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Causes of Shadow Banking -
Two Regimes of Credit Risk Transformation and its Regulation

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Abstract
This paper presents a model of banking in order to study why different agents may prefer a ‘regulation by the market’ over the regulation by a governmental agency, and it illustrates the interaction of two sectors regulated in such alternative ways. Financial intermediaries can operate either as commercial bank, which is regulated by an agency that also insures its deposits, or as uninsured ‘shadow bank’ whose leverage is constrained by the risk-aversion of investors. The analysis shows that there are exactly three possible reasons for choosing shadow banking: First, lower operational costs, second, heterogeneous beliefs about the aggregate risk, and third, the sponsoring of shadow banks by commercial banks. Heterogeneous beliefs lead to a self-selection of optimistic depositors and pessimistic intermediaries into shadow banking, with the latter profiting from the optimism of the former. Sponsored shadow banking impairs the solvency of the sponsor in downturns, but it is more profitable than independent shadow banking. It does not only allow for a shift to a system ‘regulated by markets’, but it allows for multiplicative leverage owing to the combination of two balance sheets. It is an unambiguous sign of regulatory arbitrage, as it becomes unprofitable if the regulation is adjusted to avoid contagion.
1 Introduction

The 'shadow banking sector' has emerged over the last decades as an alternative system of credit intermediation outside of the regulatory framework designed for commercial banks (Poszar et al. (2010), Gorton and Metrick (2010), Adrian, Ashcraft and Cetorelli (2013)). The current attempts to improve the regulation of banks are hence accompanied by concerns that the aim to stabilize the financial system could be undermined by an evasion of financial activity into the unregulated shadow banking sector. This concern is fostered by the experience that the activities and structures of this sector (for instance securitization and wholesale funding) played a major role in the recent crisis, see e.g. Gorton and Metrick (2012). At the same time, however, there is also strong support for shadow banking as an additional intermediation chain that can provide credit to the economy, see e.g. ECB (2014). A better understanding of the causes for and the consequences of shadow banking is thus crucial, if one wants to construct a regulatory framework whose scope and scale ensures stable financial markets without being unnecessarily restrictive.

The aim of this paper is to provide a model of banking which includes a shadow banking sector and which answers three important questions: First, what are potential reasons for the emergence of shadow banking? Second, who benefits and who looses from this 'innovation'? And third, what is the role of the regulatory framework in these respects?

A key difference between commercial banking and shadow banking is obviously that the former is subject to stricter laws and stronger supervisory authorities than the latter. These rules, however, are not set arbitrarily, but depend on the expectations of the regulatory agencies about possible losses in adverse economic states. The shadow banking sector, in comparison, is not unconstrained, either, but has to adapt to the demand of investors for safe assets by choosing their portfolio in accordance with the investors' expectations about economic developments. The assumption that shadow banking simply emerges, because it is a less restricted business model for intermediaries, trivializes the discussion and neglects the crucial underlying question: How do explicit governmental regulation and the 'regulation by markets' differ from each other, such that shadow banking becomes an attractive business model for financial intermediaries?

While 'shadow banking' has become a vague term which is sometimes used to denote all kinds of financial intermediation outside of banks, this paper focuses on those parts of the financial markets that are most similar to banking in the sense that they provide a key function of commercial banking: They finance and monitor risky projects, but refinance themselves with money-like claims which satisfy a strong risk-aversion of savers and can be used as means of payment. Prior to the financial crisis, a prominent example has been MMFs which invested in ABCP or repos backed by ABS, which were produced by the pooling and tranching of risky loans (Poszar et al. (2010), Gorton and Metrick (2010)).

\[\text{1}\]This emphasis on the provision of money-like claims as a key function of banks follows a long strand of literature reaching from Gorton and Pennacchi (1990) to recent work by Stein (2012), Gennaioli, Shleifer and Vishny (2012,2013), and DeAngelo and Stulz (2013).
The two main characteristics of money-like claims are that their nominal value is safe and that they can be withdrawn in short time\textsuperscript{2}. Financial intermediaries ensure the former by the transformation of credit risk which is shifted to the equity or other junior claims, and they ensure the latter by a maturity transformation that relies on the law of large numbers and possibilities to obtain short-term credit from other institutions. Unexpected problems with the transformation of credit risk, which means strong decreases in asset values which exceed (or at least seem to exceed) the capacity of the junior claims, are usually the trigger of critical situations in financial markets, whereas excessive maturity and liquidity transformations can further amplify crises by (shadow) bank runs and fire sales. While both dimensions are very important, this paper will start\textsuperscript{3} with the initial problem of the triggering, that means with analyzing the transformation of credit risk and the solvency of financial intermediaries.

The recent decade has provided interesting empirical evidence about the shadow banking sector, which still requires a better theoretical understanding:

1. The demand for products that belong to the shadow banking sector is driven by investors and evaluations of credit risk that are particularly optimistic, cf. Chernenko, Hanson and Sunderam (2014), Cheng, Raina and Xiong (2014), Griffin and Tang (2012), Coval, Jurek, and Stafford (2009), Gerardi et al. (2008).

2. Shadow banking is rarely independent from commercial banks, see Poszar et al. (2010) or Mandel, Morgan and Wei (2012). A common type of direct sponsoring are commercial banks which set up SPVs and provide guarantees for these. Acharya, Schnabl and Suarez (2013) illustrate the case of ABCP conduits whose losses where borne by banks which faced comparably low capital requirements for these guarantees\textsuperscript{4}. The ABCP market grew to a size of $1.3 trillion before the crisis, but strongly decreased after a readjustment of the capital requirements for these guarantees.

3. MMF shares yielded higher returns than bank deposits in all years before the crisis, see e.g. ICI (2010) [p. 34/35], although they promised a stable NAV and were perceived as safe as deposits.

4. The growth of shadow banking is positively correlated with the growing demand for money-like claims by institutional investors, see e.g. IMF (2014).

\textsuperscript{2}Both properties hold true for shares in MMFs which promise a stable NAV.

\textsuperscript{3}The regulation of maturity transformations in banks and shadow banks and the potential amplification of crises due to a lack of liquidity may be addressed in subsequent papers.

\textsuperscript{4}The guarantees were mainly structured as liquidity instead of credit guarantees in order to avoid larger requirements, but they effectively worked as credit guarantees as they ensured the full repayment of the investors at the first signs of trouble, while all losses remained with the sponsoring bank.
Based on the initial considerations, the following **structure of the model** is suggested in order to explain and analyze the mechanisms behind these empirical facts and in order to clarify the underlying question about the differences and relations between banking regulated by an agency and banking 'regulated by the markets':

Risky and riskfree projects demand loans for one period which are provided by financial intermediaries. These risk-neutral intermediaries have their own endowments, but they are mainly funded by deposits from savers that have a demand for safe claims. The financial intermediaries bear the risk of the loans and earn a spread between the average returns on these and the interest rates on deposits. This spread provides an incentive to increase the leverage. The intermediaries can diversify their portfolio, but there is always some aggregate risk\(^5\) about which each agent has a belief. If a financial intermediary operates as commercial bank, the deposits are insured by a governmental agency, which sets capital requirements in order to prevent moral hazard and to minimize its insurance payments. After the specification of this regulation but before the market opens, 'financial innovation' occurs and introduces shadow banking as an alternative operational mode. This shadow banking sector is unregulated and their 'deposits' are uninsured by the agency, so that the risk-averse savers have to examine if the equity of the shadow banks can absorb all risk in adverse states. This private monitoring implies some effort and hence some cost which has to be compensated if shadow banks want to attract depositors. Financial intermediaries choose the operational mode (commercial or shadow banking) with higher expected profit.

There are exactly **three reasons why shadow banking can emerge** for this type of banking, which means why financial intermediaries choose to operate as shadow banks despite the higher funding costs:

**I.** If shadow banking is a true innovation such that it has lower operational costs, the saved costs can be used to attract depositors. Shadow banks serve depositors who are relatively skilled in monitoring intermediaries and equally profitable commercial banks serve the less skilled ones. If deposits are relatively scarce, shadow banking of this kind is a Pareto improvement with skilled depositors as the main beneficiaries\(^6\). If deposits are relatively abundant, however, mainly the borrowers benefit due to decreased loan rates.

**II.** If there are heterogeneous beliefs about the aggregate risk and the agency has not

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\(^5\) Idiosyncratic risk as well as aggregate risk influence the risk structure of an intermediary. Concerning regulation, however, the aggregate risk seems to be the more critical one. First, synchronized insolvencies of many banks have much stronger externalities than an idiosyncratic bankruptcy. Second, in comparison to the large crises (e.g. the recent crisis or the "savings and loan crisis") which affected the stability of many financial intermediaries at the same time, only few banks have become insolvent in normal times for idiosyncratic reasons.

\(^6\) Note that I in general deliberately refrain from a welfare analysis in terms of social welfare functions. Besides a fundamental skepticism about this approach which has to be set forth elsewhere, the attempt to define a social welfare function is a particularly bold endeavor in models that deal with the stability of financial markets, because the effects of (synchronous) insolvencies of financial intermediaries are so diverse and broad that an appropriate unified accounting seems to be unfeasible.
very optimistic belief, financial intermediaries can profitably operate as shadow banks by attracting more optimistic depositors who allow for larger leverage than in the regulated banking sector. The scenario has a unique equilibrium in which pessimistic depositors stay with commercial banks, while the pessimistic intermediaries are the main operators of shadow banking. In contrast to the optimistic intermediaries, they believe that possible losses in shadow banking are partly borne by their optimistic depositors. The borrowers with risky projects are the only agents which are definitely better off due to an increased supply of loans to them, in some cases at the expense of the risk-free projects. The optimistic depositors may profit from shadow banking if their belief is correct, but can also become very unpleasantly surprised. The pessimistic depositors are unaffected in most cases, but may even gain (without any risk) from an increased demand for funds. The intermediaries gain from increased leverage, but also lose due to an increased competition that leads to a decline in loan rates and an increase in deposit rates. The relation between these two opposing effects depends on the parameters of economy in a non-trivial way.

III. A third cause of shadow banking can be the combination of both operational modes, such that guarantees for shadow banking or its equity tranches\textsuperscript{7} constitute a new kind of asset in the portfolio of the commercial bank for which appropriate capital requirements have not been imposed. This kind of shadow banking, henceforth simply denoted as ‘SPVs’ (special purpose vehicle), allows for regulatory arbitrage which is more profitable for banks than just shifting activity to a possibly less constrained sector (as it could be done by means of independent shadow banks). The key mechanism is that it allows for multiplicative leverage in which risky loans held in the SPV are levered against guarantees or equity tranches which are themselves levered against the equity of the commercial bank. This connection implies a direct channel of contagion in adverse states, such that the sponsoring bank becomes insolvent due to losses that are concentrated on its balance sheet. These have to be partly borne by the deposit insurance.

If one considers a possible adjustment of the regulation such that SPVs remain allowed, but coordinated capital requirements for the sponsoring bank and its SPV are imposed, even the requirements which would allow for the largest possible profit while ensuring solvency would eliminate the incentive for financial intermediaries to set up SPVs. This suggests that they are simply a sign of regulatory arbitrage.

The regulatory agency is the agent most negatively affected in this scenario of shadow banking, while the risky borrowers and the depositors profit from increased supply of loans and demand for deposits. Only the risk-free borrowers may lose due to a shift of funds to the risky projects. If the intermediaries gain due to larger leverage or lose due to the increased competition depends on the parameters of the economy again.

This theoretical analysis explains very well the empirical evidence about shadow bank-

\textsuperscript{7} The model is formulated in terms of equity tranches, but it is equivalent to a formulation in terms of credit guarantees, as argued in Section 2.3.
ing. In particular, cases II. and III. provide explanations of the empirical facts 1. and 2. The prediction of the model that the pessimistic financial intermediaries exploit the optimism of their 'depositors' in the shadow banking sector is difficult to test systematically, but cases like the 'Abacus' deal provide at least some anecdotal evidence. The contribution of improved cost-efficiency to the growth of shadow banking has not been empirically investigated so far, probably due to the difficulties to measure operational costs for credit intermediation with consists of many interrelated steps. The empirical facts 3. and 4. can be explained by all three scenarios of this model. Shadow banking 'deposits' have to pay a spread in comparison to deposits in a commercial bank despite being perceived as similarly safe, because they can only attract depositors if they compensate them for their private effort of assessing the safety of the deposits. And since institutional investors usually are more efficient in monitoring than single, private depositors, the size of shadow banking grows with an increasing share of this type of 'depositors'.

The introduction shall conclude with four remarks about the modeling approach:

The justification to regulate banking in the first place is based on the 'representation hypothesis' by Dewatripont and Tirole (1993,1994). It relies on the observation that the majority of debt holders in a bank are small, dispersed depositors who have little abilities to perform the monitoring functions that are usually required by debt holders in financial markets. To incorporate this idea in a concise way, the depositors in the model incur some effort costs if they have to monitor banks and they are less efficient in this task than a centralized regulatory agency. In order to account for the heterogeneity of depositors, which may range from private households that lack any financial education to large professionally managed cash pools, the size of these monitoring costs can vary. A regulatory agency which eliminates such costs by provision of encompassing deposit insurance and which protects itself against moral hazard and insurance payments by imposing capital requirements on the banks can be a Pareto improvement, as discussed in Section 3.1.

The discussion focuses on capital requirements instead of a fair pricing of the deposit insurance, because capital requirements are currently the most relevant regulatory means in practice. However, the analyzed regulatory approaches can also be understood as a combination of capital requirements and a fair pricing, in which the requirements are tried to be adjusted such that insolencies are simply prevented and the fair pricing is zero.

The analysis deviates from the assumption of rational expectations in two ways: First, due to the possibility of heterogeneous beliefs about the aggregate risk, and second, due to

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8 At the first glance, the self-selection of pessimistic intermediaries into shadow banking seems to contradict the findings of Cheng, Raina and Xiong (2014) which observe that the management of the intermediaries was also too optimistic prior to the crisis, despite their direct access to all information. However, they have investigated the behavior of the mid-level management and explicitly acknowledge the possibility that the assessment of the higher levels management could have been different, while they deliberately hired optimistic staff, as it was easier to incentivize them for their task.

9 This is probably partly due to the fact that they only require knowledge about the possible severity of adverse states, but not also knowledge about their frequency which is difficult to obtain.
the possibility of capital requirements that do not account adequately for the sponsoring of shadow banking by commercial banks. The relevance of heterogeneous beliefs for the understanding of critical aspects of financial markets has been established by an important and strongly growing strand of literature, see Xiong (2013). Moreover, there is much empirical evidence which indicates the presence of heterogeneous and distorted beliefs in the banking sector, as explained above.

The second aspect, the inadequacy of the capital requirements, is an inevitable consequence of the delay between innovation and regulation. It is the very nature of any significant innovation to be something new, whose specific characteristics cannot be foreseen. Regulators hence have to specify regulations based on the current knowledge of the world and cannot anticipate all possible changes, they can only adjust to them with some delay after they have been realized. Therefore, the regulation in the model is specified at the beginning of the period based on the business model of banking without shadow banks which are ‘invented’ afterwards. Delays of this kind will remain relevant despite the current attempts to regulate ‘shadow banking’. These address the financial structures that have been developed so far, but there will always be ‘new periods’ with unprecedented financial innovation and new, unknown types of credit intermediation.

Related Literature:

Besides the supporting empirical literature, the ‘representation hypothesis’, and the literature about heterogeneous beliefs, this paper is related to existing attempts to model shadow banking, in particular those with a focus on the transformation of credit risk. Gennaioli, Shleifer and Vishny (2013) illustrate how financial intermediaries can satisfy a demand for money and simultaneously finance risky projects by taking equity positions and using securitization. However, while the idiosyncratic risk can be eliminated by diversification, financial intermediaries remain vulnerable to aggregate risk, especially if possible adverse states are neglected. My paper modifies and extends this approach such that the consequences of regulation and the competition and possible coexistence of two different ‘regulatory regimes’ can be studied.

Hanson et al. (2014) also try to understand the coexistence of commercial banking and shadow banking, but they provide an answer with respect to liquidity transformations.

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10 This assumption is confirmed by private conversations with regulators who frankly admit that their regulatory initiatives always lag behind the ‘financial innovation’.

11 Additionally, informative surveys of shadow banking and estimates of its size can be found in annual reports like FSB (2014) or in Gallin (2013).

12 There are interesting models of shadow banking which address, for instance, liquidity and maturity transformations (Luck and Schempp (2014), Parlatore (2014)), amplifications of shocks (Meeks, Nelson and Alessandri (2013), Moreira and Savov (2014)), or monetary policy (Stein (2012)).

13 Gennaioli, Shleifer and Vishny (2013) contrast the business model of shadow banking with banking without securitization, but the latter differ from the former only in the incapability of diversification. This neglects regulatory differences and most notably leads to a biased comparison, as there are no reasons why banks should not be able to diversify to a similar extent, either directly on their balance sheet or by the use of securitization. Actually, their ‘model of shadow banking’ is in fact a ‘model of securitization’.
by arguing that the two types have comparative advantages in asset classes of different liquidity. Their paper is hence complementary to my approach. However, they do not address the different regulatory treatment of both sectors or the possible direct connection between banks and shadow banks, and they are unable to explain the four empirical facts mentioned above.

There is a recent series of models which study the shift of financial intermediation from regulated banks to shadow banks - Goodhart et al. (2013), Kim and Mangla (2013), Kolm (2013), Ordonez (2013), Gornicka (2014), Plantin (2014) - but all of them simply take it as given that commercial banks face tighter constraints. My paper, instead, analyzes the underlying reasons why and how the explicit regulation of commercial banks and the ‘market regulation’ of shadow banks can differ from each other. Furthermore, none of these papers provides explanations for the four empirical facts mentioned above. Finally, besides my paper only Gornicka (2014) clearly accounts for the wide-spread direct connection between commercial banks and shadow banks and the related contagion of credit risk. However, while Gornicka (2014) discusses implicit guarantees and the possible incentives for the sponsor to fulfil these, my model describes regulatory delays and fully committed sponsoring and it highlights the specific mechanism of multiplicative leverage, that only becomes possible by SPVs.

To sum up, the significant contributions of my paper are: First, it explains the empirical evidence about the coexistence of commercial banking and shadow banking, based on a discussion of differences and a possible interaction between banking ‘regulated by the market’ and banking regulated by an agency. Second, it demonstrates two dimensions of self-selection of depositors into coexisting types of banking. Shadow banking attracts the investors who have higher monitoring skills and who are more optimistic, commercial banking attracts the investors with lower monitoring skills or more pessimistic expectations about the economic development. Pessimistic intermediaries choose to operate as shadow banks, because they believe that their optimistic depositors bear part of the potential losses. And third, the paper provides a model of sponsored shadow banking, which highlights the concept of multiplicative leverage and explains why sponsored shadow banking is more profitable than and hence prevails over independent shadow banks. It is clearly identified as regulatory arbitrage, which becomes unprofitable if the regulation is improved.

The structure of the paper is as follows: Section 2 describes the model. Section 3 starts with a benchmark scenario of commercial banking and then discusses potential incentives for shadow banking. The possibility of heterogeneous beliefs about the aggregate risk is described in more detail. Section 4 analyzes the possible combination of both types of banking through SPVs. Section 5 discusses these results, before Section 6 concludes.

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14 In contrast to Ordonez (2013) and Freixas, Loranth, and Morrison (2007), the risk of the SPVs is really taken by the sponsoring bank and does not remain with the SPV in downturns, such that contagion as well as multiplicative leverage become possible. Luck and Schempp (2014) account for the sponsoring of shadow banking by banks, but discuss it with respect to liquidity transformations.
2 The Model

The timeline of the model is:

- At the beginning of the period:
  1. All agents form beliefs about the average default rates of risky projects in growth and downturn states. Consequently, a regulatory agency specifies capital requirements for commercial banks.
  2. Financial innovation occurs: 'Invention of shadow banking'.
  3. Financial intermediaries decide if they want to operate as commercial bank or shadow bank. Then, they take up deposits and use these and their own endowments to provide loans to borrowers with risky and risk-free projects.

- At the end of the period:
  The economy ends up in either a 'growth' or a 'downturn' state and (shadow) banks receive repayments from succeeding loans, from which they pay out their depositors. If a commercial bank is insolvent, the agency pays out its depositors.

Many convincing explanations have been given why depositors do not directly provide credit to borrowers but use financial intermediaries instead, see e.g. the overview in Freixas and Rochet (1997). Although many of these explanations could be added to this model, for instance the efficiency of centralizing the selection and monitoring of borrowers, the additional structure would not bring any new insights. Hence, the model will simply rely on these results and take the existence of intermediaries as given.

In order to emphasize how bank regulation by an agency and by the market can lead to alternative banking systems despite an equal set of information, it is assumed that both the agency and the depositors can obtain by their monitoring activity the correct information about the prices of risky and riskless loans, their volumes on the bank balances, the level of equity in each bank, and the operational costs.

In contrast to many banking models, the assets are not anonymous projects, but explicit borrowers who want to finance projects and have a demand for loans depending on their price. This enables a more comprehensive analysis of the effects of shadow banking.
Two remarks concerning notation:
For the sake of conciseness, the different variations of \( r \) and \( R \) always denote the total revenue of loans, deposits or activities, in contrast to their frequent use as denoting returns in expressions of revenues like '1 + \( r \)' and '1 + \( R \)'.
Wherever \( p_d \) and \( p_g \) are not further specified by subscripts, they refer to the beliefs of the agent that is introduced in that section or whose optimization problem is solved in that appendix.

2.1 The Borrowers

The key characteristic of borrowers is that their loan demand decreases with increasing interest rate and that the default risk depends on the aggregate state of the economy. Inspired by the example of companies, borrowers have limited liability\(^{15} \) and run projects with decreasing returns to scale which yield in case of success \( \hat{A}L^{1-\alpha} \) and 0 in case of failure, where \( L \) is the loan size and \( \alpha, \hat{A} \) represent the production technology and other input factors. In order to account for heterogeneity concerning project risk and to study the portfolio choices of intermediaries, there are risk-free as well as risky projects. While the risk-free projects always succeed, the success probability \( p \in [0, 1] \) of a risky project is \( p = p_g \) in a 'growth' state \( g \), which occurs with probability \( \omega < (0, 1) \), and \( p = p_d < p_g \) in a 'downturn' state \( d \) that has probability \( 1 - \omega \). Facing the price \( R \) for a unit of loan, a borrower with risky project chooses \( L \) such that the expected utility

\[
E[u_B] = \left( \omega p_g + (1 - \omega)p_d \right) u_B(\pi L) + \left( 1 - \omega p_g - (1 - \omega)p_d \right) u_B(0),
\]

with

\[
\pi_L = \hat{A}L^{1-\alpha} - L R_R
\]

is maximized, where \( u_B(x) \) is the utility derived from a profit \( x \) with \( u'_B > 0 \). The optimal demand \( L_R \) for credit is \( L_R^* = [(1 - \alpha)\hat{A}]^{\frac{1}{\alpha}} R_R^{\frac{1}{\alpha}} \). Since \( \frac{du_B(L_R^*)}{dR_R} < 0 \), borrowers benefit from lower rates \( R_R \) and the correspondingly larger \( L_R^* \). The expected average repayment \( R_R \) depends on the aggregate state: \( R_R \in \{ p_g R_F, p_d R_R \} =: \{ R_g, R_d \} \), which means \( E[R_R] < R_R \).

The borrower with risk-free projects solve the similar problem of choosing \( L_F \) at given price \( R_F \) in order to maximize \( E[u_B] = u_B(\pi L) \) with \( \pi_L = \hat{A}L^{1-\alpha} - L F R_F \). Their solution reads \( L_F^* = [(1 - \alpha)\hat{A}]^{\frac{1}{\alpha}} R_F^{\frac{1}{\alpha}} \), and \( \frac{du_B(L_F^*)}{dR_F} < 0 \) holds true as well. The repayment of the risk-free loans is certain, \( R_F = R_F \).

There are continua of borrowers for both types and their aggregate loan demands \( L_R^d \) with

\(^{15}\)The stability of the results in case of alternative model choices is shown in Appendix A
\( \mathcal{P} \in R, F \) can be written, by use of appropriate constants\(^{16}\) \( A_{\mathcal{P}} \), as:

\[
L_{d_{\mathcal{P}}} = A_{\mathcal{P}}^{\frac{1}{2}} \mathcal{R}_{\mathcal{P}}^{\frac{1}{2}} \iff \mathcal{R}_{\mathcal{P}} = A_{\mathcal{P}} (L_{d_{\mathcal{P}}})^{-\alpha}
\]

(1)

### 2.2 The Depositors

Depositors are agents with an endowment \( W \) that they want to store until the end of the period. A depositor can choose between three possible storages: \( D_C \) denotes cash which has a fixed value, \( D_B \) denotes bank deposits whose yield \( r_B \) is insured\(^{17}\), and \( D_S \) is the investment in uninsured deposits which require the monitoring of the respective financial intermediary.

The analysis will follow the argument of Dewatripont and Tirole (1993, 1994) that banks are special, because the majority of their debt holders, i.e. the depositors, are dispersed and have weak monitoring skills. This idea is captured in a reduced form by the monitoring ‘costs’ \( m \) which reduce the utility that is derived from the yield \( r_S \) of uninsured deposits.

Depositors may have different levels of financial education or skills and this heterogeneity is represented in a simple way by \( m \in \{m, \overline{m}\} \) with \( m < \overline{m} \).

This modeling approach can also be understand in terms of institutions like credit rating agencies, which actually perform a crucial function concerning the monitoring of the shadow banking sector. Such institutions analyze the solvency of the intermediaries and provide easily understandable information about it, but they have to be paid for this service, either directly by the interested depositors or by the shadow banks themselves. The latter has the advantage of avoiding a free-rider problem. The required payment may be small \( (m) \), but it is certainly non-zero (and it is not smaller than \( m_A \), as argued in Section 2.4). In this interpretation, the share of depositors with \( \overline{m} \) represent people who either still have costs to understand and utilize the accessible information, or who are very skeptical about the assessment of the rating agencies and only trust in their own (costly) assessment.

Furthermore, it seems to be empirically plausible that deposits cater to stronger types of risk aversion than other claims like bonds or equity shares. To account for this demand for safe claims in a stylized way which makes the analysis tractable, I follow Gennaioli, Shleifer and Vishny (2013) in their modeling of depositors as infinitely risk-averse. (The stability of the results in case of an alternative modeling choice is shown in Appendix A.)

\(^{16}\)The parameter \( \hat{A} \) can depend on \( \mathcal{P} \), but it is merged in the overall coefficient \( A_{\mathcal{P}} \), anyway. In order to make the formulation more general, one could introduce different parameters \( \alpha_R \neq \alpha_F \) as well, but this would only make the computations more tedious without changing the qualitative results.

\(^{17}\)For the sake of simplicity the insurance is defined to be comprehensive. If there were an upper bound for the insurance of a single account and a depositor had endowments in excess of this bound, her relative cost to switch to uninsured deposits would be comparably low, since the bank deposits also require some monitoring or some inconvenient partitioning. The possibility of an upper bound is hence captured in a simple way by low costs \( m \) of switching for some depositors.
The utility function $u_D$ of a price-taking depositor is hence defined as

$$u_D = E[D_C + r_B D_B + (\min_{\Omega} \{r_S\} - m)D_S] = D_C + r_B D_B + (\min_{\Omega} \{r_S\} - m)D_S$$

with $D_C + D_B + D_S = W$ and $(D_C, D_B, D_S) \geq 0$.

If depositors use shadow banking as storage, they only value the repayment $r_S$ which the shadow bank can credibly pledge independent of the state $\Omega = \{g, d\}$ of the economy. They verify the credibility of this pledge by analyzing the balance sheets of the intermediaries based on their belief about $p_g$ and $p_d$.

The depositors maximize $u_D$ by choosing $D_C$, $D_B$ and $D_S$ given the market prices $r_B$ and $r_S$ and their beliefs about $p_g$ and $p_d$. The solution is simply to use the storage with the highest safe net revenue. If this highest revenue is offered by more than storage, the endowment is split between these possibilities. With $r_{net}^B = r_B$, $r_{net}^S = \min_{\Omega} \{r_S\} - m$ and $r_{net}^C = 1$, this reads:

$$\sum_{X \in \mathcal{X}} D^*_X = W \text{ with } \mathcal{X} = \{ X | r_{net}^X = \max \{ r_{net}^B, r_{net}^S, r_{net}^C \} \}$$

and $D^*_Y = 0$ for $r_{net}^Y < \max \{ r_{net}^B, r_{net}^S, r_{net}^C \}$ (2)

It is assumed that there is a continuum of depositors, and the aggregated demands for the different money-like claims are denoted as $D_{B,d}$, $D_{S,d}$ and $D_{C,d}$ respectively. The continuum consists of depositors with monitoring costs $m$ or $\bar{m}$ and the aggregate wealth of these two groups is denoted by $\mathcal{W}$ and $\bar{\mathcal{W}}$, and the overall sum by $\mathcal{W}_D = \mathcal{W} + \bar{\mathcal{W}}$.

### 2.3 The Financial Intermediaries

There is a continuum of price-taking financial intermediaries with endowments $K$. They are risk-neutral and hence the efficient bearer of the credit risk of the loans while they offer safe claims to the depositors and earn the spread between the average loan repayments and the deposit rates.

The financial intermediaries can decide if they want to operate as regulated bank or unregulated ‘shadow bank’. (The possibility to combine both modes by means of an SPV is discussed in Section 4). They calculate the maximal expected profit that is possible in both operational modes, $E[\pi_B]$, $E[\pi_S]$, and choose the mode with higher expected profit:

$$\max_M E[\pi_M] \text{ with } M \in \{ B, S \}$$

(3)

While idiosyncratic risks can be eliminated by diversification or risk-sharing, aggregate risk is inevitable and hence more interesting concerning solvency and regulation of the banking sector. This assessment is stressed for the case of shadow banking/securitization by Gennaioli, Shleifer and Vishny (2013). It is hence assumed that intermediaries can
perfectly diversify, which could either be justified by the large size of their balance sheets compared to single loans or by the exchange of securities as in Gennaioli, Shleifer and Vishny (2013).

A financial intermediary hence expects the average, state-dependent repayment \( R_R \in \{ R_g, R_d \} := \{ p_g R_R, p_d R_R \} \) on the risky loans, given its belief about \( p_g \) and \( p_d \).

A **commercial bank** which offers the amounts \( L_F \) and \( L_R \) of risk-free and risky loans has to take up \((L_R + L_F - K)\) deposits. Due to limited liability, the bank cannot lose more than its endowment. The bank has operational costs \( c_B \) which are assigned to the size of deposits rather than the total size of loans, \( L_R + L_F \), for two reasons: First, the computations are more convenient while all qualitative results are the same. Second, credit intermediation is above all the transformation of deposits to loans, and in the limit of vanishing deposits, banks and shadow banks should be identical as they would only represent the direct investment of equity. Given these costs, the prices \( R_F = R_F, R_R \) and \( r_B \), and its belief about \( p_g \) and \( p_d \), the bank chooses \( L_F \) and \( L_R \) such that it maximizes its expected profit \( E[\pi_B] \) with

\[
\pi_B = \max \{ L_R R_R + L_F R_F - (L_R + L_F - K)(r_B + c_B) - K, -K \},
\]

subject to the regulatory constraint

\[
L_R \leq l_B K
\]

and subject to \((L_R, L_L, L_R + L_F - K) \geq 0\). The capital requirement \( l_B \) is specified by the regulatory agency which also provides the deposit insurance and whose decision problem is described in the next section. It does not set a requirement for \( L_F \), as it anticipates that the bank has no incentive to use deposits to invest in \( L_F \) if \( R_F < r_B + c_B \). Since \( R_F > r_B + c_B \) would lead to unbounded \( L_R \), any equilibrium solution is given by

\[
R_F = r_B + c_B \quad \text{and} \quad L_F^* \in [0, \infty). \tag{4}
\]

The banks make no profit with risk-free loans, but they simply use them to store the amount of deposits which exceeds their permitted investment in risky loans.

If \( l_B, R_d, r_B \) and \( c_B \) are such that \( l_B R_d \geq (l_B - 1)(r_B + c_B) \), the bank is prevented from deliberately choosing portfolios which lead to insolvency in downturns, and it simply

\[18\] They assign the possibility to diversify perfectly only to the shadow banking sector, but there is no reason why banks should not achieve the same diversification on their large balance sheets or by also using the technique of exchanging securities.

\[19\] There is the theoretical possibility of an equilibrium in which \( R_F < 1 + c_B \leq r_B + c_B \) as well as \( R_F < 1 + c_B \leq r_B + c_B \) and only the bank endowment is invested without use of any deposits, such that \( L_F + L_R = K \), but this extreme scenario will be excluded in the entire analysis by Assumption 1.
chooses maximal leverage as long as it can earn a spread on the credit intermediation:

\[ L^*_R = l_B K \quad \text{if } E[R_R] > r_B + c_B \]
\[ L^*_R = 0 \quad \text{if } E[R_R] < r_B + c_B \]
\[ L^*_R \in [0, l_B K] \quad \text{if } E[R_R] = r_B + c_B \]

(5)

However, if \( l_B R_d < (l_B - 1)(r_B + c_B) \), which allows for portfolios with insolvency in downturns, the optimal choice of \( L^*_R \) of the bank is modified such it will also choose

\[ L^*_R = l_B K \quad \text{if } \omega(l_B R_g - (l_B - 1)(r_B + c_B)) > \max\{E[R_R], R_F\} \]

(6)

The low capital requirements imply that the deposit insurance is covering some of the losses and provide an incentive for the bank to lever as much as possible even if the average revenue on credit is lower than its price, as long as the profit in the growth state and its probability is large enough. A brief derivation of (5) and (6) is given in Appendix B.

**Shadow banking** is described in this model in a simplified way as a possible operational mode of financial intermediaries that is represented by a single balance sheet. This is a useful reduction of the actual chain of financial firms that are involved in the credit intermediation of the shadow banking sector, since this paper focuses only on a particular dimension of the intermediation, the transformation of credit risk.

These representative shadow banks are very similar to banks, as they operate in the same markets and offer amounts \( S_F \) and \( S_R \) of risk-free and risky loans, which imply the demand \((S_R + S_F - K)\) for deposits. Shadow banks are not illegal, but only a business model of a firm that was ‘invented’ after the specification of the regulation, and they hence also have limited liability. This alternative business model, however, can have operational costs \( c_S \) that differ from \( c_B \). Given these costs, the market prices \( R_F = R_F, R_R \) and \( r_S \), and its belief about \( p_g \) and \( p_d \), the shadow bank chooses \( S_F \) and \( S_R \) such that it maximizes its expected profit \( E[\pi_S] \) with

\[ \pi_S = \max\{S_R R_R + S_F R_F - (S_R + S_F - K)(r_S + c_S) - K, -K\}, \]

subject to \((S_R, S_L, S_R + S_F - K) \geq 0\). There is no explicit regulatory constraint for shadow banks. But as they have no deposit insurance, their possible depositors will monitor their balance sheet and only count the value of \( r_S \) that can be guaranteed. They regard \( r_S \) as guaranteed if \( S_R p^\text{dep}_d R_R \geq (S_R - K)(r_S + c_S) \), where \( p^\text{dep}_d \) denotes the belief of the depositors which the shadow bank want to attract about \( p_d \). They realize that no intermediary has an incentive to incur losses from the risk-free projects. The requirement
of a guaranteed $r_S$ effectively imposes as a constraint that can be written as

$$S_R \leq l_SK \quad \text{with} \quad l_S := \begin{cases} \frac{r_S + c_S}{r_S + c_S - p_d^{dep}R}R & \text{for} \quad p_d^{dep}R < r_S + c_S \\ \infty & \text{else} \end{cases}$$

(7)

Note that this reformulation leads to an optimization problem for the shadow bank which has the same structure as the one for the commercial bank. The solution which is described in the equations (4), (5) and (6), and the associated remarks can be directly translated to the case of a shadow bank with the simple replacements:

$$l_B \rightarrow l_S, \quad c_B \rightarrow c_S, \quad r_B \rightarrow r_S, \quad L_R \rightarrow S_R, \quad L_F \rightarrow S_F.$$  

(8)

Having solved the optimization problem of both operational modes, the financial intermediaries choose the operation mode which yields the greatest expected profit. If the profit is the same in different modes, they are indifferent and there will be a coexistence of these modes. A coexistence can also occur if different intermediaries have different assessments about the most profitable mode (due to different beliefs $p_g$ and $p_d$).

Integrating $L_F + S_F$ and $L_R + S_R$ over the continuum of financial intermediaries yields the aggregate loan supplies $L^*_F$ and $L^*_R$. If $B$ and $S$ denote the subsets of financial intermediaries which decide to become a commercial bank or a shadow bank, the aggregate demands $D^*_B$ and $D^*_S$ for bank and shadow bank deposits are $D^*_B = \int_B L^*_F + L^*_R - K^j \ di$ and $D^*_S = \int_S S^*_F + S^*_R - K^j \ dj$. The aggregate endowments of all financial intermediaries are denoted as $K$.

### 2.4 The Regulatory Agency

The regulatory agency is set up by government\textsuperscript{21} and provides the deposit insurance, while it has the right to set and enforce regulations for the commercial banks in order to avoid moral hazard related to this insurance. The agency has sufficient financial means such that it can credibly commit itself to protect the deposits in all possible states. For simplicity, it is assumed that the government can impose lump-sum taxes at the end of the period if necessary. As explained in the Introduction, the discussion will focus on capital requirements which aim to minimize the insurance payments instead of the fair pricing of insurance premiums, which would be zero anyway if the agency specifies the regulation correctly\textsuperscript{22}.

It is assumed that the regulatory agency is at least as efficient in monitoring financial

\textsuperscript{21}In principle, the role of the agency could also be fulfilled by a private agency, but the government seems to be a more natural candidate, because it usually can promise financial support more credibly and it has greater legal power to enforce regulations.

\textsuperscript{22}Due to the discrete nature of the risk distribution the insolvency can be reduced to zero if the capital requirements are correctly set.
intermediaries as the depositors. This assumption could be justified, for instance, by its ability to hire the most skilled agents and to compensate them in a competitive labor market for their effort, while having a superior access to bank information due to its legal power. Consequently, the operational costs of the agency divided by the population of depositors is \( m_A \) with \( m_A \leq m < M \), and these costs are covered by a lump-sum tax paid by the depositors, for simplicity.

As motivated in the Introduction, the regulator specifies the regulation at the beginning of the period and has to rely on its understanding of the optimization problem of a commercial bank at that point in time. It hence expects that the payout \( P_I \) of the deposit insurance in case of a downturn is:

\[
P_I = \max \left\{ 0, \int_B E \left[ \max \{ (L_R^j + L_F^j - K^j)(r_B + c_B) - L_R^j R_d - L_F^j R_F, 0 \} \right] \right\}.
\]

Only downturns are relevant, as the revenues in growth states are larger and it can never be optimal for a bank to become insolvent and to rely on the deposit insurance in all states. Moreover, knowing that it can never be optimal for banks to incur losses from the risk-free loans, it only has to set \( l_B > 0 \) and to enforce \( L_R^j \leq l_B K^j \) for all \( j \in \mathcal{B} \).

The agency can minimize its expected payments (and set \( P_I = 0 \)) by specifying \( l_B \) depending on the market prices \( R_d, c_B, r_B \) and its belief about \( p_d \) according to

\[
l_B \leq \frac{r_B + c_B}{r_B + c_B - R_d}
\]

for \( R_d < r_B + c_B \), while there is no need for any \( l_B < \infty \) if \( R_d \geq r_B + c_B \). It is assumed that the agency does not want to restrict the agents more than necessary and chooses \( l_B \) such that the equality holds.

### 2.5 Equilibrium

**Definition:** The equilibrium is given by a set of prices \( \{r_B, r_S, R_F, R_R\} \) and the corresponding optimal choices according to (1), (2), (4), (6), (8), and (9) of all agents depending on their beliefs about \( p_g \) and \( p_d \), for which all markets clear:

\[
L_{R,d} = L_{R,s}, \quad L_{F,d} = L_{F,s}, \quad D_{B,d} = D_{B,s}, \quad D_{S,d} = D_{S,s}
\]

In the next section, equilibria with heterogeneous as well as homogeneous beliefs are discussed. In order to keep the description of scenarios with heterogeneous beliefs concise, it will be based on two types of possible beliefs about the average success rate \( p_d \) of projects in the downturn state, which is the parameter that is crucial with respect to regulation.

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23 It is assumed that the operational costs of the bank occur during the period and hence have priority to the deposit payments.

24 It is assumed that the demand for cash \( D_{C,d} \) is always satisfied by a sufficient supply of currency by the central bank.
Agents can have an optimistic belief $p^o_\text{d}$ or a pessimistic one $p^p_\text{d}$. In order to concentrate on the mechanisms of regulation, two cases will be excluded in which financial intermediaries take up so few deposits that they are always safe without any need for monitoring or regulation. This can happen either if the projects of the borrowers are so unproductive (small $A_R$, $A_F$ or $p$) that banks have small incentives to take up deposits in order to invest these, or if the depositors have little wealth in comparison to the banks, which means that the total wealth of all agents, $I_{\text{tot}} = K + W + W$, is not much larger than the endowments of the financial intermediaries. With $p := E[p] = \omega p_\text{d} + (1 - \omega)p^o_\text{d}$, the exclusion of these cases can be formalized as

**Assumption 1:**

$$0 < K < \frac{p^p_\text{d} - p^o_\text{d}}{p^p_\text{d}} \frac{1}{1 + \left(\frac{A_F}{p^p_\text{d} A_R}\right)^{\frac{1}{\alpha}}},$$

with $\hat{I} = \min \left\{ I_{\text{tot}}, \left(\frac{p^p_\text{d} A_R}{1 + \min\{c_B, m + c_S\}}\right)^{\frac{1}{\alpha}} + \left(\frac{A_F}{1 + \min\{c_B, m + c_S\}}\right)^{\frac{1}{\alpha}} \right\}$

This assumption implies that the regulator and the depositors care about a finite leverage of the intermediaries:

**Lemma 1:** For any equilibrium it holds true that: First, the regulator chooses a finite capital requirement $1 \leq l_B < \infty$ and the capital requirement $l_S$ which is effectively imposed on potential shadow banks by the risk aversion of the depositors is also finite, $1 \leq l_S < \infty$. Second, these constraints are binding for the optimal choice of a financial intermediary either operating as a commercial bank ($L^*_R = l_B K$) or as a shadow bank ($S^*_R = l_S K$).

The proof is given in Appendix C.

## 3 Commercial Banking vs. Shadow Banking

### 3.1 Benchmark Scenario and the Efficiency of Bank Regulation

In order to study the consequences of a possible emergence of shadow banking, first the benchmark scenario is presented, in which financial intermediaries can only operate as commercial banks. In this scenario the different beliefs have little importance, since the depositors can trust in the deposit insurance and the banks may have different expectations about their profits, but choose the maximal leverage $L^*_R = l_B K$ in any way, see Lemma 1. It is only the governmental agency which could incur losses in downturns in case that it has distorted beliefs.

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25 If it is optimal to operate as commercial bank with SPV, the optimal choice does not necessarily includes maximal leverage on both balances sheets.
Characteristics of the unique equilibrium

The characteristics depend on the relation between the endowments of depositors and the endowments of the financial intermediaries, which means the relation between \( I_{tot} = W_D + K \) and \( K \):

**Domain 1:** If depositors have relatively large endowments, which means

\[
\left( I_{tot} - \left( 1 + \frac{c_B}{A_F} \right)^{-\frac{1}{\alpha}} \right) \left( 1 + c_B - p_dA_R \left( I_{tot} - \left( 1 + \frac{c_B}{A_F} \right)^{-\frac{1}{\alpha}} \right)^{-\alpha} \right) \geq (1 + c_B)K ,
\]

the equilibrium is characterized by:

\[
r_B^* = 1 , \quad L_F^* = \left( 1 + \frac{c_B}{A_F} \right)^{-\frac{1}{\alpha}} \quad \text{and} \quad (1 + c_B - p_dA_R L_{R}^{*-\alpha}) L_R^* = (1 + c_B)K
\]

**Domain 2:** If depositors have comparably few endowments, which means

\[
\left( I_{tot} - \left( 1 + \frac{c_B}{A_F} \right)^{-\frac{1}{\alpha}} \right) \left( 1 + c_B - p_dA_R \left( I_{tot} - \left( 1 + \frac{c_B}{A_F} \right)^{-\frac{1}{\alpha}} \right)^{-\alpha} \right) \leq (1 + c_B)K ,
\]

the equilibrium is characterized by:

\[
A_F(I_{tot} - L_{R}^*)^{-\alpha}(L_F^* - K) - p_dA_R L_{R}^{*-\alpha} = 0 ,
\]

\[
L_F^* = I_{tot} - L_R^* , \quad \text{and} \quad r_B^* = A_F L_{F}^{*-\alpha} - c_B
\]

The probability \( p_d \) represents here the belief of the regulatory agency. In both domains the volume \( L_R \) of risky loans is only implicitly given, but it always has a unique positive value. The loan prices are accordingly given as \( R_p^* = A_P(L_p^*)^{-\alpha} \) for \( P \in \{ R, F \} \), which means they are continuously falling in increasing volumes. The transition between both domains is continuous. The derivation of this equilibrium is given in Appendix [D].

Figure 1: The equilibrium without shadow banking - the aggregate loan volumes \( L_R^* \) and \( L_F^* \), and the rate \( r_B^* \) on bank deposits depend on the ratio of the wealth of depositors to the total wealth \( w = \frac{W_D}{I_{tot}} = 1 - \frac{K}{I_{tot}} \). The graph depicts the dependency on \( w \) for a fixed \( I_{tot} \).

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26 Note that this second domain of parameters and possible solutions is restricted by Assumption 1.
Figure 1 illustrates the solution and shows the aggregate loan volumes $L^*_R$ and $L^*_F$, and the rate $r^*_B$ on bank deposits in equilibrium as a function of $w = \frac{W_D}{I_{tot}} = 1 - \frac{K}{I_{tot}}$ for a fixed $I_{tot}$, which means for varying levels of $W_D$ and $K$. If the depositors have large endowments (large $w$), its price is at the minimal level $r = 1$ and besides the maximal investment in $L_F$ (such that $R_F$ falls to $1 + c_B$) only a part of the remaining wealth is invested in risky loans due to little equity. This is a simple example of cash hoarding due to a demand for safe claims in excess of safe assets. If the relative wealth of the risk-averse depositors decreases in favor of endowments at the banks (decreasing $w$), more risky projects and more projects in total can be financed. If $w$ decreases to $w_c$ which is given by the $K$ in relation to $I_{tot}$ for which the equality in (11) holds true, the entire wealth $I_{tot}$ is invested and deposits become a scarce good and $r_B$ starts to increase. For a fixed level of endowments but larger bank equity, investment is shifted from the risk-free projects to the risky and more profitable projects.

Is the regulation of banks efficient?

While the efficiency of deposit insurance (which requires regulation in order to avoid moral hazard) has been shown with regard to liquidity transformations, see Diamond and Dybvig (1983), there can also be a justification of bank regulation with regard to credit risk transformation. It is based on the friction that the monitoring of financial intermediaries is costly for the depositors, in particular at least as costly as for a centralized agency. In order to understand if the introduction of a regulatory agency is a Pareto improvement, one can compare the benchmark scenario to a scenario without this agency, which means without explicit constraint $l_B$, but also without deposit insurance. This analysis is presented in detail in Appendix E including a description of the equilibrium without regulatory agency. The central result is that the transfer of monitoring to a regulatory agency is a Pareto improvement if deposits are relatively scarce. If they are relatively abundant, however, the skilled depositors (with $m$) can be better off in an unregulated market, because they can set themselves apart from the less skilled depositors. They can earn an interest rate premium while still being more attractive for banks than the less skilled depositors. The situation of the banks in this case depends on the specific parameters, while borrowers are always worse off without regulation since loans are more expensive due to increased funding costs of banks.

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27In that comparison, it is assumed that all agents have the same beliefs about $p_d$ and $p_g$, whereas Section 3.3 illustrates the comparison of unregulated and regulated banking in presence of heterogeneous beliefs.
3.2 The Possible Causes of Shadow Banking

**Proposition 1:** There are three possible reasons why financial intermediaries choose to engage in shadow banking, which means that in equilibrium some intermediaries choose \( S_R > 0 \) or \( S_F > 0 \) and consequently \( D_S > 0 \):

- **shadow banking has lower operational costs:** \( c_S < c_B \)
- **there are heterogeneous beliefs about the aggregate risk which allow for differences in the permitted leverage \( L_R/K \)**
- **shadow banking provides an alternative asset class \( L_S \)** (due to the possibility that intermediaries can invest into each other)

This Proposition directly follows from the observation that the profits of financial intermediaries, whether commercial bank or shadow bank, always have the form

\[
\sum_{a \in \mathcal{A}} L_a R_a - \left( \sum_{a \in \mathcal{A}} L_a - K \right) (r + c).
\]

where \( \mathcal{A} \) is the set of all possible assets/loan types, which means \( R, F \in \mathcal{A} \). Shadow banking can compete due to a lower \( c \), but never due to a lower \( r \) as it could not attract any depositors. The maximal leverage of profitable loans is restricted by \( l_B \) and \( l_S \) respectively, which both are decreasing in \( r_X \) and \( c_X \) (\( X \in \{B, S\} \)). The remaining parameter in \( l_B \) and \( l_S \) is the belief about \( p_d \) of the agency and of the potential shadow banking depositors respectively. The possibility that these two beliefs may differ is discussed in Section 3.3.

The possibility of alternative asset classes is strongly restricted, since there are only the two pools of risk-free and risky borrowers in the economy to which both types of intermediaries have the same access. Shadow banking can only make a difference and can allow for new types of assets, because it allows for loans between different kinds of financial intermediaries themselves. This possibility is discussed in Section 4.

**The simple case \( c_S < c_B \):**

The first and simplest case \( c_S < c_B \) shall only briefly be described here. In order to contrast it clearly with the other two possibilities, it is assumed in this subsection that there is no interaction between commercial banks and shadow banks and that all agents have the same belief about \( p_g \) and \( p_d \). In both operational modes the relative maximum of profit is obtained for maximal leverage, see Lemma 1. Taking the form of \( l_B \) and \( l_S \) into account, the maximal profit per \( K \) in both cases reads:

\[
\frac{\omega(R_g - R_d)(r_X + c_X)}{r_X + c_X - R_d} \quad \text{for} \quad X \in \{B, S\}
\]
Obviously, the financial intermediary chooses the sector with smaller \( r + c \), or it is indifferent between both if \( r_B + c_B = r_S + c_S \). There are three possible cases:

1. \( c_B - c_S < \overline{m} \): only commercial banks, since \( r_S + c_S > r_B + c_B \) (\( \approx \) benchmark)
2. \( c_B - c_S > \overline{m} \): only shadow banks, since \( r_B + c_B > r_S + c_S \) (\( \approx \) scenario without agency but \( c_B \rightarrow c_S \))
3. \( \overline{m} < c_B - c_S < \overline{m} \): coexistence of both types of banks

The third case describes the coexistence of commercial banks and shadow banks. Commercial banks offer \( r_B < r_S \) such that \( r_B + c_B = r_S + c_S \), but they can still attract the less skilled depositors (\( r_S - r_B < \overline{m} \)), whereas the more skilled depositors prefer shadow banks (\( \overline{m} < r_S - r_B \)). The relative size of the two banking sectors simple depends on the relative size of the endowments \( W \) and \( \overline{W} \) of more and less skilled depositors.

A more detailed description of the equilibrium in this case is given in Appendix \( \text{[F]} \) including a discussion who is better off and who is worse off with the emergence of this kind of shadow banking. The central result is that it is a Pareto improvement (with the depositors as main profiteers) for relatively scarce amounts of deposits, but not necessarily for relatively abundant deposits. In that case, the borrowers are the main profiteers, while the situation of financial intermediaries is ambiguous due to a trade-off between smaller funding costs and a decrease of the revenues \( R_B \) due to increased competition. Note that shadow banking of this kind is a Pareto improvement in the domain in which a regulation of banking is a Pareto improvement, too, as explained in Appendix \( \text{[E]} \). Thus, the regulation of shadow banking due to \( c_S < c_B \) could combine the improved cost-efficiency of the new intermediation technology with the advantage of centralized monitoring.

### 3.3 Heterogeneous Beliefs

In order to highlight the mechanisms of heterogeneous beliefs in this section, it is assumed that \( c_B = c_S \) and the possibility of SPVs as alternative assets is neglected. Since the differences between explicit regulation and ‘market regulation’ depend only on the beliefs about \( p_d \) (through \( l_B \) and \( l_S \)), but not on \( p_g \), the discussion will focus on differences in \( p_d \) only. As mentioned, it is assumed for the sake of a concise exposition that there are two types of beliefs, optimistic and pessimistic ones with \( p_o^d > p_p^d \). This model simply studies the effects of heterogeneous beliefs in a given period and remains agnostic about their causes. They could be justified in different ways, for instance in a Bayesian framework with different priors in which beliefs have not converged before the period in question, or by the realistic assumption of differences in the perception or interpretation of information combined with the belief in the superiority of one’s own assessment.

It is assumed that there are optimistic as well as pessimistic beliefs among all kinds of agents apart from the regulatory agency which has to choose a single capital requirement. The endowments of pessimistic and optimistic subgroups are denoted as \( K^p \) and \( K^o \) with
\( K = K^p + K^o \) and \( W^p \) and \( W^o \) with \( W_D = W^p + W^o \). Finally, it is assumed that \( p_d \geq \frac{1}{2} p_g \) for \( p_d \in \{ p^p_d, p^o_d \} \) in order to avoid some specific and lengthy, but economically uninteresting case distinctions concerning the size of \( m \).

The belief of the regulatory agency is crucial in this scenario. If the regulatory agency is optimistic, the equilibrium is the same as in the benchmark scenario with \( l_B \) based on \( p^o_d \). No intermediary changes to shadow banking as it does not allow for larger leverage, but has higher funding costs. The only difference to the benchmark is the possibility that the regulatory agency incurs losses in a downturn, because it has underestimated its severity and has imposed to small requirements.

In the alternative scenario with a less pessimistic regulatory agency that sets \( l_B \) based on \( p^p_d \), or more general on a belief \( p^A_d < p^o_d \), the shadow banking sector can be profitable for intermediaries if they find optimistic depositors who accept a larger leverage \( l_S > l_B \) while they can monitor the bank comparably easily and hence require only a relatively small compensation:

**Proposition 2:** In case of heterogeneous beliefs \( (p^o_d > p^p_d) \) there is always a unique equilibrium, and there are financial intermediaries which operate as shadow bank in this equilibrium if

\[
 m \leq (r^B_B + c_B) \left( \frac{(p^o_d - p^A_d)p_g + p^A_d(p_g - p^o_d)}{(p^o_d - p^A_d) \frac{r_B^B + c_B}{R^A} + p^A_d(p_g - p^o_d)} - 1 \right)
\]  

where \( p^A_d \) is the belief of the agency and \( r^B_B \) and \( R^A \) are the prices in the benchmark scenario. This equilibrium with shadow banks has the properties:

- only depositors with optimistic beliefs \( p^o_d \) provide funds for shadow banks
- \( S^*_p = 0 \), which means shadow banks focus on risky loans, and \( L_R \) increases due to shadow banking, whereas \( L_F \) can only decrease compared to the benchmark scenario

\[
\frac{d(S^*_p + D_S)}{dm} \leq 0 
\]
for \( m \in \{m, \bar{m}\} \), which means the size \( \frac{1}{r^*_S} D_S \) of the shadow banking sector increases with decreasing monitoring costs of depositors \( (\frac{1}{r^*_S} D_S = D_S + \text{share of } K \text{ invested through shadow banks}) \)

\[
\frac{d(S^*_p + D_S)}{d(p^o_d - p^p_d)} \geq 0 
\]
, which means the size of the shadow banking sector increases with an increasing disagreement \( p^p_d - p^o_d \) of the beliefs

- commercial banks and shadow banks coexist - with decreasing \( m \in \{m, \bar{m}\} \) and increasing \( p^o_d - p^p_d \) the pessimistic intermediaries switch to shadow banking first (first partly and then entirely) and only then the optimistic ones (partly) switch

The detailed description of this equilibrium and the proof of the Proposition are given in Appendix [G]. Many statements within this Proposition have already been motivated
above. Further interesting results are the explicit dependency of the size of shadow banking on the disagreement \( p_d^p - p_d^g \) of beliefs and the cost \( m \) of monitoring. Both parameters determine the potential profitability of shadow banking and they hence determine how many intermediaries change before the relative profits between both modes are equalized. It is slightly surprising at the first glance that pessimistic intermediaries are stronger attracted\(^{28}\) by shadow banking than optimistic ones. The reason is that both types of intermediaries have a similar relative evaluation of commercial banking and shadow banking, apart from the belief of pessimistic intermediaries that losses in downturns in shadow banking are partly borne by the optimistic depositors instead of themselves. They are hence attracted to shadow banking by the belief that they can exploit the optimism of their depositors.

The effects of shadow banking that emerges due to heterogeneous beliefs on the prices and quantities in equilibrium is depicted\(^{29}\) in Figure 2. As shadow banking can allow for larger leverage, an increase in \( L_R^* \) and even a possible reallocation from \( L_F^* \) to the more profitable risky projects is the direct consequence. Risky borrowers are the unambiguous profiteers of shadow banking in this scenario\(^{30}\). And while the situation of borrowers with

\[^{28}\]This means that there are parameter combinations \((m, p_g - p_d)\) such that pessimistic intermediaries engage in shadow banking while the optimistic ones do not, whereas the opposite case is not possible.

\[^{29}\]The Figure displays an example for which the parameters \((\mu, \pi\text{ and the different } c, p\text{ and } W)\) are respectively such, that the case denoted by c) in Appendix [G] holds true. This means that all pessimistic intermediaries operate as shadow banks while all optimistic ones operate as commercial banks.

\[^{30}\]Note that these borrowers do not change their behavior in presence of heterogeneous (and hence possibly wrong) beliefs about their success probability owing to the assumptions about their limited liability. However, these qualitative results remain true even with alternative assumptions as they are discussed in Appendix [A] for instance, as a possible increase in the loan demand by optimistic borrowers and a
risk-free projects does not change if there are abundant depositor endowments, they have to pay more for their loans if the endowments of depositors are relative scarce (small \( w \)). The effects on the depositors differs between the two types of belief: The situation of pessimistic depositors does not change if potential deposits are abundant (large \( w \)). If they are relatively scarce, however, the pessimistic depositors profit from shadow banking without any direct interaction owing to the general increase in investment and hence for funding which increases \( r_B \). The optimistic depositors seem to profit even more from shadow banking, if one regards the situation at beginning of the period. They are at least as valuable for the financial intermediaries as the pessimistic depositors and the intermediaries hence promise them (net) interest rates that are equal to or in many cases even larger than the already increased \( r_B \). This positive assessment remains true if the economy ends up in a growth state or if, at least, their optimistic beliefs are the downturn have been correct. But if they have been wrong and a downturn occurs which is more severe than expected, they incur losses which reduce their utility intensely. Since the emergence of shadow banking leads to stronger competition with decreases in \( R \) and increases in \( r_B \), the financial intermediaries which continue to operate as commercial banks (which tend to be the optimistic ones) have to face lower profits than in the benchmark case. Although the intermediaries that operate as shadow banks switch to this sector because they believe to earn higher profits there, it is again ambiguous, i.e. parameter-dependent, if the profit in this equilibrium actually exceeds the profit in the benchmark equilibrium. Finally, the regulatory agency is unaffected by the emergence and possible breakdown of shadow banking if it sets the regulation based on pessimistic beliefs and thus ensures that the commercial banks remain solvent even if the downturn is worse than some agents have believed. This neutral assessment of the scenario by the government would change, however, if it cared not only about its payment obligations, but also directly about the depositors which it wants to ”protect” (or about possible externalities of a breakdown of a large part of the banking sector, which are yet beyond the scope of this model). Since shadow banking in this scenario does not make either every agent better off nor every agent worth off under any convex combination of optimistic and pessimistic beliefs, its emergence is neither belief-neutral efficient nor belief-neutral inefficient according to the newly suggested welfare criterion for models with heterogeneous beliefs by Brunnermeier, Simsek and Xiong (2015).

possible decrease in the loan demand of pessimistic borrowers would neutralize each other on a aggregate level.
4 Sponsored Shadow Banking

The third and last possible reason for intermediaries to engage in shadow banking is that it provides a new profitable asset class beyond risk-free and risky loans to the borrowers. The case of commercial banks which invest in shadow banks will be studied in this section, while brief remarks about other directions of investment are given at the end of this section. As the return on deposits is always smaller than the return on direct loans, there is no incentive for commercial banks to provide deposits for shadow banks. But there may be an incentive to hold equity claims in shadow banking by setting up subsidiary shadow banks for which they provide binding guarantees to insure the deposits of the shadow bank. These subsidiaries will be denoted as SPVs (special purpose vehicles) and are modeled as being fully controlled by the sponsoring commercial bank, in accordance with the empirical facts about SPVs, see Gorton and Souleles (2006). The credit guarantees with recourse to the balances of the sponsoring bank are formally equivalent to a reassignment of loans (whose value is equal to the value of the guarantees) from the bank balance to the SPV where they constitute the equity tranche. This formulation in terms of equity tranches is more convenient and is hence used throughout.

The risky and risk-free loans in the SPV are denoted as $S_R$ and $S_F$ and the equity tranches hold by the sponsoring bank are denoted as $L_S$. Since these tranches are loans which are simply reassigned to provide recourse for the SPV, they are treated by the regulation like $L_R$, such that the capital requirement reads $L_R + L_S \leq l_B K$. This treatment represents the problem of the inadequate regulation of new types of risk transfer which have not been foreseen by regulators guided by an outdated business model of banking. (If $L_S$ were treated similar to $L_F$ instead of $L_R$, the effects would qualitatively be the same as the ones described in the following, they would only be stronger. The discussion here hence provides a lower bound for the effects of SPVs.)

Since SPVs are fully controlled by their sponsoring bank, the decision problem of a SPV can be integrated in the one of the commercial bank. The profit $\pi_{SPV}$ of a financial intermediary which operates as commercial bank but also holds $L_S$ equity tranches in a SPV read:

$$\pi_{SPV} = \max \left\{ L_R R_R + L_F R_F + L_S R_S - (L_R + L_S + L_F - K)(r_B + c_B) - K, -K \right\}$$

with $R_S = \frac{1}{L_S} \max\{(S_R + L_S)R_R + S_F R_F - (S_R + S_F)(r_S + c_S), 0\}$

In order to keep the discussion concise and to highlight the specific characteristics of sponsored shadow banking, the further analysis of SPVs will assume that all agents have the same beliefs about $p_g$ and $p_d$. The risk aversion of the depositors again imposes

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31 This formulation also implies the role of SPVs as safe harbors, which means that investors of SPVs are protected from a possible bankruptcy of the sponsor, and which is an important feature of SPVs according to Gorton and Souleles (2006).
the constraint $S_R + L_S \leq l_S L_S$ with $l_S$ as defined in (7), and it implies that $R_S = \frac{1}{L_S}((S_R + L_S)R_R + S.FR + (S_R+S.F)(r_S+c_S))$. The optimization problem of a commercial bank with SPV hence consists of the choice of non-negative $L_R, L_S, S_R, L_F$ and $S_F$ such that, given prices $R_R, R_F, r_B$ and $r_S$, beliefs $p_g$ and $p_d$, and costs $c_B$ and $c_S$, the expected profit $E[\pi_{SPV}]$ is maximized with

$$
\pi_{SPV} = \max \left\{ (L_R + L_S + S_R)R_R + (L_F + S_F)R_F - (S_R + S_F)(r_S + c_S) 
- (L_R + L_S + L_F - K)(r_B + c_B) - K, -K \right\}
$$

(15)

subject to the constraints

$$S_R \leq (l_S - 1)L_S, \text{ and } L_R + L_S \leq l_B K.$$

In spite of the linear structure of the problem, the solution is not trivial, but depends on some case distinctions. It is presented in detail in Appendix H together with an explanation of its derivation. Here, only the most relevant aspect of the solution shall be stated:

The commercial bank will only stay with its traditional business model and has no incentive to engage in shadow banking ($\rightarrow L_S = 0$) if the costs of the SPV are so much greater than the ones of commercial banks, such that $r_S + c_S > r_B + c_B$ implies $R_g < r_S + c_S$ while $E[R_R] > r_B + c_B$. This means: As long as the revenue in the growth state exceeds the costs ($R_g \geq r_S + c_S$), it is profitable for commercial banks to set up SPVs. The key point is that they allow for multiplicative leverage:

In contrast to the usual leverage of risky loans against the bank equity, the risky loans in the SPV can be levered against the equity tranche which can additionally be levered against the equity of the commercial bank. This construction is profitable for banks even if $E[R_R] < r_S + c_S$, as it yields levered revenues $l_S R_g - (l_S - 1)(r_S + c_S)$ in growth states while the loss of the equity tranche $L_S$ in downturns is not completely borne by the bank, but partly by the deposit insurance.

If the possibility of SPVs is taken into account, the choice of the financial intermediaries is extended, such that they choose the operational mode with the greatest profits
among operating as independent shadow bank (with $\pi_S$), as traditional commercial bank (with $\pi_B$) or as commercial bank with SPV (with $\pi_{SPV}$). Taking this additional option and its solution into account, the definition of the equilibrium remains the same as given in Section 2.5. The equilibrium in this scenario has the following characteristics:

**Proposition 3:** If commercial banks can set up SPVs, there is always a unique equilibrium, and $L_S > 0$ is chosen in this equilibrium (which means SPVs are actually set up) if

$$m \leq p_B R_R^b - r_B^b - c,$$

where $R_R^b$ and $r_B^b$ are the equilibrium prices in the benchmark scenario without shadow banking. The equilibrium with shadow banking has the following properties:

- $\pi_{SPV} > \pi_S$, which means it is always more profitable to engage in shadow banking through an SPV than by operating as an independent shadow bank
- $S_F^R = 0$, which means SPVs focus on risky loans, and $L_R^S$ increases due to shadow banking, whereas $L_F^S$ can only decrease compared to the benchmark scenario
- $d\left(\frac{L_s^R}{d_m}\right) \leq 0$ for $m \in \{m, \bar{m}\}$, which means the size $\frac{L_S^s}{r_S^1} D_S$ of the shadow banking sector increases with decreasing monitoring costs of depositors
- in downturns commercial banks become insolvent due to the contagion from their SPVs, $L_S^S R_S^d < (L_S^S - K)(r_B + c)$, and losses are borne by the deposit insurance

While some of these features directly follow from the solution of the optimization problem presented above, the proof of the Proposition is explicitly given in Appendix I. The effects of this kind of shadow banking on the prices and loan volumes are illustrated in Figure 3. The effects are similar than in the case of heterogeneous beliefs, but they are stronger and the actual impact on the agents is different: Again, the shadow banking sector restricts itself to risky loans and hence benefits the corresponding borrowers, while borrowers with risk-free projects may suffer from increased prices if funds are scarce and shifted to the risky projects. The depositors also gain from the emergence of SPVs and the subsequently increasing demand for funding and increasing interest rates (which are always paid owing to the deposit insurance and the shift of losses from the SPV to its sponsoring bank). The size of shadow banking increases if a larger share of the depositor endowments is hold by the depositors who are better in monitoring, and although banks have a strong incentive to set up SPVs, which are always more profitable than independent independent shadow banks (even if one allows for $c_S < c_B$ and heterogeneous beliefs), there is again a trade-off based on the adjusted expressions for the aggregated loan supplies and deposit demands: $L_F^R = \int_{B+SPV+S} L_F^R + S_F^R \, dt$, $L_R^S = \int_{B+SPV+S} L_R^S + S_R^S \, dt$, $D_B^F = \int_{B+SPV} L_F^R + L_R^S + S_R^S - K^F \, dt$, and $D_S^F = \int_S S_F^R + S_R^S - K^F \, dt + \int_{SPV} S_F^R + S_R^S \, dk$, where $S$, SPV and $B$ denote the subsets of financial intermediaries which decide to operate as independent shadow bank, or as commercial bank with or without SPV respectively.
between larger leverage and increased competition.\textsuperscript{33}

Figure 3: The equilibrium in a scenario with the possibility of SPVs - the aggregate loan volumes $L^*_R$ and $L^*_F$, and the rates $r^*_B$ and $r^*_S$ on deposits in this scenario are displayed in full colors, while the dotted graphs represent the results of the benchmark scenario. The graph depicts the dependency on $w = \frac{W^{D2}}{I^{tot}} = 1 - \frac{K}{I^{tot}}$ for fixed $I^{tot}$ and fixed $I^{tot} - \overline{W}$.

The multiplicative leverage, on which the profitability of SPVs relies, implies that the assets $L_S$ incur levered losses in downturns and are completely annihilated ($R_{S,d} = 0$) in case of the maximal leverage that is allowed by the shadow banking depositors. This contagion renders the levered sponsoring commercial bank insolvent, since the losses exceed the capital requirements which have been set in expectation of $R_R > 0$. The remaining losses are borne by the deposit insurance, and the regulatory agency, or consequently the taxpayers, are hence the agents who lose most due to the invention of SPVs.

The underlying problem is the delay between regulation and innovation. However, as soon as the regulatory agency becomes aware of new financial entities which have recourse to commercial bank balance sheets in an unprecedented way, there are different ways how the regulatory agency can react. It could simply ban the new entities, but it could also tolerate the financial innovation and simply impose a combined regulation $(l^S_{SPV}, l^A_S)$ of commercial banks and their SPVs, such that

$$\frac{1}{l^S_{SPV}} L_S + \frac{1}{l^B} L_R \leq K \text{ and } S_R \leq l^A_S L_S$$

are constraints for commercial banks and their SPVs. By choosing $(l^S_{SPV}, l^A_S)$ it can minimize the payout of the deposit insurance for commercial banks which set up SPVs, which

\textsuperscript{33}Financial intermediaries are better off with SPVs if the parameters $K$, $\overline{W}$, $\omega$, $c$, $p_d$, $p_g$, $A_R$, $A_F$, $\alpha$ are such, that $\omega(R^+_b - R^+_{g}) + \frac{\omega}{R^+_b - R^+_{d}} - \omega \frac{R^+_b + c}{R^+_b - R^+_{d}} (r^+_b + c) > \omega(R^+_g - R^+_{d}) \frac{r^+_g + c}{R^+_g - R^+_{d}}$, where the superscript $s$ denotes the prices in the SPV equilibrium and $b$ denotes the prices in the benchmark equilibrium.
is given as

\[ P^{SPV}_I = \max \left\{ 0, \int_B E \left[ \max \{ (L^j_R + L^j_S + L^j_F - K^j) (r_B + c_B) - L^j_R R_d - L^j_S R^j_{S,d} - L^j_F R_F, 0 \} \right] d\lambda \right\} \]

with \( R^j_{S,d} = \frac{1}{L^j_S} \max \{ (S^j_R + L^j_S) R_d + S^j_F R_F - (S^j_R + S^j_F) (r_S + c_S), 0 \} \).

There is one-dimensional subspace of possible combinations \((l_{SPV}, l^A_S)\) which are optimal in the sense that the corresponding constraints (17) avoid any payouts and ensure \( P^{SPV}_I = 0 \). (The payouts to commercial banks which have no SPVs are minimized by setting \( l_B \) as described in Section 2.4.) Having this freedom of choice, the agency could consider to choose the combination within this optimal subspace which allows for the highest possible bank profit \( E[\pi^{SPV}] \), if the bank solves the optimization problem of maximizing \( E[\pi^{SPV}] \) as given in (15), subject to the constraint (17). A benevolent regulatory agency of this kind has to solve the problem

\[
\max_{l_{SPV}, l^A_S} E[\pi^{SPV}_S(l_{SPV}, l^A_S)] \quad \text{s.t.} \quad P^{SPV}_I(l_{SPV}, l^A_S) = 0, \tag{18}
\]

where \( P^{SPV}_I(l_{SPV}, l^A_S) \) is the payout of the insurance in a downturn given the optimal choices \((L^*_R, L^*_S, L^*_F, S^*_R, S^*_F)\) of commercial banks with SPV which face the constraints \( l_{SPV}, l^A_S \) and \( l_B \), with \( l_B \) determined as in Section 2.4. The solution yields:

**Proposition 4:** The combined regulation \((l_{SPV}, l^A_S)\) of commercial banks and their SPVs which allow them to achieve a profit \( E[\pi^{SPV}] \) which is as large as possible while avoiding payouts of the deposit insurance \((P^{SPV}_I = 0)\) is \( l_{SPV} = l_B \) and \( l^A_S = 1 \). This effectively implies a closure of SPVs, as they are not allowed to take deposits, but only to invest \( L_S \).

The brief proof is given in part b) of Appendix I. The implication of this Proposition is that the emergence of SPVs is an unambiguous sign of regulatory arbitrage. They can only be more profitable than commercial banks without SPVs, if they exploit the deposit insurance and increase their leverage by leveraging against the implicit put of the agency. Note that the same consideration with \( c_S < c_B \) and the possibility \( r_S + c_S < r_B + c_B \) would imply \( l_{SPV} = 1 \) and \( l^A_S = l_S \) as the most favorable regulation for the intermediaries. It would be equivalent to an extension of the agency’s responsibility to shadow banks, and it would be equally profitable for intermediaries in that case to directly operate as independent shadow bank.

Finally, note that multiplicative leverage can only be exploited by the combination of a commercial bank with a subsidiary shadow bank. A risk transfer among shadow banks or the ‘sponsoring’ of a commercial bank by a shadow bank could be observed by the depositors in the markets which consequently would lower \( l_S \) sufficiently such that the equity of the sponsoring intermediary could absorb all losses from this activity. As a result, finan-
cial intermediaries could lever on two connected balance sheets, but the total product of the leverage would not be larger than in a single intermediary. The profitability of SPVs which belong to banks relies on the fact that regulatory specification precede the market activity. Whether a sponsoring between commercial banks, as the last combination, is possible would depend on whether the regulatory agency accepts guarantees by other banks as equity or whether they realize that such sponsoring allocates excessive risks to other banks that it insures.

5 Discussion

The policy implications of this analysis depend on the actual cause of shadow banking. There is no reason why it should be restricted, if it really emerges only due to lower operational costs. In order to clarify this, it would be useful to empirically identify its costs in comparison to the ones of commercial banks. However, even if this reason holds true, the extension of the regulatory framework to this kind of banking could still be a Pareto improvement due to the centralization of the monitoring, hence taking advantage of both the new technology and powerful legal institutions.

The scenario of SPVs stresses the inevitable problem of a time lag in the regulation of financial intermediaries, but it also provides a clear indicator for regulatory agencies to take action. As SPVs or any type of sponsored shadow banking are a strong sign of distorted incentives and regulatory arbitrage, the agency should forbid any new type of such 'innovation' as soon as it becomes aware of its growth. Otherwise, the taxpayer may have to incur large losses, only for the benefit of risky borrowers and increased deposit rates.

The scenario with heterogeneous beliefs provides an important explanation for the emergence of two alternative banking systems. The policy implications, however, are difficult. In the framework of this model, the regulatory agency and the government could be neutral towards the emergence of shadow banking, because some depositors may just suffer from their deliberately taken action. Only a paternalistic government would like to protect depositors from possible mistakes. The crisis, however, has shown that the government apparently cannot commit not to provide deposit insurance to shadow banking ex-post. This was not only due to the responsibility which the government felt for the savings of the depositors, but it was also due to externalities which arise from the breakdown of a significant part of the financial markets and which are beyond the scope of this model.

If the society is actually in favor of protective policies, which implies that the government effectively extends deposit insurance to shadow banks, it should regulate them in the same way as commercial banks in order to avoid costs for the public, but it then has to accept to be paternalistic towards optimistic investors.

34 It could consist of a combination of rules for the risk retention of the loan originating firm, for minimal haircuts on repos and for capital buffers of MMFs.

35 and which are in general hard to measure due to their complexity and even harder to compare to the cost of a bailout.
This paper has discussed the possible causes and the respective consequences of shadow banking due to differences in the transformation of credit risk and its regulation. The focus on this aspect of credit intermediation was chosen, because financial crises are usually triggered by problems in the credit risk transformation. Critical situations can then be further amplified by problems with maturity and liquidity transformations, like (shadow) bank runs and fire sales. An interesting extension of the analysis in this paper can hence be the discussion of differences between commercial banking and shadow banking concerning these two transformations.

6 Conclusions

In contrast to previous discussions about the regulation of banks and possible evasive responses, this paper has studied why the explicit regulation by a centralized agency and the implicit constraints imposed by the market may differ with respect to the solvency of financial intermediaries. Furthermore, it provides an analysis of the economic consequences of the emergence of an alternative banking system and identifies who loses and who benefits from shadow banking.

Concerning credit risk transformation, there are three causes for shadow banking: First, improved cost-efficiency; second, heterogeneous beliefs about the aggregate risk which lead to a self-selection of optimistic depositors into shadow banking where they are served by intermediaries who tend to be pessimistic and try to exploit their optimism; and third, the sponsoring of shadow banking by commercial banks. Sponsored shadow banking is more profitable than operating as independent shadow bank, as it allows for multiplicative leverage due to a combination of two balance sheets. But it can lead to contagion from shadow banking to commercial banks. An adjustment of the regulation in order to avoid contagion renders the sponsoring unprofitable for commercial banks. All three scenarios are characterized by a self-selection of depositors with relative high skills to monitor financial intermediaries into shadow banking, while the less skilled depositors prefer the delegation of monitoring to the agency.

While shadow banking due to cost-efficiency can be a Pareto improvement, shadow banking due to one of the other two causes always makes some agents worse off. Whereas the borrowers with risky projects enjoy an increased loan supply, the borrowers with risk-free projects sometimes face a reduced supply. The depositors in general gain by shadow banking as it increases the demand for deposits, but if they choose shadow banking because of optimistic beliefs, they may strongly suffer from unexpected losses. The regulatory agency is mainly affected by the regulatory arbitrage of sponsored shadow banking as the deposit insurance has to cover losses from sponsoring. The situation of financial intermediaries is ambiguous in most scenarios, as they profit from shadow banking due to enlarged leverage, but this leverage also decreases returns from loans and increases the cost of funding.
A  The Stability of the Results in Case of Alternative Formulations of Borrowers and Depositors

a) Alternative formulations of the borrowers:

The discussion of alternative formulations of borrowers and their decisions about debt financing is restricted to the possibility to obtain loans from banks. This paper wants to study the credit intermediation by banks and it hence relies on the fact that there are many firms and other borrowers who cannot access the bond or equity market, but have to rely on bank funding. See possible explanations see for instance Diamond (1991), Holmstrom and Tirole (1997), or Bolton and Freixas (2000).

First, the assumption of limited liability and the irrelevance of a possible default in the decision problem of a borrower shall be relaxed. If the borrower has some private wealth or income $y_w$ which has to be used to repay the loan if the project does not perform well, such that the loan only defaults if also the private liability is exhausted (for instance, if the borrower becomes unemployment in addition to the failed project), the optimization problem reads:

$$\max_L p_1u(y_w + AL^{1-\alpha} - R_R L) + p_2 u(y_w + \delta AL^{1-\alpha} - R_R L) + (1-p_1-p_2) u(y_{\text{min}})$$

with $\delta \in [0,1)$ where $y_{\text{min}}$ is a minimum consumption given for instance by social security. If one takes a CRRA utility function as example, determines the optimal choice $L^*$ and computes it derivative w.r.t. $R_R$, one finds that $\frac{dL^*}{dR_R} < 0$ and $\frac{d\mu(L^*)}{dR_R} < 0$ also holds true in this alternative description. These two features are the only relevant qualitative characteristics of the borrowers for the entire analysis.

The borrower are represented as owners of productive projects. The whole discussion, however, can also simply be extended to private borrowers who want to smoothen their consumption stream. If there are agents who borrow at the beginning of period to increase their income $y_1$ and want to repay it at the end, when they have $y_2 > y_1$ as long as they remain employed which has probability $p$. Their optimization problem reads $\max_L u(y_1 + L) + p\beta u(y_2 - LR_R) + (1-p)\beta u(y_{\text{min}})$, and assuming a CRRA utility function, for instance, one can again easily show that $\frac{dL^*}{dR_R} < 0$ and $\frac{d\mu(L^*)}{dR_R} < 0$.

Finally, there is also an alternative formulation which has the same properties as the model in this paper, but which is more similar to the usual formulation in finance in which there are possible investment possibilities with fixed returns. Assume that there are potential borrowers who have access to a project with relative yield $y$ such that they borrow if $R_R < y$, and the distribution of borrowers of such kind is given as $\rho(y) = \alpha A_R \frac{1}{R_R y^{-1-\frac{1}{\alpha}}}$ (which means that projects are the less frequent the more profitable they are). The aggregate loan demand then also reads $L_R = A_R \frac{1}{R_R^{-\frac{1}{\alpha}}}$.

The generalization of the revenue distribution from two points $R_g$ and $R_d$ to alternative description is discussed in the following subsection about the depositors.
b) Alternative formulation of the depositors:

Depositors are described in a stylized way by an infinite risk aversion. If one alternatively assumes that their risk aversion is finite and can be expressed by a CRRA utility function with large, but finite \( \theta \), their optimization is given as

\[
\max_{D_C, D_B} \frac{1}{1-\theta} \left( \omega(D_C + rD_B)^{1-\theta} + (1 - \omega)(D_C + r_d(l)D_B)^{1-\theta} \right) \quad \text{s.t. } D_C + D_B = W
\]

where \( r \) is the promised repayment on deposits and \( r_d(l) \) is the revenue in downturns which depends on the leverage of the intermediary, since it could become insolvent such that \( r \) cannot be fully paid. It then reads \( r_d(l) = \frac{l}{l-1}R_d - c \). Note that \( D_B \) and \( r \) represent both types of deposits and the respective net interest rate, either at a commercial bank or shadow bank. The optimal choice of \( D_B^* \) is given by the implicit relation

\[
(r - 1)\omega(W + (r - 1)D_B^*)^{-\theta} + (r_d(l) - 1)(1 - \omega)(W + (r_d(l) - 1)D_B^*)^{-\theta} = 0
\]

One can then deduce that the interest rate which banks have to pay to attract depositors increases with its leverage (beyond the point of ensured solvency) according to

\[
\frac{dr}{dl} = \frac{R_d}{(l-1)^2} \left( \frac{W + (r - 1)D_B}{W + (r_d(l) - 1)D_B} \right)^{\theta} \frac{1 - \omega}{1 - \theta D_B} \frac{1 - \theta D_B (r_d(l) - 1)(W + (r_d(l) - 1)D_B)}{(W + (r - 1)D_B)}
\]

Given a finite, but yet relatively large risk aversion, i.e. large \( \theta \), this derivative is dominated by the term \( \left( \frac{W + (r - 1)D_B}{W + (r_d(l) - 1)D_B} \right)^{\theta} \) for any non-zero probability of (partial) insolvency, which implies \( r_d(l) > r \).

The profit \( \pi = lE[R_R] - (l - 1)(r + c) \) of the intermediary then changes with its leverage according to \( E[R_R] - (r + c) - \frac{dr}{dl} \). It has no incentive to restrict its leverage exactly such that it always remains solvent, but due to the strong increase of \( \frac{dr}{dl} \) beyond this point for large \( \theta \), there is a maximal leverage close this benchmark at which a further increase becomes too costly. Thus, also finite risk aversion imposes a leverage restriction on intermediaries similar to the stylized description in the model.

Finally, it is easy to see that a continuous distribution of revenues on risky projects instead of \( R_d \) and \( R_g \) does not change the qualitative results, either. While the intermediaries only care about the average revenue \( E[R_R] \) anyway, it can only make a difference concerning the behavior of the depositors or regulator. In the case of infinite risk aversion, they would only be interested in the lower bound of the distribution as alternative to \( R_d \). And in the relaxed description of finite risk aversion, the argumentation would be completely analogous to the one in subsection a) above, with the only difference that all terms which include \( r_d(l) \) would then appear in an integral over the distribution of \( R_d \).
B The Optimal Choice of a Commercial Bank

The commercial bank chooses $L_R$ and $L_F$ given $c_B$, $\mathcal{R}_F = R_F$, $\mathcal{R}_R$, $r_B$, and its belief $p_g$ and $p_d$, such that it maximizes its expected profit

$$E[\pi_B] = E[\max\{L_R R_R + L_F R_F - (L_R + L_F - K)(r_B + c_B) - K, -K\}]$$

$$= \begin{cases} \\
L_R E[R_R] + L_F R_F - (L_R + L_F - K)(r_B + c_B) - K & \text{if } L_R R_d + L_F R_F \geq C_B \\
\omega(R_g - (r_B + c_B)) & \text{if } L_R R_g + L_F R_F \geq (L_R + L_F - K)(r_B + c_B) > L_R R_d + L_F R_F \\
-K & \text{if } C_B > L_R R_g + L_F R_F \\
\end{cases}$$

with $C_B = (L_R + L_F - K)(r_B + c_B)$

subject to the regulatory constraint $L_R \leq l_B K$ and $(L_R, L_L, L_R + L_F - K) \geq 0$.

It never can be optimal to be in the third domain and to always incur losses. The variation w.r.t. $L_F$ in the other two domains is $R_F - (r_B + c_B)$ and immediately leads to the solution stated in [4]. If $R_F < (r_B + c_B)$, the banks invest their initial endowment $K$ in $L_F$ only insofar as it does not yield a higher revenue in $L_R$.

The variation w.r.t. $L_R$ in the two remaining domains is

$$\begin{cases} \\
E[R_R] - (r_B + c_B) & \text{if } L_R R_d + L_F R_F \geq (L_R + L_F - K)(r_B + c_B) \\
\omega(R_g - (r_B + c_B)) & \text{if } L_R R_g + L_F R_F \geq (L_R + L_F - K)(r_B + c_B) > L_R R_d + L_F R_F \\
\end{cases}$$

If the regulation is such that $l_B R_d \geq (l_B - 1)(r_B + c_B)$ is enforced, the bank will always remain in the first domain, and the derivative $E[R_R] - (r_B + c_B)$ implies the solution stated in [5], including the maximal leverage $l_B K$ if $E[R_R] > (r_B + c_B)$. The bank will never leave the domain, because for $L_F = 0$ or $R_F = r_B + c_B$ it is in the domain for all $L_R \in [0, l_B K]$ due to the constraint $l_B R_d \geq (l_B - 1)(r_B + c_B)$. And if it left the domain via $L_F > 0$ and $R_F < r_B + c_B$, it would only lose, and it could not compensate for these losses by any choice of $L_R$. Due to the constant derivate in the second domain, it would either choose $L_R = 0$, which meant only losses from $L_F$ and no gains at all, or it would choose $L_R = l_B K$, which implies at most the same revenue on $L_R$ as in the first domain, because the constraint implies that the difference between first and second domain reads

$$(1 - \omega)(l_B K R_d - (l_B K - K)(r_B + c_B)) \geq 0.$$ 

If the regulation is weaker and $l_B R_d < (l_B - 1)(r_B + c_B)$ holds true, the bank has not bear all possible losses and the second domain could become optimal for the bank. The maximal leverage $L_R = l_B K$ is the relative maximum in this domain if $R_g > r_B + c_B$. If $E[R_R] > r_B + c_B$, this is the optimal choice in the first domain as well. But it can also be the absolute maximum for $R_g > r_B + c_B > E[R_R]$, if the related profit is higher than the maximal profit in the first domain in this scenario, which is the maximal revenue on an investment of equity only: $\omega(l_B K R_g - (l_B - 1)K(r_B + c_B)) > \max\{E[R_R], R_F\} K$. For the case $E[R_R] \leq r_B + c_B$, this relation already implies $R_g > r_B + c_B$. The boundaries
which specify the parameters for which \(L_R = l_B K\) is the optimal choice can hence be written concisely as it is stated in solution (6). The other two cases of the solution follows the usual logic.

Owing to the reformulation of the monitoring and the risk aversion of the depositors as the constraint \(S_R \leq l_S K\), the optimization problem of the shadow bank has the same structure as the one of the commercial bank, one only has to replace

\[
l_B \rightarrow l_S, \quad c_B \rightarrow c_S, \quad r_B \rightarrow r_S, \quad L_R \rightarrow S_R, \quad L_F \rightarrow S_F.
\]

The solution and its derivation are hence exactly the same as they have just been presented for the bank, taking these replacements into account.

C Relevance of leverage restriction

Note that both parts of this proof restrict possible equilibria to certain ranges of parameters. The proofs that a unique equilibrium in fact exists within these ranges in each of the discussed scenarios will be presented in the respective sections.

a) Proof of \(R_F = \min\{r_B + c_B, r_S + c_S\}\):

Assumption 1 also implies \(K < \hat{I} < \left(\frac{\hat{p}^A R}{1+\min\{c_B, m+c_S\}}\right)^{\frac{1}{\alpha}}\) and \(K < \left(\frac{A_F}{1+\min\{c_B, m+c_S\}}\right)^{\frac{1}{\alpha}}\), which can be written as

\[
A_F K^{-\alpha} > 1 + \min\{c_B, m + c_S\} < \hat{p}^A R K^{-\alpha}.
\]  

(19)

Since \(r_B = 1\) is the minimal interest rate in case that there is little demand for deposits, this relation ensures that financial intermediaries always engage in credit intermediation, because even if they already invested all their equity in one of the two project classes, the spread between their revenue and the cost of deposits would still be positive and hence motivate credit intermediation.

This result also excludes the corner solution mentioned for instance in [4], \(L_F^* = K_F\) for \(R_F < r_X + c_X\) with \(X \in \{B, S\}\). \(L_F^* = 0\) cannot be an equilibrium solution since \(R_F \rightarrow \infty\) then, and \(K_F > 0\) only for \(E[R_R] \leq R_F < r_X + c_X\) and with vanishing deposit demand, but this contradicts [19]. From Assumption 1 hence follows that in every equilibrium solution

\[
R_F = \min\{r_B + c_B, r_S + c_S\}.
\]

This statement remains true, even if there is \(L_S\) as third investment possibility which increases the demand and hence the prices for deposits. This possibility, however, if it is really used by the banks, depends on taking deposits. If their price were higher than the revenue on \(L_F\), the bank would gain by shifting funds from \(L_F\) to replace deposits,
such that $L_F = 0$ would be chosen, which yet cannot be an equilibrium. Hence, $R_F = \min\{r_B + c_B, r_S + c_S\}$ also applies in this scenario.

b) Proof of Lemma 1:

Note that $l_B$ and $l_S$ cannot be smaller than one by construction, since $R_d \geq 0$ in all cases. Assume that there could be an equilibrium with either $l_B = \infty$ or $l_S = \infty$, which means that no regulation is applied to any of the two sectors. It then must hold that $E[R_R] < R_g \leq \min\{r_B + c_B, r_S + c_S\} = R_F$ for any type of belief which underlies $E[R_R]$, since otherwise some financial intermediaries would choose an infinite leverage trying to achieve infinite profits, which cannot be an equilibrium. Taking the optimized behavior of the borrowers into account, it follows that

$$p^p A_R \mathcal{L}_R^{-\alpha} = E^p [R_R] < R_F = A_F (\mathcal{I} - \mathcal{L}_R)^{-\alpha} \Rightarrow \mathcal{L}_R \geq \frac{1}{1 + \left(\frac{A_F}{p^p A_R}\right)^{\frac{1}{\alpha}}} \mathcal{I}, \quad (20)$$

where $\mathcal{I}$ denotes the total level of investment. If $r_B > 1$ or $r_S - m^i > 1$ for each depositor $i$, the depositors will hold no cash and hence the total wealth $\mathcal{I}_{tot}$ has to be invested. If $r_B = 1$ and $r_S - m \leq 1$, not all wealth is invested, but the relation $\min\{1 + c_B, 1 + m + c_S\} \leq \min\{r_B + c_B, r_S + c_S\} = R_F = A_F (\mathcal{I} - \mathcal{L}_R)^{-\alpha}$ holds true. (The cases $r_B < 1$ and $r_S - m < 1$ are not relevant since there would be no deposits, and a scenario without any deposits would contradict part a) of this Section.) Taking $(20)$ into account, it follows that

$$\mathcal{I} \geq \left(\frac{A_F}{1 + \min\{c_B, m + c_S\}}\right)^{\frac{1}{\alpha}} + \left(\frac{p^p A_R}{1 + \min\{c_B, m + c_S\}}\right)^{\frac{1}{\alpha}}$$

Combining both possibilities leads to:

$$\mathcal{L}_R \geq \frac{1}{1 + \left(\frac{A_F}{p^p A_R}\right)^{\frac{1}{\alpha}}} \hat{\mathcal{I}} \quad \text{with} \quad \hat{\mathcal{I}} = \min\left\{\mathcal{I}_{tot}, \left(\frac{p^p A_R}{1 + \min\{c_B, m + c_S\}}\right)^{\frac{1}{\alpha}} + \left(\frac{A_F}{1 + \min\{c_B, m + c_S\}}\right)^{\frac{1}{\alpha}}\right\}$$

Assump.1 \quad $K < \frac{\mathcal{L}_R}{E^p [R_R]} \quad \Rightarrow \quad \mathcal{L}_R \geq \frac{E^p [R_R] - R_d^\alpha}{E^p [R_R]} \mathcal{L}_R$

$$\Rightarrow \mathcal{L}_R R_d^\alpha < (\mathcal{L}_R - K) E^p [R_R] \leq (\mathcal{L}_R - K) \min\{r_B + c_B, r_S + c_S\} \quad (20)$$

This relation, however, implies that either some depositors or the deposit insurance have to expect losses in downturns, even if they have optimistic beliefs. They would not accept that, which means that $E^p [R_R] > R_F = \min\{r_B + c_B, r_S + c_S\}$ has to hold in any possible equilibrium. And this is only possible with finite leverage restrictions. Furthermore, it implies that the optimal choice in a commercial bank or shadow bank mode is $L_R^* = l_B K$ and $S_R^* = l_S K$, at least in the sector with the smaller $r + c$. For the other sector, there are two possibilities in any possible equilibrium: First, if $R_d^\alpha > \max\{r_B + c_B, r_S + c_S\}$, this sector also has a finite leverage restriction, because otherwise some intermediaries
would choose infinite leverage. Or second, if \( R_g^a \leq \max\{r_B + c_B, r_S + c_S\} \), no intermediary has an incentive to operate there and a restriction is superfluous. However, if they tried, depositors would require a finite leverage as there are losses that have to borne.

### D Equilibrium in Benchmark Scenario

If there are only banks, \( \mathcal{L}_R^* = l_B K \) follows from Lemma 1 as well as a finite \( l_B \) which is chosen by the agency as \( l_B = \frac{r_B + c_B}{r_B + c_B - p_d K_R} \) depending on its belief about \( p_d \). The price \( R_R \) determines the borrower demand \( \mathcal{L}_R^d \) such that \( R_R = A_R \mathcal{L}_R^{d - \alpha} \). In equilibrium markets have to clear which means \( \mathcal{L}_R^* = \mathcal{L}_R^d = \mathcal{L}_R^s \). The combination of all this yields:

\[
(r_B^* + c_B - p_d A_R \mathcal{L}_R^{d - \alpha}) \mathcal{L}_R^* = (r_B^* + c_B) \mathcal{K}
\]

As shown in Appendix C, \( R_F = r_B + c_B \) has to hold in equilibrium and the clearing of the loan markets implies that \( R_F = A_F \mathcal{L}_F^{-\alpha} \), such that:

\[
A_F \mathcal{L}_F^{-\alpha} = r_B^* + c_B
\]

The third equilibrium equation is due to the deposit market and it is a case distinction. Deposits are only provided for \( r_B \geq 1 \). If \( r_B > 1 \), all endowments are stored at the bank and are invested, and if \( r_B = 1 \), the depositors are indifferent between cash and bank storage:

\[
\begin{align*}
\mathcal{L}_R^* + \mathcal{L}_F^* &= \mathcal{I}_tot & \text{for} \; r_B^* > 1 \\
\mathcal{L}_R^* + \mathcal{L}_F^* &= [0, \mathcal{I}_tot] & \text{for} \; r_B^* = 1
\end{align*}
\]

First, the possibility \( \mathcal{L}_R^* + \mathcal{L}_F^* = \mathcal{I}_tot \) and \( r_B^* > 1 \) shall be studied: The three equations can be combined to determine the solution for \( \mathcal{L}_R^* \), \( \mathcal{L}_F^* \) and \( r_B^* \), as stated in (13):

\[
\begin{align*}
A_F(\mathcal{I}_tot - \mathcal{L}_R^*)^{-\alpha}(\mathcal{L}_R^* - \mathcal{K}) - p_d A_R \mathcal{L}_R^{1 - \alpha} &= 0, \\
\mathcal{L}_F^* &= \mathcal{I}_tot - \mathcal{L}_R^*, \; \text{and} \; \mathcal{L}_R^* = A_F \mathcal{L}_F^{-\alpha} - c_B
\end{align*}
\]

The implicit expression for \( \mathcal{L}_R^* \) uniquely determines \( \mathcal{L}_R^* \) for the following reasons: The l.h.s. of the equation is \(-K A_F \mathcal{I}_tot^{-\alpha} < 0\) at the minimal possible value \( \mathcal{L}_R^* = 0 \) and it converges to \( \infty > 0 \) if it approaches the maximal possible value \( \mathcal{L}_R^* = \mathcal{I}_tot \). Since the l.h.s. is a continuous and differentiable function of \( \mathcal{L}_R^* \) between these points, there must be at least one root which means one solution for \( \mathcal{L}_R^* \). Being a continuous and differentiable function, there could be more than one solution only if there is at least one root at which the derivative w.r.t. \( \mathcal{L}_R^* \) is negative. The first derivative of the l.h.s., however, reads \( A_F(\mathcal{I}_tot - \mathcal{L}_R)^{-\alpha-1}(\alpha(\mathcal{L}_R - \mathcal{K}) + \mathcal{I}_tot - \mathcal{L}_R) - (1 - \alpha)p_d A_R \mathcal{L}_R^{-\alpha} \). Since \((1 - \alpha)p_d A_R \mathcal{L}_R^{-\alpha} < p_d A_R \mathcal{L}_R^{-\alpha} = A_F(\mathcal{I}_tot - \mathcal{L}_R)^{-\alpha} \) at any root, the derivative is positive.
at any root. As a consequence, there is only a single root and a single, positive solution for $L^*_R$.

Knowing that the derivative of the l.h.s. of the implicit expression for $L^*_R$ w.r.t. $L^*_R$ is positive, one can also calculate its derivative w.r.t. $K$, holding $I_{tot}$ fixed and taking into account that $L^*_R = L^*_R(K)$. One can conclude that

$$\frac{dL^*_R}{dK} \bigg|_{I_{tot}=\text{const.}} > 0$$

and due to the 2nd and 3rd equil. eq: $\frac{dr^*_B}{dK} \bigg|_{I_{tot}=\text{const.}} > 0$

These relations describes the response of $L^*_R$ and $r^*_B$ to a relative change in the distribution of the total wealth between depositors demanding money-like claims and banks providing equity, i.e. to a change of $w$ as depicted in Figure 1. The second derivative indicates that $r^*_B = 1$ is reached for a sufficiently small $K$ in comparison $I_{tot}$. For a certain $I_{tot}$ this specific $K_c$ is given as

$$K_c(1 + c_B) = \left( I_{tot} - \left( \frac{1 + c_B}{A_F} \right)^{-\frac{1}{2}} \right) \left( 1 + c_B - p_dA_R \left( I_{tot} - \left( \frac{1 + c_B}{A_F} \right)^{-\frac{1}{2}} \right)^{-\alpha} \right)$$

The parameter domain with $K < K_c$ (given a certain $I_{tot}$) is described by the second case $r^*_B = 1$ mentioned before. A combination of the three equations of this case directly leads to the solution for $L^*_R$, $L^*_F$ and $r^*_B$, as stated in (12):

$$r^*_B = 1, \quad L^*_F = \left( \frac{1 + c_B}{A_F} \right)^{-\frac{1}{2}}$$

and

$$\left( 1 + c_B - p_dA_R L^*_R^{-\alpha} \right) L^*_R - (1 + c_B)K = 0$$

Following the same logic as for the first case, it can be shown that the third expression uniquely determines $L^*_R$, since there is exactly one root. The only modification is that the interval of possible $L^*_R$ reaches from the minimal $\left( \frac{1 + c_B}{p_dA_R} \right)^{-\frac{1}{2}}$ to the maximal $I_{tot} - L_F$ this time. The l.h.s. at the maximal value, for instance, is with (21) equal to $K_c(1 + c_B) - (1 + c_B)K \geq 0$.

Taking derivatives on the l.h.s. one can also deduce that $\frac{dL^*_R}{dK} > 0$, as it is depicted in Figure 1. Since $L^*_F$ is fixed, it also means that $I = L^*_R + L^*_F$ increases with $K$ until $L^*_F + L^*_R = I_{tot}$. This holds true for $K = K_c$ as given in (24). This means the equilibrium solution is unique and continuous at the transition of the two domains.

E Is the regulation of banks efficient?

The equilibrium without regulatory agency and homogeneous beliefs about $p_g$ and $p_d$ is similar to the one in the benchmark scenario, but instead of two parameter domains there are four, as depicted in Figure 4. For a very large demand for safe storage (large $w$) even
the endowments $W$ of the relatively skilled depositors are not fully invested and $r_B = 1 + m$ (domain 1). The larger the relative size of bank endowments the more risky projects and the more projects in total are financed until $W$ is invested entirely and becomes scarce such that $r_B$ increases (domain 2). If $r_B = 1 + m$ is reached and one considers even larger sizes of bank equity, an increasing share of $W$ is stored at the bank and invested in risky projects (domain 3). Finally, if all wealth is invested and the relative share of deposits ($w$) decreases further, their price $r_B$ increases (domain 4).

Figure 4: The equilibrium in a scenario without regulation and deposit insurance - the aggregate loan volumes $L_R$ and $L_F$, and the rate $r_B$ on bank deposits in this scenario are displayed in full colors, while the dotted graphs present the results of the benchmark scenario. The graph depicts the dependency on $w = \frac{W_D}{I_{tot}} = 1 - \frac{K}{I_{tot}}$ for fixed $I_{tot}$ and fixed $I_{tot} - W$.

The solutions in domain 1 and 3 are very similar to (12), only with the replacement $1 \rightarrow 1 + m$ (domain 1) or $1 \rightarrow 1 + m$ (domain 3). The fourth domain is described by (13), and to describe the second domain one simply has to replace $I_{tot} \rightarrow I_{tot} - W$ in (13). The transition points are given by relations between $K$ and $I_{tot}$ similar to the benchmark scenario\[36\]. The proof follows the same logic as the one given in [D], just taking the mentioned replacements into account.

Figure 4 illustrates who is better off and who is worse off without regulatory agency compared to the benchmark of regulated banks. The forth domain is characterized by the same prices and quantities as the benchmark scenario. Thus, the situation is unchanged for all agents apart from the depositors who receive the same interest rate but are worse off due to the effort of monitoring which exceeds the alternatively paid tax $m_A$ for the agency. In this domain the regulation of banks is a Pareto improvement. In the first three domains, the borrowers are definitely worse off without regulation, as the relative increase

\[36\] To get the transition points one has to replace in the relation of the benchmark scenario: $1 \rightarrow 1 + m$ and $I_{tot} \rightarrow I_{tot} - W$ (domain 1/domain 2); $1 \rightarrow 1 + m$ and $I_{tot} \rightarrow I_{tot} - W$ (domain 2/domain 3); $1 \rightarrow 1 + m$ (domain 3/domain 4)
in the interest rate on deposits makes loans to risk-free as well as risky projects more expensive. For the less skilled depositors \((m = \bar{m})\), however, the larger interest can at most compensate the costs and they are better off only for the parameter range for which \(r_B < 1 + m_A\) in the benchmark scenario, since they have an effective rate of 1 and save the agency tax. The skilled depositors \((m = m)\) do not only save \(m_A\) in all three domains, but they also earn an interest rate 'premium' in domain 2 and 3, either because they can pool with the less skilled depositors and benefit from the spread \(\bar{m} - m\) (domain 3), or because they provide scarce funds when the less skilled depositors do not enter the market (domain 2). The situation of the banks is ambiguous: They have higher financing costs without regulation and the demand for loans is smaller, but the returns on these loans are higher due to decreased competition for borrowers. The bank profits change between both scenarios with increasing \(r_B\) proportional to 
\[
\frac{\alpha}{L_R} (r_B + c_B - (1 - \alpha) p_d A R_L) - \frac{p_d}{(r_B + c_B)^2}.
\]

To conclude, the transfer of monitoring to a regulatory agency is a Pareto improvement which makes the depositors better off, if there is only a moderate amount of demand for deposits. However, if many funds are available and the interest rate are just sufficient to attract depositors, the skilled depositors are better off without regulation as they can, for instance, pool with the less skilled ones and earn a premium in excess of their low monitoring costs. While the situation of the banks is ambiguous in this case, borrowers are worse off without regulation since loans are more expensive.

F Shadow banking due to \(c_S < c_B\)

This appendix describes the scenario of shadow banking which emerges due improved cost-efficiency \(c_S < c_B\), as it is described in the second part of Section 3.2. The exposition will focus on the most interesting case \(\bar{m} < c_B - c_S < m\) which implies the coexistence of commercial banks and shadow banks, and homogeneous beliefs about \(p_g\) and \(p_d\) are assumed. (In order to highlight the characteristics of this specific cause of shadow banking, it assumed that all agents have the same belief and SPVs are impossible in this subsection.) Figure 5 displays the prices and quantities in equilibrium depending on the relative share \(\frac{K}{K_{tot}} = 1 - W = 1 - w\), and again, there are four domains. Domain 3 and 4 are exactly the same as described in Domain 1 and 2 of the benchmark scenario, cf. (12) and (13). The only difference is that depositors using the shadow bank receive the interest rate \(r_S\) which is \(c_B - c_S\) larger than \(r_B\). They receive the entire surplus of the reduced costs (which they value as \(c_B - c_S - m\)) as long as their deposits are relatively scarce. This situation extends to domain 3 in which the \(\bar{m}\)-depositors become indifferent between deposits and cash while shadow bank deposits are still valuable. Thus, shadow banking which emerges due to \(c_S < c_B\) is a Pareto improvement for relatively scarce amounts of deposits, with skilled depositors being better off while all other agents are in the same situation as in the benchmark scenario. Note, however, that the regulation of this kind of shadow banking in
this parameter domain would maintain the benefits from the reduced costs while enabling a further Pareto improvement due to the mechanism described in the Appendix E. The domains 1 and 2, in contrast, depict the situation in which the endowments \( W \) are not scarce but large enough in comparison to the equity of the intermediaries to easily satisfy the demand for funding. Consequently, \( r_S \) is so small that commercial banks become relatively unprofitable and no \( m \)-deposits are needed. In this case the borrowers (providing relatively scarce investment possibilities) gain the surplus from the reduced costs, while the depositors are not better off as in the benchmark. The situation of the intermediaries in domain 1 and 2 is again ambiguous. Their profits change with decreasing funding costs proportional to 
\[
\frac{p_d}{(r_S+c_S)^2} - \frac{L_R-K}{L_R} A_R \ell_R^m (r_S+c_S - (1-\alpha)p_d A_R \ell_R^m).
\]
It hence depends on the parameters if the gains from smaller costs and larger portfolios are larger than the decrease in the revenues \( R(L_R) \) due to the increased competition.

The solution in domain 1 and 2 is given by the same equations as (12) and (13) with the replacements \( 1 \rightarrow 1 + m \), \( c_B \rightarrow c_S \) and \( I_{tot} \rightarrow I_{tot} - W \). The application of the same replacements to the domain border given in (11) yields the border between domain 1 and 2, while the border between domain 2 and 3 is obtained by only replacing \( I_{tot} \rightarrow I_{tot} - W \) in (11). The derivation of the solutions in the four parameter domains as well the description of the continuous transitions is analogous to the derivation in the benchmark case as described in Appendix D, one only has to account for the replacements concerning domain 1 and 2 as mentioned above.

![Figure 5: The equilibrium with both types of banks for \( m < c_B - c_S < m \) and homogeneous beliefs - the aggregate loan volumes \( L_R^* \) and \( L_F^* \), and the rates \( r_B^* \) and \( r_S^* \) on deposits in this scenario are displayed in full colors, while the dotted graphs represent the results of the benchmark scenario. The graph depicts the dependency on \( w = \frac{W}{I_{tot}} = 1 - \frac{K}{I_{tot}} \) for fixed \( I_{tot} \) and fixed \( I_{tot} - W \).](image-url)
G Equilibrium in the Presence of Heterogeneous Beliefs

Remember that \( c_B = c_S =: c \) is assumed in Section 3.3. Due to the monitoring costs \( r_B + c < r_S + c \) holds true and the risk-free loans are provided by the commercial banks only. Based on Lemma 1, in any possible equilibrium the financial intermediaries maximize their profit in either operational mode by choosing the maximally permitted leverage. The bank then expects the profit \( l(\omega p_g + (1 - \omega)p_d)R_R - (l - 1)(r + c) \) depending on its type and its belief. As explained, shadow banking can only emerge if the agency has not the optimistic belief but \( p_A^d < p_o^d \), and the shadow banks attract optimistic depositors. Taking the leverage restrictions (9) and (7) into account, the expected profits of the intermediaries read:

- optimist in commercial bank: \( \frac{(r_B + c)(\omega(R_g - R_d^A) + (1 - \omega)(R_o^d - R_d^A))}{r_B + c - R_d^A} \)
- optimist in shadow bank: \( \frac{(r_S + c)\omega(R_g - R_o^d)}{r_S + c - R_o^d} \)
- pessimist in commercial bank: \( \frac{(r_B + c)\omega(R_g - R_d^A)}{r_B + c - R_d^A} \)
- pessimist in shadow bank: \( \frac{(r_S + c)\omega(R_g - R_o^d)}{r_S + c - R_o^d} \)

Note that if the optimists prefer shadow banking or become indifferent between the two modes, the pessimistic intermediaries prefer it with certainty. There are five different cases:

a) all intermediaries operate as commercial banks
b) optimists operate as commercial banks while pessimists are indifferent between both types and operate as both
c) optimists operate as commercial banks and pessimists as shadow banks
d) pessimists operate as shadow banks and optimists are indifferent between both types and operate as both
e) all intermediaries operate as shadow banks

Shadow banking hence emerges if and only if

\[
\frac{(r_S + c)\omega(R_g - R_o^d)}{r_S + c - R_o^d} \geq \frac{(r_B + c)\omega(R_g - R_d^A)}{r_B + c - R_d^A}
\]

\[
\Leftrightarrow r_S - r_B \leq (r_B + c) \left( \frac{R_g(R_o^d - R_d^p) + R_d^A(R_o^d - R_d^p)}{(R_o^d - R_d^A)(r_B + c) + R_d^A(R_g - R_o^d)} - 1 \right)
\]

\[
= (r_B + c) \left( \frac{p_g(p_o^d - p_d^A) + p_d^A(p_o^d - p_d^A)}{(p_o^d - p_d^A)\frac{R_o^d}{R_R} + p_d^A(p_o^d - p_d^A)} - 1 \right)
\]

(25)
Starting from the equilibrium of the benchmark scenario as reference point, \( m \) is the smallest premium \( r_S - r_B \) that has to be promised to attract depositors to the shadow banking sector. Consequently, if \([14]\) holds true, the benchmark scenario is no longer an equilibrium and shadow banking emerges. Is there a new unique equilibrium and how does it look like? The equilibrium can be determined in a similar way as the one of the benchmark scenario, which means by \([22], [23]\), and by a modified version of \([21]\), which can be written as \( \mathcal{L}_R^* = \frac{r_B^* + c}{r_B^* + c - p_d^d A_R L_R^{* - \alpha}} K \). This modification depends on the case:

\[
\begin{align*}
b): \quad & \mathcal{L}_R^* = \left(1 - \gamma\right) \frac{r_B^* + c}{r_B^* + c - p_d^d A_R L_R^{* - \alpha}} + \gamma \frac{r_S^* + c}{r_S^* + c - p_d^d A_R L_R^{* - \alpha}} K^p + \frac{r_B^* + c}{r_B^* + c - p_d^d A_R L_R^{* - \alpha}} K^o \quad \text{with } \gamma \in (0, 1) \\
c): \quad & \mathcal{L}_R^* = \frac{r_S^* + c}{r_S^* + c - p_d^d A_R L_R^{* - \alpha}} K^p + \frac{r_B^* + c}{r_B^* + c - p_d^d A_R L_R^{* - \alpha}} K^o \\
d): \quad & \mathcal{L}_R^* = \frac{r_S^* + c}{r_S^* + c - p_d^d A_R L_R^{* - \alpha}} K^p + \left(1 - \gamma\right) \frac{r_B^* + c}{r_B^* + c - p_d^d A_R L_R^{* - \alpha}} K^p + \gamma \frac{r_S^* + c}{r_S^* + c - p_d^d A_R L_R^{* - \alpha}} K^o \quad \text{with } \gamma \in (0, 1) \\
e): \quad & \mathcal{L}_R^* = \frac{r_S^* + c}{r_S^* + c - p_d^d A_R L_R^{* - \alpha}} K \\
\end{align*}
\]

Additionally, one needs a relation for \( r_S \) to determine the equilibrium. It is given by a step-like function in \( D_S^d \). If the demand for deposits is smaller than the endowments \( W^o \) of optimistic skilled depositors, the funds can be attracted by just paying the necessary compensation \( m \) for switching. If the demand reaches \( W^o \), the competition for these deposits starts and increases \( r_S \) such that the demand is adjusted to equate \( W^o \). For \( r_S = r_B + m \) deposits of optimistic, but less skilled depositors can be attracted, too. If these are also fully used by shadow banks, the competition for the scarce funds again increases \( r_S \) such that the demand is adjusted to the maximal possible level.

\[
\begin{align*}
r_S^* &= r_B^* + m \quad \text{if } D_S^d < W^o \\
D_S^d (r_S^*) &= W^o \quad \text{if } D_S^d = W^o \\
r_S^* &= r_B^* + m \quad \text{if } W^o < D_S^d < W^o + W^o \\
D_S^d (r_S^*) &= W^o + W^o \quad \text{if } D_S^d = W^o + W^o
\end{align*}
\]

(27)

From the first line in \([25]\) it follows with \( R_g - R_d^o < R_g - R_d^A \) that for any equilibrium with shadow banking

\[
\frac{r_S + c}{r_S + c - R_d^o} > \frac{r_B + c}{r_B + c - R_d^A}
\]

holds true. This means that each term on the r.h.s. of \([26]\) is larger than the r.h.s. one case above, evaluated at the same prices. There are hence two possibilities: Either \( \mathcal{L}_R \) increases step by step from case a) down to case e) and with increasing \( \gamma \) within case b) and d), or the factor \( \frac{r_S + c}{r_S + c - R_d^o} \) or \( \frac{r_B + c}{r_B + c - R_d^A} \) decreases. The latter, however, is only possible for decreasing \( R_R \) which means increasing \( \mathcal{L}_R \), or for increasing \( r_B \) which is only possible for increasing \( \mathcal{L}_R \), too, according to \([22], [23]\). This argument applies for the
or if \( r_S \) deviate from this, the entire term \( \frac{r_S + c + m}{r_S + c - R_d} - 1 \) is \( k = D^k_S \) on the r.h.s. of (26), \( k \in \{ \gamma K^p, K^p, \gamma K^o + K^p, K^o + K^p \} \), is fixed and does not decrease (case 2 and 4 of (27)).

To sum up, the volume of risky loans \( L^*_R \) monotonously increases in the share of intermediaries which choose to operate as shadow bank.

Taking the benchmark as reference point again, financial intermediaries have an incentive to switch to shadow banking if (25) holds true. The r.h.s. of (25), however, decreases with increasing \( L^*_R \), as it can be shown by a straight-forward, but tedious computation [which can be provided on demand]. The l.h.s. of (25), in contrast, is either constant or increases continuously with a higher demand for deposits in shadow banks, i.e. with a larger share of intermediaries in this sector.

To conclude, if (14) holds true, shadow banking emerges and, having an increasing share of shadow banks from case a) down to e) including the \( \gamma \)-increases in b) and d), the new equilibrium is reached within the case at which the consequent increase in \( L_R \) has decreased the r.h.s. of (25) (and has increased its l.h.s.) such that the ‘<’ becomes an ‘=’ and no further intermediaries have an incentive to switch to shadow banking. To be precise: From case c) downwards, it depends on a modified version of (25) if more intermediaries switch to shadow banking, which means if also optimistic banks switch. This modified relation is based on \( \frac{(r_B + c)(\omega(R_g - R_d^1) + (1-\omega)(R_d^2 - R_d^3))}{r_S + c - R_d} \) on the r.h.s. in the first line. It can be solved for \( r_S - r_B \) similar to (25), and the resulting r.h.s. also decreases with increasing \( L_R \). Having determined to which case the scenario belongs, the exact description of the equilibrium is then implicitly given by the combination of the \( L_R \)-equation of the respective case, see (26), with (22) and (23). The proof of the existence, its uniqueness and the continuous transitions are then again analogous to the one in the benchmark.

### H Solution of the Optimization Problem with SPV

Given \( R_R, R_F, r_B, r_S, c_B, c_S \), and its belief about \( p_g \) and \( p_d \), a commercial bank with SPV chooses \( L_R, L_S, S_R, L_F, S_F \) to maximize \( E[\pi_{SPV}] \) with

\[
\pi_{SPV} = \max \left\{ (L_R + L_S + S_R)R_R + (L_F + S_F)R_F - (S_R + S_F)(r_S + c_S) - (L_R + L_S + L_F - K)(r_B + c_B) - K, -K \right\}
\]

subject to the constraints

\[
S_R \leq (l_S - 1)L_S, \quad \text{and} \quad L_R + L_S \leq l_B K.
\]
In the discussion of the SPV scenario, it is assumed that $c_B = c_S$ and all agents have the same beliefs, such that characteristic mechanism of the SPV can be highlighted. These two assumptions imply $r_S + c_S > r_B + c_B$ due to the compensation of monitoring costs. Furthermore, the Assumption (10) implies that $r_B + c_B = R_F$ holds true in all possible equilibria, cf. Appendix [C] for more details. As a consequence, $r_S + c_S > R_F$ and hence $S_F = 0$. The explicit form of $E[\pi_{SPV}]$ can be written as

$$E[\pi_{SPV}] = \begin{cases} 
(L_R + L_S + S_R)E[R_R] - C_{SPV} - K & \text{if } (L_R + L_S + S_R)R_d \geq C_{SPV} \\
\omega \left((L_R + L_S + S_R)R_g - C_{SPV}\right) - K & \text{if } (L_R + L_S + S_R)R_g \geq C_{SPV} > (L_R + L_S + S_R)R_d \\
-K & \text{if } C_{SPV} > (L_R + L_S + S_R)R_g 
\end{cases}$$

with $C_{SPV} = (L_R + L_S - K)(r_B + c_B) + S_R(r_S + c_S)$

The third domain in which the bank always loses can never be optimal. In the first domain, the bank will choose the maximal possible $S_R$ which is $S_R = (l_S - 1)L_S$ if $E[R_R] > r_S + c_S$, alternatively $S_R = 0$ if $E[R_R] < r_S + c_S$ and $S_R \in [0, (l_S - 1)L_S]$ if $E[R_R] = r_S + c_S$. In the second domain the same relations hold true with $R_g$ instead of $E[R_R]$. Note that $E[R_R] > r_S + c_S$ implies $R_g > r_S + c_S$. Focusing on the cases of maximal leverage first ($E[R_R] > r_S + c_S$ in the first domain, $R_g > r_S + c_S$ in the second), the expected profit in both domains respectively read:

$$(L_R + l_SL_S)E[R_R] - C_{SPV^*} - K \quad \text{for } (L_R + l_SL_S)R_d \geq C_{SPV^*}$$

$$\omega \left((L_R + l_SL_S)R_g - C_{SPV^*}\right) - K \quad \text{for } (L_R + l_SL_S)R_g \geq C_{SPV^*} > (L_R + l_SL_S)R_d$$

with $C_{SPV^*} = (L_R + L_S - K)(r_B + c_B) + (l_S - 1)L_S(r_S + c_S)$

Being restricted in the choice of $L_R + L_S$ by $l_B K$, the bank prefers to increase $L_S$ in favor of $L_R$, because $\frac{dE[\pi_{SPV}]}{dL_S} > \frac{dE[\pi_{SPV}]}{dL_R}$ holds true in both cases:

$$\frac{dE[\pi_{SPV}]}{dL_S} = l_R E[R_R] - (l_S - 1)(r_S + c_S) - (r_B + c_B) E[R_R] > r_S + c_S \quad E[R_R] - (r_B + c_B) = \frac{dE[\pi_{SPV}]}{dL_R}$$

$$\frac{dE[\pi_{SPV}]}{dL_S} = \omega(l_SR_g - (l_S - 1)(r_S + c_S) - (r_B + c_B)) > \omega(R_g - (r_B + c_B)) = \frac{dE[\pi_{SPV}]}{dL_R}$$

In the case $R_g > r_S + c_S$ but $E[R_R] \leq r_S + c_S$ while $E[R_R] > r_B + c_B$ there is a local maximum in each of the two domains. The maximum in first domain is $L_R = l_B K$, $L_S = 0$ and the relative revenue per equity that has to be hold against $L_R$ is $\omega(l_B E[R_R] - (l_B - 1)(r_B + c_B))$. For $R_g > r_S + c_S$, this is smaller than the corresponding revenue on $L_S$ per underlying $K$ at the second maximum $L_S = l_B K$, $L_R = 0$, which will be denoted by $R_S/K_S$:

$$R_S/K_S := \omega(l_S l_B R_g - (l_S - 1)l_B(r_S + c_S) - (l_B - 1)(r_B + c_B))$$
\[ E[R_R] \geq r_S + c_S \]

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<th>( R_g &gt; r_S + c_S )</th>
<th>( E[R_R] &gt; r_B + c_B )</th>
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<td>( 0 )</td>
<td>( \in [0, l_B K] )</td>
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Table 1: Solution of the optimization problem of a bank with SPV

Being the equity tranche of the SPV, \( L_S \) absorbs all losses in downturns and yields \( R_S = 0 \) for the case of the maximal leverage, but yields \( l_S R_g - (l_S - 1)(r_S + c_S) \) in growth states. For maximal leverage, \( L_S = l_B K \), the bank becomes insolvent in downturns, but has the revenues \( l_B R_S - (l_B - 1)(r_B + c_B) \) in growth states, which have the probability \( \omega \).

The argumentation so far justifies the first five lines of the solution stated in Figure 1. The only reason why the bank would choose a \( L_S^* < l_B K \) if the revenue on its endowment hold against \( L_S \) is smaller or equal to a direct investment in \( L_F \) which yields \( R_F \). This explains lines four and five.

If \( R_g = r_S + c_S \), the relative revenue \( R_S/K_S \) becomes \( \omega(l_B R_g - (l_B - 1)(r_B + c_B)) \) which is equal to relative revenue at the local maximum \( L_R = l_B K \) in the first domain that is present as long as \( E[R_R] > r_B + c_B \). In that case, given in line six, the bank is indifferent about how to choose \( L_R \) and \( L_S \) such that they add up to \( l_B K \). If \( E[R_R] = r_B + c_B \), an investment of \( K \) into \( L_F \) becomes equally profitable, see line seven, and for \( E[R_R] < r_B + c_B \) it even becomes optimal to only invest in \( L_F \), see line eight. In all these three cases the bank is indifferent about the size of \( S_R \) because of \( R_g = r_S + c_S \), as long as the expected profit is given according to the second domain of the solution, which means as long as \( S_R(r_S + c_S - R_d) \geq (L_R + L_S)(R_d - (r_B + c_B)) + K(r_B + c_B) \); \( S_R = 0 \) is an equally good choice in these two cases, but means that there is in face no SPV.

Finally, we can explain the lines nine to eleven of Figure 1. If \( R_g < r_S + c_S \), there is no maximum in the second domain, but there is the one at \( L_R = l_B K \) in the first domain for \( E[R_R] > r_B + c_B \), which is familiar from the optimization problem of a normal bank.

If \( E[R_R] \leq r_B + c_B \), however, it becomes equally or even more profitable to simply invest \( K \) in \( L_F \).

Note that the relations of prices which correspond to the cases given in lines five, eight, ten and eleven are theoretically possible, but they are not relevant in the analysis of possible equilibria in this article, as they are excluded by Assumption 1.
I Equilibrium with the Possibility of SPVs

a) Proof of Proposition 3:

Once again, the equilibrium of the benchmark scenario (with prices $R_R$ and $r_B$) will work as reference point. Remember that homogeneous beliefs about $p_g$ and $p_d$ and $c_B = c_s = c$ has been assumed in the section about SPV. Most of the statements in Proposition 4 follow from the solution of the optimization problem as it is given in Figure 4. In particular, it shows that a commercial bank sets up an SPV ($L_S^* > 0$) if

$$p_g R_R \geq r_S + c$$  \hspace{1cm} (28)

Since $r_S = r_B^h + m$ is sufficient to attract the first depositors to an SPV, commercial banks will no longer stay with the behavior in the benchmark scenario if (16) holds true.

It is also a straightforward conclusion that $\pi_{SPV} > \pi_S$ whenever SPV become profitable, as the SPV can operate exactly like an independent shadow bank, such that $R_S = \pi_S / K$.

The revenue $R_S$, however, can be levered once more against $K$, which is always profitable, since $R_S = \pi_S / K \geq r_S + c > r_B + c$ in any possible equilibrium in which there actually is an independent shadow bank ($S_R > K$), because a reduction of $S_R$ would otherwise save $r_S + c$ and would hence increase $\pi_S / K$.

Given maximal leverage $S_R = (l_S - 1)L_S$, the revenue $R_{S,d}$ on $L_S$ in downturns reads $(l_S R_d - (l_S - 1)(r_S + c))$ which is according to (7) equal to $(l_S R_d - \frac{R_d(r_S + c)}{r_S + c - R_d}) = 0$. It is then obviously $L_S^* R_{S,d} < (L_S^* - K)(r_B^* + c)$, leading to the insolvency of the sponsoring banks. This relation and the insolvency in downturns also hold true for cases six and seven as given in Figure 1, because only then the bank has an incentive to choose $S_R > 0$, as mentioned in Appendix 1.

While it was shown that the benchmark equilibrium no longer applies if (16) holds true, the unique equilibrium which emerges instead has still to be characterized. It can be determined in a similar way as in the benchmark scenario, and it is given by the combination of (22), (23), and a modified version of (21), which differs between the two possible cases: a) the financial intermediaries are indifferent between operating as commercial bank with SPV or as commercial bank without (characterized by $R_g = r_S + c$), and b) all intermediaries operate as commercial banks with SPV ($R_g > r_S + c$).

$$a): \mathcal{L}_R^* = \frac{r_B^* + c}{r_B^* + c - p_d A_R L_{R}^*} \frac{r_S^* + c}{r_S^* + c - p_d A_R L_{R}^* - \alpha} (1-\gamma)K + \frac{r_S^* + c}{r_S^* + c - p_d A_R L_{R}^* - \alpha} \frac{r_B^* + c}{r_B^* + c - p_d A_R L_{R}^* - \alpha} \gamma K \hspace{1cm} \text{with } \gamma \in (0, 1)$$

$$b): \mathcal{L}_R^* = \frac{r_B^* + c}{r_B^* + c - p_d A_R L_{R}^*} \frac{r_S^* + c}{r_S^* + c - p_d A_R L_{R}^* - \alpha} \frac{r_S^* + c}{r_S^* + c - p_d A_R L_{R}^* - \alpha} \gamma K \hspace{1cm} (29)$$

Furthermore, a specification of the equilibrium price $r_S^*$ is needed. Exactly the same logic applies that has been explained in Appendix 3, and it again provides the result (27) with

---

The fifth and eighth case in Figure 4 are not relevant, as they would imply that $\mathcal{L}_R = 0$ which cannot be an equilibrium.
the simple replacements $W^o \to W$ and $\mathcal{W}^o \to \mathcal{W}$.

Since $l_B^o > l_B$, the r.h.s. of (29) increases by switching from the benchmark case to a), by increasing $\gamma$ within a), and by switching from case a) to b) (evaluated at the same prices). This implies that $L_R^o$ increases along this ‘dimension’. The alternative possibility of decreasing factors $\frac{r+c}{r+c-R_d}$ can only be due to increasing $L_R$ as well, as explained in Appendix [G]. The effect of increasing $L_R$ on the two sides of (28) is opposite to each other: The l.h.s. is decreasing, while the r.h.s is constant or increasing.

The unique equilibrium can now be determined in the following way: If (16) holds true, SPVs emerge and, with an increasing share of SPVs following increasing $\gamma$ by increasing $l$. Since $\gamma > 0$ the simple replacements $W^o \to W$ and $\mathcal{W}^o \to \mathcal{W}$.

For these SPVs as well, the revenue of such commercial banks in case of maximal leverage reads $\omega l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$ so strongly that it becomes equal to the increased $r_B + c$ while (28) still holds true, then, the equilibrium is given by this alternative equality and the case within (29) for which this equality has been reached.

The unique equilibrium can now be determined in the following way: If (16) holds true, SPVs emerge and, with an increasing share of SPVs following increasing $\gamma$ and the switch from a) to b), the new equilibrium is reached at the case at which the consequent increase of $L_R$ has decreased the l.h.s. of (28) and has increased its r.h.s. such that the ‘$>$’ becomes an ‘$=$’ which means that no further intermediary has an incentive to set up an SPV. It is also an equilibrium, if b) is reached and ‘$>$’ still holds, since all intermediaries optimally operate as SPV then. There is one exception to this prescription: If the parameter in the economy are such that an increase of $L_R$ decreases the relative profit $R_S / K_S = \omega (l_B R_g - (l_B - 1)l_B (r_S + c) - (l_B - 1)(r_B + c))$ so strongly that it becomes equal to the increased $r_B + c$ while (28) still holds true, then, the equilibrium is given by this alternative equality and the case within (29) for which this equality has been reached.

The incentive that further intermediaries set up SPVs is gone in this case because it has become equally profitable to assign their endowment to investment in $L_F$.

Having determined to which case the scenario belongs, the exact description of the equilibrium is then implicitly given by the combination of the $L_R$-equation of the respective case, see (29), with (22) and (23). The proof of the existence and the uniqueness is then analogous to the one in the benchmark.

Finally, the increase of the size of the shadow banking sector due to decreasing monitoring costs $m \in \{m, m\}$ directly follows from the consequent decrease of the r.h.s. of (28), which determines the growth of shadow banking.

b) Proof of Proposition 4:

If the regulatory agency sets a capital requirement $L_S \leq l_{SPV} K$ for equity tranches $L_S$ in SPVs hold by commercial banks, and if it can impose a capital requirement $S_R \leq l_A^o L_S$ for these SPVs as well, the revenue of such commercial banks in case of maximal leverage reads $l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$ if payouts of the insurance shall be avoided, this revenue should be $\geq 0$ even in downturns, i.e. for $R_R = R_d$, and the capital requirements must hence be such that $l_{SPV} l_B^o \leq l_{SPV} (r_S - r_B) + r_B + c$. The expected profit in case of the least restrictive regulation of this kind (i.e. the equality binds) reads $\omega l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$, which determines the growth of shadow banking.

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If the regulatory agency sets a capital requirement $L_S \leq l_{SPV} K$ for equity tranches $L_S$ in SPVs hold by commercial banks, and if it can impose a capital requirement $S_R \leq l_A^o L_S$ for these SPVs as well, the revenue of such commercial banks in case of maximal leverage reads $l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$ if payouts of the insurance shall be avoided, this revenue should be $\geq 0$ even in downturns, i.e. for $R_R = R_d$, and the capital requirements must hence be such that $l_{SPV} l_B^o \leq l_{SPV} (r_S - r_B) + r_B + c$. The expected profit in case of the least restrictive regulation of this kind (i.e. the equality binds) reads $\omega l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$, which determines the growth of shadow banking.

b) Proof of Proposition 4:

If the regulatory agency sets a capital requirement $L_S \leq l_{SPV} K$ for equity tranches $L_S$ in SPVs hold by commercial banks, and if it can impose a capital requirement $S_R \leq l_A^o L_S$ for these SPVs as well, the revenue of such commercial banks in case of maximal leverage reads $l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$ if payouts of the insurance shall be avoided, this revenue should be $\geq 0$ even in downturns, i.e. for $R_R = R_d$, and the capital requirements must hence be such that $l_{SPV} l_B^o \leq l_{SPV} (r_S - r_B) + r_B + c$. The expected profit in case of the least restrictive regulation of this kind (i.e. the equality binds) reads $\omega l_{SPV} l_B^o R_g - (l_{SPV} l_B^o - 1)(r_S + c) + (l_{SPV} - 1)(r_B + c)$, which determines the growth of shadow banking.
and $t^A_S = 1$ would be most favorable one for commercial banks.

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