting GDP performance.

2.1.6 Policy-setting mechanism: the biggest part of society influences policy all the time

The levels of fiscal spending, $G_t$, the tax rate, $\tau_t$, and the level of debt one period ahead $B_{t+1}$, are the Nash equilibrium of a dynamic game among rent-seeking groups, which also
determines \( C_{j,t}^R \) in each period. We assume that

\[
1 - \sum_{j=1}^{N} \mu_j < \min \{ \mu_j \}_{j=1}^{N},
\]

so non rent-seekers cannot beat any rent-seeking group in a majority-voting equilibrium on these policy variables. For simplicity, we assume that all existing rent-seeking groups actively and simultaneously influence policymaking in each period, while they determine their per-member rent allocation \( \{ C_{j,1}^R \}_{j=1}^{N} \). The allocation of rents, \( \{ C_{j,1}^R \}_{j=1}^{N} \), is determined in a competitive and decentralized way, through time-consistent Nash equilibrium. The tax rate and the debt level are determined \textit{jointly through a simultaneous-move Nash equilibrium among rent-seeking groups}, as in legislative bargaining models or as in dynamic games in which different players jointly manage common-pool resources. Our Nash equilibrium concept synchronizes actions by rent-seeking groups, simplifying recursive formulations, implying that all tax/debt policies are time-consistent. The qualitative equivalence of asynchronous fiscal profligacy to a commons problem with simultaneous moves is demonstrated by Persson and Svensson (1989). Yet, such an extension should not alter our results.\(^{16}\)

Persson and Tabellini (2000, Chapter 7), present a number of applications related to the political mechanism behind the provision of club goods as rents, such as legislative bargaining, lobbying, and electoral competition. Here we abstract from such an analysis since EU core/periphery countries do not differ with respect to institutional arrangements behind these political-economy extensions. As Figure 1 illustrates, the EU core/periphery

\(^{16}\)We do not model alternating political parties and associated rent-seeking groups in power, as this would complicate the derivation of equilibrium without adding insights to the model. Having all rent-seeking groups acting simultaneously conveys the mechanics of a commons problem adequately: a rent-seeking group tends to expropriate extra rents before being crowded out by extra rents of other groups. On the one hand, each group fully internalizes the benefits of its own per-member rent-seeking good, \( C_{j}^R \). On the other hand, because financing is shared among groups, each group internalizes only one fraction of the social burden caused by higher taxes and debt. Extensions of our model employing numerical techniques may explore the role of alternating incumbent parties that are controbled by rent-seeking groups. Such extensions are beyond the scope of our analysis here, which is based on closed-form solutions.
countries differ mostly with respect to the intensity of rent-seeking/corruption.

2.2 The external creditors

We denote all external-creditor variables using a star. For simplicity, we assume that external creditors only hold bonds from one country, and maximize their total life-time utility derived from consumption,

$$\sum_{s=t}^{\infty} \beta^{s-t} \ln (c_s^*)$$

subject to the budget constraint,

$$B_{s+1}^* = (1 + r_{s+1}) (B_s^* - c_s^*)$$

Notice that the rate of time preference, $\frac{1}{\beta} - 1$, in the utility function of creditors, (13), is equal to the rate of time preference of domestic households.

The solution to the problem of maximizing (13) subject to (14) is,

$$c_t^* = (1 - \beta) B_t^*$$

$$c_s^* = (1 - \beta) \beta^{s-t} \prod_{i=t+1}^{s} (1 + r_i) B_i^*$$

which implies,

$$B_{t+1}^* = \beta (1 + r_{t+1}) B_t^*$$

Equation (15) determines the demand for bonds by external creditors in period $t + 1$.$^{17}$

Logarithmic preferences are responsible for this compact algebraic solution given by (15),

$^{17}$Notice that external creditors are a collection of individuals, institutions, etc., holding debt of the domestic economy. If that set of creditors is $I$, and if we assume that utility of any creditor $i \in I$, is given by the same utility function, $\sum_{s=t}^{\infty} \beta^{s-t} \ln (c_{i,s}^*)$, then the demand for bonds is characterized by

$$B_{i,t+1}^* = \beta (1 + r_{t+1}) B_{i,t}^*$$

for all $i \in I$. Equation (16) can be aggregated, so the aggregate demand for bonds of the domestic country is,

$$\sum_{i \in I} B_{i,t+1}^* = \beta (1 + r_{t+1}) \sum_{i \in I} B_{i,t}^*$$

which coincides with equation (15). Due to this aggregation result, there exists a representative consumer (representative creditor) with the same utility function as (13), justifying our condensed presentation of
which implies that demand for external debt depends only on the return of bonds issued in period $t$ and maturing in period $t + 1$, $r_{t+1}$.

### 2.3 Decisions of rent-seeking groups with exogenous interest rates and without the option of cooperation

We gradually unfold the mechanics of the model, ignoring, for the moment, the role of external creditors. So, for any $t \in \{0, 1, \ldots\}$, we consider that future interest rates, $\{r_s\}_{s=t+1}^{\infty}$, are exogenously given. In addition, we do not endogenize the cooperation decision yet, focusing on a noncooperative solution among $N$ rent-seeking groups. These temporary abstractions distinguish how international interest rates affect the choices of club rents, $C_{j,t}^R$, and public policies $B_{t+1}$, $G_t$, and $\tau_t$.

As explained above, fiscal policy is set $(\tau_t, G_t, B_{t+1})$ collectively by rent-seeking groups that co-determine $C_{j,t}^R$ for all $j \in \{1, \ldots, N\}$. In this section, $\{C_{j,t}^R\}_{j=1}^{N}$ is determined non-cooperatively, with each group maximizing the group’s utility, subject to the rent-seeking behavior of other rent-seeking groups (we will introduce the possibility of cooperation in a later section). We focus on time-consistent (Markovian) policies and rent-extraction strategies. For an exogenous stream of international-market interest rates, $\{r_s\}_{s=t+1}^{\infty}$, the Bellman
equation of rent-seeking group $j \in \{1, \ldots, N\}$ is given by,

$$
\hat{V}^j \left( B_t, z_t \mid \{C_i^R\}_{i=1}^N, \{r_s\}_{s=t+1}^\infty \right) = \max_{(\tau_t, C_{j,t}^R, B_{t+1})} \left\{ \theta_l \ln (1 - L) + \ln (z_t) + \ln (1 - \tau_t) + \theta_G \ln \left[ \frac{B_{t+1}}{1 + \tau_{t+1}} - \left( B_t + \omega_j C_{j,t}^R + \sum_{i=1, i \neq j}^{N} \omega_i C_i^R (B_t, z_t \mid \{r_s\}_{s=t+1}^\infty) - \tau_t Y_t \right) \right] \right. \\
\left. + \theta_R \ln \left( C_{j,t}^R \right) + \beta \hat{V}^j \left( B_{t+1}, (1 + \gamma) z_t \mid \{C_i^R\}_{i=1}^N, \{r_s\}_{s=t+2}^\infty \right) \right\}, \quad (17)
$$

in which $C_i^R (B_t, z_t \mid \{r_s\}_{s=t+1}^\infty)$ is the Markov-Perfect rent-extraction strategy of rent-seeking group $i \in \{1, \ldots, N\}$.\(^{19}\)

**Definition 1**  
*Given a stream of interest rates, $\{r_s\}_{s=t+1}^\infty$, a (Markov-Perfect) Domestic Equilibrium under No Cooperation (DENC) is a set of strategies, $\{C_i^R\}_{i=1}^N$ of the form $C_{i,t}^R = C_i^R (B_t, z_t \mid \{r_s\}_{s=t+1}^\infty)$ and a set of policy decision rules $\{T, B\}$ of the form $\tau_t = T (B_t, z_t \mid \{r_s\}_{s=t+1}^\infty)$ and $B_{t+1} = B (B_t, z_t \mid \{r_s\}_{s=t+1}^\infty)$, such that each and every rent seeking group $j \in \{1, \ldots, N\}$ maximizes (17) subject to $\{C_i^R\}_{i \neq j}$.*

\(^{19}\)There are two ways of motivating the objective function given by (17). One way is to assume that there is majority voting, and after making use of condition (12), non-rent-seeking households, if any, are powerless. So, the parametric weights $\theta_G$ and $\theta_R$ in the objective function given by (17), are simply the parametric weights of each rent-seeking-group member’s utility, given by equation (6). Policy setting is then the outcome of a dynamic bargaining game among rent-seeking groups, in which taxes, public goods, and sovereign debt, are commonly agreed policies after this bargaining for rents. Another way is to think of some legislative representation of both rent-seeking groups and the public, along the lines of legislative bargaining analyzed by Persson and Tabellini (2000, pp. 164-171). In this case, the parametric weights $\theta_G$ and $\theta_R$ in the objective function given by (17) would be endogenous (and also functions of $\mu$, among other model parameters), differing from the parametric weights $\theta_G$ and $\theta_R$ of each rent-seeking-group member’s utility, given by equation (6). Such an extension, which would take the form of a recursive contract between rent-seeking groups and non rent seekers, is beyond the scope of our analysis (we thank Fabrizio Zilibotti for suggesting this recursive-contract aspect to us). Because of the possibility of such extensions, in our analysis of rescue packages below, we also examine the determinants of policies that may find support by non rent seekers, too.
2.3.1 Exact Domestic Noncooperative solution

Proposition 1 summarizes the rent-seeking political equilibrium for a given set of interest rates.

**Proposition 1**  For all \( t \in \{0, 1, \ldots \} \), given a stream of interest rates, \( \{r_s\}_{s=t+1}^{\infty} \), there exists a symmetric DENC given by,

\[
\frac{G_t}{Y_t} = \frac{(1 - \beta) \theta_G}{1 + \theta_G + \theta_R + (N - 1) (1 - \beta) \theta_R} \left[ \frac{z_t \mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right)}{Y_t} - \frac{B_t}{Y_t} \right],
\]

in which

\[
\mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right) = \prod_{s=t+1}^{\infty} \frac{1}{1 + \bar{r}_s} + 1 + \sum_{s=t+1}^{\infty} \frac{1}{\frac{1}{\prod_{j=t+1}^{s} (1 + \bar{r}_j)}},
\]

with

\[
1 + \bar{r}_t = \frac{1 + r_t}{1 + \gamma},
\]

while

\[
\tau_t = T\left( B_t, z_t \mid \{r_s\}_{s=t+1}^{\infty} \right) = 1 - \frac{1}{\theta_G Y_t} G_t,
\]

\[
C_i^R\left( B_t, z_t \mid \{r_s\}_{s=t+1}^{\infty} \right) = \frac{1}{\omega_i} \cdot \frac{(1 - \beta) \theta_R}{1 + \theta_G + \theta_R + (N - 1) (1 - \beta) \theta_R} \cdot \frac{z_t \mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right)}{Y_t} - \frac{B_t}{Y_t},
\]

for all \( i \in \{1, \ldots, N\} \), while,

\[
\frac{B_{t+1}}{Y_{t+1}} = \frac{B\left( B_t, z_t \mid \{r_s\}_{s=t+1}^{\infty} \right)}{Y_{t+1}} = \frac{1 + r_{t+1}}{1 + \gamma} \left[ \beta_N B_t Y_t + (1 - \beta_N) \frac{z_t \mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right)}{Y_t} - 1 \right],
\]
with

\[ \beta_N = \frac{1 + \theta_G + \theta_R}{1 + \theta_G + \theta_R + (N - 1)(1 - \beta) \theta_R} \beta, \]  

(23)

\( \textbf{Proof} \)  See Appendix B.  □

2.3.2 International interest rates, chosen policies, and the ability to repay sovereign debt

The policies given by equations (18), (21), and (22) are intuitive. Notice that \( \frac{\partial W}{\partial r_s} = t + 1 = \frac{\partial r_s}{r_s} < 0 \) for all \( s \geq t + 1 \) and all \( t \in \{0, 1, \ldots\} \), according to equation (19). This reduction in economy’s worth \( (z_t \, \mathbb{W} (\{r_s\}_{s=t+1}^\infty) / Y_t) \) which occurs due to the increase in any future period’s interest rate, affects all policies. Any interest-rate increase reduces \( G_t / Y_t \) (see (18)), it increases tax rates (see (20)), and it also reduces rents (see (21)). Most importantly, any interest-rate increase reduces the economy’s ability to repay sovereign debt through collecting taxes.

The role of increasing the debt-GDP ratio is exactly the same as an interest-rate increase. Equations (22), (18), (20), and (21), reveal that future taxes must pay back the outstanding sovereign debt-GDP ratio, \( B_t / Y_t \): a higher ratio \( B_t / Y_t \) contributes to reducing \( G_t / Y_t \) and rents, and to increasing tax rates.

2.3.3 Postponed fiscal prudence and the number of rent-seeking groups: fiscal impatience due to a commons problem

Equation (22) conveys the presence of fiscal prudence in this model. Next period’s optimal debt-GDP ratio decreases if future interest rates are foreseen to increase. Since \( \frac{\partial \mathbb{W} (\{r_s\}_{s=t+1}^\infty)}{\partial r_s} < 0 \) for all \( s \geq t + 1 \), equation (22) implies that next period’s debt-GDP ratio falls, because of the foreseen increase in rolling over debt issued in the future.
Policy setting by multiple noncooperating rent-seeking groups has a profound effect on postponing fiscal prudence. Since \( \mathbb{W}(\{r_s\}_{s=t+1}^{\infty}) \) is multiplied by the factor \((1 - \beta_N)\), and \( \partial (1 - \beta_N) / \partial N > 0 \) (see equation (23)), an increase in the number of rent-seeking groups strengthens the fiscal-prudence-postponement characteristic. Postponement of fiscal prudence stems from two opposing forces. On the one hand, rent-seeking groups want to conserve the fiscal budget, in order to be able to extract more in the future. So, they exhibit fiscal prudence by having the optimal next period’s debt-GDP ratio strategy depending positively on the term \( \mathbb{W}(\{r_s\}_{s=t+1}^{\infty}) \) with \( \partial \mathbb{W}(\{r_s\}_{s=t+1}^{\infty}) / \partial r_s < 0 \) for all \( s \geq t + 1 \). On the other hand, as the number of rent-seeking groups increases, fiscal debt is issued excessively today, as is revealed by equation (21): recalling that \( C^R_{j,t} \) is the per-capita level of consumption by a member of group \( j \), and \( \omega_j C^R_{j,t} \) is the total rents extracted by group \( j \), aggregate economy-wide rents are,

\[
\sum_{i=1}^{N} \omega_i C^R_i (B_t, z_t | \{r_s\}_{s=t+1}^{\infty}) = \left[ \frac{N \cdot (1 - \beta) \theta_R}{1 + \theta_G + \theta_R + (N - 1) (1 - \beta) \theta_R} \right] \mathbb{W}(\{r_s\}_{s=t+1}^{\infty}) - B_t \]  

(24)

The fraction of economy’s net worth expropriated by all rent-seeking groups is increasing in the number of (symmetric) groups \( \Phi'(N) > 0 \) in equation (24)). Aggregate rents increase in the number of rent-seeking groups because each noncooperating rent-seeking group expropriates additional rents before being crowded out by other groups. This effect, driven by \( \Phi'(N) > 0 \), leads to collective fiscal impatience across rent-seeking groups that do not cooperate, describing a classic commons problem, in a similar fashion to problems of resource conservation. This commons problem dominates, and leads to fiscal-prudence postponement.

Yet, this dynamic game has another set of players, the external creditors. Fiscal-prudence postponement is a central reason why external creditors may require extra compensation for
debt rollover via interest rates. This mechanism is clarified after putting supply and demand together in the bonds market in order to determine international interest rates.

2.4 Determining interest-rate levels if rent-seeking groups never cooperate

The Bellman equation of rent-seeking group \( j \in \{1, ..., N\} \) under no cooperation is given by,

\[
V^{NC,j}(B_t, z_t) = \max_{(\tau_t, C^{R,NC}_{j,t}, B_{t+1})} \left\{ \theta_t \ln (1 - L) + \ln (z_t) + \ln (1 - \tau_t) \right\}
\]

\[
+ \theta_G \ln \left( \frac{B_{t+1}}{1 + R^{NC}(B_t, z_t)} \right) - \left( B_t + \omega_j C^{R}_{j,t} + \sum_{i=1, i \neq j}^{N} \omega_i C^{R,NC}_{i,t} (B_t, z_t) - \tau_t Y_t \right) \]

\[
+ \theta_R \ln \left( C^{R}_{j,t} \right) + \beta V^{NC,j}(B_{t+1}, (1 + \gamma) z_t) \left\{ \{C^{R,NC}_{i,t}\}_{i=1}^{N} \right\}
\]

(25)

in which \( r_{t+1} = R^{NC}(B_t, z_t) \) is the interest-rate rule. Definition 2 specifies international-market equilibrium under noncooperation of rent-seeking groups.

**Definition 2** An International Equilibrium under No Cooperation (IENC) is a set of strategies, \( \{C^{R,NC}_{i,t}\}_{i=1}^{N} \) of the form \( C^{R,NC}_{i,t} = C^{R,NC}_{i,t}(B_t, z_t) \) and a set of policy decision rules \( \{T^{NC}, G^{NC}, B^{NC}\} \) of the form \( \tau_t = T^{NC}(B_t, z_t), G_t = G^{NC}(B_t, z_t), \) and \( B^{NC}_{t+1} = B^{NC}_{t+1}(B_t, z_t) \), a bond-demand strategy of creditors, \( B^*_t = B^*(B_t, z_t) \), and an interest-rate rule, \( R^{NC}(B_t, z_t) \), such that \( \{T^{NC}, B^{NC}, C^{R,NC}, G^{NC}\} \) guarantee that each and every rent seeking group \( j \in \{1, ..., N\} \) maximizes (25), subject to rule \( R^{NC}(B_t, z_t) \) and subject to strategies of other rent-seeking groups \( \{C^{R,NC}_{i}\}_{i=1}^{N} \), creditors’ \( B^* \) complies with equation (15), and with \( R^{NC}(B_t, z_t) = \theta_t \) satisfying \( B^{NC}(B_t, z_t) = B^*(B_t, z_t) \), for all \( t \in \{0, 1, \ldots\} \).
2.4.1 Exact International Equilibrium Under No Cooperation

Proposition 2 conveys a crucial feature of our model.

**Proposition 2** If \( N \geq 2 \), there is no IENC equilibrium with \( B_0 > 0 \), the only possibility for IENC existence is \( B_0 = 0 \). If, \( N \geq 2 \), and \( B_0 > 0 \), are the initial conditions, then the only possible IENC as a market outcome, is immediate full debt default without return to credit markets again if debt renegotiation is not allowed.

**Proof** See Appendix B. \( \square \)

Proposition 2 offers only a building block of the full-fledged model of endogenous cooperation decisions in which accidental default occurs if there is an unexpected shock to initial conditions, an unexpected jump in GDP, \( Y_0 \), which automatically makes \( B_0/Y_0 \) to jump upwards.\(^{20}\) Proposition 2 says that financial autarky is the only Markov equilibrium without cooperation. While the proof of Proposition 2 is extensive, the key behind this result is the endogenous impatience mechanics that we have already stressed for the domestic equilibrium under no cooperation (DENC) through Proposition 1. Specifically, the endogenous discount factor \( \beta_N \), specified by equation (23), which implies \( \partial \beta_N / \partial N < 0 \), causes a mismatch in the market-clearing equation of external debt. External creditors foresee that multiple rent-seeking groups have the tendency to issue debt excessively in all periods. So, external creditors understand that the domestic economy will be unable to repay the debt asymptotically. As a result, external creditors suggest to roll over debt at a sequence of high interest rates that oblige the domestic economy to provide its total worth to creditors as-

\(^{20}\)The spirit of this unexpected shock is similar to the example employed by Kiyotaki and Moore (1997, p. 224).
ymptotically. So, if debt renegotiation is not allowed, the domestic economy can do nothing but default on its full outstanding debt, permanently exiting international credit markets.

In Section 3.1.2 we examine a bond renegotiation scheme that both creditors and rent-seeking groups may be willing to accept. According to that scheme, creditors are willing to offer a debt reduction in exchange for cooperation among rent seeking group, which results in fiscal discipline.

## 2.5 Determining interest-rate levels if rent-seeking groups always cooperate

We examine the case in which $N \geq 2$ rent-seeking groups cooperate by forming a single government coalition comprised by all existing rent-seeking groups in the economy (universal coalition). Within this universal coalition, rent-seeking groups equally share a total amount of rents, $\bar{C}_t^R$, with each group member receiving $\bar{C}_t^R / \sum_{j=1}^{N} \omega_j = \bar{C}_t^R / N$ in each period.\(^{21}\) We derive the supply of bonds decided by such a coalition and we equate it to the demand for bonds by external creditors in order to calculate international interest rates.

The Bellman equation of rent-seeking group $j \in \{1, ..., N\}$ under cooperation is given by,

$$
V^{C,j}(B_t, z_t) = \max_{(\tau, C_t^R, B_{t+1})} \left\{ \theta_l \ln (1 - L) + \ln (z_t) + \ln (1 - \tau_t) 
\right. \\
\left. + \theta_G \ln \left[ \frac{B_{t+1}}{1 + R^C(B_t, z_t)} - (B_t + \bar{C}_t^R - \tau_t Y_t) \right] + \theta_R \ln \left( \frac{\bar{C}_t^R}{N} \right) 
\right. \\
+ \beta V^{C,j}(B_{t+1}, (1 + \gamma) z_t) \right\} , \tag{26}
$$

in which the interest-rate rule, $r_{t+1} = R^C(B_t, z_t)$, is determined by equating supply and demand in the international market for bonds. Due to the symmetry of rent-seeking groups

\(^{21}\)Notice that although rent-seeking groups may be heterogeneous in size ($\mu_i \neq \mu_j$ for some $i \neq j$), under cooperation each rent-seeking group member will end up consuming the same per-capita amount from the broad-coalition club good.
there is unanimity within the universal coalition. Definition 3 specifies international-market equilibrium under cooperation of rent-seeking groups.

**Definition 3** An International Equilibrium under Cooperation (IEC) is a set of strategies, $C_{t}^{R,C}$ of the form $C_{t}^{R,C} = C_{t}(B_{t}, z_{t})$ and a set of policy decision rules $\{T^{C}, G^{C}, B^{C}\}$ of the form $\tau_{t} = T^{C}(B_{t}, z_{t})$, $G_{t} = G^{C}(B_{t}, z_{t})$, and $B_{t+1}^{C} = B^{C}(B_{t}, z_{t})$, a bond-demand strategy of creditors, $B_{t+1}^{*} = B^{*}(B_{t}, z_{t})$, and an interest-rate rule, $R^{C}(B_{t}, z_{t})$, such that $\{T^{C}, B^{C}, C^{R,C}, G^{C}\}$ guarantee that each and every rent seeking group $j \in \{1,...,N\}$ maximizes (26), subject to rule $R^{C}(B_{t}, z_{t})$, creditors’ $B^{*}$ complies with equation (15), and with $R^{C}(B_{t}, z_{t}) = r_{t+1}$ satisfying $B^{C}(B_{t}, z_{t}) = B^{*}(B_{t}, z_{t})$, for all $t \in \{0,1,...\}$.

### 2.5.1 Exact International Cooperative solution

Proposition 3 characterizes the rent-seeking political equilibrium under cooperation among rent-seeking groups (IEC).

**Proposition 3** For all $t \in \{0,1,...\}$, the IEC interest rates are constant, given by,

$$R^{C}(B_{t}, z_{t}) = r^{ss} = \frac{1 + \gamma}{\beta} - 1 , \quad t = 0,1,... ,$$

(27)

the debt-GDP ratio remains constant over time,

$$\frac{B^{C}(B_{t}, z_{t})}{Y_{t}} = \frac{B_{t}}{Y_{t}} = b_{t} = b = \frac{B_{0}}{Y_{0}} , \quad t = 0,1,... ,$$

(28)

the public-consumption-to-GDP ratio, the rents-to-GDP ratio and the tax rate,
all remain constant over time, with,
\[
\frac{G^C (B_t, z_t)}{Y_t} \equiv \hat{g}^C_t = \bar{g}^C = \frac{(1 - \beta) \theta_G}{1 + \theta_G + \theta_R} \left[ \frac{1}{1 - \beta} - \frac{b_0}{\text{Economy's worth/GDP}} \right], \quad t = 0, 1, ... ,
\]
\[
\frac{C^R, C (B_t, z_t)}{Y_t} = \frac{\theta_R}{\theta_G} \hat{g}^C, \quad \text{and} \quad T^C (B_t, z_t) = \tau_t^C = \bar{\tau}^C = 1 - \frac{1}{\theta_G} \hat{g}^C, \quad t = 0, 1, ... .
\]

Proof  See Appendix B. □

3. Debt-GDP ratios and participation in a monetary union: implications for rescue packages

Participation in a sustainable monetary union implies that fiscal debt is paid back in the common currency. The ability of each member state to issue and repay external fiscal debt is crucial for the sustainability of a banking system in which foreign banks may play the role of external creditors (EU banks are major buyers of sovereign debt issued by other EU countries).\(^{22}\) While we do not model banks explicitly, we stress that an international agreement about either, (a) entrance into a monetary union, or (b) a rescue package for debt rollover of a member state, should guarantee that rent-seeking groups which tend to act separately, have incentives to cooperate forever. Here we focus on (b), a rescue package which aims at a particular agreement: that rent-seeking groups will commit to a non-default and that they will be cooperating forever, sharing their rents.\(^{23}\)

Our analysis below can be executed for any \(N \geq 2\). We set \(N = 2\) by convention

\(^{22}\)Our formulation of external creditors reflects that banks maximize the utility of foreign bank-equity holders.

\(^{23}\)This focus of ours is inspired by the rescue packages of the post-2009 sovereign-debt crisis which emphasized the target of no defaults, because of fears of “domino effects” on previously rescued banks.
throughout the rest of this section, focusing on countries such as Greece, in which rent-seeking groups have been traditionally tied with two major political parties, such as “left-wing” versus “right-wing”, etc. In addition, without loss of generality, we assume symmetry among groups, i.e., that these two groups have the same size.

3.1 Two rent-seeking groups and Markov-perfect-Nash-equilibrium selection

Even in the one-stage, normal-form game of cooperation decisions, presented by Table 1 in the Introduction, there are multiple Nash equilibria. If cooperation is less rewarding for both players ($V_{i}^{C} < V_{i}^{NC}$, $i \in \{1, 2\}$), then a cooperation outcome is impossible. The only way to make a cooperation outcome possible is to ensure that cooperation is more rewarding for both players ($V_{i}^{C} > V_{i}^{NC}$, $i \in \{1, 2\}$). A dynamic game with infinite horizon and a free option to cooperate (or not) in each period can have multiple equilibria as well. In order to obtain clearer results whenever $V_{i}^{C} > V_{i}^{NC}$, $i \in \{1, 2\}$, we propose an equilibrium-selection assumption, which we call a “willingness refinement”. Certainly, in a dynamic game, $V_{i}^{C} > V_{i}^{NC}$, $i \in \{1, 2\}$, hinges on the debt-GDP ratio.

3.1.1 The willingness refinement mirroring international commitments within a monetary union

Following Lagunoff (2009), we restrict our attention to a self-selected dynamic politicoeconomic mechanism of cooperation. This is achieved through a Markov-perfect cooperation-decision Nash equilibrium definition which is formally given by Definition B.1 in Appendix B. In Definition B.1 of Appendix B, the implicit assumption is that whenever $V_{i}^{C} \geq V_{i}^{NC}$, $i \in \{1, 2\}$, then group $i$ always chooses to cooperate. We impose this equilibrium selection since a monetary union implies some institutional commitments which we do not wish to model explicitly in this paper. Other member states, which are not explicitly modeled here,
would dislike fiscal imbalances and would be inclined to punish countries that default due to lack of cooperation among rent-seeking groups. Yet, other member states respect the unwillingness of a polity to comply with a cooperative equilibrium, as long as this unwillingness is driven by fundamentals (even if this polity is dominated by rent-seeking groups).

For example, in the context of the Eurozone, a member state can exit the common currency after a mandate based on a referendum. For example, a left-wing Greek government, elected in the beginning of 2015, called such a referendum on June 26 2015, after a breakdown of negotiations on a new bailout deal with European partners. We believe that this rule, of allowing exit through a referendum, captures the idea that, in Definition B.1 of Appendix B, $V^C_i < V^{NC}_i$ for some $i \in \{1, 2\}$, implies no cooperation (respecting unwillingness to cooperate), whereas $V^C_i \geq V^{NC}_i$, $i \in \{1, 2\}$, always implies cooperation (forcing Euro-area obligations based on utility-based willingness to cooperate, which may be democratically expressed through a referendum). Finally, another refinement of having multiple equilibria in the case of $V^C_i \geq V^{NC}_i$, $i \in \{1, 2\}$, would be to assume i.i.d. randomizations, e.g. $\pi$ times cooperation and $1 - \pi$ times noncooperation. Such an analysis would still indicate a cutoff debt-GDP ratio level as a function of $\pi$.24

### 3.1.2 Determining cutoff debt-GDP ratios

Propositions 2 and 3, together with Table 1, illustrate that the strategies according to which two rent-seeking groups either, (i) cooperate forever, or (ii) never cooperate and default, in which case they keep not cooperating forever under a balanced fiscal budget, are both Markov-perfect cooperation-decision Nash equilibria.25

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24 We think that such a formulation would be ideal for studying an extension to our model without a currency peg to the currency of external creditors (implied or forced by participation to a monetary union). This extension could provide a tool for predicting long-term exchange-rate trends based on country corruption indicators and outstanding debt-GDP ratios.

25 A formal proof of this claim, that strategies (i) and (ii) are both Markov-perfect cooperation-decision equilibriums, appears in the proof of Proposition 4 in Appendix B.
Let’s start examining case (ii) above, i.e., default with no cooperation before and afterwards. Moving one period ahead after the full default, debt remains 0 forever (see Proposition 2), and the game is not a dynamic game anymore, but similar to the normal-form game of cooperation decisions, with the sole difference that GDP grows exogenously and sums of discounted utilities over an infinite horizon are computed. After some algebra, we find that
\[ V_{C,j}^j(B_t = 0, z_t) > V_{NC,j}^j(B_t = 0, z_t) \left( C_{i}^{R,NC} \right)^2_{i=1} \Leftrightarrow 1 + \alpha > 2^\alpha, \]  
\[ j \in \{1, 2\} \]  
in which,
\[ \alpha \equiv \frac{\theta_R}{1 + \theta_G + \theta_R}. \]
By its definition, \( \alpha \in (0, 1) \), and it is straightforward to verify that \( 1 + \alpha > 2^\alpha \) is a true statement for all \( \alpha \in (0, 1) \). So, \( V_{C,j}^j(0, z_t) > V_{NC,j}^j(0, z_t) \) for \( j \in \{1, 2\} \) and all \( t \in \{1, 2, \ldots\} \). As we have noticed above for the normal-form game, whenever cooperation is more rewarding for both players, there are two Nash equilibria, \((C, C)\) and \((NC, NC)\). So, by the unimprovability principle (cf. Kreps 1990, pp. 812-813), the strategies described by (ii) above, no cooperation in period 0, immediate default and no cooperation thereafter forever, is a Markov-perfect Nash equilibrium.

Having established that no cooperation and default is a Markov-perfect Nash equilibrium, allows us to study a sovereign-debt rescue initiative in a monetary union more formally. In the spirit of the willingness refinement discussed above, other member states may consider no cooperation among rent-seeking groups and sovereign default as being the worst possible outcome in a period that banks are fragile. The reason is that the magnitude of a full default by a sovereign state may be a big shock for banks holding external debt in the monetary union. In addition, convincing rent-seeking groups to follow a strategy of cooperation forever, in order to avoid the problems of fiscal impatience and fiscal profligacy is the most desirable outcome from the perspective of the union’s sustainability. Proposition 4 establishes that this
cooperation equilibrium is a Markov-perfect Nash equilibrium, and it identifies debt-to-GDP ratios that make its adoption and enforceability desirable by two rent-seeking groups.

**Proposition 4** If \( N = 2 \), then the strategies according to which the two rent-seeking groups cooperate forever is a Markov-perfect cooperation-decision Nash equilibrium, which holds if,

\[
V^{C,j}(B_t, z_t) \geq V^{NC,j}(B_t = 0, z_t | \left\{ \mathcal{C}_i^{R,NC} \right\}_{i \neq j}^2) \iff b_t \leq \frac{1}{1 - \beta} \left( 1 - \frac{2^\alpha}{1 + \alpha} \right) \equiv \bar{b},
\]

for all \( t \in \{0, 1, \ldots\} \), in which \( \alpha \) is given by (32).

**Proof** See Appendix B. \( \square \)

In Proposition 4 notice the converse of (33): if the debt-GDP ratio is higher than a cutoff level, \( \bar{b} \), then rent-seeking groups have higher utility by defaulting and not cooperating ever after. This is reasonable, because paying back the debt and cooperating entails a tradeoff: on the one hand, rent-seeking groups can divide the coalition rents by two, which leads to rewards in each period, as (31) reveals; on the other hand, they have to bear the cost of servicing the debt. The higher the debt-GDP ratio the lower the cooperation benefits, so default strikes as a better option.

### 3.1.3 First insights and extensions

Our model is deterministic and this simplifying aspect contributes to obtaining analytical results. A way to interpret our model’s contribution is depicted by Figure 2. Figure 2 shows that the cutoff debt-GDP ratio, \( \bar{b} \), splits the space of initial conditions into two zones, a white one of cooperation and no default, and a black one, of noncooperation and *accidental* default (which is perhaps not far from the “Graccident” concept). The key simplifying assumption
is that no shock is anticipated by creditors or rent-seeking group. So, in a deterministic world, if initial conditions are in the white area of Figure 2, it is anticipated that $b_t = b_0$ for all $t$.

Proposition 4 gives insights in case an unexpected shock on $b_0$ occurs in the same spirit as in Kiyotaki and Moore (1997, p. 224). In our model, default is a Nash-equilibrium in a deterministic framework after an unexpected shock, in the sense that the possibility of such a shock is not internalized by the creditors. For this reason, we do not include a default option in the action space of rent-seeking groups in the spirit of D’Erasmo and Mendoza (2015) in which a single player in a government may choose default strategically. Apparently this is a simplifying assumption that offers, however, useful insights for future extensions of our model. Such extensions of our model would be accommodated in our recursive framework, by incorporating anticipated shocks and would also require to include default in the strategy space of rent-seeking groups. In that case, however, corner solutions and mixed-strategy equilibria would not allow for analytical results and would require solving through numerical approaches. Our conjecture is that stochastic versions of our model would give “grey zones” of default, as depicted in Figure 2. Specifically, such grey zones would correspond to confidence intervals of default riskiness, since all variables are random in a stochastic model. We believe that this is an exciting agenda for future research, especially if it is extended beyond rational expectations, to learning about disaster risk, as in Koulovatianos and Wieland (2011).

3.1.4 Rescue packages and sovereign-debt haircuts

Monitoring the ability of a government to satisfy the conditions of a rescue package involves preventing and eliminating excessive rent seeking by groups that influence policymaking. This focus on controlling the behavior of partisan corruption is evident in IMF-report ex-
cerpts outlined in Appendix A. EU rescue packages imply monitoring of the domestic economy’s rent-seeking groups by other member states of the monetary union. Yet, in order to be proactive, it is reasonable to try to make the rescue deal palatable to the rent-seeking groups in order to achieve political sustainability and robustness of the rescue-package deal. So, if $b_t$ is larger than the threshold given by (33), $\bar{b} = [1 - 2^\alpha / (1 + \alpha)] / (1 - \beta)$, then the rescue-package deal may involve a sovereign-debt haircut of magnitude $100 \cdot (b_t - \bar{b})$ percentage points of the domestic economy’s GDP.

Another crucial aspect of rescue-package effectiveness, is the welfare change for the general public (non rent seekers). In our model, political outcomes, $(G_t, \tau_t, B_{t+1})$, are determined solely by the Nash-equilibrium decisions of rent-seeking groups. Even after a default that eliminates the burden of servicing the fiscal debt, non-rent-seekers prefer that rent-seeking groups cooperate. This happens because noncooperation implies higher total rents extracted in the form of higher $\tau$, and welfare reduction through lower $g = G/Y$. Proposition 5 shows that gains from cooperation are substantial for non-rent-seekers. Specifically, even if $b_t > \bar{b}$, and an exogenous international agreement forces rent-seeking groups to cooperate without a haircut that reduces $b_t$ to $\bar{b}$, then non-rent-seekers would benefit even if they had to service the high debt $b_t > \bar{b}$ thereafter.

**Proposition 5** There exists a cutoff debt-GDP ratio,

$$\bar{b} = \frac{1}{1 - \beta} \frac{\alpha}{1 + \alpha},$$

(34)

in which $\alpha$ is given by (32), with $\bar{b} > b$, such that, if $g^C_b$ corresponds to cooperation among rent-seeking groups together with servicing $\hat{b}$ forever, and if $g^NC_{\text{default}}$
corresponds to full default and noncooperation forever, then,

\begin{align}
(i) \quad \hat{b} &\in (\underline{b}, \bar{b}) \Rightarrow g^C_b > g^{NC}_{\text{default}}, \\
(ii) \quad \bar{b} &> \hat{b} \Rightarrow g^C_b < g^{NC}_{\text{default}}.
\end{align}

**Proof** See Appendix B. □

Proposition 5 states that attempts to convince rent-seeking groups to cooperate (see the relevant IMF-report excerpts in Appendix A) would be welcomed by the general non-rent-seeking public if the debt-GDP ratio is not too high. Non-rent-seeking households dislike excessive corruption that leads to fiscal profligacy, unless the outstanding debt GDP ratios is exceptionally high. In the following section we calibrate our stylized model in order to give a quantitative sense of \( \underline{b} \) and \( \bar{b} \).

### 3.1.5 Calibration

Our benchmark calibration focuses on matching data of the European Union (EU) periphery countries, since they are at the center of the EU crisis. Our goal is to quantify the cutoff debt-GDP ratio \( \underline{b} \), that ensures cooperation. First, we match the total-government-to-GDP spending in these countries which is an average of approximately 45%.\(^{26}\) In order to find the target value for the total-rents-GDP ratio at the cutoff debt-GDP ratio \( \underline{b} \) (denoted by \( C^R \)), we use estimates regarding the size of the shadow economy as a share of GDP reported by Elgin and Oztunali (2012). We make a simple projection of these shadow-economy estimates, assuming that these shares are uniform across the private and the public sector. In other words, the share of rents in total government spending match the size of the shadow economy as a share of GDP.

\(^{26}\)Data for \( G/Y \) are from the European Central Bank (ECB), Statistical data Warehouse, Government Finance data (Revenue, Expenditure and deficit/surplus), September 2013.
Table 2

<table>
<thead>
<tr>
<th>$C^{R,C}$ as % of government spending</th>
<th>cutoff debt-GDP ratio $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28% (EU periphery)</td>
<td>137%</td>
</tr>
</tbody>
</table>

In Table 2 we report the cutoff debt-GDP ratio, $b$, corresponding to the 28% rents-to-total-government spending ratio which is the average shadow economy share in EU-periphery countries.\textsuperscript{27} The assumed rate of time preference, $(1 - \beta)/\beta$, is 2.4%. The 137% cutoff level $b$ provides higher utility to rent-seekers if they cooperate, compared to defaulting. Interestingly, a 137% debt-GDP ratio is in the ballpark of targets of the “private sector involvement (PSI)” haircut for Greece in the period 2011-2012.\textsuperscript{28} A key factor shaping the target debt-GDP ratio of Greece during the PSI negotiations was the political sustainability of fiscal prudence. Prudence could be achieved by a coalition government, at least by the two major political parties that used to alternate in power during the previous four decades. A coalition government implies cooperation among underlying rent-seeking groups.

Finally, we find that the cutoff debt-GDP level $\bar{b}$ defined by Proposition 5 is 501%. Beyond $\bar{b}$ non-rent-seekers would support a default even with the rent-seeking groups not cooperating and exploiting excessive rents, because servicing the debt becomes too costly. Perhaps 501% is the cutoff level triggering support to social polarization among rent-seeking groups. Yet, we are not aware of any peace times during which such debt-GDP ratios have been recorded before a default.

Figure 3 depicts a sensitivity analysis of our benchmark calibration. It shows the relationship between the rate of time preference, $\rho = (1 - \beta)/\beta$ and the cutoff level $b$. We emphasize

\textsuperscript{27}So, the rents-GDP ratio is $28\% \times 45\% = 12.6\%$ in this calibration. In Appendix B we explain how calibration is achieved in this model. Specifically, we prove that calibrating $\theta_R$ and $\theta_G$ in order to match target values for the government-consumption-GDP ratio and the total-rents-GDP ratio is independent from the values of $\beta$ at the cutoff level $b$.

\textsuperscript{28}For an extensive review of the Greek sovereign crisis and an outline of PSI see Ardagna and Caselli (2014). For a study reporting the average haircut values between years 1970-2010, see Cruces and Trebesch (2012).
that varying \( \rho \) means simultaneously changing the rate of time preference of both creditors and of all agents in the domestic economy. As Proposition 3 indicates, under cooperation, international interest rates remain constant, tracking closely the rate of time preference, \( \rho \). Thus, a higher \( \rho \) implies higher cost of servicing outstanding debt, decreasing the tolerance to cooperation versus default. This is evident by Figure 3: at levels of \( \rho \) above 4%\(^{\text{a}}\), the cutoff debt-GDP ratio for cooperation versus default falls below 80%. On the contrary, more patient creditors and domestic agents (low \( \rho \)), increases the cooperation range, raising \( b \) above 160% of GDP for \( \rho \) less than 2%.

Figure 3 provides insights regarding the agreed interest rates of servicing debt under EU rescue packages (Ireland, Greece, Portugal). Since rescue packages involve long-term effective interest rates, lowering the cost of debt servicing may provide more political support in countries with corruption, by creating more incentives for rent-seeking groups to cooperate on fiscal prudence. The Greek PSI program, which involved both a reduction in interest rates and a haircut (see Ardagna and Caselli, 2014), has been followed by political consensus thereafter, providing a good example of this insight.

4. Conclusion

The EU sovereign debt crisis has painfully reminded that sustainability of debt-to-GDP ratios is of first order importance for the stability and future course of the monetary union. Rescue packages were introduced for EU periphery countries. One crucial element and a challenge behind these packages, stressed by official creditors, is the need for cooperation of political parties, in order to achieve fiscal prudence. But EU periphery politics are plagued with rent-seeking activities that overstretch fiscal budgets.

Our model studied the politics of coalition-making among rent-seeking groups, providing
a key insight. Reaching a high level of external sovereign debt-GDP ratio takes an economy beyond the perils of mere economic accounting. Beyond some debt-GDP ratio threshold which depends on the influence of rent-seeking groups in policymaking, political resistance to cooperation among rent seekers and parties on prudent policies arises. International markets respond by charging high interest rates, worsening the debt dynamics and making default immediately preferable (and unavoidable) by rent seekers. Rent seekers do not want to service a high outstanding debt, yet their noncooperation triggers the vicious circle of rapidly worsening terms of borrowing. For economies which are prone to corruption and rent-seeking phenomena, the risk of political turmoil makes the requirement of staying within a safety zone of low debt-GDP ratio tighter.

Our framework has accommodated a number of modeling elements with explicit dynamic policy setting: debt, public consumption, tax rates, and importantly, the free decision of rent-seeking groups to cooperate or not, are all determined recursively, and as functions of outstanding sovereign debt. These modeling features help us to understand what determines cutoff debt-GDP ratios which lead to political turmoil and default. The mechanism triggering the vicious circle of default is a commons problem that leads to a discrepancy between the rate of time preference of creditors and the collective rate of time preference of governments that have multiple noncooperating rent-seeking groups. While commons problems are difficult to resolve, our model points at the importance of keeping debt-GDP ratios low. The role of debt-GDP ratios should prevail in future extensions of our model (e.g., with uncertainty and productive capital) which should be easy to accommodate, given the recursive structure of the dynamic game we have suggested. Such extensions would contribute to a project of developing sovereign-default-risk indicators for countries as a function of their corruption fundamentals and debt-GDP ratios.
Our model suggests that rescue packages may use short-term tools, such as debt haircuts, or provision of low interest rates in order to convince rent-seeking groups to cooperate and to service a debt that costs less. Yet, the long-term goal of rescue packages should be to promote monitoring on reforms that are likely to eradicate rent-seeking groups.
### Appendix A – Explicit IMF reference to the need for cooperation among political parties on austerity measures

<table>
<thead>
<tr>
<th>Year</th>
<th>Country</th>
<th>IMF Country Report No.</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>Greece</td>
<td>IMF Country Report No. 12/57</td>
<td>“Staff welcomes the commitments from the political parties supporting the present coalition to continue with the objectives and policies of the new program” (IMF 2012a, p. 44)</td>
</tr>
<tr>
<td>2012</td>
<td>Italy</td>
<td>IMF Country Report No. 12/77</td>
<td>“Prospects of program success remain reasonably strong, given that substantial adjustment and significant reforms is already underway and there is strong political support” (IMF 2012b, p. 26)</td>
</tr>
<tr>
<td>2011</td>
<td>Portugal</td>
<td>IMF Country Report No. 12/179</td>
<td>“Finally, one year into the program, the authorities are building a convincing track record of meeting adjustment and reform objectives while preserving political support, and prospects of success for the program remain reasonably strong” (IMF 2012c, pp.19-20)</td>
</tr>
<tr>
<td>2011</td>
<td>Spain</td>
<td>IMF Country Report No. 12/36</td>
<td>“Structural reforms, which are critical to addressing both of these problems, lost considerable momentum during 2011. [...] Retaining broad political support for reforms will be crucial to future success” (IMF 2012a, p. 42)</td>
</tr>
<tr>
<td>2011</td>
<td>Ireland</td>
<td>IMF Country Report No. 12/204</td>
<td>“Political commitment to consolidation has been a welcome constant, as reflected in the affirmation by the new government (which took office in March 2011) of the medium-term fiscal targets in the EU-IMF supported program agreed in December 2010” (IMF 2012d, p. 20)</td>
</tr>
</tbody>
</table>

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**IMF Country Report No. 12/57**

2012 Article IV Consultation with Italy - Concluding Statement of the IMF Mission: “With broad political support, the authorities have embarked on an ambitious and wide-ranging agenda that has lifted Italy from the brink and is now seen as a model for fiscal stabilization and growth-enhancing reforms”. (IMF 2012a, p. 1)

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**IMF Country Report No. 11/173**

IMF Country Report No. 11/173: “The authorities’ welcome commitment to reduce the fiscal deficit to close to zero by 2014 needs to be accompanied by action. [...] The large size of the envisaged fiscal retrenchment requires structural changes which must be designed well in advance. This calls for a strong political consensus and careful planning”. (IMF 2011c, p. 30)

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**IMF Country Report No. 11/279**

IMF Country Report No. 11/279: “Sustained social and political support is necessary for the comprehensive structural reform program. Strong vested interests could weaken reforms, or reform fatigue could set in, and weaken growth prospects and the required adjustment in the economy”. (IMF 2011e, p. 16)

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**IMF Country Report No. 11/215**

IMF Country Report No. 11/215: “Ambitious fiscal consolidation is underway but [...] Such a comprehensive strategy would be helped by broad political and social support”. (IMF 2011d, p. 1)

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**IMF Country Report No. 10/366**

IMF Country Report No. 10/366: “The large multiyear fiscal and structural adjustment requires a decisive break from past behavior. Greece has run into fiscal problems before, which were often resolved only temporarily and by stop-gap measures. A decisive break now requires strong political will and public support. Mitigating factors include a strong mandate of the governing party and measures in the program to protect vulnerable groups.” (IMF 2010b, p. 21)

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**IMF Country Report No. 10/119**

IMF Country Report No. 10/119: “The challenge ahead will be to implement the program rigorously, while securing the necessary public consensus for reforms”. (IMF 2010a, pp. 136-9)
5. Appendix B – Proofs and formal definitions

Proof of Proposition 1  The first-order conditions of the Bellman-equation problem given by (17) lead to,

\[ G_t = \theta_G \cdot (1 - \tau_t) \cdot z_t \cdot L, \]  \hspace{1cm} (37)  

\[ C_{j,t}^R = \frac{\theta_R}{\theta_G \cdot \omega_j} G_t = \frac{\theta_R}{\omega_j} \cdot (1 - \tau_t) \cdot z_t \cdot L, \]  \hspace{1cm} (38)  

and

\[ \frac{\theta_R}{\omega_j (1 + r_{t+1}) C_{j,t}^R} = -\beta \frac{\partial \hat{V}^j}{\partial B_{t+1}} \left( B_{t+1}, z_{t+1} \mid \{ C_{i}^R \}_{i=1}^{N} \right), \]  \hspace{1cm} (39)  

together with the fiscal-budget constraint (11).

In order to identify the value function of the Bellman equation given by (17), its associated rent-seeking strategies, and the model’s decision rules, we make two guesses. We first take a guess on the functional form of the rent-seeking group consumption strategies, \( C_{i,t}^R = C_i^R (B_t, z_t \mid \{ r_s \}_{s=t+1}^{\infty}). \) Specifically,

\[ C_i^R (B_t, z_t \mid \{ r_s \}_{s=t+1}^{\infty}) = \xi_{R,i} \cdot (z_t W_{t+1} - B_t), \]  \hspace{1cm} (40)  

in which \( \xi_{R,i} \) is an undetermined coefficient, and,

\[ W_{t+1} \equiv \mathcal{W} \left( \{ r_s \}_{s=t+1}^{\infty} \right), \]  

for notational simplicity, in which \( \mathcal{W} \left( \{ r_s \}_{s=t+1}^{\infty} \right) \) is given by the expression in (19). It can be verified that the expression in (19) is the solution to the difference equation

\[ W_{t+1} = \frac{1 + \gamma}{1 + r_{t+1}} W_{t+2} + L, \]  \hspace{1cm} t = 0, 1, \ldots, \]  \hspace{1cm} (41)  

which is a recursion fully characterizing \( W_{t+1} \) in the guess given by (40). The second guess is on the functional form of the value function of player \( j \in \{ 1, \ldots, N \} \), in Bellman equation (17). Specifically,
\[ \hat{V}^j \left( B_t, z_t \mid \{ C_{i}^{R} \}_{i=1}^{N} \, \{ r_s \}_{s=t+1}^{\infty} \right) = \zeta_j + \psi_j \cdot \sum_{s=t}^{\infty} \beta^{s-t} \ln (1 + r_{s+1}) + \nu_j \cdot \ln (z_t W_{t+1} - B_t) , \]

(42)
in which \( \zeta_j, \psi_j, \) and \( \nu_j \), are undetermined coefficients, \( j \in \{1, ..., N\} \).

We substitute our guesses (40) and (42) into the Bellman equation given by (17), in order to verify whether the functional forms given by (40) and (42) are indeed correct, and also in order to calculate the undetermined coefficients \( \zeta_j, \psi_j, \nu_j, \) and \( \xi_{R,j} \). Before making this substitution, a simplifying step is to use a state-variable transformation, namely,

\[ x_t \equiv z_t W_{t+1} - B_t , \]

and to calculate the law of motion of \( x_t \), a function \( x_{t+1} = X (x_t) \), that is based on (11), the first-order conditions (37) through (42), and our guesses (40) and (42).

In order to find the law of motion \( x_{t+1} = X (x_t) \), we first combine (42) with (39) to obtain \( C_{j,t}^{R} = \theta_R x_{t+1} / [\omega_j \nu_j \beta (1 + r_{t+1})] \), and then we combine this result with (38), which leads to,

\[ (1 - \tau_t) \cdot \frac{z_t \cdot L}{\nu_j} = \frac{1}{\nu_j \beta (1 + r_{t+1})} x_{t+1} . \]

(43)

Since (43) holds for all \( j \in \{1, ..., N\} \), we conclude that

\[ \nu_j = \nu, \text{ for all } j \in \{1, ..., N\} . \]

(44)

From the fiscal-budget constraint (11) and the recursion given by (41) we obtain,

\[ z_{t+1} W_{t+2} - B_{t+1} = (1 + r_{t+1}) \left[ z_t W_{t+1} - B_t - (1 - \tau_t) Y_t - G_t - \omega_j C_{j,t}^{R} - \sum_{i=1}^{N} \omega_i C_{i,t}^{R} \right] , \]
which we combine with (37), (38), and (40), in order to get,

\[
x_{t+1} = (1 + r_{t+1}) \left[ \left( 1 - \sum_{i=1 \atop i \neq j}^{N} \omega_i \xi_{R,i} \right) x_t - (1 + \theta_R + \theta_G) (1 - \tau_t) Y_t \right] .
\] (45)

Since the choice of \( j \in \{1, \ldots, N\} \) is arbitrary, equation (45) implies,

\[
\sum_{i=1 \atop i \neq j}^{N} \omega_i \xi_{R,i} = \sum_{i=1 \atop i \neq k}^{N} \omega_i \xi_{R,i} \text{, for all } j, k \in \{1, \ldots, N\} .
\] (46)

The linear system implied by (46) has a unique solution according to which,

\[
\omega_i \xi_{R,i} = \xi_R \text{, for all } i \in \{1, \ldots, N\} .
\] (47)

Combining (45) with (47) gives,

\[
x_{t+1} = (1 + r_{t+1}) \left\{ [1 - (N - 1) \xi_R] x_t - (1 + \theta_R + \theta_G) (1 - \tau_t) Y_t \right\} .
\] (48)

After combining (48) with (43) and (44), we obtain the law of motion \( x_{t+1} = X(x_t) \), namely,

\[
x_{t+1} = \frac{1 + r_{t+1}}{1 + \frac{1 + \theta_R + \theta_G}{\nu \beta}} \left[ 1 - (N - 1) \xi_R \right] x_t .
\] (49)

With (49) at hand we return to calculating the undetermined coefficients \( \zeta_j, \psi_j, \nu, \) and \( \xi_R \). We substitute (42) into the Bellman equation given by (17) and get,

\[
\zeta_j + \psi_j \cdot \sum_{s=t}^{\infty} \beta^{s-t} \ln (1 + r_{s+1}) + \nu \cdot \ln (x_t) = \theta_t \ln (1 - L) + \ln (L)
\]

\[
+ \ln (1 - \tau_t) + \ln (z_t) + \theta_G \ln (G_t) + \theta_R \ln \left( C_{j,t}^R \right)
\]

\[
+ \beta \zeta_j + \beta \psi_j \cdot \sum_{s=t+1}^{\infty} \beta^{s-t-1} \ln (1 + r_{s+1}) + \beta \nu \ln (x_{t+1}) .
\] (50)

After combining (37), (38), and (43) with (49), we obtain,

\[
\theta_t \ln (1 - L) + \ln (L) + \ln (1 - \tau_t) + \ln (z_t) + \theta_G \ln (G_t) + \theta_R \ln \left( C_{j,t}^R \right)
\]
\[
\begin{align*}
= \theta_l \ln (1 - L) - \theta_R \ln (\omega_j) + \theta_G \ln (\theta_G) + \theta_R \ln (\theta_R) - (1 + \theta_G + \theta_R) \left[ \ln (\beta) + \ln (\nu) \right] \\
+ \ln \left( 1 + \frac{1 + \theta_G + \theta_R}{\nu \beta} \right) + (1 + \theta_G + \theta_R) \{ \ln [1 - (N - 1) \xi_R] + \ln (x_t) \}. \\
\end{align*}
\] 

(51)

In addition, equation (49) implies,

\[
\beta \nu \ln (x_{t+1}) = \beta \nu \ln (1 + r_{t+1}) + \beta \nu \left\{ \ln [1 - (N - 1) \xi_R] + \ln (x_t) - \ln \left( 1 + \frac{1 + \theta_G + \theta_R}{\nu \beta} \right) \right\}. 
\] 

(52)

Substituting (52), (38), and (51) into (50), leads to,

\[
(1 - \beta) \zeta_j = \theta_l \ln (1 - L) + \theta_G \ln (\theta_G) + \theta_R \ln (\theta_R) - (1 + \theta_G + \theta_R) \ln (\beta \nu) \\
+ (1 + \theta_G + \theta_R + \beta \nu) \left\{ \ln [1 - (N - 1) \xi_R] - \ln \left( 1 + \frac{1 + \theta_G + \theta_R}{\nu \beta} \right) \right\} - \theta_R \ln (\omega_j) \\
+ (\beta \nu - \psi_j) \ln (1 + r_{t+1}) + [1 + \theta_G + \theta_R - \nu (1 - \beta)] \ln (x_t). 
\] 

(53)

In order that the guessed functional forms given by (40) and (42) be indeed correct, equation (53) should not depend on its two variables, \(x_t\) and \(r_{t+1}\). Due to this requirement of non-dependence of equation (53) on \(x_t\) and \(r_{t+1}\), two immediate implications of (53) are,

\[
\nu = \frac{1 + \theta_G + \theta_R}{1 - \beta}, 
\] 

(54)

and \(\psi_j = \beta \nu\), so, based on (54), we obtain,

\[
\psi_j = \psi = \frac{\beta \cdot (1 + \theta_G + \theta_R)}{1 - \beta}, \text{ for all } j \in \{1, \ldots, N\}. 
\] 

(55)

Combining (43), (49), (37), and (54), we obtain,

\[
G_t = \frac{(1 - \beta) \theta_G [1 - (N - 1) \xi_R]}{1 + \theta_G + \theta_R} x_t. 
\] 

(56)

Equations (56) and (38) imply,

\[
\omega_j C_{j,t}^R = \frac{(1 - \beta) \theta_R [1 - (N - 1) \xi_R]}{1 + \theta_G + \theta_R} x_t. 
\] 

(57)
Our guess (40) concerning the exploitation strategy of group \( j \in \{1, \ldots, N\} \) is \( C_{j,t}^R = \xi_{R,j} x_t \).

So, combining (40) with (57) and (47) identifies the undetermined coefficient \( \xi_R \),

\[
\xi_R = \frac{(1 - \beta) \theta_R}{1 + \theta_G + \theta_R + (N - 1)(1 - \beta) \theta_R},
\]

which proves equation (21). Based on (58),

\[
1 - (N - 1) \xi_R = \frac{1 + \theta_G + \theta_R}{1 + \theta_G + \theta_R + (N - 1)(1 - \beta) \theta_R}.
\]

Combining (56) and (58) proves equation (18). In addition, the budget-constraint equation (22) is reconfirmed by substituting (56) and (57) into (11), and after noticing that,

\[
\beta_N \equiv \beta \left[1 - (N - 1) \xi_R\right],
\]

which proves formula (23). Equation (20) is proved directly from (37). Finally, after combining (53) with (54), (55), (58), and (59), we can identify the last undetermined coefficient, \( \zeta_j \), which is given by,

\[
\zeta_j = \frac{1}{1 - \beta} \left\{ -\theta_R \ln (\omega_j) + \theta_L \ln (1 - L) + \theta_G \ln (\theta_G) + \theta_R \ln (\theta_R)
\right. \\
+ \left. (1 + \theta_G + \theta_R) \left[ \frac{\beta}{1 - \beta} \ln (\beta) + \ln (1 - \beta) + \frac{\beta}{1 - \beta} \ln (1 + \theta_G + \theta_R) \right]
\right.
\]

\[
- \frac{1 + \theta_G + \theta_R}{1 - \beta} \ln \left[1 + \theta_G + \theta_R + (N - 1)(1 - \beta) \theta_R\right]\}
\]

completing the proof of the proposition. \( \square \)
Proof of Proposition 2

Equating demand for bonds (equation (15)) and supply of bonds (equation (22)), together with (10), leads to,

\[(\beta - \beta_N) b_t = (1 - \beta_N) \left[ \frac{W_{t+1}}{L} - \frac{1}{1 - \beta_N} \right]. \tag{61}\]

From (41) it is,

\[\frac{W_{t+2}}{L} = \frac{1 + r_{t+1}}{1 + \gamma} \left( \frac{W_{t+1}}{L} - 1 \right). \tag{62}\]

After considering equation (61) one period ahead and after substituting (62) into it, we obtain,

\[(\beta - \beta_N) b_{t+1} = (1 - \beta_N) \frac{1 + r_{t+1}}{1 + \gamma} \left( \frac{W_{t+1}}{L} - 1 \right) - 1. \tag{63}\]

After some algebra, equation (61) gives,

\[\frac{W_{t+1}}{L} - 1 = \frac{1}{1 - \beta_N} [(\beta - \beta_N) b_t + \beta_N]. \tag{64}\]

Substituting (64) into (63) gives,

\[(\beta - \beta_N) b_{t+1} = \frac{1 + r_{t+1}}{1 + \gamma} [(\beta - \beta_N) b_t + \beta_N] - 1. \tag{65}\]

Equation (22) can be expressed as,

\[b_{t+1} = \beta \left( \frac{1 + r_{t+1}}{1 + \gamma} \right) b_t, \quad \text{for all } t \in \{0, 1, \ldots\}. \tag{66}\]

Substituting (66) into (65) gives two useful equations, a linear first-order difference equation in variable $1/b_t$,

\[\frac{1}{b_{t+1}} = \frac{\beta_N}{\beta} \cdot \frac{1}{b_t} + (1 - \beta) \left( 1 - \frac{\beta_N}{\beta} \right), \tag{67}\]

and an equilibrium condition that links up $b_t$ directly with $r_t$,

\[[(1 - \beta) (\beta - \beta_N) b_t + \beta_N] \frac{1 + r_{t+1}}{1 + \gamma} = 1. \tag{68}\]
The solution to (67) is,
\[
\frac{1}{b_t} - (1 - \beta) = \left(\frac{\beta_N}{\beta}\right)^t \left[\frac{1}{b_0} - (1 - \beta)\right].
\] (69)

Combining (68) and (69) leads to,
\[
\frac{1}{1 + \bar{r}_{t+1}^*} = \frac{\beta - \beta_N}{\frac{\beta}{\beta_N} \left(\frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} + \beta_N, \quad t = 0, 1, \ldots,
\] (70)
in which \(\{\bar{r}^*_s\}_{s=1}^{\infty}\) is the sequence of international-equilibrium interest rates.

With equation (70) at hand we can identify which \(b_0\) is possible or admissible, through equating supply and demand for bonds in period 0. Recall from equation (19) that,
\[
\frac{W_1}{L} = \frac{W (\{\bar{r}^*_s\}_{s=1}^{\infty})}{L} = \prod_{s=1}^{\infty} \frac{1}{1 + \bar{r}^*_s} + 1 + \sum_{s=1}^{\infty} \frac{1}{\prod_{j=1}^{s} (1 + \bar{r}^*_j)}.
\] (71)

A direct implication of equation (70) is that \(\lim_{t \to \infty} \bar{r}^*_t = (1 - \beta) / \beta\), and consequently,
\[
\prod_{s=1}^{\infty} \frac{1}{1 + \bar{r}^*_s} = 0,
\] (72)
which is the first term of the right-hand side of (71). In particular, after incorporating (72) and (70) into (71) we obtain,
\[
\frac{W (\{\bar{r}^*_s\}_{s=1}^{\infty})}{L} = \frac{1}{1 - \beta_N} + \sum_{s=1}^{\infty} \prod_{j=1}^{s} \frac{\beta - \beta_N}{1 + \left(\frac{\beta_N}{\beta} \right)^{j-1} \left(\frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} \equiv F (b_0). \] (73)

In order to understand whether an equilibrium with default is possible in the case of \(N \geq 2\), we examine which values of \(b_0\) are possible after equating supply with demand for bonds in period 0. This market-clearing condition is obtained by substituting (73) into equation (61), after setting \(t = 0\) for the latter, which gives, \((\beta - \beta_N) b_0 = (1 - \beta_N) F (b_0) - 1\), or,
\[
H (b_0) \equiv \frac{\beta - \beta_N}{1 - \beta_N} b_0 + \frac{1}{1 - \beta_N} = F (b_0). \] (74)
In order to find solutions of (74) that reflect bond-market clearing in period 0, it is helpful to understand some properties of function $F(b_0)$. Let

$$f(b_0, j) = \frac{\beta - \beta_N}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\frac{1}{1-\beta} \cdot \frac{1}{b_0} - 1\right)}.$$  \hspace{1cm} (75)

From (75) and (73),

$$F(b_0) = \frac{1}{1 - \beta_N} + \sum_{s=1}^{\infty} \prod_{j=1}^{s} f(b_0, j) > 0 \text{, for all } b_0 \in \left[0, \frac{1}{1 - \beta}\right]. \hspace{1cm} (76)$$

Since, for all $b_0 \in [0, 1/(1 - \beta)]$,

$$f_{b_0}(b_0, j) = \frac{\beta - \beta_N}{1 - \beta} \left(\frac{\beta_N}{\beta}\right)^{j-1} \left\{\left[1 - \left(\frac{\beta_N}{\beta}\right)^{j-1}\right] b_0 + \frac{1}{1-\beta} \left(\frac{\beta_N}{\beta}\right)^{j-1}\right\}^2 > 0,$$  \hspace{1cm} (77)

an implication of (76) and (77) is,

$$F'(b_0) = f_{b_0}(b_0, 1) + \sum_{s=2}^{\infty} \sum_{j=1}^{s} f_{b_0}(b_0, j) \prod_{l=1}^{s} f(b_0, l) > 0.$$  \hspace{1cm} (78)

In addition,

$$F(0) = \frac{1}{1 - \beta_N} = H(0), \hspace{1cm} (79)$$

since $f(0, j) = 0$ for all $j \in \{1, 2, \ldots\}$,

$$F'(0) = (1 - \beta) (\beta - \beta_N) < \frac{\beta - \beta_N}{1 - \beta_N} = H'(0), \hspace{1cm} (80)$$

and

$$F\left(\frac{1}{1 - \beta}\right) = \frac{1}{1 - \beta_N} + \frac{\beta - \beta_N}{1 - (\beta - \beta_N)} < \frac{1}{1 - \beta} = H\left(\frac{1}{1 - \beta}\right). \hspace{1cm} (81)$$

Equations (78), (79), (80), and (81) show that, as $b_0$ spans the interval $[0, 1/(1 - \beta)]$, (i) function $F(b_0)$ starts from taking the value $1/(1 - \beta_N)$, and satisfying the market-clearing condition at $b_0 = 0$, (ii) it continues in the neighborhood of $b_0 = 0$ with slope which is lower
than the constant slope of $H(b_0)$, $(F'(0) < H'(0))$, meaning that $F(b_0)$ goes below function $H(b_0)$ in the neighborhood of $b_0 = 0$, (iii) $F(b_0)$ continues as a strictly increasing function all the way up to $1/(1 - \beta)$, and (iv) then at $1/(1 - \beta)$, $F(1/(1 - \beta)) < H(1/(1 - \beta))$.

Investigating concavity/convexity properties of $F(b_0)$ is a cumbersome task with, perhaps ambiguous results. Properties (i)-(iv) regarding the behavior of $F(b_0)$, reveal that, if $F(b_0)$ was either globally concave or globally convex on the interval $[0, 1/(1 - \beta)]$, then it would be immediately proved that $b_0 = 0$ (full default) would be the only value satisfying the market-clearing condition $F(b_0) = H(b_0)$. Since we do not have such a result at hand, we prove that no solutions other than default are possible, proceeding by contradiction.

Suppose that there exists some $\tilde{b}_0 \in (0, 1/(1 - \beta))$, such that,

$$F(\tilde{b}_0) = H(\tilde{b}_0). \quad (82)$$

From (67) we know that,

$$\tilde{b}_1 = \frac{1}{\beta N \frac{1}{b_0} + \xi} = \frac{\tilde{b}_0}{\alpha}, \quad (83)$$

in which $\xi \equiv (\beta - \beta_N) (1 - \beta) / \beta$ and $\alpha \equiv \beta_N / \beta + \xi \tilde{b}_0$. Since $\tilde{b}_1$ is on the equilibrium path, it should also satisfy,

$$F(\tilde{b}_1) = H(\tilde{b}_1). \quad (84)$$

From (74) and (83) it is,

$$H(\tilde{b}_1) = \frac{1}{\alpha} \frac{\beta - \beta_N \tilde{b}_0}{1 - \beta_N} + \frac{1}{1 - \beta_N},$$

and by substituting (83) into this last expression again, we obtain

$$H(\tilde{b}_1) - \frac{1}{1 - \beta_N} = \frac{1}{\alpha} \left[ H(\tilde{b}_0) - \frac{1}{1 - \beta_N} \right] = \frac{1}{\alpha} \left[ F(\tilde{b}_0) - \frac{1}{1 - \beta_N} \right] = F(\tilde{b}_1) - \frac{1}{1 - \beta_N}, \quad (85)$$

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an implication of (82) and (84). From (73) it is,

$$F(\tilde{b}_1) - \frac{1}{1 - \beta_N} = \sum_{s=1}^{\infty} \prod_{j=1}^{s} \frac{\beta - \beta_N}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\alpha \frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} ,$$

and (85) implies,

$$\sum_{s=1}^{\infty} \prod_{j=1}^{s} \frac{\beta - \beta_N}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\alpha \frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} = \frac{1}{\alpha} \sum_{s=1}^{\infty} \prod_{j=1}^{s} \frac{\beta - \beta_N}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} .$$

Subtracting the right-hand-side of (86) from the left-hand side and rearranging terms,

$$\sum_{s=1}^{\infty} (\beta - \beta_N)^s \frac{\alpha - 1}{\alpha} \prod_{j=1}^{s} \left[ \frac{1}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\alpha \frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} - \frac{1}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} \right] = 0 ,$$

or,

$$\frac{\alpha - 1}{\alpha} \sum_{s=1}^{\infty} (\beta - \beta_N)^s (1 - \alpha)^s \times$$

$$\prod_{j=1}^{s} \left[ \frac{\left(\frac{\beta_N}{\beta}\right)^{j-1} \frac{1}{1 - \beta} \frac{1}{b_0}}{1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\alpha \frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right)} \right] \left[ 1 + \left(\frac{\beta_N}{\beta}\right)^{j-1} \left(\frac{1}{1 - \beta} \cdot \frac{1}{b_0} - 1\right) \right] = 0 .$$

From (83) we know that

$$\alpha = \frac{\tilde{b}_0}{\tilde{b}_1} ,$$

and from (66) it is,

$$\frac{\tilde{b}_0}{\tilde{b}_1} = \frac{1}{\beta} \frac{1}{1 + \tilde{r}_1} .$$

Yet, it is verifiable from (70) that for all $\tilde{b}_0 < 1 / (1 - \beta)$,

$$\tilde{r}_1 > \tilde{r}^{ss} \iff \frac{1}{\beta} \frac{1}{1 + \tilde{r}_1} < \frac{1}{\beta} \frac{1}{1 + \tilde{r}^{ss}} = 1 .$$

Combining (90) with (89) and (88) implies,

$$0 < \alpha < 1 .$$
Inequality (91) implies that the left-hand side of (87) is the product of a negative term, \((\alpha - 1)/\alpha\), and an infinite summation of strictly positive terms, contradicting (87). Since the choice of \(\tilde{b}_0 \in (0, 1/(1 - \beta))\) was arbitrary, the possibility that \(N \geq 2\) and positive outstanding fiscal debt is ruled out.

Therefore, \(b_0 = 0\) is the only admissible solution. To see that \(b_0 = 0\) is admissible, notice that (66) implies \(b_t = 0\) for all \(t \in \{0, 1, \ldots\}\), so \(F(b_t = 0) = H(b_t = 0)\) is always satisfied.

To sum up, if \(N \geq 2\), domestic governments will default. After the default, all future governments will optimally cease the issuing of public deficit. This optimal behavior in our model is demonstrated by equation (66).

\[\square\]

**Proof of Proposition 3**

Interest-rate levels are determined by equating demand and supply of government bonds in international markets. In particular, the demand for bonds one period ahead, \(B_{t+1}^*\), is given by equation (15). Bond supply is obtained by combining the optimal level of government spending with the fiscal-budget constraint. From Proposition 1 (see equations (22) and (23) for \(N = 1\)) we know that the supply of bonds in period \(t + 1\) is given by,

\[B_{t+1} = \beta (1 + r_{t+1}) B_t + (1 + r_{t+1}) [(1 - \beta) z_t \mathbb{W} \left( \{r_s\}_{s=t+1}^\infty \right) - Y_t].\]  \hspace{1cm} (92)

After applying the equilibrium condition \(B_{t+1} = B_{t+1}^*\), and assuming also that \(B_t = B_t^*\) (no default in any period), equations (92) and (15) imply,

\[\mathbb{W} \left( \{r_s\}_{s=t+1}^\infty \right) = \frac{L}{1 - \beta}, \quad t = 0, 1, \ldots.\]  \hspace{1cm} (93)

In the proof of Proposition 1 we have mentioned an easily verifiable result, that the sequence \(\{W_{t+1}\}_{t=0}^\infty\) corresponding to equation (19) satisfies the recursion given by (41). Specifically,
the formula given by (19) is the solution to (41). After substituting (93) into (41), we obtain the level of interest rate $r^{ss}$ given by (27), and the implication that $r_{t+1} = r^{ss}$ for all $t \in \{0, 1, \ldots\}$.

Equations (28), (29), and (30) are derived immediately after substituting $r_{t+1} = r^{ss}$ for all $t \in \{0, 1, \ldots\}$ into (22), (18), (20), and (21). In all cases we take into account that, under cooperation, $\beta_N = \beta$. Under cooperation, all formulas are considered as if $N = 1$ with the sole exception that the aggregate rents of the coalition are equally shared among rent-seeking groups, with each rent-seeking group member receiving $\mathcal{C}_t^{R,C}(B_t, z_t) / N$. \hfill $\Box$
Definition of a Markov-perfect-cooperation-decision Nash equilibrium (MPCDNE)

Let the cooperation decision of rent-seeking group $j \in \{1, ..., N\}$ be denoted by the indicator function

$$I_{j,t} = \begin{cases} 
1 , & \text{ } j \text{ plays “cooperate” in period } t \\
0 , & \text{ } j \text{ plays “do not cooperate” in period } t 
\end{cases}.$$ 

Let the rent-consumption strategies in periods of no cooperation be denoted by $C_{j}^{NC}$ for all $j \in \{1, ..., N\}$. Let

$$S \equiv \left\{ \left( C_{i}^{NC}, I_{i} \right) \right\}_{i=1}^{N},$$

and two Bellman equations, one related to determining the value of a cooperation decision in the current period,

$$V_{C;j}^{C}(B, z | S) = \max_{(\tau, C_{j}^{R,C}, B')} \left\{ \ln (zL) + \ln (1 - \tau) + \theta_{l} \ln (1 - L) + \theta_{R} \ln \left( \frac{C_{j}^{R,C}}{N} \right) \right. 
+ \beta \left\{ \prod_{i=1}^{N} I_{i} (B', (1 + \gamma) z | S) V_{C;j}^{C} (B', (1 + \gamma) z | S) 
+ \left[ 1 - \prod_{i=1}^{N} I_{i} (B', (1 + \gamma) z | S) \right] V_{NC;j}^{C} (B', (1 + \gamma) z | S) \right\}, \tag{94}$$

and one related to determining the value of a noncooperation decision in the current period,

$$V_{NC;j}^{NC} (B, z | S) = \max_{(\tau, C_{j}^{NC}, B')} \left\{ \ln (zL) + \ln (1 - \tau) + \theta_{l} \ln (1 - L) + \theta_{R} \ln \left( c_{j}^{R,NC} \right) \right. 
+ \theta_{G} \ln \left[ \frac{B'}{1 + R (B, z | S)} - (B + C_{j}^{R,C} - \tau zL) \right] 
+ \beta \left\{ \prod_{i=1}^{N} I_{i} (B', (1 + \gamma) z | S) V_{C;j}^{C} (B', (1 + \gamma) z | S) 
+ \left[ 1 - \prod_{i=1}^{N} I_{i} (B', (1 + \gamma) z | S) \right] V_{NC;j}^{C} (B', (1 + \gamma) z | S) \right\}. \tag{95}$$
Definition B.1 focuses on global cooperation among $N$ rent-seeking groups, excluding cooperating subcoalitions. In the application of this paper we focus on a symmetric equilibrium of the case with $N = 2$, i.e., subcoalitions are impossible.

**Definition B.1** A Markov-Perfect-Cooperation-Decision Nash Equilibrium (MPCDNE) is a set of strategies, $S \equiv \left\{ \left( C_{i,t}^{R,NC}, I_i \right) \right\}_{i=1}^N$ of the form $C_{i,t}^{R,NC} = C_{i,t}^{R,NC} (B_t, z_t | S)$

$$I_{i,t} = I_i (B_t, z_t | S)$$

with

$$I_i (B_t, z_t | S) = \begin{cases} 
1 , & \text{if } V^{C,j} (B, z | S) \geq V^{NC,j} (B, z | S) \text{ and } \prod_{j=1}^N I_j (B_t, z_t | S) = 1 \\
0 , & \text{if } V^{C,j} (B, z | S) < V^{NC,j} (B, z | S) \text{ and } \prod_{j=1}^N I_j (B_t, z_t | S) = 1 , \\
or if \prod_{j=1}^N I_j (B_t, z_t | S) = 0
\end{cases}$$

and a set of policy decision rules $(T, G, B)$ of the form,

$$\tau_t = T (B_t, z_t | S) = \prod_{i=1}^N I_i (B_t, z_t | S) T^C (B_t, z_t | S)$$

$$+ \left[ 1 - \prod_{i=1}^N I_i (B_t, z_t | S) \right] T^{NC} (B_t, z_t | S) ,$$

$$B_{t+1} = B (B_t, z_t | S) = \prod_{i=1}^N I_i (B_t, z_t | S) B^C (B_t, z_t | S)$$

$$+ \left[ 1 - \prod_{i=1}^N I_i (B_t, z_t | S) \right] B^{NC} (B_t, z_t | S) ,$$

$$G_t = G (B_t, z_t | S) = \prod_{i=1}^N I_i (B_t, z_t | S) G^C (B_t, z_t | S)$$

$$+ \left[ 1 - \prod_{i=1}^N I_i (B_t, z_t | S) \right] G^{NC} (B_t, z_t | S) ,$$

a bond-supply strategy of creditors, $B_{t+1}^* = B^* (B_t, z_t | S)$, and an interest-rate rule, $R^{NC} (B_t, z_t | S)$, such that $(T^{NC}, B^{NC}, C^{R,NC}_j, G^{NC})$ guarantee that each and every rent seeking group $j \in \{1, \ldots, N\}$ solves the Bellman equation given
by (95), \( (T^C, B^C, C^{R,C}, G^C) \) solves the Bellman equation given by (94), creditors’ \( B^* \) complies with equation (15), and with \( R^{NC}(B_t, z_t | S) = r_{t+1} \) satisfying \( B(B_t, z_t | S) = B^*(B_t, z_t | S) \), for all \( t \in \{0, 1, \ldots \} \).

With this definition at hand, we proceed to formally proving Proposition 4.

**Proof of Proposition 4**

In order to calculate \( V^{C,j}(B_t, z_t) \) we substitute the results stated by Propositions 1 and 3 into the Bellman equation given by (17), after taking into account that the total rents of the coalition are divided by 2, which implies that we must subtract \( \theta_R \ln(2) / (1 - \beta) \). In the proof of Proposition 1 we have already achieved most of this calculation as we have obtained the expressions for \( \zeta \), \( \psi \), and \( \nu \) (c.f. equations (60), (55), and (54), which correspond to the value function given by (42)). From equation (27) in Proposition 2 we know that \( W_t/L = 1 / (1 - \beta) \) for all \( t \in \{0, 1, \ldots \} \), so \( V^{C,j}(B_t, z_t) \) becomes,

\[
V^{C,j}(B_t, z_t) = \frac{1}{1 - \beta} \left\{ -\theta_R \ln(2) + \theta_1 \ln (1 - L) + \theta_G \ln (\theta_G) + \theta_R \ln (\theta_R) + (1 + \theta_G + \theta_R) \ln \left( \frac{z_t L}{1 - \beta} - B_t \right) \right\}. \tag{96}
\]

In order to calculate \( V^{NC,j}(B_t = 0, z_t | \left\{ C_i^{R,NC}_{i \neq j} \right\}^2_{i=1} \) we find the static-equilibrium non-cooperative solution for \( N = 2 \), and calculate the discounted sum of lifetime utility of each group. So,

\[
V^{NC,j}(B_t = 0, z_t | \left\{ C_i^{R,NC}_{i \neq j} \right\}^2_{i=1}) = \frac{1}{1 - \beta} \left\{ \theta_1 \ln (1 - L) + \theta_G \ln (\theta_G) + \theta_R \ln (\theta_R) + (1 + \theta_G + \theta_R) [\ln(L) - \ln (1 + \theta_G + 2\theta_R)] \right\}
\]
\begin{equation}
(1 + \theta_G + \theta_R) \left[ \frac{\beta \ln(1 + \gamma)}{1 - \beta} + \ln(z_t) \right].
\end{equation}

Comparing (96) with (97) leads to the cutoff debt-GDP ratio in (33).

In order to verify that the cases in which (i) the two rent-seeking groups never cooperate, (ii) the two rent-seeking groups cooperate forever, are both \textit{Markov-Perfect-Cooperation-Decision Nash Equilibrium (MPCDNE)}, notice that, by definition B.1, (i) can be a MPCDNE, no matter what $b_t$ might be. From Proposition 2 we know that if rent-seeking groups never cooperate, then $b_t = 0$ for all $t \in \{0, 1, \ldots\}$, which still allows (i) to be an MPCDNE. To see that (ii) is also an MPCDNE, notice that, as long as (33) holds in period 0, then Proposition 3 (c.f. eq. 28) implies $b_t = b_0$, so (33) holds for all $t \in \{0, 1, \ldots\}$. So, rent-seeking groups cooperating forever is an MPCDNE, as a direct consequence of Definition B.1.

\textbf{Proof of Proposition 5}

In order to derive $\bar{b}$, notice that

\begin{equation}
g_{\text{default}}^{NC} = \frac{\theta_G}{1 + \theta_G + 2\theta_R} = \frac{\alpha \theta_G}{1 + \alpha \theta_R},
\end{equation}

and that (29) implies,

\begin{equation}g_{\text{C}}^{C} = \frac{\theta_G}{\theta_R} \alpha \left[ 1 - (1 - \beta) \bar{b} \right].
\end{equation}

Comparing (98) with (99) gives,

\begin{equation}g_{\text{C}}^{C} \geq g_{\text{default}}^{NC} \iff \bar{b} \leq \frac{1}{1 - \beta} \frac{\alpha}{1 + \alpha},
\end{equation}

proving (34), (35), and (36). To show that $\bar{b} > \underline{b}$, use (34) and (33),

\begin{equation}\bar{b} > \underline{b} \iff 2^\alpha > 1,
\end{equation}

which is a true statement, proving the proposition. \hfill \square
Proof that calibrating \( \theta_R \) and \( \theta_G \) in order to match target values for the government-consumption-GDP ratio and the total-rents-GDP ratio is independent from the values of \( \beta \) at the cutoff level \( \bar{b} \)

Let \( g \) denote the government-consumption-GDP ratio \( G/Y \) at the cutoff debt-GDP \( \bar{b} \), and let \( c_R \) denote the total-rents-GDP ratio at the cutoff debt-GDP \( \bar{b} \). Substituting the formula given by (33) for \( \bar{b} \) into (29), we obtain,

\[
g = \frac{\theta_G}{\theta_R} \alpha [1 - (1 - \beta) \bar{b}] = \frac{\theta_G}{\theta_R} \frac{\alpha 2^\alpha}{1 + \alpha},
\]

in which \( \alpha \) is given by (32). Equation (30) implies,

\[
\frac{c_R}{g} = \frac{\theta_R}{\theta_G} \Rightarrow \theta_G = \theta_R \frac{g}{c_R}.
\]

Using (101), we can express (100) as a function of parameter \( \theta_R \) alone, obtaining,

\[
g = \frac{\theta_R \frac{g}{c_R}}{1 + \theta_R(1 + \frac{g}{c_R})} \frac{\theta_R}{1 + \theta_R(1 + \frac{g}{c_R})}.
\]

Using (102) together with target calibration values for \( g \) and \( c_R \), we can find the specific value of parameter \( \theta_R^* \) by solving the nonlinear equation

\[
f(\theta_R) = 0,
\]

in which

\[
f(\theta_R) \equiv \frac{\theta_R \frac{g}{c_R}}{1 + \theta_R(1 + \frac{g}{c_R})} \frac{\theta_R}{1 + \theta_R(1 + \frac{g}{c_R})} - g.
\]

From (103) we can see that matching target calibration values for \( g \) and \( c_R \) is independent from values of \( \beta \). Finally, from (101), \( \theta_R^* = \theta_R^* g/c_R \). ☐
REFERENCES


International Monetary Fund (2011b): "Ireland: First and Second Reviews Under the Extended Arrangement and Request for Rephasing of the Arrangement—Staff Report; Letter of Intent; Memorandum of Economic and Financial Policies; Technical Memorandum of Understanding; Letter of Intent and Mem-

International Monetary Fund (2011c): "Italy—Staff Report for the 2011 Article IV Consultation; Informational Annex; Public Information Notice; Statement by the Staff Representative; and Statement by the Executive Director for Italy." IMF Country Report No. 11/173, downloadable from: http://www.imf.org/external/pubs/ft/scr/2011/cr11173.pdf


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Figure 1 Correlation between the fiscal-surplus/GDP ratio (in percentage points) and the Corruption-Perceptions Index (CPI) for Euro zone countries (t-statistics in parentheses). For Cyprus, Estonia, Malta, Slovakia and Slovenia averages are calculated since four years prior to joining the Euro zone. Sources: Eurostat, Transparency International.
Figure 2 The white and black regions defined by $b$ in the deterministic version of the model in which changes in the debt-GDP ratios occur as once-and-for-all sunspot shocks. Stochastic versions of the model with anticipated shocks will introduce grey zones of default affected by politics.
Figure 3

Rate of Time preference and cooperation bound (%)
Online Appendix

Supplementary Material

for

Political Economics of External Sovereign Defaults

by

Carolina Achury, Christos Koulovatianos and John Tsoukalas

Corruption and External Sovereign Debt Inter-linkages in Eurozone Countries
Figure S.1 depicts the evolution of external sovereign debt in years 2003, 2006, and 2009, a year before the sovereign-debt crisis broke out in the Eurozone. Figure S.1 corroborates that, perhaps due to the currency union, Eurozone countries continued to issue external debt, as the Eurozone banking system facilitated the exchange of sovereign bonds among Eurozone commercial banks.

That commercial banks had incentives to buy sovereign bonds of periphery Eurozone countries is corroborated by Figure S.2. Figure S.2 depicts the evolution of 10-year sovereign-bond returns in Eurozone-periphery countries (Greece, Portugal, Italy, Spain, and Ireland) versus Germany, before the entrance to the Eurozone and after the start of the sovereign-debt crisis. All countries that had high interest-rate spreads compared to Germany before entrance to the Eurozone suffer chronically from corruption, according to the Corruption Perceptions Index survey. Table S.1 shows that Greece, Portugal, Italy, and Spain, have been scoring low according to the Corruption Perceptions Index survey throughout the years 1995-2010. The fact that the pre-Eurozone sovereign spreads of these countries vanished rapidly and persistently between years 2001-2008, indicates that Eurozone creditors (including commercial banks) in Eurozone-core countries, bought substantial amounts of sovereign debt. According to Figure S.1, the external debt of Greece, Portugal, Italy and Spain, rose sharply between years 2003-2009, and placed these countries among the top external-sovereign-debt issuers in the world (highest debt-GDP ratios). The key message from Figure S.2 is that countries with high corruption had high sovereign-debt spreads before entering the Eurozone and afterwards, during the sovereign-debt crisis. Ireland, which exhibits low corruption, had sovereign-debt problems only during the sovereign-debt crisis, most likely due to its post-Lehman-Brothers banking crisis, which was combined with a domestic real-estate price drop.

In brief, external sovereign debt of EU periphery countries grew rapidly in the 2000s (Figure S.1), speculating that commercial banks in Eurozone-core countries may have been the main buyer of periphery sovereign debt as indicated by dynamics of 10-year-bond returns before and after the introduction of the Euro (Figure S.2). Euro area commercial banks typically hold a diversified portfolio of government bonds of several union countries and thus can be severely affected by a default through losses on these bonds. Bolton and Jeanne (2011) provide information on Euro area commercial banks foreign debt exposures as of 2010.

The overarching element before the introduction of the Euro and after the sovereign crisis broke out, distinguishing core versus periphery countries in the Eurozone, is that the latter countries always had more corruption (Table S.1). Although our framework does not explicitly model banks, we use corruption and rent seeking as the main driver of developments after the crisis, and we provide insights concerning the political sustainability of bailout plans. First, we speculate that corruption and rent seeking was responsible for having high spreads in high-corruption EU countries before the introduction of the Euro (cheap bonds due to inflationary expectations). Second, we speculate that the low prices of sovereign bonds in high-corruption countries made these bonds attractive for arbitrage by banks in low-corruption Eurozone countries in the 2000s, increasing the external sovereign
debt of high-corruption Eurozone countries dramatically. Third, since high-corruption countries held high external debt during the subprime crisis period, some consequences of sovereign default could be transmitted abroad, making the possibility of default higher, and leading bond spreads to rise dramatically (see Figure S.2). Yet, the risk of financial contagion in the Eurozone may be high, motivating bailout-package initiatives. Such a rough outline of causes and effects of the sovereign crisis in the Eurozone is what motivated us to focus our model on the interplay between external debt and corruption in order to study the political sustainability of fiscal targets set by rescue packages.

References

Figure S.1 -- Source: European Central Bank and International Monetary Fund.
Figure S.2 -- Source: European Central Bank and European Commission, secondary market yields of government bonds with maturities close to 10 years and corruption ranking according to Corruption Perception Index 2010 from Transparency International (higher ranking means lower corruption).
Table S.1 -- Corruption Perception Index

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<td>Score</td>
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<tr>
<td>Ireland</td>
<td>8.57 (10)</td>
<td>8.2 (14)</td>
<td>7.5 (18)</td>
<td>7.4 (19)</td>
<td>7.7 (16)</td>
<td>8.0 (14)</td>
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<tr>
<td>Germany</td>
<td>8.14 (11)</td>
<td>7.9 (15)</td>
<td>7.7 (16)</td>
<td>8.2 (16)</td>
<td>7.9 (14)</td>
<td>7.9 (15)</td>
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<tr>
<td>Spain</td>
<td>4.35 (24)</td>
<td>6.1 (23)</td>
<td>6.9 (23)</td>
<td>7.0 (23)</td>
<td>6.5 (28)</td>
<td>6.1 (30)</td>
</tr>
<tr>
<td>Portugal</td>
<td>5.56 (20)</td>
<td>6.5 (22)</td>
<td>6.6 (25)</td>
<td>6.5 (26)</td>
<td>6.1 (32)</td>
<td>6.0 (32)</td>
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<tr>
<td>Italy</td>
<td>2.99 (31)</td>
<td>4.6 (39)</td>
<td>5.3 (35)</td>
<td>5.0 (40)</td>
<td>4.8 (55)</td>
<td>3.9 (67)</td>
</tr>
<tr>
<td>Greece</td>
<td>4.04 (28)</td>
<td>4.9 (36)</td>
<td>4.3 (50)</td>
<td>4.3 (47)</td>
<td>4.7 (57)</td>
<td>3.5 (68)</td>
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<tr>
<td>Best-worst score</td>
<td>9.55-1.94</td>
<td>10-1.4</td>
<td>9.7-1.3</td>
<td>9.7-1.7</td>
<td>9.3-1.0</td>
<td>9.3-1.1</td>
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Source: Transparency International
Note: Higher score means lower corruption and numbers appearing in parentheses next to each score is the country’s world-corruption ranking based on the score in each particular year.
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