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Estimates of Labor-Supply Elasticities with Joint Borrowing Constraints of Couples

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Abstract

Estimates of Frisch labor-supply elasticities are larger for women than for men. We show that standard labor-supply regressions tend to overestimate this gender difference. In couples with joint borrowing constraints, wage-rate fluctuations of the secondary earner are less important for the couples' willingness to borrow. This results in smaller estimation biases for secondary earners which are mostly women, empirically. Quantitatively, our results suggest that the gender difference in true Frisch elasticities is about a fifth smaller than reported previously.

JEL classification: E24, J16, J22

Keywords: Frisch labor supply elasticity; Incomplete markets; Two-person households

1 Introduction

Estimates for the Frisch labor supply elasticity are substantially higher for women than for men, see, e.g., Blundell and MaCurdy (1999) and Keane (2011). This gender difference suggests that women react more strongly to transitory changes in the real wage rate, which is important for understanding individual labor supply reactions to, e.g., transitory tax or productivity shocks.

In this paper, we show that the difference in the true Frisch elasticities of women and men is in fact smaller than documented by previous studies. Standard approaches over-estimate the gender difference in labor-supply elasticities because they ignore the differential effect of borrowing constraints on primary and secondary earners in couples households, respectively.

The typical estimation approach to recover labor supply elasticities is regressing relative changes in hours worked on relative changes in the real wage rate, see, e.g., Blundell and MaCurdy (1999). In such regressions, relative wage changes need to be instrumented, where the typical instrument is a measure of the expected wage change. Domeij and Flodén (2006) have shown that this standard approach is problematic when idiosyncratic risk and borrowing constraints play an important role.¹ Agents with little wealth tend to work more to build some buffer against negative shocks. The true relation between changes in labor supply and changes in the wage rate includes the willingness to borrow as measured by the Kuhn-Tucker multiplier on the borrowing constraint. The econometric problem is that the willingness to borrow is correlated with the expected change in the wage rate, which is the instrument in the standard labor-supply regression. If an agent expects an increase in the wage rate, she would like to borrow against this future income and, if she cannot borrow, this drives up the shadow price of the borrowing constraint. This leads to an omitted variable bias since the multipliers are not observable and are thus part of the error term in the regression. Domeij and Flodén (2006) have shown that estimates of Frisch elasticities are biased downwards when these effects due to borrowing constraints are ignored.

Our contribution is to reconsider this estimation bias in a setting where the household consists of two decision makers, e.g. husband and wife. A two-earner setup allows us to investigate the supposed gender difference in the estimates for the Frisch labor supply elasticity.² We show that the gender difference tends to be over-estimated because the general estimation bias due to borrowing constraints discussed above is of differential importance for

¹Borrowing constraints are a substantial restriction to many households in the US. Diaz-Gimenez, Glover, and Rios-Rull 2011 have shown that about 40% of households in the US have virtually no net wealth in 2007.

²Recently, the literature has recognized the importance of the family for labor supply behavior in general and in the presence of borrowing constraints (e.g., Ortigueira and Siassi 2013).

men and women, respectively, from a quantitative point of view.

Empirically, men often tend to be primary earners in couple households, while women often tend to be the secondary earners. When the status of being primary or secondary earner is not distributed evenly among genders, also the labor-supply consequences of joint borrowing constraints differs between genders. The household's willingness to borrow is more strongly correlated with expected relative wage changes of the spouse with the higher labor earnings, i.e. with the expected relative wage change of the primary earner. One reason for this is that a given relative expected wage change for the primary earner translates into a stronger change in the couple's future income and, hence, almost mechanically, affects the couple's willingness to borrow more strongly than the same relative wage change for the spouse with the lower labor earnings (the secondary earner). Further, if the borrowing-constrained couple uses spousal labor supply to counteract negative wage shocks, it is easier to stabilize household income and consumption in reaction to wage shocks for the secondary earner using the primary earner's labor supply than vice versa. Consequently, the household's consumption and its willingness to borrow reacts more strongly to wage shocks for the primary earner.

There are two consequences to this. First, the downward bias of estimated Frisch labor supply elasticities due to borrowing constraints is less pronounced for the secondary than for the primary earner. Second, since the wife is the secondary earner in many married couples, it results that the estimated Frisch labor supply elasticity of women is less biased downwards than the estimated labor supply elasticity of men. Taken together, these findings imply that the gender difference in the true Frisch elasticities is smaller than suggested by standard labor-supply regressions that ignore borrowing constraints. In other words, part of the previous view that women tend to have higher labor-supply elasticities than men is due to the fact that the standard estimation approach yields estimation results that are less biased downwards for women than for men, simply because women tend to be secondary earners in many couple households.

Empirically, this form of household relation is the most common one in the US. In 2012, 60% of households in the US were family households (US Census Bureau). The share of married couples in the US where the wife has lower labor earnings than the husband's was 76.5%. In married couples where both spouses work, this number was still 71% (2014 BLS databook 'Women in the Labor Force'). In addition, the legal situation and banks' practices gives rise to joint borrowing constraints for married couples.³

³The legal situation in the US does distinguish between individual and joint debt in marriages. However, for debt that is taken on within marriage, joint responsibility is obligatory in any case in some federal states

To provide gender-specific estimates of labor supply elasticities, we develop a partial-equilibrium incomplete markets model (see, e.g. Deaton 1991) with endogenous labor supply and two decision makers. Both spouses, husband and wife, are subject to a joint borrowing constraint. Individual household members face idiosyncratic wage shocks and can insure against these shocks by precautionary savings in a non-state contingent asset and spousal labor supply.

We first show in a simplified version of our model that the estimation bias due to borrowing constraints is less pronounced for the secondary than for the primary earner. We then solve the full model using numerical techniques. The model is used to simulate a synthetic household panel data set featuring labor supply decisions of both partners. The model is set up in a way such that the true Frisch labor supply elasticities of women and men are known. We then estimate standard labor supply regressions on the simulated data to estimate gender-specific Frisch elasticities.

We find that the estimated Frisch elasticities are generally biased downwards, reflecting the general estimation bias due to borrowing constraints highlighted by Domeij and Flodén (2006). Our contribution is to show that this estimation bias is smaller in samples of women reflecting the lower correlation of their expected wage rate changes with the bindingness of the couple’s borrowing constraint. Quantitatively, we find that men’s Frisch elasticity is estimated to be about 58% of its true value which is similar to the results of Domeij and Flodén (2006). Further, our results indicate that the estimate of the Frisch elasticity for women is about 35% below its true value. Given the stylized observation from previous contributions that estimates for the female Frisch elasticity tend to be twice as high as those for the male elasticity, we conclude that the true female Frisch elasticity is only about 80% larger than the true male Frisch elasticity.

These results have important implications. First, they affect the calibration of incomplete market models with families (e.g., Ortigueira and Siassi 2013). In such studies, gender-specific preference parameters should not be set as differently as it would seem appropriate neglecting our results. Second, if women’s labor supply to the market is not as much more elastic than men’s as previous studies suggested, a higher proportion of the gender wage gap can be explained through monopsonistic discrimination.⁴ Further, our results imply that

and in most cases (including housing and education debt) in other states. Common financial activities of couples such as joint accounts, consolidating different debts, and refinancing, imply taking responsibility also for one’s spouse’s pre-marriage debt. Most important for our argumentation is the mutual dependency of spouses’ credit ratings. Although such ratings are individual, the spouse’s credit history is always inspected along one’s own if one applies for credit in the US. Consequently, a poor credit worthiness of one’s spouse does impact on one’s own borrowing possibilities.

⁴Employers use market power to press down wages and do more so for women over whom they have more

labor supply reactions differ by gender, wealth, and their interaction. Shocks and policies which change net real wage rates have relatively similar effects on men and women in rich households but effects are more different for men and women in wealth-poor households. In other words, in wealth-poor households, reactions to wage rate fluctuations are not solely governed by Frisch elasticities. Finally, our study points towards policy interdependencies. Policies that affect the importance of borrowing constraints, e.g. saving subsidies or public guarantees, affect the distribution of labor supply reactions to policies that affect net wage rates, such as temporary tax cuts.

The remainder of this paper is organized as follows. In Section 2, we develop a quantitative incomplete markets model with two earners to study the intertemporal behavior of a two-earner household. In Section 3, we perform a Monte-Carlo study where we estimate standard labor-supply regressions on synthetic data simulated using our model. Section 4 concludes.

2 The Model

2.1 Model set-up

The economy is populated by heterogeneous households consisting of two spouses, a female (f) and a male (m) agent each. Households differ by asset holdings and spouses' wage rates such that there is heterogeneity within as well as between households. Members of a household are subject to joint budget and borrowing constraints and take decisions collectively, in the sense of Chiappori (1988, 1992). Since intra-household decision making is collective the resulting allocations are Pareto optimal. Households are borrowing constrained and use precautionary savings and spousal labor supply to insure against bad wage rate realizations, similarly to the model of Ortigueira and Siassi (2013).

The total population is of measure two, with half of the measure being female and the other half male, such that there is a continuum of households of measure one. An agent has preferences over consumption c and hours worked n ,

$$u_{gj} = u_g(c_{gj}, n_{gj}),$$

where the index gj collects the individual's gender $g = m, f$ and the household j in which the individual lives. The utility function is increasing and concave in c_{gj} and decreasing and

market power (see, e.g., Hirsch 2009, Ransom and Oaxaca 2010). The appropriate measure of employers' market power is the inverse elasticity of labor supply to the individual firm. This elasticity is a combination of the elasticity of labor supply to the individual firm at the extensive margin (which is found to be higher for women) and the Frisch elasticity of labor supply to the market at the intensive margin. Our result that the true Frisch elasticities are not that much higher for women than for men implies that employers' market power over women is in fact larger relative to that they have over men than previously thought.

concave in n_{gj} . We assume that market consumption is a household public good, i.e. there is no rivalry between spouses, $c_{mj} = c_{fj} = c_j$, and that leisure is the only private good.⁵

Since collective intra-household decision making leads to Pareto optimal allocations, the decision process can be represented by the decisions of a household planner who maximizes a household target function v_j which is a weighted sum of members' utilities with weights μ_j and $1 - \mu_j$ for the husband and the wife, respectively,

$$v(c_j, n_{mj}, n_{fj}) = \mu_j \cdot u_m(c_j, n_{mj}) + (1 - \mu_j) \cdot u_{fj}(c_j, n_{fj}). \quad (1)$$

We assume full commitment between the two spouses. This means that, at any time, decisions reflect spouses' initial bargaining positions, i.e. the Pareto weights μ are constant. Agents discount future utility with the discount factor $\beta \in (0, 1)$ and so does the household planner future values of v_j .

Individuals' wage rates w_i are stochastic. The particular wage process is

$$\begin{aligned} \ln w_{gj} &= \psi_g + z_{gj}, \\ z'_{gj} &= \rho_g \cdot z_{gj} + \varepsilon'_{gj}, \end{aligned} \quad (2)$$

where ψ_g is a gender-specific fixed effect giving rise to a gender wage gap. z_{gj} is the stochastic wage component which follows a gender-specific AR(1) process with innovations ε_{gj} and autocorrelation ρ_g . Innovations ε have mean zero but can be correlated between spouses. The vector of both spouses' innovations is i.i.d. across time.

Asset markets are incomplete. Agents have access to a non-state-contingent risk-free asset a . This asset can be purchased at price $1/(1+r)$ per asset and pay out one unit of the consumption good per asset in the next period. Thus, r denotes the constant risk-free interest rate, which is exogenous in our model.⁶ Households are borrowing constrained, i.e., assets can not fall below a minimum level denoted by a_{\min} :

$$a'_j \geq a_{\min}. \quad (3)$$

⁵Together with additively separable preferences, the public good assumption implies that a constant marginal valuation of wealth by the household is equivalent to constant marginal utility of consumption for the individuals. The same could be achieved with purely private goods or any collection of private and public goods, where private goods are distributed among household members in a way that equalizes weighted marginal utilities, if individuals have the same risk aversion with respect to consumption.

⁶We consider a partial equilibrium set-up because we neither analyze policy nor parameter changes. As Domeij and Flodén (2006), we assume that $\beta(1+r) < 1$, which implies an equilibrium with constant aggregates.

Household j 's budget constraint is given by

$$c_j + \frac{a'_j}{1+r} \leq w_{mj}n_{mj} + w_{fj}n_{fj} + a_j, \quad (4)$$

where a' are next period asset holdings to be determined and purchased at price $1/(1+r)$ this period. Throughout, a prime denotes next period values.

Recursive Formulation. We consider the problem of a household j taking as given the Pareto weights μ_j . The household's problem can be solved with techniques of dynamic programming. The state space of the recursive maximization problem is $\Omega = [a_{\min}, a_{\max}] \times S$ where $[a_{\min}, a_{\max}]$ denotes the interval of possible asset choices, and $S = S_m \times S_f$ denotes the state space of the wage pair $\omega = (w_m, w_f)$.⁷

The household problem in recursive formulation is given by:

$$V(a_j, \omega_j) = \max_{a'_j, c_j, n_{mj}, n_{fj}} \mu_j \cdot u_m(c_j, n_{mj}) + (1 - \mu_j) \cdot u_f(c_j, n_{fj}) + \beta E[V(a'_j, \omega'_j) | \omega_j], \quad (5)$$

subject to the borrowing constraint (3) and the budget constraint (4), for exogenous wage rates ω_j and a given initial value of assets $a_{j,0}$. The solution to the maximization problem are the policy functions

$$x_j = x(a_j, \omega_j), \quad (6)$$

with $x \in X = \{c, n_m, n_f, h_m, h_f, a'_j\}$.

Equilibrium conditions. The first order conditions of the recursive problem are

$$\frac{\partial v(c_j, l_{mj}, l_{fj})}{\partial c_j} = \frac{\partial V_j(a_j, \omega_j)}{\partial a_j} = \lambda_j \quad (7)$$

$$\lambda_j - (1+r)\phi_j = (1+r)\beta E[\lambda'_j | \omega_j] \quad (8)$$

$$w_{mj} = -\mu_j \frac{\partial u_{mj} / \partial n_{mj}}{\lambda_j} \quad (9)$$

$$w_{fj} = -(1 - \mu_j) \frac{\partial u_{fj} / \partial n_{fj}}{\lambda_j} \quad (10)$$

$$\phi_j \geq 0 \quad (11)$$

$$a'_j \geq a_{\min} \quad (12)$$

$$\phi_j (a'_j - a_{\min}) = 0, \quad (13)$$

together with the budget constraint (4), given the exogenous wage rates w_{mj} and w_{fj} and the initial asset stock $a_{j,0}$. λ_j is the Lagrange multiplier on the budget constraint (4). ϕ_j

⁷The upper bound a_{\max} guarantees a bounded state space which is needed in order to describe the economy by a probability measure. It is set high enough to never be reached.

is the Kuhn-Tucker multiplier on the borrowing constraint (3). Condition (8) is the Euler equation of the household. Conditions (9) and (10) are conditions of optimal labor supply of the spouses which also reflect that an individual's labor supply depends negatively on his or her Pareto weight within the household. Conditions (11)-(13) are the Kuhn-Tucker conditions associated with the borrowing constraint (3).

From (9) and (10), it can be seen that the Frisch labor supply elasticity follows from the curvature of disutility of labor, independent of whether the household is borrowing constrained or not. Thus, the true Frisch labor supply elasticity does not depend on whether the household is borrowing constrained or not.

In the following, we focus on the behavior of a particular household and omit the household index j to save on notation. Empirical studies estimate the relation between changes in the wage rate and changes in labor supply to infer the Frisch elasticity. We use the first-order conditions above to determine the relation between these variables in our model. If we take logs of (10) and take first differences, we obtain

$$\Delta \ln w'_g = \Delta \ln (-\partial u_g (c', n'_g) / \partial n'_g) - \Delta \ln \lambda'. \quad (14)$$

The Euler equation (8) implies

$$\Delta \ln \lambda' = -\ln \beta - \ln(1+r) + \frac{\phi}{\lambda} + \xi', \quad (15)$$

where $\xi' = \ln \lambda' - E \ln \lambda'$ denotes an expectation error. Inserting (15) in (14) yields

$$\Delta \ln (-\partial u_g (c', n'_g) / \partial n'_g) = \Delta \ln w'_g + \frac{\phi}{\lambda} - \ln \beta - \ln(1+r) + \xi'. \quad (16)$$

The left-hand side of (16) is increasing in hours worked n' due to the increasing marginal disutility of labor. Hence, condition (16) states that the change in an individual's labor supply is positively related to the change in his or her wage rate but is also affected by the multiplier ratio $\frac{\phi}{\lambda}$. The latter relation leads to a bias in empirical estimations, since the multiplier ratio is unobservable.⁸ To illustrate this bias, we consider the utility function

$$u_g(c, n_g) = u_g^c(c) - \alpha_g \frac{(n_g)^{1+1/\eta_g}}{1+1/\eta_g},$$

for which (16) takes the form

$$\Delta \ln n'_g = \eta_g \cdot \Delta \ln w'_g + \eta_g \cdot \frac{\phi}{\lambda} - \eta_g \cdot \ln \beta - \eta_g \cdot \ln(1+r) + \eta_g \cdot \xi'. \quad (17)$$

⁸The remaining terms on the right-hand side of (16) can be captured by fixed effects and residuals.

Since $\Delta \ln w'_f$ is correlated with the Euler residual ξ' , this regressor needs to be instrumented. A valid instrument is its expectation, $E\Delta \ln w'_f$. In presence of borrowing constraints a further complication arises since ϕ_j/λ_j is an unobservable variable which is correlated with $E\Delta \ln w'_f$. We consider this correlation analytically in a simplified version of our model in Section 2.2. Due to its correlation with $E\Delta \ln w'_f$, omitting ϕ_j/λ_j causes a bias in the estimation. To investigate this estimation bias quantitatively, we solve the model and generate data where we know the true Frisch labor supply elasticity (see Section 2.3).

2.2 Analytical results

To investigate the correlation of ϕ/λ and $E\Delta \ln w'_g$ analytically, we use a simplified version of our model. Here, we assume that no gender differences exist except for a gender wage gap in mean wage rates, captured by

$$\psi_m > \psi_f. \quad (18)$$

In all other aspects, gender does not matter in this simplified model. In particular, the standard deviations of the wage innovations are identical:

$$std(z_m) = std(z_f). \quad (19)$$

We further assume $\rho_m = \rho_f = 0$ for simplicity. For this setup, we can show analytically that the correlation between ϕ/λ and $E\Delta \ln w'_f$ is smaller than the correlation between ϕ/λ and $E\Delta \ln w'_m$.

We define the variable ζ , which depends on w_m and w_f , as:

$$\zeta(w_m, w_f) = \frac{\phi(w_m, w_f)}{\lambda(w_m, w_f)}. \quad (20)$$

Thus, ζ is the ratio of the multipliers on the borrowing constraint and on the budget constraint, respectively. The following lemma follows from this definition of $\zeta(w_m, w_f)$.

Lemma 1 $\zeta(w_m, w_f)$ is correlated with other variables in the model, whereas the strength of the correlation depends on whether the borrowing constraint is binding ($\zeta > 0$) or not ($\phi = 0 \Leftrightarrow \zeta = 0$), $\zeta \geq 0$. Further, $\zeta(w_m, w_f)$ is weakly increasing in $\lambda(w_m, w_f)$, $\partial\zeta(w_m, w_f)/\partial\lambda(w_m, w_f) \geq 0$.

Proof. See appendix. ■

The assumption of one-off shocks ($\rho_g = 0$) implies

$$\ln w_g = \psi_g + z_g.$$

Lemma 2 ζ and z_g are weakly negative correlated:

$$\text{corr}(\zeta, z_g) \leq 0. \quad (21)$$

Proof. See appendix. ■

The expected relative wage change for an individual of gender g in the next period is:

$$E\Delta \ln w'_g = E \ln w'_g - \psi_g - z_g = -z_g. \quad (22)$$

Using the definition of the correlation, we can derive the following relation from (22):

$$\text{corr}(\zeta, E\Delta \ln w'_g) = -\text{corr}(\zeta, z_g) \geq 0,$$

where the latter inequality uses (21).

Using these results, one can show that the expected relative wage of women is less important for the household's wealth than the males' one:

Proposition 3 *The following inequality holds:*

$$\text{corr}(\zeta, E\Delta \ln w'_m) > \text{corr}(\zeta, E\Delta \ln w'_f) > 0.$$

Proof. See appendix. ■

This relation shows that the estimation bias in the estimated Frisch elasticity is less pronounced for women. Note that in this simplified model, the only gender difference was the existence of a gender wage gap. In a more general model, the difference in the quantitative importance of the estimation bias generalizes to secondary and primary earners, respectively, whereas the bias is expected to be less important for secondary earners. The data shows that, in many couple households, these are predominantly women.

2.3 Numerical solution and simulation

In this section, we solve and simulate the model presented in Section 2.1. We discuss the model's implications by presenting policy functions for the key variables. Thereafter, we use the model to simulate synthetic household panel data and estimate gender-specific Frisch elasticities using this data.

The model is solved using Value Function Iteration. To increase the precision of the numerical solution, we allow for off-grid hours and asset choices using spline interpolation. The bivariate wage process (2) is discretized using Tauchen (1986)'s algorithm (13 states per gender).

Table 1: Calibration.

Description	Parameter	Value
Discount factor	β	0.95
Relative risk aversion husband	σ_m	1.5
Relative risk aversion wife	σ_f	1.5
Frisch labor supply elasticity husband	η_m	0.5
Frisch labor supply elasticity wife	η_f	0.5
Labor disutility weight husband	α_m	25
Labor disutility weight wife	α_f	25
Pareto weight husband	μ	0.5
Risk-free interest rate	r	0.02
Fixed effect husband high	$\psi^{m,H}$	0
Fixed effect husband low	$\psi^{m,L}$	0
Fixed effect wife high	$\psi^{f,H}$	-0.3
Fixed effect wife low	$\psi^{f,L}$	-0.3
Autocorrelation wage process husband	ρ_m	0.9
Autocorrelation wage process wife	ρ_f	0.9
Standard deviation wage innovations husband	$\sigma_{\varepsilon m}$	0.21
Standard deviation wage innovations wife	$\sigma_{\varepsilon f}$	0.21
Covariance wage innovations		0
Borrowing limit	a_{\min}	0

Parameterization. The utility function is additive-separable,

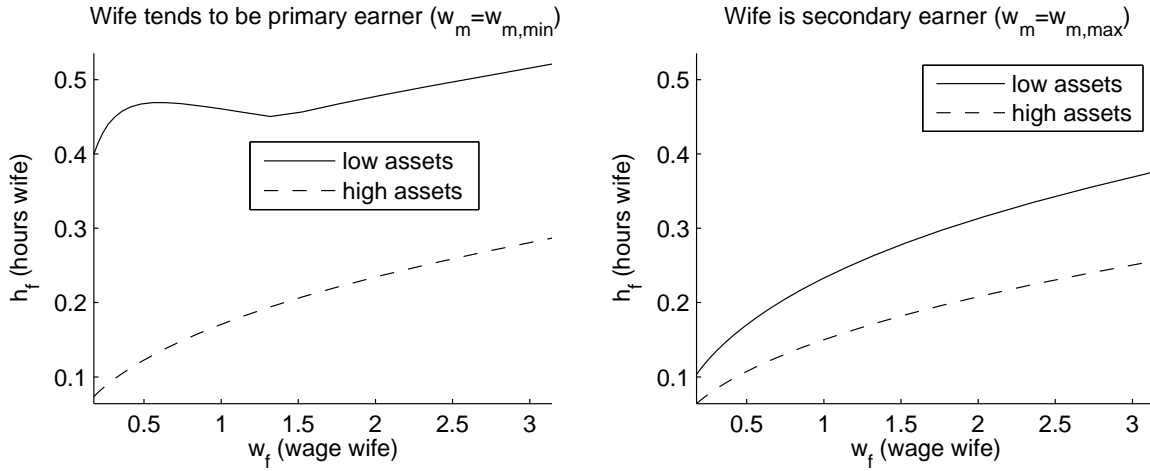
$$u_g(c, n_g) = \frac{c^{1-\sigma_g} - 1}{1 - \sigma_g} - \alpha_g \frac{(n_g)^{1+1/\eta_g}}{1 + 1/\eta_g}, \quad (23)$$

so that the true Frisch elasticities by gender are given by $\eta_g = \{ \eta_m, \eta_f \}$. In our baseline evaluations, we will assume the Frisch elasticity to be identical between genders, $\eta_m = \eta_f = \eta$, so that any deviations in the estimated Frisch elasticities from this parameter value reflect estimation biases.

Table 1 summarizes our baseline calibration. The gender wage gap is set to 0.74 (see Ortigueira and Siassi 2013). Following Domeij and Flodén (2006), we set the interest rate so that the bottom 40% of the wealth distribution own 1.4% of total wealth. Preference parameters are set so that the ratio of mean hours of market work of women and men is about 0.88.⁹ The parameters of the wage process are taken from Domeij and Flodén (2006).

⁹In 2014, women's average weekly hours worked amount to 36.0 and men work 40.9 hours on average, see BLS factbook 'Women in the Labor Force'.

Figure 1: Labor supply curves of wives, given the wage rate of the husband.



Notes: In the left panel, the wage rate of the husband is hold constant at its lowest possible realization. In the right panel, it is hold constant at the highest possible realization.

Policy functions. Figure 1 shows the labor supply curves of wives given the wage rate of the husband. The dashed line refers to a situation where the level of household wealth is high, whereas the solid line shows the decision rules when the household is borrowing-constrained (i.e. has no assets). The left panel corresponds to the case where the wife tends to be the primary earner in the household, whereas the right panel shows situations where the wife is the secondary earner.

Comparing the solid and dashed lines, respectively, one can see that assets exert a standard negative wealth effect on labor supply. The same is true is for the partner's wage rate. The labor supply curves are globally upward-sloping if wives are secondary earners (independent of whether the household is wealth-rich or wealth-poor), see the right panel in Figure 1. However, if wives tend to be primary earners (which is the case when their husbands' wages are very low), the household's asset holdings become relevant for the shape of the labor supply curves. The labor supply curve remains globally upward-sloping if the household is wealth-rich. By contrast, the labor supply curve has a downward-sloping range at small wage rates when the household is wealth-poor, see the solid line in the left panel in Figure 1. Thus, when wages are sufficiently low, a further wage decrease triggers an increase in labor supply (instead of a decrease), giving rise to a non-standard shape of the labor supply function. This shape of the labor supply curve is the result of the borrowing constraint. Consider a situation where a household is borrowing constrained and faces a negative wage rate shock. Note that it does not matter whether the negative wage shock hits the husband or the wife.

In both cases, the only possibility to stabilize household income and with it consumption is to increase labor supply. Of course, it is more efficient to increase the primary earner's labor supply than to increase hours worked of the secondary earner. Thus, a negative wage rate shock for the primary earner can lead to an increase of this spouses' labor supply.

Figure 2 shows the labor supply curves of the husband given the wage rate of the wife (upper panels) and the cross-wage labor supply functions of the wife, also taking the wife's own wage as given. Similar to the cases discussed before, the labor supply curves of husbands are globally upward sloping when wives are primary earners in the household. But, also for husbands, there is a downward-sloping range in the labor supply functions when husbands are primary earners in the household (i.e. when their wages are relatively high).

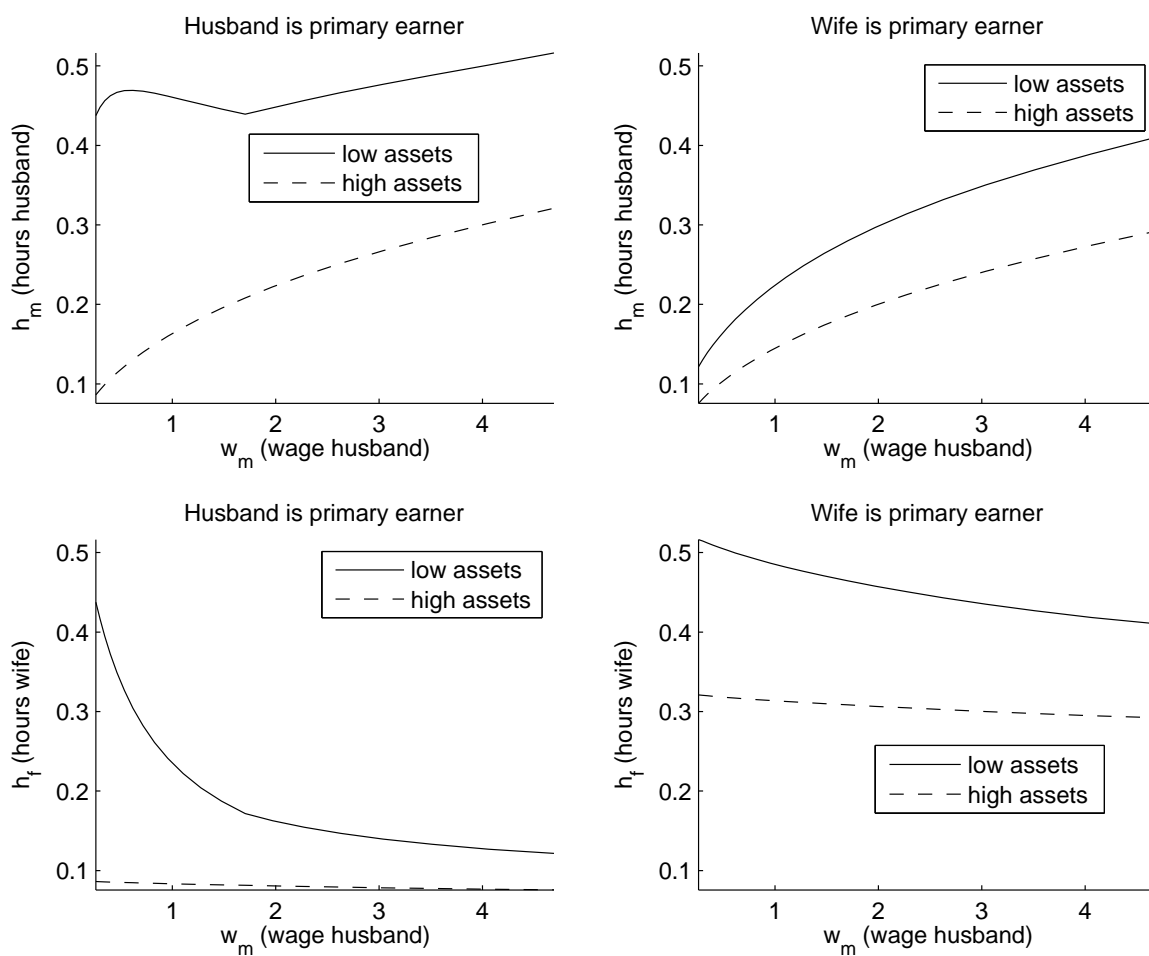
From the cross-wage labor supply relations (bottom panels in Figure 2), one can see that it is the primary earner's wage rate that impacts strongly on labor supply in wealth-poor households. Working time of the wife reacts strongly to low wage rates of the husband in wealth-poor households when the husband is the primary earner but the reaction is substantially flatter when the husband is the secondary earner.

In summary, at the borrowing constraint, stabilization of consumption is only possible through increasing labor supply. The labor supply functions shown in Figures 1 and 2 indicate that the household uses this insurance mechanism differently in reaction to the two spouses' wage shocks. In particular, the household can relatively easily smooth out wage rate shocks for the secondary earner, even when the household is at the borrowing constraint.

To inspect this point more thoroughly, we now consider the policy function for consumption. Figure 3 shows household consumption for different combinations of assets and wage rates. The left panel refers to the case where the wife has the lowest possible wage realization, i.e. the husband is the primary earner. A variation in the primary earner's wage rate affects consumption relatively little when the household is a wealth-rich (dashed line). The reason is that wealth-rich households can smooth consumption through dis-saving. By contrast, if a wealth-poor household (solid line) faces low wage rate realizations for the primary earner, consumption falls more strongly - the consumption policy function kinks downward. Although the household tries to smooth consumption by increasing labor supply (see the solid lines in the left panels of Figure 2), it cannot avoid a substantial drop in consumption. Hence, in this scenario, the household is substantially constrained by the borrowing constraint. The household has a strong wish to borrow and smooth consumption but, since it cannot, this drives up the shadow price on the budget constraint, ϕ .

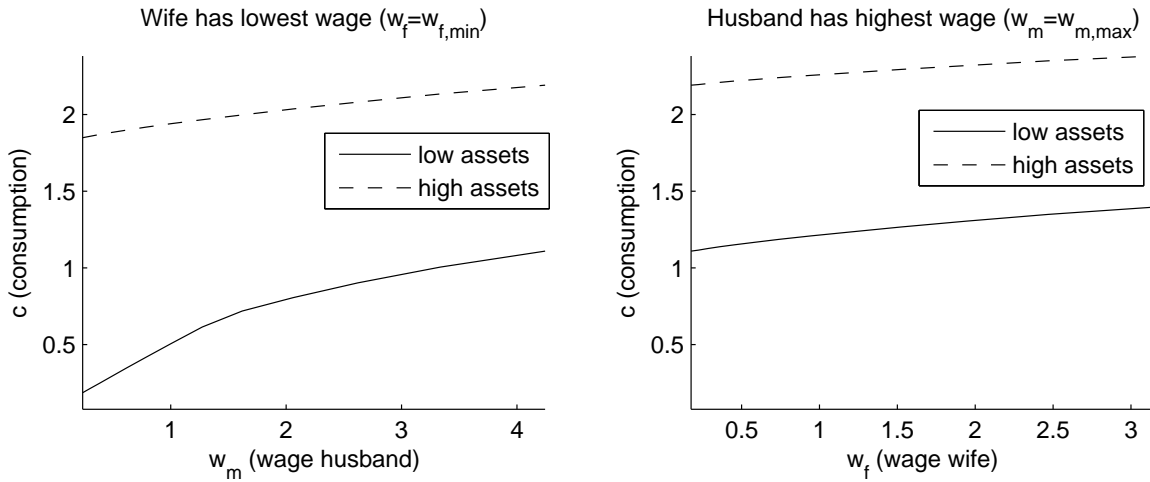
Interestingly, these effects are less pronounced in response to negative wage rate realiza-

Figure 2: Labor supply of husbands and wives, given the wage rate of the wife.



Notes: The real wage rate of the wife is held at its lowest possible realization in the left panel and at its highest possible realization in the right panel.

Figure 3: Consumption of the household for varying wage rates of primary and secondary earner.



Notes: The real wage rate of the wife is held at its lowest possible realization in the left panel. The real wage rate of the husband is held constant at its highest possible realization in the right panel.

tions of the primary earner. The right panel of Figure 3 refers to the case where the husband has the highest possible wage rate realization, i.e. the wife is the secondary earner. In this case, a variation in the secondary earner’s wage rate has only moderate impacts on consumption. This is also the case in wealth-poor households (dashed line in right panel of figure 3). These households smooth out the consumption effects of wage rate changes for the secondary earner by increasing labor supply.

To summarize the insights from Figure 3, consumption is most strongly correlated with the primary earner’s wage rate realizations in wealth-poor households. Since it is the marginal utility of consumption that determines the household’s willingness to borrow, see equation (8), the strongest correlation between the shadow price on the borrowing constraint and wage rate realizations occurs for primary earners in wealth-poor households. This has important consequences for standard labor-supply regressions that use a measure of the expected wage change as instrument: Below-average realizations of the wage rate imply positive expected wage growth such that the consumption policy functions in Figure 3 indicate that the correlation between expected wage growth and the multiplier on the borrowing constraint is most pronounced for primary earners in wealth-poor households. Thus, the (downward) estimation bias for Frisch elasticities is expected to be more substantial for primary than for secondary earners, respectively.

3 Monte-Carlo study

3.1 Estimation procedure

The Frisch labor supply elasticity can be estimated using (17). However, there are two reasons why (17) cannot be estimated with simple OLS. First, ξ' and $\Delta \ln w'_g$ are correlated. This issue can be resolved by an instrumental variable estimator. In the estimations based on synthetic data, we can use the expected relative wage change $E\Delta \ln w'_g$ to instrument for $\Delta \ln w'_g$. The expected relative wage change is

$$E\Delta \ln w'_g = Ez'_g - z_g = (\rho_g - 1) \cdot z_g. \quad (24)$$

The relative wage change is

$$\Delta \ln w'_g = z'_g - z_g = (\rho_g - 1) \cdot z_g + \varepsilon'_g. \quad (25)$$

The covariance between $E\Delta \ln w'_g$ and $\Delta \ln w'_g$ is

$$\text{cov} \left((\rho_g - 1) \cdot z_g, (\rho_g - 1) \cdot z_g + \varepsilon'_g \right) = (\rho_g - 1)^2 \cdot \text{var} (z_g) + (\rho_g - 1) \cdot \text{cov} (z_g, \varepsilon'_g) \neq 0,$$

which shows that the instrument is informative. The instrument is valid, since the covariance between $E\Delta \ln w'_g$ and ξ' is zero, $\text{cov} (E\Delta \ln w'_g, \xi') = 0$.

The second issue is the unobservability of ϕ/λ . In practise, one therefore estimates¹⁰

$$\Delta n_{gjt+1} = \text{const.} + \eta_g \cdot \Delta \ln w_{gjt+1} + u_{jt+1}, \quad (26)$$

where $\Delta \ln w_{gjt+1}$ is instrumented by $E_t \Delta \ln w_{gjt+1}$ and u_{jt+1} is the combined error term. If there was no correlation between ϕ_{jt}/λ_{jt} and $E_t \Delta \ln w_{gjt+1}$, the 2SLS estimator of η_g , $\hat{\eta}_g$, would be a consistent estimator for the Frisch labor supply elasticity. Similar as Domeij and Flodén (2006), we quantify the bias due to the neglect of borrowing constraints by estimating the regression equation on different subsamples of the simulated household data. In a subsample of close-to borrowing constrained households, $\text{corr} (\zeta, E\Delta \ln w'_m)$ is relatively large, which should lead to a substantial (downward) estimation bias of the Frisch elasticity. By contrast, in a subsample of households that are relatively wealth-rich, this correlation should be smaller, making the potential estimation bias less important quantitatively, so that we should find estimates of the Frisch elasticity that are closer to the true value η . To start off, we estimate the regression from simulated data set where the true Frisch elasticity does not differ by gender, $\eta_m = \eta_f = \eta$. Thus, any differences in the estimated labor-supply

¹⁰Here, we return to using household indices and introduce time indices in order to clarify the panel dimension of the estimation.

Table 2: Estimation results.

	(1)	(2)	(3)	(4)
	True	Estimates $\hat{\eta}_g$		
	Frisch	Full		
	LSE	Sample	$a' < 0.01\bar{a}$	$a' > \bar{a}$
Men	0.5000	0.2920 (0.0003)	0.1283 (0.0012)	0.4885 (0.0004)
Women	0.5000	0.3254 (0.0004)	0.1895 (0.0016)	0.4865 (0.0005)

Notes: Numbers in columns (2)-(4) are point estimates for coefficient $\hat{\eta}_g$ in a two-stage regression of equation (26). Numbers in parantheses are standard errors. \bar{a} denotes mean asset holdings in the economy.

elasticities that we find with respect to gender must be due to the mechanism highlighted in this paper.

In the first estimation, we use the full sample of simulated households. In the second estimation, we use a subsample of households that have asset holdings lower than one percent of mean assets holdings in the economy. Thus, this is a sample of close-to borrowing constrained households. In the third estimation, we use a subsample including households that have asset holdings higher than the mean asset holdings in the economy. Thus, these households should not be borrowing constrained even for substantial wage shocks. As a consequence, we should obtain almost unbiased estimates $\hat{\eta}_g$ in this sample.

3.2 Estimation Results

Table 2 presents the estimation results for the coefficient on $\Delta \ln w_{gjt+1}$ in our three estimations of (26). The true Frisch elasticity for men and women is given by the preference parameter $\eta_m = \eta_f = 0.5$.

The estimates $\hat{\eta}_m$ and $\hat{\eta}_f$ are clearly downward biased in the full sample, see column (2). This reflects the correlation of the expected wage change with the shadow prices as argued before. From column (3) it can be seen that the downward bias is most pronounced in the subsample of close-to borrowing constrained households. In column (3), the estimated Frisch labor supply elasticity of men is about 26% of the true Frisch labor supply elasticity. Further, the estimated Frisch labor supply elasticity of women in wealth-poor households is around 38% of the true Frisch labor supply elasticity. Recall that besides the gender wage gap, there is no further gender heterogeneity in the model parameters. The documented difference in estimation biases by gender reflects the differential importances of gender-specific wage-rate fluctuations for a couple's willingness to borrow. As discussed above, the shadow price on

the borrowing constraint is less correlated with the secondary earner's expected wage rate change. This results in a smaller estimation bias for secondary earners which are mostly women in the simulated data due to the gender gap in mean wage rates.

The estimation results in the subsample with non-borrowing constrained households, see column (4), are close to the true Frisch labor supply elasticity 0.5. In this subsample, borrowing constraints are not binding such that the shadow price ϕ_j is zero and hence uncorrelated with expected wage changes. As a consequence, the estimation bias vanishes and there are no spurious gender-differences in the estimates anymore.

For the full sample, we estimate a Frisch elasticity that lies inbetween the estimates for the two extreme subsamples of borrowing and non-borrowing constrained households, respectively. In column (2), the estimated Frisch labor supply elasticity of men is around 58% of the true Frisch labor supply elasticity, $\hat{\eta}_m = 0.58 \cdot \eta_m$. The estimated elasticity for women is about 65% of the true one, $\hat{\eta}_f = 0.65 \cdot \eta_f$. Thus, the downward biases in estimated labor supply elasticities are less pronounced for women than for men. This result is a consequence of the lower correlation of women's expected wage changes with households' willingness to borrow in borrowing constrained households, see above.

Taken together, our results imply

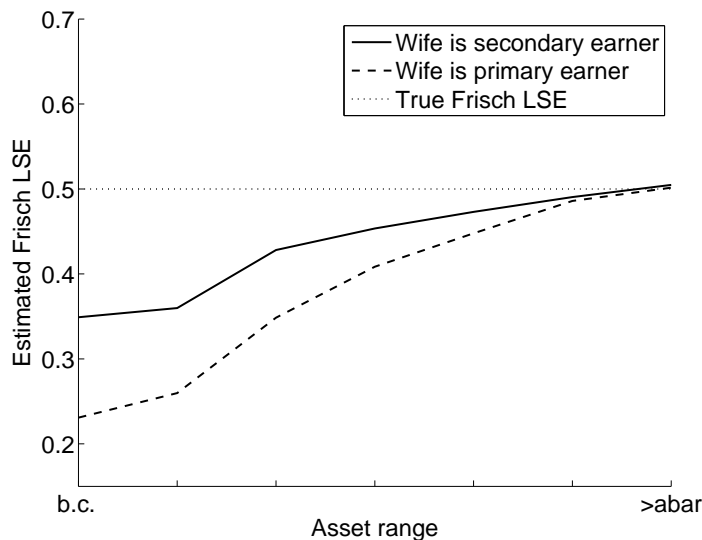
$$\frac{\eta_f}{\eta_m} = \frac{0.65}{0.58} \cdot \frac{\hat{\eta}_f}{\hat{\eta}_m} \approx 0.90 \cdot \frac{\hat{\eta}_f}{\hat{\eta}_m}, \quad (27)$$

i.e. that the gender ratio in the true Frisch elasticities is only about 90% of the gender ratio in the estimates.

In the literature, the estimated Frisch labor supply elasticities of women are about 100% larger than the estimates for men, $\hat{\eta}_f/\hat{\eta}_m \approx 2$ (Keane 2011). Using (27), this suggests that the ratio of true Frisch labor supply elasticities of women and men is around 1.79. Put differently, our results suggest that women's true Frisch elasticities are only about 80% larger than men's. Thus, about a fifth of the gender differences in Frisch labor-supply estimates is the result of the estimation bias documented in this paper.

Figure 4 illustrates the quantitative importance of the estimation bias in the female Frisch elasticity. We selected subsamples of households within different asset ranges (between the borrowing constraint $a_{\min} = 0$ and mean asset holdings \bar{a}). It can be seen that the bias in estimated Frisch elasticities of women decreases as the asset holdings of the household increase, independent of whether the wife is the primary or secondary earner. In line with the main point of our paper, it can be seen that the estimation bias is in general less pronounced when the wife is the secondary earner (the solid line lies above the dashed line).

Figure 4: Estimation bias as a function of asset holdings.



4 Conclusion

Estimates of Frisch labor-supply elasticities are typically larger for women than for men. We have shown that standard labor-supply regressions overestimate this gender difference. In couples with joint borrowing constraints, secondary earners' wage-rate fluctuations are less important for the couples' willingness to borrow which results in smaller estimation biases for secondary earners. In many households, the secondary earner is the wife. Since the estimated Frisch elasticity for secondary earners (mostly wives) is closer to the true value than the estimate for primary earners (mostly men), a spurious gender difference arises. Quantitatively, our results suggest that gender differences in true Frisch elasticities are about a fifth smaller than suggested by standard labor-supply regressions.

References

- Blundell, R. and T. MaCurdy (1999). *Labor Supply: A Review of Alternative Approaches*, Volume 3A of *Handbook of Labor Economics*, Chapter 27, pp. 1559–1695. Elsevier.
- Chiappori, P.-A. (1988). Rational Household Labor Supply. *Econometrica* 56(1), 63–90.
- Chiappori, P.-A. (1992). Collective Labor Supply and Welfare. *Journal of Political Economy* 100(3), 437–67.
- Deaton, A. (1991). Saving and Liquidity Constraints. *Econometrica* 59(5), 1221–48.
- Diaz-Gimenez, J., A. Glover, and J.-V. Rios-Rull (2011). Facts on the distributions of earnings, income, and wealth in the United States: 2007 update. *Federal Reserve Bank of Minneapolis Quarterly Review* 34(1), 2–31.

- Domeij, D. and M. Flodén (2006). The labor-supply elasticity and borrowing constraints: Why estimates are biased. *Review of Economic Dynamics* 9, 242–262.
- Hirsch, B. (2009). The Gender Pay Gap Under Duopsony: Joan Robinson Meets Harold Hotelling. *Scottish Journal of Political Economy* 56(5), 543–558.
- Keane, M. P. (2011). Labor Supply and Taxes: A Survey. *Journal of Economic Literature* 49(4), 961–1075.
- Ortigueira, S. and N. Siassi (2013). How important is intra-household risk sharing for savings and labor supply? *Journal of Monetary Economics* 60, 650–666.
- Ransom, M. R. and R. L. Oaxaca (2010). New Market Power Models and Sex Differences in Pay. *Journal of Labor Economics* 28(2), 267–289.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters* 20(2), 177–181.

A Appendix A

A.1 Proof of Lemma 1

The correlation of $\zeta(w_m, w_f)$ with other variables in the model follows from equation (15).

$\zeta(w_m, w_f)$ is greater or equal to zero depending on whether the borrowing constraint is binding ($\zeta > 0$) or not ($\phi = 0 \Leftrightarrow \zeta = 0$), $\zeta \geq 0$. If the borrowing constraint is binding, the multiplier ϕ is equal to zero. Thus, ζ is also equal to zero. If the borrowing constraint is not binding, ϕ is positive and therefore ζ is also positive (the multiplier on the budget constraint, λ , is always positive due to the assumptions on the utility function).

In the case of i.i.d. one-off wage shocks, expected wage rates in the next period are the mean wage rates. If the borrowing constraint is binding, asset holdings of the next period are zero.¹¹ Hence, the expected multiplier on the budget constraint $E\lambda'$ is always the same if the borrowing constraint is binding. At the borrowing constraint it holds that $E\lambda' = \lambda(0, (\exp(\psi_m), \exp(\psi_f))) = \lambda^*$, where the latter is a constant. From the Euler equation (8) it follows that

$$\phi = \frac{\lambda}{1+r} - \beta E[\lambda'|\omega].$$

Thus, the multiplier on the borrowing constraint satisfies

$$\phi = \max \left\{ 0, \frac{\lambda}{1+r} - \beta \cdot \lambda^* \right\}$$

¹¹Strictly speaking, the asset holdings would be at their minimum level. We assume $a_{\min} = 0$.

and hence

$$\zeta = \phi/\lambda = \max \left\{ 0, \frac{1}{1+r} - \beta \cdot \frac{\lambda^*}{\lambda} \right\}$$

ζ is therefore weakly increasing in λ , $\partial\zeta/\partial\lambda \geq 0$.

A.2 Proof of Lemma 2

λ is the multiplier on the budget constraint and therefore decreasing in wage rates w_g . Thus, ζ is also (weakly) decreasing in w_g (since ζ is weakly increasing in λ) and therefore according to (2) (weakly) decreasing in z_g . In consequence, ζ and z_g are (weakly) negative correlated:

$$\text{corr}(\zeta, z_g) \leq 0.$$

A.3 Proof of Proposition 3

To show that the correlation between ϕ/λ and $E\Delta \ln w'_f$ is smaller than the correlation between ϕ/λ and $E\Delta \ln w'_m$, it is sufficient to show that the covariance of ζ and z_m is smaller than the covariance of ζ and z_f since

$$\begin{aligned} \text{cov}(\zeta, z_m) &< \text{cov}(\zeta, z_f) \Leftrightarrow \\ \text{corr}(\zeta, z_m) &< \text{corr}(\zeta, z_f) \Leftrightarrow \\ \text{corr}(\zeta, E\Delta \ln w'_m) &> \text{corr}(\zeta, E\Delta \ln w'_f) > 0, \end{aligned}$$

where the first equivalence uses (19) and the second uses (22).

We can show that the covariance of ζ and z_m is smaller than the covariance of ζ and z_f by proving:

$$\text{cov}(\zeta, w_m) - \text{cov}(\zeta, w_f) < 0. \quad (28)$$

(28) can be rewritten as

$$E(\zeta \cdot w_m) - E(\zeta \cdot w_f) + E(\zeta) \cdot [E(w_f) - E(w_m)] < 0. \quad (29)$$

(29) is true if

$$E(\zeta \cdot w_m) - E(\zeta \cdot w_f) = E(\zeta \cdot (w_m - w_f)) < E(\zeta) \cdot [E(w_m) - E(w_f)]. \quad (30)$$

Using the Cauchy-Schwartz inequality, it can be shown that:

$$E(\zeta \cdot (w_m - w_f)) \leq \sqrt{E(\zeta^2)} \cdot \sqrt{E((w_m - w_f)^2)}. \quad (31)$$

Applying Jensen's inequality to (31) yields¹²

$$\sqrt{E(\zeta^2)} \cdot \sqrt{E((w_m - w_f)^2)} < E(\zeta) \cdot E(w_m - w_f).$$

Thus, it holds that

$$E(\zeta \cdot (w_m - w_f)) < E(\zeta) \cdot E(w_m - w_f).$$

This implies that (28) is true which means that the inequality

$$\text{corr}(\zeta, E\Delta \ln w'_m) > \text{corr}(\zeta, E\Delta \ln w'_f) > 0$$

holds.

¹²Here, we have used that the square root is convex, and ζ_j , w_m and w_f are integrable real-valued random variables.