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Cross-Border Shopping and Commodity Tax Harmonization

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Abstract

This is a continuation of Ohsawa(1998). EU corporate tax policy such as tax competition, tax harmonization and tax coordination has been a serious concern for the European Community. In this paper, we formulate multi-country commodity tax competition models where each country chooses its tax rate within a common band with an eye on strategic consideration to maximize its tax revenue. Then we explore the effect of harmonizing tax rate within a common band. In particular, sensitivity analyses concerning the common band are carried out.

JEL Classification: H71; H73; R51

Keywords: tax competition; tax harmonization; Nash equilibrium; cross-border shopper

1. Introduction

Commodity taxes vary from country to country within EU. This associated with free movements of customers induce cross-border shopping. Since cross-border shopping induced by tax differentials affects the government revenues, EU corporate tax policy such as tax competition, tax harmonization and tax coordination has been a serious concern for the European Community: see Sinn(1990), Christiansen(1994), Robson(1998). After January 1993, EU countries have experienced the origin principle such that individuals' purchases are taxed. The political trade-off between tax competition and tax coordination was discussed by Edwards and Keen (1996). As a rule, tax competition mitigates political distortions, while tax coordination limits economic distortions. Therefore, the tax harmonization including the tax competition and tax harmonization as extreme cases deserve more in-depth analysis.

Several articles have been devoted to analytical models of origin-based commodity tax competition between two governments, taking into consideration the tax-induced shopping. The first scholor to give much attention to this study by taking advantage of geographical market rather than treating a spaceless market was Kanbur and Keen(1993). They and Ohsawa(1998) formulated a simple two-country model in which each country sets its tax rate with a view to maximizing its tax revenue taking account of cross-border shopping. While in their model two countries differ by the density of customers, but have equal sizes, Ohsawa(1998) assumed that although customers are uniformly distributed over the whole market, the size of countries are different. Both works revealed that the size of countries play an important role in strategic tax design. Ohsawa(1998) also analyzed the outcomes of the tax competition among more than two countries to analyze how the position of countries affects tax rates and revenues at a Nash equilibrium.

In order to restrict the revenue losses associated with cross-border shopping, from January 1993, a standard VAT rate of at least 15 per cent was applied. As pointed out by Somers(1998), in the field of tax harmonization, there are two main approaches: one is a complete equalisation at a common tax rate: another is imposing a minimum standard rate. We shall call them *tax coordination*, *minimum standard rate policy*, respectively. Kanbur and Keen(1993) also made an important contribution on tax harmonization by examining how these two types of harmonization affect the equilibrium tax rates and revenues of two competing governments. However, the study of multi-country tax harmonization has been

strangely neglected by researchers.

In this paper, we formulate multi-country commodity tax competition models where with the customers density assumed uniform each country chooses its tax rate within a common band with the object of maximizing tax revenues. Then we analyze the effect of harmonizing tax rates within some common band. The current research extends the model in Kanbur and Keen(1993) in at least two directions. First, the number of countries is not restricted to be two. This enables us to explore the relationship between tax harmonization and the position of the countries. Second, we deal with the role of maximum standard rates. This enables us to compere several type of tax harmonization in addition to the two above-mentioned harmonization within only one framework. This research also generalizes the multi-country commodity tax competition model developed by Ohsawa(1998) in one respect; the introduction of these two standard rates. This paper focuses on the following questions: to what extent tax harmonization affects the tax rates, demands, the revenues and the volume of tax-induced cross-border shoppers at a Nash equilibrium ?; how the relative sizes and positions of the countries affect their equilibrium tax rates and revenues in tax harmonization ?; When do Nash equilibria exist ?

This paper is organized as follows. In the next section, our general tax competition model is formulated. The candidate for Nash equilibrium are also derived. In Section 3, we characterize the general effect of the tax harmonization on tax rates, cross-border shoppers, demands and revenues in equilibrium. Also, by specializing a two-country model and a multi-country model having the same size, which can be imagined more easily, we explore the effect in more detail. Section 4 provides concluding comments. For convenience, all proofs are collected in an Appendix.

2. Model

The model presented here is the same as that used in Ohsawa(1998). The assumptions used in both models are presented below for clarity. Since the notation is obviously burdensome, we try match that used by Ohsawa(1998) as much as possible. Suppose that there exists a line segment along which $N(\geq 2)$ countries divide into N contiguous and nonoverlapping line segments. They are numbered in ascending order from one end of the whole line market. Let the size for ith country be denoted by $L_i(>0)$, as shown in Figure 1. We make

the following assumptions over the liner market:

1. Customers and firms are uniformly distributed in the line segment, so that the size of a country corresponds to its areal extension.

2. Transport costs are proportional to distance.

3. There is a single homogeneous commodity that is produced by all firms at constant marginal production cost.

4. Marginal production cost is zero, because our results do not depend on them.

5. Each government levies a source-based commodity tax, denoted by p_i, on the firms located within the corresponding country.

6. Firms are non-cooperative, so that all firms in ith government would price at its tax-inclusive marginal cost, i.e., p_i.

7. Each customer purchases one unit of the commodity from the firm quoting the lowest *full price*, i.e., the mill price plus the transport cost between the firm and the customer, irrespective of its full price.

8. Each government chooses its tax rate p_i with a common band such that $\underline{p} \leq p_i \leq \overline{p}$, where $\underline{p}$ and $\overline{p}$ $(0 \leq \underline{p} \leq \overline{p})$ are maximum and minimum standard rates, respectively.

9. Each government maximizes its tax revenue by changing its tax rate, assuming that it considers the tax rates of its rivals as unchanging.

We define the *revenue* of ith government $\pi_i(p_1, \ldots, p_N)$ as the sum of the taxes from all firms within it. We call ${p^*}_i$'s in *equilibrium* if and only if $p_i^* > 0$ and $\pi_i({p^*}_1, \ldots, p_i, \ldots, {p^*}_N) \leq \pi_i({p^*}_1, \ldots, {p^*}_i, \ldots, {p^*}_N)$ $(1 \leq \forall i \leq N)$. Thus we consider a Nash equilibrium of a non-cooperative N-person game whose players are governments, and where strategies are tax rates and payoffs are revenues. We call this model *tax harmonization* (constrained tax competition) model. As special cases, tax harmonization with $\underline{p} = 0$ and $\overline{p} = \infty$ reduces to standard (unconstrained) tax competition, which has already analyzed by Ohsawa (1998). Harmonization with $\underline{p} > 0$ and $\overline{p} = \infty$ becomes minimum standard rate policies. Harmonization with $\underline{p} = \overline{p}$ correspond to tax coordination. Also, tax harmonization with $\underline{p} = 0$ and $\overline{p} < \infty$ is called *maximum standard rate policy*. To avoid misunderstanding, if and only if

$\underline{p_1} \geq \underline{p_2}$ and $\overline{p_1} \leq \overline{p_1}$ with strict inequality for at least one inequality, then tax harmonization with $\underline{p} = \underline{p_1}$ and $\overline{p} = \overline{p_1}$ is called to have a smaller common band than the one with $\underline{p} = \underline{p_2}$ and $\overline{p} = \overline{p_2}$ To make our notation simpler, we often denote $\pi_i(p^*{}_1, \ldots, p^*{}_N)$ by $\pi_i{}^*$ and $D_i(p^*{}_1, \ldots, p^*{}_N)$ by $D^*{}_i$.

As shown by Ohsawa(1998), at a Nash equilibrium all adjoining countries have to define their market boundary. Therefore, the demand of ith government, denoted by $D_i(p_1, \ldots, p_N)$, can be expressed as

$$D_i(p_1, \ldots, p_N) = \begin{cases} L_1 + \frac{1}{\gamma}(p_2 - p_1), & i = 1; \\ L_i + \frac{1}{\gamma}(p_{i-1} + p_{i+1} - 2p_i), & 2 \leq i \leq N-1; \\ L_N + \frac{1}{\gamma}(p_{N-1} - p_N), & i = N. \end{cases} \tag{1}$$

This is illustrated in Figure 1, where the horizontal axis measures distance, the vertical axis measures full prices, two standard rates. It should be noted that peripheral governments face competition from one side only; interior governments face competition from both sides. Since $\pi_i(p_1^*, \ldots, p_i, \ldots, p_N^*) = D_i(p_1^*, \ldots, p_i, \ldots, p_N^*)p_i$ is quadratic with respect to p_i, p_i^*'s have to satisfy

$$\begin{aligned} p^*{}_1 &= \mathrm{med}\{\underline{p}, \frac{\gamma L_1 + p^*{}_2}{2}, \overline{p}\}; \\ p^*{}_i &= \mathrm{med}\{\underline{p}, \frac{\gamma L_i + p^*{}_{i-1} + p^*{}_{i+1}}{4}, \overline{p}\}, \qquad 2 \leq i \leq N-1; \\ p^*{}_N &= \mathrm{med}\{\underline{p}, \frac{\gamma L_N + p^*{}_{N-1}}{2}, \overline{p}\}. \end{aligned} \tag{2}$$

where $\mathrm{med}(x, y, z)$ indicates the median of x, y and z. To simplify notations, $D_i(p_1^*, \ldots, p_N^*)$ and $\pi_i(p_1^*, \ldots, p_N^*)$ are denoted by D_i^* and π_i^*, respectively. To derive closed forms for p_i^*'s seems to be very complicated. However, it follows from the system (2) that p_i^*'s are continuous and piece-wise linear with respect to $\underline{p}$ and $\overline{p}$, so do D_i^*'s, and π_i^*'s are continuous and piece-wise quadratic with respect to them. Let $\underline{I}$ and $\overline{I}$ denote $\{i | p_i^* = \underline{p}\}$ and $\{i | p_i^* = \overline{p}\}$, respectively. Since $\pi_i^* = p_i^* D_i^*$, we have the following standard optimal conditions:

$$\gamma D_i^* \quad \begin{cases} \geq 2p_i^* & \text{for } i \in \overline{I} \\ = 2p_i^* & \text{for } i \notin \underline{I} \cup \overline{I}, \qquad 2 \leq i \leq N-1 \\ \leq 2p_i^* & \text{for } i \in \underline{I} \end{cases} \tag{3}$$

$$\gamma D_i^* \quad \begin{cases} \geq p_i^* & \text{for } i \in \overline{I} \\ = p_i^* & \text{for } i \notin \underline{I} \cup \overline{I}, \qquad i = 1 \text{ or } i = N \\ \leq p_i^* & \text{for } i \in \underline{I} \end{cases} \tag{4}$$

The following uniqueness property, which can be proved by showing that the system (2) has a unique solution based on contractive-mapping theorem, facilitates our analysis on tax rates, demands and revenues in equilibrium.

Property 1 *If a Nash equilibrium exists, it is unique.*

To ascertain that the solution to the system (2) is in equilibrium, it still remains to exclude the possibility that some governments can increase their revenues by undercutting their competitors. Some situations where there is no Nash equilibrium can be found in Ohsawa(1998). Even if a government undercut a peripheral government, it could not change its revenue discontinuously. Therefore, it suffices to show that any interior government cannot be undercut by its competitors. This also means that when the number of governments is two, there exists a Nash equilibrium.

3. Tax Harmonization

3.1. General Case

Let L_{min}, L_{max} denote the size of the smallest and largest governments, respectively. That is, $L_{min} = \min\{L_1, \cdots, L_N\}$, $L_{max} = \max\{L_1, \cdots, L_N\}$. Then we have the following proposition:

Proposition 1 *If* $\gamma L_{max} \leq \underline{p}$, *then* $p_i^* = \cdots = p_N^* = \underline{p}$ *are in equilibrium. If* $\overline{p} \leq \frac{\gamma L_{min}}{2}$, *then* $p_i^* = \cdots = p_N^* = \overline{p}$ *are in equilibrium.*

Neither very higher minimum nor very lower maximum standard rate policies is an absolute equalisation with respect to tax rates. Nevertheless, this Proposition shows that these policies establish a uniform tax structure across countries. An underlying mechanism is that very high minimum standard rates prevent any government, which would like to cut its tax rate to meet the optimality conditions (3) and (4), from doing so. A similar argument can apply to the case of very lower maximum standard rates. In both case, $D_i^* = L_i$, and in the former case $\pi_i^* = \underline{p}L_i$, in the latter case $\pi_i^* = \overline{p}L_i$.

Throughout the remainder of this paper, we deal with only the situation where $\underline{p} < \gamma L_{max}$ and $\gamma L_{min}/2 < \overline{p}$. In order to analyze their general behaviour of p_i^*, D_i^* and π_i^*, some local information such as their partial derivatives are useful. We are interested in the impacts of either raising the minimum standard rate $\underline{p}$ or lowering the maximum standard rate $\overline{p}$. Hence, we focus on the left-side partial derivatives with respect to $\underline{p}$ and the right-side one with respect to $\overline{p}$. Thus throughout this paper, the partial derivative with respect to $\underline{p}$ (resp. $\overline{p}$) should be understood to mean that the left-side (resp. right-side) partial derivative.

Property 2 *If a Nash equilibrium exists, then*

$$\frac{\partial p_i^*}{\partial \underline{p}} \begin{cases} = 0, & \text{for } i \in \overline{I}; \\ \in [0, \frac{1}{2}], & \text{for } i \notin \underline{I} \cup \overline{I}; \\ = 1, & \text{for } i \in \underline{I}, \end{cases} \qquad \frac{\partial p_i^*}{\partial \overline{p}} \begin{cases} = 1, & \text{for } i \in \overline{I}; \\ \in [0, \frac{1}{2}], & \text{for } i \notin \underline{I} \cup \overline{I}; \\ = 0, & \text{for } i \in \underline{I}. \end{cases}$$

From this Property three points become clear. First, raising minimum (resp. lowering maximum) standard rate induces equilibrium tax rates to go up (resp. go down) as a whole. Second, tax rates change less rapidly than the standard rates $\underline{p}$ and $\overline{p}$. Hence, we recognize that tax harmonization with a smaller common band results in reduction in the range of p_i^*'s, i.e., $\max\{p_1^*, \cdots, p_N^*\} - \min\{p_1^*, \cdots, p_N^*\}$. Thus, this result may justify that EU has adopted minimum standard rate policy for VAT rate in order to reduce the difference of tax rates. Finally, tax harmonization with a smaller common band decreases $|p_i^* - \underline{p}|$'s. So, there are less situation such that governments are undercut by ith government with $p_i^* \geq \underline{p}$. This means that tax harmonization tends to induce a Nash equilibrium to exist.

Let CBS denote the volume of tax-induced cross-border shoppers in the whole market, i.e., $CBS = \gamma^{-1} \sum_{i=1}^{N-1} |p_{i+1}^* - p_i^*|$.

Property 3 *If a Nash equilibrium exists, then*

$$\gamma \frac{\partial CBS}{\partial \underline{p}} \begin{cases} = 0, & \text{for } \underline{I} = \phi; \\ < -\frac{1}{2}, & \text{for } \underline{I} \neq \phi, \end{cases} \qquad \gamma \frac{\partial CBS}{\partial \overline{p}} \begin{cases} = 0, & \text{for } \overline{I} = \phi; \\ > \frac{1}{2}, & \text{for } \overline{I} \neq \phi. \end{cases}$$

Thus we arrive at the important conclusion that any tax harmonization with a smaller common band generates smaller volume of cross-border shoppers over the whole market. Thus we see that cross-border shoppers in any tax harmonization is below that in unconstrained Nash equilibrium. It should be noted that some tax harmonization with a smaller common band may generate more volume of the cross-border shoppers of *some* borders: see Example 1 in Appendix.

Property 4 *If a Nash equilibrium exists, then*

$$\gamma \frac{\partial D_i^*}{\partial \underline{p}} \begin{cases} \in [0, 1], & \text{for } i \notin \underline{I}; \\ \in [-1, 0], & \text{for } i \in \underline{I}. \end{cases} \qquad \gamma \frac{\partial D_i^*}{\partial \overline{p}} \begin{cases} \in [-1, 0], & \text{for } i \in \overline{I}; \\ \in [0, 1], & \text{for } i \notin \overline{I}. \end{cases}$$

There are at least one implications from this Property. First, it is plain that the increment in D_i^* caused by changing $\underline{p}$ or $\overline{p}$ corresponds to the one in the balance between inward and outward cross-border shoppers of ith government. Therefore, we conclude that raising $\underline{p}$ (resp. lowering $\overline{p}$) increases (resp. decreases) such balances of governments not belonging to $\underline{I}$ (resp. $\overline{I}$). Thus, although both standard rate policies reduce the volume of cross-border

shoppers, their impacts on the balances are different. Mathematically. this Property 4 yields $\frac{\partial}{\partial \underline{p}}\left((p_{i+1}^* - p_i^*) - (p_i^* - p_{i-1}^*)\right) = \gamma \frac{\partial D_i^*}{\partial \underline{p}} \geq 0$ and $\frac{\partial}{\partial \overline{p}}\left((p_{i+1}^* - p_i^*) - (p_i^* - p_{i-1}^*)\right) = \gamma \frac{\partial D_i^*}{\partial \overline{p}} \geq 0$. This implies that as $\underline{p}$ increases, so does the convexity of p_i^*'s, and as $\overline{p}$ decreases, the convexity of p_i^*'s decreases. Thus we see that raising $\underline{p}$ (resp. lowering $\overline{p}$) tends to reduce cross-border shoppers of inner (resp. outer) borders.

Property 5 *If a Nash equilibrium exists, then*

$$\frac{\partial \pi_i^*}{\partial \underline{p}} \geq 0, \text{ for } i \notin \underline{I}, \qquad \frac{\partial \pi_i^*}{\partial \overline{p}} \geq 0, \text{ for } 1 \leq \forall i \leq N.$$

This Property has at least two implications. First, the revenue derivative with respect to $\overline{p}$ are non-negative without exception. Therefore, we recognize that lowering $\overline{p}$ harms all the governments. This leads to the important conclusion that the revenue of each government in tax coordination cannot exceed that in any minimum standard rate policy with the same rate as a minimum standard. Thus the result in a two-country model developed by Kanbur and Keen (1993) are generalized to any number of governments. Second, as Kanbur and Keen (1993) pointed out, raising $\underline{p}$ may harm some governments. This can be also seen in Example 2 in Appendix. But for $i \in \underline{I}$, the revenue derivatives are non-negative. Combining this Property with Property 2 yields that the tax revenue in any minimum standard rate policy exceeds that than that in unconstrained Nash equilibrium, for the governments whose tax rate in unconstrained Nash equilibrium is greater than the minimum standard. Thus, we see that the impacts of $\underline{p}$ on π_i^*'s cannot be symmetric with the ones of $\overline{p}$.

The following Proposition gives two sufficient conditions for ensuring the existence of a Nash equilibrium. Let $\tilde{L}_{min}$ denote the size of the smallest interior country, i.e., $\tilde{L}_{min} = \min\{L_2, \cdots, L_{N-1}\}$.

Proposition 2 *If* $\min\{\overline{p}, \gamma L_{max}\} \leq \underline{p} + \gamma \tilde{L}_{min}$, *a unique Nash equilibrium exists.*

This condition has straightforward interpretations. Note that $\min\{\overline{p}, \gamma L_{max}\} \leq \underline{p} + \gamma \tilde{L}_{min}$ is equivalent to either $\overline{p} - \underline{p} \leq \gamma \tilde{L}_{min}$ or $\gamma(L_{max} - \tilde{L}_{min}) \leq \underline{p}$. The smaller difference between $\overline{p}$ and $\underline{p}$ reduces the freedom for setting tax rates. The higher $\underline{p}$ prevents governments from doing undercutting.

In Figure 2, $\underline{p}$ is measured along the horizontal scale, and $\overline{p}$ is measured along the vertical. Since $\underline{p} \leq \overline{p}$, we pay attention to only the region above 45° line. The stripped regions shown in this Figure correspond to the cases where a Nash equilibrium exists. In particular, the

vertically stripped region corresponds to the case of $p_1^* = p_2^* = \cdots = p_N^* = \underline{p}$, and the horizontally stripped region corresponds to the case is $p_1^* = p_2^* = \cdots = p_N^* = \overline{p}$, The rest regions is unclear for such existence. The shaded region corresponds to the cases of unconstrained tax competition. As special cases, in a two-country model, $\tilde{L}_{min}$ can be regarded as ∞. So, this case meet that sufficient condition. The case of $L_1 = L_2^* = \cdots = L_N$ also satisfy that sufficient conditions, so it is ensured that a Nash equilibrium exists.

3.2. Two-Country Model

To understand how the size of countries affect the magnitudes of tax rates and that of per capita revenues in equilibrium, we restrict our attention to a duopolistic case. Remember that in this case there exists a unique Nash equilibrium.

Property 6 *In a duopolistic case, when* $L_1 > L_2$,

$$p_1^* \geq p_2^*, \tag{5}$$

$$\frac{\pi_1^*}{L_1} > \frac{\pi_2^*}{L_2}. \tag{6}$$

Thus the inequality (5) states that the small country sets a lower equilibrium tax rate than the big one, irrespective of $\underline{p}$ and $\overline{p}$. The inequality (6) means that per capita revenue is larger in the small country, irrespective of $\underline{p}$ and $\overline{p}$. Property 6 which generalizes results in unconstrained Nash equilibrium developed by Ohsawa(1998), is consistent with the findings in Kanbur and Keen(1993). This may explain the fact that the government of Luxembourg sets its VAT rate to the minimum standard rate.

3.3 Multi-country Model with Identical Size

To examine how the position of countries affect on the ranking of tax rates and revenues in equilibrium, we focus attention to more than two governments with restriction that the size of all countries are the same, i.e., $L_1 = L_2 = \cdots = L_N (\equiv L)$. In this case, based on Proposition 2, there exists a unique Nash equilibrium.

Define the median M by $M = (N+1)/2$ if N is odd, $M = N/2$ or $M = N/2+1$ otherwise. Of course, p_i^*'s, D_i^*'s and π_i^*'s are symmetric with respect to the median M, i.e., $p_i^* = p_{N+1-i}^*$, $D_i^* = D_{N+1-i}^*$ and $\pi_i^* = \pi_{N+1-i}^*$ for $1 \leq \forall i \leq M$. In addition, the following three properties

also characterize them more precisely:

Property 7

$$\begin{aligned} p^*_{i+1} - p^*_i \quad &> \quad p^*_i - p^*_{i-1}, \qquad \textit{for } i \notin \underline{I} \cup \overline{I} \\ p^*_1 > p^*_2 \geq \cdots \geq \quad p^*_{\frac{N+1}{2}} \quad &\leq \cdots \leq p^*_{N-1} < p^*_N, \qquad \textit{if } N \textit{ is odd,} \\ p^*_1 > p^*_2 \geq \cdots \geq \quad p^*_{\frac{N}{2}} = p^*_{\frac{N}{2}+1} \quad &\leq \cdots \leq p^*_{N-1} < p^*_N, \qquad \textit{if } N \textit{ is even.} \end{aligned}$$

The inequality $p^*_i \geq p^*_{i+1}$ ($p^*_{i-1} \leq p^*_i$) strictly holds if and only if $p^*_i > \underline{p}$.

A close look at this Property reveals at least three points. First, Ohsawa(1998) showed that in unconstrained Nash equilibrium, U-shaped rate structure is established. An important property of Property 7 is that U-shaped rate structure is established, irrespective of $\underline{p}$ and $\overline{p}$. This U-shaped structure may answer the question of why the Scandinavian government set higher VAT rate. As Ohsawa (1989) pointed out, the intuitive explanation of the U-shaped structure is simple. Peripheral countries enjoy a local monopoly, so they set the highest tax rate. The farther the country lies from the market boundary, the smaller its advantage due to competing with one peripheral country, and the larger its disadvantage due to competing with more interior countries. Second, from Property 7, it is interesting to note that while the cardinality of $\overline{I}$ is at most two, that of $\underline{I}$ may exceed two. This means that while some tax harmonization can eliminate the cross-border shoppers of inner borders, any tax harmonization, that is not absolute equalization, cannot eliminate that of outermost borders. Finally, this U-shaped tax rate structure enables us to derive the following two Properties. In fact, this U-shaped tax rate structure implies p^*_i's have unimodal. Therefore, if $\underline{I} \neq \phi$, then $CBS = 2(\min\{p^*_1, \overline{p}\} - \underline{p})$. If $\overline{I} \neq \phi$, then $CBS = 2(\overline{p} - \max\{p^*_M, \underline{p}\})$.

Property 8

$$-2 \leq \gamma \frac{\partial CBS}{\partial \underline{p}} \leq -1, \ \textit{for } \underline{I} \neq \phi, \qquad 1 \leq \gamma \frac{\partial CBS}{\partial \overline{p}} \leq 2, \ \textit{for } \overline{I} \neq \phi.$$

Moreover, CBS is convex with respect to both $\underline{p}$ and $\overline{p}$.

Thus it can be concluded from the second claim of this Property that in order to reduce the volume of cross-border shoppers, raising $\underline{p}$ is more effective than lowering $\overline{p}$ if the same size is changed.

The following Property compares the revenues in tax coordination with that in unconstrained Nash equilibrium.

Property 9 *$p_1^*, \cdots, p_N^*$ are the tax rates in unconstrained Nash equilibrium. There exists τ_0 between p_{i-1}^* and p_i^* such that if $\tau \geq \tau_0$, tax coordination improves the ith government $(1 < i < M)$, and otherwise, tax coordination harms it. There exists τ_0 between p_1^* and p_2^* (resp. between p_{M-2}^* and p_{M-1}^*) such that if $\tau \geq \tau_0$, tax coordination improves first (resp. M-th) government, and otherwise, tax coordination harms it.*

Property 10

$$\begin{aligned} &D_1^* < D_2^* \geq \cdots \geq \quad D_{\frac{N+1}{2}}^* \quad \leq \cdots \leq D_{N-1}^* > D_N^*, \qquad \text{if } N \text{ is odd,} \\ &D_1^* < D_2^* \geq \cdots \geq \quad D_{\frac{N}{2}}^* = D_{\frac{N}{2}+1}^* \quad \leq \cdots \leq D_{N-1}^* > D_N^*, \qquad \text{if } N \text{ is even.} \end{aligned}$$

The inequality $D_i^* \geq D_{i+1}^*$ $(D_{i-1}^* \leq D_i^*)$ strictly holds if and only if $p_i^* > \underline{p}$.

Property 11

$$\begin{aligned} &\pi_1^* < \pi_2^* \geq \cdots \geq \quad \pi_{\frac{N+1}{2}}^* \quad \leq \cdots \leq \pi_{N-1}^* > \pi_N^*, \qquad \text{if } N \text{ is odd,} \\ &\pi_1^* < \pi_2^* \geq \cdots \geq \quad \pi_{\frac{N}{2}}^* = \pi_{\frac{N}{2}+1}^* \quad \leq \cdots \leq \pi_{N-1}^* > \pi_N^*, \qquad \text{if } N \text{ is even.} \end{aligned}$$

The inequality $\pi_i^* \geq \pi_{i+1}^*$ $(\pi_{i-1}^* \leq \pi_i^*)$ strictly holds if and only if $p_i^* = \underline{p}$.

Ohsawa(1998) revealed that in unconstrained Nash equilibrium both D_i^*'s and π_i^*'s have inverted W-shaped structures. A noteworthy characteristics of Properties 10 and 11 is that $\underline{p}$ and $\overline{p}$ unaffect these structures. Thus, it can be concluded that although any tax harmonization is introduced, the best location is either the second and $N-1$th positions.

As we have seen in Property 5, the sign of the revenue derivative $\frac{\partial \pi_i^*}{\partial \underline{p}}$ for $i \in \underline{I}$ depends on the spatial configuration. The following Property ensures that when $L_1 = L_2 = \cdots = L_N$, that sign of all governments are non-negative.

Property 12

$$\frac{\partial \pi_i^*}{\partial \underline{p}} \begin{cases} = 0, & p_{i-1}^* = p_i^* = p_{i+1}^*; \\ > 0, & \textit{otherwise,} \end{cases} \qquad \frac{\partial \pi_i^*}{\partial \overline{p}} \begin{cases} = 0, & p_{i-1}^* = p_i^* = p_{i+1}^*; \\ > 0, & \textit{otherwise.} \end{cases}$$

This Property means although higher minimum standard rate is adopted, the harmonized tax rate structure generates a Pareto-improvement for all the countries as a whole.

Which country enjoys the consequence of tax harmonization? Which country revenue suffers the consequence of harmonization? In order to see these, we shall take up three

simple tax policies where $N = 10$, $\gamma = 1$ and $L = 1$. In the first tax policy, although $\overline{p}$ is fixed at 1.0, $\underline{p}$ ranges from 0.5 to 1.0 at 0.05 intervals. The corresponding harmonization tax rates in equilibrium are illustrated in Figure 3. Their equilibrium revenues are given in Figure 4. In the second tax policy, for fixed $\underline{p} = 0.5$, $\overline{p}$ ranges from 1.0 to 0.5 at 0.05 intervals. Clearly, we have symmetric equilibrium The corresponding tax rates and revenues in equilibrium are displayed in Figures 5 and 6, respectively. In the last tax policy, $\underline{p}$ ranges from 0.5 to 0.6 at 0.01 intervals and $\overline{p}$ ranges from 1.0 to 0.6 at 0.04 intervals. The corresponding tax rates and revenues in equilibrium are presented in Figures 7 and 8, respectively. They were solved by the procedure which we have given in Appendix. The results in these Figures are consistent with the theoretical results such as Properties 2, 5, 7, 11 and 12.

In addition, per capita tax rate $\frac{1}{10}\sum_{i=1}^{N}\pi_i^*$ and the volume of cross-shoppers per government $\frac{1}{10}\sum_{i=1}^{N-1}|p_{i+1}^* - p_i^*|\left(=\frac{1}{5}|p_1^* - p_M^*|\right)$ are plotted in Figures 9 and 10, respectively. These plots are function with respect to the increment of the sum of $\underline{p}$ and $\overline{p}$, denoted by Δ. Thus, the plots for the first policy are function with respect to only the increment of $\underline{p}$, and the ones for the second policy are function with respect to only the increment of $\overline{p}$. From Figure 10, we can confirm the characteristics derived in Property 8.

6. Conclusions

Within very simple models of tax harmonization such that a common band on tax rates are imposed, tax rates, demands, revenues and the number of cross-border shoppers in equilibrium of some strategic tax design were characterized. We generalized some results in Kanbur and Keen(1993) and in Ohsawa(1998) in many respects. We derive at least the following four conclusions, which may be useful to discuss the possible future of EU corporate tax policy.

1. It was proved that extreme standard rate policies may result in an uniformity across countries. It was also demonstrated that tax coordination induce revenues to decline than minimum standard rate policy imposing the same rate as a minimum standard.

2. It was verified that tax harmonization with a narrow common band reduces the range of tax rates, and generate less volume of cross-border shoppers in equilibrium. It was also observed that this harmonization tends to induce a Nash equilibrium to exist.

3. It was shown that raising minimum rate and lowering maximum standard rate have symmetric impacts in many respects. For example, while raising minimum standard

rate induces equilibrium tax rates to go up, loweing maximum standard rate induces equilibrium tax rates to go down, Another example is that their impacts on the balances between inward and outward cross-border shoppers are different.

4. Both also have asymmetric impacts in several respects. For example, whereas lowering maximum rate always harms all governments, raising minimum rate may harm some countries and improve others simultaneously. Another example is that CBS is convex with respect to both $\underline{p}$ and $\overline{p}$. One more example is that in the case of identical size, while the former may eliminate any cross-border shopper of some border, the latter cannot eliminate the cross-border shoppers of any border.

5. In the two-country model, the smaller government sets a lower tax rate and obtain more per capita revenue than the bigger one. Also, in the case of identical size, any tax harmonization establishes U-shaped rate and inverted W-shaped demand, revenue structures. It should be concluded, from what has been said above, that the size and position of countries play a central role in tax harmonization.

Appendix

Proof of Property 1

Here, we express the system (2) in terms of a mapping of the form $\mathbf{p} = G(\mathbf{p})$. Any point satisfying $\mathbf{p} = G(\mathbf{p})$ is called a fixed point of G. We shall verify that the system (2) has a unique solution by showing that G has a unique fixed point. $\underline{\mathbf{p}}$ and $\overline{\mathbf{p}}$ stand for the vectors having all their entries equal to $\underline{p}$ and $\overline{p}$, respectively. For fixed $\underline{\mathbf{p}}$ and $\overline{\mathbf{p}}$, define the closed set D_0 by $D_0 = \{\mathbf{q} \in \mathbf{R}^N | \underline{\mathbf{p}} \leq \mathbf{q} \leq \overline{\mathbf{p}}\}$. Now for $\forall \mathbf{p}, \forall \mathbf{q} \in D_0$ and $\forall i$, let e_i denote ith element of the vector $G(\mathbf{p}) - G(\mathbf{q})$. Then one easily verifies that

$$e_i^2 \leq \begin{cases} \frac{1}{4}(p_2 - q_2)^2, \\ \frac{1}{16}(p_{i-1} - q_{i-1} + p_{i+1} - q_{i+1})^2, \quad 2 \leq i \leq N-1; \\ \frac{1}{4}(p_{N-1} - q_{N-1})^2. \end{cases}$$

Elementary manipulations show that for all $\mathbf{p}, \mathbf{q} \in D_0$, $\|G(\mathbf{p}) - G(\mathbf{q})\|^2 = \sum_{i=1}^N e_i^2 < \frac{1}{2}\sum_{i=1}^N (p_i - q_i)^2 = \frac{1}{2}\|\mathbf{p} - \mathbf{q}\|^2$. This states that for all $\mathbf{p}, \mathbf{q} \in D_0, \frac{\|G(\mathbf{p}) - G(\mathbf{q})\|}{\|\mathbf{p} - \mathbf{q}\|} < \frac{1}{\sqrt{2}} < 1$. Thus we recognize that the mapping G is contractive on the closed set D_0. In addition, it is evident from the system (2) that $G(D_0) \subset D_0$. It follows from the contractive-mapping theorem that G has a unique fixed point in D_0: see Ortega and Rheinboldt(1970). Thus, it can be concluded that the system (2) has a unique solution.□

Proof of Property 2

Define the matrix A, the vectors $\mathbf{x}$ and $\mathbf{b}$ by

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 & \dots & 0 & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & \ddots & 0 \\ 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} \\ 0 & 0 & \dots & 0 & -\frac{1}{4} & \frac{1}{2} \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} p_1^* \\ p_2^* \\ \vdots \\ p_N^* \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} \frac{\gamma}{4} L_1 \\ \frac{\gamma}{4} L_2 \\ \vdots \\ \frac{\gamma}{4} L_N \end{bmatrix}.$$

Let $A_{I \cup J}$ denote the matrix which arises from A by replacing ith row with unit vector for $\forall i \in I \cup J$, and let $\mathbf{b}_{I,J}$ denotes the vector which arise from $\mathbf{b}$ by replacing ith element with $\underline{p}$ for $\forall i \in I$, and by replacing the jth element with $\overline{p}$ for $\forall j \in J$. Since the system (2) has a unique solution, this solution has to coincide with $\mathbf{x} = A^{-1}_{\underline{I} \cup \overline{I}} \mathbf{b}_{\underline{I},\overline{I}}$.

For the identity matrix E, define the matrix $B_{\underline{I} \cup \overline{I}}$ by $E - A_{\underline{I} \cup \overline{I}}$. Since the absolute value of all eigenvalues of $B_{\underline{I} \cup \overline{I}}$ are less than unity, we have $A^{-1}_{\underline{I} \cup \overline{I}} = E + B_{\underline{I} \cup \overline{I}} + B^2_{\underline{I} \cup \overline{I}} + B^3_{\underline{I} \cup \overline{I}} + \cdots$. Since the matrix $B_{\underline{I} \cup \overline{I}}$ is non-negative, the matrix $A^{-1}_{\underline{I} \cup \overline{I}}$ is also non-negative. This ensures that $\frac{\partial p_i^*}{\partial \underline{p}} \geq 0$ and $\frac{\partial p_i^*}{\partial \overline{p}} \geq 0$.

In addition, let a_{ij} denote i-row and jth column element of the matrix $A^{-1}_{I \cup J}$. Some simple calculations show that if $i \in \underline{I} \cup \overline{I}$, then $a_{ii} = 1$, $a_{ij} = 0$ for $\forall j \in \underline{I} \cup \overline{I} - \{i\}$; Otherwise $0 \leq a_{i,j} \leq 1/2$ for $\forall j \in \underline{I} \cup \overline{I}$. □

Proof of Proposition 1

First let us prove the first claim. Suppose in the contrary that $p_j^* > \underline{p}$ for some j. Define $k = \max_{i \in \{1,\cdots,N\}} \{p_i^* - \underline{p}\}$. It follows that $D_k(p_1^*, \dots, p_N^*) < L_k \leq L_{max}$. Combining this with $p_k^* > \underline{p}$ yields $\gamma D_k(p_1^*, \dots, p_N^*) \leq \gamma L_{max} \leq \underline{p} < p_k^*$. This contradicts one of the optimality conditions (3) and (4) of the government k. Similar arguments lead to the second claim. □

Proof of Property 3

Let $j_1, \cdots, j_t$ be the indices such j_kth government experiences only inward cross-border shoppers, and $j_k < j_{k+1}$, $(k = 1, \cdots, t-1)$. For $1 \leq \forall k \leq t-1$, if $p^*_{j_k} = p^*_{j_k+1} = \cdots = p^*_{j_{k+1}}$, then $\sum_{i=j_k}^{j_{k+1}-1} |p^*_{i+1} - p^*_i| = 0$. Otherwise, there exists uniquely sth government with $j_k < s < j_{k+1}$ that experiences only outward cross-border shoppers. In this case, we have $\sum_{i=j_k}^{j_{k+1}-1} |p^*_{i+1} - p^*_i| = 2p^*_s - p^*_{j_k} - p^*_{j_{k+1}}$. On the other hand, we know $\frac{\partial p^*_s}{\partial \underline{p}} \leq \frac{1}{4} \left(\frac{\partial p^*_{j_k}}{\partial \underline{p}} + \frac{\partial p^*_{j_{k+1}}}{\partial \underline{p}} \right)$.

Combining these yields

$$\frac{\partial}{\partial \underline{p}} \sum_{i=j_k}^{j_{k+1}-1} |p^*_{i+1} - p^*_i| = \frac{\partial}{\partial \underline{p}}(2p^*_s - p^*_{j_k} - p^*_{j_{k+1}}) \leq -\frac{1}{2}\left(\frac{\partial p^*_{j_k}}{\partial \underline{p}} + \frac{\partial p^*_{j_{k+1}}}{\partial \underline{p}}\right).$$

Similarly, if $1 < j_1$, then $\frac{\partial}{\partial \underline{p}} \sum_{i=1}^{j_1-1} |p^*_{i+1} - p^*_i| \leq -\frac{1}{2}\frac{\partial p^*_{j_1}}{\partial \underline{p}}$. If $j_t < N$, then $\frac{\partial}{\partial \underline{p}} \sum_{i=j_t}^{N-1} |p^*_{i+1} - p^*_i| \leq -\frac{1}{2}\frac{\partial p^*_{j_t}}{\partial \underline{p}}$. Thus, we have $\gamma^{-1}\frac{\partial CBS}{\partial \underline{p}} = \sum_{i=1}^{N-1} \frac{\partial |p^*_{i+1} - p^*_i|}{\partial \underline{p}} \leq -\frac{1}{2}\sum_{i=j_1}^{j_t} \frac{\partial p^*_i}{\partial \underline{p}}$. It should be noted that there exists u such that $p^*_{j_u} = \underline{p}$, i.e., $\frac{\partial p^*_{j_u}}{\partial \underline{p}} = 1$. Hence, $\gamma^{-1}\frac{\partial CBS}{\partial \underline{p}} \leq -\frac{1}{2}$, as required. Similar argument leads to the second claim. □

Proof of Property 4

Combining the optimal conditions (3) and (4) with Property 2 yields this claim. □

Example 1

Take the case of $N = 3$, $\gamma = 1$, $3L_1 < L_2$, $L_3 = 0$, and $\overline{p} = \infty$. It follows from Proposition 2 that there exists a unique Nash equilibrium. From the system (2), when $(L_1 + 2L_2)/3 \leq \underline{p} \leq (4L_1 + L_2)/6$, the equilibrium tax rates are $p^*_1 = (4L_1 + L_2 + \underline{p})/7$, $p^*_2 = (L_1 + 2L_2 + 2\underline{p})/7$, and $p^*_3 = \underline{p}$. So we have $p^*_3 < p^*_1 < p^*_2$. The volume of cross-border shoppers from second government to first government is proportional to $p^*_2 - p^*_1 = (-3L_1 + L_2 + \underline{p})/7$. Thus we see that as $\underline{p}$ increases, so does this volume.

Proof of Property 5

Combining Property 2 and this Property yields $\frac{\partial \pi^*_i}{\partial \overline{p}} \geq 0$ for $i \notin \overline{I}$. However, the sign of $\frac{\partial \pi^*_i}{\partial \overline{p}} \geq 0$ for $i \in \overline{I}$ is unclear. Suppose that $p^*_i = \overline{p}$. If $2 \leq i \leq N-1$, since $\pi^*_i = \overline{p}(L_i + \frac{p^*_{i-1} + p^*_{i+1} - 2\overline{p}}{\gamma})$, we have $\gamma\frac{\partial \pi^*_i}{\partial \overline{p}} = \overline{p}\frac{\partial p^*_{i-1}}{\partial \overline{p}} + \overline{p}\frac{\partial p^*_{i+1}}{\partial \overline{p}} + 4(\frac{p^*_{i-1} + p^*_{i+1} + \gamma L_i}{4} - \overline{p})$. The substitution of $\frac{\partial p^*_{i-1}}{\partial \overline{p}} \geq 0$, $\overline{p}\frac{\partial p^*_{i+1}}{\partial \overline{p}} \geq 0$ and $\frac{p^*_{i-1} + p^*_{i+1} + \gamma L_i}{4} > \overline{p}$ into this partial derivative yields $\frac{\partial \pi^*_i}{\partial \overline{p}} > 0$. Otherwise, likely we can verify that inequality.

Combining Property 2 and this Property yields $\frac{\partial \pi^*_i}{\partial \underline{p}} \geq 0$ for $i \notin \underline{p}$. □

Proof of Proposition 2

When $\gamma L_{max} > \underline{p}$ and $\overline{p} > \frac{\gamma L_{min}}{2}$, it is easy to check that $1/2 < \sum_{j=1}^{N} a_{ij} < 1$. This means that $L_{min}/2 < p^*_i = \sum_{j=1}^{N} a_{ij} L_j < L_{max}$. Thus we have

$$\frac{\gamma L_{min}}{2} < p^*_i < \gamma L_{max}, \qquad (1 \leq \forall i \leq N) \tag{7}$$

This inequalities indicate the lower and upper bounds on p_i^*'s, collectively.

Based on the upper bound (7), we have $p_i^* \leq \gamma L_{max}$ for $1 \leq \forall i \leq N$, Hence, $\min\{\overline{p}, \gamma L_{max}\} - \underline{p} \leq \gamma \tilde{L}_{min} \Rightarrow \min\{\overline{p}, \gamma L_{max}\} \leq \underline{p} + \gamma \tilde{L}_{min} \Rightarrow p_i^* \leq \underline{p} + \gamma \tilde{L}_{min}, (1 \leq \forall i \leq N) \Rightarrow p_i^* \leq p_{i-1}^* + \gamma \tilde{L}_{min}$ and $p_i^* \leq p_{i+1}^* + \gamma \tilde{L}_{min} (2 \leq \forall i \leq N-1) \Rightarrow p_i^* \leq p_{i-1}^* + \gamma L_i$ and $p_i^* \leq p_{i+1}^* + \gamma L_i (2 \leq \forall i \leq N-1)$. These two inequalities state that any interior government cannot be undercut by its two neighbours. □

Proof of Property 6

The inequality (5) is immediate from the system (1). Let us verify the inequality (6). The inequality (5) yields $D_2^* \geq L_2$. This leads to $\pi_2^* = p_2^* D_2^* < p_2^* L_2$. On the other hand, since p_1^* is a maximizer of $\pi_1(p_1, p_2^*)$, if the first government sets the same tax rate with the second one, then it would get a lower revenue and its demand coincides with its size. In symbols, $\pi_1^* > \pi_1(p_2^*, p_2^*) = p_2^* L_1$. Combining these two inequalities results in $\frac{\pi_1^*}{L_1} > p_2^* > \frac{\pi_2^*}{L_2}$, as required.

Proof of Property 7

Substituting $L_{min} = L$ into the lower bounds (7) yields $\gamma L < 2p_i^*$. This together with the system (2) gives $(p_{i+1}^* - p_i^*) - (p_i^* - p_{i-1}^*) = 2p_i^* - \gamma L > 0$, as required.

It follows from $\gamma L < 2p_i^*$ that $p_{i-1}^* + p_{i+1}^* \leq 2\overline{p} < 4\underline{p} - \gamma L$, so we have $\frac{\gamma L + p_{i-1}^* + p_{i+1}^*}{4} < \underline{p}$. Thus we see that

$$p_i^* < \overline{p}, \qquad (2 \leq \forall i \leq N-1). \tag{8}$$

On the other hand, it follows from the system (2) that $1, N \notin \underline{I}$. Assume that $k \notin \underline{I}$ and $k+1 \in \underline{I}$. Let us show that $p_i^* > p_{i+1}^*$, $(1 \leq i \leq k)$. Since $p_k^* > p_{k+1}^*$, Property 7 implies that $p_{k-1}^* - p_k^* > 0$. This argument together with (8) also establishes that $p_j^* > p_{j+1}^*$ for all $1 \leq j < k$. A symmetric property ensures that $p_i^* < p_{i+1}^*, (N-k+1 \leq i \leq N)$ and $p_{k+1}^* = p_{k+2}^* = \cdots = p_{N-k}^* = p_{N-k+1}^* = \underline{p}$. □

Proof of Property 8

In the former case, if $p_1^* > \overline{p}$, the claim holds obviously. Otherwise, $\frac{\partial CBS}{\partial \underline{p}} = \frac{\partial p_1^*}{\partial \underline{p}} - 2$. In the later case, if $p_M^* < \underline{p}$, the claim holds obviously. Otherwise, $\frac{\partial CBS}{\partial \overline{p}} = 2 - \frac{\partial p_M^*}{\partial \overline{p}}$. Combining thiese equations with Property 2 yields the bounds on CBS derivatives, i.e., in the former case, $-2 \leq \frac{\partial CBS}{\partial \underline{p}} \leq -1$, and in the later case $1 \leq \frac{\partial CBS}{\partial \overline{p}} \leq 2$. Also, similarly as for the proof

with Property 2 that $\frac{\partial p_1^*}{\partial \underline{p}}|_{p_i^*=q_1} \geq \frac{\partial p_1^*}{\partial \underline{p}}|_{p_i^*=q_2}$ for $q_1 < q_2$. On the other hand, $\frac{\partial p_M^*}{\partial \underline{p}} = 0$ if $\underline{I} = \phi$, and $\frac{\partial p_M^*}{\partial \underline{p}}|_{p_i^*=q_2}$ is constant otherwise. Thus, $\frac{\partial p_1^*}{\partial \underline{p}}$ is non-increasing function with respect to $\underline{p}$, but $\frac{\partial p_M^*}{\partial \overline{p}}$ is non-decreasing function with respect to $\overline{p}$. This implies that both $\frac{\partial CBS}{\partial \underline{p}}$ and $\frac{\partial CBS}{\partial \overline{p}}$ are non-increasing function with respect to $\underline{p}$ and $\overline{p}$, respectively. Thus, we see that CBS is convex with respect to both $\underline{p}$ and $\overline{p}$. □

Proof of Property 9

It suffices to prove this Property for $1 \leq i \leq M$ becsuse of the symmetric property. We consider four cases separetely: *Case 1*: $i = 1$; *Case 2*: $i = 2$; *Case 3*: $2 < i < M$; *Case 4*: $i = M$. In the first case, an argument which is same with the roof of Proposition in Kanbur and Keen(1993) can be applied. When $\tau_0 = p_2^*$, its revenue is below that in unconstrained Nash equilibrium because p_1^* is its best reply against p_2^*. When $\tau_0 = p_1^*$, its revenue is above that in unconstrained Nash equilibrium because it cannot experience any cross-border shopping.

In the second case, when $\tau_0 = p_2^*$, its revenue is less than that in unconstrained Nash equilibrium because the U-shaphed structure implies that the volume of inward cross-border shoppers exceeds that of outward ones in unconstrained Nash equilibrium. When $\tau_0 = p_1^*$, its revenue is $p_1^* L$. It follws from the system (2) that $p_1^* = (L + p_2^*)/2$. On the other hand, its revenue in unconstained Nash equilibrium is $2(p_2^*)^2$. Since N is greater than four, we know that $p_2^* < (1 + \sqrt{12})L/8 (< 0.640)$ by Ohsawa(1998). Thus we have $p_1^* L - 2(p_2^*)^2 = (-1/2)(4(p_2^*)^2 - Lp_2^* - L^2) > 0$.

In the third case, when $\tau_0 = p_i^*$, its revenue gets fewer as is the case with the second case. When $\tau_0 = p_{i-1}^*$, its revenue is $p_{i-1}^* L$. Following the system (2), we have $p_{i-1}^* = 4p_i^* - p_{i+1}^* - L > 3p_i^* - L$. On the other hand, its revenue in the unconstained Nash equilibrium is $2(p_i^*)^2$. Accounting for that $L/2 < p_i^* < L$ in the inequalities (7), we have $p_{i-1}^* L - 2(p_i^*)^2 > -2(p_i^*)^2 + 3Lp_i^* - L^2 = -(2p_i^* - L)(p_i^* - L) > 0$.

In the last case, when $\tau_0 = p_{M-1}^*$, its revenue gets fewer because p_M^* is best reply against p_{M-1}^*. When $\tau_0 = p_{M-2}^*$, its revenue is $p_{M-1}^* L$. Based on the system (2), routine calculations show that $p_{M-2}^* = 7p_M^* - 3L$. On the other hand, its revenue in the unconstained Nash equilibrium is $2(p_M^*)^2$. Making use of $L/2 < p_i^* < L$, we have $p_{M-2}^* L - 2(p_M^*)^2 = -2(p_M^*)^2 + 7Lp_M^* - 3L^2 = -(p_M^* - 3L)(2p_M^* - L) > 0$. □

Proof of Property 10

The U-shaped tax rate structure with the optimal conditions (3) and (4) guarantees that for $2 \leq i \leq \lfloor \frac{N-1}{2} \rfloor$, $D_i^* \geq D_{i+1}^*$. Thus, because of symmetry, it suffices to show that $D_1^* < D_2^*$.

If $p_2^* = \underline{p}$, Property 7 means that $p_1^* > p_2^* = p_3^*$, so we have $D_1^* < D_2^*$. Otherwise, using the lower bounds (7), we have $\gamma L < p_1^* + p_2^*$, so we obtain $D_1^* = L + \frac{p_2^* - p_1^*}{\gamma} < \frac{2p_2^*}{\gamma} = D_2^*$. □

Proof of Property 11

As is the case with the proof of Property 10, it suffices to show that $\pi_1^* < \pi_2^*$. We consider four cases separately: *Case 1*; $\overline{p} > p_1^* > p_2^* > \underline{p}$; *Case 2*; $\overline{p} = p_1^* > p_2^* > \underline{p}$; *Case 3*; $\overline{p} > p_1^* > p_2^* = \underline{p}$; *Case 4*; $\overline{p} = p_1^* > p_2^* = \underline{p}$.

For the first case, since $\frac{\partial p_i^*}{\partial \underline{p}} \geq 0$, $\pi_1^* < \pi_2^*$, as we have seen in Section 5.

For the second case, $\gamma(\pi_2^* - \pi_1^*) = 2(p_2^*)^2 - \overline{p}(\gamma L + p_2^* - \overline{p})$. Hence, substituting $2p_2^* \geq \overline{p}$, $\frac{\partial p_2^*}{\partial \overline{p}} \geq \frac{1}{2}$ and $p_2^* + \overline{p} \geq \gamma L$ into $\frac{\partial(\pi_2^* - \pi_1^*)}{\partial \overline{p}}$ yields $\gamma \frac{\partial(\pi_2^* - \pi_1^*)}{\partial \overline{p}} = \frac{\partial p_2^*}{\partial \overline{p}}(2p_2^* - \overline{p}) + \left(2\frac{\partial p_2^*}{\partial \overline{p}} p_2^* + \overline{p} - \gamma L\right) + (\overline{p} - p_2^*) > 0$. On the other hand, if $\overline{p} = \frac{\gamma L + p_2^*}{2}$, then $\pi_2^* - \pi_1^* > 0$. If $\overline{p} = p_2^*$, i.e., $\overline{p} = \gamma L$, then $\pi_2^* - \pi_1^* = 0$. Thus we see that for $\frac{\gamma L + p_2^*}{2} \leq \overline{p} < \gamma L$, $\pi_1^* < \pi_2^*$.

For the third case, it follows from $p_1^* = \frac{\gamma L + \underline{p}}{2}$ that $\gamma(\pi_2^* - \pi_1^*) = \underline{p}(\gamma L + p_1^* - \underline{p}) - (p_1^*)^2 = -\frac{3}{4}(\underline{p} - \frac{2}{3}\gamma L)^2 + \frac{(\gamma L)^2}{4} > 0$ for $\frac{\gamma L}{2} < \underline{p} < \gamma L$.

For the last case, unless $\overline{p}$ is imposed upon the tax rate of the first government, $L + \frac{p_1^* - \overline{p}}{\gamma}$ is optimal for it. However, its revenue is below π_2^*. Thus, we have $\pi_1^* < \pi_2^*$. □

Proof of Property 12

First we shall prove the first claim. Clearly the claims hold if $p_{i-1}^* = p_i^* = p_{i+1}^*$. Therefore, using symmetrical property and Property 7, we may confine our attention to the case where either $p_{i-1}^* > p_i^* = \underline{p}$ or $\underline{p} = p_i^* < p_{i+1}^*$. Two cases may arise: *Case 1*: either N is even or N is odd but $i \neq M$; *Case 2*: N is odd and $i = M$. It should be noted that in the first (resp. second) case, cross-border shoppers are induced by tax rate differentials in only one side (resp. in both sides).

For the first case, $\pi_i^* = \underline{p}(L + \frac{p_{i-1}^* - \underline{p}}{\gamma})$. Hence, substituting $\frac{\partial p_{i-1}^*}{\partial \underline{p}} > 0$ and $\gamma L > \underline{p}$ into $\frac{\partial \pi_i^*}{\partial \underline{p}}$ yields $\gamma \frac{\partial \pi_i^*}{\partial \underline{p}} = \underline{p}\frac{\partial p_{i-1}^*}{\partial \underline{p}} + (\gamma L - \underline{p}) + (p_{i-1}^* - \underline{p}) > 0$.

For the second case, taking advantage of the fact that $p_{M-1}^* = p_{M+1}^*$, we get

$$\pi_M^* = \begin{cases} \underline{p}L, & p_{M-1}^* = p_M^* = p_{M+1}^* = \underline{p}; \\ \frac{1}{\gamma}\underline{p}(\gamma L + 2p_{M-1}^* - 2\underline{p}), & \text{otherwise.} \end{cases}$$

As a result, since $\frac{\partial \pi_M^*}{\partial \underline{p}} = L > 0$ for $p_{M-1}^* = p_M^* = p_{M+1}^* = \underline{p}$, is suffices to examine the case where $p_{M-1}^* = p_{M+1}^* > \underline{p}$. When $N = 3$, i.e., $M = 2$, differentiating π_2^* with respect to $\underline{p}$ yields $\gamma \frac{\partial \pi_2^*}{\partial \underline{p}} = \underline{p}(2\frac{\partial p_1^*}{\partial \underline{p}} - 1) + (\gamma L - \underline{p}) + 2(p_1^* - \underline{p})$, provided that $p_1^* = p_3^* > \underline{p}$. Combining $\frac{\partial p_1^*}{\partial \underline{p}} > \frac{1}{2}$ and $\gamma L > \underline{p}$ with this gives $\frac{\partial \pi_2^*}{\partial \underline{p}} > 0$. For $N = 5$, it is easy to check that $p_2^* = p_3^* = p_4^* \Leftrightarrow \underline{p} \geq \frac{3\gamma L}{5}$. Moreover, p_{M-1}^*, p_M^* and p_{M+1}^* are decreasing function with respect to N. Hence, when $N \geq 5$, if $\underline{p} \geq \frac{3\gamma L}{5}$, then $p_{M-1}^* = p_M^* = p_{M+1}^*$. Otherwise, the insertion of $\frac{\partial p_{M-1}^*}{\partial \underline{p}} > \frac{1}{4}$ and $\frac{3\gamma L}{5} \geq \underline{p}$ in $\frac{\partial \pi_M^*}{\partial \underline{p}}$ gives $\gamma \frac{\partial \pi_M^*}{\partial \underline{p}} = \underline{p}(2\frac{\partial p_{M-1}^*}{\partial \underline{p}} - \frac{1}{2}) + \frac{3}{2}(\frac{2}{3}\gamma L - \underline{p}) + 2(p_{M-1}^* - \underline{p}) > 0$, as required.

Next, we shall verify the second claim. As is the case with the proof of the first claim, it suffices to prove $\frac{\partial \pi_1^*}{\partial \overline{p}} > 0$ for $\pi_1^* = \overline{p}$. Since $\pi_1^* = \overline{p}(L + \frac{p_2^* - \overline{p}}{\gamma})$, we have $\gamma \frac{\partial \pi_1^*}{\partial \overline{p}} = \overline{p}\frac{\partial p_1^*}{\partial \overline{p}} + 2(\frac{p_2^* + \gamma L}{2} - \overline{p})$. The substitution of $\frac{\partial p_2^*}{\partial \underline{p}} > 0$ and $\frac{p_2^* + \gamma L}{2} > \overline{p}$ into this yields $\frac{\partial \pi_1^*}{\partial \underline{p}} > 0$. □

Example 2

Take the case of $N = 2$, $\gamma = 1$, $L_1 \geq L_2$ and $\overline{p} = \infty$. It follows from Proposition 2 that there exists a unique Nash equilibrium. Based on the system (2), the equilibrium tax rates are

$$
\begin{aligned}
p_1^* &= \begin{cases} (2L_1 + L_2)/3, & \underline{p} \leq (L_1 + 2L_2)/3; \\ (L_1 + \underline{p})/2, & (L_1 + 2L_2)/3 \leq \underline{p} \leq L_1; \\ \underline{p}, & L_1 \leq \underline{p}, \end{cases} \\
p_2^* &= \begin{cases} (L_1 + 2L_2)/3, & \underline{p} \leq (L_1 + 2L_2)/3; \\ \underline{p}, & (L_1 + 2L_2)/3 \leq \underline{p}. \end{cases}
\end{aligned}
$$

Hence, we have

$$
\pi_2^* = \begin{cases} (L_1 + 2L_2)^2/9, & \underline{p} \leq (2L_1 + L_2)/3; \\ \underline{p}(L_1 + 2L_2 - \underline{p})/2, & (2L_1 + L_2)/3 \leq L_1; \\ \underline{p}L_2, & L_1 \leq \underline{p}. \end{cases}
$$

This function is plotted in Figure 11. By noting that $(L_1 + 2L_2)^2/9 > L_1 L_2 \Leftrightarrow L_1 > 4L_2$, this Figure makes it clear that when $L_1 > 4L_2$, imposing $\underline{p}$ with $(2/3)(L_1 + 2L_2) < \underline{p} < (1/9)(L_1 + 2L_2)^2/L_2 (\leq (L_1 + 2L_2)/3)$ harms the second government. The intuition of this result is as following. For $\underline{p} \leq (L_1 + 2L_2)/3$, the revenue from its citizen is $L_2(L_1 + L_2)/3$, and the revenue from cross-border shoppers is $\pi_2^* - L_2(L_1 + L_2)/3$. For $(L_1 + 2L_2)/3 \leq \underline{p}$, because of $p_2^* = \underline{p}$ the revenue from its citizen is $\underline{p}L_2$, and the revenue from cross-border shoppers is $\pi_2^* - \underline{p}L_2$, i.e., the vertical distance from π_2^* to 45° line. As is evident on referring to Figure 11, as $\underline{p}$ increases, the revenue from its citizen increases and the revenue from cross-border shoppers decreases. When L_2 is very small, in unconstrained Nash equilibrium, the revenue from its citizen is below the ones from cross-border shoppers. Therefore, in this case, the

revenue in unconstrained Nash equilibrium is less than that at the smallest tax rate which the second government cannot get any cross-border shopper.

Solution Method

For solving the system (2), i.e., finding a fixed point of G, several algorithms have been developed: for example, see Dai and Yamamoto(1994). Although such algorithms may run efficiently, making the program for such algorithms is very complicated. Therefore, rather than using these algorithms, we propose a simple algorithm for finding the solution to the system (2), i.e, the sets I and J satisfying $\underline{\mathbf{p}} \leq A_{I\cup J}^{-1}\mathbf{b}_{I,J} \leq \overline{\mathbf{p}}$. The solution technique is summarized below.

1. Set $\underline{q} = 0$, $\overline{q} = \infty$, $I = \phi$ and $J = \phi$. Calculate $A_{I\cup J}^{-1}\mathbf{b}_{I,J}$.
2. Decrease $\overline{q}$ by $\delta > 0$ small and carry out Step 4 until $\overline{q} = \overline{p}$.
3. Increase $\underline{q}$ by $\delta > 0$ and do Step 4 until $\underline{q} = \underline{p}$.
4. Compute $A_{I\cup J}^{-1}\mathbf{b}_{I,J}$. If there exists $i \notin I$ such that $p_i^* \geq \overline{q}$, then update I by $I \cup \{i\}$. If there exists $j \notin J(\underline{q}, \overline{q})$ such that $p_j^* \leq \underline{q}$, then update J by $J \cup \{j\}$.
5. Stop.

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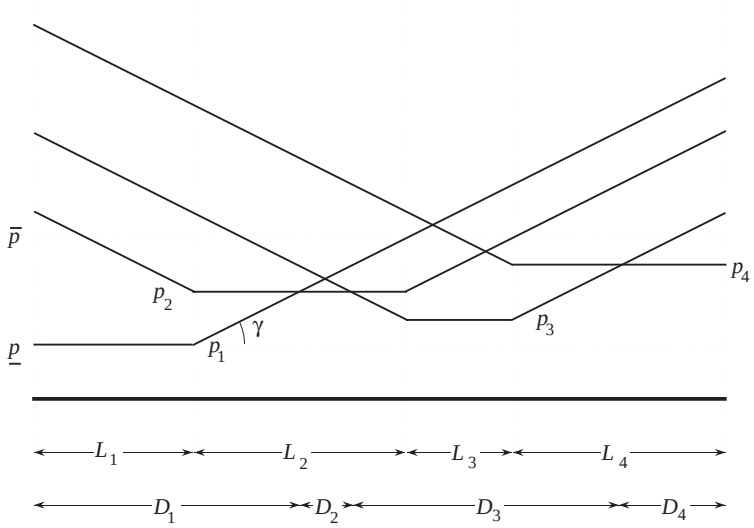

Figure 1: Market

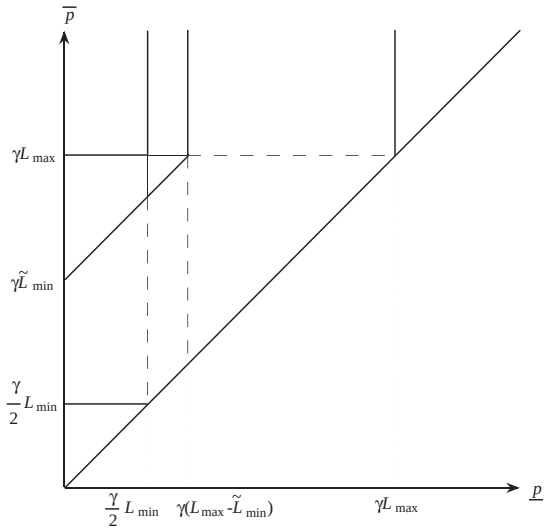

Figure 2: Existence of Nash equilibria

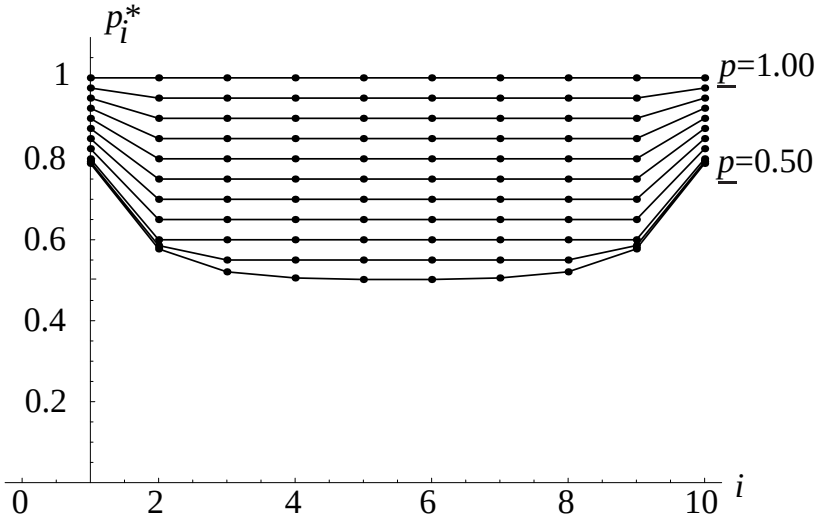

Figure 3: Equilibrium rates for $\overline{p} = 1.0$ and $\underline{p}$ ranging from 0.5 to 1.0 at 0.05 intervals

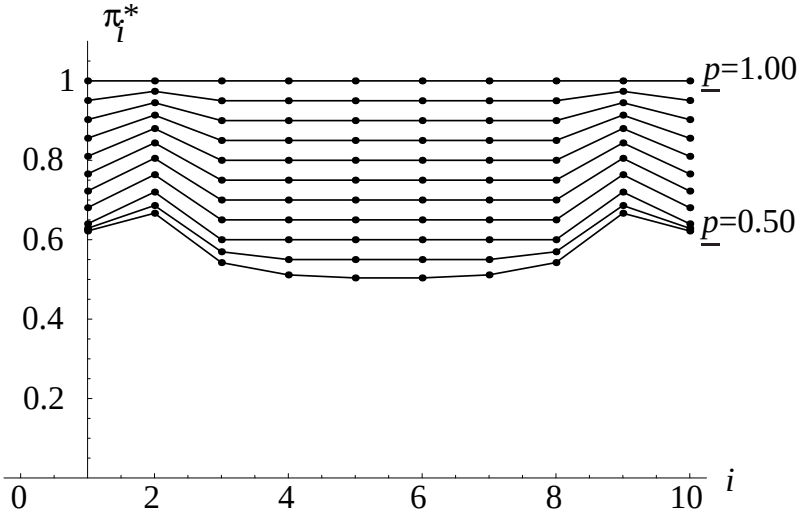

Figure 4: Equilibrium revenues for $\overline{p} = 1.0$ and $\underline{p}$ ranging from 0.5 to 1.0 at 0.05 intervals

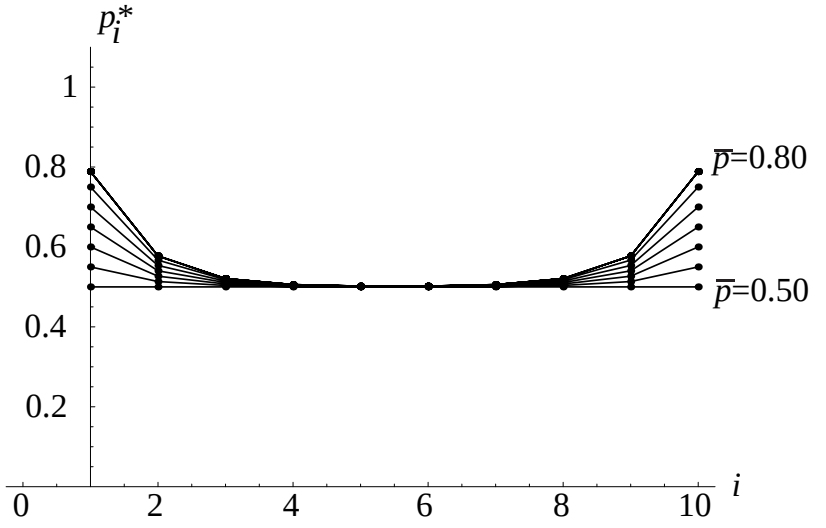

Figure 5: Equilibrium rates for $\underline{p} = 0.5$ and $\overline{p}$ ranging from 1.0 to 0.5 at 0.05 intervals

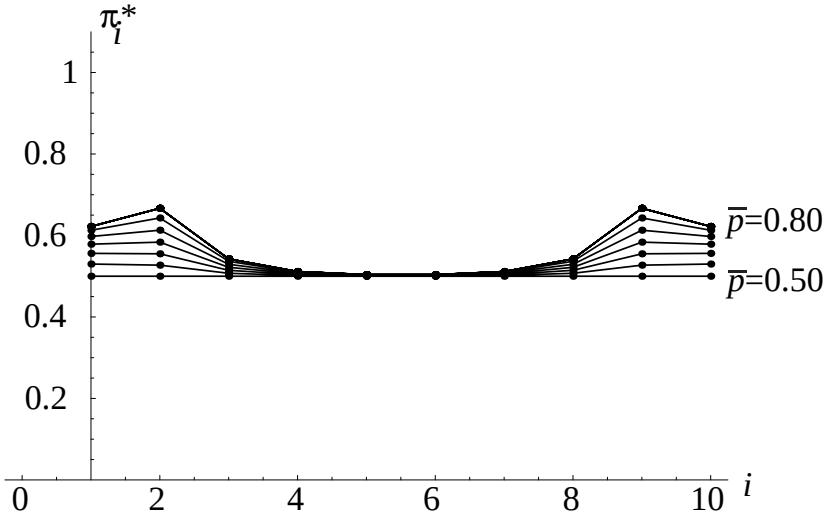

Figure 6: Equilibrium revenues for $\underline{p} = 0.5$ and $\overline{p}$ ranging from 1.0 to 0.5 at 0.05 intervals

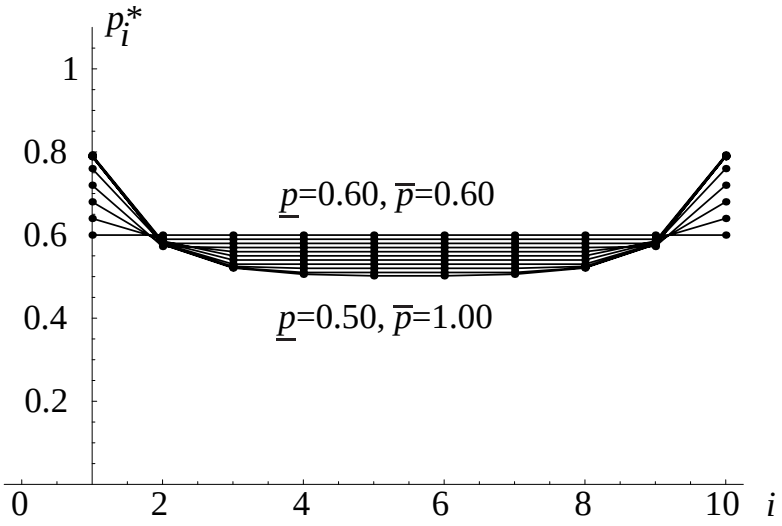

Figure 7: Equilibrium rates for $\underline{p}$ ranging from 0.5 to 0.6 at 0.01 intervals and $\overline{p}$ ranging from 1.0 to 0.6 at 0.04 intervals

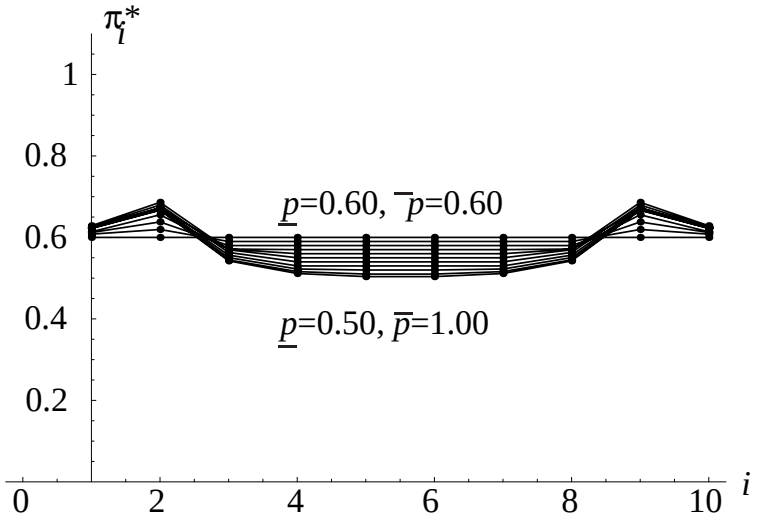

Figure 8: Equilibrium revenues for $\underline{p}$ ranging from 0.5 to 0.6 at 0.01 intervals and $\overline{p}$ ranging from 1.0 to 0.6 at 0.04 intervals

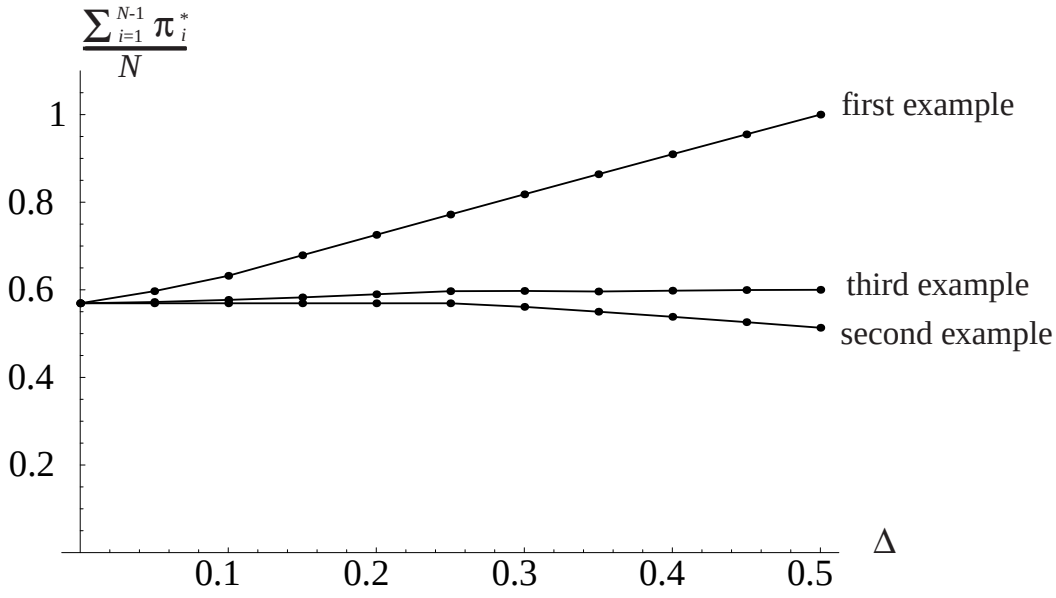

Figure 9: Revenue per government in equilibrium

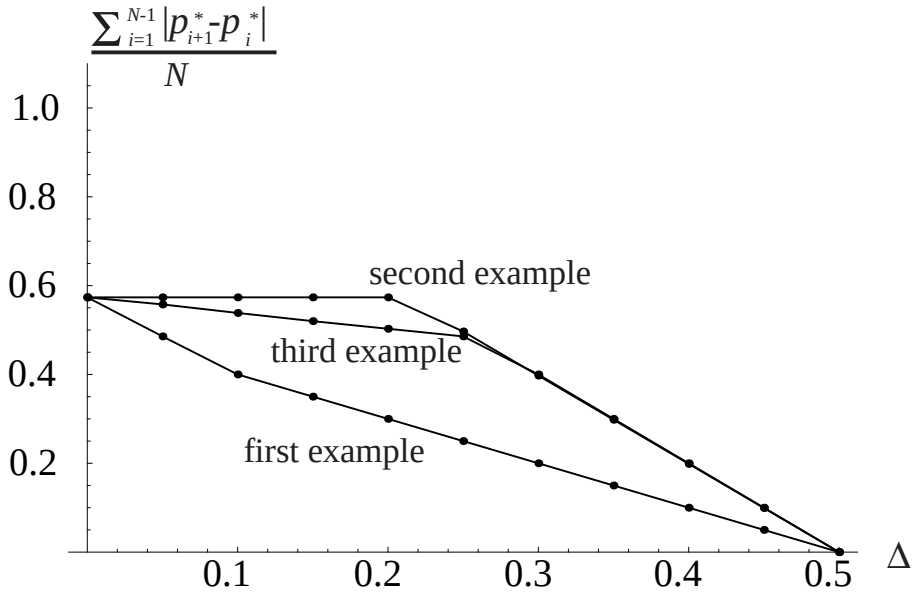

Figure 10: Volume of cross-shoppers per government in equilibrium

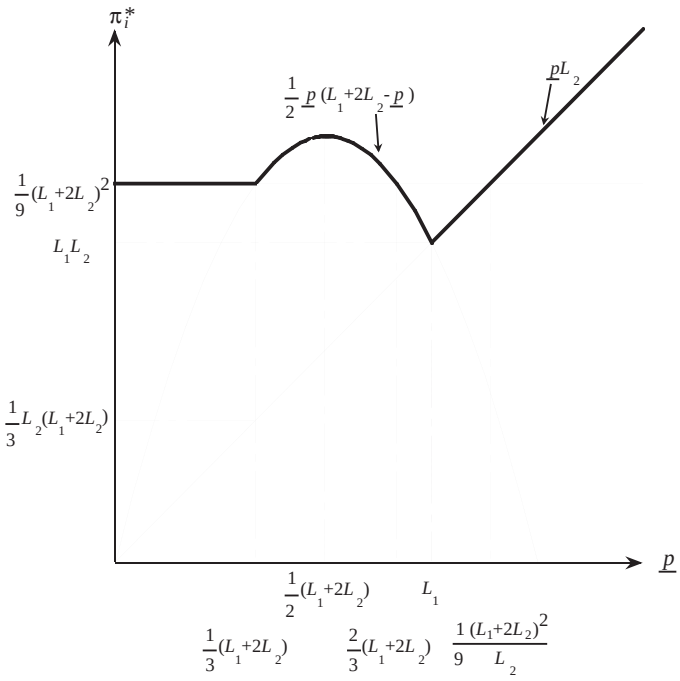

Figure 11: Equilibrium revenue of the second government