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## Information-processing, technological progress and regional retail networks dynamics

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## **Information-Processing, Technological Progress, and Regional Retail Networks Dynamics**

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In regions where old, traditional industries have lost their importance, retail firms play an important role in job creation and provide local governments with a large part of their tax revenues. The hypothesis in this paper is that the existence of regional retail networks may not necessarily be determined by spatial factors and increasing returns in retailing, but can be explained by the rational behaviour of firms operating in a stochastic environment. It is shown that demand uncertainty can serve as an independent source of retail trade, and, consequently, the ability of firms to process information and predict demand (i.e., to decrease demand uncertainty) may affect the characteristics of retail networks. The results indicate that risk-averse firms always devote resources to demand forecasting; producers are better off trading with retailers than with final consumers; and the volume of output supplied through retail networks is greater than it would be if producers traded directly with consumers (thus benefiting social welfare). Furthermore, the paper shows that technological progress in data-processing, which allows for cheaper and better predictions of market demand, increases the size of regional retail networks. This in turn, increases the number of jobs available in regional job markets (decreases employment) and increases tax revenues.

Keywords: retail trade, regional retail networks, demand uncertainty, information-processing, technological progress.

JEL Classification: D73.

## 1. Introduction

The rapid development of information and communications technologies is considered one of the major driving forces in changes in economies and lifestyles. Technological progress in information and communications techniques alters the spatial character of production, organization and competition. The characteristics of the new information based economy affect urban structures and the role and the importance of cities. In particular, new information technologies have an effect on organizations in the economy world wide, as well as the kinds of jobs available and their geographical distribution. New information technologies cause the loss of jobs in established resource-intensive and area-consuming industries, and, consequently, lead to high unemployment even in industrialized regions. In regions in which old industries have lost their importance, existing urban infrastructures with developed retail networks remain an important source of jobs and provide local governments with a large part of their tax revenues. However, as in other sectors of the economy, the development of information technologies does not leave regional retail networks unaffected. The hypothesis in this paper is that technological progress in data processing increases the size of regional retail networks, and, as a result, reduces the negative effects of advances in information technology on job markets in regional economies dominated by declining resource-intensive industries.

The existence of regional retail networks and the role of retail firms are traditionally explained by spatial factors and increasing returns to scale in transportation, storage or in the acquisition and dissemination of information about the quality, range, and prices of products available (see, e.g., Heal, 1980 and Wilson, 1975). In many cases, such as retail trade in services or in goods which cannot be transported or stored, most of these factors are irrelevant. The main limitation of the earlier literature on regional retail networks is that it attempts to see the retail trade only from the supply side, and the models used have no explicit reference to demand, especially to demand uncertainty, which is natural in most markets. This paper shows that retail trade (or at least part of it) may not be connected with economies of scale and that it can be explained exclusively by the rational behaviour of firms operating in a stochastic environment. Since retailers are, by definition, intermediaries between consumers and suppliers, they can serve as a buffer between suppliers and a market with demand uncertainty, and, in particular, they can bear the risk associated with demand fluctuations. Thus, to analyze retailing convincingly one needs to explicitly model the interaction between consumers and suppliers in an uncertain environment. Only this type of model can credibly explain why there is a need for retailers as middlemen and what determines the characteristics of the retail networks.

As has been already recognized in the literature, in real life firms are never sure about a number of variables such as factor prices, the exact shape of the production function or the market

demand curve. Even if firms are certain of their cost structures, they very rarely (if ever) know precisely which demand conditions they face. The behaviour of firms operating in markets with uncertain demand has been analyzed in several studies (see Sandmo, 1971; Leland, 1972; Lim, 1980). However, in most of these papers, the firms' beliefs about demand are summarized in a subjective probability distribution which cannot be changed by the firms' actions. The fact that the firm may be able to predict changes in demand, or at least to decrease the range of possible variations, is usually neglected in the standard studies of economic behavior under uncertainty. Nevertheless, the ability of the firm to predict demand, although not always perfect, may affect a number of parameters of economic equilibrium (see, e.g., Nelson, 1960, for an analysis of uncertainty and prediction in competitive markets). The conjecture in this paper is that market analysis, information processing and demand forecasting activities not only affect the characteristics of economic equilibrium when the producer sells goods directly to the final consumers, but also influence the equilibrium characteristics of retail markets.

In the analysis which follows we assume that the relationship between quantities demanded and market prices randomly varies from period to period, and that demand analysis is both costly and time consuming. In particular, we focus on the market where the total demand originates from a large (but finite) number of sources. The demand curve in each individual source changes randomly from period to period, but in any time period demand changes are assumed to be correlated with the changes prior to this period, reflecting a certain inertness in consumer behavior. Since gathering and processing information requires time, the sum of individual demands (i.e., the total demand) cannot be instantaneously determined (in particular, we assume that the results of the market analysis are available only after the end of the period). Consequently, the firms' output-price decisions have to be made based not on the current demand but on its prediction. In each period the profit-maximizing firms set their volumes of output, since it has a high commitment value within a period of time (i.e., the output decisions are irreversible within the time unit). The price is assumed to be more flexible and can change to some extent due to real market conditions. However, firms operating in the market are still assumed to be unable to learn the true demand function during the period of time, and, consequently, have to rely only on the results of the demand analysis.<sup>1</sup> Since demand forecasts are based on past data, a prediction error appears, and, consequently, firm's output decisions always deviate from what is optimal.

To focus solely on the role of uncertainty and data processing (to avoid the problem of inventories, transportation and storage), one can think about a supplier of services, such as a sightseeing tour operator, operating in the market with demand depending on the weather in a season. We suppose that the supplier can analyze the market in order to decrease the variance of demand fluctuations. Moreover, the supplier is assumed to set capacity before the season (i.e.,

before real demand becomes known) and has two options: to sell services directly to final consumers during the season (at an uncertain price), or to sell services forward (before the season) to retail firms (at a fixed price, lower, however, than the expected price to consumers). In such a set-up, we examine how technological progress in information processing can affect the size of regional retail networks.

The paper is organized as follows. Market demand is characterized in Section 2. Section 3 shows how forecasts of actual demand can be computed. Section 4 provides an analysis of the optimal demand-forecasting strategy in the monopolistic supplier and the retail firms. In Section 5 the alternative methods of distribution of the output produced (i.e., with and without retail firms) are considered. The implication of technological progress in data processing on the size of regional retail networks is presented in Section 6. Section 7 concludes.

## 2. Uncertain Demand

Consider a market where total demand originates from a large number of identical sources  $N$  (one can think of these sources as consumers). Suppose that demand in each individual source  $i$  ( $i=1,2,\dots,N$ ) at any period of time  $t$  ( $t$  is an integer number,  $-\infty < t < +\infty$ ) is linear with an additive random term  $\epsilon_{i,t}$  (for the sake of simplicity, assume that random variables  $\epsilon_{i,t}$  are identically distributed with zero mean and finite variance  $\sigma_{\epsilon}^2 = \sigma_{\epsilon,t}^2$ ). Total inverse demand at period  $t$  is

$$P_t(Q_t, \epsilon_{1,t}, \epsilon_{2,t}, \dots, \epsilon_{N,t}) = a - bQ_t + \sum_{i=1}^N \epsilon_{i,t}, \quad (1)$$

where  $Q_t = \sum_{i=1}^N q_{i,t}$  is the total quantity demanded at price  $P_t$  ( $P_t > 0$ ),  $a$  and  $b$  are positive constants.

The random variables  $\epsilon_{i,t}$  can move up or down in response to changes in the variables omitted from a correct demand specification, such as, for instance, interest rates, inflation, personal income, prices of other goods, etc. Much of this movement, however, might be due to factors which are hard to capture, such as, for example, changes in the weather or in consumer tastes. Thus, in many cases it may be difficult (or even impossible) to explain fluctuations in demand through the use of a structural model. Moreover, it might happen that, even if statistically significant regression equations can be estimated, the result will not be useful for forecasting purposes (for example, when explanatory variables which are not lagged must themselves be forecasted). In such situations, an alternative means of obtaining predictions of  $\epsilon_{i,t}$  have to be used. The easiest way is to predict changes in  $\epsilon_{i,t}$  based on the analysis of their movements in the past.

Such forecasts, however, are possible only if the random variables  $\epsilon_{i,t}$  are observable and if they are correlated with their previous values.

### 3. Demand Forecasting

To simplify the analysis, assume that random deviations  $\epsilon_{i,t}$  ( $i=1,2,\dots,N$ ) from the expected values of individual demands are independent<sup>2</sup> and described by identical stationary stochastic processes with a memory (e.g., by autoregressive processes of any order).<sup>3</sup> In other words, assume that for any individual demand, variances and covariances of random variables,  $\epsilon_{i,t}$ , are invariant with respect to displacement in time (note that, by definition, mean values of random variables  $\epsilon_{i,t}$  are equal to zero,  $E(\epsilon_{i,t})=0$ ), i.e.,  $Var(\epsilon_{i,t})=Var(\epsilon_{i,t-s})= \sigma^2 > 0$ , and  $Cov(\epsilon_{i,t}, \epsilon_{i,t+s}) = 0$ , for  $s=0,1,\dots, i=1,2,\dots,N$ , and integer valued  $t$  ( $-\infty < t < +\infty$ ).

Since immediate computations are not possible and the firm's output-price decisions have to be made prior to the knowledge of the market price, the result computed in period  $t$  can be used only in subsequent periods, i.e., deviations  $\tilde{\epsilon}_t = \sum_{i=1}^N \epsilon_{i,t}$  can be estimated based on the results computed in the past, and, consequently, always with certain error. It has to be stressed, however, that the variance of the error in the estimation increases with the time elapsed from observations of individual demands to the moment when decisions are made (see Radner and Van Zandt, 1992, for a discussion). Therefore, the monopolist faces not only the rather standard problem of finding appropriate estimations of demand but also the problem of finding the optimal cost of these estimations, since data processing is inherently costly and the acquisition and analysis of more pieces of information (and in particular, more recent information) has to be weighed against the increasing costs of such an endeavor.

In general, the firm may find it advantageous to compute in subsequent periods (say,  $t-m$ ,  $m=1,2,\dots$ ) deviations from the mean values of random variables  $\epsilon_{i,t-m}$  coming from different subsets of sources (say,  $S_{t-m}$ ,  $m=1,2,\dots$ ) and use them for the estimation of the total deviation from the expected demand in period  $t$  (rational strategy requires that sources of demand should be analyzed cyclically one after the other).<sup>4</sup>

Denote the result computed in subsequent periods as  $\tilde{\epsilon}_{t-m}, \dots, \tilde{\epsilon}_{t-1}$ . If the subsets  $\{S_{t-m}, \dots, S_{t-1}\}$  contain  $n_{t-m}, \dots, n_{t-1}$  ( $n_{t-m} > 0, \dots, n_{t-1} > 0$ ) sources of individual demand, then there exists integer number  $K$  ( $N \geq K \geq 1$ ) such that  $\sum_{i=1}^K n_{t-i} \geq N > \sum_{j=1}^{K+1} n_{t-j}$ . Thus, the estimation of total deviation  $\tilde{\epsilon}_t$  can be computed as

$$\tilde{~}_t = \sum_{i=1}^K \tilde{~}_{S_{t-i}} + \frac{N - \sum_{i=1}^K n_{t-i}}{n_{t-K-1}} \tilde{~}_{S_{t-K-1}}, \quad (2)$$

where  $\tilde{~}_{S_{t-m}}$  is a forecast (for period  $t$ ) of the sum of the deviations from the mean values of random variables coming from the sources included in the set  $S_{t-m}$  ( $m=1,2,\dots,K+1$ ).

Since all the available predictions of partial deviations ( $\tilde{~}_{S_{t-m}}$ ,  $m=1,2,\dots,K+1$ ) can be represented as linear combinations of the true values of corresponding partial deviations in past, the expected error in the prediction of total deviation equals zero. Furthermore, its variance (assuming that deviations from the expected values of individual demands,  $\tilde{~}_{i,t}$ , are independent, identically distributed, and time invariant) is

$$\sigma_t^2 = \sum_{i=1}^K n_{t-i} \sigma_{t,i}^2 + (N - \sum_{i=1}^K n_{t-i}) \sigma_{t,K+1}^2, \quad (3)$$

where

$$\sigma_{im}^2 = E \left\{ \left[ \tilde{~}_{i,t} - \tilde{~}_{i,t}(m) \right]^2 \right\}, \quad (4)$$

is the variance of error in the estimation (with lag  $m$ ,  $m=1,2,\dots,K+1$ ) of the deviation of the random variable  $\tilde{~}_{i,t}$  ( $i=1,2,\dots,N$ ) from its mean value, and  $\tilde{~}_{i,t}(m)$  denotes the estimation with lag  $m$  ( $m=1,2,\dots,K+1$ ) of the deviation of the random variable  $\tilde{~}_{i,t}$  ( $i=1,2,\dots,N$ ) from its mean value.

The forecast of the inverse demand can be specified as  $\tilde{P}_t(Q_t) = P(Q_t) + \tilde{~}_t$ , where  $P_t(Q_t) = a - bQ_t$  denotes the expected demand curve in period  $t$  ( $- < t < +$ ). The prediction error  $\tilde{~}_t$  is given by expression (2) and its variance  $\sigma_t^2$  by an expression (3).

#### 4. A monopolistic supplier and retail firms

Taking into account that the variability of demand decreases the quality of output-price decisions (i.e., price-output decisions deviate from the optimal decision that would be made if the variance were equal to zero) and that the results of demand analysis can be used only after the end of the period in which they were computed, the smallest variance of the prediction error corresponds to the case when all sources of demand are analyzed in the preceding period. The analysis of the total demand in each period, however, requires a number of economic resources to be devoted to data-processing in the firm, i.e., it induces significant costs that cannot always be offset by the expected benefit from the output-price decision with a lower risk of error. Thus, instead of examining the demand coming from all sources in each period, the firm can sequentially analyze the demand coming from certain subsets of sources. In this case, however,

the firm has to determine the optimal number of sources of demand that should be analyzed in subsequent periods.

Suppose now that there are two types of firms operating in the market: a monopolistic supplier ( $s$ ) of a single type of service and perfectly competitive retail firms ( $r$ ) that can resell services and freely enter or exit the market. Suppose that firms (the supplier and retailers) are managed according to the wishes of their owners who are typical asset holders, and the decisions in each firm are made by a group of decision-makers with sufficiently similar preferences to guarantee the existence of a group-preference function, representable by a von Neuman-Morgenstern utility function. Given these conditions we assume risk aversion, so that the utility functions of the supplier ( $U_s$ ) and retail firms ( $U_r$ ) are concave and differentiable functions of profits. The objective of both the supplier and the retail firms is to maximize the expected utility from profit (we assume that the firms set the volume of output supplied).

Assume that the firms are able to analyze market data and predict demand. Taking into account that the life of the firm is unlimited, the optimization task of firm  $x$  ( $x \in \{s, r\}$ ) can be represented as the following infinite-horizon, discounted, dynamic programming problem:

$$\max_{Q_{x,t}, n_{x,t}} \sum_{t=0}^{\infty} \beta^t E\{ U_x[ \pi_{x,t}(Q_{x,t}, \epsilon_{x,t}, \epsilon_{x,t}^2) ] \}, \quad (5)$$

where  $\pi_{x,t+1} = f(\pi_{x,t}, n_{x,t})$ , with  $\pi_{x,0} = \pi_0 = (N \sigma^2)^{1/2}$ ,

$E$  is an expectation operator,

$U_x(\cdot)$  denotes the utility function of the firm ( $x \in \{s, r\}$ ),

$\pi_{x,t}(\cdot)$  is the profit of the firm in the period  $t$ ,  $t=0, 1, \dots$ , ( $x \in \{s, r\}$ )

$Q_{x,t}$  is a quantity supplied by the firm  $x$  ( $x \in \{s, r\}$ ) in the period  $t$ ,  $t=0, 1, \dots$ ,

$n_{x,t}$  denotes the number of individual sources of demand analyzed in firm  $x$  ( $x \in \{s, r\}$ ) in the period  $t$ ,  $t=0, 1, \dots$

$\epsilon_{x,t}$  is the standard deviation of the error in the prediction of the total deviation of the random variables  $\epsilon_{i,t}$  ( $i=1, 2, \dots, N$ ) in firm  $x$  ( $x \in \{s, r\}$ ) in the period  $t$ ,  $t=0, 1, \dots$ ,

$N$  is the total number of sources of demand,

$\sigma^2$  is the variance of the stochastic process underlying each individual demand around its mean,

$\beta$  is the discount factor,  $\beta \in (0, 1)$ .

The cost of gathering and processing information in a given period and the benefits from this activity in future periods (i.e., smaller variance of the prediction error) specify the link which



connects the present with the future. In other words, in the model considered, there is an intertemporal trade off between higher costs of data processing today and future benefits in the form of a higher expected utility. Thus, along the optimal path the disutility from the analysis of one additional source of demand in a period  $j$  ( $j=0,1,\dots$ ) has to be equalized with the sum of the discounted marginal benefits in all future periods. If  $V(n_{x,j})$  denotes the cost of data processing in firm  $x$  ( $x \in \{s,r\}$ ) in period  $j$  when  $n_{x,j}$  sources of demand are analyzed, then, along the optimal path<sup>5</sup>

$$\frac{E\{U_x [Q_{x,j}, n_{x,j}]\}}{n_{x,j}} = \sum_{t=j+1}^{\infty} \frac{E\{U_x [Q_{x,t}, n_{x,t}]\}}{n_{x,t}} \cdot \frac{1}{(1+r)^{t-j}} \quad (6)$$

Assuming that all the parameters of the model are stationary over time, the optimal solution to an infinite-horizon, discounted, dynamic programming problem is time-invariant (see, e.g., Sargent, 1987). Thus, in the problem considered, the optimal output and demand-predicting strategy is stationary, i.e.,  $Q_{x,0}^* = Q_{x,1}^* = Q_{x,2}^* = \dots = Q_x^*$  and  $n_{x,0}^* = n_{x,1}^* = n_{x,2}^* = \dots = n_x^*$  ( $x \in \{s,r\}$ ). This implies that the optimal value of the standard deviation ( $\sigma_x^*$ ) of the error in the prediction of the total deviation of the random variables  $\epsilon_{i,t}$  ( $i=1,2,\dots,N$ ) from their means is stationary and depends only on the number of individual demands analyzed in every period,  $\sigma_x^* = \sigma_x(n_x^*)$ . It follows that the unique one-period cost of data-processing can be related to each value of the standard deviation  $\sigma_x(n_x^*)$ , i.e., the costs of data processing in each period can be represented as a function of the standard deviation in the steady state,  $g[\sigma_x(n_x^*)] = V(n_x^*)$ . Since the stationary standard deviation,  $\sigma_x(n_x^*)$ , decreases if the number of individual demands analyzed in each period increases, the cost of data processing is a decreasing function of the steady state standard deviation (differentiating the cost of data processing  $V(n_x^*)$  with respect to  $n_x^*$  gives

$dV(n_x^*)/dn_x^* = g'[\sigma_x(n_x^*)] d\sigma_x(n_x^*)/dn_x^* > 0$ , since  $dV(n_x^*)/dn_x^* > 0$  and  $d\sigma_x(n_x^*)/dn_x^* < 0$ , it follows that  $g'[\sigma_x(n_x^*)] < 0$ ). Moreover, the shape of the function  $V(n_x^*)$  ( $dV(n_x^*)/dn_x^* > 0$ ,  $d^2V(n_x^*)/dn_x^{*2} > 0$ ) (see Cukrowski, 1996), implies that  $g$  is a convex function of  $\sigma_x$ , i.e.,  $d^2g[\sigma_x(n_x^*)]/d\sigma_x^2 > 0$ .

Assume, for simplicity, that a standard deviation is the following function of the cost of data processing  $\sigma_x = \sigma_0 e^{-g}$ , where  $g$  denotes the cost of data processing, and  $\sigma_0$  ( $> 0$ ) is a parameter describing the current state of information processing technology. Consequently, the cost of data processing is specified as  $g(\sigma_x) = -(\ln \sigma_x - \ln \sigma_0) / \beta$ .

The consideration above shows that the optimization problem of the firm  $x$  ( $x \in \{s,r\}$ ) can be solved in two steps. First, the optimal quantity  $Q_x^*$  and the optimal value of standard deviation

$Q_x^*$  can be determined, and, second, knowing  $Q_x^*$ , the optimal size of the cohorts of data summarized in each period can be found.

Thus, in the first stage the firm  $x$  ( $x \in \{s, r\}$ ) chooses the steady-state quantity of output  $Q_x$  and the value of the standard deviation  $\sigma_x$  which maximize the following objective function

$$\max_{Q_x, \sigma_x} E\{U_x[Q_x, \sigma_x]\}. \quad (7)$$

To simplify the analysis, assume that the steady-state error in prediction of the total demand is a normally distributed random variable with zero mean and variance  $\sigma_x^2$  (this corresponds to the case when random deviations follow stochastic processes with normally distributed random terms such as, for example, the autoregressive process of any order).<sup>6</sup> Since the distribution of the total random deviation from the mean value of demand is normal, the total deviation can take positive and negative values, each having probability  $\frac{1}{2}$  (the expected value of positive deviation equals  $\sigma_x / (2)^{1/2}$  and the expected value of negative deviation equals  $-\sigma_x / (2)^{1/2}$ ).<sup>7</sup> Consequently, the total inverse random demand in any period  $t$

( $- \infty < t < + \infty$ ) can be approximated as  $\tilde{P}(Q_x, \sigma_x) = a - bQ_x + (\epsilon_x)$ , where  $(\epsilon_x)$  is a random factor (not known ex ante) which with probability  $\frac{1}{2}$  equals  $(\epsilon_x)$  and with probability  $\frac{1}{2}$  equals  $-(\epsilon_x)$ , where  $(\epsilon_x) = \sigma_x / (2)^{1/2}$ . Consequently, with probability  $\frac{1}{2}$  market demand is  $\underline{P}(Q_x, \sigma_x) = a - bQ_x - (\epsilon_x)$ , and with probability  $\frac{1}{2}$  is  $\overline{P}(Q_x, \sigma_x) = a - bQ_x + (\epsilon_x)$ . The expected market demand curve is determined as  $P(Q_x) = a - bQ_x$ .

Using this approximation, the optimization problem of firm  $x$  ( $x \in \{s, r\}$ ) can be represented as

$$\max_{Q_x, \sigma_x} \left\{ \frac{1}{2} U_x[\overline{P}(Q_x, \sigma_x)] + \frac{1}{2} U_x[\underline{P}(Q_x, \sigma_x)] \right\}, \quad (8)$$

where  $\overline{P}(Q_x, \sigma_x) = Q_x \overline{P}(Q_x, \sigma_x) - F_x(Q_x, \sigma_x)$ , and  $\underline{P}(Q_x, \sigma_x) = Q_x \underline{P}(Q_x, \sigma_x) - F_x(Q_x, \sigma_x)$ ,  $F_x(Q_x, \sigma_x)$  denotes a cost function of the firm  $x$ .

## 5. Distribution of output

First consider the case when a monopolistic supplier trades directly with final consumers. Assume that the supplier's cost function is  $F_s(Q_s, \sigma_s) = cQ_s + g(\sigma_s) + B$ , where  $Q_s$  denotes the volume of output produced,  $g(\sigma_s)$  denotes the cost of data processing,  $c$  is the marginal cost, and

$B$  is the fixed cost. To simplify the analysis assume that the exact shape of the utility function  $U_s$  is specified as follows:

$$U_s(p_s) = \begin{cases} u_1 - p_s, & \text{if } p_s < \frac{a-c}{2}, \\ u_2 - p_s + (u_1 - u_2) \frac{a-c-p_s}{a-c}, & \text{if } p_s \geq \frac{a-c}{2}, \end{cases} \quad (9)$$

where  $u_1 > u_2 > 0$  and  $\frac{a-c}{2} < \frac{a-c}{2} < \frac{a-c}{2}$ .<sup>8</sup>

The interior solution to this optimization problem (8) exists if

$$\frac{8b}{(a-c)^2} \quad (10)$$

(see Appendix for details).

Assuming that the primitives of the model:  $a, b, c$ , satisfy the condition above, the optimal steady state values of the volume of output supplied  $Q_s^*$  and the standard deviation of the demand  $\sigma_s^*$  are determined as

$$Q_s^* = \frac{a-c}{4b} + \sqrt{\frac{a-c}{4b}^2 - \frac{1}{2b}}, \quad (11)$$

$$\sigma_s^* = \frac{a-c}{2} - \sqrt{\frac{a-c}{2}^2 - \frac{2b}{k_s}} \frac{\sqrt{2}}{k_s}, \quad (12)$$

where  $k_s = (u_1 - u_2) / (u_1 + u_2)$ ,  $k_s \in (0, 1)$  for the risk averse firm<sup>9</sup> (see Appendix for the proof).

Assume now that the monopolist can sell the output not to final consumers but to perfectly competitive retail firms which can freely enter and exit the market. Each individual retail firm operates in the market only if its expected utility from profit is at least equal to the utility of some benchmark activity ( $U_r = 0$ ). Since for a risk averse retail firm earning random profit ( $\pi_r$ ) the following is true  $U_r[E(\pi_r)] > E[U_r(\pi_r)] = 0$ , where  $U_r$  denotes utility function of the retail firm and  $E$  is an expectation operator, the expected value of profit of a single retailer operating in the market is positive. This implies that the retail market can be established only if the expected value of the profit of the retail sector as a whole is positive, i.e., if the supplier sells services to retail firms at a lower price than the expected price to final consumers.

**PROPOSITION 1.** *Rational behaviour of the risk averse monopolistic supplier under uncertainty of demand implies that the supplier is always willing to sell services to retail firms at a lower price than the expected price to consumers.*

Proof. Under demand uncertainty the risk averse monopolistic supplier trading directly with consumers earns random profit with the expected value  $E[U_s(Q_s^*, s^*)]$ , such that  $E[U_s(Q_s^*, s^*)] < E[U_s(Q_0^*, 0)]$ , where  $Q_0^*$  is the optimal monopolistic output without uncertainty (i.e., if  $\sigma = 0$ ). Taking into account that the optimal volume of output supplied to consumers by retail firms, and, consequently, demanded from the supplier, is deterministic (see Appendix), and that a risk averse firm always prefers deterministic profit to random profit with the same (or even slightly higher) expected value, the deterministic price  $P_0(Q)$  at which the supplier would be willing to sell the volume of output  $Q$  to retail firms should satisfy the following condition:

$$E\{U_s[P_0(Q)Q - cQ - B]\} = E\{U_s[Q_s^*, s^*]\}. \quad (13)$$

Since,  $E\{U_s[P_0(Q)Q - cQ - B]\} = U_s\{E[P_0(Q)Q - cQ - B]\}$  and for a risk averse firm  $U_s\{E[Q_s^*, s^*]\} < E\{U_s[Q_s^*, s^*]\}$ , inequality (13) is satisfied for any  $P_0(Q)$ , such that<sup>10</sup>

$$P_0(Q)Q - cQ - B \geq E[Q_s^*, s^*]. \quad (14)$$

Expression (14) states that the deterministic profit of the supplier (when he trades with retail firms) should be at least equal to the expected value of profit that the supplier would earn if he sold services directly to final demanders. Note that for any  $s^*$ ,  $E[Q_s^*, s^*]$  is a continuous, strictly concave, function of  $Q$ , positive for  $Q \in (0, Q_C)$ , where  $Q_C$  is the optimal competitive output without uncertainty, achieving its maximum for  $Q = Q_0^*$ . Since  $E[Q_s^*, s^*] < E[Q_0^*, s^*]$ , there exists an interval (say,  $(Q_A, Q_B)$ , where  $Q_A = Q_s^*$  and  $Q_0^* < Q_B < Q_C$ ), in which  $E[Q_s^*, s^*] < E[Q, s^*]$ . Plugging  $E[Q_s^*, s^*] = P_0(Q)Q - cQ - B$  and  $E[Q, s^*] = P(Q)Q - cQ - B$ , where  $P(Q)$  is an expected price if quantity  $Q$  is supplied to consumers, into the above inequality and rearranging, we get that  $P_0(Q) < P(Q)$ , for any  $Q \in (Q_A, Q_B)$ . QED.

Suppose now that the supplier trades with retail firms, but it cannot (or it is not legally allowed to) impose any vertical restraints, i.e., assume that the supplier is willing to sell any given volume of output  $Q_R$  at price  $P_0(Q_R)$  to perfectly competitive retail firms. Assume also that there exists an interval, say  $(Q, Q)$ , where  $Q_A < Q < Q < Q_B$ , such that, for any volume of output supplied to retail market  $Q_r$  in this interval, retail market is organized (i.e., the number of retail firms operating in the market  $n$  is greater or equal to 1), and for any  $Q \in (Q, Q)$  the retail market cannot be organized ( $n < 1$ ).

If the supplier trades with retail firms the cost function of a single retail firm is  $F_r(Q_r) = Q_r P_0(Q_R) + g(\sigma_r)$ , where  $Q_r$  is the volume of output supplied to final demanders by a single retail firm,  $P_0(Q_R)$  is the price to retail firms if the volume of output  $Q_R$  is supplied to the retail market (to focus directly on the problem no additional cost is assumed), and  $g(\sigma_r)$  is the steady state cost of data processing, which corresponds to the standard deviation  $\sigma_r$ . Consequently, the optimization problem of each individual retail firm can be represented as<sup>11</sup>

$$\max_{Q_r, \sigma_r} E\{U_r[\pi_r(Q_r, \sigma_r)]\} = \max_{Q_r, \sigma_r} \left\{ \frac{1}{2} U_r[\bar{\pi}_r(Q_r, \sigma_r)] + \frac{1}{2} U_r[\underline{\pi}_r(Q_r, \sigma_r)] \right\}, \quad (15)$$

where  $\bar{\pi}_r(Q_r, \sigma_r) = Q_r [\bar{P}(nQ_r) - P_0(nQ_r)] - g(\sigma_r)$ ,  $\bar{P}(Q_r, \sigma_r) = a - bnQ_r - (\sigma_r)$ ,

$\underline{\pi}_r(Q_r, \sigma_r) = Q_r [\underline{P}(nQ_r) - P_0(nQ_r)] - g(\sigma_r)$ ,  $\underline{P}(Q_r, \sigma_r) = a - bnQ_r - (\sigma_r)$ ,

$(\sigma_r) = \sigma_r / \sqrt{2}$ ,  $\sigma_r$  denotes steady state standard deviation of total demand and  $Q_r$  denotes the output supplied to final demanders by a single retail firm ( $n$  is the number of retail firms operating in the market).<sup>12</sup>

To simplify the analysis assume that the exact shape of the utility function  $U_r$  is specified as follows:

$$U_r(\sigma_r) = \begin{cases} y_1 - \sigma_r, & \text{if } \sigma_r < \frac{0}{r}, \\ y_2 - \sigma_r + (y_1 - y_2) \frac{0}{r}, & \text{if } \sigma_r \geq \frac{0}{r}, \end{cases} \quad (16)$$

where  $y_1 > y_2 > 0$  and  $\underline{\sigma}_r < \frac{0}{r} < \bar{\sigma}_r$ .<sup>13</sup>

Thus each individual retail firm considers maximization problem (8). The interior solution to this optimization problem exists if

$$\frac{8bn}{(a-c)^2} \quad (17)$$

(see Appendix). Assuming that the primitives of the model satisfy inequality (17), the optimal steady state values of the volume of output supplied to final consumers by each individual retail firm  $Q_r^*$  and steady state value of the standard deviation of demand  $\sigma_r^*$  are determined as

$$Q_r^* = \frac{a-c}{4bn} + \sqrt{\frac{a-c}{4bn}^2 - \frac{1}{2bn}}, \quad (18)$$

$$\sigma_r^* = \frac{a-c}{2} - \sqrt{\frac{a-c}{2}^2 - \frac{2bn}{k_r}} \frac{\sqrt{2}}{k_r}, \quad (19)$$

where  $k_r = (y_1 - y_2) / (y_1 + y_2)$ ,  $k_r \in (0, 1)$  for risk averse firm<sup>14</sup> (see Appendix for the proof).

Since the expected value of profit of each retail firm operating in the market is positive (greater or equal to  $>0$ ) and the maximum expected value of profit of the retail sector equals  $E[U_r(Q_0^*, 0)] - E[U_r(Q_s^*, s^*)]$ , where  $Q_0^*$  is the optimal monopolistic output without uncertainty (i.e., if  $\sigma=0$ ), in equilibrium only a finite number of perfectly competitive retail firms can operate in the market. Assuming that the number of firms  $n$  is a continuous number instead of an integer, in market equilibrium the expected utility of profit of a single retail firm is

$$E\{U_r\{Q_r^*[P(nQ_r^*) - P_0(nQ_r^*)]\}\} = > 0. \quad (20)$$

Since, for the risk averse firm  $U_r\{E[Q_r^*(Q_r^*, s^*)]\} > E\{U_r[Q_r^*(Q_r^*, s^*)]\}$  the inequality above is satisfied only if  $U_r\{E[Q_r^*(Q_r^*, s^*)]\} > 0$ , i.e., if  $E[Q_r^*(Q_r^*, s^*)] > 0$ . Taking into account that  $E[P(nQ_r^*, s^*)] - P_0(nQ_r^*) > 0$  if  $nQ_r^* \in (Q_A, Q_B)$ , where  $Q_A = Q_s^*$  and  $Q_0^* < Q_B < Q_C$  ( $Q_C$  is a perfectly competitive output), the condition (20) is satisfied only if  $Q_s^* < nQ_r^* < Q_C$ , i.e., if the quantity of output supplied through the retail network is greater than it would be if the supplier traded directly with final consumers.

An important implication of the result above is that retail trade under uncertainty of demand changes the distribution of welfare in the economy. In particular, it decreases the expected value of the deadweight loss (i.e., the volume of output is higher than it would be without the retail market) and increases the expected value of consumer surplus (consumers consume more and at a lower price). The monopolistic supplier is also better off since the supplier changes random profit to deterministic profit with the same expected value.

## 6. Progress in information processing technology and the size of retail networks

Since the optimal volumes of output supplied to final consumers by both the monopolistic supplier and the retail firms (see expressions (11) and (18), respectively), as well as the steady state value of the standard deviations of the demand (expressions (12) and (19)) depend on the parameter describing the current state of information processing technology  $\sigma$ , technological progress in data processing, which makes predictions cheaper (increases  $\sigma$ ), changes the characteristics of market equilibrium. The optimal output of the monopolistic supplier shifts closer to the optimal monopolistic output without uncertainty of demand, and consequently, the offer curve to retail firms ( $P_0(Q)$ ) shifts upward. This decreases the difference between the expected price to consumers and the price to retail firms. The expected profit of the retail sector as a whole and the expected profit of each particular retail firm both decrease, and consequently, the number of retail firms operating in the market tends to decrease. At the same time retail firms are also able

to make better predictions (i.e., decrease the variance of demand), and, as a result, are able to increase the expected value of profit for any particular volume of output supplied. Therefore, other things being constant, more retail firms can operate in the market. The total effect of technological change on the number of firms in market equilibrium is characterized by the proposition below.

**PROPOSITION 2.** *Technological progress in information processing increases the number of retail firms operating in the market.*

Proof. Note that for any fixed  $\theta$  both  $q_r^*$  and  $\hat{U}_r^*$  can be considered as functions of  $n$ . Assume for the time being that the number of retail firms  $n$  is continuous (rather than integer valued) and consider function

$$G(q_r^*(n), q_r^*(n)) = \theta + G(q_r^*(n), q_r^*(n)), \quad (21)$$

where  $\theta > 0$ . Taking into account that in market equilibrium the expected utility from profit of each individual retailer operating in the market must be at least equal to  $\theta$  (0), in market equilibrium  $G(q_r^*(n), Q_r^*(n)) = \theta$ . Rearranging the latest expression we get

$$Q_r^*(n)[a - bnQ_r^*(n) - c] - \frac{E[\ln(Q_s^*)] + B}{n} - \frac{\ln \theta - \ln q_r^*(n)}{n} = \frac{2}{y_1 + y_2} + \frac{k_r q_r^*(n)}{\sqrt{2}} - 2k_r q_r^{\theta}. \quad (22)$$

The left hand side of the expression (22) can be represented as

$$\frac{n Q_r^*(n)[a - bnQ_r^*(n) - c] - \frac{\ln \theta - \ln q_r^*(n)}{n} - B - E[\ln(Q_s^*)]}{n}. \quad (23)$$

The numerator in the expression above describes the total profit of the retail sector. Therefore, the expression (23) describes the profit of a single retail firm, which in equilibrium has to be equal to  $\theta$ . Consequently, in market equilibrium

$$\frac{2}{y_1 + y_2} + \frac{k_r q_r^*(n)}{\sqrt{2}} - 2k_r q_r^{\theta} = \theta. \quad (24)$$

Taking into account expression (19) and rearranging an equilibrium condition can be represented as

$$\frac{2}{y_1 + y_2} - I + \frac{a - c}{2} - \sqrt{\frac{a - c}{2}^2 - \frac{2bn}{2}} - 2k_r q_r^{\theta} = \theta. \quad (25)$$

Assume now that  $\theta$  is not constant but can fluctuate. Define a function

$$H(d, n) = \frac{2}{y_1 + y_2} - 1 + \frac{a - c}{2} - \sqrt{\frac{a - c}{2}^2 - \frac{2bn}{2}} - 2k_r \frac{d}{r} > 0. \quad (26)$$

Since  $H(d, n)$  is a continuously differentiable function of  $d$  and  $n$ , according to the implicit function theorem, the first derivative of  $n$  with respect to  $d$  is  $\frac{dn}{dd} = -\frac{H/d}{H/n}$ .  $H/d$  is always positive, and  $H/n$  is negative. Therefore,  $dn/dd > 0$ , i.e., the number of retail firms operating in the market increases with  $d$  (i.e., with technological progress in data processing). QED.

The most important implication of the proposition above is that in the period of transition from an industrial to an information society, accompanied by rapid progress in information processing technologies, workers who will undoubtedly lose their jobs in old resource-intensive industries will have a chance to find positions in growing regional retail networks.

## 7. Conclusion

Today, at the beginning of the period of transition from an industrial to an information society, the problem of growing unemployment in regions dominated by old area-intensive and resource-intensive industries seems to be especially important. Economists agree that very soon traditional “old-industries” (e.g., mining, metallurgy, heavy industry, machine building industries) will lose their importance. This process will be accompanied by the development of new information-intensive industries. Unfortunately, it is also expected that in many industrialized regions declining industries will not be replaced by new ones. This will create problems in regional job markets. The analysis presented in this paper shows that the decline of some industries due to the progress in information processing technologies might be accompanied by the growth of regional retail networks, which could create a number of jobs in new retail firms.

Using a simple model, we have tried to explain the possible impact of the progress in information processing technology on the size of regional retail networks. The analysis focused on a single commodity market with uncertain demand. It has been shown that demand uncertainty can be considered an independent source of retail trade, and, consequently, the ability of firms to process information and predict demand may affect the characteristics of retail networks. The results derived show that risk-averse firms always devote resources to demand forecasting, producers are better off trading with retailers than with final consumers, and the volume of output supplied through the retail network is always greater than it would be if producers traded directly with consumers (i.e., it increases welfare). Finally, we proved that technological progress in data processing (which improves predictions and/or makes them cheaper, decreases uncertainty about



demand in retail firms much more than in the supplier's firm) increases the size of retail networks. Growing retail networks (a source of new jobs) can, to some extent, reduce unemployment and increase tax revenues in regions dominated by declining industries.

The results above have been derived based on a set of simplifying assumptions concerning demand structure and data information in demand forecasting problems. However, similar results could be obtained from more complicated and sophisticated models. Such models make the analysis more difficult, but the general result concerning the pattern of changes in the size of regional retail networks in response to changes in information processing technology remains the same.

### *Appendix.*

#### The maximization problem of the supplier

The objective function of the monopolistic supplier trading with final demanders can be approximated as

$$\begin{aligned} \max_{Q_s, \sigma_s} & \left\{ \frac{1}{2} u_1 \left[ Q_s \left( a - bQ_s - \frac{\sigma_s}{\sqrt{2}} \right) - cQ_s - B - \left( \frac{\ln \sigma_0}{\sigma_s} - \frac{\ln \sigma_s}{\sigma_s} \right) \right] \right. \\ & \left. + \frac{1}{2} \left\{ u_2 \left[ Q_s \left( a - bQ_s + \frac{\sigma_s}{\sqrt{2}} \right) - cQ_s - B - \left( \frac{\ln \sigma_0}{\sigma_s} - \frac{\ln \sigma_s}{\sigma_s} \right) \right] + (u_1 - u_2) \frac{\sigma_s^2}{\sigma_s} \right\} \right\}, \end{aligned} \quad (A.1)$$

where  $Q_s$  denotes the volume of output supplied, and  $\sigma_s$  is the steady state standard deviation of demand. The first order conditions to the above optimization problem can be represented as

$$\frac{\partial \pi_s(Q_s, \sigma_s)}{\partial Q_s} = \frac{1}{2} u_1 \left( a - 2bQ_s - \frac{\sigma_s}{\sqrt{2}} - c \right) + \frac{1}{2} u_2 \left( a - 2bQ_s + \frac{\sigma_s}{\sqrt{2}} - c \right) = 0, \quad (A.2)$$

$$\frac{\partial \pi_s(Q_s, \sigma_s)}{\partial \sigma_s} = \frac{1}{2} u_1 \left( -\frac{Q_s}{\sqrt{2}} + \frac{1}{\sigma_s} \right) + \frac{1}{2} u_2 \left( \frac{Q_s}{\sqrt{2}} + \frac{1}{\sigma_s} \right) = 0. \quad (A.3)$$

The second order conditions to this maximization problem require the Hessian of the objective function

$$\begin{aligned} & \frac{\partial^2 \pi_s(Q_s, \sigma_s)}{\partial Q_s^2} & \frac{\partial^2 \pi_s(Q_s, \sigma_s)}{\partial Q_s \partial \sigma_s} \\ & \frac{\partial^2 \pi_s(Q_s, \sigma_s)}{\partial Q_s \partial \sigma_s} & \frac{\partial^2 \pi_s(Q_s, \sigma_s)}{\partial \sigma_s^2} \end{aligned} \quad (A.4)$$

to be negative definite (it guarantees that the objective function is strictly concave). This Hessian is negative-definite (the objective function is strictly concave) iff

$$\frac{\partial^2 \pi(Q_s, \sigma_s)}{\partial Q_s^2} < 0 \text{ and } \frac{\partial^2 \pi(Q_s, \sigma_s)}{\partial Q_s^2} \frac{\partial^2 \pi(Q_s, \sigma_s)}{\partial \sigma_s^2} - \left( \frac{\partial^2 \pi(Q_s, \sigma_s)}{\partial Q_s \partial \sigma_s} \right)^2 > 0. \quad (\text{A.5})$$

Taking derivatives and rearranging, we conclude that the second order conditions are satisfied iff

$$b < \left( k_s \sigma_s / \sqrt{8} \right)^2. \quad (\text{A.6})$$

Rearranging the first order conditions, we obtain two possible values of output which maximize the objective function considered

$$Q_{s,1} = \frac{a-c}{4b} + \sqrt{\frac{a-c}{4b}^2 - \frac{1}{2b}}, \quad (\text{A.7})$$

$$Q_{s,2} = \frac{a-c}{4b} - \sqrt{\frac{a-c}{4b}^2 - \frac{1}{2b}}, \quad (\text{A.8})$$

assuming that  $[(a-c)/4b]^2 - 1/(2b) > 0$ , i.e.,

$$8b < (a-c)^2. \quad (\text{A.9})$$

If the cost of data processing goes to zero ( $\sigma_s$  goes to infinity), the firm knows demand almost perfectly, and the optimal volume of output goes to optimal volume of output without uncertainty  $(a-c)/2b$ . Consequently, the optimal quantity of output supplied  $Q_s^*$  is determined by the first expression i.e.,  $Q_s^* = Q_{s,1}$ .

Similarly, rearranging the first order conditions, we get two possible values of steady state standard deviation which maximize the objective function considered<sup>15</sup>

$$\sigma_{s,1} = \frac{a-c}{2} - \sqrt{\frac{a-c}{2}^2 - \frac{2b}{k_s}} \frac{\sqrt{2}}{k_s}, \quad (\text{A.10})$$

$$\sigma_{s,2} = \frac{a-c}{2} + \sqrt{\frac{a-c}{2}^2 - \frac{2b}{k_s}} \frac{\sqrt{2}}{k_s}. \quad (\text{A.11})$$

Here again, if the cost of data processing goes to zero ( $\sigma_s$  goes to infinity), the firm knows demand almost perfectly ( $\sigma_s$  goes to zero). Consequently, the optimal value of the standard deviation  $\sigma_s^*$  is determined by the expression (A.10), i.e.,  $\sigma_s^* = \sigma_{s,1}$ . Taking into account condition (A.9), plugging (A.10) into inequality (A.6) and rearranging we get the following condition

$$\frac{2b}{\frac{a-c}{2}^2} < 2 \frac{2b}{\left( \frac{a-c}{2} - \sqrt{\frac{a-c}{2}^2 - \frac{2b}{k_s}} \right)^2}, \quad (\text{A.12})$$

which is always satisfied if  $8b/(a-c)^2$  (i.e.,  $Q_s^*$  and  $Q_r^*$  corresponds to the maximum of the objective function).

### The maximization problem of the retail firm

The objective function of the retail firm can be approximated as

$$\begin{aligned} \max_{Q_r, \sigma_r} \pi_r(Q_r, \sigma_r) &= \frac{1}{2} y_1 [Q_r(a - bnQ_r - \frac{r}{\sqrt{2}}) - \frac{E(Q_s^*) + B}{nQ_r} + c] Q_r - (\frac{\ln \sigma_0}{2} - \frac{\ln \sigma_r}{2}) \\ &+ \{ \frac{1}{2} y_2 [Q_r(a - bnQ_r + \frac{r}{\sqrt{2}}) - \frac{E(Q_s^*) + B}{nQ_r} + c] Q_r - (\frac{\ln \sigma_0}{2} - \frac{\ln \sigma_r}{2}) \}. \end{aligned} \quad (A.13)$$

where  $Q_r$  denotes the volume of output supplied, and  $\sigma_r$  is the steady state standard deviation of demand. The first order conditions to the above optimization problem can be represented as

$$\frac{\partial \pi_r(Q_r, \sigma_r)}{\partial Q_r} = \frac{1}{2} y_1 (a - 2bnQ_r - \frac{r}{\sqrt{2}} - c) + \frac{1}{2} y_2 (a - 2bnQ_r + \frac{r}{\sqrt{2}} - c) = 0, \quad (A.14)$$

$$\frac{\partial \pi_r(Q_r, \sigma_r)}{\partial \sigma_r} = \frac{1}{2} y_1 (-\frac{Q_r}{\sqrt{2}} + \frac{1}{\sigma_r}) + \frac{1}{2} y_2 (\frac{Q_r}{\sqrt{2}} + \frac{1}{\sigma_r}) = 0. \quad (A.15)$$

The second order conditions to this maximization problem require the Hessian of the objective function

$$\begin{aligned} \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial Q_r^2} & \quad \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial \sigma_r^2} \\ \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial Q_r \partial \sigma_r} & \quad \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial \sigma_r \partial Q_r} \end{aligned} \quad (A.16)$$

to be negative definite (it guarantees that the objective function is strictly concave). This Hessian is negative-definite (the objective function is strictly concave) iff

$$\frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial Q_r^2} < 0 \text{ and } \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial Q_r^2} \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial \sigma_r^2} - \left( \frac{\partial^2 \pi_r(Q_r, \sigma_r)}{\partial Q_r \partial \sigma_r} \right)^2 > 0. \quad (A.17)$$

Taking derivatives and rearranging we get that the second order conditions are satisfied iff

$$8b < nb / \left( k_r \sigma_r / \sqrt{8} \right)^2. \quad (A.18)$$

Rearranging the first order conditions we get two possible values of output which maximize the objective function considered

$$Q_{r,1} = \frac{a-c}{4bn} + \sqrt{\frac{a-c}{4bn}^2 - \frac{1}{2bn}}, \quad (A.19)$$

$$Q_{r,2} = \frac{a-c}{4bn} - \sqrt{\frac{a-c}{4bn}^2 - \frac{1}{2bn}}, \quad (A.20)$$

assuming that  $[(a-c)/4bn]^2 - 1/(2bn) \geq 0$ , i.e.,

$$8bn/(a-c)^2 \geq 1. \quad (A.21)$$

If cost of data processing goes to zero ( $k_r$  goes to infinity) the firm knows demand almost perfectly, and the optimal volume of output goes to the optimal volume of output without uncertainty  $(a-c)/2bn$ . Consequently, the optimal quantity of output supplied  $Q_r^*$  is determined by the first expression i.e.,  $Q_r^* = Q_{r,1}$ .

Similarly, by rearranging the first order conditions, we get two possible values of steady state standard deviation which maximize the objective function considered<sup>16</sup>

$$r_{,1} = \frac{a-c}{2} - \sqrt{\frac{a-c}{2}^2 - \frac{2bn}{k_r}} \frac{\sqrt{2}}{k_r}, \quad (A.22)$$

$$r_{,2} = \frac{a-c}{2} + \sqrt{\frac{a-c}{2}^2 - \frac{2bn}{k_r}} \frac{\sqrt{2}}{k_r}. \quad (A.23)$$

If cost of data processing goes to zero ( $k_r$  goes to infinity) the firm knows demand almost perfectly ( $r_{,}$  goes to zero). Consequently, the optimal value of the standard deviation  $r_{,}^*$  is determined by the expression (A.22), i.e.,  $r_{,}^* = r_{,1}$ . Taking into account condition (A.21), plugging (A.22) into inequality (A.18) and rearranging we get the following condition

$$\frac{2bn}{\frac{a-c}{2}^2} < 2 \frac{2bn}{\frac{a-c}{2} - \sqrt{\frac{a-c}{2}^2 - \frac{2bn}{k_r}}}, \quad (A.24)$$

which is always satisfied if  $8bn/(a-c)^2 \geq 1$  (i.e.,  $Q_r^*$  and  $r_{,}^*$  corresponds to the maximum of the objective function).

## Endnotes

<sup>1</sup> Note that by allowing for a small price adjustment, we avoid the problem of inventories and any potential losses connected with them (see Zabel, 1970, for an analysis of the behavior of the firm in a multiperiod model with inventories).

<sup>2</sup> In general, specifications of stochastic processes describing individual demands should also include a “common noise” which could reflect aggregate demand shocks (i.e., which could equally affect all sources of demand), but to simplify the exposition we will disregard this common component.

<sup>3</sup> A similar structure of demand was assumed by Radner and Van Zandt (1992).

<sup>4</sup> See Cukrowski (1996) for details.

<sup>5</sup> It has been shown that  $V$  can be considered as a twice differentiable, strictly increasing and convex function of  $n_{x,j}$  (see Cukrowski, 1996).

<sup>6</sup> It should be stressed that, although the assumption of the normal distribution of the random deviations from the expected demand corresponds to the wide class of stochastic processes that would govern stochastic demand, it is chosen solely for simplicity and clarity, and no attempt is made at generality. We believe, however, that many of the qualitative results would hold also in more general, and, consequently, more complicated models.

<sup>7</sup> Expected values of positive and negative deviations are computed as  $\int_0^{\infty} \frac{1}{\sqrt{2\pi}x} e^{-\frac{\tilde{x}^2}{2x^2}} d\tilde{x}$  and  $-\int_{-\infty}^0 \frac{1}{\sqrt{2\pi}x} e^{-\frac{\tilde{x}^2}{2x^2}} d\tilde{x}$ , respectively.

<sup>8</sup> Note that a function defined is concave and twice differentiable if  $\frac{\partial^2 V}{\partial n_{x,j}^2} < 0$ .

<sup>9</sup> Coefficient  $k_s$  characterizes the attitude towards risk and increases with risk aversion.

<sup>10</sup> Note that  $Q$  is deterministic variable, and, consequently,  $P_0(Q)Q - cQ - B$  is deterministic as well.

<sup>11</sup> Note that if the optimal value supplied by each individual retail firm ( $Q_r^*$ ) exists, it is not a random but a deterministic variable.

<sup>13</sup> Recall that under uncertainty of demand the number of firms operating in perfectly competitive market is finite (see Ghosal 1996, for empirical evidence)

<sup>14</sup> Note that a function defined is concave and twice differentiable if  $\frac{\partial^2 V}{\partial n_r^2} < 0$ .

<sup>15</sup> Coefficient  $k_r$  characterizes the attitude towards risk and increases with risk aversion.

<sup>16</sup> The square root in the expressions (A.10) and (A.11) is non negative if  $8b/(a-c)^2$ .

<sup>17</sup> The square root in the expressions (A.22) and (A.23) is non negative if  $8bn/(a-c)^2$ .

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