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Conference Paper

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2015: Ökonomische Entwicklung - Theorie und Politik - Session: Firms, credit and the cycle, No. D03-V2

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Wulff, Alexander; Heinemann, Maik (2015) : Idiosyncratic Risk, Borrowing Constraints and Financial Integration - A Discussion of Ambiguous Results, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2015: Ökonomische Entwicklung - Theorie und Politik - Session: Firms, credit and the cycle, No. D03-V2, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft

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Idiosyncratic Risk, Borrowing Constraints and Financial Integration - A Discussion of Ambiguous Results*

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May, 2015

Abstract

This paper examines the effects of financial market imperfections in the context of financial integration. We employ a general equilibrium model with heterogeneous entrepreneurs and address the question of cross-border capital flows from poor to rich as well as focus on aggregate capital accumulation, output and welfare. The motivation is based on the mixed results from the existing literature. We add to this debate by discussing the effects from a general perspective in an environment where entrepreneurs face capital risk, earn risky profits and receive riskless wage income. Moreover, borrowing constraints impede consumption smoothing as well as restrict the access to external funds for scaling up individual production. In order to separate the distinct effects, we consider several scenarios. In the first scenario without binding borrowing constraints and without persistent effects of shocks, we overcome the restriction that no analytical solution is available by deriving two rules of thumb, predicting the outcome of the model with high accuracy. These rules explain under which condition the less financially developed country features lower levels of capital and output in the autarchic steady state, implying capital flows from poor to rich, as well as under which condition an increase in the interest rate implied by financial integration unambiguously leads to larger levels of capital and output in the integrated steady state. For plausible parametrizations, we find that both conditions are likely to hold. Subsequently, we add tight borrowing constraints and increase the persistence of shocks. Both changes strongly affect the results derived in the first scenario.

JEL classification: D52; E22; F41; G11

Keywords: Incomplete markets; Borrowing constraints; Financial integration

*Financial support from the German Research Foundation (Deutsche Forschungsgemeinschaft) within the priority program 1578 "Financial Market Imperfections and Macroeconomic Performance" is gratefully acknowledged. The authors would like to especially thank Christiane Clemens, Marius Clemens, Frank Heinemann, Jean Imbs, Vahagn Jerbashian, Tom Krebs and Lutz Weinke for valuable comments and suggestions. We also thank various participants at the Royal Economic Society conference in Manchester, the ZEW conference in Mannheim, the ISNE conference in Galway, the DFG workshop in Konstanz, the Brown Bag Seminar at HU Berlin and the Potsdam Research Seminar. Any remaining errors are ours.

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1 Introduction

The topic of international integration has clearly become more important over the last decades. Among others, Prasad et al. (2006, 2007) and Mendoza et al. (2009a) document a persistent increase in the volume of cross-border capital flows reflecting the profound developments in the process of financial integration. However, existing discrepancies between the observed patterns on the one hand and the predictions from standard theory on the other hand have not been wiped away by the larger amount of capital flows and further striking empirical findings have emerged over time. In this regard, the so-called 'Lucas paradox' is one of the most prominent examples. In his influential paper, Lucas (1990) points out that quantities of capital flows from rich to poor countries are significantly smaller compared to the predictions of the frictionless, neoclassical model. In recent times, this paradox has even intensified. Prasad et al. (2006, 2007) show that since the end of the 20th century, the average income per capita of countries running current account surpluses has fallen below the average income per capita of countries running current account deficits, implying capital flows from poor to rich countries. Furthermore, in addition to the direction of capital flows, empirical observations on economic development in the light of capital outflows challenge conventional wisdom. While the neoclassical model predicts permanently lower levels of capital and output compared to the situation of financial autarky if the interest rate increases under financial integration, triggering capital outflows, Prasad et al. (2006, 2007) find periods of capital outflows from high-growth nonindustrial countries. Similarly, Sandri (2010) notes improvements in the current accounts of developing countries during periods of high per capita income growth. Furthermore, Gourinchas and Jeanne (2013) find a negative correlation between total factor productivity growth and net capital inflows to GDP for developing countries, calling this phenomenon the 'allocation puzzle'.

In recent times, financial market imperfections have attracted more attention in the context of financial integration. Mendoza et al. (2009a) emphasize that countries still differ significantly in the degree of financial development even though the integration process has intensified over time. However, regarding the implications for the direction of capital flows and the consequences for capital accumulation, output and wages, the results are very diverse and strongly vary with the respective specification of financial frictions and other model assumptions. This paper contributes to the debate by aiming at unifying essential parts of the different approaches and by adding new aspects to the discussion of financial market imperfections and financial integration. Our approach is related to the line of research including, among others, Gertler and Rogoff (1990), Matsuyama (2005), Aoki et al. (2009), Buera and Shin (2009), Mendoza et al. (2009a,b), Sandri (2010), Angeletos and Panousi (2011), Song et al. (2011), Clemens and Heinemann (2013) and von Hagen and Zhang (2014).

We apply a general equilibrium model with heterogeneous entrepreneurs and consider the effects of uninsurable idiosyncratic risk and borrowing constraints. Both types of frictions are widely studied in the literature. Our model economy features two sectors of production. The final good is produced under perfect competition, with intermediate goods and labor as input factors. In the intermediate goods sector, firms operate under monopolistic competition and each firm, producing a single intermediate good, is owned and managed by one entrepreneur. The economy is populated with a continuum $[0,1]$ of infinitely-lived households. Each household consists of one entrepreneur and is endowed with one unit of labor, being supplied inelastically at the perfectly competitive labor market. Entrepreneurs can invest in the own but in no other firm. They can trade a riskless bond but face a borrowing constraint. Idiosyncratic risk is introduced by stochastic fluctuations in entrepreneur's productivity, capturing different kinds of business risk. Since investment has to be

chosen before the shock is realized, entrepreneurs face capital risk on the individual level. Moreover, profits earned by the entrepreneurs fluctuate as well, constituting a risky income component. In the initial situation of financial autarky, the bond market has to clear on the country-wide level whereas in case of financial integration bonds can be traded on an international level. We take the perspective of a less financially developed and small country. The latter aspect means that the interest rate under financial integration is determined by the rest of the world, also denoted as the large country. However, this assumption is less restrictive since we show how differences in financial development translate into differences in steady state interest rates under financial autarky, being crucial to pin down capital flows. Finally, financial integration takes place without financial development and entrepreneurs cannot bypass the domestic borrowing restrictions in the case of financial integration.

Contrary to main parts of the literature, we do not apply a specific calibration strategy, but focus on the different implications of financial market imperfections from a general perspective. From an economic point of view, we discuss the macroeconomic effects which have to be expected in case of financial integration between countries with different levels of financial development. From a rather technical point of view, we show under which conditions financial frictions can be applied in order to explain parts of the empirical findings and how these conditions may change with different model assumptions. In sum, our research questions can be stated as follows:

- i.) Under which conditions does the existence of uninsured idiosyncratic risk lead to capital flows from less financially and initially poor countries to more financially developed and initially rich countries and how do these conditions change if entrepreneurs additionally face occasionally binding borrowing constraints and if shocks have persistent effects?*
- ii.) Considering these frictions, what are the plausible long-run consequences of financial integration on domestic capital accumulation, output and the wage and how do these predictions change between the different scenarios?*

In the first scenario, we only focus on uninsured risk, abstracting from binding borrowing constraints and from persistent effects of shocks. Given capital risk and profits constituting a risky income component, we find that for plausible parametrizations the less financially developed country is also the poor one in terms of capital and output in the steady state under financial autarky and capital does flow from poor to rich. Furthermore, from the perspective of the less financially developed country and for plausible parameter values, the increase of the interest rate implied by financial integration necessarily leads to larger levels of capital and output in the steady state under financial integration in comparison to the situation of financial autarky.¹ The main contribution in this scenario is the derivation of two rules, predicting the described outcome with a high degree of accuracy. In general, as in many other applications, analytical solutions are not available in our case, but building on Angeletos (2007) and Angeletos and Panousi (2011) as well as by starting with a simplified framework, we partly overcome this restriction. Similar to Angeletos (2007), the first condition describes the threshold level of the elasticity of intertemporal substitution where the less financially developed economy switches from over- to under-accumulation of capital in the autarchic steady state. Similar to Angeletos and Panousi (2011), the second rule describes the threshold level of the elasticity of intertemporal substitution from which on an increase of the interest rate above its autarchic steady state level necessarily leads to larger long-run levels of capital and output compared to the situation of financial autarky. Due to the flexibility of the approach used to derive these conditions, which we regard as rules of thumb, the procedure can

¹In the following, we also refer to the steady state under financial autarky (integration) as the autarchic (integrated) steady state.

also be applied to other model types considered in the literature.

In the next scenario, where solutions are solely derived from simulations, borrowing constraints, occasionally, bind. In our case, borrowing constraints affect the demand for the riskless asset as well as influence entrepreneur's risky investment decisions. On the one hand, borrowing constraints make it even more difficult for agents to smooth consumption and lead to an increase in aggregate demand for the safe asset. On the other hand, borrowing constraints restrict the access to external funds for scaling up individual production and, even if not currently binding, discourage risky investment. The latter effects dampen the upward trend of the aggregate capital stock associated with lower interest rates. In sum, the overall effect of borrowing constraints is generally ambiguous. However, we find that the drop of the interest rate needed to clear the bond market under financial autarky is the dominant effect and tight borrowing constraints lead to stronger restrictions regarding the elasticity of intertemporal substitution in order to observe lower levels of capital and output in the less financially developed country under financial autarky as well as to necessarily observe larger long-run levels of capital and output in case of financial integration. Especially, if differences in the tightness of borrowing constraints between countries are large, the critical values of the elasticity of intertemporal substitution are much bigger compared to the case without this dimension of financial market imperfections.

In the final scenario, we consider the case where shocks have persistent effects while keeping the unconditional variance at a constant level. Larger persistence leads to a further increase in the demand for the riskless asset and, consequently, to a further reduction of the autarchic steady state interest rate. In almost all exercises, we find that the existence of persistence alone leads to larger critical values of the elasticity of intertemporal substitution and, thereby, to stronger parameter restrictions compared to the situation where the current productivity level is unaffected by past developments.

From a welfare perspective, we find significant differences between the case where the capital stock, output and the wage eventually reach larger levels under financial integration compared to the situation of financial autarky and the case where levels of these variables are permanently lower under financial integration in comparison to the autarchic steady state. In the former case, we find robust welfare gains for the less financially developed country on the aggregate level.

The remainder of this paper is organized as follows. Section 2 briefly reviews the relevant literature and relates our approach to this line of research. Section 3 describes the model structure and section 4 describes the benchmark parametrization. Section 5 introduces the different scenarios and the derivation of the rules of thumb. Section 6 presents the transitory dynamics between the autarchic and the integrated steady state and discusses the welfare implications. Section 7 concludes and the appendix collects relevant proofs.

2 Literature review

The overview mainly focuses on some of the different results arising due to variations in the source of the underlying risk and in the specification of borrowing constraints. A detailed discussion of financial frictions and macroeconomic performance is, for example, provided by Brunnermeier et al. (2012). Gourinchas and Rey (2013) focus on several issues in the context of international integration.

The standard incomplete markets model, being widely studied in the literature, builds on the so-called income fluctuation problem (Heathcote et al., 2009; Ljungqvist and Sargent, 2012). The individual agent has to decide on an optimal consumption/savings path but faces stochastic fluctuations in the income process. The set of instruments to insure against the income risk is restricted

to a riskless asset and agents can only borrow up to an exogenous debt limit. This kind of problem is studied in various settings and is embedded into the general equilibrium model of a neoclassical production economy by Aiyagari (1994) where income fluctuations arise due to labor endowment shocks. As a consequence of the existence of uninsurable income risk and the borrowing constraint, savings increase and the interest rate in steady state is lower compared to the complete markets case where agents can completely insure against income fluctuations.² However, uninsurable income risk and borrowing constraints which do not directly affect the production side of the economy do not break the equality between the interest rate and the marginal product of capital. Consequently, the economy suffering from financial market imperfections features a lower interest rate and larger levels of capital and output in the steady state under financial autarky in comparison to the case of complete markets.

Applied to an open economy setting to study the welfare implications of financial globalization, Mendoza et al. (2009b) consider a general equilibrium model with heterogeneous agents facing uninsurable labor income risk and a borrowing constraint.^{3,4} Differences in financial development are captured by differences in the tightness of the borrowing constraint, and the stronger restrictions in the less financially developed country lead to a lower interest rate in the steady state under financial autarky. Consequently, if financial integration takes place, interest rates become equalized across countries and the less financially developed country builds up a positive net foreign asset position. However, if countries only differ in the tightness of the borrowing constraint or, alternatively, in the amount of uninsured risk, the less financially developed country is the rich and not the poor one in terms of capital and output in the initial autarchic steady state. Mendoza et al. (2009b) assume differences in productivity levels between countries which eliminate this effect. The increase in the interest rate from the perspective of the less financially developed country means a corresponding increase in the marginal product of capital. Consequently, while the country builds up its net foreign asset position, levels of capital, output and wage are necessarily lower compared to the situation of financial autarky. From a welfare perspective, the poorest agents in the less financially developed country are hurt twice; they suffer from the lower wage level as well as from the larger interest rate since they are borrowers in the autarchic steady state. Only the relatively rich agents in this economy gain from financial globalization.

Contrary to the assumption of fluctuations in labor income as the underlying source of idiosyncratic risk, Angeletos (2007) emphasizes the importance of investment or capital risk and augments the neoclassical growth model to study the macroeconomic consequences of market incompleteness.^{5,6} While assuming away binding borrowing constraints, Angeletos (2007) shows that the aggregate capital stock and output may be lower compared to the case of complete markets even though the interest rate is lower as well. The fundamental difference compared to income risk is that capital risk also affects the demand for investment, breaking the equality between the interest rate and the marginal product of capital and leading to an ambiguous overall effect on capital accumulation. The elasticity of intertemporal substitution is crucial to pin down the sign of the ultimate effect and, as argued by Angeletos (2007), the case where uninsured capital risk leads to lower levels of capital and output compared to the complete markets case is the relevant one for empirically plausible parametrizations. Focusing on the effects of financial integration in the presence of unin-

²See Huggett (1993) for similar results in an endowment economy.

³Contrary to Aiyagari (1994), production takes place on the individual level. However, since the allocations of production inputs are the same for all agents in the baseline model, the production side is similar to Aiyagari (1994).

⁴See also Mendoza et al. (2009a) where investment risk is additionally included but without aggregate capital accumulation.

⁵See Phelps (1962) and Levhari and Srinivasan (1969) for early discussions of the saving effect of risky returns and Sandmo (1970) for a comparison with income risk.

⁶See also Angeletos and Calvet (2005, 2006) for a discussion with CARA preferences and endowment risk.

surable idiosyncratic entrepreneurial risk, Angeletos and Panousi (2011) show that, similar to the case of income risk, the less financially developed country runs a series of current account surpluses along the transition from the autarchic to the integrated steady state.⁷ However, in this case and contrary to the case of income risk, the less financially developed country is also the poor one in terms of capital and output in the initial autarchic steady state, meaning that capital does flow from the less financially developed and initially poor country to the more financially developed and initially rich country. In other words, uninsurable capital risk may provide an explanation of the 'uphill' phenomenon, i.e. of capital flows from poor to rich countries. Regarding the joint dynamics of capital outflows and domestic economic development, the effects may be quite different to the case of income risk as well, at least in the long-run. Angeletos and Panousi (2011) show that the accumulation of wealth stimulates capital accumulation since agents' willingness to take risk increases. Depending on the elasticity of intertemporal substitution, this wealth effect may eventually dominate and leads to larger levels of capital and output in the integrated compared to the autarchic steady state. Clearly, this result also influences the distribution of winners and losers of financial integration.

Our approach is most closely related to Angeletos (2007) and Angeletos and Panousi (2011), but shares some features with the standard incomplete markets approach as well. With Angeletos (2007) and Angeletos and Panousi (2011) we share the feature that entrepreneurs face capital risk and receive riskless wage income. However, while their approach has the advantage of analytical tractability, we additionally consider the case with occasionally binding borrowing constraints as an additional dimension of financial market imperfections and the case with persistent effects of shocks. Furthermore, risky profits exist in our model.

Level effects of uninsurable investment risk within a closed economy are also considered by Covas (2006) and Meh and Quadrini (2006). Meh and Quadrini consider different risk-sharing environments where market incompleteness endogenously arises due to agency problems and study the welfare implications of improvements in insurance opportunities. In case where agents cannot perfectly insure against investment risk, they find lower levels of capital compared to the complete markets case. In contrast, Covas (2006) often finds over-accumulation of capital in his model, reflecting the sensitivity of the capital risk effect. Tight borrowing constraints and larger persistence of shocks widen the gap between the complete and the incomplete markets steady state capital stock even further. However, Covas abstracts from any type of riskless income component beyond the safe asset. During the discussion of our rules of thumb and the other scenarios, we will revisit parts of his results.⁸ Goldberg (2013) considers the dynamics of a credit crunch in a model with uninsured idiosyncratic investment risk and collateral constraints and shows that the response of the steady state aggregate capital stock to tighter constraints is generally ambiguous.⁹ However, depending on the tightness of the collateral constraint, firms in Goldberg's model can trade state-contingent promises, unlike our model where full insurance is not obtainable even if borrowing constraints do not bind.

The implications of risky returns are also widely studied in the context of long-term endogenous growth, either based on analytically tractable models, e.g., Devereux and Smith (1994), Obstfeld (1994), Krebs (2003), Clemens (2009) and/or based on numerical results, e.g., Jones et al. (2005). In all cases, the risk effect on growth rates is ambiguous and in most cases the critical value of

⁷See also Corneli (2009, 2010) for a related discussion as well as Sandri (2010).

⁸See also Covas and Fujita (2011) for a discussion of idiosyncratic and aggregate risk.

⁹He shows that whether the aggregate capital stock decreases in response to tighter constraints also crucially depends on the elasticity of intertemporal substitution.

the elasticity of intertemporal substitution equals unity.¹⁰ In our model, where entrepreneurs additionally receive riskless wage income, the critical value may be much lower, depending on the strength of the borrowing constraint and the persistence of shocks.

Our approach also relates to the literature emphasizing the role of financing constraints on the production side of the economy in order to explain capital flows from poor to rich countries and/or further empirical findings in the context of financial integration, e.g., Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2005), Aoki et al. (2009), Buera and Shin (2009), Song et al. (2011), Benhima (2013), Clemens and Heinemann (2013), and von Hagen and Zhang (2014).¹¹ Interestingly, as shown by Buera and Shin (2009) and Clemens and Heinemann (2010, 2013), financing constraints restricting the amount entrepreneurs can borrow in order to scale up production may help to overcome the result that the less financially developed country is the rich one in Aiyagari-type income risk models. Studying the joint dynamics of capital flows and total factor productivity, Buera and Shin (2009) embed entrepreneurship, occupational choice, idiosyncratic distortions and collateral constraints in an Aiyagari-type model and show that tighter collateral constraints lead to a lower interest rate and lower aggregate output in the steady state under financial autarky. Consequently, capital flows out from the poorer country with stronger financial frictions in case of financial integration. Moreover, Buera and Shin (2009) show that if free cross-border capital flows are accompanied by a reform removing idiosyncratic distortions, total factor productivity and, thereby, aggregate output increase significantly.¹² The model economy considered by Clemens and Heinemann (2010) also features entrepreneurship, occupational choice and collateral constraints and additionally has a vertically connected sectoral structure with capital accumulation in both sectors. Clemens and Heinemann (2010) show that tighter financial constraints lead to lower interest rates and reduce aggregate output even though the aggregate capital stock in the final good sector increases. Focusing on financial integration, Clemens and Heinemann (2013) further show that capital flows out of the less financially developed and poorer country, and that if credit constraints are sufficiently tight, aggregate output increases despite a reduction of the aggregate capital stock in the final good production. The adjustment of domestic output also strongly affects the distribution of winners and losers and Clemens and Heinemann (2013) show that the share of agents gaining from financial integration is the largest in case where aggregate output and wages increase.

In our case, tight borrowing constraints also restrict the access to external financing, but increase the demand for the riskless asset as well.

3 Model

3.1 Overview

In the final good sector, a large number of perfectly competitive firms produce a homogeneous good which can be used for consumption and investment. Input factors are intermediate goods and labor. In the intermediate goods sector, firms operate under monopolistic competition. Each firm is owned and managed by one entrepreneur and produces a single intermediate good.

The economy is populated with a continuum $[0,1]$ of infinitely-lived households. Each household consists of one entrepreneur and is endowed with one unit of labor supplied inelastically at the per-

¹⁰See Clemens (2009) for different critical values due to endogenous labor supply and labor income risk. See also Jones et al. (2005).

¹¹von Hagen and Zhang (2014) additionally distinguish between financial capital and foreign direct investments and von Hagen and Zhang (2011) compare the effects of limited commitment and incomplete markets.

¹²See also Buera and Shin (2011) for a discussion of the effects of increasing shock persistence, Buera et al. (2011) for a multi-sector analysis as well as Buera and Shin (2013).

fectly competitive labor market. In the following, we refer to the household and the entrepreneur interchangeably. Intermediate goods are produced with the help of capital invested by the corresponding owner. In addition, entrepreneur's productivity is subject to idiosyncratic shocks; a shortcut to capture different kinds of business risk.

Depending on the scenario, entrepreneurs suffer from two types of financial market imperfections commonly assumed in the literature: idiosyncratic risk is not completely insurable since the only financial asset available is a riskless bond and, in addition, entrepreneurs may face occasionally binding borrowing constraints. In general, financial frictions may arise due to many reasons, e.g., market incompleteness occurs in case of moral hazard or adverse selection. However, we abstract from deeper micro-foundations and focus on the consequences instead.¹³ The strength of the two frictions and, thereby, the level of financial development is captured by the unconditional variance of the underlying productivity process and the debt limit, respectively. A larger variance c.p. means a larger portion of risk that cannot be insured through financial markets and, thus, remains with the entrepreneurs. In other words, the variability of the productivity process measures the residual risk that rests on agents after using risk-sharing opportunities provided by the financial sector.¹⁴ A tighter constraint c.p. means a reduction in the amount entrepreneurs can borrow in comparison to the case with a less restrictive debt limit. Depending on the scenario, the less financially developed country is characterized by a larger amount of risk remaining with the entrepreneur and/or a more restrictive debt limit.

3.2 Final good sector

The representative firm produces the homogeneous good under perfect competition with labor, L_t , and intermediate goods, x_{it} , as inputs. Production takes place according to

$$Y_t = L_t^{1-\alpha} \int_0^1 x_{it}^\alpha di, \quad 0 < \alpha < 1. \quad (1)$$

The price of Y_t is normalized to unity so that profits are given by

$$\Pi_t^F = Y_t - w_t L_t - \int_0^1 p_{it} x_{it} di. \quad (2)$$

Optimization yields the standard result that each factor is paid according to its marginal product

$$w_t = (1 - \alpha) \frac{Y_t}{L_t} \quad (3)$$

$$p_{it} = \alpha x_{it}^{\alpha-1} L_t^{1-\alpha}. \quad (4)$$

3.3 Household/Entrepreneur sector

Technology and Budgets

The entrepreneur's risky technology is given by

$$x_{it} = \theta_{it} k_{it}, \quad (5)$$

where k_{it} is the physical capital stock and θ_{it} denotes entrepreneur's individual productivity. θ_{it} is assumed to be uncorrelated across agents but may be correlated over time. The worst possible

¹³See, for example, Meh and Quadrini (2006) where market incompleteness arises endogenously based on asymmetric information.

¹⁴See Angeletos and Calvet (2006) and Angeletos and Panousi (2011) for a more detailed discussion of modeling financial markets.

realization of θ_{it} means that x_{it} is (arbitrarily close to) zero.

The budget constraint is given by

$$k_{it+1} + b_{it+1} + c_{it} = p_{it}x_{it} + (1 - \delta)k_{it} + R_t b_{it} + w_t, \quad (6)$$

where b_{it+1} denotes safe investment and $R_t \equiv (1 + r_t)$ is the gross riskless interest rate. w_t denotes the wage and δ the constant depreciation rate. $p_{it}x_{it}$ describes the entrepreneur's income from selling the intermediate good at chosen price p_{it} . The monopolistic profit maximization problem is simple in this case since the capital stock is already installed at the beginning of period t . Consequently, the amount of the intermediate good produced and sold to the final good sector in period t is fixed after the realization of the individual productivity shock is observed. Using the demand function described in (4) and using (5), $p_{it}x_{it}$ can be expressed as $p_{it}x_{it} = \alpha \bar{L}_t \tilde{\theta}_{it} k_{it}^\alpha$ with $\tilde{\theta}_{it} \equiv \theta_{it}^\alpha$ and $\bar{L}_t \equiv L_t^{1-\alpha}$. The budget constraint simplifies to

$$k_{it+1} + b_{it+1} + c_{it} = \alpha \bar{L}_t \tilde{\theta}_{it} k_{it}^\alpha + (1 - \delta)k_{it} + R_t b_{it} + w_t. \quad (7)$$

The representation in (7) illustrates that the household essentially solves a consumption/ savings as well as a portfolio choice problem between a risky and a riskless asset.

Preferences

Each household has preferences over consumption and maximizes discounted expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}). \quad (8)$$

E_0 is the expectation operator conditional on information at date $t = 0$ and $0 < \beta < 1$ is the discount factor. Preferences display constant relative risk aversion

$$U(c) = \begin{cases} \frac{c^{1-\rho}}{1-\rho} & \rho > 0, \rho \neq 1 \\ \ln(c) & \rho = 1. \end{cases} \quad (9)$$

Let entrepreneur's period t net worth, ω_{it} , be defined as $\omega_{it} \equiv \alpha \bar{L}_t \tilde{\theta}_{it} k_{it}^\alpha + (1 - \delta)k_{it} + R_t b_{it} + w_t$. Given the deterministic sequence of prices and aggregate employment, $\{w_t, R_t, L_t\}_{t=0}^{\infty}$, the single entrepreneur's problem is described by the following program¹⁵

$$V_t(\omega_t, \tilde{\theta}_t) = \max_{b_{t+1}, k_{t+1}} \left\{ U(c_t) + \beta E \left[V_{t+1}(\omega_{t+1}, \tilde{\theta}_{t+1}) \mid \tilde{\theta}_t \right] \right\} \quad (10)$$

$$s.t. \quad c_t + b_{t+1} + k_{t+1} = \omega_t \quad (11)$$

$$\omega_{t+1} = \alpha \bar{L}_{t+1} \tilde{\theta}_{t+1} k_{t+1}^\alpha + (1 - \delta)k_{t+1} + R_{t+1} b_{t+1} + w_{t+1} \quad (12)$$

$$c_t \geq 0 \quad (13)$$

$$k_{t+1} \geq 0 \quad (14)$$

$$b_{t+1} \geq -\bar{b}. \quad (15)$$

The time subscript attached to the value function indicates that the program is not only defined at steady states. Agents may hit the borrowing limit defined in (15) in case when \bar{b} is more stringent than the 'Natural Debt Limit' (NDL; henceforth) specified in the next section.

The first-order conditions of the individual problem are given by

¹⁵The subscript i is dropped in this definition for notational ease.

$$U'(c_{it}) = \beta R_{t+1} E_t [U'(c_{it+1})] + \lambda_{it} \quad (16)$$

$$U'(c_{it}) = \beta E_t [R_{it+1}^r U'(c_{it+1})], \quad (17)$$

where λ_{it} is the nonnegative Lagrange multiplier associated with the borrowing constraint (15) and $R_{it+1}^r \equiv 1 - \delta + \alpha^2 \bar{L}_{t+1} \tilde{\theta}_{it+1} k_{it+1}^{\alpha-1}$ is the return of the risky asset, i.e. the gross return on capital. Combining the two equations yields

$$E_t R_{it+1}^r - R_{t+1} = -\frac{\text{Cov}(U'(c_{it+1}), R_{it+1}^r)}{E_t U'(c_{it+1})} + \frac{\lambda_{it}}{\beta E_t U'(c_{it+1})}. \quad (18)$$

Equation (18) describes the wedge between the expected return of the risky asset and the riskless interest rate, induced by uninsurable capital risk and potentially binding borrowing constraints. The first term on the right-hand side describes the risk premium demanded by the entrepreneurs for bearing the uninsurable investment risk. Since $\text{Cov}(U'(c_{it+1}), R_{it+1}^r)$ is negative, this expression is positive. The second term on the right-hand side additionally appears in case when the borrowing constraint binds in period t . Since λ_{it} is nonnegative, both terms positively contribute to the wedge and play a key role in the further analysis.

Before this section concludes with the definition of the general equilibrium under financial autarky as well as in the case of financial integration, we briefly revisit the entrepreneur's budget constraint in order to identify the different income components and corresponding sources of risk more precisely. Using the definition of the risky return presented above, entrepreneur's net worth, ω_{it} , can be decomposed as

$$\omega_{it} = R_{it}^r k_{it} + \pi_{it} + R_t b_{it} + w_t, \quad (19)$$

where $R_{it}^r k_{it}$ is capital income and $\pi_{it} \equiv (1 - \alpha) \alpha \bar{L}_t \tilde{\theta}_{it} k_{it}^\alpha$ describes risky profits, arising due to the monopolistic structure in the intermediate goods sector. Hence, in terms of Sandmo (1970), entrepreneurs face both capital and income risk, where the latter is induced by the existence of risky profits. Clearly, in our model, capital and income risk are not independent because risky return and profits both depend on the same stochastic process describing entrepreneur's productivity. However, this separation will play an important role later on, when the influence of risk is discussed more intensively and different model types are being compared. In particular, the ratio and the interaction of capital income, profits and the riskless wage income are key factors of the underlying mechanism.

Definition 1 *Given the initial distribution of agents, $\Psi_0(\omega, \tilde{\theta})$, a general competitive equilibrium under financial autarky is defined by*

a) a sequence of policy functions $\{c_t(\omega, \tilde{\theta}), k_{t+1}(\omega, \tilde{\theta}), b_{t+1}(\omega, \tilde{\theta})\}_{t=0}^\infty$, b) a sequence of value functions $\{V_t(\omega, \tilde{\theta})\}_{t=0}^\infty$, c) a sequence of prices $\{R_t, w_t, p_t(i)\}_{t=0}^\infty$, and d) a sequence of distributions $\{\Psi_t(\omega, \tilde{\theta})\}_{t=1}^\infty$, such that, for all t

1. *The policy functions described above solve the entrepreneur's decision problem.*
2. *Intermediate goods and labor are paid according to their marginal product.*
3. *Aggregate quantities of consumption, capital, labor and bonds are the aggregation of individual quantities. For given prices markets clear, especially $B_t = 0$ and $L_t = 1$.*
4. *The sequence of distributions is consistent with the initial distribution, entrepreneur's policy functions and the stochastic process for productivity.*

A competitive equilibrium for a small open economy is defined in a similar fashion. However, the bond market clearing condition is replaced by a fixed world interest rate, R^* , and B_t represents the net foreign asset position of the small country.

In the following analysis, we will initially focus on steady state relations while transitory dynamics between the autarchic and the integrated steady state are discussed more detailed in section 6. In the steady state under financial autarky as well as under financial integration, the distribution of agents is invariant and aggregate prices are constant.

4 Parametrization

To numerically solve the model we have to assign seven parameter values $\{\alpha, \beta, \rho, \delta, \bar{b}, \rho_\theta, \sigma\}$. σ is defined as the unconditional standard deviation of $\ln(\tilde{\theta})$ and ρ_θ is the serial correlation parameter specified below. Corresponding to our previous discussion, σ and \bar{b} are the 'financial' parameters defining a country's level of financial development. We directly target the properties of $\tilde{\theta}$ since it shows up as the relevant term in the budget constraint (7). Depending on the scenario, a larger value of σ and/or a lower value of \bar{b} means a lower level of financial development.

In each scenario, we initially define a benchmark economy featuring standard values from the literature. Subsequently, we consider variations in several directions to identify the conditions, i.e. the sets of parameter values, for which the different model predictions apply. The benchmark economy values of $\{\alpha, \beta, \rho, \delta, \sigma\}$ are the same in all scenarios. The discount factor, β , equals 0.96 and α is set to 0.4, implying a labor income share of 0.6. The elasticity of intertemporal substitution, $\vartheta = 1/\rho$, is set to 2/3 which means that the parameter of relative risk aversion, ρ , equals 1.5. The depreciation rate denoted by δ equals 0.08. In general, the productivity process is first-order Markov and defined as

$$\ln \theta_{t+1} = -\frac{\alpha}{1 + \rho_\theta} \frac{\sigma_\epsilon^2}{2} + \rho_\theta \ln \theta_t + \epsilon_{t+1}, \quad \epsilon \sim N(0, \sigma_\epsilon^2). \quad (20)$$

The specification of the constant term in (20) leads to the normalization $E(\tilde{\theta}) = 1$. σ_ϵ^2 is adjusted accordingly to ensure a value of σ equal to 0.4 which is comparable to Covas (2006), Angeletos (2007) and Angeletos and Panousi (2011).

The parameter values of the benchmark economies in the different scenarios only differ with respect to \bar{b} and ρ_θ . The lower bound of \bar{b} is given by $\bar{b}_{min} = 0$ and means that no borrowing is allowed. The upper bound, \bar{b}_{max} , is described by the NDL, defined as the maximum amount of repayable debt consistent with nonnegative consumption in the worst case scenario.¹⁶ In the case where \bar{b} equals the NDL, as assumed in the first scenario, the borrowing constraint does not bind and thus plays no crucial role in the analysis. Finally, ρ_θ controls the persistence of shocks or risk and corresponding effects are considered in the third scenario. In the numerical procedure applied to solve the model, we use the method proposed by Rouwenhorst (1995) to approximate the productivity process in (20) with a five-state Markov chain.

Table 1 summarizes the common parameter values of the benchmark economies.

¹⁶In steady state we have

$$\bar{b}_{max} \equiv NDL \equiv \frac{w}{R-1}. \quad (21)$$

Table 1: Common parameter values

Parameter		Value
discount factor	β	0.96
curvature of production (final good sector)	α	0.4
depreciation rate	δ	0.08
elasticity of intertemporal substitution	$\vartheta = 1/\rho$	2/3
uncond. standard deviation of $\ln(\tilde{\theta})$	σ	0.4

5 Results

5.1 Overview

In this section we consider three scenarios. Initially, we consider the case with $\bar{b} = NDL$ and $\rho_\theta = 0$ in order to focus exclusively on the effects of uninsurable risk. In the second scenario, borrowing constraints become an important factor of the analysis and in the last scenario the assumption of $\rho_\theta = 0$ will be relaxed additionally. Apart from these differences, the focus is essentially the same and relates to the questions of interest, i.e. the direction of capital flows and the long-run consequences of financial integration from the perspective of a small and less financially developed country. In the first scenario, where our approach closely relates to Angeletos (2007) and Angeletos and Panousi (2011), we derive two rules of thumb, predicting the outcome of the model with a high degree of accuracy. Due to the flexibility of the approach used to derive these rules, the procedure can be applied to other model types as well. In particular, we discuss a specific variant of the model considered by Covas (2006).

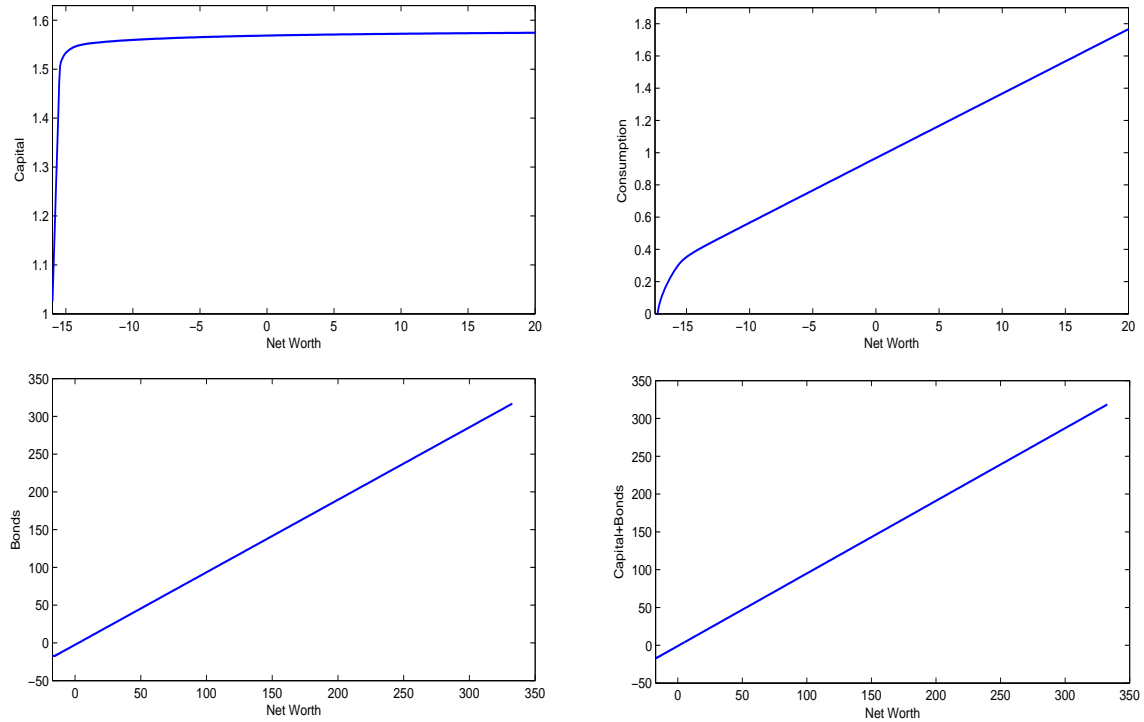
5.2 Scenario 1: Uninsurable risk

5.2.1 Policy functions

Figure 1 shows the entrepreneur’s policy functions of consumption, capital and bonds associated with the autarchic steady state of the benchmark economy.¹⁷ Since productivity follows a simple i.i.d. shock in this scenario, the decision rules do not differ between productivity levels. The upper left panel shows that the individual capital stock increases with net worth, albeit at a decreasing rate. The former property follows from diminishing absolute risk aversion. At larger levels of net worth, agents are willing to increase investment in the risky asset, i.e. to further build up the individual capital stock. The second observation, that at first agents are willing to build up the capital stock very fast but then investment slows down, follows from the fact that the expected return itself is a decreasing function of the capital stock. Initially, a high expected return triggers fast capital accumulation, but the implied reduction slows investment down in the following. The upper bound of the individual capital stock is determined by the interest rate, since no agent will accept an expected return of the risky asset being lower than the one of the riskless bond.

The lower left panel shows that entrepreneurs borrow to finance both consumption and the capital stock if today’s level of net worth is very low or even negative. This observation already indicates the opposing effects of borrowing constraints discussed in the second scenario.

Figure 1: Policy functions



5.2.2 Aggregate results

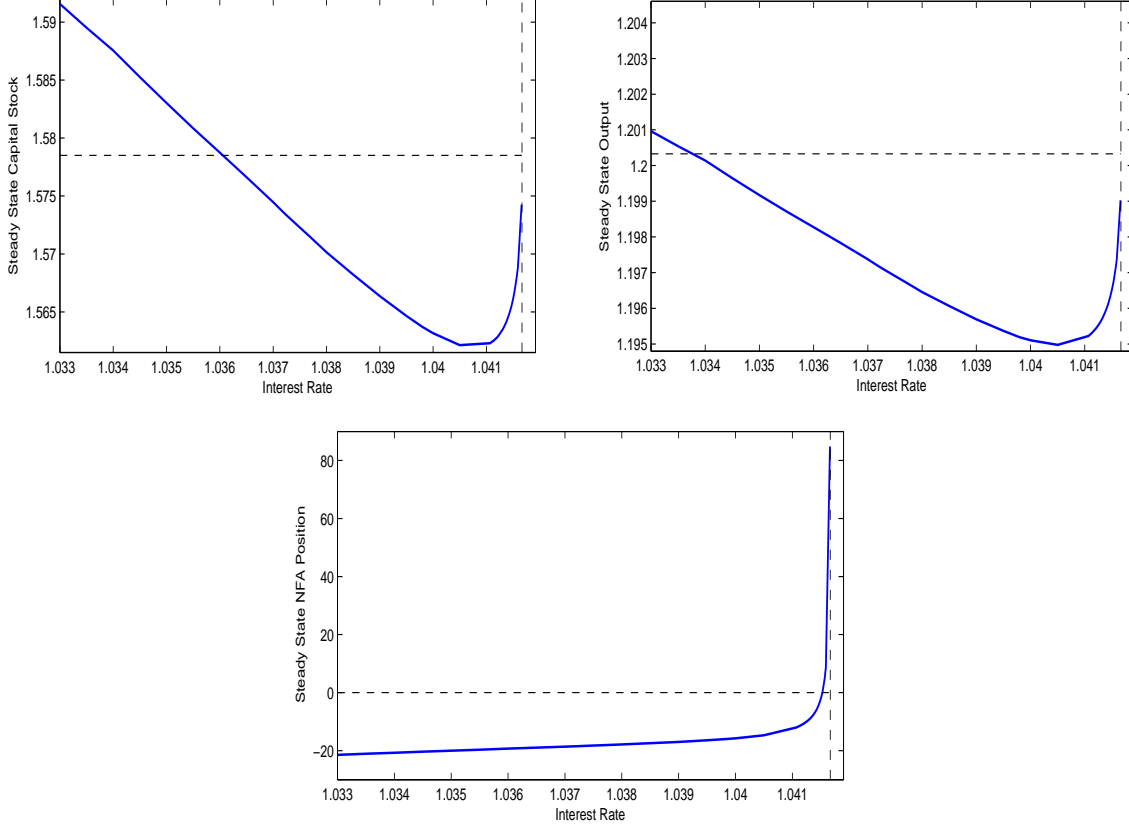
Figure 2 shows how individual behavior translates into long-run aggregate outcome. The blue line in the upper left panel shows the behavior of the long-run aggregate capital stock as a function of the interest rate. The corresponding output function is presented in the upper right panel and aggregate bond holdings or, equivalently, the net foreign asset position are shown in the lower panel. The presented functions can be interpreted in two ways, both being applied in the following. On the one hand, figure 2 shows the adjustment of aggregate variables, expressed as changes in long-run levels, toward the steady state under financial autarky where the bond market has to clear on the country-wide level. The corresponding interest rate is determined by the intersection point of the blue and the dashed horizontal zero line in the lower panel. On the other hand, figure 2 shows a broad range of possible steady state equilibria under financial integration and includes the initial autarchic steady state as special case. This main interpretation follows from the fact that the bond market clearing condition is replaced by a world market interest rate in the case of financial integration.

The example of a more financially developed country initially used in this exercise in order to compare steady states under financial autarky is the case of complete markets where full insurance against the underlying idiosyncratic risk is provided. The corresponding autarchic steady state interest rate is indicated by the vertical dashed lines in figure 2 and levels of the aggregate capital stock, output and bonds are indicated by the horizontal dashed lines. Based on our previous discussion, the case of complete markets corresponds to the highest level of financial development. In a second step where the implications are explicitly applied to the question of capital flows and the long-run consequences of financial integration, we consider the general case with frictions on both sides.

In a nutshell, two characteristics of the capital and the output function as well as the behavior of

¹⁷In the autarchic steady state, R is equal to 1.0415 and w is equal to 0.718. See also figure 2.

Figure 2: Aggregate results



the autarchic steady state interest rate are important. The first important property follows from the comparison of the blue line with the horizontal complete markets line in the upper left panel. At high interest rates, the blue line lies below the dashed line so that the existence of uninsurable risk in the less financially developed benchmark economy may lead to a lower aggregate capital stock in the autarchic steady state compared to the more financially developed country. The second important property is the U-shaped form of the capital function implying that an increase in the interest rate may lead to a larger long-run capital stock. Particularly the second property crucially depends on a sufficiently large level of the autarchic steady state interest rate but, as shown in the lower panel, the autarchic steady state interest rate of the less financially developed benchmark economy is necessarily lower compared to the case of complete markets. This implies a kind of race between a lower interest rate and the exact position of the autarchic steady state capital stock on the capital function. With respect to output, the same conclusions can be derived from the upper right panel.

The mechanisms underlying the results are seen more clearly by firstly comparing the complete with the incomplete markets capital stock at the largest interest rate for which the incomplete markets model is solved. This interest rate is slightly smaller compared to the complete markets autarchic steady state, but the deviation is so small that the corresponding complete markets capital stock is appropriately described by the horizontal dashed line.¹⁸ Here there are two important observations. Firstly, the blue line lies below the horizontal line which means a drop in the aggregate capital stock by switching from the complete markets case to the benchmark economy with incomplete markets. Secondly, as shown in the lower panel, a large excess demand for the riskless asset exists

¹⁸ R_{max} is equal to 1.04166 and, therefore, very close to the autarchic steady state interest rate under complete markets $R = 1/\beta = 1.041667$.

in the benchmark economy. The first observation follows from the fact that the expected return of the risky asset must exceed the riskless interest rate since agents demand a risk premium if full insurance is not provided. A larger expected return requires a reduction of the individual capital stock which translates into a smaller capital stock on the aggregate level. In other words, agents flee from the risk until the increase of the expected return provides sufficient compensation. The underlying intuition behind the second observation, i.e. the large excess demand for the riskless asset in comparison to the complete markets case, is seen most clearly in the case when the riskless return exactly equals its autarchic steady state value under complete markets. In this case, βR equals unity and the corresponding bond Euler equation reduces to $U'(c_{it}) = E_t U'(c_{it+1})$.¹⁹ Since $U'(\cdot)$ is convex, Jensen's inequality implies that expected consumption has to grow permanently which requires a corresponding growth of savings in order to finance the consumption stream. As the maximum individual capital stock which agents are willing to build up is finite, the demand for the riskless asset becomes unbounded. Regarding small deviations of the interest rate from its complete markets steady state value as considered here, this precautionary saving effect still leads to a large excess demand for the riskless asset as shown in the lower panel. Consequently, the autarchic steady state interest rate of the benchmark economy has to be smaller compared to the complete markets case.

Moving to the left along the blue line in the upper left panel shows that the long-run capital stock initially falls if the interest rate goes down, but finally starts to recover. This U-shaped form of the capital function is based on two opposing effects as discussed by Angeletos and Panousi (2011). On the one hand, investing in the risky asset becomes more attractive when the interest rate decreases, which is the usual opportunity-cost effect also being present in the complete markets case. On the other hand, an additional effect exists under incomplete markets which is based on agents' willingness to take risk as discussed earlier. As shown in the upper left panel in figure 1, entrepreneurs who become richer increase investment in the individual capital stock due to diminishing absolute risk aversion. However, as shown in the lower panel in figure 2, the long-run average level of wealth is an increasing function of the interest rate. Hence, contrary to the opportunity-cost effect, the wealth effect discourages investment if the interest rate decreases and stimulates capital accumulation if the interest rate increases. According to the upper left panel in figure 2, the wealth effect dominates at high interest rates close to the complete markets steady state level since the long-run capital stock increases with the interest rate. At lower levels of the interest rate, however, the opportunity-cost effect is the dominant force. The relation between the two effects is in line with the following observation: the demand for bonds and, consequently, wealth strongly reacts to small changes in the interest rate if the interest rate is close to its complete markets steady state level, but changes become less pronounced at lower interest rates.

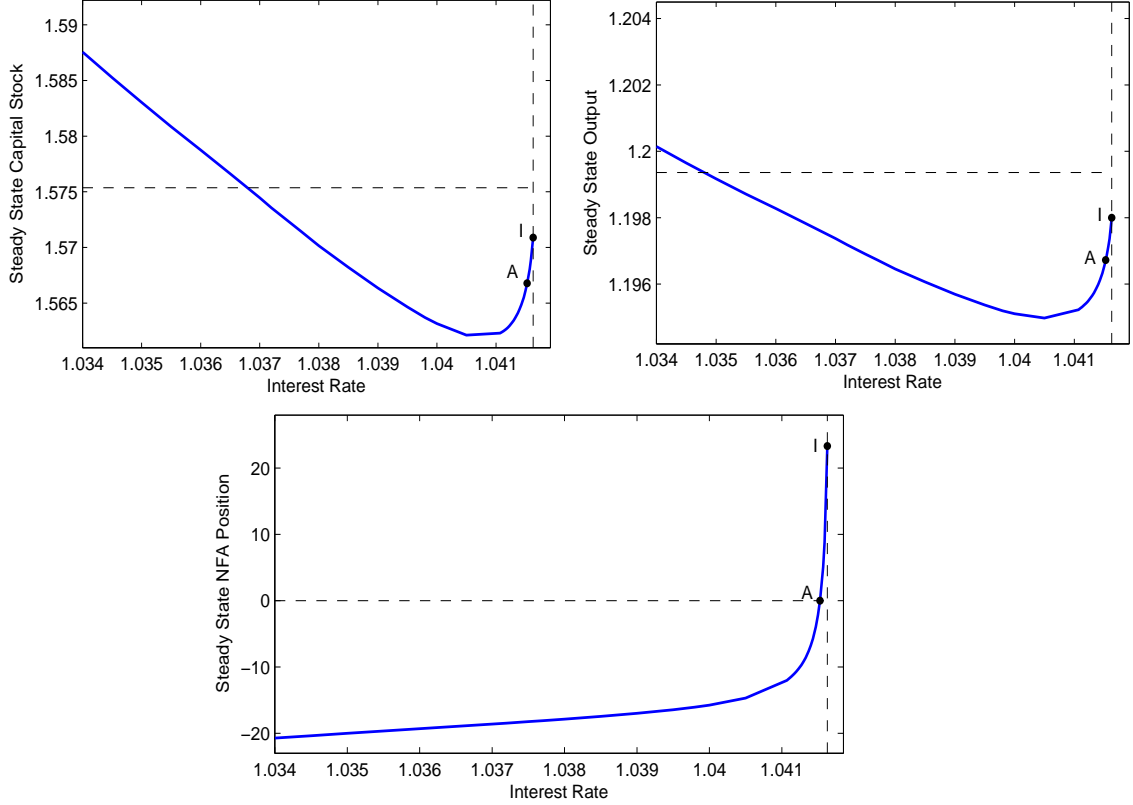
Turning to the upper right panel, the output function has almost the same shape as the capital function presented in the upper left panel. This co-movement is intuitive since production of the final good ultimately depends on capital as can be inferred from plugging (5) into (1). However, there are some differences between the two functions being related to the level of production efficiency. According to the upper right panel, the long-run level of output is the same in the benchmark economy with incomplete markets and in the economy with complete markets at the interest rate where the blue line intersects the horizontal dashed line. At this interest rate, however, the aggregate capital stock of the benchmark economy is larger than its complete markets counterpart, meaning that production is less efficient in the former case. The mechanism underlying this difference in total factor productivity (TFP; henceforth) builds on the distribution of capital across firms in the intermediate goods sector. In the complete markets case, all entrepreneurs choose the

¹⁹See Chamberlain and Wilson (2000) for a related and extensive discussion of the optimal consumption program.

same investment level, whereas in the benchmark economy, the investment levels differ between entrepreneurs due to the dependence on the individual level of net worth. Since production is most efficient in the former case, a larger amount of capital is needed in the economy with incomplete markets to compensate for the lower TFP level.

5.2.3 Financial integration

Figure 3: Aggregate results with two-sided frictions



Turning to the general case where both countries, i.e. the less and the more financially developed country suffer from financial market imperfections does not qualitatively change the characteristics of the capital and the output function as well as the behavior of the autarchic steady state interest rate. The blue lines in figure 3 are identical to the previous case and again refer to the benchmark economy with $\sigma = 0.4$. The corresponding autarchic steady state is now explicitly indicated by point A. The horizontal and vertical dashed lines, however, do not refer to the case of complete markets as in figure 2, but indicate the autarchic steady state levels of the capital stock, output, bonds and interest rate for the case of $\sigma = 0.2$. In other words, entrepreneurs in the more financially developed country considered here cannot perfectly insure against the underlying risk but the amount they have to bear is considerably smaller as in the less financially developed benchmark economy. Based on this parametrization, we consider the effects of a financial market liberalization reform removing trading barriers between the two countries. In particular, we consider the following exercise. Initially, both countries stay in the steady state under financial autarky. The corresponding interest rate of the less financially developed benchmark economy is indicated by point A in the lower panel ($R_l^A = 1.0415$), whereas the autarchic steady state interest rate of the more financially developed country is indicated by the vertical dashed line ($R_m^A = 1.04163$). Then, an unanticipated liberalization reform removes the barriers to trading the riskless asset. Since the

more financially developed country is assumed to be sufficiently large, the common interest rate under financial integration equals the autarchic steady state interest rate of that country.²⁰

The long-run effects of the liberalization reform regarding cross-border capital flows are presented in the lower panel since the riskless bond is the only financial asset being traded on the international level. Evaluating the net foreign asset position of the less financially developed economy at the interest rate under financial integration shows that the less financially developed economy features a substantial and positive asset position in the integrated steady state (point I). This result follows from the lower interest rate in the initial autarchic steady state compared to the more financially developed country, the upward adjustment of the interest rate implied by financial integration and the upward sloping long-run demand curve for the riskless asset. The lower autarchic steady state interest rate is driven by the larger portion of risk to be borne by agents in the less financially developed country. A larger amount of risk increases the demand for the riskless asset and forces the interest rate to fall in order to clear the bond market under financial autarky. Consequently, as the interest rate under financial integration is determined by the more financially developed country, the interest rate increases from the perspective of the less financially developed country once existing trading barriers are removed. Combined with the upward sloping long-run demand curve for the riskless asset, this adjustment of the interest rate finally leads to the positive net foreign asset position of the less financially developed country. Though the general pattern of the transitory dynamics between the autarchic and the integrated steady state is discussed more extensively in section 6, it essentially shows that the evolution of the asset position toward the integrated steady state is a gradual and monotone process. In particular, in all exercises being considered, we find that the less financially developed country facing an increase in the interest rate in case of financial integration runs a persistent series of current account surpluses along the transition from the autarchic to the integrated steady state. This pattern is robust across the different scenarios and arises generally as long as the interest rate under financial integration exceeds the autarchic steady state level. Consequently, we find that the direction of capital flows between countries differing in the level of financial development can directly be inferred from the corresponding differences in autarchic steady state interest rates and that capital does flow out of the country with the lower autarchic steady state interest rate.

The link between capital outflows from the less financially developed country and the aspect of capital flows from poor to rich can be inferred from the upper panels and is determined by the initial differences in the level of economic development between the two countries. In this first scenario, the ability of financial frictions to serve as an explanation of capital flows from poor to rich countries crucially depends on the question whether the larger portion of risk entrepreneurs have to bear in the less financially developed country leads to a lower level of economic development in the autarchic steady state in comparison to the more financially developed country. According to the upper panels in figure 3, this is exactly the case in the specific exercise considered here. Comparing the position of point A (autarchic steady state less financially developed country) with the position of the horizontal dashed line (autarchic steady state more financially developed country) shows that the less financially developed country features a lower aggregate capital stock as well as a lower output level in the autarchic steady state. Since capital does flow out of the less financially developed country in the case of financial integration, it follows that capital does flow from the initially poor to the initially rich country. Essentially, the co-existence of a lower autarchic steady state interest rate and a lower aggregate capital stock leads to this result.

²⁰In a two-country framework with similar weight on each economy, one would expect that the common interest rate under financial integration settles at a level between both autarchic steady state interest rates. However, qualitatively, the effect that the interest rate increases from the perspective of the less financially developed country is the same as in the exercise considered here.

Turning to the second question of interest, i.e. the long-run consequences of the liberalization reform on domestic capital accumulation, output and the wage from the perspective of the less financially developed country, the relations presented in the upper panels in figure 3 indicate an increase in capital and output compared to the situation of financial autarky. Since the autarchic steady state capital stock of the benchmark economy (point A) is located on the increasing portion of the capital function, i.e. on the part of the capital function where the wealth effect dominates the opportunity-cost effect, the increase in the interest rate implied by financial integration necessarily leads to a larger aggregate capital stock and, consequently, to a larger output and wage level in the integrated steady state (point I). During the transition, the dynamics are somewhat different since accumulation of wealth happens gradually. However, after an initial drop driven by the upward adjustment of the interest rate, the aggregate capital stock steadily increases and finally reaches its larger level in the integrated steady state. In sum, we find that capital flows from poor to rich in the exercise considered here and the less financially developed and initially poor country finally ends up with larger levels of the wage, capital and output despite the increase of the interest rate. However, the relations presented in figure 3 also indicate the existence of two alternative situations leading to significantly different model implications:

The first situation refers to the case where the autarchic steady state capital stock and the corresponding output level of the less financially developed country are located on the decreasing part of the capital and the output function, respectively. Here, a small increase in the interest rate implied by financial integration leads to lower instead of larger levels of capital and output in the integrated steady state since long-run levels of capital and output decrease if the interest rate increases. Pushing this argument a step further leads us to conclude that the model may even fail to explain capital flows from poor to rich. This situation arises in case when the less financially developed country features a larger capital stock and a larger output level in the autarchic steady state compared to the more financially developed country. According to the upper panels, the existence of this situation arises from the fact that the capital and the output function may lie above the horizontal dashed line.

What follows from the discussion of these alternative situations is that the results derived from the specific example of the two economies do not constitute the only possible model outcome, but depend on certain conditions. In the following steps, we derive two rules of thumb in order to describe these conditions more precisely. The first rule is related to the direction of capital flows and describes the condition under which the less financially developed country features a lower capital stock and, consequently, a lower output level in the autarchic steady state compared to the more financially developed country. Since capital flows out of the less financially developed country in the case of financial integration due to the lower autarchic steady state interest rate, it then follows that capital flows from poor to rich. Initially, the more financially developed country is described by the complete markets case but as shown in the following, the rule can also be applied to the general case with frictions on both sides. As part of the derivation, we exploit the fact that bond holdings have to sum up to zero in the autarchic steady state so that from the average agent's perspective, risk cannot simply be avoided by reallocating savings in favor of the riskless asset. The second rule describes the condition guaranteeing that in the neighborhood of the autarchic steady state, the wealth effect of a larger interest rate dominates the opportunity-cost effect. According to the upper left panel, this means that the autarchic steady state capital stock is located on the increasing portion of the capital function. If this condition holds, then capital does not only flow from poor to rich but the increase in the interest rate also necessarily leads to a larger capital, output and wage level in the integrated steady state from the perspective of the less financially developed country.

5.2.4 Two rules of thumb

Since analytical solutions are not available in our case, several steps are necessary to derive the first rule of thumb.²¹ Initially, we simplify our model by reducing the time horizon to two periods and by assuming that all entrepreneurs start with the same level of net worth. Up to a certain extent, the first simplification is related to the discussion of Krusell and Smith (2006), emphasizing the usefulness of the two-period setup for conducting pilot studies within the class of incomplete markets models with heterogeneous agents. The second assumption leads to the simplification that choices of capital and bonds are identical across entrepreneurs. Consequently, aggregation is easily obtained and differences in TFP do not appear. In general, this assumption may be problematic since the ultimate aim is to predict the behavior of aggregate variables in the autarchic steady state of the underlying infinite horizon model where a nontrivial distribution of agents exists under incomplete markets. Based on the policy functions presented in figure 1, however, this simplification seems to be less harmful in our case since the policy functions for consumption and bond holdings are quite linear in the level of net worth. Moreover, with respect to the range of net worth being relevant in the autarchic steady state, there is only little variation in risky investment.²² Within this simplified framework the incomplete markets outcome is compared to the outcome under complete markets and a condition leading to under-accumulation of capital in the former case is derived. Subsequently, the same steps are applied to a simplified model version sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), i.e. existence of capital risk and riskless wage income, but no further risky income component.²³ This step allows us to compare the conditions of both types of models. Finally, the results taken from this comparison and the condition described in Angeletos (2007) are used to derive the final condition regarding the autarchic steady state of the underlying infinite horizon model.

The simplified entrepreneur's optimization problem in the case of incomplete markets, i.e. in the less financially developed country, is described as follows

$$\max_{k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{t+1}) \quad (22)$$

$$s.t. \quad c_t = \omega_t - k_{t+1} - b_{t+1} \quad (23)$$

$$c_{t+1} = \omega_{t+1} \quad (24)$$

$$\omega_{t+1} = R_{t+1} b_{t+1} + R_{it+1}^r k_{t+1} + w_{t+1} + \pi_{it+1}. \quad (25)$$

The subscript i is dropped whenever optimal choices are identical across agents. By assumption, ω_t is equal across all entrepreneurs while the individual level of net worth in the final period $t+1$ depends on the realization of the idiosyncratic shock. The decomposition of ω_{t+1} is in line with the expression in (19) and again emphasizes the existence of capital risk and profits constituting a risky income component. Capital income is defined by $R_{it+1}^r k_{t+1}$ with risky return defined as $R_{it+1}^r \equiv 1 - \delta + \alpha^2 \tilde{\theta}_{it+1} k_{t+1}^{\alpha-1}$ and risky profits are described by $\pi_{it+1} \equiv (1 - \alpha) \alpha \tilde{\theta}_{it+1} k_{t+1}^\alpha$.²⁴ Since the optimal choice of risky investment is the same across all entrepreneurs, k_{t+1} denotes the individual as well as the aggregate capital stock. Combined with the fact that $E(\tilde{\theta}) = 1$, the expression for the wage reduces to $w_{t+1} = (1 - \alpha) k_{t+1}^\alpha$. Finally, since market clearing of the bond

²¹See Covas (2006) and Meh and Quadrini (2006) for related examples where analytical solutions are not available.

²²See Krusell and Smith (1998, 2006) for a general discussion. In particular, the discussion in Krusell and Smith (2006) supports our reasoning. The authors consider a two-period version of an Aiyagari-type income risk model as well as a simplified version of the model considered by Covas (2006) and show that the amount of initial wealth dispersion does not play an important role, for a plausible amount of risk and without strict borrowing constraints at least.

²³In Angeletos and Panousi (2011) the specification of investment risk differs slightly from Angeletos (2007) and time is continuous, but the main results we refer to are comparable. See also Angeletos and Panousi (2009).

²⁴Here and in the following we exploit the fact that $L_{t+1} = 1$. See Appendix A1 and A2 for details.

market implies $\int_0^1 b_{t+1} di = 0$ and b_{t+1} is the same across all agents, the capital Euler equation becomes a function of k_{t+1} alone

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1} + \pi_{it+1})]. \quad (26)$$

In the case where markets are complete, as it is initially assumed in the more financially developed economy, a full set of contingent claims can be traded. Since no aggregate risk exists, agents fully insure against the idiosyncratic risk and the individual consumption level becomes constant across all states in period $t + 1$. The resulting capital Euler equation is given by

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(k_{t+1} E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}]), \quad (27)$$

where the subscript i attached to the return and to profits is dropped in order to emphasize that the expected value of the individual return and of individual profits is identical across agents, i.e. $E_t[R_{it+1}^r] = E_t[R_{t+1}^r]$ and $E_t[\pi_{it+1}] = E_t[\pi_{t+1}]$. Equation (27) shows that the complete markets case corresponds to a deterministic setting where the riskless return and riskless profits are equal to their expected value. Hence, the comparison between (26) and (27) corresponds to the discussion of how agents respond to the presence of risk in case the mean of each risky component equals its deterministic counterpart.

To simplify the comparison between (26) and (27), the right-hand side of (26) is approximated using a second-order Taylor expansion around the point $(E_t[R_{t+1}^r], E_t[\pi_{t+1}])$. This procedure could be simplified in our case since return and profits both depend on the same stochastic process. The more general approach, however, helps to separate the effects of the different sources of risk and captures a broader set of model types. For small risks, (26) simplifies to

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(\cdot) \quad (28)$$

$$+ \frac{1}{2} \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] \sigma_{R_{t+1}^r}^2 \quad (29)$$

$$+ \frac{1}{2} \beta E_t[R_{t+1}^r] U'''(\cdot) \sigma_{\pi_{t+1}}^2 \quad (30)$$

$$+ \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1} + U''(\cdot)] \sigma_{R_{t+1}^r \pi_{t+1}}, \quad (31)$$

with $(\cdot) \equiv (k_{t+1} E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}])$. $\sigma_{R_{t+1}^r}^2$ is the conditional variance of the return, $\sigma_{\pi_{t+1}}^2$ is the conditional variance of profits and $\sigma_{R_{t+1}^r \pi_{t+1}}$ is the conditional covariance between the two. The term on the right-hand side in (28) is equal to the corresponding expression under complete markets presented in (27) and the additional terms in (29)-(31) capture the influence of risk. Based on this representation, the following relation between the equilibrium levels of k_{t+1} in the two cases can be derived.

Lemma 1 *Let k_{t+1}^* denote the solution in the incomplete markets case. Then, the capital stock is the same under complete and incomplete markets if and only if the additional terms in (29)-(31) evaluated at k_{t+1}^* sum up to zero. Moreover, the capital stock in the incomplete markets case is lower (larger) than in the complete markets case if and only if the additional terms in (29)-(31) evaluated at k_{t+1}^* sum up to a value strictly lower (larger) than zero.*

Before we proceed with the derivation of the final condition leading to a lower capital stock and, consequently, to a lower output level in the economy with incomplete markets, we use the expressions described in (29)-(31) to briefly discuss the differences in the influence of the underlying sources of risk. Initially, we focus exclusively on the effect of income risk being captured by the expression in (30). In our model, income risk is implied by the existence of risky profits but the

same expression also occurs if, for simplicity, π_{t+1} is treated as a stochastic endowment component with variance $\sigma_{\pi_{t+1}}^2$. According to (30) and Lemma 1, the well-known result applies that uninsurable income risk generates precautionary savings and, consequently, leads to over-accumulation of capital if $U'''(\cdot) > 0$ (see e.g., Leland, 1968; Kimball, 1990). Since this condition holds in our case, uninsurable income risk alone would necessarily lead to larger levels of capital and output in the less financially developed country with incomplete markets.

Next, we focus exclusively on capital risk so that only (29) shows up in the incomplete markets case and the model economy becomes closely related to Levhari and Srinivasan (1969) and Sandmo (1970).²⁵ Inspecting (29) shows that the ultimate effect of uninsurable capital risk on the choice of k_{t+1} is ambiguous. On the one hand, a precautionary saving effect exists as well, but on the other hand, the term $2U''(\cdot) k_{t+1}$ is negative if agents are risk-averse.²⁶ Intuitively, and in contrast to income risk, agents can reduce the extent to which their resources are exposed to potential losses by consuming more and saving less in case where risk is associated with risky returns. Consequently, capital risk leads to lower levels of capital and output in the economy with incomplete markets if the latter effect, to which we refer as substitution effect, dominates.

Finally, we assume that both types of risk exist simultaneously and if $\sigma_{R_{t+1}\pi_{t+1}} \neq 0$, all terms in (29)-(31) show up in the approximation. In general, the appearance of the covariance in (31) implies another ambiguous effect on the choice of k_{t+1} and the sign of the covariance reflects the extent to which the risky asset can be used to hedge against the underlying income risk.²⁷ In our case, $\sigma_{R_{t+1}\pi_{t+1}}$ is positive and second moments, evaluated at k_{t+1}^* , are described as follows

$$\begin{aligned}\sigma_{R_{t+1}}^2 &= \alpha^4 (k_{t+1}^*)^{2\alpha-2} \sigma_{\theta_{t+1}}^2 \\ \sigma_{\pi_{t+1}}^2 &= (1-\alpha)^2 \alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 \\ \sigma_{R_{t+1}\pi_{t+1}} &= (1-\alpha) \alpha^3 (k_{t+1}^*)^{2\alpha-1} \sigma_{\theta_{t+1}}^2,\end{aligned}\tag{32}$$

where $\sigma_{\theta_{t+1}}^2$ denotes the variance of $\tilde{\theta}_{t+1}$. Based on these expressions and on Lemma 1, the following final condition can be derived.

Lemma 2 *Levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if*

$$\vartheta > \frac{\phi}{2-\phi}, \quad \phi \equiv \frac{\alpha + (1-\delta) \frac{K_{t+1}^*}{\alpha Y_{t+1}^*}}{1 + (1-\delta) \frac{K_{t+1}^*}{Y_{t+1}^*}},\tag{33}$$

where production simplifies to $Y_{t+1}^* = (K_{t+1}^*)^\alpha$.

The first part of (33) shows that the elasticity of intertemporal substitution, ϑ , has to exceed a certain threshold level which is then described more precisely in the second part. Although the individual equals the aggregate capital stock in this simplified model version, capital letters are used to explicitly indicate aggregate variables. Note that the condition described in the first part of (33) is defined for the interesting case of $(2-\phi) > 0$. If ϕ turns out to be larger than 2 for some parameter values and initial endowments, levels of capital and output are necessarily larger in the economy with incomplete markets in comparison to the complete markets case since lower levels

²⁵See also Rothschild and Stiglitz (1971) and for a separate analysis of both types of risk see Eeckhoudt and Schlesinger (2008).

²⁶In general, the sign of $2U''(\cdot) k_{t+1}$ also depends on whether agents are savers ($k_{t+1} > 0$) or borrowers ($k_{t+1} < 0$). In our case, however, only $k_{t+1} > 0$ is relevant.

²⁷For a more rigorous and simultaneous treatment of both types of risk in cases where income and the return are exogenously given, see Li (2012) and Baiardi et al. (2014).

of capital and output require ϑ to be negative in that case.

To facilitate the discussion of Lemma 2, we firstly derive a respective condition for the case when the underlying model shares the main features of Angeletos (2007) and Angeletos and Panousi (2011).²⁸ In this case and comparable to our model economy, each household owns a private firm, receives riskless wage income by supplying one unit of labor inelastically and can freely trade a riskless bond. However, no intermediate goods sector exists, but all firms produce the same final good using capital and labor as inputs. Capital is invested by the respective firm owner and optimal employment in each period is chosen after the capital stock has been installed and the contemporaneous idiosyncratic shock has been observed. Assuming a constant returns to scale production function on the individual level, capital income becomes linear in the capital stock and output is completely divided between capital and labor income. In sum, individual net worth can be described as follows

$$\omega_{it} = R_{it}^r k_{it} + R_t b_{it} + w_t, \quad (34)$$

where $R_{it}^r \equiv 1 - \delta + \theta_{it}\alpha \left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}}$ is the risky return in the case when individual technology is described by a Cobb-Douglas production function, and w_t denotes the wage rate. In terms of the previous discussion, agents face capital risk and receive riskless wage income, but no further risky income component exists. Consequently, in the simplified two-period model, only the term capturing the influence of capital risk as generally described in (29) shows up in the approximation of the capital Euler equation. Applying the same steps as above finally leads to the following condition guaranteeing lower levels of capital and output in the less financially developed country.

Lemma 3 *In the simplified two-period model sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if*

$$\vartheta > \frac{\kappa}{2 - \kappa}, \quad \kappa \equiv \frac{\alpha + (1 - \delta) \frac{K_{t+1}^*}{Y_{t+1}^*}}{1 + (1 - \delta) \frac{K_{t+1}^*}{Y_{t+1}^*}}, \quad (35)$$

where aggregate production simplifies to $Y_{t+1}^* = (K_{t+1}^*)^\alpha$.

Similar to Lemma 2, the first part of (35) shows that the elasticity of intertemporal substitution has to exceed a certain threshold level which is described more precisely in the second part. Note that κ is less than or equal to unity for plausible parameter values and, thereby, less than 2.

The motivation to refer to the elasticity of intertemporal substitution as the key parameter of the model even though this concept is inextricably linked to agents' attitudes toward risk in our case, is based on Angeletos (2007) and Angeletos and Panousi (2011). While the previous discussion has shown that uninsurable capital risk leads to two opposing effects on saving if agents are risk-averse, Angeletos (2007) shows that the ultimate effect most critically depends on agents' attitudes toward intertemporal substitution.²⁹ However, in our case, the plausibility to observe a specific outcome may also be evaluated from the perspective of the critical value of the parameter of relative risk aversion being implied by the critical value of the elasticity of intertemporal substitution.

The intuition underlying the threshold level described in (35) appears most obvious when $\alpha = 1$

²⁸Regarding Angeletos (2007) we refer to the model without extensions (p.5f.).

²⁹The author applies a preference structure allowing to separate the concept of risk aversion from the concept of intertemporal substitution. Intuitively, from the perspective of a risk-averse agent, an increase in the variance reduces the risk adjusted return. Consequently, agents' responses to this change are comparable to their responses when the interest rate changes in a deterministic setting where the elasticity of intertemporal substitution is the key parameter. See also Weil (1990) for a discussion.

where the model matches the standard 'Ak'-case and labor plays no role. Since κ is equal to unity in this case, the well-known result applies that the aggregate capital stock and, consequently, the output level, is lower in the economy with incomplete markets if and only if the elasticity of intertemporal substitution is larger than unity. In terms of the previous discussion this means the substitution effect dominates if $\vartheta > 1$.

When α is strictly less than unity, however, riskless wage income exists. According to (35), the value of κ as well as the final threshold level the elasticity of intertemporal substitution has to exceed in order to guarantee lower levels of capital and output in the less financially developed economy become less than unity. Intuitively, the existence of the riskless income component reduces the percentage drop of consumption in times when the risky asset pays off poorly and, therefore, weakens the need for precautionary savings. Consequently, the substitution effect already dominates at lower levels of the elasticity of intertemporal substitution compared to the case where no riskless income component exists.

With this result at hand, we return to the discussion of Lemma 2 and the condition described in (33). Our model essentially reduces to the standard 'Ak'-case as well if $\alpha = 1$ since labor plays no role and production of the final good described in (1) may be simply reinterpreted as the aggregation of individual production. However, a value of α less than unity does not only lead to the existence of riskless wage income, but entrepreneurs also earn risky profits. To study the consequences of this additional effect, we firstly consider the case of complete depreciation, i.e. of $\delta = 1$. According to (33) and (35), ϕ and κ are equal to α in this case which at first glance is surprising since profits constitute an additional source of risk in our model. However, this result follows from two effects where the first refers to the ratio between the risky and the riskless income components. In the simplified version of the model sharing the main features of Angeletos (2007) and Angeletos and Panousi (2011), aggregate consumption in the final period equals aggregate production if capital completely depreciates. The corresponding share of capital as the risky income component from the individual perspective is α and the riskless wage share is $(1 - \alpha)$. In our case, the capital share is only α^2 while the share $\alpha(1 - \alpha)$ is profit income. However, since the total share of the risky components sums up to α as well and the remaining share $(1 - \alpha)$ is riskless wage income, the ratio between the risky and the riskless income components is the same in both models. Secondly, in our case, a strong co-movement between the risky return and risky profits exists since they are perfectly correlated. In sum, since ϕ and κ are both equal to α in the case of complete depreciation, differences in the individual shares of the risky components between the models do not play a role in this special case.

To stress this point even further, we reconsider the variant of our model in which production of the final good reduces to $Y_t = \int_0^1 x_{it} di$ as α is assumed to be equal to unity. As outlined above, the model may be reinterpreted as a one-sector model in which each entrepreneur produces the same final good and Y_t simply represents the aggregation of individual production. However, instead of reproducing the 'Ak'-case, we additionally change entrepreneur's technology described in (5). Individual production now takes place according to $y_{it} = \theta_{it} k_{it}^\alpha$ with $\alpha < 1$. In this case, the model essentially matches the one-sector model considered by Covas (2006) where each entrepreneur produces the same final good according to $y_{it} = \theta_{it} k_{it}^\alpha$. Separating capital income from profits leads to the following expression of individual net worth

$$\omega_{it} = R_{it}^r k_{it} + \pi_{it} + R_t b_{it}, \quad (36)$$

where $R_{it}^r \equiv 1 - \delta + \alpha \theta_{it} k_{it}^{\alpha-1}$ is the risky return and risky profits are given by $\pi_{it} \equiv (1 - \alpha) \theta_{it} k_{it}^\alpha$.³⁰

³⁰Profits may also be interpreted as entrepreneur's risky labor income with $l_{it} = 1$.

The expression in (36) shows that agents face capital risk and profits constitute a risky income component. However, no riskless wage income exists. Focusing on the simplified two-period model version without persistence in risk and without binding borrowing constraints leads to the following condition guaranteeing lower levels of capital and output in the less financially developed country.

Lemma 4 *In the simplified two-period model version of Covas (2006), levels of the aggregate capital stock and output are lower in the economy with incomplete markets compared to the complete markets case if and only if*

$$\vartheta > \frac{\nu}{2 - \nu}, \quad \nu \equiv \frac{1 + (1 - \delta) \frac{K_{t+1}^*}{\alpha Y_{t+1}^*}}{1 + (1 - \delta) \frac{K_{t+1}^*}{Y_{t+1}^*}}, \quad (37)$$

where aggregate production simplifies to $Y_{t+1}^* = (K_{t+1}^*)^\alpha$.

Similar to Lemma (2) and (3), the condition described in the first part of (37) is defined for the interesting case of $(2 - \nu) > 0$. If ν turns out to be larger than 2 for some parameter values and initial endowments, levels of capital and output are necessarily larger in the economy with incomplete markets in comparison to the complete markets case since lower levels of capital and output require ϑ to be negative in that case.

In contrast to ϕ described in (33) and κ described in (35), the first term in the numerator of the definition of ν is equal to unity. Consequently, the non-existence of riskless wage income and, therefore, the stronger influence of the precautionary saving motive tend to lead to larger critical values compared to our condition described in (33). Moreover, in the case of complete depreciation, the threshold level implied by (37) is equal to unity and thus equal to the 'Ak'-case with $\alpha = 1$. The intuition underlying the latter result is based on the same effects as described above and again emphasizes the total share of the risky income components as the important statistic. In the 'Ak'-case, aggregate consumption equals aggregate production in period $t + 1$ if capital completely depreciates and the corresponding capital share as the risky income component from the individual perspective is equal to unity. In contrast, in the model considered by Covas (2006), the capital share is equal to α while the remaining share $(1 - \alpha)$ is profit income. However, since profits are risky as well, the total share of the risky components also adds up to unity. In addition, return and profits are perfectly correlated so that the differences in the capital share do not play a role in the case of $\delta = 1$ where the same condition applies to both models.

Returning to the comparison of (33) with (35) shows, however, that ϕ and κ differ from each other if $\delta < 1$. Consequently, in the more important case without complete depreciation, the differences in the model structure matter and lead to different conditions in order to observe a lower capital stock as well as a lower output level in the economy suffering from incomplete markets.

While the discussion so far focuses on the simplified model structure, the next step compares the results described in Lemma 3 with the corresponding condition explaining the behavior of aggregate variables in the autarchic steady state of the underlying infinite horizon model.³¹ According to Angeletos (2007, p.11) autarchic steady state levels of capital, output and consumption are lower in the economy with incomplete markets than in the case of complete markets if and only if

$$\vartheta > \frac{\hat{\kappa}}{2 - \hat{\kappa}}, \quad \hat{\kappa} \leq \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \quad (38)$$

³¹In contrast to our model, the model structure in Angeletos (2007) and in Angeletos and Panousi (2011) admits exact aggregation. See also Angeletos and Panousi (2009) for a discussion of the steady state effects of uninsurable investment risk.

where variables are evaluated at the autarchic steady state. $\hat{\kappa}$ represents the fraction of private equity in total wealth and α denotes the income share of capital. Assuming a Cobb-Douglas specification on the individual level as in the simplified model version leads to $Y^* = (K^*)^\alpha$. According to Angeletos (2007), the condition described in the first part of (38) is exact only in the continuous-time limit of the model, but is an extremely good approximation for the discrete-time model as well. In addition, the upper bound for $\hat{\kappa}$ described in the second part of (38) becomes binding only in the case when risk is completely absent, but it also is nearly binding for all plausible parameter values. Focusing on plausible parameter values, the condition to observe lower levels of capital and output in the less financially developed country with incomplete markets can be summarized as follows:

In the model considered by Angeletos (2007), autarchic steady state levels of capital and output are lower in the economy with incomplete markets than in the case of complete markets if and only if

$$\vartheta > \frac{\hat{\kappa}}{2 - \hat{\kappa}}, \quad \hat{\kappa} \cong \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \quad (39)$$

where $Y^* = (K^*)^\alpha$ in case of a Cobb-Douglas specification.

Even though condition (39) refers to the steady state of the infinite horizon model while condition (35) explains the influence of uninsurable capital risk in the simplified two-period model version, they obviously share a similar structure.³² In fact, the only significant difference, which appears in the expressions of κ and $\hat{\kappa}$, indicates a different role played by the depreciation rate δ . This difference, however, seems intuitive due to the different nature of the steady state and of the last period in the two-period model. In the two-period model, aggregate consumption in the final period equals aggregate production and κ is equal to α if capital completely depreciates. The corresponding wage share equals $(1 - \alpha)$ and the income share of capital is equal to α . However, to obtain the same relation in the steady state of the infinite horizon model and to get the same value of $\hat{\kappa}$, the depreciation rate has to converge to zero so that no additional resources are needed to keep the aggregate capital stock on its steady state level. Hence, the 'natural' counterpart of $\delta = 1$ in the two-period version is the case of $\delta = 0$ in the steady state of the infinite horizon version. Since the comparison of (35) with (39) identifies the general difference between the simplified two-period world and the underlying infinite horizon model, we use the results to derive the final condition describing the autarchic steady state of our model economy. The last step involves the derivation of a new variable, $\hat{\phi}$, such that the difference between $\hat{\phi}$ and ϕ reflects the difference between $\hat{\kappa}$ and κ . Applying this step finally leads to the following rule, explaining capital accumulation in our model economy.

Rule of thumb 1 *In the autarchic steady state, levels of the aggregate capital stock and output are lower in the economy with incomplete markets than in the case of complete markets if and only if*

$$\vartheta > \frac{\hat{\phi}}{2 - \hat{\phi}}, \quad \hat{\phi} \equiv \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \quad (40)$$

where production is approximated by $Y^* = (K^*)^\alpha$.

Since this last step is mainly based on a model comparison and differences in TFP are being neglected, we refer to the condition described in (40) as a rule of thumb. However, as shown in

³²In Angeletos (2007), capital risk also includes depreciation risk. However, taking account of this feature in the two-period version does not change the condition described in (35). Consequently, the implications derived from comparing (35) with (33) and (35) with (39) also remain unaffected.

Appendix A7, the same general relation between the simplified two-period version and the underlying infinite horizon model also appears in the model sharing the main features of Angeletos and Calvet (2006), which further supports our reasoning. In addition, since borrowing constraints and persistence in shocks do not play a role, the differences in individual investment levels are rather small in the first scenario considered here. Finally, note that for all parametrizations being considered, we observe that $\hat{\phi}$ is less than unity. Consequently, possible problems with the interpretation of the first as well as of the second rule of thumb which may arise in case of dealing with negative threshold levels do not appear.

To evaluate the performance of the first rule of thumb, we compare the corresponding predictions with our actual model solutions and the predictions derived from applying the condition described in (39) to our model. The first comparison gives some indication of the degree of accuracy and the second shows how strongly the difference between $\hat{\phi}$ and $\hat{\kappa}$ affects the corresponding results. The exercise is based on the benchmark parametrization but shows some variation in the depreciation rate. The results are presented in table 2. The numbers without brackets in the third column represent the values of ϑ satisfying $\vartheta = \hat{\phi}/(2 - \hat{\phi})$, i.e. condition (40) holds with equality. Consequently, the capital stock should be the same under complete and incomplete markets for these values of the elasticity of intertemporal substitution. Larger values of ϑ then lead to $\vartheta > \hat{\phi}/(2 - \hat{\phi})$, such that, according to our first rule of thumb, levels of capital and output should be lower in the economy suffering from incomplete markets. The corresponding critical values derived from (39) are presented in the second column while the actual solutions are shown in the last column. Finally, the numbers within brackets in the second and third column indicate the differences between the predicted and the actual critical values.

Table 2: Under-accumulation of capital; critical values

Depreciation rate	Angeletos's rule	First rule of thumb	Model solutions
0.001	0.248 (0.01)	0.244 (0.006)	0.238
0.04	0.211 (0.089)	0.125 (0.003)	0.122
0.08	0.198 (0.122)	0.083 (0.007)	0.076

In the fourth column, the numbers indicate that changes in the depreciation rate have a strong impact and lead to significant differences between the actual critical values. In terms of the parameter of relative risk aversion, ρ , the critical value is close to 4 if the depreciation rate is close to zero, but increases to 13 when the depreciation rate equals 8 percent. However, the results presented in the third column show that the predictions derived from our first rule capture these differences quite well since the reported prediction error is rather small in all three cases. Further simulations suggest that this result is also robust to changes in α and to changes in the discount factor β . Apart from its small size, the prediction error in our case is additionally less problematic since the numbers in the third column are uniformly larger than the actual critical values in the fourth column. Consequently, as long as ϑ exceeds the predicted critical value, the economy suffering from incomplete markets indeed features lower levels of capital and output compared to the complete markets case. Turning to the comparison of the second and the third column, both conditions predict almost the same critical value for our model if δ is extremely close to zero, but

the differences become more pronounced at larger values of the depreciation rate. The former result constitutes the counterpart of $\delta = 1$ in the two-period setup, whereas the latter confirms that, in general, the different conditions may not be used interchangeably between the models. Note, however, that the predicted critical values presented in the second column do not reproduce the results of Angeletos (2007) since they are derived from applying his rule to our model and steady state levels of the aggregate capital stock and output may differ between the models.

The considered exercise and, especially, the precise predictions derived from our rule of thumb also indicate a simple way to quickly identify the model properties given a chosen set of parameter values. The idea is to plug the complete markets values of capital and output into the definition of $\hat{\phi}$ and to compute the corresponding expression $\hat{\phi}/(2 - \hat{\phi})$. Since the complete markets values are easily derived, this procedure yields a fast and accurate initial guess of the actual critical value of the elasticity of intertemporal substitution.

Finally and most important, the numbers in the fourth column suggest a very low threshold level of the elasticity of intertemporal substitution for plausible values of the depreciation rate, which is in line with Angeletos (2007). Even if the wage share is reduced to 0.4, which implies that entrepreneurs are more vulnerable to the underlying risk, the critical value does not exceed 0.15 in case when the depreciation rate equals 8 percent. Since a significant share of the empirical literature suggests a value of the elasticity of intertemporal substitution close to unity, as discussed in Angeletos (2007), our model predicts lower levels of capital and output in the less financially developed country for plausible parametrizations. However, it seems worth noting that the empirical results are far from being uniform. Dacy and Hasanov (2011), for example, find a much smaller value of the elasticity of intertemporal substitution of around 0.2.³³ Consequently, changes in the critical values as arising in the next scenarios are not only of theoretical interest, even if based on a rather low initial level. In addition, the critical values referring to the long-run consequences of financial integration on capital and output as considered in the following are already significantly larger than the ones presented in table 2.

The derivation of the second rule of thumb builds on Angeletos and Panousi (2011) and the comparison of (39) with (40) considered above. Revisiting the upper left panel in figure 3, the second rule describes the condition guaranteeing that in the economy with incomplete markets, the autarchic steady state capital stock is located on the increasing part of the capital function so that long-run levels of capital and output necessarily increase with the interest rate. Regarding the model considered by Angeletos and Panousi (2011), the following condition ensures that in the neighborhood of the autarchic steady state the wealth effect of a higher interest rate dominates the opportunity-cost effect:³⁴

$$\vartheta > \frac{\hat{\kappa}}{1 - \hat{\kappa}}, \quad \hat{\kappa} \cong \frac{\alpha - \delta \frac{K^*}{Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \quad (41)$$

where $Y^* = (K^*)^\alpha$ in case of a Cobb-Douglas specification.

Comparing (39) with (41) shows that both conditions share a similar structure, but the latter leads to larger critical values of the elasticity of intertemporal substitution. The stronger restrictions regarding ϑ reflect the stronger restrictions regarding the autarchic steady state interest rate compared to the case where the focus is exclusively on lower levels of capital and output in the less financially developed country. Adjusting the condition described in (40) to match the difference

³³See also Gomes and Paz (2013) for an extensive discussion.

³⁴We slightly change the presentation compared to Angeletos and Panousi (2011, p.874) to facilitate the further discussion.

between (39) and (41) leads to the following condition, describing the behavior of long-run levels of capital and output in our model.

Rule of thumb 2 *Based on the autarchic steady state of the economy with incomplete markets, long-run levels of capital and output necessarily increase with the interest rate if and only if*

$$\vartheta > \frac{\hat{\phi}}{1 - \hat{\phi}}, \quad \hat{\phi} \equiv \frac{\alpha - \delta \frac{K^*}{\alpha Y^*}}{1 - \delta \frac{K^*}{Y^*}}, \quad (42)$$

where production is approximated by $Y^* = (K^*)^\alpha$.

The evaluation of the performance of the second rule takes place in a similar way as in the previous case and is again based on the benchmark parametrization. The numbers without brackets in the third column of table 3 represent the values of ϑ satisfying $\vartheta = \hat{\phi}/(1 - \hat{\phi})$, i.e. condition (42) holds with equality. Consequently, the autarchic steady state capital stock should exactly be equal to the minimum of the corresponding capital function at these values of the elasticity of intertemporal substitution. Larger values of ϑ then lead to $\vartheta > \hat{\phi}/(1 - \hat{\phi})$, such that, according to the second rule of thumb, long-run levels of capital and output should necessarily increase if the interest rate rises above its autarchic steady state level. The respective critical values derived from applying (41) to our model are presented in the second column. The actual critical values are presented in the fourth column and the numbers within brackets in the second and third column indicate the prediction errors.

Table 3: Long-run effects capital and output; critical values

Depreciation rate	Angeletos's and Panousi's rule	Second rule of thumb	Model solutions
0.001	0.662	0.645	0.556
	(0.106)	(0.089)	
0.04	0.538	0.287	0.286
	(0.252)	(0.001)	
0.08	0.493	0.184	0.182
	(0.311)	(0.002)	

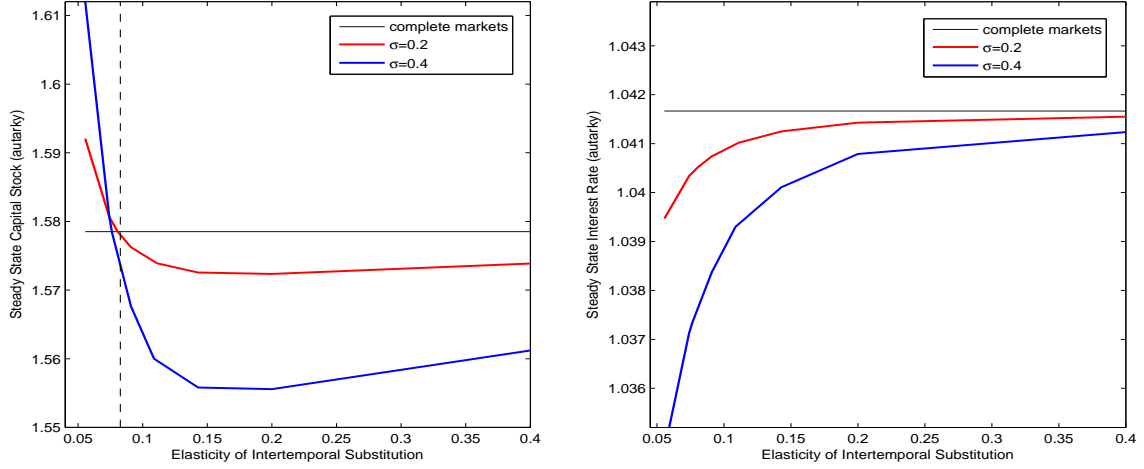
Apart from level differences, the results draw a similar picture as in the previous case. Firstly, the prediction errors reported in column three are rather small, implying a high degree of accuracy of the second rule of thumb. Secondly, the predictions derived from both conditions are almost the same for $\delta = 0.001$, but significantly different at larger values of the depreciation rate. The actual critical values presented in the last column, however, are roughly twice as large as the ones in table 2 and reflect the stronger restrictions regarding ϑ . As long as the depreciation rate is below 8 percent, they are significantly larger than 0.2, but stay below this value for larger values of δ .

5.2.5 Applying the rules and returning to financial integration

In this section, we examine the effects of changes in the amount of risk and apply the rules to the general case with frictions on both sides. Since, in contrast to the second rule, the first rule of thumb builds on the comparison with the complete markets case, we want to analyze its performance when the more financially developed country also suffers from financial market imperfections. For this purpose, we additionally consider the example of $\sigma = 0.2$, lying in between the case of complete

markets and the benchmark economy. Finally, coming back to financial integration, we summarize the main implications of the first scenario.

Figure 4: Critical values, changes in risk and financial integration



The left panel of figure 4 provides a graphical presentation of the first two aspects while the right panel becomes important in the last part of this section. The horizontal black and the colored lines in the left panel indicate the relation between the elasticity of intertemporal substitution, ϑ , and the corresponding autarchic steady state capital stock for the different economies which only differ in the portion of uninsured risk. The horizontal black line refers to the complete markets case where the autarchic steady state capital stock is independent of ϑ . The blue line corresponds to the benchmark economy with $\sigma = 0.4$ and the red line corresponds to the case in between, with $\sigma = 0.2$. The dashed vertical line constitutes a graphical presentation of the prediction of the first rule of thumb. The value of ϑ determined by this line is the predicted critical value listed in the third row, third column of table 2. Consequently, according to the first rule of thumb, autarchic steady state levels of capital and output should be lower in the benchmark economy compared to the complete markets case if ϑ exceeds this threshold level. The actual critical value, in turn, is determined by the intersection point of the blue line and the horizontal complete markets line. As already discussed, the difference between the predicted and the actual critical value is quite small. Since the economies only differ in the portion of uninsurable risk, the effects of changes in risk are reflected by the differences in the corresponding critical values of ϑ . Comparing the critical value being determined by the intersection point of the red line and the horizontal complete markets line to the critical value being determined by the intersection point of the blue line and the horizontal complete markets line leads to two observations. Firstly, both critical values are almost identical so that, as long as the absolute level of risk does not become too large, differences in the amount of risk have rather small effects. Further simulations suggest that small changes in risk also hardly affect the critical values in table 3. In particular, small deviations of σ from its benchmark economy level barely move the numbers presented in the fourth column. Secondly, a larger amount of risk leads to a lower critical value. While the effect, as just described, is negligible for small risks, it becomes more pronounced at larger values of σ and increases the likelihood to observe lower levels of capital and output in the less financially developed country compared to the complete markets case.

The observation that the effects associated with changes in the amount of risk are rather small also drives the results in the general case with frictions on both sides. Since both the blue line and the

red line in the left panel of figure 4 intersect the horizontal complete markets line at almost the same value of ϑ , the intersection point of the lines itself implies a similar critical value. In other words, the results derived in the case where the more financially developed country is characterized by complete markets largely carry over to the case where agents in the more financially developed country suffer from incomplete markets as well. In particular, for small risks, this means a broader interpretation of the first rule of thumb. In all cases considered, we find that if ϑ exceeds the critical value implied by the first rule of thumb, the less financially developed country features lower levels of capital and output in the autarchic steady state compared to any better financially developed country, and not only compared to the ideal case of complete markets.

With these results at hand, we finally return to the discussion of financial integration. As already mentioned in the discussion of figure 3, whether uninsurable idiosyncratic risk can contribute to explain capital flows from poor to rich depends on the level of economic development in the less financially developed country. Since the lines in the right panel in figure 4 show that the autarchic steady state interest rate decreases with the amount of risk, the less financially developed country necessarily features a lower interest rate in the autarchic steady state. A lower interest rate, in turn, leads to capital outflows in case of financial integration so that, in order to explain capital flows from poor to rich, lower autarchic steady state levels of capital and output are required in the economy with stronger financial frictions. In the first scenario, we find that a value of ϑ slightly larger than 0.08 is already sufficient, at least for small risks and the benchmark values of $\{\alpha, \delta, \beta\}$. Since this low critical value of ϑ stays within empirically plausible bounds, the model predicts capital flows from poor to rich for plausible parameter values. One of the most important factors underlying the low threshold level is the existence of riskless wage income or in general, the existence of any type of riskless income component. Even if the wage share is reduced significantly, the critical value is still considerably smaller than unity.

Turning to the second question of interest, we find that a larger value of ϑ is generally required in order to additionally explain larger levels of capital and output in the integrated steady state from the perspective of the less financially developed country and compared to the autarchic steady state. Regarding the standard parameter values assigned in this first scenario, the critical value more than doubles from 0.08 to approximately 0.18, but still stays below 0.2. Consequently, in addition to capital flows from poor to rich, financial integration leads to larger long-run levels of capital and output in the less financially developed country for plausible parameter values. Although this result is generally in line with Angeletos (2007) and Angeletos and Panousi (2011), the rules of thumb have proven to be a useful tool for identifying the properties of our model more easily and for describing possible differences between the models.

In the next section, we will additionally impose tight borrowing constraints. While the general shape of the capital and the output function as presented in figure 2 will be preserved, borrowing constraints may strongly influence the conditions to observe the same outcome as described above.

5.3 Scenario 2: Borrowing constraints

In this scenario, the assumption that entrepreneurs may in principle borrow up to the NDL is replaced by a more restrictive debt limit. As it can already be inferred from the policy functions presented in figure 1, borrowing constraints affect entrepreneurs in two different ways. Since, in bad times, agents need to borrow to finance both consumption and the capital stock of the individual firm, borrowing constraints impede consumption smoothing and limit the access to external funds for scaling up production. Consequently, the approach combines consumer credit restrictions known from standard incomplete markets models with credit market frictions which

affect the production side of the economy. Since the corresponding effects tend to work in opposite directions as discussed below, the ultimate impact of borrowing constraints is generally ambiguous and borrowing constraints may either tighten or weaken the conditions derived in the first scenario. Initially, the more financially developed country is again characterized by complete markets while, subsequently, we discuss the general case with financial frictions on both sides.

Figure 5: Aggregate results: borrowing constraints

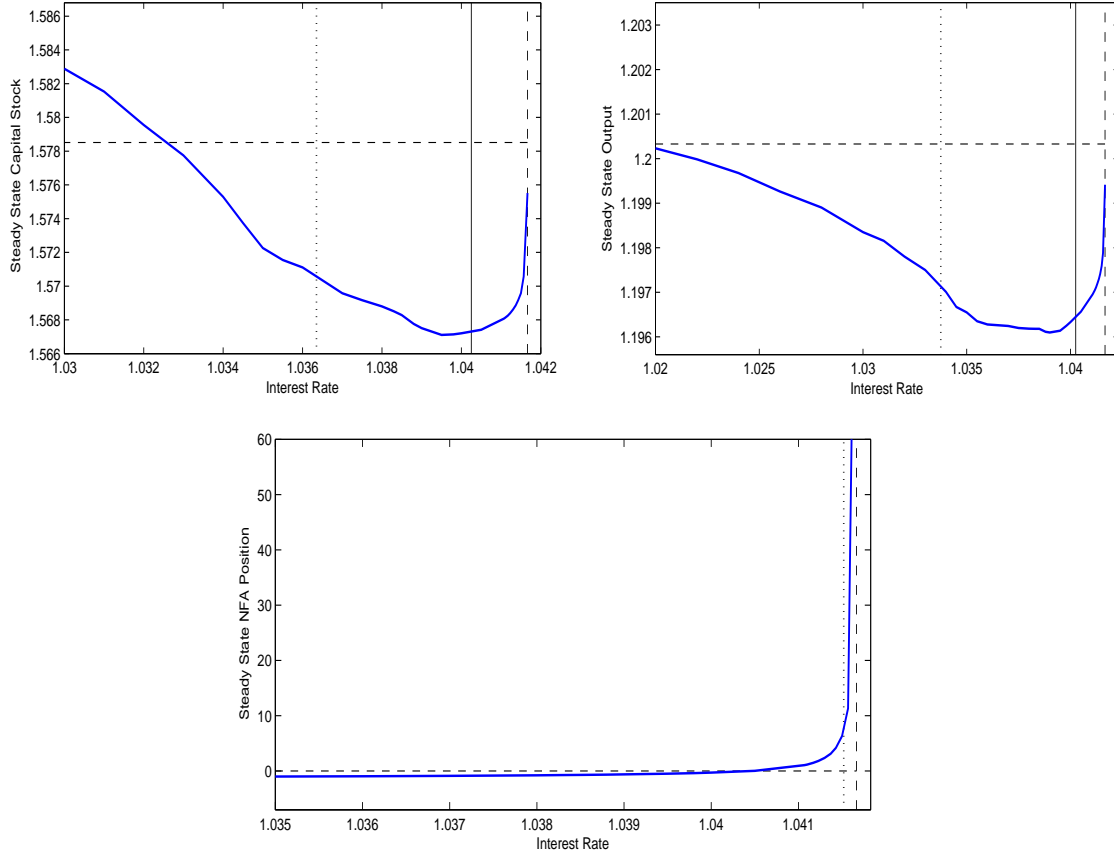


Figure 5 provides a graphical presentation of the aggregate effects. Similar to figure 2, the blue lines indicate the steady state relations between aggregate variables and the interest rate in the less financially developed benchmark economy. However, contrary to the previous scenario, entrepreneurs in the benchmark economy considered here face a borrowing limit of $\bar{b} = 1.2$ which is roughly in line with the average net income in the steady state under financial autarky (avg. net income = 1.1965).³⁵ The autarchic steady state levels of aggregate variables in the case of complete markets are indicated by the horizontal dashed lines and the corresponding level of the interest rate is indicated by the vertical dashed lines.

At first view, the main characteristics discussed in the first scenario carry over to the case where borrowing constraints play a significant role. Despite a lower interest rate level, the less financially developed country may feature lower levels of capital and output in the autarchic steady state compared to the more financially developed country. Consequently, the model may still explain capital flows from poor to rich in case of financial integration. Moreover, since the U-shaped form of the capital and the output function, i.e. the blue lines in the upper panels, is preserved, the less financially developed country may additionally end up with larger levels of capital and output

³⁵Huggett (1993) suggests a debt limit of one year's average endowment to be reasonable.

in the integrated steady state compared to the situation of financial autarky. However, in order to see how borrowing constraints influence the conditions leading to the described outcome, the corresponding saving and the corresponding investment effect both have to be considered.

Aggregate demand for the riskless asset is presented in the lower panel and the vertical dotted line indicates the autarchic steady state interest rate level in case where only the NDL is imposed in order to facilitate the comparison with the previous scenario. Comparing this interest rate to the one clearing the bond market in case where agents face the tight borrowing constraint, i.e. the interest rate where the blue line intersects the horizontal zero line, it is obvious that the latter is considerably smaller. This relative drop of the interest rate level is driven by a larger aggregate demand for the riskless asset. In a mechanical way, borrowing constraints reduce the supply of the riskless asset on the individual level as they limit the amount entrepreneurs can borrow. Moreover, entrepreneurs who are currently not constrained but whose level of net worth is rather low save more compared to the case where only the NDL is imposed. This larger individual demand for the riskless asset mainly follows from a stronger necessity to self-insure against income fluctuations. Since the lower supply and the larger demand on the individual level lead to an increased demand for the riskless asset on the aggregate level, the autarchic steady state interest rate has to decrease in case of tight borrowing constraints.

The response of the aggregate demand for the riskless asset is comparable to Huggett (1993) and Aiyagari (1994) where tight borrowing constraints as well lead to a lower interest rate in the autarchic steady state. Furthermore, if the interest rate effect is the only consequence of borrowing constraints, there is a clear answer to the question of interest. Everything else equal, a lower interest rate level pushes the location of the corresponding autarchic steady state capital stock to the left on the capital function. Consequently, it becomes less likely that the less financially developed country ends up with lower autarchic steady state levels of capital and output compared to the more financially developed country. Moreover, it becomes also less likely that the autarchic steady state capital stock of the less financially developed country is even located on the increasing part of the capital function. In terms of the previous discussion this means that tight borrowing constraints unambiguously lead to larger critical values of the elasticity of intertemporal substitution compared to the numbers in table 2 and 3 and thus to stronger parameter restrictions. However, in our case, borrowing constraints also influence entrepreneurs' risky investment decisions and the capital function presented in the upper left panel in figure 5 differs from its counterpart in figure 2.

The main changes in the relation between the long-run aggregate capital stock and the interest rate implied by borrowing constraints can be identified with the help of the solid and the dotted vertical line in the upper left panel. Both lines refer to the benchmark economy considered in the previous scenario where only the NDL is imposed. The solid line indicates the interest rate level where the sign of the slope of the capital function changes, i.e. where the long-run aggregate capital stock starts to increase if the interest rate increases. The dotted line indicates the level of the interest rate where the capital function hits the horizontal complete markets line. Comparing these interest rate levels with the corresponding ones of the economy with the tight borrowing constraint shows that the latter are considerably lower. Consequently, with tight borrowing constraints, it is possible to observe a lower interest rate while the aggregate capital stock is still located on the increasing part of the capital function and to observe a lower interest rate while levels of capital and output are still lower compared to the complete markets case. In other words, the interest rate effect of borrowing constraints may be compensated by the weakening of the upward trend of the long-run aggregate capital stock associated with low interest rates and borrowing constraints do not necessarily have to lead to larger critical values of ϑ . The intuition underlying this observation

is most clearly seen on the individual level. As borrowing constraints not only affect consumption smoothing but also limit the entrepreneurs' options to scale up production using external funds, constrained entrepreneurs invest less compared to the case where only the NDL is imposed. Moreover, entrepreneurs who are currently not constrained but whose level of net worth is rather low, choose a lower capital stock as well. Intuitively, the riskless asset is more suitable than the risky asset in order to transfer resources to those states where borrowing constraints may bind and entrepreneurs are willing to adjust their portfolio in favor of the riskless asset. Consequently, individual investment levels are lower in case of tight borrowing constraints, either because constraints are binding today or are expected to bind in the future.³⁶

In sum, what can be inferred from figure 5 are ambiguous effects of borrowing constraints. On the one hand, borrowing constraints lead to a lower interest rate in the autarchic steady state compared to the case where only the NDL is imposed. On the other hand, the interest rate level from which on the long-run capital stock starts to increase if the interest rate increases and the interest rate level where the capital stock of the less financially developed economy reaches the same level as in the economy with complete markets may be lower as well. Consequently, taking account of borrowing constraints may in principle lead to either stronger or weaker parameter restrictions in order to observe lower autarchic steady state levels of capital and output in the less financially developed country as well as to additionally observe that, starting from the autarchic steady state, long-run levels of capital and output necessarily increase with the interest rate. To identify the dominant effect, we consider several debt limits and compare the corresponding critical values of the elasticity of intertemporal substitution. The results are presented in table 4 and table 5.

Table 4: Under-accumulation of capital - borrowing constraints

Debt Limit	Critical Value
NDL	0.076
twice average net income	0.111
average net income	0.154
20 percent of average net income	0.417

Similar to table 2, the numbers in table 4 represent the critical values of the elasticity of intertemporal substitution leading to the same autarchic steady state capital stock in the economy with financial frictions as in the economy with complete markets. Consequently, if ϑ is larger than the respective threshold level, the economy suffering from financial market imperfections is poorer in terms of capital and output in the autarchic steady state compared to the case of complete markets. While production of the final good is additionally negatively affected by the impact of borrowing constraints on the TFP level, we stick to our approach and focus on the critical values of ϑ describing the behavior of the aggregate capital stock. Firstly, a lower aggregate capital stock in the less financially developed country also means a lower output level. Secondly, changes in the efficiency of the final good production do not directly influence the entrepreneur's individual expected return.

The critical value presented in the first row of table 4 refers to the NDL considered in the previous scenario and is already reported in table 2. The second debt limit corresponds roughly to twice the average net income in the autarchic steady state ($\bar{b}=2.4$). This specification is in line with Covas

³⁶Angeletos and Panousi (2011) also mention the effects of borrowing constraints on capital accumulation and the interaction with changes in the wealth level, but borrowing constraints do not play a crucial role in their theoretical model.

(2006). The critical value in the third row refers to the debt limit considered in the benchmark economy described above ($\bar{b} = 1.2$), and the most restrictive debt limit corresponds to approximately 20 percent of the average net income in the autarchic steady state ($\bar{b} = 0.24$). The latter debt limit indicates the effects arising when the financial sector is almost completely collapsed. Though the discussion above has shown that the effects of borrowing constraints are generally ambiguous, we observe a monotone increase of the critical value. Consequently, the saving and interest rate effect dominates and tighter borrowing constraints lead to stronger parameter restrictions in order to observe lower levels of capital and output in the economy with financial frictions.³⁷ Compared to the NDL, the critical value is twice as large if the debt limit is equal to the average net income in the autarchic steady state and becomes roughly 5 times larger when entrepreneurs can only borrow up to approximately 20 percent of the average net income in the autarchic steady state.³⁸ Expressed in terms of the parameter of relative risk aversion, ρ , borrowing constraints reduce the critical value from approximately 13 when only the NDL is imposed to 2.4 when the debt limit is equivalent to 20 percent of the average net income in the autarchic steady state. Similarly, if α is set to 0.6, the critical value of the elasticity of intertemporal substitution monotonically rises from 0.147 when only the NDL is imposed to 0.455 when the debt limit corresponds to 20 percent of the average net income in the autarchic steady state.

Table 5: Long-run effects capital and output - borrowing constraints

Debt Limit	Critical Value
NDL	0.182
twice average net income	0.435
average net income	0.556
20 percent of average net income	2.5

Turning to the results presented in table 5, we observe a similar effect of tightening borrowing constraints. The critical value of ϑ means that the autarchic steady state capital stock is equal to the minimum level of the corresponding capital function. Consequently, if ϑ is larger than the threshold level, the autarchic steady state capital stock is located on the increasing part of the capital function and the economy with financial frictions does not only feature lower levels of capital and output in the autarchic steady state compared to the complete markets case but an increase in the interest rate in case of financial integration also necessarily leads to larger levels of capital and output in the integrated steady state compared to the situation of financial autarky. Based on the same borrowing limits considered before and in line with the results presented in table 4, we observe that tighter borrowing constraints lead to a larger critical value. If α is set to 0.6, the results are quite similar. The critical value monotonically increases from 0.385 when only the NDL is assumed to 2.5 when the debt limit approximately equals 20 percent of the average net income in the autarchic steady state. Before these results are discussed more in detail, however, we turn to the general case with frictions on both sides.

5.3.1 The general case with borrowing constraints

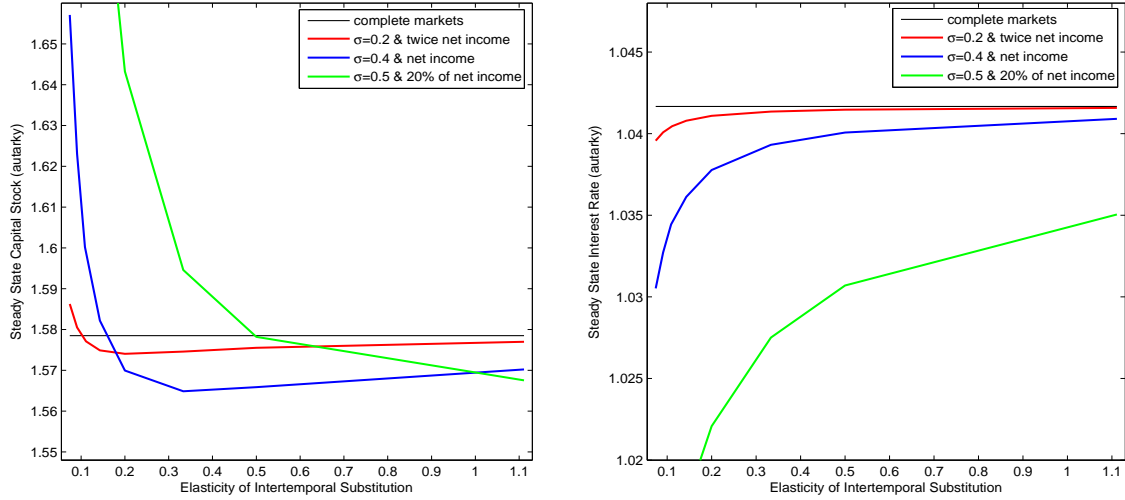
Since the level of financial development is defined by the amount of uninsurable risk and by the tightness of the borrowing constraint in this scenario, we consider several economies which differ

³⁷The observation of stronger parameter restrictions is in line with Covas (2006).

³⁸If the borrowing limit is approximately equal to 10 percent of the average net income in the autarchic steady state, the critical value is even around 0.8.

along both dimensions. The results are presented in figure 6.

Figure 6: Critical values, changes in risk and debt limits



Similar to figure 4, the left panel of figure 6 shows the relationship between the elasticity of intertemporal substitution and the corresponding capital stock level in the autarchic steady state for each of the economies. Apart from differences in the level of financial development, the economies share the same values of $\{\alpha, \beta, \delta\}$ as the benchmark economy. The horizontal black line corresponds to the case of complete markets while the economy represented by the red line is associated with $\sigma = 0.2$ and a debt limit approximately equivalent to twice the average net income in the autarchic steady state ($\bar{b} = 2.4$).³⁹ The blue line refers to the benchmark economy with $\sigma = 0.4$ and a borrowing limit approximately equal to the average net income in the autarchic steady state ($\bar{b} = 1.2$). Finally, the case where entrepreneurs can only borrow up to roughly 20 percent of the average net income in the autarchic steady state ($\bar{b} = 0.24$) and where σ equals 0.5 is represented by the green line. The corresponding interest rate levels in the autarchic steady state are shown in the right panel.

The positions of the lines in the right panel show that the autarchic steady state interest rate monotonically decreases with the portion of uninsurable risk and the tightness of the borrowing constraint. Since both, a larger amount of risk and a tighter debt limit, leads to an increased demand for the riskless asset, the differences in the interest rates between the economies are quite substantial. The lower interest rate in the less financially developed country means that capital flows from the less financially developed to the more financially developed country in case of financial integration. Turning to the question under which condition the less financially developed country is also the initially poor country, as it is necessary to observe capital flows from poor to rich, the relations presented in the left panel largely reflect the effects of borrowing constraints shown in table 4. In particular, all of the critical values of the elasticity of intertemporal substitution determined by the intersection points of the different lines are larger compared to the previous scenario where borrowing constraints play no role. For example, ϑ has to be larger or at least equal to 0.2 in order to guarantee that the benchmark economy represented by the blue line has a lower capital stock and a lower output level in the autarchic steady state compared to the relatively better financially developed country represented by the red line. Comparing the green line and the blue line shows that the corresponding critical value is even equal to 1. In the previous

³⁹Note that \bar{b} remains fixed at its indicated level within each economy and does not vary with ϑ .

scenario, a value of ϑ slightly larger than 0.08 already leads to capital flows from the initially poor to the initially rich country.

The results presented in the left panel of figure 6 also show that the effects of borrowing constraints are robust to a simultaneous increase of the amount of risk. As discussed in the previous scenario, a larger value of σ tends to reduce the threshold level if borrowing constraints play no role. However, with borrowing constraints, we observe that the largest critical value of ϑ refers to the economy with the largest value of σ due to the tight borrowing constraint in that country.

In sum, if borrowing constraints do not only limit the access to external funds for scaling up production, the main mechanism widely used to explain capital flows from poor to rich, but also affect agents through the consumer credit channel, we observe stronger parameter restrictions compared to the case where borrowing constraints play no important role. Whether these restrictions may turn out to be too strong depends on the question of interest and the tightness of the borrowing constraint. By exclusively focusing on the direction of capital flows, uninsurable risk and borrowing constraints may still explain capital flows from poor to rich for plausible parameter values as long as the borrowing constraint is not too tight in both countries. The intersection point of the blue line and the red line, for example, still means a critical value of ϑ around 0.2. However, as the green line shows, the critical value easily rises to 0.6 or larger when borrowing constraints are very tight. Turning to the question of larger levels of capital and output in the integrated steady state despite an increase in the interest rate, we observe similar effects of borrowing constraints. When only the NDL is imposed, and given the benchmark values of $\{\alpha, \beta, \delta\}$, a value of ϑ slightly larger than 0.182 ensures that the less financially developed country necessarily features larger levels of capital and output in the integrated steady state compared to the situation of financial autarky. However, table 5 shows that the critical value already becomes larger than 0.5 in case where entrepreneurs face a debt limit approximately equal to the average net income in the autarchic steady state. Tighter debt limits may even require that ϑ is larger than 2 and, thus, at least twice as large as suggested by main parts of the empirical literature. In any case, borrowing constraints as specified in our model weaken rather than strengthen the ability of financial market imperfections to serve as an explanation for the pattern of capital flows and the positive correlation between domestic economic development, larger interest rates and capital outflows.

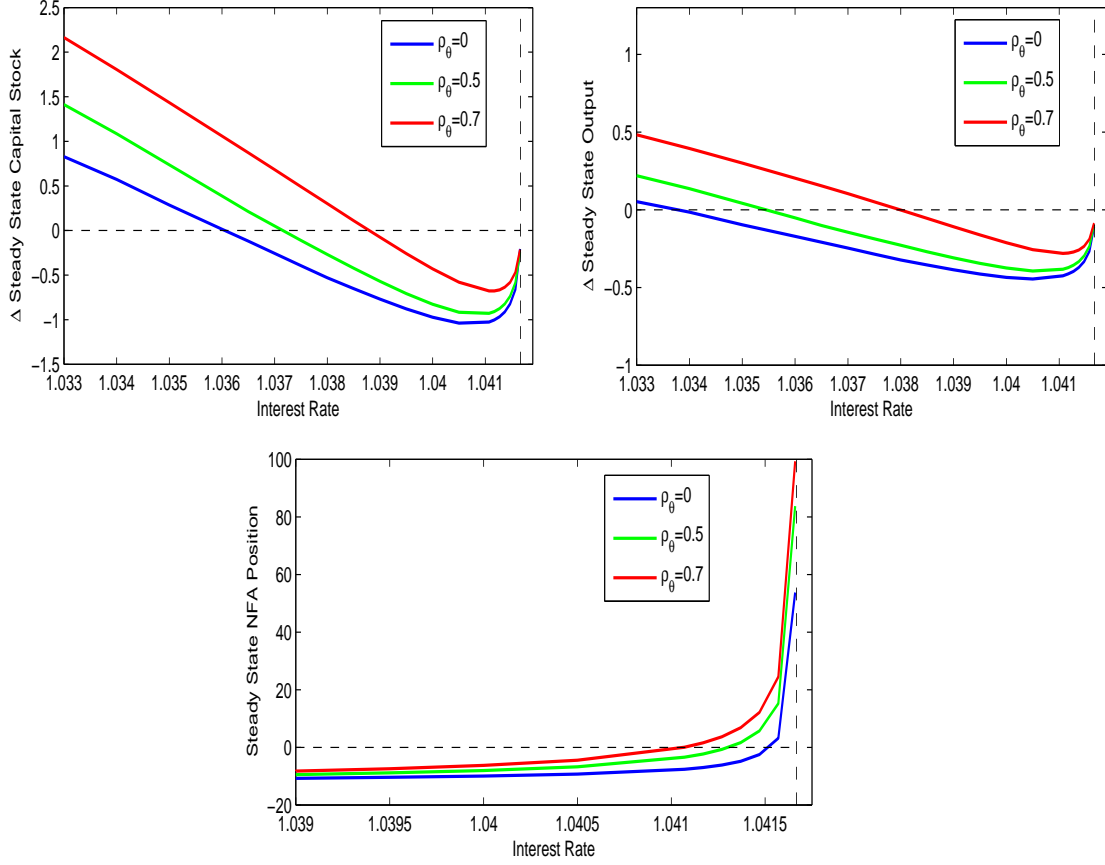
5.4 Scenario 3: Persistence

The last scenario focuses on the effects associated with changes in the properties of the productivity process. So far, productivity is described by a simple i.i.d. shock and current levels of the individual return and of profits do not affect entrepreneurs' expectations regarding future developments. However, though this assumption is widely used for analytical reasons, the empirical literature emphasizes the existence of substantial persistence in labor earnings risk and business income risk. Focusing on labor earnings, Storesletten et al. (2004), for example, find idiosyncratic risk to be highly persistent with an annual autocorrelation coefficient of 0.95. Even more related to our framework are the findings by DeBacker et al. (2012). The authors analyze the properties of business income risk from privately held businesses and find persistence in business income to be significant, even if somewhat less pronounced compared to labor income.

In order to take account of these empirical findings, we drop the assumption of $\rho_\theta = 0$ in the following. We focus on the question how the presence of persistent effects of shocks may influence the results from the previous scenarios. First insight into the effects is provided by figure 7.

Each of the colored lines in the upper left panel shows the percentage deviation of the long-run aggregate capital stock under incomplete markets from the corresponding level of the aggregate

Figure 7: Aggregate results: persistence



capital stock in the autarchic steady state under complete markets for a specific value of ρ_θ . If the difference becomes zero, the long-run aggregate capital stock of the less financially developed economy is at the same level as in the autarchic steady state under complete markets. The blue line refers to the first scenario with $\rho_\theta = 0$ and mirrors the relation between the capital function and the horizontal complete markets line presented in the upper left panel of figure 2. The green line represents the case of $\rho_\theta = 0.5$ and the red line represents the case of $\rho_\theta = 0.7$. Apart from differences in the persistence parameter, the economies share the same parameter values as the benchmark economy of the first scenario. Especially, σ is equal to 0.4 and borrowing constraints do not play a role yet. Differences in aggregate output are presented in the upper right panel and aggregate demand for the riskless asset, normalized by the respective level of the autarchic steady state capital stock under complete markets, is presented in the lower panel.

The effects of persistence on aggregate demand for the riskless asset are similar to those discussed by Covas (2006) and known from incomplete markets models with uninsurable labor income risk (Aiyagari, 1994). The positions of the colored lines in the lower panel show that the demand for the riskless asset increases with the persistence parameter. A larger value of ρ_θ means that, on average, recovering from bad times takes longer and agents try to protect themselves against a long series of low consumption levels. Consequently, the interest rate has to decrease at larger levels of ρ_θ in order to clear the bond market in the autarchic steady state.

Turning to the effects on aggregate capital accumulation presented in the upper left panel, we observe that the level of the interest rate where the long-run capital stock under incomplete markets reaches the same level as in the autarchic steady state under complete markets, i.e. where the

colored lines hit the horizontal zero line, increases with ρ_θ . Combined with the interest rate effect, this change in aggregate capital accumulation suggests that the existence of persistent effects of shocks unambiguously leads to stronger parameter restrictions in order to observe lower levels of capital and output in the less financially developed country. A larger value of ρ_θ leads to a lower interest rate in the autarchic steady state and, simultaneously, increases the critical level of the interest rate from which on the economy with incomplete markets features lower levels of capital and output in comparison to the complete markets case.

Table 6: Under-accumulation of capital - persistence

Autocorrelation Coefficient (ρ_θ)	Critical Value
0	0.076
0.3	0.110
0.5	0.159
0.7	0.278

The critical values of the elasticity of intertemporal substitution presented in table 6 confirm this reasoning. In the first scenario with $\rho_\theta = 0$, a value of ϑ slightly larger than 0.076 already ensures that the less financially developed benchmark economy features lower levels of capital and output in the autarchic steady state compared to the complete markets case and, consequently, capital does flow from poor to rich. If ρ_θ is equal to 0.5, however, the critical value doubles, and if ρ_θ is equal to 0.7, which is still a rather moderate level based on the empirical literature, the critical value even increases to approximately 0.3. Consequently, if shocks have persistent effects, we observe stronger parameter restrictions in order to explain capital flows from poor to rich countries with the help of uninsurable risk. The monotone increase of the critical value of ϑ is also in line with the intuition that the average agent, who does not hold bonds in equilibrium under financial autarky, has to choose a larger capital stock in response to the increased importance of precautionary saving.

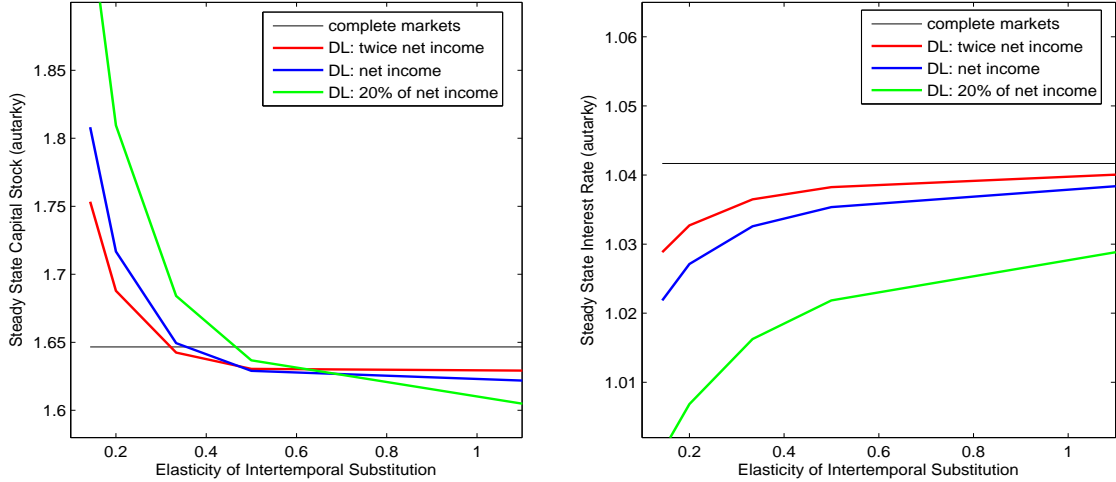
Table 7: Long-run effects capital and output - persistence

Autocorrelation Coefficient (ρ_θ)	Critical Value
0	0.182
0.3	0.278
0.5	0.417
0.7	0.667

Regarding the critical value of the elasticity of intertemporal substitution guaranteeing that the economy with incomplete markets additionally ends up on the increasing part of the capital function in the autarchic steady state, the relations presented in figure 7 suggest similar effects of persistence. The colored lines in the upper left panel show that the level of the interest rate where the slope of the capital function changes the sign increases with ρ_θ . Even though the effect is rather weak, the drop in the interest rate needed to clear the bond market in the equilibrium under financial autarky means that the point where the autarchic steady state capital stock is equal to the minimum level of the corresponding capital function is reached much faster, i.e. at larger values of the elasticity of intertemporal substitution, compared to the case without persistence. Table 7 confirms this reasoning. The number in the first row refers to the first scenario while the other results show

that the critical value of ϑ monotonically increases if ρ_θ becomes larger. If ρ_θ is equal to 0.5, the elasticity of intertemporal substitution has to be larger than approximately 0.4 to ensure that the autarchic steady state capital stock is located on the increasing part of the capital function and that an increase of the interest rate in the case of financial integration necessarily leads to larger long-run levels of capital and output. The critical value of ϑ associated with $\rho_\theta = 0.7$ is even equal to 0.667, which emphasizes the possibly strong effects of persistence.

Figure 8: Critical values, persistence and changes in debt limits



Turning to the interplay of persistence and borrowing constraints, the corresponding effects are presented in figure 8. The colored lines in the left panel show the relationship between the elasticity of intertemporal substitution and the corresponding autarchic steady state capital stock for several economies which only differ in the tightness of the borrowing constraint.⁴⁰ The corresponding levels of the autarchic steady state interest rate are presented in the right panel. Each of the economies is characterized by the same values of $\{\alpha, \beta, \delta, \sigma\}$ as the benchmark economy of the first scenario and ρ_θ is equal to 0.7.⁴¹ Similar to the discussion of persistence and uninsurable risk presented above, we expect that the existence of persistence alone, even if ρ_θ does not differ between the economies, leads to different results compared to the case where persistence is neglected.

The positions of the colored lines in the right panel show that the interest rate monotonically decreases with the tightness of the borrowing constraint. This result is similar to the second scenario where persistence plays no role and means that capital does flow from the less financially developed to the more financially developed country in the case of financial integration. However, compared to figure 6 and especially when entrepreneurs suffer from a very tight borrowing constraint, we observe much lower levels of the interest rate in the autarchic steady state since persistence increases the demand for the riskless asset.

The implied pressure on the interest rate also drives the results presented in the left panel. The intersection points of the colored lines with the horizontal complete markets line show that the critical values of the elasticity of intertemporal substitution are uniformly larger compared to the numbers in table 4 where persistence plays no role. For example, ϑ has to be approximately larger than 0.35 to guarantee that the less financially developed country represented by the blue line in figure 8 also features lower levels of capital and output in the autarchic steady state compared

⁴⁰We choose the same values for \bar{b} as in the previous scenario.

⁴¹In order to focus exclusively on the interplay of persistence and borrowing constraints, we do not assume that the economies also differ in the amount of uninsurable risk. See figure 6 for a discussion of that case.

to the complete markets case and capital does flow from poor to rich. Without persistence, the corresponding critical value is 0.154. When the debt limit is less restrictive as it is demonstrated by the red line, the critical value increases from 0.111 in the case where persistence is absent to approximately 0.31 in the case where ρ_θ is equal to 0.7.⁴²

In the general case with frictions on both sides, the results are somewhat mixed but on average the critical value also increases. The intersection point of the red line and the blue line, for example, implies a critical value of ϑ approximately equal to 0.5. Even though we neglect differences in the portion of risk between countries in the exercise considered here, this value is more than two times larger compared to the corresponding threshold level in figure 6 where persistence plays no role in the two economies.

In sum, by focusing on the interaction of persistence and borrowing constraints, we find that taking persistent effects of shocks into account also leads to stronger parameter restrictions in most of the cases. In particular, we observe a significant increase of the critical value of ϑ at moderate levels of the borrowing constraint if persistence is taken into account.

Since this section completes the discussion of the different scenarios considered in this paper, we employ a simple numerical example in order to summarize some of the main results. Initially, we consider two economies, both characterized by the same values of $\{\alpha, \beta, \delta, \bar{b}, \rho_\theta\}$ as the benchmark economy of the first scenario, but with differences in the amount of risk agents have to bear. In the large and more financially developed country, determining the common riskless interest rate under financial integration, a lower amount of risk remains with the entrepreneurs compared to the small and less financially developed country. Differences in the portion of risk between countries and the absolute level of risk in both economies are assumed to be rather small. Under this set of assumptions, we observe a lower autarchic steady state interest rate in the less financially developed country since the larger amount of risk leads to a larger demand for the riskless asset. Consequently, if financial integration takes place, the interest rate increases from the perspective of the less financially developed economy and the country builds up a positive net foreign asset position. Since capital does flow from the less to the more financially developed country, capital flows from poor to rich require that the less financially developed country is also the poor one in the initial autarchic steady state. In this first scenario considered here, where borrowing constraints and persistence do not play a role yet, we find that the larger amount of risk in the less financially developed country leads to a lower level of economic development in the autarchic steady state and capital flows from poor to rich if the elasticity of intertemporal substitution is slightly larger than 0.08.

In the next step, we assume that borrowing constraints additionally play a role in both countries, but are more severe in the less financially developed economy. Entrepreneurs in the less financially developed country face a debt limit which is approximately equal to the average net income in the autarchic steady state while entrepreneurs in the more financially developed country can borrow up to roughly twice the average net income in the autarchic steady state. The tighter constraint in the less financially developed country means a stronger drop of the autarchic steady state interest rate so that the direction of capital flows between the less and the more financially developed country remains unaffected. However, the threshold level which has to be exceeded by the elasticity of intertemporal substitution in order to ensure that the less financially developed country is still the poor one in the initial autarchic steady state changes with this additional dimension of financial

⁴²For $\alpha = 0.6$ and very tight borrowing constraints, we find a slightly lower critical value if ρ_θ is equal to 0.7 compared to the case with $\rho_\theta = 0$. For most of the debt limits, however, accounting for persistence also leads to stronger parameter restrictions.

frictions. Compared to the previous case, the critical level more than doubles and requires that ϑ is equal or larger than 0.2.

Finally, we assume that ρ_θ is equal to 0.7 in both of the economies. In this case, the larger amount of risk and the tighter borrowing constraint in the less financially developed country still lead to a lower interest rate in the autarchic steady state compared to the more financially developed country. However, the critical value of ϑ guaranteeing capital flows from poor to rich again more than doubles from 0.2 to approximately 0.5.⁴³

Regarding the long-run consequences of the integration process on domestic capital accumulation and output, we observe similar changes in the parameter restrictions between the scenarios. In the first scenario of this example, a value of ϑ slightly larger than 0.18 already ensures that the less financially developed country is not only the poor one in the autarchic steady state, but necessarily features larger levels of capital and output in the integrated steady state. However, in the last scenario, the critical value is more than three times larger compared to the case where borrowing constraints and persistence play no role.

This simple example shows how differences in the specification of financial frictions and differences in the properties of the underlying stochastic process may lead to different parameter restrictions and, consequently, may alter the model predictions for plausible parameter values. Though the critical values of ϑ mainly stay below unity, we observe a significant increase in the last two scenarios where borrowing constraints and persistence are taken into account. Since parts of the empirical literature also support values of the elasticity of intertemporal substitution less than unity, these changes are not only of theoretical interest. Furthermore, the critical values in the first scenario of this example are rather small. A slightly larger value of α or a slightly lower depreciation rate may lead to larger critical values of ϑ in the first and, consequently, in the other scenarios.

6 Transitory dynamics and welfare effects

In this section, we briefly consider the transitory dynamics between steady states and discuss the corresponding welfare effects of financial market liberalization. Since our calibration strategy mainly captures standard values from the literature and serves to discuss the different model implications, we focus on the general pattern in this section.

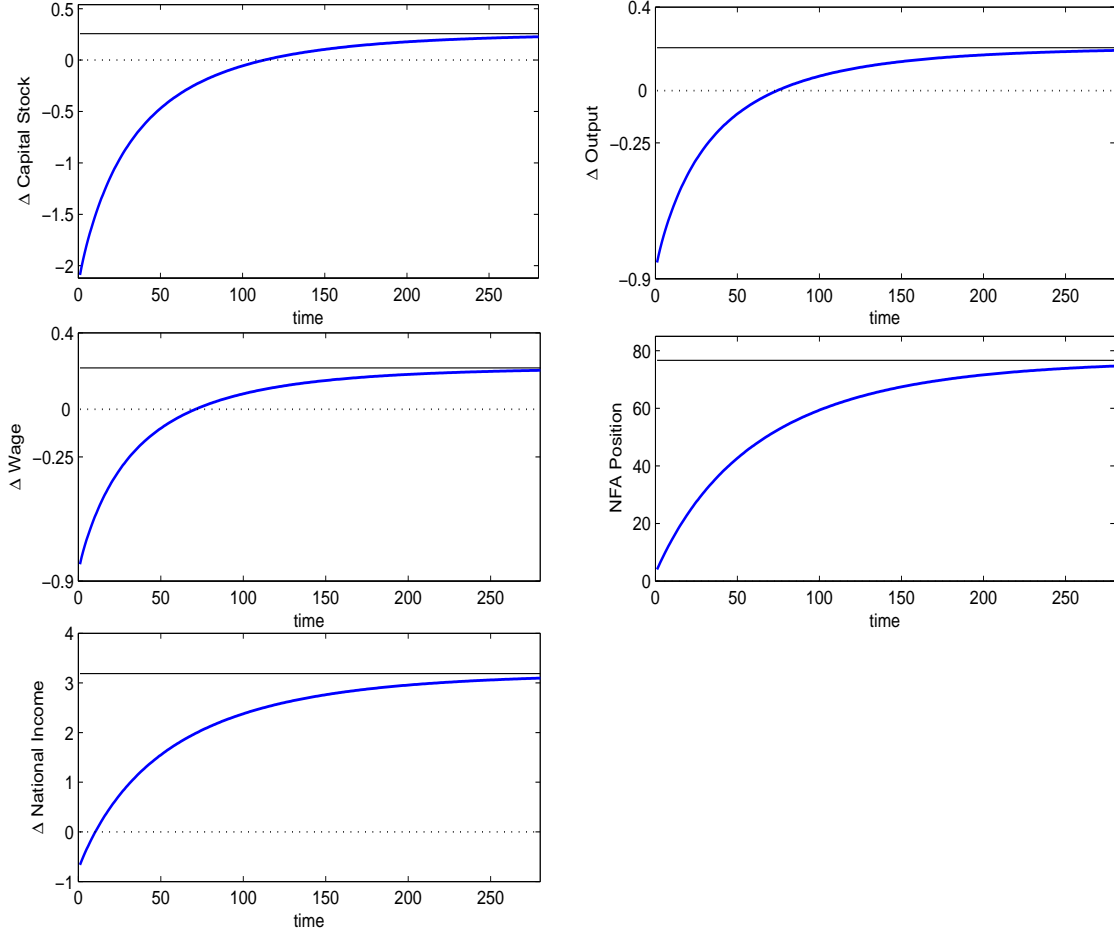
6.1 Transitory dynamics

Figure 9 shows the transition dynamics of selected macroeconomic variables from the steady state under financial autarky to the integrated steady state. The exercise takes the perspective of the less financially developed country considered at the end of the previous section. The debt limit faced by the entrepreneurs is approximately equal to the average net income in the autarchic steady state, σ is equal to 0.4 and ρ_θ is equal to 0.7. The other parameter values are identical to the benchmark economy of the first scenario. The dynamics of aggregate variables over time are presented by showing the percentage deviation from the autarchic steady state. A positive number means a larger level and a negative number means a lower level compared to the autarchic steady state. The integrated steady state is indicated by the solid lines.

The interest rate under financial integration ($R^* = 1.039$) is determined by the large and more financially developed country. Entrepreneurs in the more financially developed country can borrow up to roughly twice the average net income in the autarchic steady state which leads to a 6

⁴³Note that we assume rather small differences in the amount of risk between economies and refer to the critical value shown in figure 8.

Figure 9: Transitory dynamics



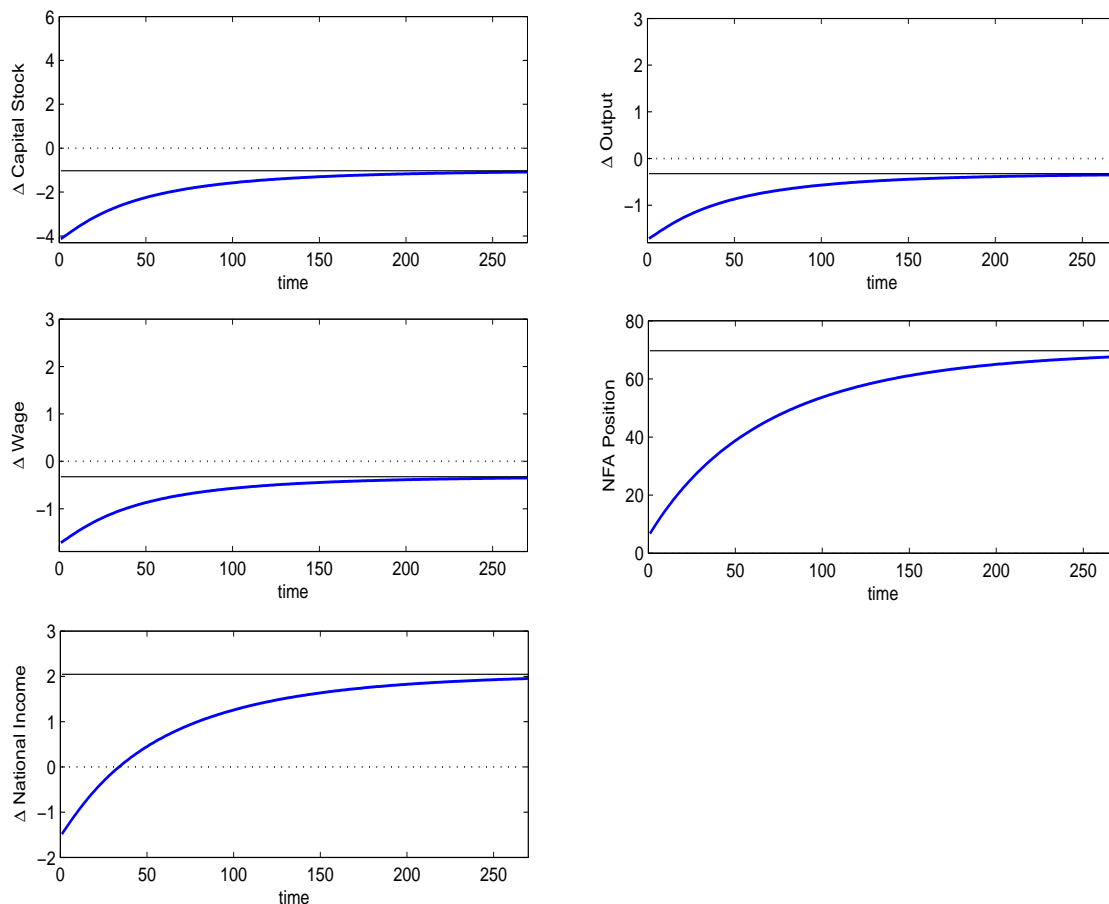
percent larger net interest rate in the initial autarchic steady state compared to the less financially developed country. When barriers to trading the riskless asset are removed, this gap is closed immediately and the less financially developed country builds up a positive net foreign asset position as shown in the middle right panel of figure 9. The monotone increase of the net foreign asset position means that the less financially developed country runs a series of current account surpluses along the transition path and finally reaches its long-run asset position equivalent to roughly 80 percent of the initial output level. As mentioned earlier, this pattern does not only occur in the exercise considered here but generally arises if financial integration is associated with an increase of the interest rate. Since the less financially developed country is also the poor one in the autarchic steady state, capital flows from the less to the more financially developed country and also from poor to rich.

The evolution of the net foreign asset position also influences the short-run dynamic of the domestic aggregate capital stock. When trading barriers are removed, the capital stock in the less financially developed country initially falls below its autarchic steady state level but immediately starts to recover. Since the elasticity of intertemporal substitution is just large enough in order to ensure that the long-run capital stock increases with the interest rate based on the autarchic steady state, the capital stock eventually reaches a larger level compared to the situation of financial autarky. The observed short-run behavior of the aggregate capital stock is similar to Angeletos and Panousi (2011) who consider the case without binding borrowing constraints and without persistence and is driven by the different timing of the opportunity-cost and wealth effect. The upward adjustment

of the interest rate immediately reduces entrepreneur's incentive to invest in the own firm, while accumulation of wealth, stimulating investment in the following, is a gradual process. Output and wage largely follow the described adjustment of the aggregate capital stock, but both variables are back at their autarchic steady state levels somewhat faster. The faster recovering is driven by the improvement in production efficiency since the accumulation of wealth helps entrepreneurs to partly overcome the borrowing constraint and reduces differences in the risk premium implied by uninsured capital risk. Hence, financial integration leads to an increase in total factor productivity. The short-run dynamic of national income reflects the adjustment of output and of the net foreign asset position. The initial drop is driven by the reduction in output, but interest earnings lead to a fast and substantial increase in the following.

In order to prepare the ground for the following welfare analysis, we also consider a second exercise which demonstrates the case of lower long-run levels of capital and output under financial integration compared to the situation of financial autarky. We obtain such a long-run development by reducing the elasticity of intertemporal substitution from 0.67 to 0.25. We also assume a larger increase of the net interest rate from the perspective of the less financially developed country (+14%) when trading barriers are removed in order to obtain a similar long-run net foreign asset position as in the main exercise. However, a smaller increase of the interest rate leads to similar results and does not change the general pattern of this alternative exercise.

Figure 10: Transitory dynamics, alternative exercise



The key difference compared to the main exercise is that the elasticity of intertemporal substitution is now less than the corresponding critical value. Consequently, from the perspective of the less

financially developed country, the increase of the interest rate implied by financial integration does not lead to larger long-run levels of capital and output. Instead, figure 10 shows that the blue lines representing the dynamics of the capital stock, output and the wage stay permanently below the corresponding horizontal zero line, indicating the initial autarchic steady state. From this it follows that the less financially developed country features a lower capital stock, a lower output level and a lower wage level in the integrated steady state compared to the steady state under financial autarky. Since this long-run development is quite different compared to the main exercise where the wage, capital and output reach larger levels in the integrated compared to the autarchic steady state, we will revisit this alternative exercise during the following welfare analysis.

6.2 Welfare implications

The individual welfare effect of financial integration is measured as the proportional change in consumption required to leave each entrepreneur indifferent between the steady state under financial autarky and the case of financial market liberalization. The latter case includes the transition from the autarchic to the integrated steady state. Formally, for each entrepreneur being identified by corresponding values of $(\omega, \tilde{\theta})$, the individual welfare effect, $g(\omega, \tilde{\theta})$, solves

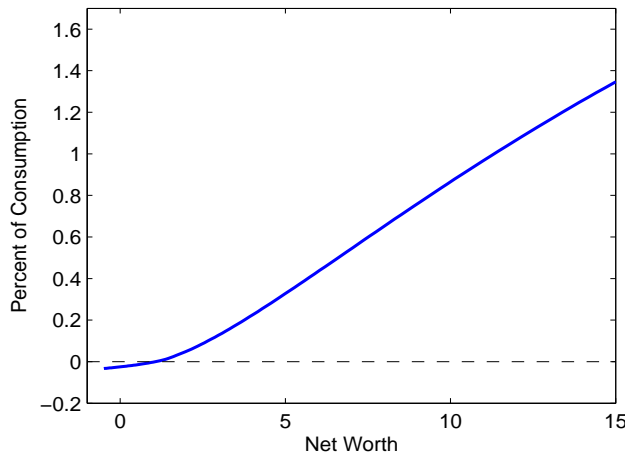
$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^A (1 + g(\omega, \tilde{\theta}))) = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^I), \quad (43)$$

where c_t^A denotes consumption under financial autarky and c_t^I denotes consumption in the case of financial market liberalization. Starting from the autarchic steady state, the liberalization reform takes place in period 0. Simplifying (43) leads to

$$(1 + g(\omega, \tilde{\theta}))^{1-\rho} = V^I(\omega, \tilde{\theta}) / V^A(\omega, \tilde{\theta}), \quad (44)$$

where $V^I(\omega, \tilde{\theta})$ and $V^A(\omega, \tilde{\theta})$ are the value functions under financial integration and financial autarky, respectively.⁴⁴ A positive value of g means a welfare gain.

Figure 11: Individual welfare effects



Initially, we focus on the main exercise where transitory dynamics and the long-run positions are presented in figure 9 and financial integration leads to larger long-run levels of capital and output compared to the autarchic steady state from the perspective of the less financially developed

⁴⁴In the case of log-utility, (44) reads $1 + g(\omega, \tilde{\theta}) = \exp((1 - \beta)(V^I(\omega, \tilde{\theta}) - V^A(\omega, \tilde{\theta})))$.

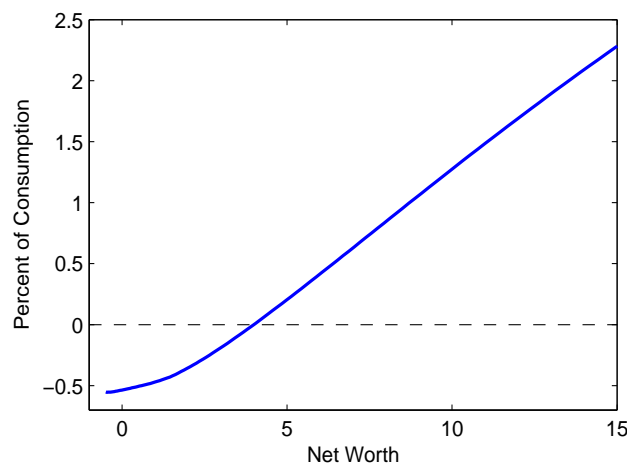
country. The corresponding welfare effects as a function of initial net worth for entrepreneurs with average productivity level are presented in figure 11.

For low or negative levels of net worth we observe a welfare loss while financial integration turns out to be beneficial for entrepreneurs with larger levels of net worth. The largest welfare loss is rather small, roughly 0.05 percent of consumption, whereas welfare gains are more substantial for rich entrepreneurs. In sum, the poor lose while the rich win. This result also applies to other productivity levels and is similar to Mendoza et al. (2009b), Angeletos and Panousi (2011), and depending on the tightness of the credit constraint, Clemens and Heinemann (2013). However, especially in comparison to Mendoza et al. (2009b), some interesting differences emerge by having a closer look at the individual effects.

In general, the fact that welfare effects vary with the level of net worth is a consequence of the different impact of the implied interest rate and wage effect. In the main exercise considered here, the interest rate increases from the perspective of the less financially developed country once trading barriers are removed while, as figure 9 shows, the wage initially drops, but exceeds its autarchic steady state level in the following. The larger interest rate is beneficial for savers but simultaneously means an increase in the cost of borrowing. Since the poorest agents in the autarchic steady state borrow a substantial amount, they strongly suffer from the upward adjustment of the interest rate and experience a welfare loss.

Moving to larger levels of net worth, financial integration turns out to be beneficial. According to figure 11, entrepreneurs with a net worth larger than 1.1 in the autarchic steady state are better off in the case of financial market liberalization. This result is remarkable because not all entrepreneurs are savers and gain from the larger interest rate. In fact, up to a level of net worth equal to 2.65, entrepreneurs still borrow although at a decreasing rate. Consequently, since borrowing costs increase under financial integration, the welfare gains of these entrepreneurs must crucially be influenced by the development of the capital stock, output and especially, the wage, eventually reaching larger levels under financial integration than under financial autarky. In order to examine this aspect more in detail, we revisit our alternative exercise where financial integration leads to permanently lower levels of the capital stock, output and the wage compared to the autarchic steady state. The corresponding welfare effects are presented in figure 12 and, again, refer to entrepreneurs with average productivity level.

Figure 12: Individual welfare effects, alternative exercise



Though a one-to-one comparison of individual numbers between the two exercises is restricted due to the change in the elasticity of intertemporal substitution, we observe some interesting differences

between the groups of agents gaining from financial integration. Figure 12 shows that entrepreneurs in the alternative scenario have to be about four times richer to benefit from financial market liberalization. Moreover, we do not observe entrepreneurs who are borrowers in the autarchic steady state and gain from financial integration as in the main exercise, but we do observe entrepreneurs who save and lose from financial market liberalization. This observation follows from the fact that in the alternative scenario entrepreneurs with net worth larger than 2.86 already become savers whereas a positive welfare effect of financial integration only associates with a level of net worth larger than 4. Consequently, in the second exercise, only entrepreneurs saving a substantial amount benefit from financial integration while in the main exercise, where long-run levels of the capital stock, output and the wage are larger under financial integration than under financial autarky, a larger group of agents experience a welfare gain if trading barriers are removed. Since differences in the long-run development of the capital stock, output and the wage constitute the main change in macroeconomic outcome between the two exercises, we conclude that these differences may have significant welfare effects.

A similar conclusion can also be drawn from comparing our findings with the results of Mendoza et al. (2009b). In their model, and similar to our alternative exercise, financial integration leads to permanently lower levels of the capital stock, output and the wage in the less financially developed country compared to the autarchic steady state. Though their model differs from ours and the short-run dynamics are somewhat different, their results as well suggest that borrowers and even savers with low levels of net worth suffer from financial integration.

In order to address the question whether individual gains or losses dominate on the aggregate level, we take the perspective of a social welfare planner, assigning equal weight to each agent. Following Mendoza et al. (2009b), the aggregate welfare effect, G , solves

$$(1 + G)^{1-\rho} = \int_{\omega, \tilde{\theta}} V^I(\omega, \tilde{\theta}) \Psi^A(\omega, \tilde{\theta}) \ / \ \int_{\omega, \tilde{\theta}} V^A(\omega, \tilde{\theta}) \Psi^A(\omega, \tilde{\theta}), \quad (45)$$

where $\Psi^A(\omega, \tilde{\theta})$ is the stationary distribution over entrepreneurs under financial autarky. G is the required proportional change in autarky consumption equal for all agents, that makes the social planner indifferent between financial autarky and financial market liberalization. For our main exercise, we find a positive aggregate welfare effect of 0.13 percent of consumption while we observe an aggregate welfare loss of 0.24 percent in our alternative exercise. In other words, we observe an aggregate welfare gain for the less financially developed country when the increase of the interest rate implied by financial integration leads to larger long-run levels of the capital stock, output and the wage compared to the autarchic steady state, while we observe an aggregate welfare loss when financial integration leads to permanently lower levels of the capital stock, output and the wage compared to the situation of financial autarky.

The negative welfare effect observed in the second exercise is also in line with Mendoza et al. (2009b) who report a welfare loss of 0.4 percent for the less financially developed country. Moreover, this result is not driven by the relatively larger increase of the interest rate. If we assume that the net interest rate only increases by 6 percent from the perspective of the less financially developed country, as in the main exercise, we still observe an aggregate welfare loss of 0.11 percent.⁴⁵

We conclude this section with a sensitivity analysis, focusing on the robustness of aggregate welfare gains for the less financially developed country. We consider variations of the main exercise with

⁴⁵Similarly, we still observe a welfare gain in the main exercise if we consider a larger increase of the interest rate. If, for example, r increases by 12% from the perspective of the less financially developed country, we observe an aggregate welfare gain of 0.39 percent.

respect to the borrowing constraint, the persistence parameter and the wage share. However, in all cases, the less financially developed country features larger levels of the aggregate capital stock, output and the wage in the integrated steady state compared to the autarchic steady state.

Table 8: Robustness of aggregate welfare gains

Parametrization	Welfare Effect (% of Consumption)
DL = twice average net income	0.05
DL = twice average net income, $\alpha = 0.6$	0.37
$\rho_\theta = 0.3$	0.04

We begin by loosening the borrowing constraint entrepreneurs face in the less financially developed country and consider a debt limit of roughly twice the average net income in the autarchic steady state. A looser borrowing constraint leads to a larger maximum amount of debt and may lead to larger welfare losses for poor entrepreneurs in the case of financial integration. In fact, table 8 shows a smaller corresponding aggregate welfare effect compared to the main exercise, although the increase of the net interest rate is smaller (+3%) to prevent the long-run net foreign asset position from becoming too large. However, financial market liberalization is still beneficial on the aggregate level.

We further examine the effects associated with a less restrictive debt limit by simultaneously increasing α from 0.4 to 0.6. A larger value of α means a lower wage share so that entrepreneurs become more vulnerable to uninsured risk, but it also leads to an upward level shift in aggregate variables. The aggregate welfare effect of financial integration is still positive, which indicates that welfare gains for the less financially developed country do not depend on particularly tight borrowing constraints.

Finally, we reduce the level of persistence compared to the main exercise by reducing ρ_θ from 0.7 to 0.3. Table 8 shows that the corresponding aggregate welfare effect is smaller but still positive. In sum, we find that the existence of positive welfare effects for the less financially developed country in case financial integration stimulates long-run economic development is a quite robust result. Changes in the level of financial development or changes in other model parameters may weaken, but do not necessarily cancel out the positive effects.

7 Conclusion

In this paper, we examine the effects of uninsurable idiosyncratic risk and borrowing constraints in the context of financial integration. We employ a general equilibrium model with heterogeneous entrepreneurs and focus on the direction of cross-border capital flows as well as on domestic capital accumulation and output. The effects of uninsurable idiosyncratic risk and/or borrowing constraints are already widely studied in the literature, however, the results are rather diverse: depending on the specific source of the underlying risk, the restrictions implied by borrowing constraints and the existence of persistent effects of shocks, the implications differ drastically and range from under- to over-accumulation of capital under financial autarky and from lower to larger levels of capital and output in the steady state under financial integration.

We contribute to this ongoing debate by discussing the ambiguous effects from a general perspective in an environment where entrepreneurs face capital risk, earn risky profits and receive riskless wage income. Moreover, borrowing constraints impede consumption smoothing and limit

the access to external funds for scaling up production. In order to separate the different effects, we consider different scenarios. In the first scenario, where borrowing constraints play no crucial role and shocks do not have persistent effects, we partly overcome the restriction that no analytical solution is available by deriving two rules of thumb. Our rules explain under which condition the less financially developed country features lower levels of capital and output in the autarchic steady state, implying that it is in fact the initially poor country which builds up a positive net foreign asset position under financial integration. Furthermore, these rules also explain under which condition long-run levels of capital and output necessarily increase with the interest rate, implying that the less financially developed country features larger levels of capital and output in the integrated compared to the autarchic steady state. We find that for plausible parametrizations both conditions are likely to hold. Moreover, the general discussion is further enriched by the fact that the procedure of deriving these conditions is also applicable to other model types considered in the literature. In the next scenarios, we additionally consider tight borrowing constraints and increase the level of persistence. Both aforesaid changes strongly affect the previously obtained results and mainly lead to tighter parameter restrictions in comparison to the first scenario. From a welfare perspective, we find aggregate welfare gains for the less financially developed country in case the domestic capital stock, output and the wage eventually reach larger levels under integration than under autarky.

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Appendix

A1. Two-period model - incomplete markets

The entrepreneur's maximization problem reads

$$\max_{k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{it+1}) \quad (46)$$

$$s.t. \ c_t = \omega_t - k_{t+1} - b_{t+1} \quad (47)$$

$$c_{it+1} = \omega_{it+1} \quad (48)$$

$$\omega_{it+1} = b_{t+1}R_{t+1} + k_{t+1}R_{it+1}^r + w_{t+1} + \pi_{it+1}, \quad (49)$$

where the subscript i is dropped whenever optimal choices are the same across agents. The corresponding first-order conditions are

$$U'(c_t) = \beta E_t [R_{it+1}^r U'(c_{it+1})] \quad (50)$$

$$U'(c_t) = \beta R_{t+1} E_t [U'(c_{it+1})]. \quad (51)$$

Since bond market clearing implies $\int_0^1 b_{t+1} di = b_{t+1} = 0$, the capital Euler equation becomes

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1} + \pi_{it+1})]. \quad (52)$$

In addition, the expression for the wage simplifies to

$$w_{t+1} = (1 - \alpha) \int_0^1 \tilde{\theta}_{it+1} k_{t+1}^\alpha di = (1 - \alpha) k_{t+1}^\alpha \int_0^1 \tilde{\theta}_{it+1} di = (1 - \alpha) k_{t+1}^\alpha E(\tilde{\theta}_{t+1}) = (1 - \alpha) k_{t+1}^\alpha. \quad (53)$$

A2. Two-period model - complete markets

In the complete markets case, a full set of contingent claims (Arrow securities) can be traded. In a first step, we show that each agent consumes the same amount in all states in period $t + 1$ and due to the same initial conditions, it follows that the consumption level is also the same across all agents.

The existence of Arrow securities implies the well-known result that the marginal rate of substitution between consumption in any two states (denoted by w and u) in period $t + 1$ is equalized across all agents

$$\frac{U'(c^i(s_w^{t+1}))}{U'(c^i(s_u^{t+1}))} = \frac{U'(c^j(s_w^{t+1}))}{U'(c^j(s_u^{t+1}))}, \quad (54)$$

where $c^i(s_w^{t+1})$, $(c^j(s_w^{t+1}))$ denotes consumption of agent i , (j) in period $t + 1$ in state w . Since $U'(x) = x^{-\rho}$, this simplifies to

$$\frac{c^i(s_w^{t+1})}{c^i(s_u^{t+1})} = \frac{c^j(s_w^{t+1})}{c^j(s_u^{t+1})}. \quad (55)$$

The relation presented in (55) shows that if agent i consumes different amounts in state w and u , the same must be true for all other agents and, consequently, aggregate consumption will differ across states. However, since no risk exists on the aggregate level, aggregate resources, i.e. output and capital net of depreciation, which can be used for consumption, do not differ across states. Consequently, aggregate consumption does not vary and each agent consumes the same amount in

each state in period $t + 1$. The constant consumption level is given by

$$c_{t+1} = k_{t+1} E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}], \quad (56)$$

where the subscript i attached to the return and to profits is dropped in order to emphasize that the expected value of the individual return and of individual profits is identical across all agents, i.e. $E_t[R_{it+1}^r] = E_t[R_{t+1}^r]$ and $E_t[\pi_{it+1}] = E_t[\pi_{t+1}]$. Finally, since expenditures in the initial period to finance the corresponding asset bundle sum up to zero and consumption in $t + 1$ is constant, the capital Euler equation becomes

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(k_{t+1} E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}]). \quad (57)$$

A3. Proof of Lemma 1

The first step applies a second-order Taylor expansion to $\beta R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1} + \pi_{it+1})$ around the point $(E_t[R_{t+1}^r], E_t[\pi_{t+1}])$. For small risks this leads to

$$\begin{aligned} \beta R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1} + \pi_{it+1}) &= \beta E_t[R_{t+1}^r] U'(\cdot) \\ &+ \beta [E_t[R_{t+1}^r] U''(\cdot) k_{t+1} + U'(\cdot) [R_{it+1}^r - E_t[R_{t+1}^r]] \\ &+ \beta [E_t[R_{t+1}^r] U''(\cdot) [\pi_{it+1} - E_t[\pi_{t+1}]] \\ &+ \frac{1}{2} \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] [R_{it+1}^r - E_t[R_{t+1}^r]]^2 \\ &+ \frac{1}{2} \beta [E_t[R_{t+1}^r] U'''(\cdot) [\pi_{it+1} - E_t[\pi_{t+1}]]^2 \\ &+ \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1} + U''(\cdot) [R_{it+1}^r - E_t[R_{t+1}^r]] [\pi_{it+1} - E_t[\pi_{t+1}]]], \end{aligned} \quad (58)$$

with $(\cdot) \equiv (k_{t+1} E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}])$. Applying the conditional expectation operator yields the expressions described in (28)-(31).

For notational ease, we introduce the following definitions

$$f(k_{t+1}) \equiv U'(\omega_t - k_{t+1}) \quad (59)$$

$$g(k_{t+1}) \equiv \beta E_t[R_{t+1}^r] U'(\cdot) \quad (60)$$

$$\begin{aligned} h(k_{t+1}) &\equiv \frac{1}{2} \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] \sigma_{R_{t+1}^r}^2 \\ &+ \frac{1}{2} \beta E_t[R_{t+1}^r] U'''(\cdot) \sigma_{\pi_{t+1}}^2 \\ &+ \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1} + U''(\cdot)] \sigma_{R_{t+1}^r \pi_{t+1}}, \end{aligned} \quad (61)$$

with $(\cdot) \equiv (k_{t+1} E_t[R_{t+1}^r] + w_{t+1} + E_t[\pi_{t+1}])$ and where it is explicitly indicated that all functions depend on k_{t+1} . Then, (28)-(31) can be paraphrased as

$$f(k_{t+1}) = g(k_{t+1}) + h(k_{t+1}), \quad (62)$$

while in the complete markets case, according to (27) we have

$$f(k_{t+1}) = g(k_{t+1}). \quad (63)$$

Let k_{t+1}^* denote the solution in the incomplete and \hat{k}_{t+1} denote the solution in the complete markets case.

If $h(k_{t+1}^*) = 0$ then $f(k_{t+1}^*) = g(k_{t+1}^*)$. Since \widehat{k}_{t+1} satisfies $f(\widehat{k}_{t+1}) = g(\widehat{k}_{t+1})$ as well, it follows that $k_{t+1}^* = \widehat{k}_{t+1}$.

If $h(k_{t+1}^*) < 0$ then $f(k_{t+1}^*) < g(k_{t+1}^*)$. Since $f(k_{t+1})$ is continuous and strictly increasing in k_{t+1} and $g(k_{t+1})$ is continuous and strictly decreasing in k_{t+1} , it follows that $f(k_{t+1}^*) < g(k_{t+1}^*)$ and $f(\widehat{k}_{t+1}) = g(\widehat{k}_{t+1})$ implies $k_{t+1}^* < \widehat{k}_{t+1}$.

If $h(k_{t+1}^*) > 0$ then $f(k_{t+1}^*) > g(k_{t+1}^*)$. Since $f(k_{t+1})$ is continuous and strictly increasing in k_{t+1} and $g(k_{t+1})$ is continuous and strictly decreasing in k_{t+1} , it follows that $f(k_{t+1}^*) > g(k_{t+1}^*)$ and $f(\widehat{k}_{t+1}) = g(\widehat{k}_{t+1})$ implies $k_{t+1}^* > \widehat{k}_{t+1}$.

The other way around it follows from $k_{t+1}^* = \widehat{k}_{t+1}$ that $h(k_{t+1}^*) = 0$ since $f(k_{t+1}^*) = g(k_{t+1}^*)$.

If $k_{t+1}^* < \widehat{k}_{t+1}$, it follows that $f(k_{t+1}^*) < g(k_{t+1}^*)$ since $f(k_{t+1})$ is continuous and strictly increasing in k_{t+1} , $g(k_{t+1})$ is continuous and strictly decreasing in k_{t+1} and $f(\widehat{k}_{t+1}) = g(\widehat{k}_{t+1})$. Consequently, since k_{t+1}^* satisfies $f(k_{t+1}^*) = g(k_{t+1}^*) + h(k_{t+1}^*)$, it follows that $h(k_{t+1}^*) < 0$.

If $k_{t+1}^* > \widehat{k}_{t+1}$, it follows that $f(k_{t+1}^*) > g(k_{t+1}^*)$ since $f(k_{t+1})$ is continuous and strictly increasing in k_{t+1} , $g(k_{t+1})$ is continuous and strictly decreasing in k_{t+1} and $f(\widehat{k}_{t+1}) = g(\widehat{k}_{t+1})$. Consequently, since k_{t+1}^* satisfies $f(k_{t+1}^*) = g(k_{t+1}^*) + h(k_{t+1}^*)$, it follows that $h(k_{t+1}^*) > 0$.

A4. Proof of Lemma 2

Based on Lemma 1 and second moments described in (32), levels of capital and output are lower in the incomplete compared to the complete markets case if and only if

$$\begin{aligned} & [E_t [R_{t+1}^r] U'''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^*] \alpha^4 (k_{t+1}^*)^{2\alpha-2} \sigma_{\theta_{t+1}}^2 \\ & + E_t [R_{t+1}^r] U'''(\cdot) (1-\alpha)^2 \alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 \\ & + 2[E_t [R_{t+1}^r] U'''(\cdot) k_{t+1}^* + U''(\cdot)] (1-\alpha) \alpha^3 (k_{t+1}^*)^{2\alpha-1} \sigma_{\theta_{t+1}}^2 < 0, \end{aligned} \quad (64)$$

where $(\cdot) \equiv (k_{t+1}^* E_t [R_{t+1}^r] + w_{t+1} + E_t [\pi_{t+1}])$.

Since $k_{t+1}^* > 0$, getting rid of $\alpha^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 > 0$ and collecting terms yields

$$E_t [R_{t+1}^r] U'''(\cdot) [\alpha^2 + 1 - 2\alpha + \alpha^2 + 2\alpha - 2\alpha^2] + 2U''(\cdot) (k_{t+1}^*)^{-1} [\alpha^2 + \alpha - \alpha^2] < 0, \quad (65)$$

and finally

$$-\frac{1}{\alpha} \frac{U'''(\cdot)}{U''(\cdot)} (E_t [R_{t+1}^r] k_{t+1}^*) < 2. \quad (66)$$

Applying $U''(x) = -\rho x^{-\rho-1}$ and $U'''(x) = \rho(\rho+1)x^{-\rho-2}$ leads to

$$\left(\frac{1}{\vartheta} + 1\right) \frac{1}{\alpha} \frac{k_{t+1}^* E_t [R_{t+1}^r]}{k_{t+1}^* E_t [R_{t+1}^r] + w_{t+1} + E_t [\pi_{t+1}]} < 2, \quad (67)$$

where ϑ is the elasticity of intertemporal substitution ($\vartheta = 1/\rho$). Since production of the final good simplifies to $Y_{t+1}^* = (k_{t+1}^*)^\alpha$ and $K_{t+1}^* \equiv \int_0^1 k_{t+1}^* di = k_{t+1}^*$, we get $w_{t+1} + E_t [R_{t+1}^r] k_{t+1}^* + E_t [\pi_{t+1}] = Y_{t+1}^* + (1-\delta)K_{t+1}^*$ and $k_{t+1}^* E_t [R_{t+1}^r] = \alpha^2 Y_{t+1}^* + (1-\delta)K_{t+1}^*$. Plugging these expressions into (67) leads to the final condition in (33).

A5. Proof of Lemma 3

Essentially, we apply the same steps as to derive the condition described in (33). Initially, the simplified two-period model is described and the corresponding Euler equations in case of complete

and incomplete markets are derived. Comparing these equations leads to the condition described in (35).

In the model economy considered here, individual production in period $t + 1$ takes place according to $y_{it+1} = \theta_{it+1}^\alpha k_{it+1}^\alpha l_{it+1}^{1-\alpha}$. Assuming that $\ln(\theta) \sim N(-\sigma^2/2, \sigma^2)$ and that all agents start with the same initial level of net worth, the entrepreneur's maximization problem in the incomplete markets case reads

$$\max_{k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{it+1}) \quad (68)$$

$$s.t. \quad c_t = \omega_t - k_{t+1} - b_{t+1} \quad (69)$$

$$c_{it+1} = \omega_{it+1} \quad (70)$$

$$\omega_{it+1} = b_{t+1} R_{t+1} + \pi_{it+1} + w_{t+1} + (1 - \delta) k_{t+1}, \quad (71)$$

where the subscript i is dropped whenever optimal choices are the same across agents. δ is the depreciation rate, k_{t+1} and b_{t+1} denote risky and riskless investment, respectively, and w_{t+1} is the wage rate. π_{it+1} is defined as

$$\pi_{it+1} \equiv \max_{l_{it+1}} \theta_{it+1}^\alpha k_{it+1}^\alpha l_{it+1}^{1-\alpha} - w_{t+1} l_{it+1}, \quad (72)$$

where optimal employment maximizes π_{it+1} state by state. Since $l_{it+1} = \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}} \theta_{it+1} k_{t+1}$, ω_{it+1} simplifies to

$$\omega_{it+1} = b_{t+1} R_{t+1} + k_{t+1} R_{it+1}^r + w_{t+1}, \quad (73)$$

where $R_{it+1}^r \equiv 1 - \delta + \theta_{it+1} \alpha \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1-\alpha}{\alpha}}$ is the risky return. Hence, in this case, agents only face capital risk where capital income, $k_{t+1} R_{it+1}^r$, is linear in the individual capital stock.

Bond market clearing implies $\int_0^1 b_{t+1} di = b_{t+1} = 0$ and the capital Euler equation in the incomplete markets case becomes

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{it+1}^r U'(k_{t+1} R_{it+1}^r + w_{t+1})]. \quad (74)$$

In addition, since labor market clearing implies $\int_0^1 l_{it+1} di = 1$, we get $w_{t+1} = (1 - \alpha) k_{t+1}^\alpha$.

In the complete markets case, full insurance means that agents' consumption levels in period $t + 1$ are constant across states and given by

$$c_{t+1} = k_{t+1} E_t [R_{t+1}^r] + w_{t+1}, \quad (75)$$

where the subscript i attached to the return is dropped in order to emphasize that the expected value of the individual return is identical across all agents, i.e. $E_t [R_{it+1}^r] = E_t [R_{t+1}^r]$. Finally, since expenditures in the initial period to finance the corresponding asset bundle sum up to zero and consumption in $t + 1$ is constant, the capital Euler equation in the complete markets case reads

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{t+1}^r] U'(k_{t+1} E_t [R_{t+1}^r] + w_{t+1}). \quad (76)$$

Since only capital risk exists in this case, the right-hand side of equation (74) is approximated using a second-order Taylor expansion around the point $(E_t [R_{t+1}^r])$. For small risks this leads to

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{t+1}^r] U'(\cdot) \quad (77)$$

$$+ \frac{1}{2} \beta [E_t [R_{t+1}^r] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] \sigma_{R_{t+1}^r}^2, \quad (78)$$

with $(\cdot) \equiv k_{t+1} E_t [R_{t+1}^r] + w_{t+1}$.

Let k_{t+1}^* denote the solution in the incomplete markets case. By analogy with Lemma 1, levels of capital and output are lower in the incomplete compared to the complete markets case if and only if

$$[E_t [R_{t+1}^r] U'''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^*] \sigma_{R_{t+1}^r}^2 < 0. \quad (79)$$

Since $\sigma_{R_{t+1}^r}^2 k_{t+1}^* > 0$, this simplifies to

$$-\frac{U'''(\cdot)}{U''(\cdot)} E_t [R_{t+1}^r] k_{t+1}^* < 2. \quad (80)$$

Applying $U''(x) = -\rho x^{-\rho-1}$ and $U'''(x) = \rho(\rho+1)x^{-\rho-2}$ leads to

$$\left(\frac{1}{\vartheta} + 1\right) \frac{E_t [R_{t+1}^r] k_{t+1}^*}{E_t [R_{t+1}^r] k_{t+1}^* + w_{t+1}} < 2, \quad (81)$$

where ϑ is the elasticity of intertemporal substitution ($\vartheta = 1/\rho$). Since aggregate production simplifies to $Y_{t+1}^* = (k_{t+1}^*)^\alpha$ and $K_{t+1}^* \equiv \int_0^1 k_{t+1}^* di = k_{t+1}^*$, it follows that $E_t [R_{t+1}^r] k_{t+1}^* = (1 - \delta)K_{t+1}^* + \alpha Y_{t+1}^*$ and $E_t [R_{t+1}^r] k_{t+1}^* + w_{t+1} = (1 - \delta)K_{t+1}^* + Y_{t+1}^*$. Plugging these expressions into (81) yields the final condition described in (35).

A6. Proof of Lemma 4

In this case, individual production in the final period $t+1$ takes place according to $y_{it+1} = \theta_{it+1} k_{it+1}^\alpha$. Assuming that $\ln(\theta) \sim N(-\sigma^2/2, \sigma^2)$ and that all agents start with the same initial level of net worth, the entrepreneur's maximization problem in the incomplete markets case reads

$$\max_{k_{t+1}, b_{t+1}} U(c_t) + \beta E_t U(c_{it+1}) \quad (82)$$

$$s.t. \ c_t = \omega_t - k_{t+1} - b_{t+1} \quad (83)$$

$$c_{it+1} = \omega_{it+1} \quad (84)$$

$$\omega_{it+1} = b_{t+1} R_{t+1} + k_{t+1} R_{it+1}^r + \pi_{it+1}, \quad (85)$$

where the subscript i is dropped whenever optimal choices are the same across agents. k_{t+1} and b_{t+1} denote risky and riskless investment, respectively. $R_{it+1}^r \equiv 1 - \delta + \alpha \theta_{it+1} k_{t+1}^{\alpha-1}$ is the risky return, and risky profits are given by $\pi_{it+1} \equiv (1 - \alpha) \theta_{it+1} k_{t+1}^\alpha$. Bond market clearing implies $\int_0^1 b_{t+1} di = b_{t+1} = 0$ and leads to the following capital Euler equation in the incomplete markets case

$$U'(\omega_t - k_{t+1}) = \beta E_t [R_{it+1}^r U'(k_{t+1} R_{it+1}^r + \pi_{it+1})]. \quad (86)$$

In the complete markets case, full insurance means that consumption in period $t+1$ is riskless and given by

$$c_{t+1} = k_{t+1} E_t [R_{t+1}^r] + E_t [\pi_{t+1}], \quad (87)$$

where the subscript i attached to the return and to profits is dropped in order to emphasize that the expected value of the individual return and of individual profits is identical across all agents,

i.e. $E_t[R_{it+1}^r] = E_t[R_{t+1}^r]$ and $E_t[\pi_{it+1}] = E_t[\pi_{t+1}]$. Finally, since expenditures in the initial period to finance the corresponding asset bundle sum up to zero and consumption in $t + 1$ is constant, the following Euler equation pins down k_{t+1} in the complete markets case

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(k_{t+1} E_t[R_{t+1}^r] + E_t[\pi_{t+1}]). \quad (88)$$

Since agents face capital risk and profits constitute a risky income component, the right-hand side of equation (86) is approximated using a second-order Taylor expansion around the point $(E_t[R_{t+1}^r], E_t[\pi_{t+1}])$. For small risks this leads to

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(\cdot) \quad (89)$$

$$+ \frac{1}{2} \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1}^2 + 2 U''(\cdot) k_{t+1}] \sigma_{R_{t+1}^r}^2 \quad (90)$$

$$+ \frac{1}{2} \beta E_t[R_{t+1}^r] U'''(\cdot) \sigma_{\pi_{t+1}}^2 \quad (91)$$

$$+ \beta [E_t[R_{t+1}^r] U'''(\cdot) k_{t+1} + U''(\cdot)] \sigma_{R_{t+1}^r \pi_{t+1}}, \quad (92)$$

with $(\cdot) \equiv (k_{t+1} E_t[R_{t+1}^r] + E_t[\pi_{t+1}])$.

Second moments are given by $\sigma_{R_{t+1}^r}^2 = \alpha^2 k_{t+1}^{2\alpha-2} \sigma_{\theta_{t+1}}^2$, $\sigma_{\pi_{t+1}}^2 = (1 - \alpha)^2 k_{t+1}^{2\alpha} \sigma_{\theta_{t+1}}^2$ and $\sigma_{R_{t+1}^r \pi_{t+1}} = \alpha(1 - \alpha) k_{t+1}^{2\alpha-1} \sigma_{\theta_{t+1}}^2$ where $\sigma_{\theta_{t+1}}^2$ denotes the variance of θ_{t+1} .

Let k_{t+1}^* denote the solution in the incomplete markets case. By analogy with Lemma 1, levels of capital and output are lower in the incomplete compared to the complete markets case if and only if

$$\begin{aligned} & [E_t[R_{t+1}^r] U'''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^*] \alpha^2 (k_{t+1}^*)^{2\alpha-2} \sigma_{\theta_{t+1}}^2 \\ & + E_t[R_{t+1}^r] U'''(\cdot) (1 - \alpha)^2 (k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 \\ & + 2[E_t[R_{t+1}^r] U'''(\cdot) k_{t+1}^* + U''(\cdot)] \alpha(1 - \alpha) (k_{t+1}^*)^{2\alpha-1} \sigma_{\theta_{t+1}}^2 < 0. \end{aligned} \quad (93)$$

Since $k_{t+1}^* > 0$, getting rid of $(k_{t+1}^*)^{2\alpha} \sigma_{\theta_{t+1}}^2 > 0$ and collecting terms yields

$$E_t[R_{t+1}^r] U'''(\cdot) [\alpha^2 + 1 - 2\alpha + \alpha^2 + 2\alpha - 2\alpha^2] + 2U''(\cdot) (k_{t+1}^*)^{-1} [\alpha^2 + \alpha - \alpha^2] < 0, \quad (94)$$

and finally

$$-\frac{1}{\alpha} \frac{U'''(\cdot)}{U''(\cdot)} (E_t[R_{t+1}^r] k_{t+1}^*) < 2. \quad (95)$$

Applying $U''(x) = -\rho x^{-\rho-1}$ and $U'''(x) = \rho(\rho + 1)x^{-\rho-2}$ leads to

$$\left(\frac{1}{\vartheta} + 1\right) \frac{1}{\alpha} \frac{E_t[R_{t+1}^r] k_{t+1}^*}{E_t[R_{t+1}^r] k_{t+1}^* + E_t[\pi_{t+1}]} < 2, \quad (96)$$

where ϑ is the elasticity of intertemporal substitution ($\vartheta = 1/\rho$). Since aggregate production simplifies to $Y_{t+1}^* = (k_{t+1}^*)^\alpha$ and $K_{t+1}^* \equiv \int_0^1 k_{t+1}^* di = k_{t+1}^*$, it follows that $E_t[R_{t+1}^r] k_{t+1}^* + E_t[\pi_{t+1}] = Y_{t+1}^* + (1 - \delta) K_{t+1}^*$ and $E_t[R_{t+1}^r] k_{t+1}^* = \alpha Y_{t+1}^* + (1 - \delta) K_{t+1}^*$. Plugging these expressions into (96) yields the final condition described in (37).

A7. Two-period and steady state comparison - specification based on Angeletos and Calvet (2006)

In this case, each entrepreneur produces the same final good according to $y_{it} = \theta_{it} k_{it}^\alpha$ with $\theta \sim$

$N(1, \sigma^2)$. Preferences are of CARA-type with

$$U(c_t) = -\frac{\exp(-\rho c_t)}{\rho}. \quad (97)$$

In the simplified two-period model with identical initial conditions, the capital Euler equation under incomplete markets finally reads

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{it+1}^r U'(k_{t+1} R_{it+1}^r + \pi_{it+1})], \quad (98)$$

where $R_{it+1}^r \equiv 1 - \delta + \alpha \theta_{it+1} k_{t+1}^{\alpha-1}$ is the risky return, risky profits are given by $\pi_{it+1} \equiv (1 - \alpha) \theta_{it+1} k_{t+1}^\alpha$ and bond market clearing implies $\int_0^1 b_{t+1} di = b_{t+1} = 0$.

In the complete markets case, the capital Euler equation reads

$$U'(\omega_t - k_{t+1}) = \beta E_t[R_{t+1}^r] U'(k_{t+1} E_t[R_{t+1}^r] + E_t[\pi_{t+1}]), \quad (99)$$

where the subscript i attached to the return and to profits is dropped in order to emphasize that the expected value of the individual return and of individual profits is identical across all agents, i.e. $E_t[R_{it+1}^r] = E_t[R_{t+1}^r]$ and $E_t[\pi_{it+1}] = E_t[\pi_{t+1}]$.

Let k_{t+1}^* denote the solution in the incomplete markets case. Based on a second-order Taylor expansion around the point $(E_t[R_{t+1}^r], E_t[\pi_{t+1}])$ and by analogy with Lemma 1, levels of capital and output are lower in the incomplete compared to the complete markets case if and only if

$$\begin{aligned} & [E_t[R_{t+1}^r] U'''(\cdot) (k_{t+1}^*)^2 + 2 U''(\cdot) k_{t+1}^*] \alpha^2 (k_{t+1}^*)^{2\alpha-2} \sigma_{t+1}^2 \\ & + E_t[R_{t+1}^r] U'''(\cdot) (1 - \alpha)^2 (k_{t+1}^*)^{2\alpha} \sigma_{t+1}^2 \\ & + 2[E_t[R_{t+1}^r] U'''(\cdot) k_{t+1}^* + U''(\cdot) \alpha (1 - \alpha) (k_{t+1}^*)^{2\alpha-1} \sigma_{t+1}^2] < 0, \end{aligned} \quad (100)$$

with $(\cdot) \equiv (k_{t+1}^* E_t[R_{t+1}^r] + E_t[\pi_{t+1}])$.

Getting rid of $(k_{t+1}^*)^{2\alpha} \sigma_{t+1}^2 > 0$ and collecting terms yields

$$-\frac{1}{\alpha} \frac{U'''(\cdot)}{U''(\cdot)} (E_t[R_{t+1}^r] k_{t+1}^*) < 2. \quad (101)$$

Applying $U''(x) = -\rho \exp(-\rho x)$ and $U'''(x) = \rho^2 \exp(-\rho x)$ leads to

$$\frac{1}{\alpha} E_t[R_{t+1}^r] k_{t+1}^* < \frac{2}{\rho}. \quad (102)$$

With $\gamma \equiv 1/\rho$ and dividing both sides by $2E_t[c_{t+1}^*] > 0$ where $E_t[c_{t+1}^*] = k_{t+1}^* E_t[R_{t+1}^r] + E_t[\pi_{t+1}]$ yields

$$\frac{\gamma}{E_t[c_{t+1}^*]} > \frac{E_t[R_{t+1}^r] k_{t+1}^*}{2\alpha E_t[c_{t+1}^*]}. \quad (103)$$

Since aggregate production simplifies to $Y_{t+1}^* = (k_{t+1}^*)^\alpha$ and $K_{t+1}^* \equiv \int_0^1 k_{t+1}^* di = k_{t+1}^*$, it follows that $E_t[R_{t+1}^r] k_{t+1}^* + E_t[\pi_{t+1}] = Y_{t+1}^* + (1 - \delta) K_{t+1}^*$ and $E_t[R_{t+1}^r] k_{t+1}^* = \alpha Y_{t+1}^* + (1 - \delta) K_{t+1}^*$. Moreover, $C_{t+1}^* = E_t[c_{t+1}^*]$ where C_{t+1}^* denotes aggregate consumption. Plugging these expressions into (103) leads to the final condition

$$\frac{\gamma}{C_{t+1}^*} > \frac{1 + (1 - \delta) \frac{K_{t+1}^*}{\alpha Y_{t+1}^*}}{2[1 + (1 - \delta) \frac{K_{t+1}^*}{Y_{t+1}^*}]}. \quad (104)$$

Here, the close relation to the 'Ak'-case becomes obvious again. If $\delta = 1$, the condition simplifies

to

$$\frac{\gamma}{C_{t+1}^*} > \frac{1}{2}, \quad (105)$$

for all $0 < a \leq 1$, which is exactly the same condition as in the 'Ak'-case with $\alpha = 1$.

Concerning the underlying infinite horizon model and according to Angeletos and Calvet (2006, p.1113), the following condition guarantees that for small risks, the steady state aggregate capital stock decreases with σ^2 ⁴⁶

$$\frac{\gamma}{C^*} > \frac{1 - \delta \frac{K^*}{\alpha Y^*}}{2[1 - \delta \frac{K^*}{Y^*}]}, \quad (106)$$

where α is the income share of capital and aggregate variables are evaluated at the incomplete markets steady state. γ/C^* is the elasticity of intertemporal substitution at the steady state.

Comparing (104) with (106), it becomes obvious that the same difference of the influence of the depreciation rate appears as between (35) and (39). This result supports our reasoning to apply the same kind of adjustment in our case.

A8. Computational Issues

Steady state, financial autarky

1. We start with an initial guess for the wage and for the interest rate.
2. The state space of net worth is approximated by a grid of N points with a larger amount of points being allocated to lower levels of net worth. The productivity process described in (20) is approximated by a five-state Markov chain.
3. Policy functions are computed by value function iteration with interpolation between grid points.
4. The stationary distribution of agents is computed using the decision rules and the transition matrix of the productivity process. Aggregate quantities are calculated by adding up weighted individual demands.
5. Within a first loop, the guess for the wage, and within a second loop, the guess for the interest rate are updated. The procedure is repeated until the market clearing conditions (labor and bond market) are satisfied, except for a tolerably small approximation error.

Steady state, financial integration

The procedure to compute the long-run equilibrium under financial integration for the small and less financially developed country is similar to the steps outlined above. However, the bond market clearing condition is replaced by a fixed interest rate determined by the large and more financially developed country.

Transitional Equilibrium

1. In a first step, we solve for the initial (autarchic) and the final (integrated) steady state, following the steps outlined above.

⁴⁶Note that we abstract from further sources of risk to simplify the comparison. Consequently, if the steady state aggregate capital stock decreases with σ^2 , the capital stock is lower under incomplete markets than under complete markets. For a more detailed discussion see Angeletos and Calvet (2006).

2. The number of transition periods, T , is chosen. T is set sufficiently large to ensure that the integrated steady state is, approximately, reached in T periods.
3. Based on an initial guess for the time path of the wage and the computation of the integrated steady state from step 1, the policy functions are computed by iterating backwards in time.
4. The sequence of distributions is computed with the help of the policy functions and the initial autarchic steady state distribution. Aggregate quantities are calculated.
5. The initial guess for the time path of the wage is updated and the procedure is repeated from step 3 until the labor market clearing condition is satisfied at each point in time, except for a tolerably small approximation error.