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# The (Un)Level Playing Field: How Color-Blind Educational Tracking Leads to Unequal Access

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## Abstract

Educational tracking seeks to group students by unobserved ability using measures of observable acquired skills. In a model where individuals have differential skills prior to beginning formal education due to differences in early childhood development (e.g. linguistic, cultural, or nutritional disadvantages), we show that color-blind tracking systematically underplaces minorities. As a result, minorities have, in expectation, higher abilities than non-minorities assigned to the same track—regardless of track. A counterintuitive empirical implication of the model is that, conditional on tracking score and track, minorities will outperform non-minorities in subsequent testing following tracking. Affirmative action policies seeking to equalize post-tracking outcomes share similar flaws to color-blind standards in that the average ability of minorities assigned to the upper track remains higher than for non-minorities.

Keywords: Educational tracking, discrimination, early childhood disadvantage, equality of opportunity.

JEL Numbers: D21, D63, D83, J71, J78

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# 1 Introduction

A common standard for non-discrimination is achieving equality of opportunity—minorities of a given ability should have the same access to opportunities as non-minorities. Educational tracking, the sorting of students into advanced and vocational paths, is a critical early determinant of this access. These decisions are sometimes made at a very young age, as early as 10 years old in Germany and Austria, where early childhood experiences can play an important role.<sup>1</sup> While the goal of tracking to sort students by ability, the main estimate of ability is an assessment of the individual's state of knowledge at the time of the tracking decision.

Tracking instruments consist of standardized tests, like the SAT, as well as grades achieved in school up to the time of the tracking decision. Critics have argued that these performance measures are biased against minorities by virtue of the discourse system employed and types of learning that are valued, both of which typically reflect the views of non-minorities. We show, theoretically, that even were such instruments able to perfectly measure an individual's knowledge at the time of the tracking decision and regardless of minority status, then the use of color-blind standards leads to unequal access for minorities.

To be precise, consider the situation of two individuals with equal long-run ability, one of whom is a minority. We claim that, under color-blind admissions, the minority is more likely to be assigned to a lower track than the non-minority. The reason is the following: Knowledge, at the time of the tracking decision, consists of two parts—that portion gained in school, and initial knowledge, determined in early childhood, and outside of the control of the individual. It has been well-documented that minorities face a number of unique early childhood challenges: linguistic hurdles from parents who are non-native speakers, less preschool exposure, malnutrition, neglect, and so on.<sup>2</sup> At the conclusion of early childhood, an individual's state of knowledge then evolves according to her ability. The model (optimistically) assumes that the disadvantages of early childhood do not affect this evolution, only its starting point. The knowledge of individuals with the same ability assigned to the same track evolves at the same rate, regardless of minority status. The goal of tracking is to sort students by ability, so that subsequent knowledge may be developed more effectively through specialized pedagogy, differential access to resources, or peer effects.

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<sup>1</sup>See Table 1 of Brunello and Checchi (2006) for a list of tracking ages across OECD countries.

<sup>2</sup>See, e.g. Duncan, *et al.* (1994).

It is not altogether surprising that color-blind tracking disadvantages minorities owing to worse early childhood experiences. The more subtle insight of the model concerns the implications for the ability and performance of minorities post-tracking. Our main result shows that color-blind tracking leads to a situation where, in expectation, minorities have *higher* ability than non-minorities *in all tracks*. On its face, this would seem to be impossible given that average ability is the same for minorities and non-minorities. The distortions in selection resolve this apparent puzzle—minorities of medium abilities are shunted to a lower track while similarly able non-minorities are assigned to a higher track; thus producing a situation where minorities have higher ability, on average, in both tracks.

The model also offers important empirical implications. Controlling for test score and track, minorities are predicted to outperform non-minorities in subsequent performance evaluations following the tracking decision. Evidence that minorities have “caught up” in performance is sometimes used to justify an end to affirmative action policies. In fact, the opposite implication is demanded—the greater the degree of “catch up,” the larger the distortion of equality of opportunity in the track assignment. This statistic has the important additional advantage of being readily testable from existing data though, to the best of our knowledge, no one has yet performed this comparison.

Finally, we investigate alternatives to color-blind tracking, such as affirmative action schemes designed to achieve equality of outcomes as measured by subsequent testing. We show that these schemes have many of the same problems as a color-blind tracking. In particular, this procedure still leads to a situation where, in expectation, minorities have higher ability than non-minorities assigned to the upper track. Indeed, the model suggests that focus on equality of outcomes is misplaced. Under a first-best scheme that perfectly sorts individuals by ability, minorities will exhibit a persisting, though possibly diminishing, performance gap owing to their disadvantage in early childhood. Thus, far from representing a failure of policy, the presence of such a gap may signal success in sorting by ability.

We now place our work in the context of the extant literature. While the effects of tracking have been much studied, the methodology is almost entirely empirical. One key finding (Waldinger, 2009) is that tracking amplifies the importance of family background, including the effects of early childhood, on educational achievement. Family background is more important the earlier is the tracking decision (Brunello and Checchi, 2006; Schuetz, *et al.* 2008), which is consistent with our model. Many of these studies seek to measure peer

effects, and, in particular, how tracking influences their impact. (See, e.g. Hanushek, *et al.*, 2003). Brunello and Checchi (2006) share our concern with the connection between equality of opportunity and tracking; however, their empirical approach permits only indirect identification of these effects. In particular, they view significant coefficients on family background variables as *per se* evidence of failure of equality of opportunity. By contrast, our theoretical approach permits us to examine, in detail, the connection between tracking and underlying ability, which is typically unobservable empirically. There is a small theoretical literature on tracking, but none focuses on issues of discriminatory outcomes.<sup>3</sup> To the best of our knowledge, we are the first to do so.

From a broader perspective, our paper contributes to the literature on statistical discrimination. The usual problem confronted in these models is a situation where a payoff relevant, but unobservable, variable correlates with minority status and, as a consequence, employers treat minorities differently from non-minorities.<sup>4</sup> By contrast, our concerns center on discriminatory situations that arise from utilizing an identical standard for all individuals equally despite group differences. That is, we highlight discriminatory outcomes arising from apparently non-discriminatory processes.

The remainder of the paper is as follows: Section 2 studies a model of color-blind tracking in a setting where both abilities and early childhood experiences are heterogeneous. While the former distribution is identical for minority and non-minority populations, the latter favors non-minorities in a likelihood-ratio sense. Section 3 amends the model to study the effects of affirmative action. Finally, section 4 concludes.

## 2 A Model of Color-Blind Tracking

Children are born with unobservable, innate ability  $\theta \in \mathbb{R}^+$  distributed independently according to  $F$  with associated continuous density  $f$  and having a strictly increasing hazard rate. Ability represents a combination of raw talent, effort and desire. Children also have

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<sup>3</sup>For example, Lazear (2001) views tracking as a production decision in a model of educational output. Epplé, Newlon, and Romano (2002) offer a model of tracking as an equilibrium. political economy response to the threat of exit (private school) by rich parents.

<sup>4</sup>The seminal paper in this field is Phelps (1972), which has been extended many times and in many directions. See, e.g. Aigner and Cain (1977), Coate and Loury (1993), Lundberg and Startz (1998), and Morgan and Vardy (2010).

initial knowledge  $y_0 \in \mathbb{R}^+$ , which is distributed independently according to  $G_s$  with associated continuous density  $g_s$ , where  $s \in \{m, M\}$  denotes an individual's minority status. Here,  $m$  is a mnemonic for minority and  $M$  for majority (non-minority). Initial knowledge is gained in early childhood through a combination of parental inputs, exposure to enriching stimuli, preschool attendance, and nutrition. Consistent with various studies indicating minority disadvantage in early childhood, we assume that  $g_M \succ_{LR} g_m$ ; that is, initial knowledge for non-minorities exceeds that for minorities in the sense of the likelihood ratio ordering. Among other things, this implies that, on average, non-minorities have an advantage in initial knowledge. To summarize, while the distribution of ability is independent of minority status, initial knowledge is not.

Let  $t = 0$  denote the first date of “school,” *i.e.* the process at which incremental knowledge acquisition based on ability begins. Up to time  $\hat{t}$ , the point at which tracking begins, all students share the same curriculum and gain knowledge at a rate that depends on their ability. An individual's *performance* (*i.e.* measured knowledge) at time  $t$  occurring during this period is  $Y(t, y_0, \theta) = y_0 + y(t, \theta) \in \mathbb{R}^+$ . We assume that knowledge acquisition is (1) strictly positive for all abilities over time ( $\frac{d}{dt}y(t, \theta) > 0$ ), (2) strictly greater for more able individuals ( $\frac{d}{d\theta}y(t, \theta) > 0$ ), (3) concave in ability.

At time  $\hat{t}$ , the tracking decision occurs. Student performance  $Y(\hat{t}, y_0, \theta) \equiv y_{\hat{t}}$  is measured at this point, and educators assign those performing above some threshold,  $\hat{y}$ , to the upper track. They assign the remainder to the lower track. Throughout, we shall refer to  $y_{\hat{t}}$  as a student's score at the point of tracking.

Tracking is color-blind: Minorities must exceed the same threshold as non-minorities. Moreover, for each level of initial knowledge,  $y_0$ , the minimum ability needed to attain the upper track is the same for minorities and non-minorities. However, because minorities are disadvantaged in early childhood, it follows that, a minority of a given ability  $\theta$  is less likely to be assigned to the upper track than a non-minority of the same ability. That is, color-blind tracking produces systematically fewer opportunities for minorities. To formalize this intuition, let  $\tau \in \{l, u\}$  denote the track to which a given student is assigned, where  $u$  denotes the upper track and  $l$  the lower. Of interest is  $\Pr[u|\theta, s]$ , the probability that an individual with ability  $\theta$  and minority status  $s$  is assigned to the upper track. Equality of opportunity would imply that this probability is independent of minority status. The following proposition shows that a color-blind tracking policy does not have this property.

**Proposition 1** *A color-blind tracking policy systematically favors less able non-minorities at the expense of more able minorities. Formally, for all  $\theta$*

$$\Pr[u|\theta, M] > \Pr[u|\theta, m]$$

**Proof.** Fix an ability level  $\theta$ . Then the threshold initial knowledge required to achieve the threshold  $\hat{y}$  is

$$\hat{y}_0 = \hat{y} - y(\hat{t}, \theta)$$

Thus,

$$\Pr[u|\theta, s] = 1 - G_s(\hat{y}_0)$$

The likelihood ratio ordering implies that, for all  $\hat{y}_0 > 0$ ,  $G_m(\hat{y}_0) > G_M(\hat{y}_0)$ . Since this inequality holds for all  $\theta$ , the distribution of which is independent of minority status, this establishes the claim. ■

Since Proposition 1 holds for each ability level and the distribution of abilities is the same for minorities and non-minorities, it then immediately follows that  $\Pr[u|m] < \Pr[u|M]$ ; that is, non-minorities gain preferential access to the upper track compared with minorities. This is not altogether surprising given the disadvantage minorities face in early childhood, which is not accounted for by the selection rule. Our next two results, Propositions 2 and 3, which represent the main contribution of the paper, highlights more subtle implications of color-blind tracking on the distribution of abilities in each track and post-tracking performance.

Since minorities must overcome early childhood disadvantages to achieve a score placing them on the upper track, they must compensate for these shortcomings through higher ability. As a result, the minority population in the upper track has higher average ability than non-minorities. The same is true of the lower track, though for different reasons. Here, it is the relatively talented, but excluded minority population that provides the ability boost. Able individuals suffering from moderate to severe early childhood disadvantages are shunted to the lower track. Since these individuals are more numerous among minorities than non-minorities, such mistaken tracking afflicts them more severely and, as a consequence, their average ability is higher than non-minorities in the lower track as well. Thus, despite the fact that the ability distribution is the same for minorities and non-minorities, color-blind tracking produces minority populations with superior ability to non-minorities in *both* tracks. Formally,

**Proposition 2** *Conditional on track  $\tau$ , the expected ability of minority students exceeds that of non-minorities.*

**Proof.** We first establish the claim for the upper track. It is useful to define the decumulative distribution

$$\bar{F}(y_0) \equiv 1 - F(y^{-1}(\hat{y} - y_0))$$

and the unscaled expectation conditional on being in the upper track

$$\bar{X}(y_0) \equiv \int_{y^{-1}(\hat{y}-y_0, t)}^{\infty} \theta f(\theta) d\theta$$

and hence,

$$\begin{aligned} E[\theta|u, m] &= \frac{\int_0^{\infty} g_m(\sigma) \bar{X}(\sigma) d\sigma}{\int_0^{\infty} g_m(y_0) \bar{F}(y_0) dy_0} \\ E[\theta|u, M] &= \frac{\int_0^{\infty} g_M(\sigma) \bar{X}(\sigma) d\sigma}{\int_0^{\infty} g_M(y_0) \bar{F}(y_0) dy_0} \end{aligned}$$

Cross-multiplying and differencing, we obtain the condition:

$$\int_0^{\infty} \int_0^{\infty} \bar{X}(\sigma) \bar{F}(y_0) [g_m(\sigma) g_M(y_0) - g_M(\sigma) g_m(y_0)] dy_0 d\sigma > 0$$

or, equivalently

$$\begin{aligned} & \int_0^{\infty} \bar{X}(\sigma) \int_{\sigma}^{\infty} \bar{F}(y_0) [g_m(\sigma) g_M(y_0) - g_M(\sigma) g_m(y_0)] dy_0 d\sigma \\ & > \int_0^{\infty} \bar{X}(\sigma) \int_0^{\sigma} \bar{F}(y_0) [g_m(y_0) g_M(\sigma) - g_M(y_0) g_m(\sigma)] dy_0 d\sigma \end{aligned}$$

We will rewrite the RHS, so that we can readily combine it with the left. Changing the order of integration of the RHS

$$= \int_0^{\infty} \bar{X}(\sigma) \int_{y_0}^{\infty} \bar{F}(y_0) [g_m(y_0) g_M(\sigma) - g_M(y_0) g_m(\sigma)] d\sigma dy_0$$

Exchanging  $\sigma$  and  $y_0$

$$= \int_0^{\infty} \bar{X}(y_0) \int_{\sigma}^{\infty} \bar{F}(\sigma) [g_m(\sigma) g_M(y_0) - g_M(\sigma) g_m(y_0)] dy_0 d\sigma$$

Differencing the left and right hand side

$$\int_0^{\infty} \int_{\sigma}^{\infty} (\bar{X}(\sigma) \bar{F}(y_0) - \bar{X}(y_0) \bar{F}(\sigma)) [g_m(\sigma) g_M(y_0) - g_M(\sigma) g_m(y_0)] dy_0 d\sigma$$



Notice that, since  $\sigma < y_0$ , it follows by MLRP that  $g_m(\sigma)g_M(y_0) - g_M(\sigma)g_m(y_0) > 0$ . It remains to show that for all  $\sigma < y_0$  in the support,

$$\bar{X}(\sigma)\bar{F}(y_0) - \bar{X}(y_0)\bar{F}(\sigma) > 0$$

or, equivalently

$$E[\theta|\theta \geq y^{-1}(\hat{y} - \sigma, \hat{t})] > E[\theta|\theta \geq y^{-1}(\hat{y} - y_0, \hat{t})]$$

which holds for all  $\sigma < y_0$ .

To establish the claim for the lower track. Substitute the cumulative distribution

$$F(y_0) \equiv F(y^{-1}(\hat{y} - y_0))$$

and the unscaled expectation conditional on being in the lower track

$$X(y_0) \equiv \int_0^{y^{-1}(\hat{y} - y_0, \hat{t})} \theta f(\theta) d\theta$$

everywhere in the above steps for  $\bar{F}(\cdot)$  and  $\bar{X}(\cdot)$  respectively, resulting in the desired condition

$$E[\theta|\theta \leq y^{-1}(\hat{y} - \sigma, \hat{t})] > E[\theta|\theta \leq y^{-1}(\hat{y} - y_0, \hat{t})]$$

which holds for all  $\sigma < y_0$ . ■

At first blush, the result contained in Proposition 2 seems impossible. If minorities and non-minorities have the same ability overall, how can it be the case that the expected ability of minorities is higher in *both* tracks? The intuition behind this apparent contradiction can readily be seen by conditioning the expected ability of minority and non-minority students over the two educational tracks:

$$E[\theta] = E[\theta|m] = E[\theta|u, m] \Pr[u|m] + E[\theta|l, m] (1 - \Pr[u|m])$$

$$E[\theta] = E[\theta|M] = E[\theta|u, M] \Pr[u|M] + E[\theta|l, M] (1 - \Pr[u|M])$$

which highlights that it is the product  $E[\theta|\tau, s] \times \Pr[\tau|s]$ , summed over all  $\tau$ , that must be equal. We showed that  $E[\theta|u, m] > E[\theta|u, M]$  and  $E[\theta|l, m] > E[\theta|l, M]$ , but this can only occur when  $\Pr[u|m] < \Pr[u|M]$ , which we established in Proposition 1. Thus, the peculiar ranking of average abilities displayed by minorities under a color-blind scheme is, in part, a consequence of their being overrepresented in the lower track and underrepresented in the upper track.

Proposition 2, while surprising, is somewhat difficult to measure. Indeed, it is precisely the inability to accurately measure ability that causes the problem with color-blind tracking in the first place. Our next set of results show how post-tracking performance may be used to estimate the distortions of color-blind tracking on minorities. Before proceeding, however, we need to extend the model to account for knowledge acquisition post tracking. The performance at time  $t > \hat{t}$  of an individual with ability  $\theta$  who achieved a score  $y_{\hat{t}}$  and was assigned to track  $\tau$  is

$$Y(t, \theta, \tau, y_{\hat{t}}) = y_{\hat{t}} + \tau(t - \hat{t}, \theta)$$

We slightly abuse notation by treating  $\tau(t - \hat{t}, \theta)$  as a knowledge acquisition function that depends on the track to which a student is assigned. The function  $\tau(t - \hat{t}, \theta)$  has the same properties as  $y(t, \theta)$ : it is strictly increasing over time and over ability.

Our next lemma is primarily technical—it shows that the likelihood ratio ordering property of initial knowledge is inherited, but in reverse, in the distribution of abilities for students with status  $s$  achieving score  $y_{\hat{t}}$  in the tracking exam. For future reference, define the function  $h_s(\theta|y_{\hat{t}})$  to be the conditional density of ability for an individual of minority status  $s$  achieving score  $y_{\hat{t}}$ . That is,

$$h_s(\theta|y_{\hat{t}}) = \frac{f(\theta) g_s(y_{\hat{t}} - y(t, \theta))}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}} - y_0, \hat{t})) g_s(y_0) dy_0}$$

Formally,

**Lemma 1** *For a given score  $y_{\hat{t}}$ ,  $h_m(\theta|y_{\hat{t}}) \succ_{LR} h_M(\theta|y_{\hat{t}})$ . That is, the conditional distribution of abilities satisfies MLRP.*

**Proof.** We will establish that  $h_m(\theta|y_{\hat{t}})/h_M(\theta|y_{\hat{t}})$  is strictly increasing in  $\theta$ . Suppose

$$\frac{\frac{f(\theta)g_m(y_{\hat{t}}-y(t,\theta))}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}}-y_0,\hat{t}))g_m(y_0)dy_0}}{\frac{f(\theta)g_M(y_{\hat{t}}-y(t,\theta))}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}}-y_0,\hat{t}))g_M(y_0)dy_0}} < \frac{\frac{f(\theta')g_m(y_{\hat{t}}-y(t,\theta'))}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}}-y_0,\hat{t}))g_m(y_0)dy_0}}{\frac{f(\theta')g_M(y_{\hat{t}}-y(t,\theta'))}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}}-y_0,\hat{t}))g_M(y_0)dy_0}}$$

for all  $\theta' > \theta$ .

Rewriting

$$\begin{aligned} & \frac{f(\theta) g_m(y_{\hat{t}} - y(t, \theta))}{f(\theta) g_M(y_{\hat{t}} - y(t, \theta))} \times \frac{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}} - y_0, \hat{t})) g_M(y_0) dy_0}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}} - y_0, \hat{t})) g_m(y_0) dy_0} \\ & < \frac{f(\theta') g_m(y_{\hat{t}} - y(t, \theta'))}{f(\theta') g_M(y_{\hat{t}} - y(t, \theta'))} \times \frac{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}} - y_0, \hat{t})) g_M(y_0) dy_0}{\int_0^{\hat{y}_t} f(y^{-1}(y_{\hat{t}} - y_0, \hat{t})) g_m(y_0) dy_0} \end{aligned}$$

Simplifying

$$\frac{g_m(y_{\hat{t}} - y(t, \theta))}{g_M(y_{\hat{t}} - y(t, \theta))} < \frac{g_m(y_{\hat{t}} - y(t, \theta'))}{g_M(y_{\hat{t}} - y(t, \theta'))}$$

where the inequality follows from the fact that  $y(t, \theta)$  is strictly increasing in  $\theta$  and  $g_s$  satisfies MLRP. ■

Lemma 1 allows us to readily compare the performance of minorities and non-minorities after tracking has been imposed. Suppose that, at date  $t > \hat{t}$  an students take an exam to estimate their current knowledge. Then,

**Proposition 3** *Conditional on achieving a given tracking score,  $y_{\hat{t}}$ , the expected subsequent performance of minorities exceeds that of non-minorities assigned to a given track.*

**Proof.** Recall that subsequent performance of minorities at time  $t > \hat{t}$  is simply

$$\begin{aligned} E[y_{\hat{t}} + \tau(t - \hat{t}, \theta) | m, y_{\hat{t}}] &= y_{\hat{t}} + \int_0^{y_{\hat{t}}} \tau(t - \hat{t}, \theta) h_m(\theta | y_{\hat{t}}) d\theta \\ &> y_{\hat{t}} + \int_0^{y_{\hat{t}}} \tau(t - \hat{t}, \theta) h_M(\theta | y_{\hat{t}}) d\theta \\ &= E[y_{\hat{t}} + \tau(t - \hat{t}, \theta) | M, y_{\hat{t}}] \end{aligned}$$

where the inequality follows from the fact that  $\tau(t - \hat{t}, \theta)$  is strictly increasing in ability and the likelihood ratio ordering property of Lemma 1. ■

Proposition 3 is important for its empirical content. It suggests that, controlling for score, a regression of subsequent performance on minority status will yield a *positive* coefficient. One might interpret this as implying that minorities need no additional help from affirmative action in that they are already outperforming non-minorities. In fact, just the opposite implication is the correct one to draw—the larger is the magnitude of this coefficient, the greater the degree of inequality of opportunity as the differential gains accruing to minorities are proportional to their underrepresentation in the upper track. Moreover, the result is independent of the tracking location—minorities assigned to the lower track will outperform non-minorities controlling for score, as will minorities assigned to the upper track. This is, of course, a direct consequence of Proposition 2—minorities, on average, are more able than non-minorities in the same track and display this prowess in post-tracking performance measures.

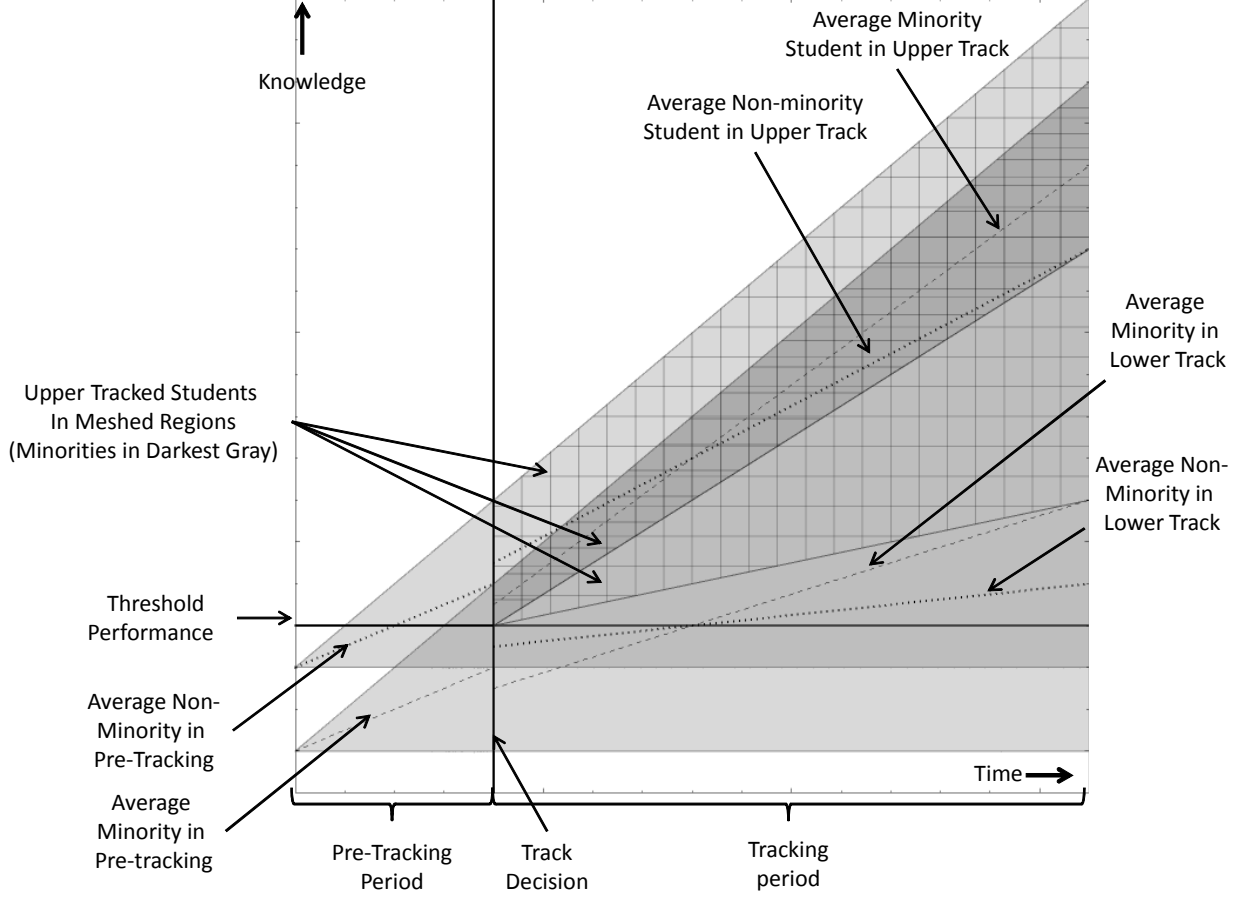


Figure 1: Plot of distribution of knowledge among students over time. Assumptions:  $\theta \sim U[0, 1]$ ,  $Y = y_{0s} + \theta t$ . All individuals of a given status have the same initial knowledge, where  $y_{0m} < y_{0M}$ .

Figure 1 illustrates the dynamics of color-blind tracking. Prior to the tracking event, minorities, suffering from early childhood disadvantage, lag the performance of non-minorities in the acquisition of knowledge. This is reflected in the tracking decision: Notice the much larger fraction of non-minorities assigned to the upper track compared to minorities. Of necessity, the reverse is true of the lower track. The figure then traces the knowledge path of the average student of each status in a given track. In the upper track, minorities continue to trail immediately after the tracking decision, though the gap is reduced compared to pre-tracking owing to the selection implicit in color-blind tracking. As time passes, the gap continues to shrink and is eventually erased entirely or even reversed. Again, this follows from the ranking of abilities between minorities and non-minorities in the same track.

The figure also illustrates the dispersion of abilities in each track. In the upper track, owing to stringent selection, minority students have relatively similar abilities. They begin near the bottom of the class and gradually move up. Non-minorities, by contrast, display a much wider range of abilities, accounting for both the most and least knowledgeable students in the track. This situation is reversed in the lower track, where minorities have more dispersed abilities. The model suggests that minorities will account for the most exceptional students in that track—some will perform far above non-minorities on the same track while others will struggle continually throughout their time in school.

### 3 Affirmative Action

We have demonstrated that color-blind tracking has several undesirable properties. It produces a biased separation in terms of ability, systematically favoring less able majorities while excluding some more able minorities. It also produces perverse peer effects—minorities are, on average, more talented than non-minorities regardless of track. Finally, the extent of these difficulties is measurable—the greater the distortion from color-blind track, the larger the measured performance difference (favoring minorities) between minorities and non-minorities in post-tracking exams.

What can be done to remedy these defects? One possibility is to adjust the threshold to minorities to be assigned to the upper track in some way. Such a solution is typical of affirmative action schemes. As a practical matter, this might be done by adding to the tracking score based on minority status or simply having different thresholds for admission to the upper track. In terms of the model, let  $\hat{y}_s$  denote the tracking threshold used for an individual with status  $s$ . The following lemma highlights that, regardless of the particular adjustment employed to benefit minorities, those selected to the upper track will still suffer from lower initial knowledge, in expectation. Formally,

**Lemma 2** *Suppose that  $\hat{y}_m \leq \hat{y}_M$ . Then  $E[y_0|u, m] < E[y_0|u, M]$ .*

**Proof.** It is useful to define the decumulative distribution

$$\bar{F}_s(y_0) \equiv 1 - F(y^{-1}(\hat{y}_s - y_0))$$

and note that it is increasing in its argument.

$$E[y_0|m, u] = \frac{\int_0^\infty g_m(y_0) \bar{F}_m(y_0) y_0 dy_0}{\int_0^\infty g_m(y_0) \bar{F}_m(y_0) dy_0}$$

$$E[y_0|M, u] = \frac{\int_0^\infty g_M(y_0) \bar{F}_M(y_0) y_0 dy_0}{\int_0^\infty g_M(y_0) \bar{F}_M(y_0) dy_0}$$

To establish the desired inequality, cross-multiply the above expressions to obtain

$$\int_0^\infty g_m(y_0) \bar{F}_m(y_0) y_0 dy_0 \int_0^\infty g_M(\sigma) \bar{F}_M(\sigma) d\sigma < \int_0^\infty g_M(y_0) \bar{F}_M(y_0) y_0 dy_0 \int_0^\infty g_m(\sigma) \bar{F}_m(\sigma) d\sigma$$

Rewriting

$$\begin{aligned} & \int_0^\infty \int_0^{y_0} g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma) y_0 d\sigma dy_0 \\ & + \int_0^\infty \int_{y_0}^\infty g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma) y_0 d\sigma dy_0 \\ & < \int_0^\infty \int_0^{y_0} g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma) y_0 d\sigma dy_0 \\ & + \int_0^\infty \int_{y_0}^\infty g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma) y_0 d\sigma dy_0 \end{aligned}$$

or

$$\begin{aligned} & \int_0^\infty \int_{y_0}^\infty (g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma) - g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma)) y_0 d\sigma dy_0 \\ & < \int_0^\infty \int_0^{y_0} (g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma) - g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma)) y_0 d\sigma dy_0 \end{aligned}$$

Let us rearrange the RHS to make it compatible with the LHS by changing the order of integration and renaming the variables.

$$\begin{aligned} & \int_0^\infty \int_0^{y_0} (g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma) - g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma)) y_0 d\sigma dy_0 \\ & = \int_0^\infty \int_{y_0}^\infty (g_M(\sigma) \bar{F}_M(\sigma) g_m(y_0) \bar{F}_m(y_0) - g_m(\sigma) \bar{F}_m(\sigma) g_M(y_0) \bar{F}_M(y_0)) \sigma d\sigma dy_0 \end{aligned}$$

Subtracting the LHS from the RHS,

$$\int_0^\infty \int_{y_0}^\infty (g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma) - g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma)) (\sigma - y_0) d\sigma dy_0 > 0$$

It remains to show that, for all  $\sigma \geq y_0$

$$g_m(y_0) \bar{F}_m(y_0) g_M(\sigma) \bar{F}_M(\sigma) \geq g_M(y_0) \bar{F}_M(y_0) g_m(\sigma) \bar{F}_m(\sigma)$$

Rearranging, this amounts to the condition

$$\frac{g_m(y_0) \bar{F}_m(y_0)}{g_M(y_0) \bar{F}_M(y_0)} \geq \frac{g_m(\sigma) \bar{F}_m(\sigma)}{g_M(\sigma) \bar{F}_M(\sigma)}$$

which holds owing to MLRP of  $g$  and since monotonicity  $\frac{\bar{F}_m(\cdot)}{\bar{F}_M(\cdot)}$  is strictly decreasing in its argument (see claim and proof below). (with strict inequality a.e.).

Differentiating  $\frac{\bar{F}_m(\cdot)}{\bar{F}_M(\cdot)}$  and simplifying yields the required property

$$\frac{f(y^{-1}(\hat{y}_m - y_0))}{1 - F(y^{-1}(\hat{y}_m - y_0))} < \frac{f(y^{-1}(\hat{y}_M - y_0))}{1 - F(y^{-1}(\hat{y}_M - y_0))} \frac{y'(y^{-1}(\hat{y}_m - y_0, \hat{t}))}{y'(y^{-1}(\hat{y}_M - y_0, \hat{t}))}$$

which holds since  $y^{-1}(\hat{y}_m - y_0, t) < y^{-1}(\hat{y}_M - y_0, t)$  along with increasing hazard rates and concave  $y(\theta, t)$ . ■

### Equality of Outcomes

A fundamental question any affirmative action procedure must address is how to determine the degree of benefit to accord. The goal of many such schemes is to achieve equality of outcome as well as equality of opportunity for minorities and hence, policy makers sometimes resort to outcome-based metrics. In a tracking context, one proposed strategy is to adjust the threshold until post-tracking performance is equalized between minorities and non-minorities. For instance, the threshold adjustment might depend on equalizing graduation rates, GPA, or achievement test scores at some point in the education process. Our next proposition shows that, even were such an adjustment made perfectly, so that equality of outcome in the desired metric is achieved, the resulting tracking assignments would still be biased against minorities. To see this, suppose that post-tracking performance is assessed at date  $t^* > \hat{t}$ , where  $y_{t^*}$  denotes the score received at this assessment. Assume that  $\hat{y}_m$  adjusts to achieve equality of outcome in this metric. Then

**Proposition 4** *Fix  $\hat{y}_M$  and suppose that  $\hat{y}_m$  is adjusted to equalize post tracking performance at time  $t^* > \hat{t}$ . Then, in expectation, minorities assigned to the upper track have higher ability than non-minorities.*

**Proof.** We will show that, using the affirmative action policy of equating post-tracking performance implies that  $E[\theta|u, m] > E[\theta|u, M]$ . Under such a rule

$$\begin{aligned} E[y_{t^*}|u, m] &= E[y_{\hat{t}}|u, m] + E[u(\theta, t^* - \hat{t})|u, m] \\ &= E[y_{\hat{t}}|u, M] + E[u(\theta, t^* - \hat{t})|u, M] = E[y_{t^*}|u, M] \end{aligned}$$

Therefore,  $E[u(\theta, t^* - \hat{t}) | u, m] > E[u(\theta, t^* - \hat{t}) | u, M]$  iff  $E[y_{\hat{t}} | u, m] < E[y_{\hat{t}} | u, M]$ . Furthermore, since  $u(\theta, t^* - \hat{t})$  is a strictly increasing function of  $\theta$ , if  $E[u(\theta, t^* - \hat{t}) | u, m] > E[u(\theta, t^* - \hat{t}) | u, M]$  then  $E[\theta | u, m] > E[\theta | u, M]$ . We will examine all possible cases of tracking scores and show that each leads to the same ordering of expected abilities.

**Case 1:**  $E[y_{\hat{t}} | u, m] = E[y_{\hat{t}} | u, M]$ . By Lemma 2,  $E[y_0 | u, m] < E[y_0 | u, M]$ , and hence  $E[y(\theta, \hat{t}) | u, m] > E[y(\theta, \hat{t}) | u, M]$ , but this is equivalent to  $E[\theta | u, m] > E[\theta | u, M]$ .

**Case 2:**  $E[y_{\hat{t}} | u, m] < E[y_{\hat{t}} | u, M]$ . To equate post-tracking performance requires that  $E[u(\theta, t^* - \hat{t}) | u, m] > E[u(\theta, t^* - \hat{t}) | u, M]$ , but this is equivalent to  $E[\theta | u, m] > E[\theta | u, M]$ .

**Case 3:**  $E[y_{\hat{t}} | u, m] > E[y_{\hat{t}} | u, M]$ . This is equivalent to

$$E[y_0 | u, m] + E[y(\theta, \hat{t}) | u, m] > E[y_0 | u, M] + E[y(\theta, \hat{t}) | u, M]$$

which implies

$$E[y_0 | u, m] - E[y_0 | u, M] > E[y(\theta, \hat{t}) | u, M] - E[y(\theta, \hat{t}) | u, m]$$

By Lemma 2,  $E[y_0 | u, m] - E[y_0 | u, M] < 0$ . Hence,  $E[y(\theta, \hat{t}) | u, m] > E[y(\theta, \hat{t}) | u, M]$ , but this is equivalent to  $E[\theta | u, m] > E[\theta | u, M]$ . ■

The policy described in Proposition 4 seeks to redress the failure of equality of opportunity by applying an equality of outcome metric to level the playing field. As the Proposition shows, enforcing equality of outcomes at time  $t^*$  fails in its intended purpose—the early childhood disadvantage experienced by minorities is only partially ameliorated as measured by ability in the upper track. The reason this procedure fails is that, for minorities to “catch up” at time  $t^*$ , they must be gaining knowledge at a faster rate than non-minorities, and this can only happen if, on average, they have higher ability.

### First-Best Policy

Finally, let us consider what first-best policy, which would occur were ability observable. In that case, a color-blind *ability* threshold would be selected and, since the ability distribution is independent of minority status, minorities would be admitted to the upper track at the same rate as non-minorities, *i.e.*  $\Pr[u | m, \theta] = \Pr[u | M, \theta]$  for all  $\theta$ . Post-tracking performance measures would show a persistent “achievement gap” where minority performance would lag owing to worse early childhood experiences. Over time, this gap will close as early childhood outcomes mattered ever less. The broader point is that policies seeking to erase post-tracking achievement differences are, of necessity, biased against minorities in the same way as a color-blind standard, though not to the same extent.



## 4 Conclusions

The three main lessons to emerge from our analysis: (1) In the face of early childhood disadvantage, a color blind tracking policy systematically excludes more able minorities while including less able non-minorities. (2) As a result, the average ability of minorities exceeds that of non-minorities on the same track, regardless of track. (3) The distortive effects of such a policy can be readily measured. Controlling for tracking score and track, minorities are predicted to outperform non-minorities in subsequent knowledge acquisition. The magnitude of the performance difference is proportional to the degree to which equality of opportunity is distorted. To the best of our knowledge, no one has yet conducted such an empirical test, but the simplicity and starkness of the prediction certainly merit attention.

While the setting of the model emphasizes the importance of early childhood outcomes in initial, and highly consequential, tracking decisions such as those used in Europe to determine secondary school assignments, the conceptual point applies with equal force to later tracking decisions. So long as differences outside the control of the individual continue to impinge on learning outcomes and disparately affect those of minority status, color-blind tracking will produce the same result wherever in the process it is imposed. For at-risk groups in the US, this is certainly a factor influencing another key tracking decision, admission to college. For instance, certain minorities in the US are overwhelmingly more likely to suffer from deficiencies in the fulfillment of basic life needs—safety, adequate nutrition, and even access to heat and electricity—with profound effects even beyond early childhood. In addition, stark differences in the quality of primary education, which often correlate with minority status, further exacerbate initial disadvantages, particularly at the critical level of college admissions. For members of such groups, a “level playing field” in terms of evaluation produces a decidedly unlevel outcome in terms of access to the appropriate track for the most able members of the population.

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