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The impossibility of informationally efficient markets when forecasts are self-defeating

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The impossibility of informationally efficient markets when forecasts are self-defeating

February 17, 2015

Abstract

A policy maker (PM) needs information that only financial market traders know in order to implement his optimal policy, and traders may aggregate this information in asset prices. In such a setting, prices can become uninformative, because the PM reacts to information contained in prices, thereby changing asset values and possibly punishing traders for revealing the information. I derive a necessary and sufficient condition for the possibility of fully revealing and accurate prices in large financial markets. Non-fulfillment of this condition means prices cannot be informationally efficient. The condition is also necessary and sufficient for the existence of fully revealing REE, and explains results from several applications in the literature. The condition implies that assets whose values are invertible in the underlying are superior in terms of information revelation compared to assets whose values are non-invertible.

Keywords: Asymmetric Information, Efficient Market Hypothesis, Endogenous Asset Values, Forecasts, Rational Expectations Equilibrium, Self-Defeating Prophecy

JEL Classification: D53, D82, D84, G10

1 Introduction

Suppose a large bank might or might not collapse within the next month, and the regulator wants to bail out if and only if the bank collapses without aid. Suppose further the bank issued a bond whose payment is due in two months, but which it cannot pay if it collapses. Unlike the regulator, informed financial market traders know the condition of the bank and the necessity for a bail out perfectly, and the regulator knows that these traders know, and so on. The regulator may therefore infer information about the condition of the bank from the market price of the bond. Suppose, finally, that the bank will in fact collapse without bail out.

Can traders profit from their information, and how would they invest to do so? If traders sell bonds, the market price will fall. The regulator infers from the market price that traders think the bank will collapse,¹ and rescues it, therefore falsifying the trader forecast of a collapsing bank. The bank can therefore honor its financial obligations, but since traders sold the bond below value they lose money. If, on the other hand, traders buy bonds, the market price will rise. The regulator infers from the price that traders think the bank will not collapse, and not rescue it, which leads to bank collapse, the bank cannot make the bond payments, and traders lose money.

This is one of many possible situations where traders cannot make money off of their information in the market if they correctly anticipate the policy maker reaction. Although traders know all fundamentals that influence the asset value, they lose money when they trade. But if informed traders have no incentive to trade, then their information will not be incorporated into market prices. And if prices do not reveal the trader information, then imperfectly informed policy makers (here the regulator) are not able to implement their first best policy.

The difficulty in the general setting is that a policy maker can react to information contained in asset prices, and his reaction in turn affects asset values. Thus, unlike standard models in financial economics, asset values are endogenous. This paper provides a sufficient and necessary condition for the possibility of informative and accurate prices, i.e., characterizes all situations where prices can be informative and traders do not lose money by revealing it if a policy maker reacts to prices. The condition puts constraints on the underlying of the asset as well as policy maker preferences and information, all of which influence the policy maker reaction to revealing market prices. The condition is also necessary and sufficient for the existence of fully revealing rational expectations equilibria and (under some conditions) Perfect Bayesian Nash equilibria in large markets. It therefore unifies several models considered in the literature, and provides the general reason why a fully revealing equilibrium does or does not exist.

¹Clearly, the regulator expectations about how fundamentals of bank health map into prices is endogenous in equilibrium. In this example, the equilibrium candidate is that traders buy if the bank does not need rescuing, and sell if it does, which leads to the expectations described.

Similar problems to the above example have been investigated for specific applications in finance and macroeconomics. For example, the problem applies to monetary policy (e.g., [Woodford, 1994](#), [Bernanke and Woodford, 1997](#)), bank supervision and management ([Birchler and Facchinetti, 2007](#), [Bond et al., 2010](#)), or contingent capital, where the policy maker is in fact a debt conversion rule (e.g., [Prescott, 2012](#), [Sundaresan and Wang, forthcoming](#)). In general terms, the problem is as follows. A policy or decision maker wants to implement his optimal policy, which depends on the realization of a state variable $\theta \in \Theta$ that he does not perfectly know. However, traders in a financial market do know the realization of the state θ , and the policy maker may infer information about it from asset prices. Prices refer to an asset whose value depends on an outcome/underlying represented by function $f(\theta, i)$ —bank solvency in the example—which depends both on the state θ (bank health absent intervention) and the policy of the policy maker i (bail out or not). Clearly, if prices reveal something about θ , then policy i will react to them. This “double endogeneity” ([Birchler and Facchinetti, 2007](#)), i.e., prices react to policy and policy reacts to prices, may preclude existence of equilibrium, or at least existence of a revealing equilibrium, as in most of the referenced papers.

The general setting permits identification of the problem common to all of these applications. The policy maker has preferences that lead him to falsify the forecast implied by prices if prices are fully revealing, hence the (asset value) forecasting problem is self-defeating. More specifically, the necessary and sufficient condition for the existence of a fully revealing rational expectations equilibrium (REE) if the policy maker is uninformed is that the outcome preferred by the policy maker under full information leads to an invertible asset value function $v(\theta)$. If the policy maker receives informative but imperfect signals, then the condition requires that asset values under full information are such that accurate asset prices and policy maker signals can jointly reveal the state. Non-fulfillment of the condition implies that a price function cannot reveal θ and price accurately at the same time.

The setting described here is one where strong informational efficiency of markets (in a REE sense) may not be possible even under otherwise standard assumptions. The reason is not informational frictions (e.g., [Grossman and Stiglitz, 1980](#)), but the behavior of the policy maker, who de facto ‘prefers to falsify forecasts.’

2 The model

2.1 Set-up

Consider a financial market with a single riskless asset with rate normalized to 1, and a single risky asset. Let $f(\theta, i) : \Theta \times I \rightarrow F$, be the outcome function (possibly stochastic),

determined by state θ and policy i , and let $a(f(\cdot, \cdot)) : F \rightarrow A$ be a function that maps all outcomes into values of the risky asset, with $\Theta, I, F, A \subseteq \mathbb{R}$.

The financial market consists of N traders with a common prior q . Every trader j receives an informative i.i.d. signal s_j on the realization of the state variable θ , resulting in signal vector (s_1, s_2, \dots) . Since different signal vectors can have the same information content, denote the summary statistic of (s_1, s_2, \dots) by \mathbf{s} , and the set of all possible realizations of the summary statistic by \mathbf{S} . The following are three examples of commonly used information structures for traders with corresponding summary statistic.

Example.

1. Traders receive perfect signals, i.e., $s_j = \theta$ for all j , as for example in [Bernanke and Woodford \(1997\)](#) or [Bond et al. \(2010\)](#). The summary statistic is $\mathbf{s} = \theta$.
2. State $\theta \in \{0, 1\}$ is binomially distributed, $s_j \in \{0, 1\}$, and traders receive imperfect signals, i.e., $1 > \Pr(s_j = 1 | \theta = 1) = \Pr(s_j = 0 | \theta = 0) > 1/2$. The summary statistic is $\mathbf{s} = \sum s_j$.
3. The state space is the entire real line, $\theta \in \mathbb{R}$, $s_j \sim \mathcal{N}(\theta, \sigma^2)$, and traders receive imperfect signals, i.e., $\sigma^2 > 0$. The summary statistic is $\mathbf{s} = N^{-1} \sum s_j$.

Let $p(\mathbf{s}) : \mathbf{S} \rightarrow A$ be the prediction or price function of expected asset value $\mathbb{E}_{f,\theta}[a(f(\theta, i)) | \mathbf{s}]$ given trader information. For example, $f(\theta, i)$ may represent the value of a company, while $p(\mathbf{s})$ is the price of the publicly traded company stock.² The timing of decisions is illustrated in Figure 1: first, traders determine $p(\mathbf{s})$ via trading, then, observing the price, the policy maker sets i .

The policy maker receives an imperfect signal $s \in S$ on the realization of θ . In a special case the PM receives a completely uninformative signal, i.e., $|S| = 1$, which is identical to the PM receiving no signal at all. Utility function u represents the rational preference ordering of the policy maker over outcomes $f(\theta, i)$. Thus, if trader signals \mathbf{s} were known to the PM, he would choose policy

$$i^*(\mathbf{s}) \in \arg \max_i \mathbb{E}_{f,\theta}[u(f(\theta, i)) | \mathbf{s}, s].$$

To simplify the exposition, I will assume throughout this paper that i^* is single-valued. Results can be straightforwardly extended for the case where the policy maker is indifferent between several policies leading to different expected outcomes. Define $v(\mathbf{s}) : \mathbf{S} \rightarrow A$, the expected asset value for outcomes chosen by the policy maker if \mathbf{s} were known to the PM,

$$v(\mathbf{s}) := \mathbb{E}_{f,\theta}[a(f(\theta, i^*(\mathbf{s}))) | \mathbf{s}].$$

²Throughout this paper, “traders” may be replaced by “forecasters,” and “prices” may be replaced by “forecasts,” because the problem applies to all situations where a policy maker reacts to forecasts and forecasters care about the accuracy of their predictions.

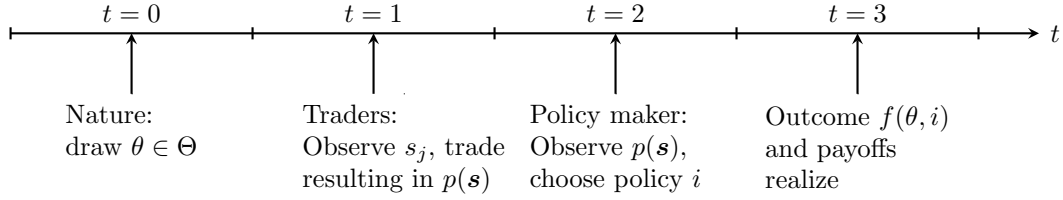


Figure 1: Timeline with traders and policy maker.

In the introductory example $v(\mathbf{s}) = v(\theta)$ is the bond value under full information about bank health, depending on bank health θ . The objects $i^*(\mathbf{s})$ and $v(\mathbf{s})$ are defined *assuming* \mathbf{s} is common knowledge, even though it is not. The reason is that once prices are fully revealing (see definition 2 below), then \mathbf{s} is known to the PM. Hence, $i^*(\mathbf{s})$ and $v(\mathbf{s})$ are reactions that forward looking traders are going to anticipate if prices are fully revealing.

Given this information structure, policy i cannot be conditioned on \mathbf{s} directly, only on $p(\mathbf{s})$, so the outcome is $f(\theta, i(p(\mathbf{s})), s)$. Equilibrium (to be formally defined below) will require policy given beliefs to be optimal. Policy maker behavior, in particular u , is common knowledge. For non-triviality, I assume the policy maker needs to know θ in order to implement his optimal policy, so he is interested in additional information.

The next two definitions define properties of price functions.

Definition 1. A price function $p(\mathbf{s})$ is accurate if and only if

$$p(\mathbf{s}) = \mathbb{E}_{f,\theta}[a(f(\theta, i(p(\mathbf{s})))) | \mathbf{s}] \text{ for all } \mathbf{s} \in \mathbf{S}.$$

In words, an accurate price function prices the asset at its expected value for every signal vector realization; there is no systematic mispricing. This definition of accurate prices resembles a rational expectations assumption, as will be made clear below. However, it is important beyond REE, because inaccurate prices imply that the expected asset value and its price differs, which creates incentives to change investment behavior to exploit the difference in any rational equilibrium concept.

In this setting, \mathbf{s} can only be indirectly revealed via price $p = p(\mathbf{s})$ in combination with the PM signal s . If the policy maker knows the price function (knows trader behavior), as he will in any standard equilibrium concept, then \mathbf{s} can always be inferred from $(p = p(\mathbf{s}), s)$ if the Bayesian posterior puts a positive probability on at most one $\mathbf{s} \in \mathbf{S}$.

Definition 2. A price function $p(\mathbf{s})$ and PM signals are jointly fully revealing if and only if

$$|\{t \in \mathbf{S} : \Pr(\mathbf{s} = t | p(\mathbf{s}) = p, s) > 0\}| \leq 1 \quad \forall s, \forall p.$$

According to this definition, prices need not be fully revealing to traders or outsiders, only to the PM with additional information s . If policy maker signals are uninformative, i.e., $|S| = 1$, then full revelation reduces to invertibility of the price function in \mathbf{s} , because

prices alone have to reveal \mathbf{s} . In this case, the information is fully revealed to anybody who observes the price and knows the price function.³

Finally, we are going to need a definition to characterize the PM signal structures that allow for full revelation. First, define the full inverse $v^{-1}(p)$, i.e., the set of $\mathbf{s} \in \mathbf{S}$ for which the expected asset value under full information equals p as

$$I(p) := \{\mathbf{s} \in \mathbf{S} : v(\mathbf{s}) = p\},$$

and the set of p for which $I(p)$ contains more than one element as

$$X := \{p \in \text{Image}(v(\mathbf{s})) : |I(p)| > 1\}.$$

If X is empty, then $v(\mathbf{s})$ is invertible. Any element in X is a price—assuming accurate prices—for which the price alone does not reveal \mathbf{s} .

The following condition states that if $v(\mathbf{s})$ is not invertible, that is, set $I(p)$ contains more than one element for some p in the image of $v(\mathbf{s})$, then the probability of receiving any signal s must be zero in all $\mathbf{s} \in I(p)$ except for at most one.

Condition 1 (Excluding signal structure in case of non-invertibility of v).

$$|X| > 0 \implies \Pr(s|\mathbf{s} = t) \cdot \Pr(s|\mathbf{s} = t') = 0 \quad \forall s \in S, \forall t \neq t' \in I(p), \forall p \in X.$$

The condition has similarity to what [Cabrales et al. \(2014\)](#) call ‘excluding signal structure,’ but it is not identical, because in their meaning it is sufficient for signals to exclude one state, whereas here PM signals might have to exclude several.

2.2 The possibility of information revelation via prices

This section asks if a price function $p(\mathbf{s})$ exists which allows for both full revelation and accurate prices. There is no microfoundation for this price function yet, i.e., the section does not explain how the price function arises in some specified trading game or equilibrium concept. This foundation will be provided in subsequent sections. The analysis is separated in this way to highlight that the impossibility of fully revealing equilibria does not depend on the microfoundation. Instead, under some conditions it is mathematically impossible to find a price function that is both fully revealing and accurate.

When is it possible for a price function to reveal \mathbf{s} to the policy maker (definition 2) *and*

³Note that even if full revelation is not possible, say because the PM still puts positive probability on two states after observing the price, he may still be able to implement his optimal policy if the optimal policy is identical in both states. Optimal policy without full revelation would not be possible if we imposed a strong non-triviality assumption on u and f , so that $i^*(\mathbf{s}) \neq i^*(\mathbf{s}') \quad \forall \mathbf{s} \neq \mathbf{s}' \in \mathbf{S}$, i.e., the PM’s optimal policy is different in every possible information realization. Since PM preferences depend on the application, I will focus on the possibility of full revelation in general terms.

price accurately (definition 1) at the same time? Without full revelation, the policy maker has inferior information and may implement worse policies, and without accurate prices, traders would lose money, hence might be better off not trading.

Given correct policy maker beliefs about $p(\mathbf{s})$, Theorem 1 shows that it is possible if and only if condition 1 is satisfied. The result is not specific to an equilibrium concept, as correct beliefs are fulfilled in all standard concepts (e.g., REE or Perfect Bayesian Nash equilibrium).

Theorem 1 (possibility of full revelation and accurate prices). *Suppose the policy maker knows $p(\mathbf{s})$ and maximizes expected utility. Then full revelation and accurate prices are possible if and only if condition 1 holds.*

Proof. Necessity: full revelation by definition implies

$$|\{t \in \mathbf{S} : \Pr(\mathbf{s} = t | p(\mathbf{s}) = p, s) > 0\}| \leq 1 \quad \forall s, \forall p.$$

Rewriting,

$$\Pr(\mathbf{s} = t | p(\mathbf{s}) = p, s) = \frac{\Pr(\mathbf{s} = t) \cdot \Pr(p(\mathbf{s}) = p | \mathbf{s} = t) \cdot \Pr(s | \mathbf{s} = t)}{\Pr(p(\mathbf{s}) = p) \cdot \Pr(s)}.$$

If $\Pr(p(\mathbf{s}) = p | \mathbf{s} = t) > 0$ for exactly one $t \in \mathbf{S}$ for all $p \in \text{Image}(v(\mathbf{s}))$, then $v(\mathbf{s})$ is invertible and the price alone fully reveals \mathbf{s} . Hence $\nexists p \in \text{Image}(v(\mathbf{s})) : |I(p)| > 1$, i.e., the antecedent of condition 1 is false. If the antecedent of condition 1 is true, then $\Pr(p(\mathbf{s}) = p | \mathbf{s} = t) > 0$ for all $t \in I(p)$ and $p \in X$. Since there is full revelation, we must have $\Pr(s | \mathbf{s} = t) \cdot \Pr(s | \mathbf{s} = t') = 0 \quad \forall s \in S, \forall t \neq t' \in I(p), \forall p \in X$, so that $\Pr(\mathbf{s} = t | p(\mathbf{s}) = p, s) > 0$ for at most one $t \in I(p)$ for each $p \in X$. Thus, condition 1 holds.

Sufficiency: If condition 1 holds, then, taking $p(\mathbf{s}) = v(\mathbf{s})$, $\Pr(\mathbf{s} = t | p(\mathbf{s}) = p, s) > 0$ for at most one $t \in \mathbf{S}$ for all p and s . To see this, note that either the antecedent is false, in which case $\Pr(p(\mathbf{s}) = p | \mathbf{s} = t) > 0$ for exactly one $t \in \mathbf{S}$ for each $p \in \text{Image}(v(\mathbf{s}))$, hence the price alone is fully revealing. Or the antecedent is true, and then $\Pr(s | \mathbf{s} = t) \cdot \Pr(s | \mathbf{s} = t') = 0$ whenever $\Pr(p(\mathbf{s}) = p | \mathbf{s} = t) \cdot \Pr(p(\mathbf{s}) = p | \mathbf{s} = t') > 0$, hence $\Pr(\mathbf{s} = t | p(\mathbf{s}) = p, s) > 0$ for exactly one $t \in \mathbf{S}$. Therefore, there is full revelation and prices are accurate. \square

Corollary 2. *Suppose the policy maker knows $p(\mathbf{s})$ and maximizes expected utility. Then prices can be fully revealing and accurate if $v(\mathbf{s})$ is invertible.*

Thus, full revelation and accurate prices are possible either if $v(\mathbf{s})$ is invertible (for any PM signal structure), or if $v(\mathbf{s})$ is not invertible but the PM signal structure is excluding in the sense of condition 1. The requirement on the signal structure is rather strong, as it requires that any \mathbf{s} which is not ruled out by the “price signal” is ruled out by the imperfect signal of the policy maker. For example, if traders have perfect information ($\mathbf{s} = \theta$), if $\theta \in \mathbb{R}$ and $s \sim \mathcal{N}(\theta, \sigma^2)$ as in most finance models, then $\Pr(s | \theta = t) = \phi\left(\frac{s-t}{\sigma}\right) > 0$ for all $t \in \mathbb{R}$

and $s \in S$, i.e., condition 1 is not fulfilled. Hence, full revelation with normally distributed signals is possible if and only if $v(\theta)$ is invertible, because signals never rule out any state.

Similarly, in the special case where the policy maker does not receive an informative signal, $\Pr(s) = \Pr(s|\mathbf{s} = t) = 1 \forall t \in \mathbf{S}$, hence invertibility of $v(\mathbf{s})$ is necessary and sufficient for full revelation and accurate prices, as stated in the following corollary.

Corollary 3. *Suppose the policy maker knows $p(\mathbf{s})$, does not receive informative signals ($|S| = 1$), and maximizes expected utility. Then prices can be fully revealing and accurate if and only if $v(\mathbf{s})$ is invertible.*

This simple condition—non-invertibility of the ‘expected asset value given optimal policy’ function $v(\mathbf{s})$ —explains the general difficulty of finding fully revealing equilibria in this setting. Prices cannot always reveal \mathbf{s} and at the same time be accurate. Thus, traders might have the incentive to make the forecasts implicit in their trades less revealing (rather than wrong). Mathematically, the result obtains because a price function cannot at the same time be invertible (as required by full revelation) and non-invertible (as required by accuracy if $v(\mathbf{s})$ is non-invertible).

Theorem 1 implies that accurate prices and full revelation are possible if condition 1 holds. Traders simply predict the expected asset value given optimal policy and trade accordingly so that $p(\mathbf{s}) = v(\mathbf{s})$, and the policy maker happily fulfills this forecast. Hence, condition 1 is also the condition that determines whether the forecasting problem is self-defeating or self-fulfilling.

2.3 Examples

In all of the aforementioned papers, traders have perfect information about the state variable θ . It can easily be verified that invertibility of $v(\theta)$ is not fulfilled in these papers, see Figure 2. For example (adapting their notation), in [Bernanke and Woodford \(1997\)](#)’s static model, $v(\theta) = 0$. In [Bond et al. \(2010\)](#) and [Prescott \(2012\)](#), $v(\theta)$ is at first strictly increasing in θ , but has a discontinuous downward jump at some threshold $\hat{\theta}$ (see Figure 2). Hence, the underlying problem—non-invertibility of $v(\theta)$ —is the same in all of these papers. But despite the same problem, preferences of the policy maker differ. In [Bernanke and Woodford \(1997\)](#), a central bank wants to minimize the variance of inflation $f(\theta, i)$, and in [Bond et al. \(2010\)](#) a board of directors wants to maximize firm value $f(\theta, i)$ minus intervention cost. In [Prescott \(2012\)](#), the policy is determined by a capital conversion rule. This shows the problem of full revelation with self-defeating prophecies is not due to specific policy maker goals.

Despite non-invertibility of $v(\theta)$ in [Bond et al. \(2010\)](#), they show that full revelation can occur, exactly because the informative signal of the policy maker—which is uniformly distributed on a bounded support—rules out all states that are not ruled out by the price.

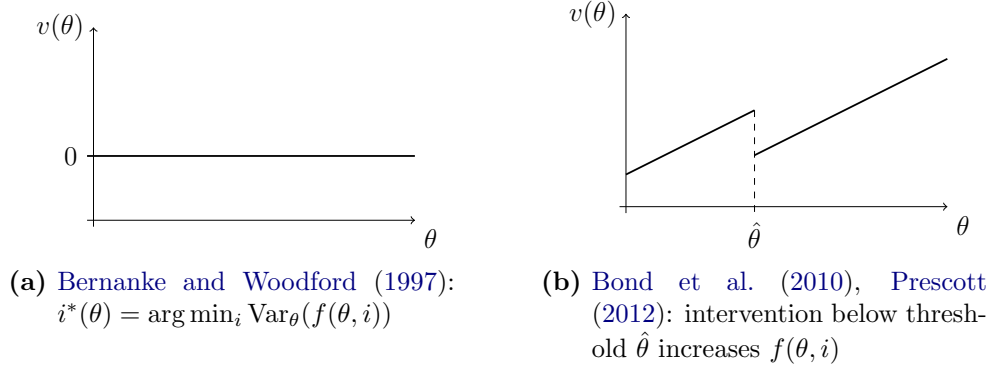


Figure 2: Examples for non-invertible asset values at the optimal policy $v(\theta)$ under full information

In the other models, the PM does not receive informative signals, and invertibility of $v(\theta)$ alone determines whether full revelation and accurate prices are possible.

2.4 Non-existence of fully revealing rational expectations equilibria

This section investigates whether accurate prices and full revelation can be implemented as a rational expectations equilibrium if every trader only observes his private signal s_j . That is, under which conditions can the financial market aggregate *and* reveal private information to the policy maker in equilibrium?

A rational expectations equilibrium in this setting is defined as follows (for a similar definition in the perfect information case, see Bond et al., 2010), where both functions are known in equilibrium.

Definition 3. A rational expectations equilibrium consists of

1. A forecasting/price function $p(\mathbf{s}) = \mathbb{E}_{f,\theta}[a(f(\theta, i(p(\mathbf{s})))) | \mathbf{s}]$ for all $\mathbf{s} \in \mathbf{S}$, and
2. an optimal policy function $i(p)$ given knowledge of $p(\mathbf{s})$, i.e.,

$$i(p) = \arg \max_i \mathbb{E}_{f,\theta}[u(f(\theta, i)) | p(\mathbf{s}) = p, \mathbf{s}].$$

Condition 1 (REE condition) of definition 3 requires that the price equals the expected value of the asset given trader information. Condition 2 requires that the policy maker acts optimally given his beliefs.

It turns out that the same condition that determines the possibility of full revelation and accurate prices is also necessary and sufficient for the existence of a fully revealing REE.

Corollary 4. A fully revealing rational expectations equilibrium exists if and only if condition 1 holds.

Proof. Condition 1 of definition 3 implies that the REE has to price accurately (cf. definition 1). Thus, the result follows from Theorem 1. \square

Thus, according to the REE concept, financial markets can both aggregate and reveal all trader information \mathbf{s} , but only if condition 1 holds. Hence, there are situations where markets *cannot* be strong form informationally efficient. Even in the most extreme case, where all traders perfectly know the state of the world ($s_j = \theta$) and perfectly know PM behavior, prices cannot reflect fundamental values of the asset if condition 1 fails to hold. The problem is not an informational one—traders know everything. Instead, accurate prices and full revelation are mutually exclusive, because the PM prefers policies that undermine incentives to reveal information via trading, i.e., he “prefers to falsify trader forecasts.” This result is in strong contrast to the standard case, where asset values are exogeneous and existence of fully revealing REE is generic (see for example Radner, 1979 or Allen, 1981).

While Corollary 4 explains the non-existence of *fully revealing* equilibria, the conditions in Corollary 4 and the following Proposition 5 jointly explain the REE non-existence results (fully revealing or otherwise) for example in Bernanke and Woodford (1997) or Prescott (2012), where the outcome $f(\theta, i)$ changes monotonically in θ .

Proposition 5. *Suppose the policy maker does not receive any signals ($|S| = 1$). If $\mathbb{E}_{f,\theta}[a(f(\theta, i))|\mathbf{s}] \neq \mathbb{E}_{f,\theta}[a(f(\theta', i))|\mathbf{s}'] \forall \mathbf{s} \neq \mathbf{s}' \in \mathcal{S}$, then there exists no ‘not fully revealing’ rational expectations equilibrium.*

Proof. Suppose there exists a not fully revealing REE. This implies there exist $\mathbf{s} \neq \mathbf{s}'$, $\mathbf{s}, \mathbf{s}' \in \mathcal{S}$, such that $p(\mathbf{s}) = p(\mathbf{s}')$. Since the policy maker cannot distinguish the two states, he will take the same action i for \mathbf{s} and \mathbf{s}' . But then for at least one of these two states prices must be inaccurate, since $\mathbb{E}_{f,\theta}[a(f(\theta, i))|\mathbf{s}] \neq \mathbb{E}_{f,\theta}[a(f(\theta', i))|\mathbf{s}']$, which contradicts this being an REE (condition 1 of definition 3). \square

The last two results imply that the fully revealing REE is the unique REE if condition 1 holds, the PM is uninformed, and $\mathbb{E}_{f,\theta}[a(f(\theta, i))|\mathbf{s}]$ is non-invertible in \mathbf{s} .

A very well known problem with the REE concept without noise is that it does not explain how the information is incorporated into prices (e.g., Dubey et al., 1987). Thus, the next section investigates whether these rational expectations equilibria are implementable as Perfect Bayesian Nash equilibria in a typical trading environment.

2.5 Full revelation in Perfect Bayesian Nash equilibria in large markets

To define the Perfect Bayesian Nash equilibrium (PBNE), we need to specify the trading environment. The financial market consists of a continuum (of mass 1) of traders with

utility function over wealth u_i , $u'_i > 0$, $u'' < 0$, who have a Pareto-optimal monetary $\omega_j > 0$ and asset endowment $x_j > 0$, with $\int x_j dj > 0$.⁴

Denote the set of all possible excess demand functions by \mathcal{X} . Unlike REE, we have to specify exactly how prices are set in this trading environment. The following definition describes a class of tâtonnement processes, which rules out an omniscient Walrasian auctioneer, who for every $\mathbf{s} \in \mathbf{S}$ proposes the fully revealing REE price and clears the market with it. Any price setting algorithm that conditions on demand functions (e.g., market orders, limit orders, etc.) in determining the price fulfills the definition. Examples include double auctions or Dutch auctions.

Definition 4. A generic price finding rule $\Gamma : \mathcal{X}^N \rightarrow \mathbb{R}$ maps all profiles of excess demand functions $\mathbf{x} := (x_1(p, s_1), x_2(p, s_2), \dots)$ into a price p^* .

Note that price $p^* = \Gamma(\mathbf{x})$ need not be a market clearing price, since a market clearing price need not exist for all profiles of excess demand functions \mathbf{x} . However, the following definition requires market clearing in equilibrium.

Definition 5. A Perfect Bayesian Nash equilibrium (PBNE) in this game requires

- i. an expected utility maximizing excess demand function $x_j(p, s_j)$ given \mathbf{x}_{-j} for all traders j at $t = 1$,
- ii. market clearing in the financial market at $t = 1$, i.e., a price function $p(\mathbf{s}) = \Gamma(\mathbf{x})$ generated by a generic price finding rule, such that

$$\int x_j(p^* = p(\mathbf{s}), s_j) dj = 0 \quad \forall \mathbf{s} \in \mathbf{S},$$

- iii. policy maker believes $\mu(t|p^*) = \Pr(\mathbf{s} = t | p^* = p(\mathbf{s}), s)$, $\sum_{t \in \mathbf{S}} \mu(t|p^* = p(\mathbf{s}), s) = 1$, about financial market behavior, derived by Bayes' rule whenever possible,
- iv. and an expected utility maximizing policy $i(s, p^*, \mu) \in I$ for the policy maker at $t = 2$.

Beliefs μ refer to the financial market as a whole, represented by the market clearing price p^* , rather than to actions of individual traders. This is a realistic way to model the interaction, as a policy maker usually cannot track investments of specific traders in anonymous markets, especially not in large markets as considered here. The price function $p(\mathbf{s})$ is known in equilibrium via Bayes' rule. There is no need to specify off equilibrium path beliefs, because no trader can influence the price alone, i.e., the price with a single deviation can never be off the equilibrium path.

⁴The continuum assumption implies that individual traders cannot move the price. Results will differ with finitely many traders, because in such a strategic setting any single trader can change the message sent to the policy maker by changing the investment strategy. A market with finitely many ("big") traders is beyond the scope of this paper.

The following proposition distinguishes two cases: All traders receive the same signal, which includes the perfect information case $s_j = \theta$ or any other information structure with perfectly correlated signals (*i.*), and the case where not all traders receive the same signal (*ii.*). The crucial difference is that information in (*i.*) does not have to be aggregated, since all traders hold the same information. In (*ii.*), on the other hand, information is dispersed, and its aggregate is more informative than any single piece.

Proposition 6. *i. If there is a continuum of traders and trader signals are perfectly correlated ($s_j = c \forall j, \mathbf{s} = c$), then a fully revealing PBNE exists if and only if condition 1 is fulfilled.*

ii. If there is a continuum of traders and trader signals are drawn independently, then there exists no fully revealing PBNE.

Proof. *i.* Necessity: if a fully revealing PBNE exists, it implies that $p(s_j = \mathbf{s}) = v(s_j = \mathbf{s})$. Suppose not ($p(\mathbf{s}) \neq v(\mathbf{s})$), then given market clearing there exists at least one trader who can profit by deviating from the candidate strategy x_j to $\hat{x}_j > x_j$ if $v(\mathbf{s}) > p(\mathbf{s})$ or to $\hat{x}_j < x_j$ if $v(\mathbf{s}) < p(\mathbf{s})$, since $v(\mathbf{s})$ is the expected value of the asset given full revelation, and the deviation in a continuum does not change prices. Thus, full revelation implies either that $v(\mathbf{s})$ is invertible, so that $p(\mathbf{s})$ perfectly reveals the state, or that $p(\mathbf{s})$ together with PM signal s always reveals the state, i.e., condition 1 holds.

Sufficiency: Define $\bar{x}_j(p, \mathbf{s}) > 0$ as j 's best response to $v(\mathbf{s}) > p$ and $\underline{x}_j(p, \mathbf{s}) < 0$ as j 's best response to $v(\mathbf{s}) < p$. If condition 1 holds, then the following strategy for all j forms a fully revealing PBNE:

$$x_j(p, \mathbf{s}) = \begin{cases} \bar{x}_j(p, \mathbf{s}) > 0 & \text{if } v(\mathbf{s}) > p \\ \underline{x}_j(p, \mathbf{s}) < 0 & \text{if } v(\mathbf{s}) < p \\ 0 & \text{if } v(\mathbf{s}) = p. \end{cases}$$

Clearly, market clearing occurs iff $p = v(\mathbf{s})$ for all $\mathbf{s} \in \mathbf{S}$. Due to condition 1, either $v(\mathbf{s})$ and therefore the equilibrium price function is fully revealing, or equilibrium prices together with PM signals s are fully revealing. There is no profitable deviation, since x_j best responds to mispricing by going long if $p < v(\mathbf{s})$ and short if $p > v(\mathbf{s})$. At $p = v(\mathbf{s})$, no deviation changes the payoff, since individual traders cannot influence the price and the asset is priced at its expected value. The PM obtains his first best given all available information, so he has no incentive to deviate either.

ii. Suppose there is a fully revealing equilibrium with price function $p(\mathbf{s})$. Since it is an equilibrium, we must have

$$p^* = \mathbb{E}_{f, \theta}[a(f(\theta, i(p^{-1}(p^*)))) | p^{-1}(p^*)] \text{ for all } p^* = p(\mathbf{s}),$$

otherwise market clearing implies there exists at least one trader who can profit by deviating at some price. The no-trade theorem (e.g., [Milgrom and Stokey, 1982](#)) implies that the risk averse traders respond with $x_j = 0$. But then the profile of excess demand functions and thus $\Gamma(\mathbf{x})$ are independent of \mathbf{s} , and since $p(\mathbf{s}) = \Gamma(\mathbf{x})$ in equilibrium, so is $p(\mathbf{s})$, which contradicts this being a fully revealing equilibrium. \square

Thus, if traders hold the same information, then condition 1 determines the possibility of full revelation in both major equilibrium concepts used in financial economics, REE and PBNE, in large markets. However, if traders have differential information, then the fully revealing REE equilibrium is not implementable as PBNE using a generic price finding rule, as is well known in financial economics [Vives \(2010\)](#). Thus, this is a problem with fully revealing REE in general, and not with this setting in particular.

To see why the REE cannot be implemented in the trading game as PBNE, consider the case where the asset return $a(f(\theta, i))$ is stochastic even under full revelation (say, because f is stochastic). Suppose a PBNE implements the fully revealing REE, then the price of the risky asset equals its expectation. But since traders are strictly risk averse, they strictly prefer the riskless asset with the same return. Thus, traders demand no assets in the equilibrium, but then the private information of traders cannot be incorporated into prices to make them fully revealing, which would require the demand function to change with the realization of the private information.

2.6 Asset design and selection

The results of the previous sections state that accurate and fully revealing prices, and the existence of fully revealing equilibria, require an invertible $v(\mathbf{s})$ function. From an asset design or asset selection point of view, we can ask what kind of asset supports full revelation. Formally, assuming the same underlying outcome $f(\theta, i)$ for the asset,⁵ differences in assets are captured by the asset payoff function $a(\cdot)$, the function that maps the underlying $f(\theta, i)$ into an asset value. For example, the function may be continuous or invertible, or it might be discontinuous and non-invertible, such as an Arrow security:

$$a(f(\theta, i)) = \begin{cases} 1 & \text{if } f(\theta, i) \geq T, \\ 0 & \text{if } f(\theta, i) < T, \end{cases}$$

for some threshold value $T \in \mathbb{R}$. Such asset payoff functions can be observed for example with credit default swaps, which have positive value if and only if the debtor solvency $f(\theta, i)$ is below a certain threshold that does not allow him to repay his debt. Bonds arguably also have this structure; they have positive value if and only if the issuing entity can repay.

⁵Another implication of the previous sections is clearly that if there are several outcomes to choose as underlying of the asset, then the one whose expectation is invertible in \mathbf{s} under full information is preferable in terms of information revelation.

Moreover, prediction markets (e.g., [Wolfers and Zitzewitz, 2004](#), [Siemroth, 2014](#)) often use this payoff function with so called ‘winner take all’ securities.

The following proposition shows that invertible asset payoff functions are never worse, but may be better suited to promote fully revealing prices than non-invertible ones. This is because a non-invertible asset payoff function may ‘bunch’ several states in a single asset value, thereby making it impossible to infer the state from the asset value, i.e., precluding invertibility of $v(\mathbf{s})$. Denote the set of all invertible functions $a : F \rightarrow A$ by \mathcal{A} and the set of non-invertible functions by \mathcal{A}' .

Proposition 7. *Any invertible function $a \in \mathcal{A}$ is weakly preferable to a non-invertible function $a' \in \mathcal{A}'$ in terms of promoting full revelation and accurate prices. That is, if a non-invertible $a' \in \mathcal{A}'$ leads to full revelation and accurate prices, then so does any invertible $a \in \mathcal{A}$, and $a \in \mathcal{A}$ sometimes leads to full revelation and accurate prices while $a' \in \mathcal{A}'$ does not, all else equal.*

Proof. Define $v(\mathbf{s}, a) := \mathbb{E}_{f,\theta}[a(f(\theta, i^*(\mathbf{s})))|\mathbf{s}]$. We need to show that there is no pair $\mathbf{s} \neq \mathbf{s}'$ such that $v(\mathbf{s}, a') \neq v(\mathbf{s}', a')$ but $v(\mathbf{s}, a) = v(\mathbf{s}', a)$, i.e., whenever a pair of states is revealed solely via prices under $a' \in \mathcal{A}'$, then it must also be revealed solely by prices under all $a \in \mathcal{A}$. Formally,

$$v(\mathbf{s}, a') \neq v(\mathbf{s}', a') \implies v(\mathbf{s}, a) \neq v(\mathbf{s}', a) \quad \forall a \in \mathcal{A}. \quad (1)$$

If this condition holds, then it implies that a function $a \in \mathcal{A}$ can never be worse in terms of information revelation than some non-invertible $a' \in \mathcal{A}'$, since the same states are revealed via prices, and if there are some states that are revealed via PM signals under a' , then they will also be revealed under a . That is, if (1) holds, then if condition 1 holds under a' , it will also hold under a .

To show (1) holds, note that for any non-invertible $a' \in \mathcal{A}'$,

$$\begin{aligned} \mathbb{E}_{f,\theta}[a'(f(\theta, i^*(\mathbf{s})))|\mathbf{s}] &\neq \mathbb{E}_{f,\theta}[a'(f(\theta, i^*(\mathbf{s}')))|\mathbf{s}'] \\ \implies \mathbb{E}_{f,\theta}[f(\theta, i^*(\mathbf{s}))|\mathbf{s}] &\neq \mathbb{E}_{f,\theta}[f(\theta, i^*(\mathbf{s}'))|\mathbf{s}']. \end{aligned} \quad (2)$$

Moreover, because \mathcal{A} is the set of invertible functions,

$$\begin{aligned} \mathbb{E}_{f,\theta}[f(\theta, i^*(\mathbf{s}))|\mathbf{s}] &\neq \mathbb{E}_{f,\theta}[f(\theta, i^*(\mathbf{s}'))|\mathbf{s}'] \\ \implies \mathbb{E}_{f,\theta}[a(f(\theta, i^*(\mathbf{s})))|\mathbf{s}] &\neq \mathbb{E}_{f,\theta}[a(f(\theta, i^*(\mathbf{s}')))|\mathbf{s}'] \quad \forall a \in \mathcal{A}. \end{aligned} \quad (3)$$

Combining (2) and (3) results in (1).

We still need to show that $a \in \mathcal{A}$ sometimes allows for full revelation and accurate prices while it would not with $a' \in \mathcal{A}'$. This immediately follows from the fact that invertibility of a is necessary for invertibility of $v(\mathbf{s})$, but not sufficient. And according to Corollary 2, full revelation and accurate prices are possible if $v(\mathbf{s})$ is invertible. \square

Consequently, in this setting, we would expect assets with invertible asset payoff function $a \in \mathcal{A}$ to be more informative, all else equal, compared to assets with non-invertible asset payoff function. Proposition 7 thus provides an empirical implication of the theory, which can be tested with prices for assets that have the same underlying $(f(\theta, i))$ but different payoff functions.

2.7 Summary

If all traders have perfect information about the realization of the state variable θ , then a single condition explains why prices can or cannot be informative in equilibrium. If condition 1 is not fulfilled, then policy maker and traders cannot both get what they want: either prices are not fully revealing, so the PM cannot always implement his preferred policy. Or there is mispricing, which implies that traders lose money. If the condition holds, then the goals of traders and policy maker are not mutually exclusive, and full revelation can be an equilibrium outcome. This result holds beyond a specific equilibrium concept such as REE. It generalizes several models from the literature (e.g., [Bernanke and Woodford, 1997](#) and [Bond et al., 2010](#)) and explains why their no equilibrium or no fully revealing equilibrium results occur.

The condition states that the expected value of the asset under full information (i.e., after revelation) $v(\theta)$ has to be invertible in the state variable θ , or if it is not, then the PM has to receive informative signals which allow him to rule out all states that are not ruled out by the asset price. Hence, the condition demands a lot from the signal structure of the policy maker if $v(\theta)$ is non-invertible, as the PM's posterior always has to put zero probability on any state that is consistent with the asset price but one. Normally distributed signals, for example, do not satisfy this condition, whereas uniformly distributed signals might (see [Bond et al., 2010](#) for an example).

If traders receive differential information, then the financial market's purpose is not only to communicate trader information, but also to aggregate it. In the absence of noise, this can be achieved in a rational expectations equilibrium if condition 1 is fulfilled. But the fully revealing REE outcome is not implementable as Perfect Bayesian Nash equilibrium of a well-specified trading game. Thus, in applications, the possibility of full revelation of trader information in financial markets will depend on the choice of equilibrium concept.

3 Conclusion

This paper presents a general model where traders have information which a policy maker needs in order to implement his optimal policy. Traders may reveal their information via trading in asset prices, but since policy influences asset values, they may not have an incentive to do so. For example, there may be situations where buying the asset raises

the asset price, which communicates information to the policy maker and leads to a policy that reduces asset values. In short, the asset value forecast implicit in asset prices may be a self-defeating prophecy, because the policy maker falsifies it.

I derive a necessary and sufficient condition for the possibility of fully revealing and accurate prices. If prices fulfill only one of these two properties, then either the policy maker does not learn from traders what he needs to know for optimal policy making, or assets are mispriced, which implies traders have an incentive to invest differently (or not at all). Indeed, reflecting this, the condition is also necessary and sufficient for the existence of fully revealing rational expectations equilibria.

If the policy maker does not receive informative signals on the realization of the state variable, then the necessary and sufficient condition for the possibility of accurate and fully revealing prices is very simple. It requires the asset values given the optimal policy for the policy maker ($v(\theta)$) to be invertible in the trader information. Intuitively, since traders know policy maker preferences, they can anticipate which policy he is going to implement once he knows the trader information. The invertibility condition ensures that prices can equal fundamental values of the asset and at the same time reveal trader information. Thus, traders can just set a price that equals the asset value at the optimal policy, and the policy maker indeed sets this policy—a self-fulfilling prophecy. If the invertibility condition is not fulfilled, then the policy maker would implement a policy that leads to mispricing for some realizations of the trader information if prices were fully revealing, and this is exactly the situation where traders lose money. Hence, the invertibility condition characterizes all situations where full revelation and accurate pricing are consistent with each other.

If the policy maker receives informative (but imperfect) signals, then the invertibility condition is still sufficient—after all, prices alone can reveal the state—but not necessary any more. In this case, the new necessary and sufficient condition broadly states that if asset values given the optimal policy for the policy maker are not invertible in the trader information, then the informative but imperfect information of the policy maker has to rule out all possible realizations of trader information but one which are consistent with asset prices. This condition is very demanding on the signal structure of the policy maker. Normally distributed signals, for example, do not fulfill this requirement, since these put positive density on every state on the real line. Hence, if the policy maker receives normally distributed signals, which never rule out any state, then invertibility of $v(\theta)$ is necessary and sufficient for prices to be fully revealing and accurate.

Using these results, I show that assets whose value is invertible in the underlying are weakly more favorable to full revelation in this setting. This means that if there are two kinds of assets with the same underlying (say, the value of a firm), then the asset whose value always changes monotonically with a change in the value of the firm should be more informative than the asset whose value is not. Non-invertible assets include those whose value depends on threshold values, such as the Arrow securities used in prediction markets.

In the setting described here, the impossibility of fully revealing prices in case condition 1 fails to hold is not due informational frictions—traders know the realization of the state variable and policy maker behavior perfectly. Rather, strong informational efficiency is impossible, because the policy maker reacts to the forecasts implicit in asset prices and falsifies them.

References

- ALLEN, B. (1981): “Generic existence of completely revealing equilibria for economies with uncertainty when prices convey information,” *Econometrica*, 1173–1199.
- BERNANKE, B. S. AND M. WOODFORD (1997): “Inflation forecasts and monetary policy,” *Journal of Money, Credit, and Banking*, 29, 653–684.
- BIRCHLER, U. W. AND M. FACCHINETTI (2007): “Self-destroying prophecies? The endogeneity pitfall in using market signals as triggers for prompt corrective action,” Working paper.
- BOND, P., I. GOLDSTEIN, AND E. S. PRESCOTT (2010): “Market-based corrective actions,” *Review of Financial Studies*, 23, 781–820.
- CABRALES, A., O. GOSSNER, AND R. SERRANO (2014): “The Inverse Demand for Information and the Appeal of Information Transactions,” Working paper.
- DUBEY, P., J. GEANAKOPOLOS, AND M. SHUBIK (1987): “The revelation of information in strategic market games: A critique of rational expectations equilibrium,” *Journal of Mathematical Economics*, 16, 105–137.
- GROSSMAN, S. J. AND J. E. STIGLITZ (1980): “On the impossibility of informationally efficient markets,” *American Economic Review*, 70, 393–408.
- MILGROM, P. AND N. STOKEY (1982): “Information, trade and common knowledge,” *Journal of Economic Theory*, 26, 17–27.
- PRESCOTT, E. S. (2012): “Contingent capital: the trigger problem,” *Federal Reserve Bank of Richmond Economic Quarterly*, 98, 33–50.
- RADNER, R. (1979): “Rational expectations equilibrium: Generic existence and the information revealed by prices,” *Econometrica*, 655–678.
- SIEMROTH, C. (2014): “Why prediction markets work: The role of information acquisition and endogenous weighting,” Working paper, University of Mannheim.
- SUNDARESAN, S. AND Z. WANG (forthcoming): “On the design of contingent capital with market trigger,” *Journal of Finance*.
- VIVES, X. (2010): *Information and learning in markets: The impact of market microstructure*, Princeton University Press.

WOLFERS, J. AND E. ZITZEWITZ (2004): “Prediction Markets,” *Journal of Economic Perspectives*, 18, 107–126.

WOODFORD, M. (1994): “Nonstandard Indicators for Monetary Policy: Can Their Usefulness Be Judged from Forecasting Regressions?” in *Monetary Policy*, ed. by N. G. Mankiw, University of Chicago Press.