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Self-enforcing Intergenerational social contract as a source of Pareto improvement and Emission mitigation

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Abstract

We consider, in a general equilibrium overlapping generations (OLG) model with environmental externalities, a contract between successive generations, whereby agents of the current working-age generation privately invest a share of their labor income in pollution mitigation in exchange for a transfer to their old-age capital income paid by the next generation. We show that the existence of a contract which is Pareto-improving compared to an equilibrium without contract requires a minimum level of income and we characterize the set of Pareto-improving mitigation-transfer combinations as well as the Pareto frontier. Nash bargaining yields unique solutions for the mitigation share and transfer rate respectively which increase in income. We prove that Nash bargaining leads to a steady state with lower emission stock and higher income compared to the steady state without a contract and we study transitional dynamics. Notably, simulation shows that delaying the implementation of a social contract for too long may have considerable welfare costs: Income inevitably falls below the threshold in finite time so that Pareto improving mitigation is no longer possible and the economy converges to a steady state with high emission stock and low income. In the second part of the paper, we study a non-cooperative setting, taking into account that credibly committing to a contract might not be possible. We show that there exist mitigation transfer schemes which are both Pareto improving and give no generation an incentive to deviate from the provisions of the contract. Incentive compatible Nash bargaining leads to a lower mitigation investment and transfer than bargaining in the cooperative setting.

Keyword(s): OLG models, mitigation, Pareto-improvement, social contract.

1 Introduction

Protecting environmental stocks like the climate or a natural resource affects generations unequally. It requires action of current generations while benefits accrue largely to future

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generations. This makes it difficult for policymakers to make environmental policy measures appealing to their voters. Still, it has been pointed out e.g. by Bovenberg and Heijdra (1998) and Gerlagh and Keyzer (2001) that environmental protection can be achieved in a Pareto-improving way, making also those alive today better off. The reason is that the benefits from environmental protection can be distributed over current and future generations such that each generation enjoys higher welfare.⁴ Foley (2007) even argues that protection can be achieved without any loss of consumption for any generation. Bovenberg and Heijdra (1998) and Foley (2007) suggest taxes to deal with the environmental externality and debt policy for intergenerational redistribution, while Gerlagh and Keyzer (2001) propose to establish a trust fund, giving equal claims over a natural resource to all current and future generations.

We study instead a contract between successive generations, whereby agents of the current working-age generation privately invest a share of their labor income in pollution mitigation in exchange for a transfer to their old-age capital income they receive from the next generation when retired. For this purpose, we set up a dynamic general equilibrium OLG-model with an environmental externality: The stock of emissions in a period reduces next period's total factor productivity. We show that the existence of a contract which is Pareto-improving compared to an equilibrium without contract requires a minimum level of income and we characterize the set of Pareto-improving mitigation-transfer combinations as well as the Pareto frontier. Nash bargaining yields unique solutions for the mitigation share and transfer rate respectively which increase in income. We prove that steady state emissions under the Nash bargaining outcome are lower and steady state income is higher than without a contract. This is not trivial result because in our model, the long run effects of a social contract on the environment are priory ambiguous. The reason is that mitigation has an ambiguous effect on income and thereby capital accumulation. Note that the effects of policies on capital accumulation are neglected in the related models by Gerlagh and Keyzer (2001) as well as Karp and Rezai (2014). Another important result of our model is that delaying the implementation of a social contract for too long may have considerable welfare costs: Simulation shows that income inevitably falls below the threshold in finite time so that Pareto improving mitigation is no longer possible and the economy converges to a steady state with high emission stock and low income.

Our work builds to some extent on a sizable literature studying the role of government intervention in the market economy for eliminating the dynamic inefficiency of overlapping generation (OLG) economies with environmental externalities. An incomplete list of papers includes John and Pecchenino (1994), Ono (1996), Jouvét et al. (2000), Gutierrez (2008), Goenka et al. (2012), Fodha and Seegmüller (2014), Dao and Davila (2014), Dao and Edenhofer (2014). These models do not consider intergenerational redistribution of the gains from mitigation efforts. Of course, in our model, intergenerational contracts involving a large num-

⁴Karp and Rezai (2014) argue that environmental protection may, through a positive effect on productivity, lead to an appreciation in the price of capital and thereby benefit current generations. The conclusion is similar: Through intergenerational redistribution, a Pareto-improvement can be obtained.

ber of agents and global externalities require a large extent of coordination, information and surveillance. We suppose that these tasks are carried out by the government. The government gathers information, e.g. on emission accumulation, which it also shares with the public. It then proposes a mitigation-transfer combination based on this information and puts it to a vote.

This however may raise another question concerning the feasibility of environmental protection: Even if policies voted for by generations currently alive improve the welfare of all generations involved, future generations may still have an incentive to abandon them if given the chance to vote about continuation in each period. Talking in terms of our model, once a generation has invested in mitigation, the succeeding generation might be even better off if it does not pay the transfer to the old generation's capital income. Knowing that future generations might not want to fulfill their obligations from the contract and that policy makers might not be able or willing to enforce them, current generations will not be willing to offer a contract in the first place. Neither of the papers cited above takes this commitment problem into account. To do so, a non-cooperative setup is required.

In the last part of our paper, we develop such a setup. We characterize the conditions under which a contract is self-enforcing or incentive compatible in the sense that no generation has an incentive to deviate. We study the existence of a set of incentive compatible combinations of mitigation share and transfer and we show under which condition there exist combinations of mitigation share and transfer, for which a series of contracts is simultaneously Pareto-improving and self-enforcing. For such combinations, institutions are needed to provide information, coordinate payments and surveil compliance with the contract but agents of each generation would in every period voluntarily vote to continue the contract. Rangel (2003), Cigno (1993, 2006a, 2006b) and Anderberg and Balestrino (2003) analyze the scope for providing a public good (education, the environment) by linking its provision to pay-as-you-go transfers in a game theoretic setup with non-altruistic agents. They show that a necessary condition to sustain the provision of the public good is that the return to the compensating intergenerational transfer must be sufficiently larger than the return to private savings. With a constant pay-as-you-go transfer, sufficiently fast population growth is needed to satisfy this condition. A crucial difference of our model is that we assume transfers are paid from labor income and redistributed to the old as a subsidy on capital income rather than pay-as-you-go pensions. In our model, a mitigation-transfer scheme can be sustainable in the absence of population growth if the capital income share is sufficiently below the labor income share. Further, the game-theoretic models above typically consider a partial equilibrium or small open economy with fixed interest rate, while we consider a general equilibrium setup.

We describe the setup of the model in section 2. In Section 3 we explain our understanding of a social contract, study the existence of a set of Pareto-improving combinations of mitigation investment and transfer and characterize this set. We then derive the Pareto-frontier in

section 4 and prove that Nash bargaining yields a unique solution for the mitigation share and transfer which is increasing in income. Further, we compare the dynamic behavior of the model without social contracts and with social contracts determined by Nash bargaining in each period. We prove that under period-by-period social contracts, there exists a steady-state with lower emission stock and higher income compared to the steady-state which is reached without social contracts. Finally, in section 6, we investigate the dynamic sustainability of a mitigation-transfer scheme when agents are allowed to deviate from a contract ex-post. Section 7 concludes the paper.

2 The model

2.1 Emission dynamics

We assume that emission dynamics are given by

$$E_t = \bar{E} + (1 - \delta)(E_{t-1} - \bar{E}) + P_t$$

where $E_t \in \mathbb{R}$ is an index of the emissions stock, which measures the carbon concentration in the atmosphere, in period t ; P_t is the pollution flow released in the atmosphere due to human economic activities (say production and mitigation) in period t and \bar{E} is the natural state of carbon concentration in the atmosphere, i.e. the state of the ecological system without any human activity. $\delta \in [0, 1]$ is the decay rate of emissions which measures the convergence speed of the emissions stock to the natural state \bar{E} . For simplicity, we normalize $\bar{E} = 0$. Therefore, we hereafter rewrite the dynamics of emissions as

$$E_t = (1 - \delta)E_{t-1} + P_t$$

We assume that the pollution flow P_t is determined by

$$P_t = \xi K_t - \gamma M_t,$$

where ξK_t is the pollution from capital K_t , and γM_t is pollution abatement resulting from the mitigation effort M_t ; $\xi, \gamma > 0$ are pollution and mitigation coefficients of physical capital and mitigation respectively. Without loss of generality, we normalize $\xi = 1$. So hereafter, the dynamics of emission stock is

$$E_t = (1 - \delta)E_{t-1} + K_t - \gamma M_t$$

2.2 Production

Aggregate production is given by

$$Y_t = z(E_{t-1})F(K_t, L_t)$$

where $z(E_{t-1})$ is total factor productivity in period t which depends on the index of the emissions stock in the previous period. Note that last period's index of emissions stock affects current productivity, reflecting the long-run effect of emission stocks. For simplification without loss of generality, in this paper we consider the following functional form of $z(E)$:

$$z(E) = \begin{cases} Ae^{-E} & \text{if } E \geq 0 \\ Ae^E & \text{if } E < 0 \end{cases}; A > 0$$

The damage function $z(E)$ above tells us that total factor productivity gets maximal when the carbon concentration in the atmosphere is at the natural state. In the following, we consider a production function of the Cobb-Douglas type, so that $z(E_{t-1})F(K_t, L_t) = z(E_{t-1})K_t^\alpha L_t^{1-\alpha}$, $\alpha \in (0, 1)$. We normalize each period to be one unit of time, and we assume that the capital stock fully depreciates during each period. The returns to capital and labor in period t are determined through profit maximization of the producing firms as follows

$$R_t = z(E_{t-1})F_K(K_t, L_t) = z(E_{t-1})\alpha k_t^{\alpha-1} \quad (1)$$

$$w_t = z(E_{t-1})F_L(K_t, L_t) = z(E_{t-1})(1 - \alpha)k_t^\alpha \quad (2)$$

where $k_t = K_t/L_t$.

2.3 Agents and the intergenerational social contract

For simplification, we assume that each agent has one parent and one offspring and lives for three periods $t - 1$, t , and $t + 1$. In the first period of life (childhood) $t - 1$, a generation of identical agents are children who cannot make choices to influence their respective utility levels. In the second period of life (young age) t , they are identical adults and each of them is endowed with 1 unit of labor. They supply their labor endowments inelastically to the market to get the labor incomes. During this period they allocate their net income between young-age consumption c_t^y and savings k_{t+1} . Savings are consumed in the third period of life (old age) so as to maximize lifetime utility. Hereafter, we denote by agent t or/and generation t the agent and/or generation entering working age in period t . At the beginning (end) of period t , this generation negotiates with the preceding (succeeding) generation for pollution mitigation and an intergenerational transfer and signs a contract. We do not assume altruism between generations, so that each generation seeks to maximize its own welfare. In particular, the negotiation at the beginning of each period $t + 1$ is the following: The young generation at $t + 1$ (i.e. generation $t + 1$) offers the contemporary old generation (i.e. generation t) to sign a contract according to which each agent t leave a portion $m_t \in [0, 1)$ of his young-age income

for mitigation. In exchange, each agent t will receive a transfer at rate $\tau_{t+1}^o \geq 0$ to his gross capital income when old. Note that the case $(m_t, \tau_{t+1}^o) = (0, 0)$ implies the contract between the young and old generations at the beginning of period $t + 1$ is not signed.

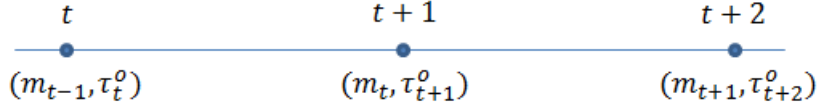


Figure 1: Negotiation between the young and old generations at the beginning of each period

A young agent's problem

We consider the behavior of an agent t in period t under the intergenerational social contract (m_t, τ_{t+1}^o) at the beginning of period $t + 1$ with the succeeding generation. His/her problem is

$$\max_{c_t^y, k_{t+1}, c_{t+1}^o} \ln c_t^y + \beta \ln c_{t+1}^o \quad (3)$$

subject to

$$c_t^y + k_{t+1} \leq I_t(1 - m_t) \quad (4)$$

$$c_{t+1}^o \leq R_{t+1}^e k_{t+1}(1 + \tau_{t+1}^o) \quad (5)$$

given real income I_t , the perfectly foreseen return to capital $R_{t+1}^e = R_{t+1}$ and the social contract (m_t, τ_{t+1}^o) ; $\beta \in (0, 1)$ denotes the agent's time preference parameter.

The agent's optimal choices are

$$c_t^y = \frac{1}{1 + \beta} I_t(1 - m_t) \quad (6)$$

$$k_{t+1} = \frac{\beta}{1 + \beta} I_t(1 - m_t) \quad (7)$$

$$c_{t+1}^o = \frac{R_{t+1}\beta}{1 + \beta} I_t(1 - m_t)(1 + \tau_{t+1}^o) \quad (8)$$

2.4 Equilibrium

Given an intergenerational social contract (m_t, τ_{t+1}^o) , the competitive equilibrium in the economy is characterized by: (i) utility maximization of each agent (3) under the budget constraints (4) and (5); (ii) the law of motion of capital $K_{t+1} = k_{t+1}$; (iii) the maximization problem of the final good producing firm determining the returns of production factors (1) and (2); (iv)

the dynamics of the environment; and (v) the balanced budget constraint of intergenerational transfers. Therefore, the set $\{c_t^y, k_{t+1}, c_{t+1}^o, E_t, P_t\}_t$, which fully characterizes the competitive equilibrium of the economy, is the solution to the following system of equations:

$$c_t^y = \frac{I_t(1 - m_t)}{1 + \beta} \quad (9)$$

$$k_{t+1} = \frac{\beta}{1 + \beta} I_t(1 - m_t) \quad (10)$$

$$c_{t+1}^o = \frac{z(E_t)F_K(k_{t+1}, 1)\beta}{1 + \beta} I_t(1 - m_t)(1 + \tau_{t+1}^o) \quad (11)$$

$$E_t = (1 - \delta)E_{t-1} + P_t \quad (12)$$

$$P_t = k_t - \gamma M_t \quad (13)$$

given k_t , E_{t-1} , and M_t , where

$$I_t = z(E_{t-1})(1 - \alpha)k_t^\alpha(1 - \tau_t^y) \quad (14)$$

is the income of agent t after subtracting the transfer at rate τ_t^y to the contemporary old generation. τ_t^y and τ_t^o satisfy

$$\tau_t^y z(E_{t-1})(1 - \alpha)k_t^\alpha = \tau_t^o z(E_{t-1})\alpha k_t^\alpha; \quad \text{i.e.} \quad \tau_t^y = \frac{\alpha}{1 - \alpha} \tau_t^o$$

which reflects the balanced intergenerational transfer constraint between generations t and $t + 1$.

3 Intergenerational social contract

We now define the perfectly foreseen indirect utilities of agents at equilibrium and at the timing of negotiation for the intergenerational social contract. Since the damage effect is larger (i.e. lower $z(E_t)$) when the carbon concentration index is farther from the natural state ($E = 0$), rational agents will never reach a contract under which the carbon concentration is negative. Hence, hereafter, we always impose implicitly or explicitly a non-negativity constraint on the carbon concentration index:

$$E_{t+1} = (1 - \delta)E_t + \frac{\beta I_t(1 - m_t)}{1 + \beta} - m_t I_t \geq 0 \quad \forall t \quad (15)$$

This constraint is equivalent to

$$m_t \leq \frac{(1-\delta)(1+\beta)E_t/I_t + \beta}{\beta + \gamma + \gamma\beta} \equiv \bar{m}(I_t, E_t) = \bar{m}_t \quad \forall t$$

3.1 Agent t at $t + 1$

The perfectly foreseen indirect utility of working-age agent t under the contract (m_t, τ_{t+1}^o) with $m_t \leq \bar{m}_t$ at the beginning of time $t + 1$ is

$$\begin{aligned} V_t^{t+1}(I_t, m_t, \tau_{t+1}^o) &= \ln \left[\frac{1}{1+\beta} I_t (1 - m_t) \right] + \beta \ln \left[\frac{R_{t+1}\beta}{1+\beta} I_t (1 - m_t) (1 + \tau_{t+1}^o) \right] \\ &= \Phi + \beta \ln[z(E_t)\alpha k_{t+1}^{\alpha-1}] + (1+\beta) [\ln I_t + \ln(1 - m_t)] + \beta \ln(1 + \tau_{t+1}^o) \end{aligned} \quad (16)$$

where $\Phi = \ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta}$.

We can also derive indirect utility of this agent without a social contract, which is

$$V_t^{t+1}(I_t, 0, 0) = \Phi + \beta \ln[z(E_t)\alpha \tilde{k}_{t+1}^{\alpha-1}] + (1+\beta) \ln I_t \quad (17)$$

where \tilde{k}_{t+1} is capital per capita if there is no social contract. From equation (10), we find that $\tilde{k}_{t+1} = \frac{\beta}{1+\beta} I_t = \frac{k_{t+1}}{1-m_t}$.

So, from (16) and (17), the foreseen surplus that agent t gains from the social contract (m_t, τ_{t+1}^o) is

$$\Delta V_t^{t+1} = V_t^{t+1}(I_t, m_t, \tau_{t+1}^o) - V_t^{t+1}(I_t, 0, 0) = \beta \ln \frac{k_{t+1}^{\alpha-1}}{\tilde{k}_{t+1}^{\alpha-1}} + (1+\beta) \ln(1 - m_t) + \beta \ln(1 + \tau_{t+1}^o) \quad (18)$$

Knowing that $\tilde{k}_{t+1} = \frac{k_{t+1}}{1-m_t}$ we can rewrite the difference to be

$$\Delta V_t^{t+1} = (1 + \alpha\beta) \ln(1 - m_t) + \beta \ln(1 + \tau_{t+1}^o) \quad (19)$$

We say the agent t does not suffer a welfare-loss from the social contract (m_t, τ_{t+1}^o) if $\Delta V_t^{t+1} \geq 0$.

3.2 Agent $t + 1$ at $t + 1$

The perfectly foreseen indirect utility of working-age generation $t + 1$ under the contract (m_t, τ_{t+1}^o) with $m_t \leq \bar{m}(I_t, E_t)$ at the beginning of time $t + 1$ is

$$\mathcal{V}_{t+1}^{t+1}(I_{t+1}, m_{t+1}^e, \tau_{t+2}^{o,e}) = \Phi + \beta \ln \left[z(E_{t+1})\alpha \left(\frac{\beta I_{t+1}(1 - m_{t+1}^e)}{1 + \beta} \right)^{\alpha-1} \right]$$

$$+(1+\beta) [\ln I_{t+1} + \ln(1 - m_{t+1}^e)] + \beta \ln(1 + \tau_{t+2}^{o,e}) \quad (20)$$

where $(m_{t+1}^e, \tau_{t+2}^{o,e})$ is the foreseen social contract that the young generation $t+1$ will sign with the succeeding young generation $t+2$ at the beginning of period $t+2$, and $I_{t+1} = z(E_t)(1-\alpha)k_{t+1}^\alpha(1-\tau_{t+1}^y)$ is the income of the young agent $t+1$ in period $t+1$ after paying the intergenerational transfer $\tau_{t+1}^o R_{t+1} k_{t+1} = z(E_t)(1-\alpha)k_{t+1}^\alpha \tau_{t+1}^y$ under the social contract (m_t, τ_{t+1}^o) .

The perfectly foreseen indirect utility of working-age generation $t+1$ without the contract (m_t, τ_{t+1}^o) at the beginning of time $t+1$ is

$$\begin{aligned} \tilde{\mathcal{V}}_{t+1}^{t+1}(\tilde{I}_{t+1}, m_{t+1}^e, \tau_{t+2}^{o,e}) &= \Phi + \beta \ln \left[z(\tilde{E}_{t+1}) \alpha \left(\frac{\beta \tilde{I}_{t+1} (1 - m_{t+1}^e)}{1 + \beta} \right)^{\alpha-1} \right] \\ &+ (1+\beta) [\ln \tilde{I}_{t+1} + \ln(1 - m_{t+1}^e)] + \beta \ln(1 + \tau_{t+2}^{o,e}) \end{aligned} \quad (21)$$

where $\tilde{I}_{t+1} = z(E_t)(1-\alpha)\tilde{k}_{t+1}^\alpha$ and \tilde{E}_{t+1} are the income of the young agent $t+1$ and the stock of emissions, respectively, in period $t+1$ in the case of no contract (m_t, τ_{t+1}^o) with the preceding generation at the beginning of period $t+1$.

So, from (20) and (21), the foreseen surplus that the agent $t+1$ gains from the social contract (m_t, τ_{t+1}^o) is

$$\Delta \mathcal{V}_{t+1}^{t+1} = \mathcal{V}_{t+1}^{t+1}(I_{t+1}, m_{t+1}^e, \tau_{t+2}^{o,e}) - \tilde{\mathcal{V}}_{t+1}^{t+1}(\tilde{I}_{t+1}, m_{t+1}^e, \tau_{t+2}^{o,e}) = (1+\alpha\beta) \ln \frac{I_{t+1}}{\tilde{I}_{t+1}} + \beta \ln \frac{z(E_{t+1})}{z(\tilde{E}_{t+1})} \quad (22)$$

We know that

$$\frac{I_{t+1}}{\tilde{I}_{t+1}} = \frac{k_{t+1}^\alpha (1 - \tau_{t+1}^y)}{\tilde{k}_{t+1}^\alpha}$$

where $k_{t+1} = \tilde{k}_{t+1}(1-m_t) = \frac{\beta I_t}{1+\beta}(1-m_t)$, $\tilde{P}_{t+1} = \tilde{k}_{t+1}$, $P_{t+1} = k_{t+1} - \gamma m_t I_t$, and $\tau_{t+1}^y = \frac{\alpha}{1-\alpha} \tau_{t+1}^o$

Therefore,

$$\frac{I_{t+1}}{\tilde{I}_{t+1}} = \left(1 - \frac{\alpha \tau_{t+1}^o}{1-\alpha} \right) (1 - m_t)^\alpha$$

We also know that

$$\frac{z(E_{t+1})}{z(\tilde{E}_{t+1})} = e^{\tilde{E}_{t+1} - E_{t+1}} = e^{\tilde{P}_{t+1} - P_{t+1}} = e^{\frac{\beta + \gamma + \gamma\beta}{1+\beta} m_t I_t}$$

We say that agent $t+1$ does not suffer from the social contract (m_t, τ_{t+1}^o) if

$$\Delta \mathcal{V}_{t+1}^{t+1} = (1 + \alpha\beta) \ln \left[\left(\frac{1 - \alpha(1 + \tau_{t+1}^o)}{1 - \alpha} \right) (1 - m_t)^\alpha \right] + \frac{\beta(\beta + \gamma + \gamma\beta)}{1 + \beta} m_t I_t \geq 0 \quad (23)$$

3.3 Existence of a Pareto-improving intergenerational social contract

We consider a social contract (m_t, τ_{t+1}^o) a Pareto improvement over the situation without contract if under the contract, both generations t and $t + 1$ are better off. In this subsection, we will derive a condition that guarantees the existence of such a social contract. We define the set \mathcal{S}_{t+1}^o such that

$$\mathcal{S}_{t+1}^o = \{(m_t, \tau_{t+1}^o) \in [0, \bar{m}(I_t, E_t)] \times \mathbb{R}, m_t < 1 : \Delta V_t^{t+1} > 0\} \quad (24)$$

i.e. the set consists of all pairs (m_t, τ_{t+1}^o) which make generation t better off.

And we define the set \mathcal{S}_{t+1}^y such that

$$\mathcal{S}_{t+1}^y = \{(m_t, \tau_{t+1}^o) \in [0, \bar{m}(I_t, E_t)] \times \mathbb{R}, m_t < 1 : \Delta \mathcal{V}_{t+1}^{t+1} > 0\} \quad (25)$$

All pairs $(m_t, \tau_{t+1}^o) \in \mathcal{S}_{t+1}^y$ increase welfare of generation $t + 1$.

Lemma 1. *The sets \mathcal{S}_{t+1}^o and \mathcal{S}_{t+1}^y are convex.*

Proof. We define the boundary of \mathcal{S}_{t+1}^o as

$$\underline{\mathcal{S}}_{t+1}^o = \{(m_t, \tau_{t+1}^o) \in [0, \min\{1, \bar{m}_t\}] \times \mathbb{R}, m_t < 1 : \Delta V_t^{t+1} = 0\}$$

i.e.

$$\tau_{t+1}^o = \left(\frac{1}{1 - m_t} \right)^{\frac{1}{\beta} + \alpha} - 1 \equiv \Omega(m_t) \quad \text{with} \quad m_t \in [0, \min\{1, \bar{m}_t\}], m_t < 1 \quad (26)$$

All pairs $(m_t, \tau_{t+1}^o) \in [0, \min\{1, \bar{m}_t\}] \times \mathbb{R}$ above the boundary $\underline{\mathcal{S}}_{t+1}^o$ belong to the set \mathcal{S}_{t+1}^o . In order to prove that \mathcal{S}_{t+1}^o is convex, it is sufficient to prove that $\Omega(m_t)$ is convex in $m_t \in [0, \min\{1, \bar{m}_t\}]$. This proof is straightforward from the last equation.

Similarly we define the boundary of \mathcal{S}_{t+1}^y as

$$\underline{\mathcal{S}}_{t+1}^y = \{(m_t, \tau_{t+1}^o) \in [0, \min\{1, \bar{m}_t\}] \times \mathbb{R}, m_t < 1 : \Delta \mathcal{V}_{t+1}^{t+1} = 0\}$$

i.e.

$$\tau_{t+1}^o = \frac{1 - \alpha}{\alpha} \left[1 - e^{-\frac{(\beta + \gamma + \gamma\beta)\beta m_t I_t}{(1 + \beta)(1 + \alpha\beta)}} (1 - m_t)^{-\alpha} \right] \equiv \psi(m_t, I_t) \quad \text{with} \quad m_t \in [0, \min\{1, \bar{m}_t\}], m_t < 1 \quad (27)$$

All pairs $(m_t, \tau_{t+1}^o) \in [0, \min\{1, \bar{m}_t\}] \times \mathbb{R}$ below the boundary $\underline{\mathcal{S}}_{t+1}^y$ belong to the set \mathcal{S}_{t+1}^y . So in order to prove that \mathcal{S}_{t+1}^y is convex, it is sufficient to prove that $\psi(m_t, I_t)$ is concave in $m_t \in [0, \min\{1, \bar{m}_t\}]$. Indeed, we have

$$\psi_{mm}(m_t, I_t) = (\alpha - 1) \frac{e^{-am_t I_t}}{(1 - m)^{\alpha}} \left[\frac{1}{(1 - m_t)^2} + \left(\frac{a I_t}{\sqrt{\alpha}} - \frac{\sqrt{\alpha}}{1 - m_t} \right)^2 \right] < 0 \quad \forall m_t \in [0, 1)$$

where $a = \frac{\beta(\beta+\gamma+\gamma\beta)}{\alpha(1+\beta)(1+\alpha\beta)}$.

□

Define

$$\mathcal{P}_{t+1}^t = \mathcal{S}_{t+1}^o \cap \mathcal{S}_{t+1}^y$$

The set \mathcal{P}_{t+1}^t is the Pareto-improvement set between generations t and $t + 1$, i.e. any pair $(m_t, \tau_{t+1}^o) \in \mathcal{P}_{t+1}^t$ makes both generations t and $t + 1$ better off compared to the pair $(m_t, \tau_{t+1}^o) = (0, 0)$. The condition for the existence of this set is stated in proposition 1 below.

Proposition 1. *In the economy set up above, there exists a threshold \hat{I} of real income such that $\mathcal{P}_{t+1}^t \neq \emptyset$ if, and only if the income of agent t exceeds that threshold. i.e. $I_t > \hat{I}$.*

Proof. We see from (26) and (27) that

$$\Omega(0) = \psi(0, I_t) = 0 \tag{28}$$

Now we take into account the slopes of $\Omega(m_t)$ and $\psi(m_t, I_t)$ as m_t approaches 0 from the right. We have

$$\Omega'(0^+) = \lim_{m_t \rightarrow 0^+} \frac{1 + \alpha\beta}{\beta} \left(\frac{1}{1 - m_t} \right)^{1 + \alpha + \frac{1}{\beta}} = \frac{1}{\beta} + \alpha$$

and

$$\begin{aligned} \psi_m(0^+, I_t) &= \frac{1 - \alpha}{\alpha} \lim_{m_t \rightarrow 0^+} \left[\frac{(\beta + \gamma + \gamma\beta)\beta I_t}{(1 + \beta)(1 + \alpha\beta)} - \frac{\alpha}{1 - m_t} \right] e^{-\frac{(\beta + \gamma + \gamma\beta)\beta m_t I_t}{(1 + \beta)(1 + \alpha\beta)}} (1 - m_t)^{-\alpha} \\ &= (1 - \alpha) \left[\frac{(\beta + \gamma + \gamma\beta)\beta I_t}{\alpha(1 + \beta)(1 + \alpha\beta)} - 1 \right] \end{aligned}$$

It comes from the convexity of $\Omega(m_t)$, the concavity of $\psi(m_t, I_t)$, and (28) that $\mathcal{P}_{t+1}^t \neq \emptyset$ if, and only if,

$$\Omega'(0^+) = \frac{1}{\beta} + \alpha < (1 - \alpha) \left[\frac{(\beta + \gamma + \gamma\beta)\beta I_t}{\alpha(1 + \beta)(1 + \alpha\beta)} - 1 \right] = \psi_m(0^+, I_t) \quad (29)$$

$$\iff I_t > \frac{\alpha(1 + \alpha\beta)(1 + \beta)^2}{(1 - \alpha)(\beta + \gamma + \gamma\beta)\beta^2} = \hat{I} \quad (30)$$

□

Proposition 1 states the condition on real income of agent t under which a set of Pareto-improving pairs (m_t, τ_{t+1}^o) exists. The existence of such a set depends on the slopes of $\Omega(m_t)$ and $\psi(m_t, I_t)$ at the origin in the (m_t, τ_{t+1}^o) -plane. These slopes reveal the incentive of the old and the young to engage in an intergenerational social contract. For the old generation, say generation t , the higher $\Omega'(0^+)$, the weaker their incentive to engage in the social contract, while for the young generation, say generation $t + 1$, the higher $\psi_m(0^+, I_t)$, the stronger their incentive to engage in the social contract. So, for an intergenerational social contract to exist, the net incentive as defined by $\psi_m(0^+, I_t) - \Omega'(0^+)$ (i.e. the domination of the strong incentive of the young over the weak incentive of the old), must be strictly positive. The existence and nonexistence of \mathcal{P}_{t+1}^t are depicted in the following figure.

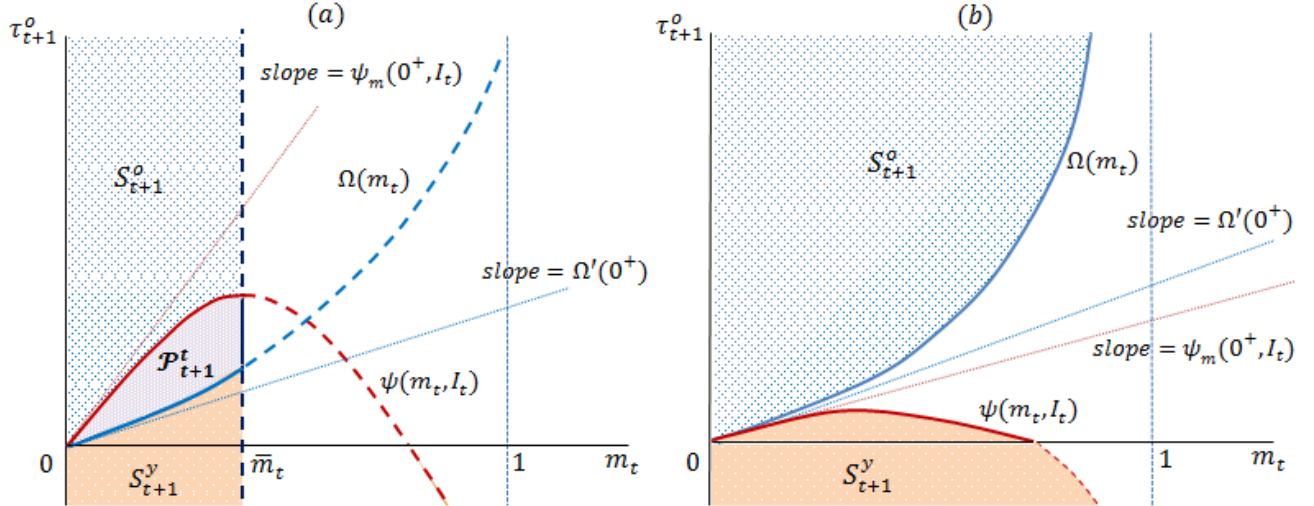


Figure 2: The existence and nonexistence of \mathcal{P}_{t+1}^t : (a) the case $I_t > \hat{I}$; and (b) the case $I_t \leq \hat{I}$.

Lemma 2. $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$ constitutes a compact set, where $\underline{\mathcal{P}}_{t+1}^t$ is the boundary of \mathcal{P}_{t+1}^t .

Proof. This statement is trivially true for the case $\mathcal{P}_{t+1}^t = \emptyset$. We now just focus on the case $\mathcal{P}_{t+1}^t \neq \emptyset$. We have

$$\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t = (\mathcal{S}_{t+1}^o \cup \underline{\mathcal{S}}_{t+1}^o) \cap (\mathcal{S}_{t+1}^y \cup \underline{\mathcal{S}}_{t+1}^y)$$

Since $\mathcal{S}_{t+1}^o \cup \underline{\mathcal{S}}_{t+1}^o$ and $\mathcal{S}_{t+1}^y \cup \underline{\mathcal{S}}_{t+1}^y$ are closed sets, $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$ is closed. Moreover, since $\psi(m_t, I_t)$

is continuous over $m_t \in [0, 1)$ and $\psi(0, I_t) = 0$, $\lim_{m_t \rightarrow 1^-} \psi(m_t, I_t) = -\infty$ for all $I_t < +\infty$, it is true that $\exists N < +\infty$ such that $\psi(m_t, I_t) < N$ for all $m_t \in [0, 1)$ and $I_t < +\infty$. Thus $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$ is always bounded by the ball $B_R(0, 0)$ of center $(0, 0)$ and some finite radius $R \geq \sqrt{1 + N^2}$. Therefore, $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$ is compact. \square

4 Bargaining problem, dynamics and steady state

4.1 Pareto efficiency frontier

Now that we have proven that there exist pairs (m_t, τ_{t+1}^o) which are a Pareto improvement compared to the situation without contract for the two successive generations t and $t + 1$, the question arises which of these pairs lie on the Pareto frontier. Along the frontier, one generation cannot be made better off by a change dm_t in mitigation along with a change $d\tau_{t+1}^o$ in the transfer without making the other worse off. Figure 2 shows two indifference curves, one for each generation, which intersect. As the indifference curves for generation t are strictly convex and the indifference curves for generation $t + 1$ are strictly concave, we can derive the Pareto efficiency frontier by finding the set of points at which the indifference curves for the two generations are tangent to each other. We therefore first derive the slope of an arbitrary indifference curve for each generation and then set equal the slopes.

Implicit differentiation of equation (16) yields

$$0 = dV_t^{t+1} = \beta \frac{d\tau_{t+1}^o}{1 + \tau_{t+1}^o} - (1 + \beta) \frac{dm_t}{1 - m_t} - (1 - \alpha)\beta \frac{dk_{t+1}}{k_{t+1}}$$

where $k_{t+1} = \frac{\beta}{1+\beta} I_t(1 - m_t)$ and $dk_{t+1} = -\frac{\beta}{1+\beta} I_t dm_t$. The slope of the indifference curve is:

$$\frac{d\tau_{t+1}^o}{dm_t} = \frac{1 + \alpha\beta}{\beta} \frac{1 + \tau_{t+1}^o}{1 - m_t}$$

In the same way, implicit differentiation of equation (20) yields

$$0 = dV_{t+1}^{t+1} = \beta \frac{z'(E_{t+1})}{z(E_{t+1})} dE_{t+1} + (1 + \alpha\beta) \frac{dI_{t+1}}{I_{t+1}}$$

where $dI_{t+1} = \frac{\partial I_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial m_t} dm_t + \frac{\partial I_{t+1}}{\partial \tau_{t+1}^o} \frac{\partial \tau_{t+1}^o}{\partial \tau_{t+1}^o} d\tau_{t+1}^o = -\alpha z(E_t) k_{t+1}^{\alpha-1} \left[(1 - \alpha) \frac{\beta}{1+\beta} \left(1 - \frac{\alpha}{1-\alpha} \tau_{t+1}^o \right) I_t dm_t + k_{t+1} d\tau_{t+1}^o \right]$ and $dE_{t+1} = dP_{t+1} = dk_{t+1} - \gamma dM_t = -\frac{\beta + \gamma(1+\beta)}{1+\beta} I_t dm_t$. From this equation, the slope of an indifference curve for generation $t + 1$ is found to be

$$\frac{d\tau_{t+1}^o}{dm_t} = \left[\beta \frac{\beta + \gamma(1 + \beta)}{1 + \beta} I_t - \frac{\alpha(1 + \alpha\beta)}{(1 - m_t)} \right] \frac{1 - \alpha(1 + \tau_{t+1}^o)}{\alpha(1 + \alpha\beta)}.$$

Setting equal the slopes and rearranging, we derive the Pareto frontier:

$$\tau_{t+1}^o = \frac{\beta^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} (1-m_t) I_t - \alpha \beta (1+\alpha\beta)}{\alpha \left(\beta^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} (1-m_t) I_t + (1+\alpha\beta) \right)} - 1 =: PF(m_t; I_t) \quad (31)$$

The Pareto frontier can easily be shown to be a decreasing, convex function of m_t :

$$\begin{aligned} \frac{\partial PF(m_t; I_t)}{\partial m_t} &= - \frac{\beta^2 (1+\alpha\beta)^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} I_t}{\alpha \left(\beta^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} (1-m_t) I_t + (1+\alpha\beta) \right)^2} < 0 \\ \frac{\partial^2 PF(m_t; I_t)}{\partial m_t^2} &= -2 \frac{\beta^4 (1+\alpha\beta)^2 \left(\frac{\beta+\gamma(1+\beta)}{1+\beta} I_t \right)^2}{\alpha \left(\beta^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} (1-m_t) I_t + (1+\alpha\beta) \right)^3} < 0 \end{aligned}$$

At $m_t = 0$, the value of τ_{t+1}^o which leads to a Pareto efficient contract is given by

$$PF(0; I_t) = \frac{(1-\alpha) \beta^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} I_t - \alpha (1+\beta) (1+\alpha\beta)}{\alpha \left(\beta^2 \frac{\beta+\gamma(1+\beta)}{1+\beta} I_t + (1+\alpha\beta) \right)} > 0 \text{ as } PF(m_t; I_t) > 0 \forall m_t < \hat{m}_t$$

Considering the limit of the Pareto efficient transfer as $m_t \rightarrow 1$, we find that

$$\lim_{m_t \rightarrow 1} PF(m_t; I_t) = -(1+\beta)$$

Obviously, while $\lim_{m_t \rightarrow 1} PF(m_t)$ is independent of I_t , both the vertical intercept and the horizontal intercept, that is \hat{m}_t increase with income. The frontier shifts upwards and its curvature increases, with $\lim_{I_t \rightarrow \infty} PF(m_t; I_t) = \frac{1-\alpha}{\alpha} \forall m_t < 1$, $\lim_{I_t \rightarrow \infty} \left(\frac{\partial PF(0; I_t)}{\partial m_t} \right) = 0$ and $\lim_{I_t \rightarrow \infty} \left(\lim_{m_t \rightarrow 1} \frac{\partial PF(m_t; I_t)}{\partial m_t} \right) = -\infty$.

4.2 Bargaining problem

Indeed, any pair $(m_t, \tau_{t+1}^o) \in P_{t+1}$ makes both generations t and $t+1$ better off and even if we focus on Pareto efficiency, there are infinitely many Pareto efficient combinations of m_t and τ_{t+1}^o . Which $(m_t, \tau_{t+1}^o) \in \mathcal{P}_{t+1}^t$ will be the outcome of the negotiations between generations? The agreement (m_t, τ_{t+1}^o) in general depends on the bargaining power of each generation. Unfortunately, the bargaining theory has so far said relatively little about the determinant of bargaining power. In the following, we will consider the so-called Nash bargaining problem. In his seminal paper, Nash (1950) introduced a *two-person bargaining theory* and provided an normative approach to finding a solution to the Nash bargaining problem. Note that the Nash bargaining solution is Pareto efficient. In our context, the Nash solution (m_t^*, τ_{t+1}^{o*}) , given it exists, is

$$(m_t^*, \tau_{t+1}^{o*}) \in \arg \max_{(m_t, \tau_{t+1}^o) \in \mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t} \{ \Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1} \} \quad (32)$$

Proposition 2. For all $I_t > \hat{I}$, then there exists a unique point $(m_t^*, \tau_{t+1}^{o*}) \in \mathcal{P}_{t+1}^t$ solving the Nash bargaining problem (32). If (m_t^*, τ_{t+1}^{o*}) is an interior point in \mathcal{P}_{t+1}^t , then it is the unique stationary point for the function $\Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1}$ defined over the set $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$.⁵

Proof. Since both ΔV_t^{t+1} and $\Delta \mathcal{V}_{t+1}^{t+1}$ are well-defined and continuous over the set $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$, the product $\Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1}$ is continuous over $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$. So the existence of (m_t^*, τ_{t+1}^{o*}) is guaranteed by the compactness of $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$ which has been proven in lemma 2. It is trivial to rule out the case where at least one of the constraints $\Delta V_t^{t+1} \geq 0$ and $\Delta \mathcal{V}_{t+1}^{t+1} \geq 0$ is binding because in this case the objective function $\Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1} = 0$ while any other point $(m_t, \tau_{t+1}^o) \in \mathcal{P}_{t+1}^t$ gives $\Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1} > 0$. So at the maximum none of these two constraints is binding. Hence, the Lagrangian for the optimization is

$$\mathcal{L}(m_t, \tau_{t+1}^o, \mu) = \Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1} + \mu(m_t - \bar{m}_t)$$

where $\mu \geq 0$ is the Lagrangian multiplier for the constraint $m_t - \bar{m}_t \leq 0$. The first-order Kuhn-Tucker conditions at the optimal point $(m_t, \tau_{t+1}^o) \in \mathcal{P}_{t+1}$ are

$$\mathcal{L}_m(m_t^*, \tau_{t+1}^{o*}, \mu) = \mu - \frac{1 + \alpha\beta}{1 - m_t^*} \Delta \mathcal{V}_{t+1}^{t+1,*} + \Delta V_t^{t+1,*} \left[\frac{\beta + \gamma + \gamma\beta}{1 + \beta} \beta I_t - \frac{\alpha(1 + \alpha\beta)}{1 - m_t^*} \right] = 0 \quad (33)$$

$$\mathcal{L}_\tau(m_t^*, \tau_{t+1}^{o*}, \mu) = \frac{\beta}{1 + \tau_{t+1}^{o*}} \Delta \mathcal{V}_{t+1}^{t+1,*} - \Delta V_t^{t+1,*} \frac{\alpha(1 + \alpha\beta)}{1 - \alpha(1 + \tau_{t+1}^{o*})} = 0 \quad (34)$$

$$\mu(m_t^* - \bar{m}_t) = 0$$

(i) If $m_t^* = \bar{m}_t = \frac{(1-\delta)(1+\beta)E_t/I_t+\beta}{\beta+\gamma+\gamma\beta}$, then from (34) we have

$$\begin{aligned} Q(\tau_{t+1}^{o*}) \equiv & \frac{\beta}{1 + \tau_{t+1}^{o*}} \left\{ (1 + \alpha\beta) \ln \left[\left(\frac{1 - \alpha(1 + \tau_{t+1}^{o*})}{1 - \alpha} \right) (1 - \bar{m}_t)^\alpha \right] + \frac{\beta + \gamma + \gamma\beta}{1 + \beta} \beta \bar{m}_t I_t \right\} \\ & - \left\{ (1 + \alpha\beta) \ln(1 - \bar{m}_t) + \beta \ln(1 + \tau_{t+1}^{o*}) \right\} \frac{\alpha(1 + \alpha\beta)}{1 - \alpha(1 + \tau_{t+1}^{o*})} = 0 \end{aligned} \quad (35)$$

where $Q(\tau_{t+1}^{o*})$ is decreasing in τ_{t+1}^{o*} , and

$$Q(0) = \beta^2 \frac{\beta + \gamma + \gamma\beta}{1 + \beta} \bar{m}_t I_t - \frac{\alpha(1 + \alpha\beta)^2}{1 - \alpha} \ln(1 - \bar{m}_t) > 0$$

⁵The uniqueness of the solution and the stationary point is important because it helps us to rule out the case of bifurcation when we study the dynamic system.

and

$$\lim_{\tau_{t+1}^{o*} \rightarrow (\frac{1-\alpha}{\alpha})^-} Q(\tau_{t+1}^{o*}) = -\infty$$

Hence, there always exists a unique $\tau_{t+1}^{o*} \in (0, \frac{1-\alpha}{\alpha})$ solving (35), implying that there always exists a unique $(m_t, \tau_{t+1}^o) = (\bar{m}_t, \tau_{t+1}^{o*})$ solving the Nash bargaining problem in case the constraint $m_t \leq \bar{m}_t$ is binding.

(ii) If $m_t^* < \bar{m}_t$, then $\mu = 0$ and from (33) and (34) we find that

$$\begin{aligned} \frac{1 + \tau_{t+1}^{o*}}{1 - m_t^*} \frac{1 + \alpha\beta}{\beta} &= \left[\beta \frac{\beta + \gamma + \gamma\beta}{1 + \beta} I_t - \frac{\alpha(1 + \alpha\beta)}{1 - m_t^*} \right] \frac{1 - \alpha(1 + \tau_{t+1}^{o*})}{\alpha(1 + \alpha\beta)} \\ \implies 1 + \tau_{t+1}^{o*} &= \frac{\beta^2 \frac{\beta + \gamma + \gamma\beta}{1 + \beta} I_t (1 - m_t^*) - \alpha\beta(1 + \alpha\beta)}{\alpha \left(\beta^2 \frac{\beta + \gamma + \gamma\beta}{1 + \beta} I_t (1 - m_t^*) + 1 + \alpha\beta \right)} \end{aligned} \quad (36)$$

Since $\tau_{t+1}^{o*} > 0$, it follows from (36) that

$$1 - m_t^* > \frac{\alpha(1 + \alpha\beta)(1 + \beta)^2}{(1 - \alpha)(\beta + \gamma + \gamma\beta)\beta^2 I_t} \quad \text{i.e.} \quad m_t^* < 1 - \frac{\alpha(1 + \alpha\beta)(1 + \beta)^2}{(1 - \alpha)(\beta + \gamma + \gamma\beta)\beta^2 I_t} = \hat{m}_t \quad (37)$$

Substituting (36) into (33), we find m_t^* as the solution to

$$\begin{aligned} \hat{Q}(m_t; I_t) &= \ln \left[(1 - m_t)^{1 + \alpha\beta} \left(\frac{\beta b I_t (1 - m_t) - \alpha\beta(1 + \alpha\beta)}{\alpha(\beta b I_t (1 - m_t) + 1 + \alpha\beta)} \right)^\beta \right]^{b I_t - \frac{\alpha(1 + \alpha\beta)}{1 - m_t}} \\ &\quad - \frac{1 + \alpha\beta}{1 - m_t} \left\{ \ln \left[\frac{(1 - \alpha^2 \beta^2)(1 - m_t)^\alpha}{(1 - \alpha)(\beta b I_t (1 - m_t) + 1 + \alpha\beta)} \right]^{1 + \alpha\beta} + b m_t I_t \right\} = 0 \end{aligned} \quad (38)$$

where $b = \frac{\beta(\beta + \gamma + \gamma\beta)}{1 + \beta}$.

The existence of a solution to (38) is proved by the existence of a solution (m_t, τ_{t+1}^o) to the Nash bargaining problem. The uniqueness of m_t^* is guaranteed by the monotonicity of the function $\hat{Q}(m_t^*; I_t)$. We now prove that for all $I_t > \hat{I}$, $\hat{Q}(m_t^*; I_t)$ is decreasing in m_t^* . From (38), it is sufficient to prove that $\frac{(1 - m_t)^\alpha}{\beta b I_t (1 - m_t) + 1 + \alpha\beta}$ is increasing in m_t for all $I_t > \hat{I}$ and $m_t < \hat{m}_t$. In effect, its derivative with respect to m_t is

$$\frac{(1 - \alpha)(1 - m_t)\beta b I_t - \alpha(1 + \alpha\beta)}{(1 - m_t)^{1 - \alpha} [\beta b I_t (1 - m_t) + 1 + \alpha\beta]^2} > 0 \quad \forall m_t < \hat{m}_t, \forall I_t > \hat{I}$$

Therefore, there exists a unique $m_t^* = m(I_t) \in (0, \hat{m}_t)$ that solves (38), i.e., in this case there exists a unique interior pair $(m_t^*, \tau_{t+1}^{o*}) \equiv (m(I_t), \tau(I_t)) \in \mathcal{P}_{t+1}^t$ that solves the Nash bargaining

problem. Since (m_t^*, τ_{t+1}^{o*}) is the unique pair solving the first order conditions (33) and (34), it is the unique stationary point of the function $\Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1}$ defined over the set $\mathcal{P}_{t+1}^t \cup \underline{\mathcal{P}}_{t+1}^t$. \square

It is interesting to find a condition under which the constraint $m_t \leq \bar{m}_t$ is binding. In other word, we wish to find a condition under which

$$m_t^* = \frac{(1-\delta)(1+\beta)E_t + \beta I_t}{(\beta + \gamma + \gamma\beta)I_t} \equiv \bar{m}_t \quad (39)$$

where $m_t^* = \arg \max_{m_t} \Delta V_t^{t+1} \Delta \mathcal{V}_{t+1}^{t+1}$ subject to $\Delta V_t^{t+1} \geq 0$ and $\Delta \mathcal{V}_{t+1}^{t+1} \geq 0$.⁶ This condition allows us to characterize an area such that when the economy enters this area, the stock of emission will converge to the natural state, i.e. $E = 0$. We call this area the “Ideal area”.

Because such an area is not a focus of our paper and for simplification, without substantial loss of generality, we consider the area under the following assumption

Assumption 1. $\frac{\beta+\gamma+\gamma\beta}{\alpha(1+\beta)} \left[\frac{\beta(1-\alpha)}{1+\beta} \right]^{\frac{2-\alpha}{1-\alpha}} < 1$.

This assumption is a sufficient condition to guarantee that m_t^* in (39) is increasing in I_t . This assumption can hold in practice when the mitigation efficiency parameter γ is not too high. Note that in this set up we normalize the pollution effect of capital to 1, so it is plausible that the mitigation efficiency parameter is around this value. Assumption 1 holds when we set plausible values $\alpha = 0.3$, $\beta = 0.6$ and $\gamma = 1$.

Similar to the proof of Proposition 2 this optimization problem has an unique interior solution and it holds that

$$\begin{aligned} \hat{Q}(m_t^*; I_t) = \ln \left[(1 - m_t^*)^{1+\alpha\beta} \left(\frac{\beta b I_t (1 - m_t^*) - \alpha\beta(1 + \alpha\beta)}{\alpha(\beta b I_t (1 - m_t^*) + 1 + \alpha\beta)} \right)^\beta \right]^{bI_t - \frac{\alpha(1+\alpha\beta)}{1-m_t^*}} \\ - \frac{1 + \alpha\beta}{1 - m_t^*} \left\{ \ln \left[\frac{(1 - \alpha^2\beta^2)(1 - m_t^*)^\alpha}{(1 - \alpha)(\beta b I_t (1 - m_t^*) + 1 + \alpha\beta)} \right]^{1+\alpha\beta} + b m_t^* I_t \right\} = 0 \end{aligned}$$

where $\hat{Q}_m(m_t^*; I_t) < 0$ and, under assumption 1, $\hat{Q}_I(m_t^*; I_t) > 0$ for all $I_t \in (\hat{I}, \bar{I})$, in which $\bar{I} = [A(1 - \alpha)]^{\frac{1}{1-\alpha}} \left(\frac{\beta}{1+\beta} \right)^{\frac{\alpha}{1-\alpha}}$ is conditional steady state income in the case of no social contract and emission stock is set at $E = 0$. By applying the implicit function theorem we have

$$\forall I_t \in (\hat{I}, \bar{I}), \quad m_t^* = m(I_t) \quad \text{and} \quad m'(I_t) > 0$$

⁶Note that in this optimization we ignore the constraint $m_t \leq \bar{m}_t$ in order to find the condition under which this constraint is just binding.

Lemma 3. *If $m(\bar{I}) > \frac{\beta}{\beta+\gamma+\gamma\beta}$, then there exists $\underline{I} \in (\hat{I}, \bar{I})$ and $\hat{E}_t = \hat{E}(I_t)$ for $I_t \in (\underline{I}, \bar{I})$ such that $m_t^* = \bar{m}_t$ (or equivalently $E_{t+1} = 0$) if, and only if $E_t \leq \hat{E}(I_t)$. Moreover, $\hat{E}(\underline{I}) = 0$, and $\hat{E}'(I_t) > 0$ for $I_t \in [\underline{I}, \bar{I}]$.*

Proof. Since $m(\hat{I}) = 0$, $m(\bar{I}) > \frac{\beta}{\beta+\gamma+\gamma\beta}$ and $m'(I_t) > 0$ for all $I_t \in (\hat{I}, \bar{I})$ then there exists a unique $\underline{I} \in (\hat{I}, \bar{I})$ such that $m(\underline{I}) = \frac{\beta}{\beta+\gamma+\gamma\beta}$. From (39) we have

$$m(I_t) - \frac{(1-\delta)(1+\beta)E_t + \beta I_t}{(\beta + \gamma + \gamma\beta)I_t} = 0 \quad (40)$$

implies for all $I_t \in [\underline{I}, \bar{I}]$, there exists a unique \hat{E}_t solving (40), and

$$\hat{E}_t = \frac{(\beta + \gamma + \gamma\beta)m(I_t) - \beta I_t}{(1-\delta)(1+\beta)} I_t \equiv \hat{E}(I_t)$$

where $\hat{E}(\underline{I}) = 0$ and $\hat{E}'(I_t) > 0$ for all $I_t \in [\underline{I}, \bar{I}]$. □

Lemma 3 reveals the existence of an area in the North-East of the $I - E$ plane such that under period-by-period social contracts an economy starting from any point in this area may converge to a steady state with $E_* = 0$ and $I_* \in (\underline{I}, \bar{I})$, that is, a steady state in the ideal area. The proof for existence of this steady state is fairly straightforward because starting from any point (I_t, E_t) in the ideal area leads to a social contract with $m_t = \frac{(1-\delta)(1+\beta)E_t + \beta I_t}{(\beta+\gamma+\gamma\beta)I_t}$. Hence, from $t+1$ onwards $E = 0$ and $m = \frac{\beta}{\beta+\gamma+\gamma\beta}$. The existence and uniqueness of the transfer τ_{t+1}^{o*} is proved in Proposition 2. Since $I_{t+1} = z(E_t) \left(\frac{\beta(1-m_t)I_t}{1+\beta} \right)^\alpha [1 - \alpha(1 + \tau_{t+1}^o)]$, the steady state is characterized by

$$I_*^{1-\alpha} - A \left(\frac{\gamma\beta}{\beta + \gamma + \gamma\beta} \right)^\alpha [1 - \alpha(1 + \tau(I_*))] = 0$$

which always guarantees the existence and uniqueness of the steady state.

4.3 Dynamics and steady states

We first consider the steady state of the economy in the case of no social contract. With $(m_t, \tau_{t+1}^o) = (0, 0)$ for all t , the dynamic system is fully characterized by the evolution of real income and stock of emissions as follows

$$I_{t+1} = A e^{-E_t} \left[\frac{\beta I_t}{1+\beta} \right]^\alpha (1 - \alpha) \quad (41)$$

$$E_{t+1} = (1 - \delta)E_t + \frac{\beta}{1+\beta} I_t \quad (42)$$

Proposition 3. *Without intergenerational social contracts in all periods the economy converges to a globally stable steady state.*

Proof. Under the dynamic system (41)-(42), the steady state is characterized by the following function

$$\varphi(E) \equiv E - \frac{1}{\delta} \left[\frac{\beta(1-\alpha)}{1+\beta} A e^{-E} \right]^{\frac{1}{1-\alpha}} = 0 \quad (43)$$

We have

$$\varphi'(E) = 1 + \frac{1}{(1-\alpha)\delta} \left[\frac{\beta(1-\alpha)}{1+\beta} A e^{-E} \right]^{\frac{1}{1-\alpha}} > 0$$

and

$$\varphi(0) = -\frac{1}{\delta} \left[\frac{\beta(1-\alpha)}{1+\beta} \right]^{\frac{1}{1-\alpha}} < 0 \quad \text{and} \quad \lim_{E \rightarrow +\infty} \varphi(E) = +\infty$$

Hence, there exists a unique steady state which is characterized by (43). The steady state income of the agent in this case is

$$\tilde{I} = A^{\frac{1}{1-\alpha}} e^{-\frac{\tilde{E}}{1-\alpha}} \left(\frac{\beta}{1+\beta} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)^{\frac{1}{1-\alpha}}$$

The Jacobian matrix associated with the dynamic system (41)-(42) evaluated at the steady state (\tilde{I}, \tilde{E}) is as follows:

$$\tilde{J} = \begin{pmatrix} \alpha & -\frac{1+\beta}{\beta} \delta \tilde{E} \\ \frac{\beta}{1+\beta} & 1 - \delta \end{pmatrix}$$

Its determinant and trace are

$$\det(\tilde{J}) = \alpha(1-\delta) + \delta \tilde{E} > 0; \quad \text{Tr}(\tilde{J}) = \alpha + 1 - \delta > 0$$

and the characteristic function is

$$C(\lambda) = \lambda^2 - \text{Tr}(\tilde{J})\lambda + \det(\tilde{J})$$

(i) If $\text{Tr}(\tilde{J})^2 > 4 \det(\tilde{J})$, there are two distinct real eigenvalues λ_1, λ_2 . We have

$$C(-1) = 1 + \text{Tr}(\tilde{J}) + \det(\tilde{J}) > C(1) = \delta(1-\alpha) + \delta \tilde{E} > 0$$

Hence, in this case we have two distinct eigenvalues $\lambda_1, \lambda_2 \in (0, 1)$. The steady state (\tilde{I}, \tilde{E}) is a stable node.

- (ii) If $\text{Tr}(\tilde{J})^2 = 4\det(\tilde{J})$, we have a pair of repeated real eigenvalues $\lambda = \frac{\alpha+1-\delta}{2} \in (0, 1)$. The steady state (\tilde{I}, \tilde{E}) is stable.
- (iii) If $\text{Tr}(\tilde{J})^2 < 4\det(\tilde{J})$, we have two complex eigenvalues. It is obvious that

$$\varphi\left(\frac{1-\alpha(1-\delta)}{\delta}\right) > 0 \implies \tilde{E} < \frac{1-\alpha(1-\delta)}{\delta}$$

Hence

$$\det(\tilde{J}) = \alpha(1-\delta) + \delta\tilde{E} < 1$$

Therefore, in this case, the steady state (\tilde{I}, \tilde{E}) is a spiral sink. \square

We next study the dynamic system with the social contract under the Nash bargaining problem. The dynamic system is now characterized by the following equations

$$I_{t+1} = Ae^{-E_t} \left[\frac{\beta I_t(1-m_t^*)}{1+\beta} \right]^\alpha [1 - \alpha(1 + \tau_{t+1}^{o*})] \quad (44)$$

$$E_{t+1} = (1-\delta)E_t + \frac{\beta - (\beta + \gamma + \gamma\beta)m_t^*}{1+\beta} I_t \quad (45)$$

given $I_0, E_0 > 0$. Note that for the case $I_t \leq \hat{I}$, we set $(m_t^*, \tau_{t+1}^{o*}) = (0, 0)$.

We next consider the dynamics, characterized by the system (44)-(45), and the steady states of the economy under the intergenerational social contract (m_t^*, τ_{t+1}^{o*}) . In order to guarantee the existence of a steady state with intergenerational social contracts, it is necessary to introduce the following assumption

Assumption 2. $\bar{I} > \hat{I}$, i.e. $A > \left[\frac{\alpha(1+\alpha\beta)}{\beta+\gamma+\gamma\beta} \right]^{1-\alpha} \left[\frac{1+\beta}{\beta(1-\alpha)} \right]^{2-\alpha}$.

This assumption implies that the economy's level of technology A should be sufficiently high. Otherwise the economy always converges to the steady state (\tilde{I}, \tilde{E}) , i.e. the steady state without social contract.

The first property of the steady state under the intergenerational social contract (m_t^*, τ_{t+1}^{o*}) is stated in the next proposition

Proposition 4. *Under the intergenerational social contracts $(m_t^*, \tau_{t+1}^{o*}) \in \mathcal{P}_{t+1}^t$, when $I_t > \hat{I}$, and if $EE(m_t^*, \tau_{t+1}^{o*}) \cap II(m_t^*, \tau_{t+1}^{o*}) \neq \emptyset$, the emission stock at a steady state with social contract is lower than that at the steady state without social contract.*

Proof. From the dynamic system (44)-(45), a steady state with social contract is characterized by

$$I^{1-\alpha} = A e^{-E} \left[\frac{\beta(1-m)}{1+\beta} \right]^\alpha [1 - \alpha(1+\tau)] > \hat{I}^{1-\alpha}$$

$$\delta E = \frac{\beta - (\beta + \gamma + \gamma\beta)m}{1+\beta} I \geq 0$$

where $(m, \tau) \in \mathcal{P} \neq \emptyset$ is the social contract at the steady state. Since $\hat{I} > 0$, it follows from two last equations that $\tau < \frac{1-\alpha}{\alpha}$ and $m \leq \frac{\beta}{\beta+\gamma+\gamma\beta}$.

The stock of emission at the steady state with social contract is characterized by

$$e^{\frac{E}{1-\alpha}} E = A^{\frac{1}{1-\alpha}} \frac{\beta - (\beta + \gamma + \gamma\beta)m}{\delta(1+\beta)} \left[\frac{\beta(1-m)}{1+\beta} \right]^{\frac{\alpha}{1-\alpha}} [1 - \alpha(1+\tau)]^{\frac{1}{1-\alpha}} \quad (46)$$

It is straightforward from (46) that E is decreasing in both m and τ so that

$$E = E(m, \tau) < E(0, 0) = \tilde{E}$$

□

Now it would be interesting to illustrate dynamics and convergence to the steady states intuitively. We focus on the most interesting dynamics which will be supported by our simulations in the later section. We study two distinct cases, $\hat{I} < \tilde{I}$ and $\hat{I} > \tilde{I}$, and present the dynamics through the phase diagrams. Define the sets II and EE corresponding to the dynamic system given by (44) and (45) as follows

$$II \equiv \{(I_t, E_t) \in \mathfrak{R}_+^2 : I_{t+1} = I_t\}$$

$$\text{i.e.} \quad E_t = \ln A + \alpha \ln \left[\frac{\beta(1-m(I_t))}{1+\beta} \right] + \ln [1 - \alpha(1+\tau(I_t))] - (1-\alpha) \ln I_t \equiv \Gamma(I_t)$$

and

$$EE \equiv \{(I_t, E_t) \in \mathfrak{R}_+^2 : E_{t+1} = E_t\}$$

$$\text{i.e.} \quad E_t = \frac{\beta - (\beta + \gamma + \gamma\beta)m(I_t)}{\delta(1+\beta)} I_t \equiv \Lambda(I_t)$$

Lemma 4. *For the dynamic system $(I_t, E_t)_t$ characterized by equations (44)-(45), it holds that:*

$$(i) \quad I_{t+1} - I_t \begin{cases} > 0 & \text{if } 0 < E_t < \Gamma(I_t) \\ = 0 & \text{if } E_t = \Gamma(I_t) \\ < 0 & \text{if } E_t > \Gamma(I_t) \end{cases} \quad \text{and} \quad (ii) \quad E_{t+1} - E_t \begin{cases} > 0 & \text{if } 0 < E_t < \Lambda(I_t) \\ = 0 & \text{if } E_t = \Lambda(I_t) \\ < 0 & \text{if } E_t > \Lambda(I_t) \end{cases}$$

Proof. The proof for this Lemma is fairly straightforward. □

In the following phase diagrams, the dashed lines $II(0,0)$ and $EE(0,0)$ are locus II and EE without social contract (m_t^*, τ_{t+1}^{o*}) in all periods. The dashed directions of motion also present the dynamics in the case of no social contracts. Similarly, the bold lines $II(m_t^*, \tau_{t+1}^{o*})$, $EE(m_t^*, \tau_{t+1}^{o*})$ and the bold directions of motion represent corresponding locus and dynamics in the case of having social contracts (m_t^*, τ_{t+1}^{o*}) .

In the case $\hat{I} < \tilde{I}$, under the period-by-period intergenerational social contracts $(m_t^*, \tau_{t+1}^{o*})_t$ the economy has a chance to converge to steady state characterized by (I, E) in which $E < \tilde{E}$ as depicted in Figure 3. Interestingly, in this case, under period-by-period social contracts $(m_t^*, \tau_{t+1}^{o*})_t$ determined through Nash bargaining, the steady state (\tilde{I}, \tilde{E}) disappears and the dynamics of motion change substantially.

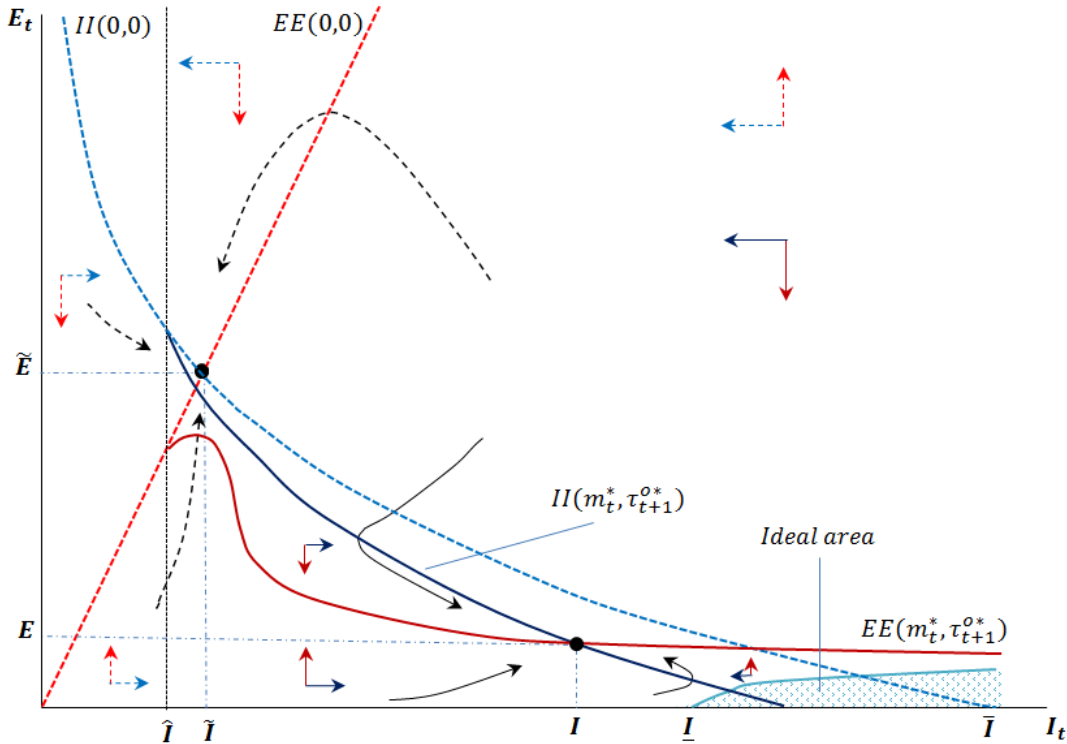


Figure 3: Dynamics and steady states in the case $\hat{I} < \tilde{I}$

In the case $\hat{I} > \tilde{I}$ and the cardinal $\|EE(m_t^*, \tau_{t+1}^{o*}) \cap II(m_t^*, \tau_{t+1}^{o*})\| \geq 1$, then under the period-by-period intergenerational social contracts $(m_t^*, \tau_{t+1}^{o*})_t$ the economy has a chance to converge to a better steady state characterized by (I, E) in which not only $E < \tilde{E}$ but also $I > \tilde{I}$ as depicted in Figure 4. Interestingly, in this case, unlike when $\hat{I} < \tilde{I}$, under the period-by-period social contracts (m_t^*, τ_{t+1}^{o*}) the steady state without social contracts does not vanish. Because the steady state (\tilde{I}, \tilde{E}) is stable, as proved in proposition 3, in the case $\hat{I} > \tilde{I}$, there always exists an area such that when the economy enters this area, it will converge to the stable steady state (\tilde{I}, \tilde{E}) and the social contracts in this setup cannot help the economy escape this trap. For this reason, we call this area the “*dangerous area*”. Figure 4 depicts the dynamic behavior of the economy and it also characterizes the *dangerous area* (shaded) if the convergence behavior to the steady state without social contract, (\tilde{I}, \tilde{E}) , is not spiral. This result reveals an important economic implication: If the economy delays signing intergenerational social contracts when its income is still sufficiently high, the costs of this delay may be extremely high, particularly when the economy enters the *dangerous area*.

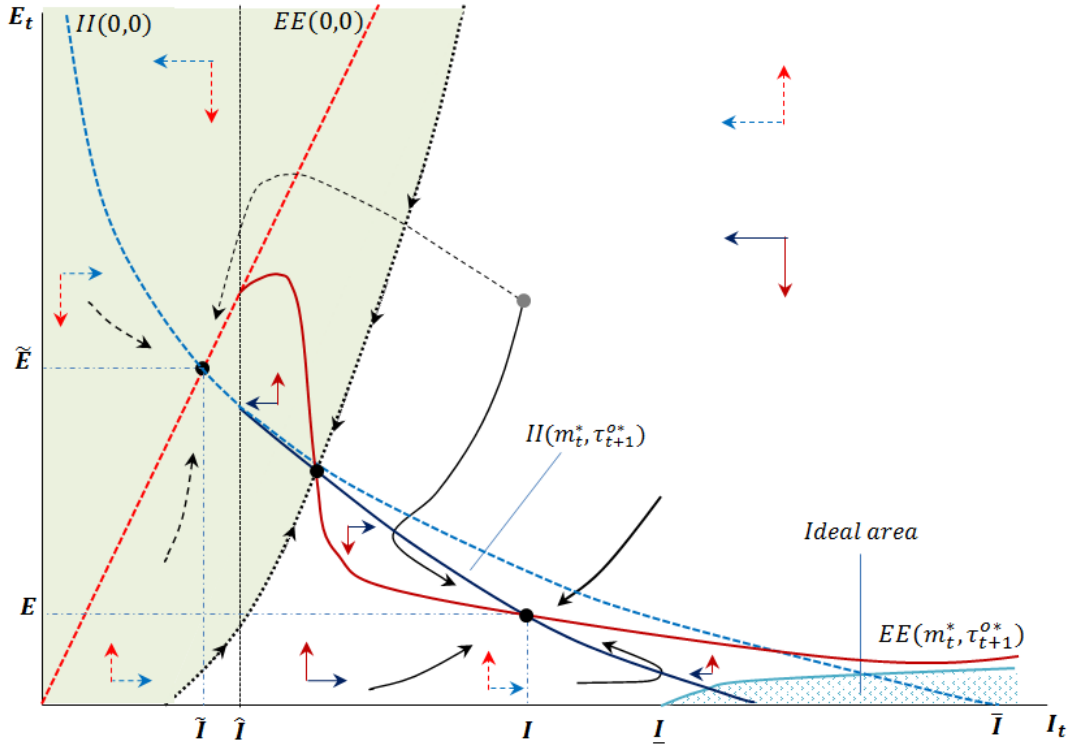


Figure 4: Dynamics and steady states in the case $\hat{I} > \tilde{I}$

5 Simulation

In the previous section, we argue that through Pareto-improving intergenerational social contracts, an economy may have a chance to converge to a better steady state with lower stationary emission stock and higher stationary income compared to the case of no social contracts. In this section we provide numerical simulation results to illustrate the argument in the previous section. The share of capital, α , is set to 0.3 which is quite well known from the literature. The rate of time preference, β , is set to 0.7 which gives us a plausible savings/income rate of households around 40%. Without loss of generality we set the effectiveness of mitigation to $\gamma = 1$. Note that these parameter values guarantee that the condition in assumption 1 holds. With the above parameters values we compute $\hat{I} = \frac{\alpha(1+\alpha\beta)(1+\beta)^2}{(1-\alpha)(\beta+\gamma+\gamma\beta)\beta^2} \simeq 1.2744$. We run the simulation with different levels of technology A corresponding to the two distinct cases $\tilde{I} < \hat{I}$ and $\tilde{I} > \hat{I}$ in the following subsections.

5.1 The case $\hat{I} < \tilde{I}$

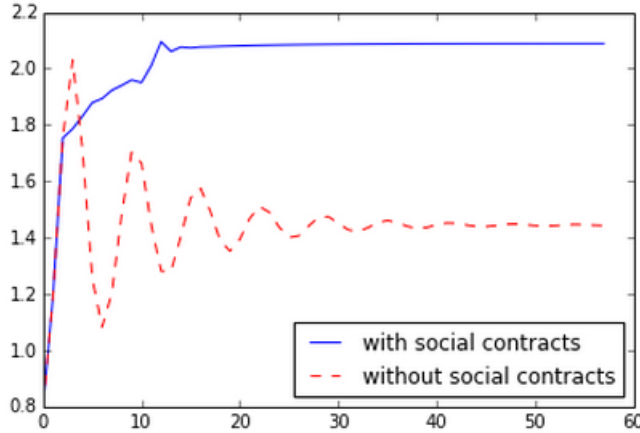


Figure. Income dynamics with and without social contracts in the case $\hat{I} < \tilde{I}$

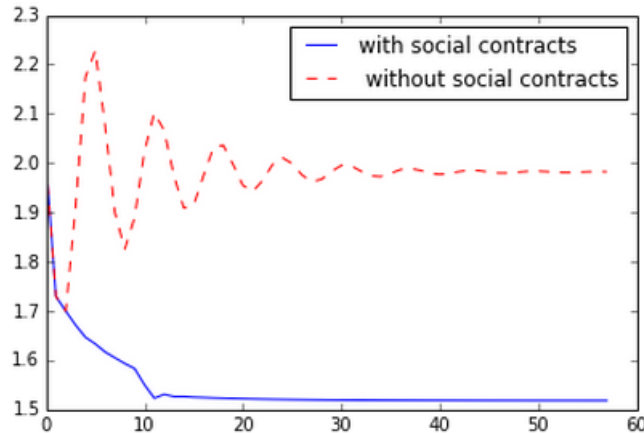
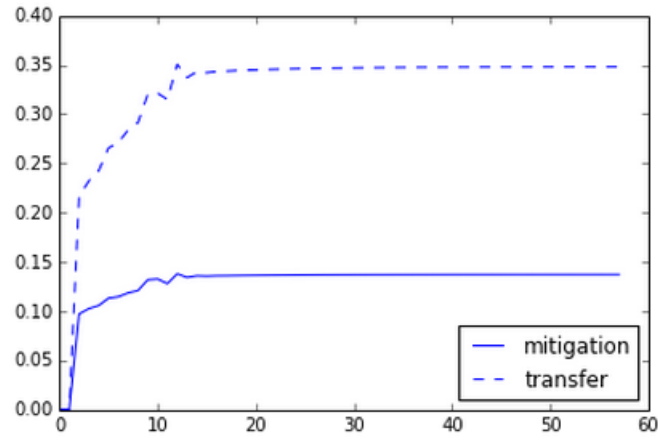


Figure. Emission dynamics with and without social contracts in the case $\hat{I} < \tilde{I}$ Figure. Mitigation and transfer in the case $\hat{I} < \tilde{I}$

5.2 The case $\hat{I} > \tilde{I}$

Early signing of social contracts

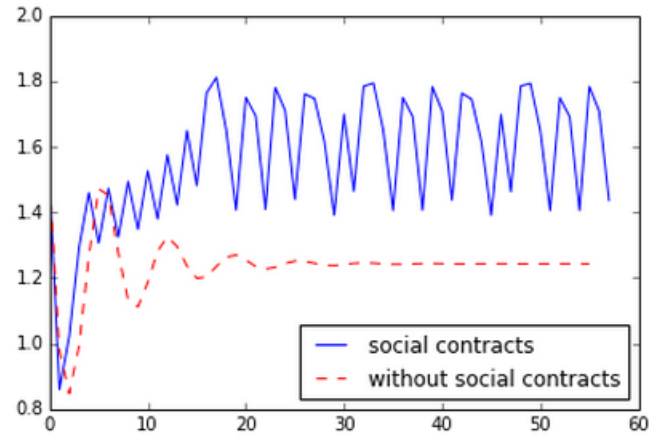
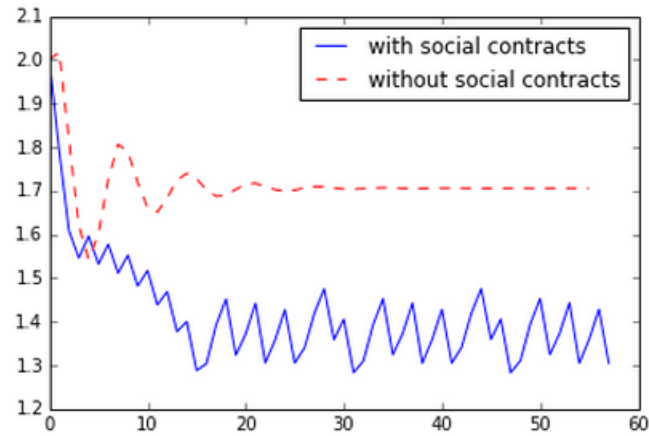
Figure. Income dynamics with and without social contracts in the case $\hat{I} > \tilde{I}$ 

Figure. Emission dynamics with and without social contracts in the case $\hat{I} > \tilde{I}$

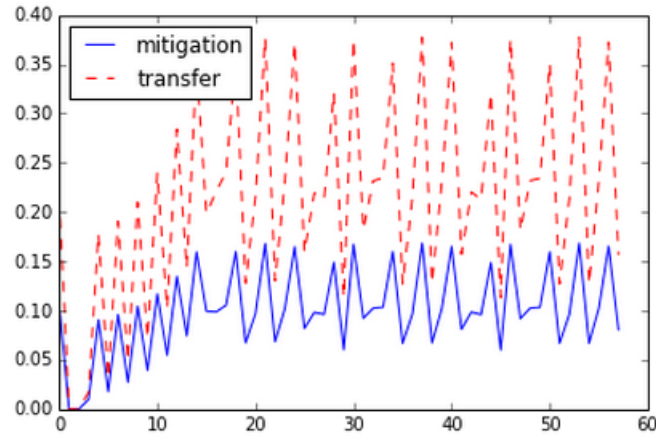


Figure. Mitigation and transfer in the case $\hat{I} > \tilde{I}$

Delayed signing of social contracts

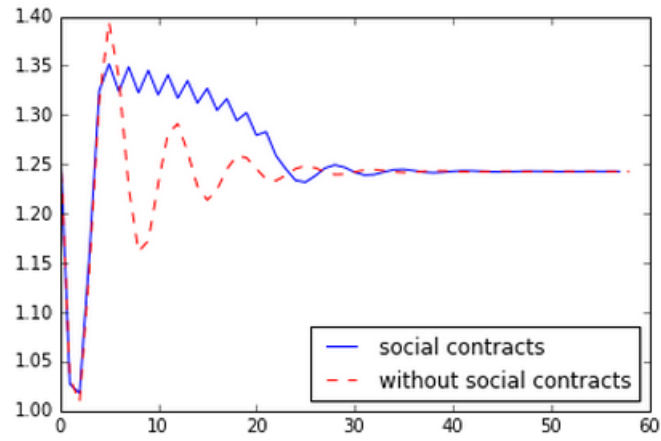


Figure. Income dynamics when delaying to sign social contracts in the case $\hat{I} > \tilde{I}$

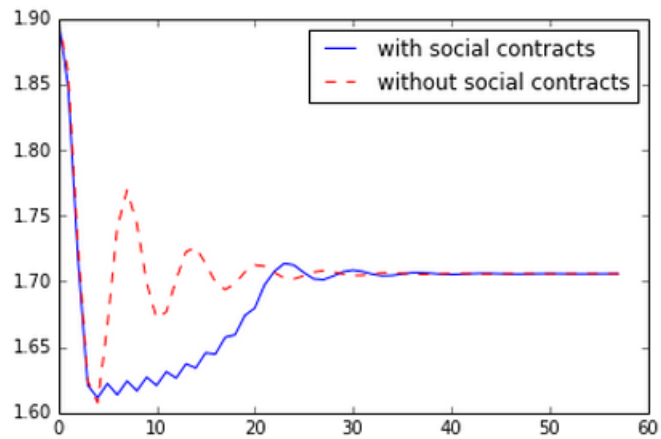
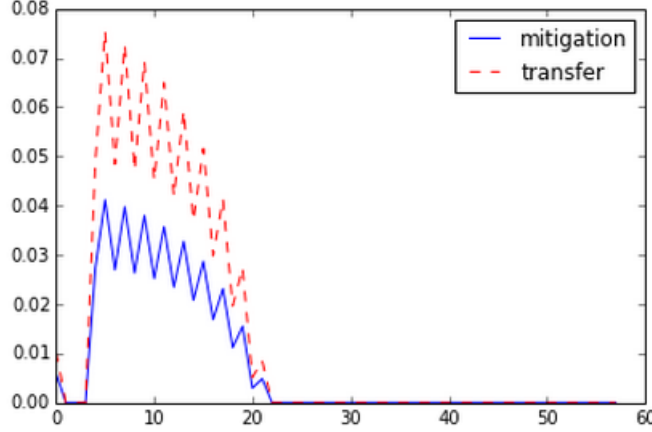


Figure. Emission dynamics when delaying to sign social contracts in the case $\hat{I} > \tilde{I}$ Figure. Mitigation and transfer in the case $\hat{I} > \tilde{I}$

In the previous section, when $\hat{I} > \tilde{I}$ we ignored the case $EE(m_t^*, \tau_{t+1}^{o*}) \cap II(m_t^*, \tau_{t+1}^{o*}) = \emptyset$. In this case, even under period-by-period social contracts, at the end the economy converges to the globally stable steady state (\tilde{I}, \tilde{E}) . This case implies that the social contracts are not strong enough to help the economy avoid the steady state (\tilde{I}, \tilde{E}) . In this paper, we do not focus on this case in both analytical analysis and simulation because, in our point of view, this case can be ruled out when we extend the model by allowing for technological progress. Technical progress can increase conditional steady state income \tilde{I} above the threshold \hat{I} .

6 A non-cooperative setup

So far, it has been assumed that two generations t and $t+1$ determine a contract (m_t^*, τ_{t+1}^{o*}) through negotiation at the beginning of period $t+1$. An agent in his working-age in period t leaves part of his labor income to the next generation to cover mitigation expenses. In exchange, he expects a subsidy on his old-age capital income. This setup presumes cooperation and neglects any commitment problems of agents. One could object, however, that the mitigation investment m_t is determined **within** period t , **before** the young agent in period $t+1$ decides about paying the transfer τ_{t+1}^o . This has crucial implications for the sustainability of a contract if effective enforcement or other forms of credible commitment to the contract are not possible. If agents of the young generation have an incentive to default on their obligations from the contract once the mitigation level is set, a contract between generations - even if Pareto-improving for both - will not be sustainable.

If we consider the contract between the adult generations in t and $t+1$ and there is no possibility of another contract after period $t+1$, the working-age generation in $t+1$ will always want to default on its contract with generation t if effective enforcement is not in place. The

mitigation level m_t is given from its perspective and the transfer it is obliged to pay is only a cost. Having perfect foresight, generation t will not offer a contract in the first place and choose not to mitigate. For there to be a chance of continuation after period $t + 1$, there has to be a return from complying with the contract. This can only be the case if generation $t + 1$ expects the next generation to sign a similar contract and thus to receive a transfer themselves. The transfer system thus offers a return for mitigation investment, which has to be sufficiently large (see Rangel (2003), Anderberg and Balestrino (2003)).

In this section, we first set up conditions under which the working-age generation in a period t will not default on a social contract forcing it to invest an amount \widehat{m}_t in mitigation and transfer a share $\widehat{\tau}_t^y$ of its income to provide for the subsidy $\widehat{\tau}_t^o$ to the capital return of the old in t . These conditions naturally depend on the expected transfer $\tau_{t+1}^{o,e}$ and on whether previous generations chose to comply with the contract. We then analyze the existence of an equilibrium path with positive transfer payment and mitigation investment in each period satisfying these conditions. In the next step, we examine whether and under which conditions the set of mitigation-transfer combinations which do not lead to default on the contract intersects the set of Pareto-improving mitigation-transfer pairs.

6.1 Conditions for self-sustainability

If a mitigation-transfer path imposed by a series of contracts gives no generation an incentive to default, it is dynamically self-sustainable without external enforcement (by a government). The following definition clarifies the terminology:

Definition 1. An equilibrium path with positive mitigation investment and a positive intergenerational transfer imposed by a series of contracts is self-enforcing or self-sustainable if there does not exist a time T such that any of the generations alive in that period has an incentive to default on the obligations from either of the contracts it is involved in.

Clearly, a mitigation-transfer path is self-enforcing or self-sustainable if, for each generation and **given the history of actions** taken by previous generations, complying with the contract yields higher lifetime-utility than defaulting. While the old generation in a period t has no obligations but only benefits from a contract, the young generation faces a trade-off.

To make compliance possible, two ingredients are crucial: (1.) There may not be a known terminal date of compliance: As explained above, a single contract between two generations t and $t + 1$ will not be sustainable if both generations know future generations at some time $T > t + 1$ will not sign another contract.⁷ (2.) There has to be credible punishment for deviation⁸: Generation t 's reward for compliance is the transfer it receives from generation $t + 1$ when old. But it will only have an incentive to comply, if it does not receive this transfer

⁷As Boldrin and Rustichini (2000) point out, if the time is not known with certainty, generations may choose compliance even if they are aware that the mitigation-transfer scheme will not be sustained forever.

⁸This point has, amongst others, been pointed out by Rangel (2003).

also if it deviates. Generation $t + 1$ must thus have an incentive to punish generation t for defaulting.

In the following, we will define simple trigger strategies under which each deviation is punished in the next - but only in the next - period. We then derive conditions for a strategy profile of these simple trigger strategies to be an equilibrium of the infinitely repeated game between generations. As Rangel (2003) proves in his paper, a path of mitigation investment and transfers can be sustained as a (subgame perfect) equilibrium in an infinitely repeated game if and only if it can be sustained by simple trigger strategies.

Definition 2. The history h_t of the game denotes the vector of actions $((m_{t-1}, \tau_{t-1}^y), \dots, (m_1, \tau_1^y))$ chosen up to time t . A strategy s_t for generation t is a plan which assigns a choice (m_t, τ_t^y) to every given history $h_t = ((m_{t-1}, \tau_{t-1}^y), \dots, (m_1, \tau_1^y))_t$. A strategy profile is a set $\{s_t\}_{t=1}^\infty$ containing the strategies chosen in each period t .

The following trigger strategy $s_t(h_t)$ is a slight modification of the one in Rangel (2003), where $(\hat{m}_t, \hat{\tau}_t^y)$ denotes some given combination of mitigation investment and transfer payment to the current old:

$$s_t(h_t) = \begin{cases} (\hat{m}_t, \hat{\tau}_t^y) & \text{if } p(h_t) = C \\ (\hat{m}_t, 0) & \text{if } p(h_t) = P \end{cases}$$

where

$$p(h_t) = \begin{cases} C & \text{if } (m_{t-1}, \tau_{t-1}^y) = (\hat{m}_{t-1}, \hat{\tau}_{t-1}^y) \text{ and } p(h_{t-1}) = C \\ C & \text{if } (m_{t-1}, \tau_{t-1}^y) = (\hat{m}_{t-1}, 0) \text{ and } p(h_{t-1}) = P \\ P & \text{if } t = 1 \\ P & \text{otherwise} \end{cases}$$

$p(h_t) = C$ indicates that the game is in a compliance phase. This phase occurs (i) if generation $t - 1$ has invested in mitigation and paid the transfer to generation $t - 2$ and generation $t - 2$ has itself complied with the contract and (ii) if generation $t - 1$ has invested in mitigation but not paid the transfer because generation $t - 2$ has deviated. $p(h_t) = P$ occurs if generation $t - 1$ has deviated, so that the game is in a punishment phase.

If a generation t wants to deviate, the best possible deviation is to choose $(m_t, \tau_t^y) = (0, 0)$. This is because if it chooses either the transfer to the currently old or mitigation different from the level prescribed by the contract, it will be punished by the next adult generation. It should thus expect not to receive a transfer when old and therefore choose both mitigation and the transfer payment equal to zero.

Denote the expected indirect utility from compliance as $V^c(\hat{m}_t, \hat{\tau}_t^y | p(h_t), \hat{\tau}_{t+1}^{o,e})$ and expected indirect utility from deviation as $V^d(0, 0 | p(h_t), 0)$. Generation t compares $V^d(0, 0 | p(h_t), 0)$ to $V^c(\hat{m}_t, \hat{\tau}_t^y | p(h_t), \hat{\tau}_{t+1}^{o,e})$ for any given history h_t , that is for the case that the previous generation complied and for the case that it deviated.

Incentive constraint when the game is in a state of compliance

If generation t complies with the contract when the game is in a compliance phase ($p(h_t) = C$), indirect utility is:

$$V^c(\hat{m}_t, \hat{\tau}_t^y | p(h_t) = C, \hat{\tau}_{t+1}^{o,e}) = \Phi + \beta \ln \left[z(\hat{E}_t) \alpha \left(\frac{\beta \hat{I}_{t|p(h_t)=C} (1 - \hat{m}_t)}{1 + \beta} \right)^{\alpha-1} \right] \\ + (1 + \beta) \left[\ln \hat{I}_{t|p(h_t)=C} + \ln(1 - \hat{m}_t) \right] + \beta \ln(1 + \hat{\tau}_{t+1}^{o,e})$$

where $\hat{\tau}_{t+1}^{o,e}$ is the foreseen transfer that generation t will receive from generation $t + 1$ when old, and $\hat{I}_{t|p(h_t)=C} = z(E_{t-1})(1 - \alpha)\hat{k}_t^\alpha(1 - \hat{\tau}_t^y)$ is the income of the young agent in period t after paying the intergenerational transfer $z(E_{t-1})(1 - \alpha)\hat{k}_t^\alpha\hat{\tau}_t^y = \hat{\tau}_t^o\hat{R}_t\hat{k}_t$, given a history of compliance up to period t . E_{t-1} is the emission stock in period $t - 1$ which is given in period t . While this stock differs depending on whether generation $t - 2$ mitigated or not, we will see below that it does not affect generation t 's decision to comply and we will neglect the case differentiation here. \hat{E}_t is the stock of emissions after a history of compliance up to date t .

If generation t deviates, indirect utility is:

$$V^d(0, 0 | p(h_t) = C, 0) = \Phi + \beta \ln \left[z(\hat{E}_t) \alpha \left(\frac{\beta I_{t|p(h_t)=C}}{1 + \beta} \right)^{\alpha-1} \right] + (1 + \beta) \ln I_{t|p(h_t)=C}$$

where $I_{t|p(h_t)=C} = z(E_{t-1})(1 - \alpha)\hat{k}_t^\alpha$ is income if generation t does not pay the transfer ($\hat{\tau}_t^y = 0$) after a history of compliance. Note that like the emission stock E_{t-1} , the pollution flow \hat{P}_t and the capital stock \hat{k}_t are given at time t and income is therefore, except for the transfer, given and observable for generation t .

The difference between indirect utilities is

$$\Delta V(\hat{m}_t, \hat{\tau}_t^y | p(h_t) = C, \hat{\tau}_{t+1}^{o,e}) = \beta(\alpha - 1) \ln \left(\frac{\hat{I}_{t|p(h_t)=C}(1 - \hat{m}_t)}{I_t} \right) \\ (1 + \beta) \left[\ln \hat{I}_{t|p(h_t)=C} + \ln(1 - \hat{m}_t) - \ln I_{t|p(h_t)=C} \right] + \beta \ln(1 + \hat{\tau}_{t+1}^{o,e}) \\ = (1 + \alpha\beta) \ln((1 - \hat{m}_t)(1 - \hat{\tau}_t^y)) + \beta \ln(1 + \hat{\tau}_{t+1}^{o,e})$$

There is no incentive to deviate, if and only if

$$\Delta V(\hat{m}_t, \hat{\tau}_t^y | p(h_t) = C, \hat{\tau}_{t+1}^{o,e}) \geq 0$$

or

$$[(1 - \hat{m}_t)(1 - \hat{\tau}_t^y)]^{1+\alpha\beta} (1 + \hat{\tau}_{t+1}^{o,e})^\beta \geq 1$$

Using the relation $\hat{\tau}_t^o \hat{R}_t \hat{k}_t = z(E_{t-1})(1 - \alpha) \hat{k}_t^\alpha \hat{\tau}_t^y$ between the transfer received by the old generation in t and the transfer paid by the working-age generation in t , this condition is equivalent to

$$\left(1 - \frac{\alpha}{1 - \alpha} \hat{\tau}_t^o\right)^{1+\alpha\beta} (1 - \hat{m}_t)^{1+\alpha\beta} (1 + \hat{\tau}_{t+1}^{o,e})^\beta \geq 1 \quad (47)$$

Note that the constraint for Pareto improvement of generation t does not depend on the size of the transfer or mitigation shares before period t .⁹

Incentive constraint given a history of deviation

Now consider the decision of generation t if generation $t - 1$ deviated from the contract, choosing $(m_{t-1}, \tau_{t-1}^y) = (0, 0)$. In this case, generation t should punish generation $t - 1$ with the choice $(\hat{m}_t, 0)$. We therefore have to check whether $(\hat{m}_t, 0)$ is (weakly) preferred over $(0, 0)$.

Indirect utility from compliance with the contract when the game is in a punishment phase ($p(h_t) = P$) after a history of deviation is:

$$\begin{aligned} V^c(\hat{m}_t, 0 | p(h_t) = P, \hat{\tau}_{t+1}^{o,e}) &= \Phi + \beta \ln \left[z(E_t) \alpha \left(\frac{\beta \hat{I}_{t|p(h_t)=P}(1 - \hat{m}_t)}{1 + \beta} \right)^{\alpha-1} \right] \\ &\quad + (1 + \beta) \left[\ln \hat{I}_{t|p(h_t)=P} + \ln(1 - \hat{m}_t) \right] + \beta \ln(1 + \hat{\tau}_{t+1}^{o,e}) \end{aligned}$$

where $\hat{I}_{t|p(h_t)=P} = z(\hat{E}_{t-1})(1 - \alpha) \hat{k}_t^\alpha$ is income if the agent born in t chooses compliance when the game is in a state of punishment.

Indirect utility from deviation is:

$$V^d(0, 0 | p(h_t) = P, 0) = \Phi + \beta \ln \left[z(E_t) \alpha \left(\frac{\beta I_{t|p(h_t)=P}}{1 + \beta} \right)^{\alpha-1} \right] + (1 + \beta) \ln I_{t|p(h_t)=P}$$

⁹In an earlier version of the model, we assumed that mitigation in period $t - 1$ positively affects potential income of generation t as it lowers health costs from the emission flow which this generation has to bear. In this case, the higher the mitigation investment m_{t-1} , the easier is the constraint satisfied. Because labor income is higher, a smaller share $\hat{\tau}_t^y$ of income is sufficient to provide a given transfer level $\hat{\tau}_t^o \hat{R}_t \hat{k}_t$ to the current old and the cost of compliance with the contract is lower.

where $I_{tp(h_t)=P} = z(\hat{E}_{t-1})(1 - \alpha)k_t^\alpha$. Note that income is the same whether generation t complies with the contract or deviates because in a punishment phase, generation t will not pay the transfer even if it chooses compliance.

There is no incentive for deviation if and only if:

$$\Delta V(\hat{m}_t, 0|p(h_t) = P_t, \hat{\tau}_{t+1}^{o,e}) = (1 + \alpha\beta) \ln(1 - \hat{m}_t) + \beta \ln(1 + \hat{\tau}_{t+1}^{o,e}) \geq 0$$

This condition is equivalent to

$$(1 - \hat{m}_t)^{1+\alpha\beta} (1 + \hat{\tau}_{t+1}^{o,e})^\beta \geq 1. \quad (48)$$

Not surprisingly, the minimum expected transfer $\hat{\tau}_{t+1}^{o,e}$ required by (48) to make the adult agent comply with the contract for a given m_t is smaller than that of (47) whenever the contract prescribes a positive transfer $\hat{\tau}_t^o$ in period t : To receive a transfer in period $t + 1$ when the history of the game is compliance, the adult agent has to pay a share $\hat{\tau}_t^y = \frac{\alpha}{1-\alpha} \hat{\tau}_t^o$ of his income in addition to mitigation costs, while he does not have to pay the transfer in the punishment phase. Therefore, the expected transfer $\hat{\tau}_{t+1}^{o,e}$ which is needed to make the agent just indifferent between compliance and default his higher after a history of compliance.

The insights of this paragraph yield the following proposition:

Lemma 5. *A path with a positive mitigation investment $\hat{m}_t > 0$ compensated for by a positive transfer $\hat{\tau}_{t+1}^o > 0$ in every period $t = 1 \dots \infty$ can be sustained as a subgame perfect equilibrium for the game with initial state $p(h_1) = P$ if and only if condition (48) is satisfied in period $t = 1$ and (47) is satisfied in every period $t > 1$, with $\hat{\tau}_{t+1}^{o,e} = \hat{\tau}_{t+1}^o$.*

Proof. The proof is essentially contained in the text. The condition $\hat{\tau}_{t+1}^{o,e} = \hat{\tau}_{t+1}^o$ merely states that in equilibrium, expectations must be realized. □

Define the set of all tuples $(m_t, \tau_t^o, \tau_{t+1}^{o,e})$ which satisfy condition (47) as

$$S_t^{y,IC} = \left\{ (m_t, \tau_t^o, \tau_{t+1}^{o,e}) \in [0, 1) \times [0, \tau^{o,max})^2 : \left(1 - \frac{\alpha}{1-\alpha} \tau_t^o \right)^{1+\alpha\beta} \geq \frac{1}{(1 - m_t)^{1+\alpha\beta} (1 + \tau_{t+1}^{o,e})^\beta} \right\} \quad (49)$$

We defined as $\tau^{o,max} \equiv \frac{1-\alpha}{\alpha}$ the maximum transfer reconcilable with non-negative income (i.e. $\tau_t^y \leq 1$) of the working-age agent in period t . Define the minimum expected transfer $\tau_{t+1}^{o,e}$ which is required so that the working-age (young) generation t in a period $t > 1$ will not default¹⁰ as a function of the mitigation share m_t for any given transfer τ_t^o it has to provide for the current old generation as

¹⁰We neglect condition (48) for the first generation here, as it is less strict than condition (47).

$$IC^y(m_t; \tau_t^o) = \left(1 - \frac{\alpha}{1-\alpha} \tau_t^o\right)^{-\frac{1+\alpha\beta}{\beta}} \frac{1}{(1-m_t)^{\frac{1+\alpha\beta}{\beta}}} - 1 \quad (50)$$

We will in the following refer to this curve as the IC -curve or 'incentive compatibility constraint'. The function is strictly increasing and strictly convex, with $IC^y(0; 0) = 0$.

6.2 Existence of self-enforcing mitigation-transfer schemes

Having defined the conditions for an equilibrium path with mitigation and an intergenerational transfer to be self-enforcing, the question arises if there exist, for every period t , any pairs (m_t, τ_{t+1}^o) with $\tau_{t+1}^{o,e} = \tau_{t+1}^o$ and $m_t > 0, \tau_{t+1}^o > 0$ which satisfy the constraints.

The first conclusion with this regard must be that along a path with self-enforcing contracts, the intergenerational transfer and the mitigation share cannot increase forever but must converge to some constant values τ^o, m . This is intuitive because at the latest if all income has to be given up to pay the transfer or invest in mitigation, the contract scheme will break down.

We can characterize the set of dynamically sustainable mitigation-transfer schemes with stationary transfer $\tau_{t+1}^o = \tau_t^o = \tau^o$ for all t by setting $\tau_{t+1}^o = \tau_t^o$ in (49):

$$S^{IC} = \left\{ (m, \tau^o) \in [0, 1) \times [0, \tau^{o,max}) : m \leq 1 - \frac{1}{(1 - \frac{\alpha}{1-\alpha} \tau^o) (1 + \tau^o)^{\frac{\beta}{1+\alpha\beta}}} \right\} \quad (51)$$

In what follows, we call this set the 'stationary' incentive compatible set and its boundary, given by

$$m^{IC} = 1 - \frac{1}{(1 - \frac{\alpha}{1-\alpha} \tau^o) (1 + \tau^o)^{\frac{\beta}{1+\alpha\beta}}} \quad \forall t \quad (52)$$

the 'stationary' incentive compatibility constraint.

Lemma 6. *Along an equilibrium path with dynamically self-sustainable or self-enforcing contracts, the series $(m_t, \tau_{t+1}^o)_{t=1}^\infty$ must converge to a constant (m, τ^o) . The set of sustainable pairs (m, τ^o) is characterized by (51).*

The social contracts proposed in this model uphold mitigation investment by linking it to a scheme of intergenerational transfers. The second conclusion with respect to the existence of a sustainable equilibrium path with positive transfer payment and mitigation investment stems from the observation that investing in the transfer scheme however to some extent competes with other forms of investment, in our model with the accumulation of physical capital. The incentive compatibility constraint weighs these two forms of investment against each other. A necessary and sufficient condition for the existence of self-enforcing contracts is therefore following: In a situation without a social contract (so that $m_t = \tau_t^o = \tau_{t+1}^o = 0$) the net

return of marginally increasing τ_t^o to invest in a transfer scheme with **stationary** transfers $\tau_t^o = \tau_{t+1}^o = \tau^o$ in addition to investing in capital must be strictly larger than the return that could be obtained by investing solely in capital instead. Formally, this comes down to demanding that the derivative of the stationary incentive constraint at $m = \tau^o = 0$ be positive. This is guaranteed by the following condition:

$$\frac{\alpha}{1-\alpha} \frac{1+\alpha\beta}{\beta} < 1 \quad (53)$$

Proposition 5. *There exist pairs $(m_t, \tau_{t+1}^{o,e})$ with $\tau_{t+1}^{o,e} = \tau_{t+1}^o$ and $m_t > 0, \tau_{t+1}^o > 0$ for every $t = 1 \dots \infty$ such that the associated equilibrium path is self-sustainable as a subgame perfect equilibrium if and only if condition (53) is satisfied. The maximum mitigation share which can be sustained is $m^{max} = 1 - \left(\frac{\alpha}{1-\alpha} \frac{1+\alpha\beta}{\beta} \right)^{\frac{1+\alpha\beta}{\beta}} \left((1-\alpha) \frac{1+\frac{1+\alpha\beta}{\beta}}{\frac{1+\alpha\beta}{\beta}} \right)^{1+\frac{1+\alpha\beta}{\beta}} < 1$.*

Proof. A sustainable path with $m_t > 0, \tau_{t+1}^o > 0$ for every $t = 1 \dots \infty$ exists, if and only if $S^{IC} \neq \emptyset$ with S^{IC} given by (51).

As can be seen from (52), the function $m^{IC}(\tau^o)$ delineating the boundary of S^{IC} is concave, with $m^{IC}(0) = 0$ and $m^{IC}(\bar{\tau}^o) = 0$ for some positive $\bar{\tau}^o < \tau^{o,max}$ as well as $\lim_{\tau^o \rightarrow \tau^{o,max}} m^{IC}(\tau^o) = -\infty$. Hence, both for $\tau^o = 0$ and for $\bar{\tau}^o < \tau^o < \tau^{o,max}$, no positive m can be sustained. $S^{IC} \neq \emptyset$ if and only if slightly raising the transfer τ^o from $\tau^o = 0$ allows to also slightly raise m . Formally, this is expressed by the condition $\frac{\partial m^{IC}}{\partial \tau^o}(0) = \frac{\beta}{1+\alpha\beta} - \frac{\alpha}{1-\alpha} > 0$ which results in condition (53). Because the IC-curve is continuous in τ^o for $\tau^o \in [0, \bar{\tau}^o]$, a sustainable equilibrium path with positive transfer payments and positive mitigation investment in each period will exist under condition (53) for sufficiently small mitigation levels m_t .

If m_t becomes too large, however, the transfer scheme needed to compensate for mitigation investment would be increasing over time and thus unsustainable. The maximum sustainable transfer is derived by setting $\frac{\partial m^{IC}}{\partial \tau^o} = 0$, solving for τ^o and substituting the solution back into (52). □

Note that for the existence of a sustainable path, it is crucial that the ratio of capital- to labor income at $m_t = \tau_t^o = \tau_{t+1}^o = 0$ which is $\frac{\alpha}{1-\alpha}$, be strictly smaller than one. Contrary to the previously cited models by Rangel (2003), Cigno (1993, 2006a, 2006b) and Anderberg and Balestrino (2003), we do not assume positive population growth. However, population growth is typically the only mechanism guaranteeing a positive rate of return to intergenerational transfers if they are granted lump sum. In our model, the transfer is subtracted from labor income of the working-age generation and given to the current old as a subsidy on capital returns. Therefore, the transfer rate τ^y the working-age generation has to pay to provide (and later receive) a given transfer rate τ^o on old-age income is the smaller, the smaller the old generation's capital income compared to the working-age generation's labor income. In

particular, as long as $\frac{\alpha}{1-\alpha} < 1$, the transfer rate paid is lower than the transfer rate received, i. e. $\tau^y = \frac{\alpha}{1-\alpha}\tau^o < \tau^o$. This gives rise to a positive net return of investing in a system of intergenerational transfers even in the absence of population growth! For the usual values $\alpha = 1/3$, $1 - \alpha = 2/3$ of the capital and labor elasticities in the production function, $\frac{\alpha}{1-\alpha}$ is indeed below one so that condition (53) can be satisfied. The second term in the condition, $\frac{1+\alpha\beta}{\beta}$, is greater than one, decreases in β and increases in α . It stems from the effects of a social contract on the total return to savings which are the following: (1.) A social contract increases the return to savings for any given future capital stock k_{t+1} and interest rate R_{t+1} via the transfer τ^o . This effect is the stronger, the higher the future capital stock k_{t+1} and thus the savings rate $\frac{\beta}{1+\beta}$ and β (2.) A social contract lowers savings and therefore the future capital stock and the total return to savings for a given interest rate. (3.) Reduced capital accumulation increases the equilibrium interest rate R_{t+1} . This last effect is more pronounced if α is smaller.

We can look at the effects just described in more detail, deriving condition (53) by explicitly comparing returns from pure capital investment and capital investment combined with the transfer scheme: Marginally increasing the transfer τ_t^o to the current old generation yields a perfectly foreseen return of $R_{t+1}^e k_{t+1} d\tau_{t+1}^{o,e} = R_{t+1} k_{t+1} d\tau_{t+1}^o$ in period $t + 1$. Further, an agent of generation t takes into account the equilibrium effect of slower capital accumulation due to lower income on the perfectly foreseen return to capital, $\frac{\partial R_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \tau_t^o} d\tau_t^o > 0$, which is associated with a total change in return of $\frac{\partial R_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \tau_t^o} d\tau_t^o k_{t+1} = (1 - \alpha) \frac{\beta}{1+\beta} R_{t+1} R_t k_t d\tau_t^o$. On the other hand, increasing the transfer τ_t^o reduces income in period t by $R_t k_t d\tau_t^o$. Investing this income in physical capital instead would yield an expected return of $R_{t+1} R_t k_t d\tau_t^o$ (as $\tau_{t+1}^{o,e} = 0$ without social contract). The agent will prefer to combine capital investment with investing in the transfer system and the social contract over investing only in capital if and only if

$$R_{t+1} k_{t+1} d\tau_{t+1}^o + (1 - \alpha) \frac{\beta}{1 + \beta} R_{t+1} R_t k_t d\tau_t^o - R_{t+1} R_t k_t d\tau_t^o > 0$$

Assuming a stationary transfer system, so that $d\tau_t^o = d\tau_{t+1}^o$, this condition becomes

$$R_{t+1} k_{t+1} + (1 - \alpha) \frac{\beta}{1 + \beta} R_{t+1} R_t k_t - R_{t+1} R_t k_t > 0.$$

After some simplification, taking into account $\tau_t^y = 0$, so that $R_t = \alpha z(E_{t-1}) k_t^{\alpha-1}$ and $k_{t+1} = \frac{\beta}{1+\beta} z(E_{t-1}) (1 - \alpha) k_t^\alpha$, the condition is equivalent to

$$1 - \frac{\alpha}{(1 - \alpha) \frac{\beta}{1+\beta}} \left(1 - (1 - \alpha) \frac{\beta}{1 + \beta} \right) > 0$$

$$1 - \frac{\alpha}{1 - \alpha} \frac{1 + \alpha\beta}{\beta} > 0$$

which is the condition given in condition (53).

6.3 Self-enforcing mitigation and Pareto improvement

We have shown that there exist combinations of mitigation investment and a transfer scheme which, when implemented, lead to a Pareto-improvement between successive generations. We have also shown that there exist equilibrium paths with positive mitigation investment and positive transfers in each period which are self-sustainable if commitment problems are taken into account. We are now interested in the question if and under which condition(s) a series of Pareto-improving contracts¹¹ between successive generations is also self-enforcing and self-sustainable over time.

Along an equilibrium path with self-enforcing Pareto improving contracts, conditions (19) and (23) for Pareto improvement as well as condition (47) for incentive compatibility have to be satisfied in every period t . Reinspection of the conditions shows that condition (19), which gives the lower boundary of the Pareto improvement set, is equivalent to condition (48). We know already that this condition, which ensures that generation t has no incentive to deviate from its strategy of choosing $(\hat{m}_t, 0)$ when the infinitely-repeated game is in a state of punishment, is less strict than condition (47), which ensures compliance of generation t when the game is in a state of compliance. It can therefore be ignored. We only have to check whether conditions (23) and (47), i. e. conditions

$$\left(1 - \frac{\alpha}{1 - \alpha} \tau_{t+1}^o\right)^{1+\alpha\beta} (1 - m_t)^{\alpha(1+\alpha\beta)} e^{\frac{\beta+\gamma(1+\beta)}{1+\beta} \beta m_t I_t} \geq 1$$

$$(1 - m_t)^{1+\alpha\beta} (1 + \tau_{t+1}^o)^\beta \left(1 - \frac{\alpha}{1 - \alpha} \tau_t^o\right)^{1+\alpha\beta} \geq 1$$

can be satisfied in any period t .¹²

Both conditions depend on τ_t^o and τ_{t+1}^o : Condition (23) is affected by τ_t^o via income I_t and additionally depends on gross income before transfer payment, $I_t^g = w_t$. As only paths with stationary mitigation-transfer schemes are sustainable, we can concentrate on these pairs. The set of stationary incentive compatible pairs, S^{IC} , is given in (51). The upper boundary of the set of stationary Pareto improving mitigation transfer combinations is implicitly given by setting $\tau_{t+1}^o = \tau_t^o$ in (23). It can be verified from the condition that for any given m_t , there still exists a unique τ_{t+1}^o so that (23) is satisfied with strict equality. The upper boundary with stationary transfers is still a continuous function of m_t , which we denote by $\bar{\psi}(m_t; I_t^g)$. Through implicit differentiation, we find that $\bar{\psi}(m_t; I_t^g)$ is also still concave, at least in the

¹¹Note that we require every single contract of the series to be a Pareto-improvement. As every generation signs two contracts, we could also allow welfare gains in one contract to compensate welfare losses in the other for the series of contracts to be Pareto-improving.

¹²In fact, in period $t = 1$, any Pareto-improving contract between generations 1 and 2 never gives generation 1 an incentive to deviate. The reason is that $\tau_1^o = 0$ according to our definition of strategies and condition (48) reduces to (19) for $\tau_t^o = 0$.

upward sloping part.¹³

The following figure shows the set of Pareto improving stationary combinations $(m_t, \tau_{t+1}^o; \tau_{t+1}^o = \tau_t^o)$ together with the stationary incentive compatible set, in (m_t, τ_{t+1}^o) -space, assuming a concave upper bound $\bar{\psi}(m_t; I_t^g)$:

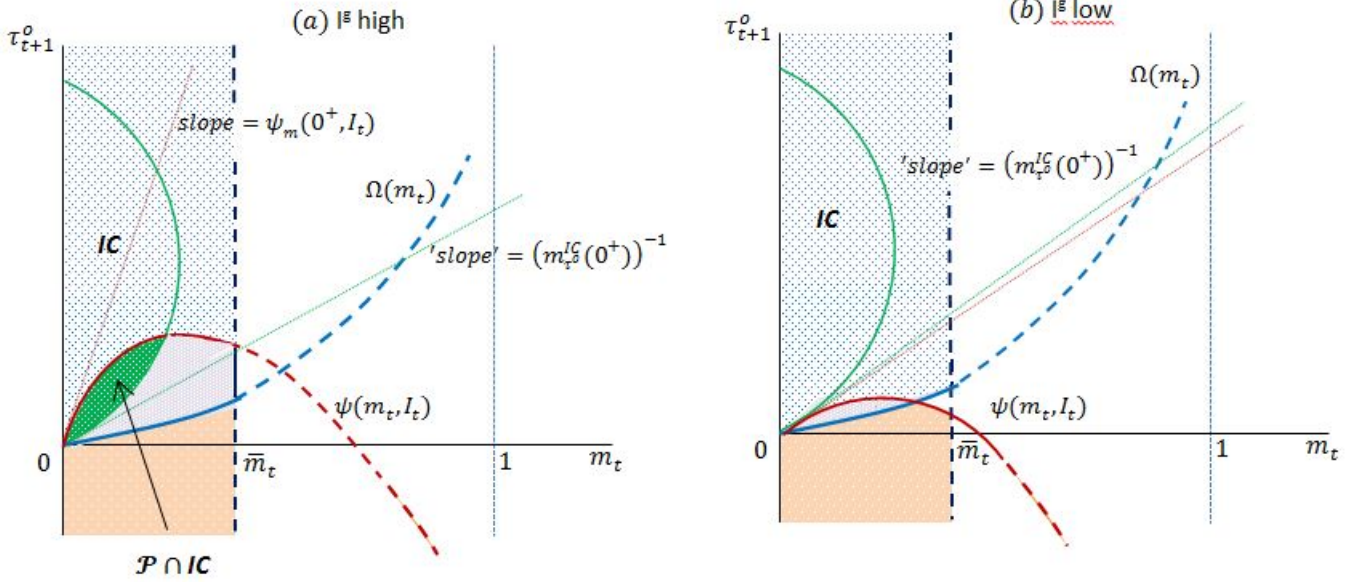


Figure 5: Pareto-improvement and incentive compatibility

We need to show now that at $m_t = \tau_{t+1}^o = 0$, the stationary IC-curve is less steep than the upper boundary of the stationary Pareto improvement area. We will prove below that this is guaranteed by the following condition on gross income:

$$I_t^g > \frac{(1-\alpha) \left(1 + \frac{\alpha\beta}{1+\beta}\right)}{1 - \frac{\alpha}{1-\alpha} \frac{1+\alpha\beta}{\beta}} \hat{I} > \hat{I} \text{ for } \frac{\alpha}{1-\alpha} \frac{1+\alpha\beta}{\beta} < 1 \quad (54)$$

Intuitively, the condition states that starting from $(m_t, \tau_t^o) = (0, 0)$, a slight increase in m_t going along with an equal increase $d\tau^o$ both in the transfer paid to the current old, τ_t^o , and the transfer received τ_{t+1}^o , such that agents of the transfer-providing generation are just kept from deviating will also improve these agents' welfare compared to the situation without a contract. There may still exist sustainable mitigation-transfer schemes not satisfying (54), but any transfer scheme with an initial mitigation-transfer combination from the Pareto-improvement set will be increasing over time and thus not be sustainable in the long run. A necessary condition for the existence of self-enforcing Pareto improving mitigation-transfer schemes is of course that the set IC of stationary self-enforcing schemes is non-empty, which is guaranteed by condition (53).

¹³This is enough to derive a necessary and sufficient condition for the existence of a set of sustainable, Pareto improving mitigation-transfer schemes. In figure 5 we assume $\bar{\psi}(m_t; I_t^g)$ is concave throughout.

Under conditions (53) and (54), we can prove the existence of mitigation-transfer schemes which are Pareto-improving and self-enforcing.

Proposition 6. *There exist paths $(m_t, \tau_{t+1}^o)_{t=1}^\infty$ which are both Pareto-improving compared to an equilibrium path without a contract and dynamically self-sustainable with (m_t, τ_{t+1}^o) being incentive compatible in every t if and only if conditions (53) and (54) are satisfied.*

Proof. First, note that condition (53) is needed to ensure the existence of a self-sustainable mitigation-transfer path and is therefore necessary but not sufficient for the existence of a sustainable and Pareto-improving path.

We need to show that at $m = \tau^o = 0$, the slope of the boundary of the set of self-sustainable contracts is steeper than the slope of $\Omega(m)$, the lower boundary of the set of stationary Pareto-improving contracts, and flatter than the slope of $\bar{\psi}(m; I_t^g)$, the upper bound of the set of stationary Pareto-improving mitigation-transfer schemes, if in addition condition (54) is satisfied.

The change along the boundary of the set IC of self-sustainable contracts starting from $m = \tau^o = 0$ is found by taking the inverse of

$$\frac{\partial m^{IC}}{\partial \tau^o}(\tau^o) = \frac{\frac{\beta}{1+\alpha\beta}}{(1 - \frac{\alpha}{1-\alpha}\tau^o)^2 (1 + \tau^o)^{1+\frac{\beta}{1+\alpha\beta}}} \left(1 - \frac{\alpha}{1-\alpha} \frac{1+\alpha\beta}{\beta} - \frac{\alpha}{1-\alpha} \left(\frac{1+\alpha\beta}{\beta} + 1 \right) \tau^o \right)$$

which is the derivative of (52), and evaluate at the origin:

$$\left(\frac{\partial m^{IC}}{\partial \tau^o}(0) \right)^{-1} = \frac{\frac{1+\alpha\beta}{\beta}}{1 - \frac{\alpha}{1-\alpha} \frac{1+\alpha\beta}{\beta}}$$

From (19), this slope can be easily verified to be steeper than that of $\Omega(m)$. Also, we have already noted before that for any $m > 0$, the set IC lies above the lower boundary $\Omega(m)$ of the Pareto improvement set.

The slope of the upper boundary $\bar{\psi}(m; I_t^g)$ can be found by setting $\tau_{t+1}^o = \tau_t^o$ in (23) and using implicit differentiation. It is given by

$$\bar{\psi}'(m; I_t^g) = \frac{(1-\alpha) \left(\frac{\beta+\gamma(1+\beta)}{\alpha(1+\beta)(1+\alpha\beta)} \beta I_t^g \left(1 - \frac{\alpha}{1-\alpha} \bar{\psi}(m; I_t^g) \right) - \frac{1}{1-m} \right)}{e^{\frac{(\beta+\gamma(1+\beta))\beta}{(1+\beta)(1+\alpha\beta)}} m I_t^g \left(1 - \frac{\alpha}{1-\alpha} \bar{\psi}(m; I_t^g) \right) (1-m)^\alpha + \frac{(\beta+\gamma(1+\beta))\beta}{(1+\beta)(1+\alpha\beta)} m I_t^g}$$

Now compare $\left(\frac{\partial m^{IC}}{\partial \tau^o}(0)\right)^{-1}$ to $\bar{\psi}'(0; I_t^g)$ which, using $\bar{\psi}(0; I_t^g) = 0$ can be shown to be $\bar{\psi}'(0; I_t^g) = (1 - \alpha) \left(\frac{\beta + \gamma(1 + \beta)}{\alpha(1 + \beta)(1 + \alpha\beta)} \beta I_t^g - 1\right)$: The relation

$$\left(\frac{\partial m^{IC}}{\partial \tau^o}(0)\right)^{-1} < \bar{\psi}'(0; I_t^g)$$

is satisfied if and only if

$$\begin{aligned} \frac{\frac{1 + \alpha\beta}{\beta}}{1 - \frac{\alpha}{1 - \alpha} \frac{1 + \alpha\beta}{\beta}} &< (1 - \alpha) \left(\frac{\beta + \gamma(1 + \beta)}{\alpha(1 + \beta)(1 + \alpha\beta)} \beta I_t^g - 1\right) \\ \Leftrightarrow I_t^g &> \frac{(1 - \alpha) \left(1 + \frac{\alpha\beta}{1 + \beta}\right)}{1 - \frac{\alpha}{1 - \alpha} \frac{1 + \alpha\beta}{\beta}} \underbrace{\frac{\alpha(1 + \beta)^2(1 + \alpha\beta)}{(1 - \alpha)\beta^2(\beta + \gamma(1 + \beta))}}_{\hat{I}_t} > \hat{I}_t \end{aligned} \quad (55)$$

Therefore $\left(\frac{\partial m^{IC}}{\partial \tau^o}(0)\right)^{-1} < \bar{\psi}'(0; I_t^g)$ if and only if condition (54) holds along with condition (53). This is necessary and sufficient for $IC \cap \mathcal{P} \neq \emptyset$. \square

Note that condition (54) is satisfied the easier, the smaller α and the higher the efficiency of the mitigation technology as measured by γ . The intuition for the effect of α is twofold: (1.) On the one hand, mitigation in period t reduces income in t and, because less capital is accumulated, also capital income in period $t + 1$ for a given interest rate. On the other hand, the negative effect of mitigation on capital accumulation increases the equilibrium interest rate in $t + 1$. This second positive effect on agent t 's old age income is the larger, the smaller α . Therefore, the minimum transfer τ^o required for compliance of generation t will be lower. Generation $t + 1$ faces similar effects on its income in $t + 1$ and expected income in $t + 2$ from paying the transfer τ^o so that the maximum transfer τ^o consistent with a welfare-improvement for generation $t + 1$ will be larger for relatively smaller α . (2.) The ratio of capital to labor income, $\alpha / (1 - \alpha)$, is the smaller, the smaller α . The smaller capital income compared to labor income, the lower the share of income that has to be given up to achieve a given subsidy rate τ^o to the old generation of a period t . A marginal increase in the intergenerational transfer τ^o of every period can therefore be obtained at lower cost. This decreases the minimum transfer τ^o required for compliance of generation t and increases the maximum transfer τ^o consistent with a welfare-improvement for the transfer-providing generation $t + 1$.

The more efficient the mitigation technology (the higher γ), the larger the decrease in emissions and associated positive effect on productivity and thus the interest rate in $t + 2$ achieved by an increase in mitigation in t . Therefore the maximum transfer generation $t + 1$ can pay without suffering a welfare loss increases. γ is not relevant for incentive compatibility because emission

reductions from mitigation in previous periods are always taken as given by every generation. The effect of an increase in the rate of time preference β is ambiguous: On the one hand, compliance is easier to achieve, as pointed out in the previous section, because the positive effect of a transfer on capital income in $t+1$ is the more pronounced, the higher the capital stock and thus the savings rate which increases in β . On the other hand, a welfare improvement may be harder or easier to achieve with higher β : Because of the higher savings rate, a marginal increase in mitigation m has a stronger negative effect on capital accumulation. This lowers income of the young generation in $t+1$. But slower capital accumulation is also reflected in smaller emissions which increases the expected interest rate for period $t+2$.

Note that neither is every path $(m_t, \tau_{t+1}^o)_{t=1}^\infty$ which is Pareto-improving compared to the situation without contract also self-enforcing, nor is every self-enforcing path a Pareto-improvement. The first observation follows from the fact that the stationary *IC*-curve lies above the lower boundary $\Omega(m)$ of the Pareto-improvement area for $m > 0$, so that for any given $m > 0$, there exists a stationary transfer $\tau^o > 0$ which satisfies the conditions for Pareto-improvement but not condition (47) for incentive compatibility. The second part of the claim can be verified by noting that mitigation-transfer schemes close to the vertical intercept at $(0, \bar{\tau}^o)$ with $m > 0$ but close to zero and a transfer smaller than but close to $\bar{\tau}^o$ are incentive compatible but not Pareto improving.

7 Conclusion

Climate change mitigation is a question of intergenerational distribution of costs and benefits and of intergenerational cooperation. While the burden of mitigation mainly falls on current generations, benefits largely accrue to generations not yet born. This is one reason why policy makers focussing more on current generations of voters are reluctant to impose environmental policy even if they believe that the correction of the environmental externality is desirable from a social planner's perspective. In this paper, we have studied the scope for mitigation to be supported by all generations in a general equilibrium OLG model with non-altruistic agents.

We have considered a social contract between successive generations according to which young generations invest a share of their labor income in mitigation in return for a subsidy to their old-age capital income which is paid by the next young generation. We have derived a condition for the existence of contracts which are Pareto-improving over the equilibrium without contract and characterized the set of Pareto-improving combinations of mitigation share and transfer, as well as the efficiency frontier and the Nash bargaining solution. While in our model the long-run effects of a social contract on the environment are a priori ambiguous because of its diverse effects on capital accumulation, we have proven that steady-state emissions are in fact lower and steady state income is higher under period-by-period social

contracts with Naks bargaining.

Even if a contract improves the welfare of all generations involved, generations may have an incentive to default on the provisions of the contract in the absence of legal enforcement. We have therefore also characterized the conditions under which a series of contracts is self-enforcing or dynamically self-sustainable in the sense that no generation has an incentive to deviate in a non-cooperative setting. We have shown under which condition there exists a set of incentive compatible combinations of mitigation share and transfer. Finally, we have shown it is possible to find combinations of the mitigation share and transfer, for which a contract scheme is simultaneously Pareto-improving and self-enforcing. For such combinations, mitigation will be beneficial for and supported by voters in each period so that political pressure is minimized. Institutions are still needed though to provide information, coordinate payments and surveil compliance with the contract.

Two strengths of our work are the general equilibrium perspective of the model and the full dynamic analysis given. Further, a contribution to existing literature is the accounting for commitment problems in this setup. Our model could be extended in several ways. In particular, studies on how technical change and population growth affect the results of our paper are under way.

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