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On the Distributional Implications of Demographic Change

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On the Distributional Implications of Demographic Change*

Christian Geppert◊

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Abstract

The ongoing aging process in all major industrialized countries gives rise to scarcity of labor relative to capital. Standard models suggest that this increases wages and provides incentives for higher education. This paper asks what that implies for the German economy by quantifying the impact of demographic change on the distributions of income, skills, and welfare across and within generations. The results suggest that demographic change causes a decline in the skill premium of 15 percentage points by the year 2050 while the college educated share of the workforce increases by about 3 percentage points. Welfare effects of demographic change are substantial and vary between -3% and $+2\%$ of consumption in every period of lifetime depending on skill group and generation. All currently living generations lose. Despite the drop in the skill premium, demographic change benefits skilled over unskilled households.

A secondary result shows that past skill-biased technological change will put additional downward pressure on the future skill premium by increasing the stock of skilled workers in the future. This induces strong welfare losses for college households of up to 8% of consumption in every period of lifetime.

JEL classification: J11, J24, J22, I24, C68, D91

Keywords: demographic change; human capital; skill premium; distribution of welfare; overlapping generations

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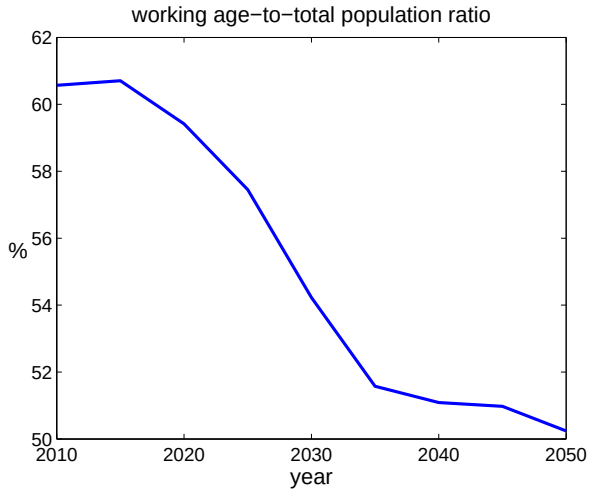
1 Introduction

"Skills are the currency for the 21st century – they will be decisive for economies, societies and the prospects of people." (OECD 2013)

Like all other major economies, Germany faces severe population aging within the next decades. Figure 1 depicts the expected evolution of the working age-to-total population ratio by United Nations (2013) holding fix the retirement age at 65. The graph shows a severe drop of more than 10 percentage points until 2050. Even when accounting for a step-wise increase of the statutory retirement age to 67, as implemented by the German government in 2007, the drop in the working age-to-total population ratio remains substantial.[1]

Figure 1: Expected Demographic Change in Germany



Source: Own calculations based on United Nations (2013) using medium fertility prospects and holding the retirement age fix at 65. Note that taking account of the step-wise increase of the statutory retirement age to 67 as implemented by the German government in 2007 would lead to a working age-to-total population ratio in 2050 of about 53%.

These strong changes in the population structure will have important implications for the macroeconomic composition of capital and labor in the production as Ludwig, Schelkle, and Vogel (2012) remark. From a theoretical macroeconomic perspective, labor becomes ceteris paribus a scarce factor in an aging economy if labor market participation, education, or human capital formation do not increase strongly. In this case, labor is partially substituted by physical capital which, in turn, leads to increasing wages and decreasing interest rates. Furthermore, in the

[1] In that case, the working age-to-total population ratio would decrease to about 53 percent by the year 2050 using medium fertility prospects.

presence of capital-skill complementarity the abundance of physical capital benefits some workers more than others. Krusell, Ohanian, Ríos-Rull, and Violante (2000) show that the latter is key for the explanation of the trend rise in the skill premium[2] in the U.S. over a course of thirty years. Hence, the aging process gives rise to an increase in *intra*-generational inequality in addition to the often discussed shift in *inter*-generational inequality. The latter arises from changes in the overall wage level, the interest rate, and the generosity of pay-as-you-go (PAYG) financed pension systems over time. However, the evolution of the aggregate of labor services[3] depends heavily on individual household behavior. Adjustments of the latter with respect to labor market participation (along the intensive and the extensive margin) and labor productivity might counteract or even overturn the labor scarcity along with the aforementioned resulting effects in equilibrium.

Against that background, this paper investigates the impact of demographic change on the distributions of income, skills, and welfare in the German economy, along the inter- and the intra-generational dimension. Therefore, it builds on an overlapping generations structure in the tradition of Auerbach and Kotlikoff (1987) with households being heterogeneous in their (innate) ability for studying in college. The model accounts for capital-skill complementarity and several endogenous household choices. Households initially choose whether to receive tertiary education which splits them into *high-school* and *college* types thereby determining their degree of substitutability against capital in production. On a period-by-period basis, households decide on consumption as well as on the time they spend both, in the labor market and on skill formation on-the-job.

The major contribution of the paper is to show quantitatively the impact of demographic change on different skill groups and to reveal how households can react to the altered market conditions arising from demographic change. Furthermore, the paper highlights the relevance of past skill-biased technological change for the future dynamics of the income and skill distribution.

The quantitative experiments reveal the following effects of demographic change comparing year 2010 to year 2050: 1) The skill premium declines by about 15 percentage points while the college educated share in the workforce increases by about 3 percentage points. 2) The interest rate falls by 1 percentage point while the average wage increases by about 20% induced by a substitution of labor by capital and the aforementioned skill increase in the production. 3) The replacement rate falls massively by about 40 percentage points if the contribution rate to the public pension system is held fix.

[2]The authors define the *skill premium* as the ratio of wages paid to college workers to wages paid to non-college workers while this paper refers to the corresponding ratio of earnings.

[3]Throughout the paper, the terms "effective hours of labor supply", "effective labor supply", and "labor services" all refer to the productivity weighted hours of labor supply.

Welfare effects of demographic change[4] are substantial and vary between −3% and +2% of consumption in every period of lifetime depending on skill group and generation. All currently living generations lose. Despite the drop in the skill premium, demographic change benefits skilled over unskilled households. This is mainly due to the co-incident decline of the interest rate which makes borrowing for education less costly. While less able households benefit strongly from equilibrium effects arising from a higher college share in the workforce, more able households rather benefit from higher idiosyncratic human capital investments on the job.

As a secondary result, the quantitative experiments show that past skill-biased technological change will depress the future skill premium by additional 15 percentage points due to an ongoing increase in the relative supply of college workers. This causes strong welfare losses for college households of up to 8% of consumption in every period of lifetime. Note that the prediction of a strongly declining earnings premium and the associated welfare consequences could be turned around in case of ongoing skill-biased technological change in the future. However, this paper remains agnostic with respect to the direction of future technological change in the sense that all future change in technology is assumed to be skill-neutral.

After a brief literature review in the next section the theoretical model in use is described in section 3. Section 4 elaborates on the quantitative approach of the paper and the calibration of the model. Results from simulations are shown in section 5. Finally, section 6 concludes.

2 Relation to the Literature

This paper relates to three strands of the literature.

The first strand deals with the welfare consequences of demographic change. Here, the focus has been on the sustainability of PAYG financed public social security systems and the related inter-generational effects. This paper is closest in relation to the part of that literature which highlights the importance of changes in household behavior in response to the altered economic conditions arising from demographic change. Among those De Nardi, Imrohoroglu, and Sargent (1999) and Ludwig, Schelkle, and Vogel (2012) are in a closed economy setting. The former paper raises the problem of excess burden due to distortionary government policies which try to maintain past welfare levels. The latter finds that equilibrium effects on wages and interest rates which arise in aging economies induce higher incentives for human capital formation. The authors show that those higher human capital investments mitigate the effects of demographic change on

[4]Note that all welfare measures throughout the paper exclude welfare gains from increasing survival probabilities over time which are exogenous in this model.

macroeconomic aggregates and prices, and reduce welfare losses of middle aged agents substantially. The results complement similar findings with respect to endogenous labor supply by Börsch-Supan, Ludwig, and Winter (2006). Based on those findings, this paper adds to the literature on the welfare consequences of demographic change by accounting for the dimension of intra-cohort inequality and by investigating the role of tertiary education.

Based on the importance of interest rate dynamics for the welfare consequences of demographic change Börsch-Supan, Ludwig, and Winter (2006), Krüger and Ludwig (2007), and Attanasio, Kitao, and Violante (2007) extend the investigation to an open economy setting in which different regions of the world age at different paces. The authors show that capital flows evolve from more to less strongly aging regions of the world and evaluate the associated consequences for social security systems, a task that this paper leaves for future research.

Within a second related strand of the literature, researchers claim that reductions in mortality rates have positive incentives for education and human capital formation from a theoretical point of view. Among those are De La Croix and Licandro (1999), Kalemli-Ozcan, Ryder, and Weil (2000), Boucekkine, de la Croix, and Licandro (2002), Boucekkine, de la Croix, and Licandro (2003), Lagerlöf (2003), Soares (2005), and Cervellati and Sunde (2005). Moreover, Cervellati and Sunde (2013) disprove Hazan (2009)'s claim that a necessary condition for positive incentives to arise is an increase in lifetime labor supply. These theoretical findings are confirmed by an empirical literature which suggests a positive causal effect of higher life expectancy on educational attainment, cf., e.g., Bleakley (2007), Jayachandran and Lleras-Muney (2009), and Oster, Shoulson, and Dorsey (2013).

The third strand of the literature is concerned with the past evolution of the skill premium and the wage distribution. Dustmann, Ludsteck, and Schönberg (2009) as well as Fuchs-Schündeln, Krueger, and Sommer (2010) elaborate on recent empirical trends in Germany and are important sources for the calibration of the model in this paper. In a seminal paper, Katz and Murphy (1992) set up a simple supply and demand model of the labor market. They show that the model, together with some latent time trend in relative demand for skilled labor, is able to explain the evolution of the U.S. skill premium from 1963 to 1987. The results of Katz and Murphy (1992) have led many economists to search for the economic forces behind the measured time trend interpretable as latent skill-biased technological change. Among those, Krusell, Ohanian, Ríos-Rull, and Violante (2000) show that a negative time trend in the price of capital equipment relative to the price of capital structures can explain the overall rise in the U.S. skill premium between 1963 and 1992. The result is based on the complementarity between capital equipment and skilled labor. However, in both papers the relative supply of aggregate effective labor hours by skilled versus

unskilled households is key for the throughout explanation of the evolution of the skill premium. This is true in particular for the decline of the latter in the 1970s. While Krusell, Ohanian, Ríos-Rull, and Violante (2000) neglect the efficiency part and account only for the relative supply of labor hours, Heckman, Lochner, and Taber (1998) do the opposite. In fact, they set up a model with endogenous productivity along two margins. First, households decide on tertiary education at the beginning of the life cycle and, second, they choose the time that they spend on on-the-job skill formation on a period-by-period basis.

This paper adds to that strand of the literature by accounting for both, endogenous hours and productivity of labor supply, and by investigating their relative importance for the evolution of the skill premium in earnings. From a technical point of view the education decision is modeled based on Willis and Rosen (1979) and Keane and Wolpin (1997). The endogenous decisions on human capital and labor supply are in line with Becker (1967) and Ben-Porath (1967).

3 Model

3.1 Time and Demographics

Time is discrete and runs from $t = 0, 1, \ldots, \infty$. In every period, the economy is populated with $J + 1$ overlapping generations and the population structure[5] is time-dependent and exogenous. Households enter the economy at the age of $j = 0$, have the possibility to go to college[6] at ages $j = 0, ..., j_w - 1$, retire at the age of $j = j_r$, and live at most until turning $j = J + 1$ years. The population of age j in time period t is denoted by $N_{t,j}$ and the total population in time period t equals $N_t = \sum_{j=0}^{J} N_{t,j}$. Households face mortality risk represented by exogenous survival probabilities. $\varsigma_{t,j}$ is the probability of a household at age j and time t to survive until the next period.

3.2 Innate Ability and Endowments

A household enters the economically relevant time of life at age $j = 0$ being equipped with an idiosyncratic innate ability for tertiary education which is fully observable. The ability, indicated by superscript a, is represented exclusively by the amount of time per period, $\bar{i}$, which the household will have to spend on studying if it chooses to accomplish tertiary education. Technically speaking, upon entering the economy, a household draws from an ability distribution which

[5] I use the terms *demographic distribution* and *population structure* interchangeably throughout the paper indicating the distribution of the population by age.

[6] I use the terms *tertiary education*, *formal education*, *schooling*, and *college (C)* interchangeably throughout the paper.

is independent and identical across all newborn households in the course of time:

$$\bar{i} \overset{iid}{\sim} \mathcal{D}(\mu, \sigma^2) \tag{1}$$

where $\mathcal{D}$ is some distribution with mean μ and variance σ^2. Moreover, a newborn household is endowed with a positive initial level of human capital, $h^a_{t,0} = h_0 > 0$, but neither physical capital, $k^a_{t,0} = k_0 = 0$, nor claims to the public pension system, $b^a_{t,0} = b_0 = 0$, for all t. Note that all endowments are time-independent and identical across households.

3.3 Optimal Education and Subsequent Choices

At the beginning of life, a household faces the decision of accomplishing tertiary education or not. In the following, I will speak of college ($\mathcal{C}$) and high-school ($\mathcal{H}$) households respectively. Attending college implies a time investment as was described in section 3.2.[7] In return, the household accumulates human capital and joins the tertiary educated labor force upon graduation. Trading off costs against benefits leads to the optimal educational choice given by

$$\mathcal{S}^a_{t,0} = arg \max_{\mathcal{S} \in \{\mathcal{H}, \mathcal{C}\}} \left\{ v^{a,\mathcal{S}}_{t,0}(k_0, h_0, b_0) \right\} \tag{2}$$

where $v^{a,\mathcal{S}}_{t,0}(\cdot)$ is the lifetime utility of a household with ability a from schooling $\mathcal{S}$ in period t at age $j = 0$.

In each single period, a household chooses consumption, c, hours spent on on-the-job human capital development[8], i, and hours supplied to the labor market, l, based on an utilitarian preference function, $u(c, 1-i-l)$. $u(\cdot)$ fulfills standard assumptions, further specified in section 4.2, and features that a household gains utility from consumption and leisure, $1-i-l$. Please see section A.1 for a detailed derivation of the solution to the household problem.

3.4 Wealth Accumulation

Over the course of life, a household accumulates physical capital, human capital, and pension benefit entitlements as specified below.

The dynamic budget constraint is given by

$$c^a_{t,j} + k^a_{t+1,j+1} = e^a_{t,j} + (1 + r^K_t) \cdot k^a_{t,j} \tag{3}$$

[7] Note that tertiary education does not involve any pecuniary private costs other than foregone wages which comes close to the German university system.

[8] Note that currently enrolled college students are excluded from on-the-job human capital development by assumption such that their i equals $\bar{i}^a$ as described in section 3.2.

where

$$e^a_{t,j} := \begin{cases} (1-\tau_t) \cdot r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j} & if \quad j < j_r \\ \mathcal{P}_t(b^a_{t,j}) & else \end{cases}$$

are net earnings and pension benefits respectively. $r^{\mathcal{S}}$ is the return on labor services, $h \cdot l$, of an agent with education level $\mathcal{S}$. *Labor services* denote the human capital (productivity) weighted hours of labor supply. τ is the contribution rate to the pension system, and $\mathcal{P}(\cdot)$ is the pension benefit function. Note that the gross hourly wage of an agent with ability a and education level $\mathcal{S}$, which is the observable variable in the data, equals $r^{\mathcal{S}}_t \cdot h^a_{t,j}$.

Human capital represents the idiosyncratic labor productivity of a household. It accumulates according to the following law of motion:

$$h^a_{t+1,j+1} = \begin{cases} h_0 & if \quad j+1 < j_w \quad \wedge \quad \mathcal{S} = \mathcal{C} \\ (1+\bar{h}) \cdot h_0 & if \quad j+1 = j_w \quad \wedge \quad \mathcal{S} = \mathcal{C} \\ \varphi^{\mathcal{S}}(h^a_{t,j}, i^a_{t,j}) & else. \end{cases} \tag{4}$$

While studying in college, households remain at the initial human capital level, h_0. Upon graduation, they receive a fix markup, $\bar{h}$, on their human capital stock and join the tertiary educated labor force.[9] Households which are not currently enrolled in college are able to accumulate human capital by on-the-job time investments, i. The accumulation evolves according to the well-known production function $\varphi^{\mathcal{S}}(h, i)$ going back to Ben-Porath (1967). Note that $\varphi^{\mathcal{S}}(\cdot)$ differs by education type reflecting the difference in the evolution of labor productivity over the life cycle between educational groups observed in the data.[10]

Households collect benefit claims to the public pension system through their working life cycle:

$$b^a_{t+1,j+1} = \begin{cases} \vartheta_t(b^a_{t,j}, e^a_{t,j}) & if \quad j < j_r \\ b^a_{t,j} & else \end{cases} \tag{5}$$

where $\vartheta(\cdot)$ is an increasing function in both of its arguments which implies that new benefit claims are earnings related.

[9] The jump in the human capital profile upon graduation indicates the newly achieved possibility to apply for jobs which require a formal tertiary degree and is in line with the approach of Kindermann (2014).

[10] Note that limited data availability inhibits a further break down into ability classes among those groups as was already pointed out by Kindermann (2014, p. 14).

3.5 Production

Production takes place with a constant returns to scale production function which is based on Krusell, Ohanian, Ríos-Rull, and Violante (2000). It features that labor services of different education types are not perfect substitutes:

$$\begin{aligned}\mathcal{F}_t(K_t, L_t^{\mathcal{H}}, L_t^{\mathcal{C}}) =& \{\alpha_2 \cdot \left[\Upsilon_t \cdot L_t^{\mathcal{H}}\right]^{\rho_2} \\ &+ (1-\alpha_2) \cdot \left[\left(\alpha_1 \cdot (K_t)^{\rho_1} + (1-\alpha_1) \cdot (\Upsilon_t \cdot L_t^{\mathcal{C}})^{\rho_1}\right)^{\frac{1}{\rho_1}}\right]^{\rho_2}\}^{\frac{1}{\rho_2}} \quad (6)\end{aligned}$$

where $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$, $\rho_1 < 1$, and $\rho_2 < 1$. K is the aggregate stock of physical capital. $L^{\mathcal{C}}$ and $L^{\mathcal{H}}$ denote the aggregate inputs of labor services by college graduates and high-school households respectively. Υ_t denotes the labor augmenting technology which improves at the exogenous fix rate g, i.e., $\Upsilon_{t+1} = (1+g)\cdot\Upsilon_t$ for all t. Note that this represents skill-neutral technological progress in the economy and captures trend growth in output per capita.[11] $1/(1-\rho_1)$ is the elasticity of substitution between college labor, $L_t^{\mathcal{C}}$, and capital, K_t, while $1/(1-\rho_2)$ is the elasticity of substitution between college labor (or capital) and high school labor, $L_t^{\mathcal{H}}$, holding fix the relative price between college labor and capital. If ρ_1 is smaller than ρ_2 the economy features capital-skill complementarity (CSC).

Perfect competition among firms leads to standard first order conditions of the firm problem stating that prices equal marginal products minus depreciation:

$$r_t^K = \frac{\partial \mathcal{F}_t}{\partial K_t} - \delta^K, \; r_t^{\mathcal{H}} = \frac{\partial \mathcal{F}_t}{\partial L_t^{\mathcal{H}}}, \; r_t^{\mathcal{C}} = \frac{\partial \mathcal{F}_t}{\partial L_t^{\mathcal{C}}} \quad (7)$$

3.6 Government

The government plays a twofold role in this model.

First, it runs a pay-as-you-go (PAYG) pension system. In any period t, earnings of workers are taxed at the rate τ_t whereas households in retirement ($j \geq j_r$) receive a pension. The condition for a balanced budget of the PAYG system in every period is:

$$\tau_t \cdot \sum_{j=0}^{j_r-1} N_{t,j} \cdot \int r_t^{\mathcal{S}} \cdot h_{t,j}^a \cdot l_{t,j}^a \cdot \zeta(a)\, da = \sum_{j=j_r}^{J} N_{t,j} \cdot \int \mathcal{P}_t(b_{t,j}^a) \cdot \zeta(a)\, da \quad (8)$$

where $\zeta(a)$ denotes the fraction of the population with ability, a.

Second, the government taxes accidental bequests of physical capital by departed households at 100% and uses it for government consumption:

$$G_t = \sum_{j=1}^{J} (1 - \varsigma_{t-1,j-1}) \cdot N_{t-1,j-1} \cdot \int (1 + r_t^K) \cdot k_{t,j}^a \cdot \zeta(a)\, da \quad (9)$$

[11] Note that growth is only balanced in the model in the case of a stationary demographic distribution.

3.7 Equilibrium

Given the exogenous distributions of the population weights, $\{\{N_{t,j}\}_{j=0}^{J}\}_{t=0}^{T}$, and the survival rates, $\{\{\varsigma_{t,j}\}_{j=0}^{J}\}_{t=0}^{T}$, the exogenous time-constant ability distribution $\mathcal{D}$, as well as initial stocks of human capital, physical capital, and pension benefit entitlements, h_0, k_0, b_0, a competitive equilibrium consists of sequences of individual variables, $\{\{\{c^a_{t,j}, l^a_{t,j}, i^a_{t,j}, h^a_{t+1,j+1}, k^a_{t+1,j+1}, b^a_{t+1,j+1}\}_a\}_{j=0}^{J}\}_{t=0}^{T}$, sequences of aggregate variables, $\{L^{\mathcal{H}}_t, L^{\mathcal{C}}_t, K_{t+1}, Y_t, G_t, C_t, I^K_t\}_{t=0}^{T}$, government policies $\{\tau_t, \vartheta_t(\cdot), \mathcal{P}_t(\cdot)\}_{t=0}^{T}$, and prices, $\{r^{\mathcal{H}}_t, r^{\mathcal{C}}_t, r^K_t\}_{t=0}^{T}$, such that[12]

1. households behave optimally as according to equations (2) and (25–27),
2. firms behave optimally as according to the equations in (7),
3. factor markets clear:

$$K_{t+1} = \sum_{j=0}^{J} N_{t,j} \cdot \int k^a_{t+1,j+1} \cdot \zeta(a)\ da \tag{10}$$

$$L^{\mathcal{C}}_t = \sum_{j=0}^{j_r-1} N_{t,j} \cdot \int_{\underline{\bar{a}}} l^a_{t,j} \cdot h^a_{t,j} \cdot \zeta(a)\ da \tag{11}$$

$$L^{\mathcal{H}}_t = \sum_{j=0}^{j_r-1} N_{t,j} \cdot \int^{\bar{\bar{a}}} l^a_{t,j} \cdot h^a_{t,j} \cdot \zeta(a)\ da \tag{12}$$

   where $\underline{\bar{a}}$ indicates the marginal ability type being indifferent of going to college or not and $\zeta(a)$ is the probability density function of a,
4. the PAYG pension budget clears as according to (8),
5. accidental bequests finance government consumption as according to (9),
6. and the aggregate resource constraint holds:[13]

$$Y_t = C_t + I^K_t + G_t \tag{13}$$

4 Calibration and Numerical Solution

4.1 Solution Strategy

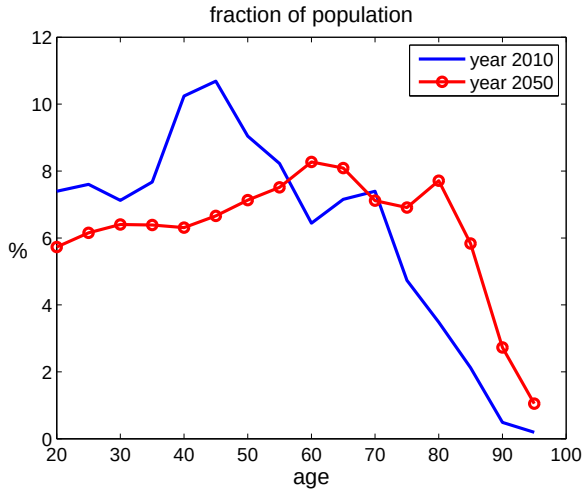
The solution of the model involves outer and inner loop iterations and follows the approach taken by Ludwig, Schelkle, and Vogel (2012). The outer loop solves for

[12] The presentation of the equilibrium definition follows Ludwig, Schelkle, and Vogel (2012).
[13] Please see section A.2 for a detailed derivation.

equilibrium by iterating on the aggregate capital stock, K, the aggregate college labor services, $L^{\mathcal{C}}$, the aggregate non-college labor services, $L^{\mathcal{H}}$, as well as the aggregate pension payments for all periods $t = 0, ..., T$. The inner loop solves for the household policy functions[14] in all periods $t = 0, ..., T$. In each outer loop iteration, household level variables are aggregated in order to update the aggregate stocks using a simple Gauss-Seidel algorithm as explained in Ludwig (2007).

The exogenous driving process[15] in the model is demographic change between 1950 and 2100 represented by both, a time-varying age structure of the population and increasing survival rates. Demographic data and projections are taken from United Nations (2013) under the medium fertility assumption.[16] Figure 2 shows the population age structure of the economy in the years 2010 and 2050. It reflects exemplarily the prediction of a strongly aging German adult population.

Figure 2: Demography: Data 2010 vs. Projections 2050



Source: Own calculations based on United Nations (2013), medium fertility prospects. *Notes:* The graphs show fractions of the adult population by 5-year age bins which begin with the indicated age. The adult population in the data corresponds to the total population in the model.

For computational reasons, the solution of the model begins in year 1750 ($t = 0$) in which an artificial initial steady state (in per efficient capita units)

[14] Please see section B.1 in the appendix for details.

[15] Note that there is an additional exogenous driving force in the model, skill-biased technological change between 1975 and 2010, which serves calibration purposes. Please see section 4.2 for details.

[16] For a detailed description of data and estimates I refer to the author.

with the demographic structure of year 1950 is assumed.[17] I then compute the transitional dynamics to an artificial final steady state in year 2500 ($t = T$) with the fix demographic structure of year 2100.[18] According to data availability the calibration period runs from 1975 to 2010. The main period of projection is 2010 to 2050.

4.2 Calibration

The calibration of the model follows a two-step procedure. In the first step, model parameters are set exogenously in line with empirical evidence and the literature. In the second step, model parameters are set in order to match key moments in the data.

The period length in the model is five years. Accordingly, all references to a specific year or a specific age throughout the paper stand for the respective 5-year period starting with the indicated year or age. Individuals are assumed to enter their economically relevant life at the age of 20 ($j = 0$), graduate potentially when turning 25 ($j_w = 1$), retire when turning 65 ($j_r = 9$), and die, the latest, when turning 100 ($J + 1 = 16$).

Preferences over consumption and leisure follow Ludwig and Abiry (2015):[19]

$$u(c^a_{t,j}, 1 - i^a_{t,j} - l^a_{t,j}) := \begin{cases} \ln(c^a_{t,j}) + \gamma \frac{1}{1-\upsilon} \left((1 - i^a_{t,j} - l^a_{t,j})^{1-1/\upsilon} - 1 \right) & if\ \theta = 1, \\ \frac{(c^a_{t,j})^{1-\theta}}{1-\theta} \cdot \left(1 + \gamma \frac{1-\theta}{1-\upsilon} \left((1 - i^a_{t,j} - l^a_{t,j})^{1-1/\upsilon} - 1 \right) \right)^{\theta} & else, \end{cases} \tag{14}$$

where I restrict υ, without loss of generality, to the empirically relevant case of being smaller than unity. γ denotes the utility weight of leisure which is calibrate in order to match the average number of labor hours in the data. $1/\theta$ is the elasticity of inter-temporal substitution. The λ-constant Frisch elasticity of labor supply equals $\upsilon \cdot (1 - i^a_{t,j} - l^a_{t,j})/l^a_{t,j}$ and, thus, varies along with leisure and labor over the life cycle.[20] Note that from this it follows that labor supply elasticities differ also by skill group. As college households tend to supply more labor and consume less leisure than non-college households during most of their working lifetime they exhibit smaller labor supply elasticities. This is in line with the empirical literature on labor supply elasticities (cf. Browning, Hansen, and

[17] The phase-in period with fix demographics until 1950 assures fully rational anticipation of changing market conditions arising from demographic change which takes place as of 1950.

[18] The phase-out period with fix demographics beyond 2100 assures that the transitional dynamics of the model, indeed, lead to the final steady state.

[19] Note that the considered preferences exhibit a jump at $\theta = 1$. This comes from the need of a homothetic utility function for computing consumption equivalent variation as in section 5. Therefore, I primarily consider the case $\theta > 1$ in the quantitative experiments.

[20] Please see Ludwig and Abiry (2015) and Ludwig, Schelkle, and Vogel (2012) for a discussion on implications arising from the life cycle variation of the Frisch elasticity.

Heckman (1999)) without imposing preference heterogeneity across skill groups. υ is set to 0.21 thereby pinning down the average (hours weighted) elasticity in the model which equals 0.475 in year 2010. This is in line with the micro-evidence on labor supply elasticities (cf. Domeij and Flodén (2006) for models without borrowing constraints).

On-the-job skill formation follows the well-known Ben-Porath (1967) human capital production function

$$\varphi^{\mathcal{S}}(h^a_{t,j}, i^a_{t,j}) := (1-\delta^h)\cdot h^a_{t,j} + \varpi^{\mathcal{S}}\cdot (h^a_{t,j}\cdot i^a_{t,j})^{\varrho}. \tag{15}$$

$0 < \varrho < 1$ governs the complementarity between old human capital and time investments in the accumulation of new human capital and δ^h is the depreciation rate of human capital. $\varpi^{\mathcal{S}}$ is calibrated in order to match the earnings increase over the life cycle in the data and differs accordingly by education type.

Pension entitlements accumulate according to a purely earnings related scheme which comes close to the actual German public pension system:

$$\vartheta_t(b^a_{t,j}, r^{\mathcal{S}}_t \cdot h^a_{t,j}\cdot l^a_{t,j}) := b^a_{t,j} + r^{\mathcal{S}}_t \cdot h^a_{t,j}\cdot l^a_{t,j}/\bar{e}_t \tag{16}$$

where $\bar{e}_t := (r^{\mathcal{H}}_t \cdot L^{\mathcal{H}}_t + r^{\mathcal{C}}_t \cdot L^{\mathcal{C}}_t)/(\sum_{j=0}^{j_r-1} N_{t,j})$ are average earnings in period t. The contribution rate of the pension system is set exogenously as according to data from DRV (2014, p. 262) and held constant in future periods.[21] Pension benefits

$$\mathcal{P}_t(b^a_{t,j}) := \nu_t \cdot b^a_{t,j} \tag{17}$$

are payed proportional to the accumulated benefit stock. ν_t is the so-called *actual pension amount* payed to the household in period t for each point of accumulated benefit entitlements. It adjusts in every period so that the pension budget clears. Note that ν_t grows over time along with wages due to exogenous (skill-neutral) technological progress in Υ_t. This implies that the growth of the pension payment over the retirement spell of a household keeps track with wage growth which is (broadly) consistent with the German public pension scheme.

Time spent on studying in college (ability), $\bar{i}^a$, is assumed to be uniformly distributed. That distribution is then approximated by ten different ability groups of equal size[22] where

$$\bar{i}^a := \mu + \sigma\cdot[1, \frac{7}{9}, \frac{5}{9}, \frac{3}{9}, \frac{1}{9}, -\frac{1}{9}, -\frac{3}{9}, -\frac{5}{9}, -\frac{7}{9}, -1].$$

21

	−1965	1970	1975	1980	1985	1990	1995	2000	2005	2010−
τ	0.140	0.170	0.180	0.182	0.189	0.182	0.198	0.193	0.197	0.194

[22] Note that this does not hold necessarily for the two marginal ability groups around the cut-off value, $\underline{a}$. Their size is adjusted using interpolation techniques such that the tertiary educated share of the population observed in the data can be matched precisely. Hence, the selected number of groups matters for computational accuracy. Ten clusters showed to be sufficient as the use of 18 different ability types had only a minor impact on results.

μ determines the average time per period spent in college and is calibrated such that the college educated share of the 25+ workforce in the model matches its counterpart in the data. σ governs the standard deviation of time per period spent in college and is set to 0.32 such that the model leads to empirically reasonable elasticities of the college decision as will be shown next. Therefore, I compute the percentage change of the college educated share to 1) a yearly college grant of US$1000 and 2) a 1% increase in the skill premium. The change under 1) turns out to be 0.43 in 2010 which is in line with the micro elasticity for Germany estimated in Steiner and Wrohlich (2011)[23] and literature cited therein. Results from the quasi-experimental literature for the U.S. find higher numbers for an increase in college enrollment in the magnitude of 3 to 5 percentage points (c.f., e.g., Kane (2006), Deming and Dynarski (2009)). However, the key driver for those high estimates are borrowing constraints which are absent in this model. Johnson (2013) and Findeisen and Sachs (2014) find a 2.4 and 4.2 percentage point increase in the college share, respectively, when abolishing borrowing constraints. Under 2), the experiment results in an increase of 0.60 percentage points in 2010 which is slightly higher than what is found in the literature (cf., e.g., Fredriksson (1997) for Sweden). Section 5.1 shows that the model is able to match the past evolution of the college workforce share in the data. $\bar{i}$ turns out to lie in the interval $[0.34, 0.98]$.

Along with Heckman, Lochner, and Taber (1998), I assume exogenous skill-biased technological change in the calibration period, 1975 to 2010. It is imposed by a downward trend in the production function parameter α_2[24] and induces an upward trend in the skill return premium $r_t^{\mathcal{C}}/r_t^{\mathcal{H}}$. I set the constant yearly increase in $(1-\alpha_2)/\alpha_2$ in the period $1975-2010$ to 2%. While Heckman, Lochner, and Taber (1998) choose a value of 3.6% which is in line with the literature on the past evolution of the skill premium in the U.S. (cf., e.g., Katz and Murphy (1992)), the results in Dustmann, Ludsteck, and Schönberg (2009, p. 852) suggest a smaller value for Germany of about 60% of the U.S. value which comes close to the considered 2%. Section 5.1 shows that the model is able to trace jointly the past evolution of the skill premium and the college workforce share in the data.

[23] Steiner and Wrohlich (2011) estimate that the share of high-school graduates enrolling for college increases by 1.5 percentage points in response to a yearly college grant of €1000 while Autorengruppe Bildungsberichterstattung (2012) reports a high-school graduate share of 41% in the $30-35$ year old population.

[24] Note that this implies an upward drift in the capital income share and the capital-output ratio during the calibration period. The upward drift is consistent with the findings of Piketty and Zucman (2014) for Germany who recently challenged the predominant view in the literature of the constancy of the aforementioned capital measures with a new data set.

Table 1: First and Second Stage Parameters

Parameter	Value	Target	Source, Comment
Firm sector			
Skill-neutral techn. progress: g	0.01	1^{st} stage	OECD (2014b): $g^{Y/N}$
Skill-biased techn. change: $g_{1975-2010}^{(1-\alpha_2)/\alpha_2}$	0.02	1^{st} stage	DLS, period: $1975-2010$
ES($L^{\mathcal{H}}, L^{\mathcal{C}}$)=ES($L^{\mathcal{H}}, K$): $1/(1-\rho_2)$	1.59	1^{st} stage	DPP
ES($K, L^{\mathcal{C}}$): $1/(1-\rho_1)$	0.64	1^{st} stage	DPP
Weight on L^H: α_2	0.34	$\bar{e}^{\mathcal{C}}/\bar{e}^{\mathcal{H}} = 1.71$	OECD (2014a)
Weight on K: α_1	0.51	$\frac{(r^{\mathcal{H}}L^{\mathcal{H}}+r^{\mathcal{C}}L^{\mathcal{C}})}{Y} = 0.65$	literature
Depreciation rate of K: δ^K	0.059	$I^K/Y = 0.18$	OECD (2014b)
Preferences			
EIS, $1/\theta$	0.5	1^{st} stage	literature, LSV
Leisure elasticity: υ	0.21	1^{st} stage	labor supply elast., see text
Time discount factor: β	0.999	$K/Y = 2.9$	literature
Weight of leisure: γ	0.34	aver. $l = 0.285$	OECD (2014b)
Endowment and ability			
Endowment: $\{h_0, k_0, b_0\}$	$\{1.0, 0.0, 0.0\}$	1^{st} stage	normalization
Std. time effort college: σ	0.32	1^{st} stage	$\mathcal{C}$ share elast., see text
Human capital accumulation			
Depreciation rate of h: δ^h	0.008	1^{st} stage	LSV
On-the-job h accum.: ϱ	0.65	1^{st} stage	BHH
Mean time effort college: μ	0.66	$\mathcal{C}$ share $= 0.27$	OECD (2014b)
h markup from college: $\bar{h}$	0.58	$r^{\mathcal{C}} = r^{\mathcal{H}}$	normalization
On-the-job h accum. $\mathcal{H}$: $\varpi^{\mathcal{H}}$	0.75	$\bar{e}^{\mathcal{H}}_{55-60}/\bar{e}^{\mathcal{H}}_{25} = 1.35$	OECD (2014a)
On-the-job h accum. $\mathcal{C}$: $\varpi^{\mathcal{C}}$	0.84	$\bar{e}^{\mathcal{C}}_{55-60}/\bar{e}^{\mathcal{C}}_{25} = 1.78$	OECD (2014a)
Pension system			
Contribution rate: τ	see text	1^{st} stage	German public pension sys.

Notes: The indicated values are converted to annualized rates where applicable and refer to year 2010 if not stated differently. ES($L^{\mathcal{H}}, L^{\mathcal{C}}$) $\hat{=}$ elasticity of substitution between high school and college labor holding the relative price of college labor to capital fix. ES($K, L^{\mathcal{C}}$) $\hat{=}$ elasticity of substitution between college labor and physical capital. EIS $\hat{=}$ elasticity of inter-temporal substitution. BHH $\hat{=}$ Browning, Hansen, and Heckman (1999). DLS $\hat{=}$ Dustmann, Ludsteck, and Schönberg (2009). LSV $\hat{=}$ Ludwig, Schelkle, and Vogel (2012). DPP $\hat{=}$ Duffy, Papageorgiou, and Perez-Sebastian (2004). The average labor supply in the data refers to the number of hours per worker (incl. part time) divided by 5000.

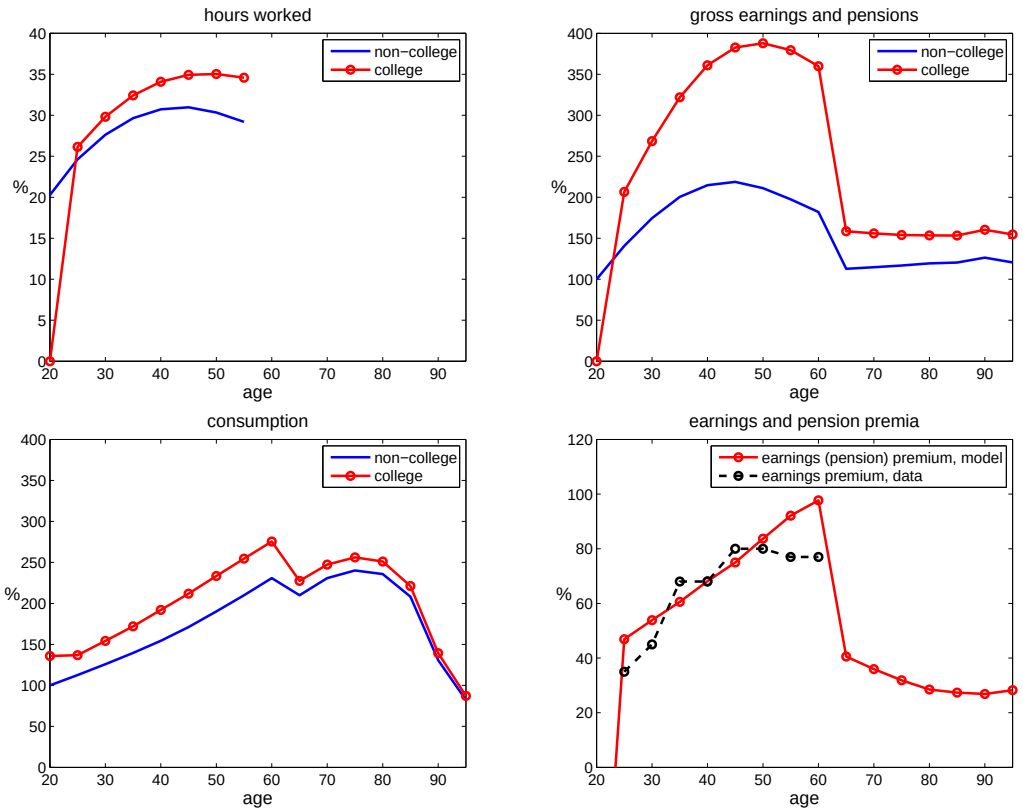
Table 1 summarizes first and second stage parameters. Note that the elasticity of substitution between physical capital and high-school labor services is higher than the one between physical capital and college labor services which implies capital-skill complementarity. This is in line with empirical estimates and the selected values lie in the range considered in the literature (cf., e.g., Duffy, Papageorgiou, and Perez-Sebastian (2004), Krusell, Ohanian, Ríos-Rull, and Violante (2000), and Heckman, Lochner, and Taber (1998)).

5 Results

5.1 Cross-Sectional Profiles in 2010 and the Past Trend in the Skill Premium

Figure 3 shows resulting cross-sectional age profiles of the model economy in year 2010 by educational group. College households choose not to work besides study-

Figure 3: Cross-Sectional Age Profiles By Education Type in 2010

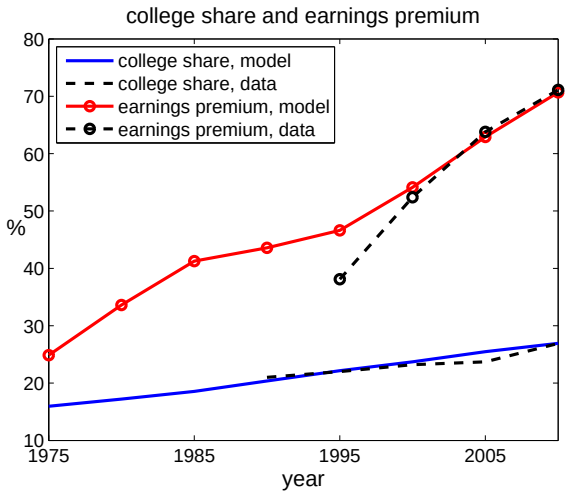


Source: Baseline model in year 2010: Selected average cross-sectional age profiles by education type. Earnings premium data is taken from OECD (2014a).

ing and benefit from the markup in their human capital stock upon graduation which leads to a jump in their earnings profile (top right panel). Furthermore, college graduates are more efficient in developing human capital on-the-job. Together with higher relative working hours at old ages that leads to a steeper age-earnings profile of college households compared to high-school households, or, equivalently, to an increase in the earnings premium over the working life cycle (bottom right panel). Both is consistent with empirical evidence from OECD (2014a) and the German Socio-Economic Panel (cf. Kindermann (2014)). Note that the pension premium is at a lower level than the earnings premium albeit the absence of a re-distributive mechanism in the pension benefit function. This comes from the strong rise in the earnings premium in the past (cf. figure 4) and shows that the pension premium serves as a lagged indicator for the earnings premium. Consumption (bottom left panel) and hours worked (top left panel) show the typical hump-shaped pattern with college households supplying more labor than high-school households. The closer to retirement, the larger is that difference. Note that the peak in the consumption profile is too late compared to the data which is a common problem in deterministic life cycle models.

Figure 4 reflects the good fit of the model to the data with respect to the evolution of the college share in the 25+ workforce and the average earnings premium of college graduates. The rise in the two variables is a result of both,

Figure 4: Past Evolution of College Share and Earnings Premium



Source: Baseline model vs. data in the calibration period, 1975 – 2010. Data on the evolution of the college share in the 25+ workforce is taken from Statistisches Bundesamt (2014) and scaled to the target value of the college share in 2010 taken from OECD (2014b), cf. table 1. Earnings premium data comes from OECD (2014a).

demographic change and a past trend in the direction of technological progress as

observed in the data (cf. section 4.2). Note that the model slightly underestimates the increase in the earnings premium between 1995 and 2000. That is likely due to other factors being absent in the model which had affected the earnings premium at that time. For example, Dustmann, Ludsteck, and Schönberg (2009, pp. 859ff.) estimate that the decrease in the unionization rate of the labor force in the considered period can account for 28% of the increase in the 50-15 earnings gap while it is less important at the upper end of the earnings distribution.

5.2 Aggregate Dynamics Beyond 2010

Let us now turn to the future dynamics in the considered model economy. Figure 5 shows the evolution of key aggregate measures in the main period of projection, i.e., year 2010 to year 2050.

The top-left panel depicts the evolution of the working age-to-population ratio (WAPR). This is the exogenous driving force in future periods of the model and a key measure for the demographic structure of the economy. It reflects the strong aging process which is expected to hit the German economy.

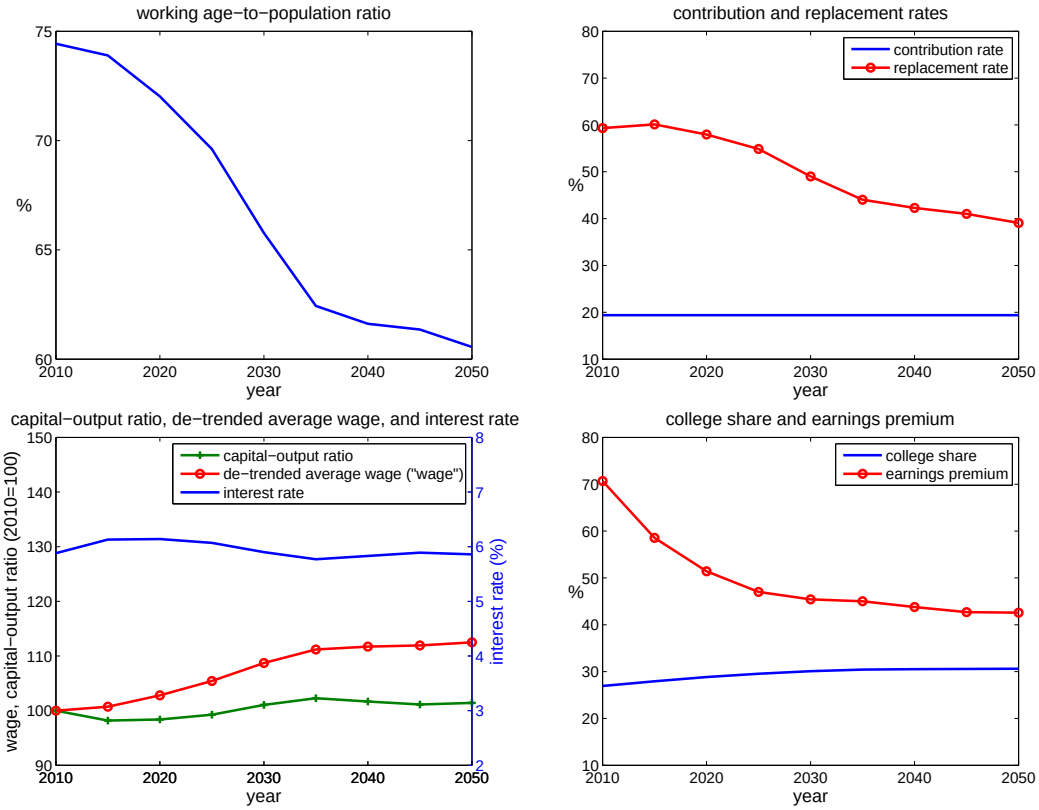
The top-right panel depicts the contribution and replacement rates of the public pension system and confirms the expected decline in the generosity of the pension system. Note that this scenario assumes that the government will hold the contribution rate fix in all future periods which results in a strong decline in the replacement rate of about 20 percentage points by 2050. This is within the range of estimates for pay-as-you-go financed pension systems like the German one with defined contributions.

The bottom left panel shows an increase in the capital-output ratio with the associated increase in the de-trended average gross wage[25] and the decline in the interest rate. It reveals the substitution of the scarce production factor, labor, by the one in abundance, capital, and is in line with the results of Ludwig, Schelkle, and Vogel (2012) for the US. However, while the rise in the capital-output ratio and the decline in the interest rate remain rather small the rise in the wage is more pronounced and equals 12.5% by 2050. This points to a second substitution effect taking place in the aging economy: Labor input of non-college households is substituted by labor input of college households. The re-composition of the labor force elevates the average wage because the latter households are more productive than the former and thus earn a higher wage.

The bottom right panel confirms the re-composition of the labor force by showing a rise in the college share of about 4 percentage points by 2050. Furthermore, it reveals that the rise in the college share is associated with a strong decline in the average earnings premium of college households. These developments are mainly driven by two effects.

[25]The average gross wage follows a trend in time arising from exogenous skill-neutral technological change in Υ_t. The trend is removed for the sake of meaningful comparison between different time periods.

Figure 5: Aggregate Dynamics Beyond 2010



Source: Baseline model in the main period of projection, 2010 to 2050: Selected aggregate measures. *Notes:* The replacement rate in the top right panel refers to the average ratio of the pension payment to a 65 year old household to the sum of net earnings (adjusted by wage inflation) over its working life. The average gross hourly wage in the bottom left panel is shown net of the trend growth arising from exogenous skill-neutral technological change in Υ_t over time. Capital-output ratio and de-trended average wage are normalized to 100 in 2010. The bottom right panel shows the average earnings premium of college graduates and the share of college graduates in the 25+ workforce in percent.

The first effect arises from the scarcity of labor as an input in the production which follows ceteris paribus from the process of aging beyond 2010. Its scarcity makes labor relative to capital ceteris paribus more expensive and leads to a substitution of labor by capital. Due to the complementarity of capital and college labor input in the production the substitution of labor by capital is associated with a rise in the relative demand for college workers compared to non-college workers. This elevates ceteris paribus the earnings premium. However, in anticipation of this and other effects described below more households choose to go to college which, in turn, induces an increase in the relative supply of college labor and lowers ceteris paribus the earnings premium.

The second effect stems from skill-biased technological change in the calibration period due to which the share of newborn households selecting college has risen strongly in the period until 2010 (cf. figure 4). From this it follows that less educated retiring generations of workers are substituted gradually by better educated new generations of workers also beyond 2010. This implies a higher relative supply of college workers which lowers ceteris paribus the earnings premium.

As mentioned above, the bottom right panel shows a strong and steady overall decline in the earnings premium beyond 2010 and, thereby, reveals that the supply side effects overcompensate clearly the demand side effect. The decrease in the earnings premium occurs most strongly in the period 2010 to 2030 amounting to more than 20 percentage points. This is when the aforementioned second effect is at its peak. Note that it becomes optimal to pursue a college degree for a higher fraction of households despite the decline in the earnings premium. Reasons for that are the co-incident decline in the interest rate which makes borrowing for education less costly (and savings less attractive) as well as the increase in life expectancy. The latter prolongs the expected return-on-investment period of education (inter alia via an earnings related pension system) and induces a need for higher lifetime earnings. Education still shows to be the most efficient way of achieving it.

As an indicator for the development of overall economic inequality in the economy table 2 displays the change in the Gini coefficients of net total income ("income Gini") and consumption ("consumption Gini") from 2010 to 2050.[26] First, consider the top row. The number in the top left corner suggests that overall economic inequality rises strongly in the period 2010 to 2050 as the income

[26] The model clearly underestimates the level of income inequality in the economy. However, note that income inequality in the data is driven by both, variation in initial conditions and differences in shocks over the course of life whereof the latter are absent in this model. Huggett, Ventura, and Yaron (2011) estimate for the U.S. that about two third of lifetime inequality stems from variation in initial conditions. Assuming ad-hoc the same decomposition for Germany allows to compute the Gini of net total income which corresponds to the model. Using data from OECD (2014b) it amounts to $0.286 \cdot 2/3 = 0.191$ which is very close to the actual value in the model in 2010, 0.194.

Table 2: Measures of Economic Inequality: The Change in Ginis from 2010 to 2050

	net total income	consumption
$Gini(2050) - Gini(2010)$, baseline	+6.1	−3.4
$Gini(2050) - Gini(2010)$, fix demographics	−7.6	−0.2

Source: Baseline model in 2010 and 2050. *Notes:* The numbers show percentage point changes. The top row displays the total change in Gini coefficients while the bottom row shows the corresponding values if the Gini coefficients in 2050 are recomputed using the age distribution of the population in year 2010. Net total income is the sum of capital income and net earnings, respectively pension income of a household.

Gini increases by 6.1 percentage points. However, the consumption Gini shows a decline of 3.4 percentage points over the same period indicating the opposite. Note that the change in a Gini coefficient can be induced by a change in the income (respectively consumption) distribution given the same age distribution of the population or by a change in the age distribution of the population itself. Here, both effects apply at the same time.

In order to isolate the first effect the bottom row of table 2 shows the change in Gini coefficients if the Gini coefficients in 2050 are recomputed using the age distribution of the population in year 2010. The number in the bottom left corner now indicates a strong drop in the income Gini. Recalling the dynamics displayed in figure 5 suggests that this stems primarily from receding income inequality between ability groups represented by the drop in the earnings premium. Hence, the overall increase in the income Gini (top row) stems primarily from a change in the population distribution, more precisely, a higher fraction of income poor (but asset rich) old households. Now, consider the number in the bottom right corner. It shows a rather stable consumption Gini which indicates rather stable lifetime income prospects of a household between 2010 and 2050 because the consumption decision depends on lifetime income rather than current temporary income. Again, the overall decrease in the consumption Gini (top row) stems primarily from a change in the population distribution, more precisely, a smaller fraction of consumption poor young households.

5.3 Welfare Effects Within and Across Generations

I now turn to the welfare effects arising from the dynamics described in section 5.2. Following the literature I measure welfare effects by consumption equivalent variation (cf. Ludwig, Schelkle, and Vogel (2012) and the literature cited therein). A household's welfare is affected in two ways. The first effect arises from changes in survival probabilities which are exogenous in this model. The second effect

stems from changing good and time allocations induced by changes in wages, interest rates and average pension amounts.

I want to isolate the second effect and, therefore, conduct the following auxiliary computation using the same (time- and age-dependent) survival probabilities throughout the entire welfare calculation. I compute a partial equilibrium version of the model given a particular exogenous vector of wages, interest rates, and average pension amounts for the entire transition which I will call *price vector* in the following. The price vector equals the corresponding vector from the equilibrium path of the baseline case up until year 2010 and keeps prices fixed from then onward. This represents an auxiliary world in which the future price changes arising from demographic and past skill-biased technological change do not evolve.[27] Welfare effects are then measured as the percentage change in consumption in every period of lifetime that a household must be compensated with in order to be indifferent between the auxiliary world and the baseline case.[28]

In this model, total future price changes and their welfare effects are induced by the combination of demographic and past skill-biased technological change. In order to decompose total welfare effects I redo the entire welfare calculation described above under the assumption that no skill-biased technological change has taken place up until 2010. Note that, except for the mentioned change, parameters of the model are not re-calibrated in order to have a reasonable case of comparison.[29] Figure 6 presents resulting average welfare effects by educational class. The top row shows welfare effects which arise purely from demographic change while the bottom row displays the effects from the combination of demographic and past skill-biased technological change. The left panels depict welfare effects on all generations alive in year 2010 while the right panels display welfare effects on generations born[30] in years 2010 to 2050.

Consider first the panels in the top row of figure 6 showing the average welfare effects from demographic change. Section B.3 in the appendix presents the associated future dynamics of key aggregate measures in detail. The central findings can be summarized as follows: While 1) the replacement rate, 2) the interest rate, and 3) the earnings premium decline gradually over time, 4) the average wage level increases. Note that while effect 3) obviously benefits high-school households, effect 2) rather benefits college households because their advantage from

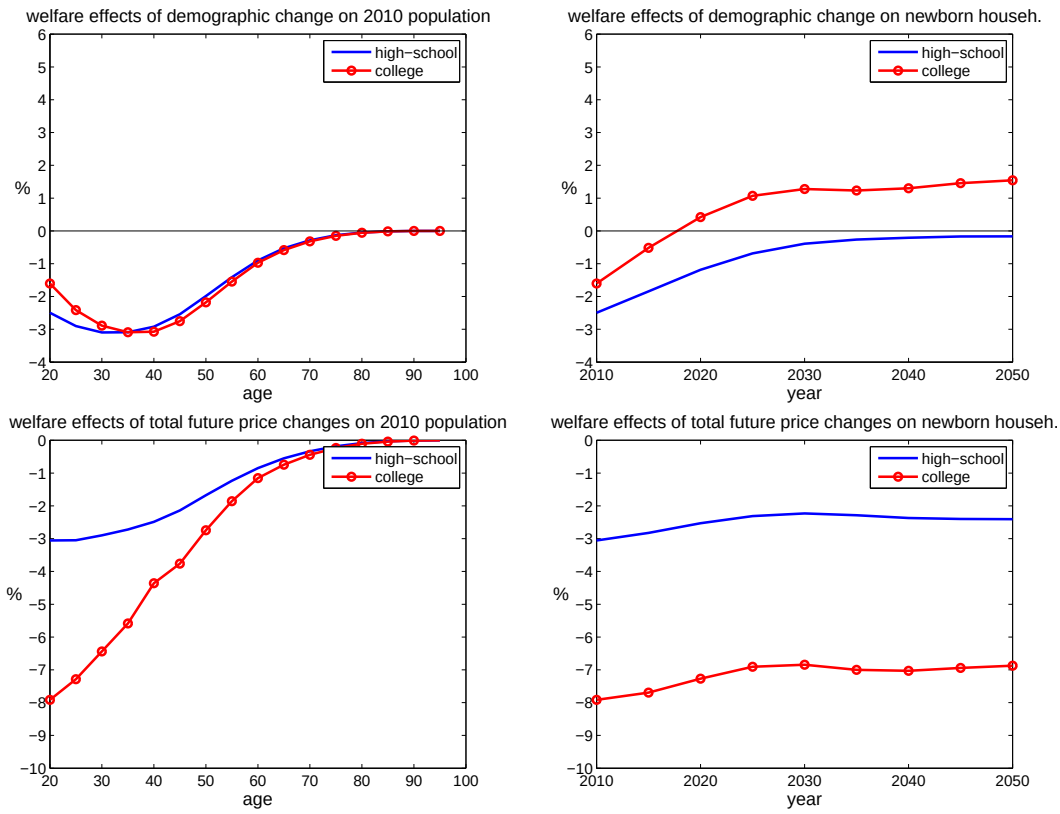
[27] Note that household choices are fully rational given the exogenous price vector while the procedure does not involve the general equilibrium as described in section 3.7.

[28] Based on the homotheticity of the value function, consumption equivalent variation can be measured as $cev_t := (\frac{v_{t,0}}{v^A_{t,0}})^{\frac{1}{1-\theta}} - 1$, where $v_{t,0}$ and $v^A_{t,0}$ are the lifetime values of the generation born in period t in the baseline case and the auxiliary world respectively.

[29] I.e., the parameter values of table 1 still apply except for $g^{(1-\alpha_2)/\alpha_2}_{1975-2010}$ which equals 0.

[30] *Birth* refers to the creation of the household which occurs when the household head turns 20.

Figure 6: Welfare Effects of Demographic Change (Top Panels) and Total Future Price Changes (Bottom Panels) on Current and Future Populations



Source: Baseline model in the main period of projection, 2010 to 2050. *Notes:* Welfare effects measured as consumption equivalent variation. Negative values indicate welfare losses. Top row: Welfare effects from demographic change. Bottom row: Welfare effects from future price effects induced by both, demographic and past skill-biased technological change. Left panels: Welfare effects on all generations alive in year 2010. Right panels: Welfare effects on generations born in years 2010 to 2050.

smaller borrowing costs during the college years shows to dominate the loss from smaller capital income at wealthy old ages. Developments 1) and 4) affect college and high-school households similarly.

The top left panel shows the net welfare effects on all generations alive in 2010 and features an u-shaped pattern for both education types. This was already found by Ludwig, Schelkle, and Vogel (2012) in an overlapping generations model without differentiation by education. The top right panel displays welfare effects of newborn generations in years 2010 to 2050 and shows that welfare losses from demographic change on newborn households vanish in the course of time and turn even positive for both educational groups at some point in the future. Taken both panels together, one can make the following two major observations: First, the later in time a household is born, the less it suffers from demographic change irrespective of education. This is due to the developments described above, in particular, rising wages. However, that first observation does not apply to households which are more than about 35 years old in 2010. The reason is that they benefit less from rising wages due to a shorter expected remaining working life and from decreasing interest rates for financing education whereof the latter only applies to college households. Furthermore, welfare effects run off with increasing age per definition because the household's expected remaining lifetime decreases. As a net effect, welfare losses are severest for the generation aged $30 - 40$ in 2010 and amount to about 3% of consumption in every period of lifetime. This holds for college as well as for high-school households. Second, the later in time a household is born, the more it benefits from going to college. This shows that effect 2) of dropping interest rates becomes more and more important over time while the opposite holds true for effect 3), the decline in the earnings premium.

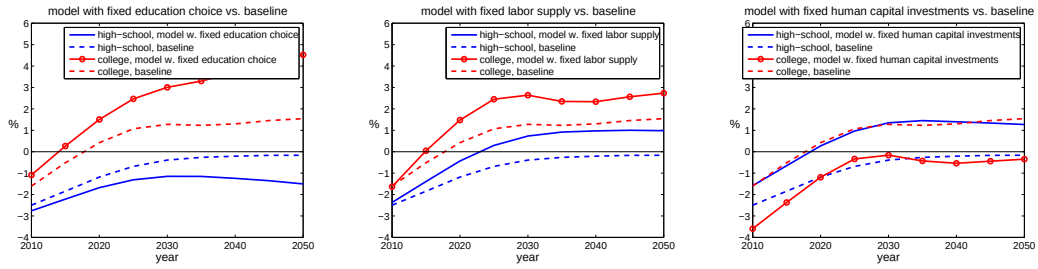
Let us now move on to the bottom panels. Here, welfare effects arise from total future price changes which are induced by the combination of demographic change and past skill-biased technological change. Comparing the graphs in the bottom panels to the respective graphs in the top panels reveals the central welfare effect arising from past skill-biased technological change: A strong negative effect for college households. This holds for all considered generations. Why is this the case? Note that the baseline economy matches the past evolution of the college share in the workforce which is characterized by a stronger upward trend (cf. figure 4) than in the case of demographic change only. From this it follows that, the increase in the college share in the future, in particular until 2040, is also more severe for the baseline economy. This is due to a larger difference between the college shares of newborn households and substituted retiring households in that period. The stronger future increase of the college share has two consequences. First, the decline in the earnings premium is larger in the baseline economy, and, second, the decline in the interest rate is smaller. Both induces negative welfare effects on college households.

5.4 The Role of Changes in Household Behavior

This section investigates the quantitative importance of changes in household behavior in response to altering market conditions arising from demographic change[31] along three dimensions: tertiary education, working hours, and human capital investments. Correspondingly, I run three experiments, each of them recomputes the general equilibrium path of the model economy and evaluates the associated welfare effects (following the approach described in section 5.3) under exactly one restriction: From period 2010 onward, the choice of tertiary education, working hours, or human capital investments respectively is restricted as specified in more detail below.[32]

Figure 7 shows the welfare effects of demographic change on future generations in the respective model variant by education. As implications are similar for

Figure 7: The Role of Changes in Household Behavior



Source: Variants of the model with restricted household choices (solid lines) vs. baseline model (dashed lines, cf. figure 6): Welfare effects of demographic change of newborn generations in the main period of projection, 2010 to 2050. *Notes:* Welfare effects measured as consumption equivalent variation. Negative values indicate welfare losses of demographic change.

current generations they are omitted for the sake of brevity, here.

The left panel depicts the welfare effects for the model variant in which the share of newborn households selecting tertiary education is held fix artificially from 2010 onward. This dampens the supply side effect on the labor market which was described in detail in section 5.2. It leads to a smaller drop in the earnings premium with positive (negative) welfare implications for college (high-school) households. Note that welfare advantages of college households from restricting the college choice become larger in time because the aforementioned

[31] Here, I focus on the case without past skill-biased technological change in order to answer more precisely the question how well households can react to challenges from demographic change. However, at least qualitatively, results are similar to the baseline case.

[32] Note that each experiment assures the general equilibrium under an additional constraint in the household problem.

dampening of the supply side effect applies to every period past 2010 and thus accumulates.

The panel in the center shows welfare effects of demographic change if hours worked are held fix artificially from 2010 onward. This implies that the age-profiles of hours worked by ability group remain time-constant as of 2010. The graphs show that the increase of working hours is an important adjustment channel for households to the altered market conditions arising from demographic change. In particular, it enables households to benefit more strongly from rising wages (per hour). Note that the restriction of adjustments along the working hours margin has similar welfare costs for both education types.

The right panel contains the corresponding graphs for the model variant in which the hours spent on human capital formation are held fix artificially from 2010 onward. This implies that the age-profile of human capital by ability group remains constant as of 2010. College households exhibit a higher productivity in human capital formation on-the-job than high-school households. The picture shows clearly that this turns into a true asset in the demographic transition. It enables college households to benefit from higher returns to human capital investments at a lower cost than high-school households.

To sum up, upward adjustments of hours worked affect all households similarly, increased human capital investments rather benefit college households, and the opposite of the latter is induced by more tertiary education. While increased human capital investments directly elevate the idiosyncratic productivity and in turn the idiosyncratic wage of a household, the increase in the tertiary educated share in the work force depresses the market-wide return premium to college labor input. Hence, the latter acts through an equilibrium effect.

6 Concluding Remarks

This paper investigates the effects of demographic change on the distributions of income, skills, and welfare in the German economy. The dynamic overlapping generations model features heterogeneity along both, the inter- and the intra-generational dimension whereof the latter is given by an idiosyncratic innate ability for tertiary education. Besides consumption, the model features three choices which can help households to overcome the altered economic conditions arising from demographic change: tertiary education, on-the-job skill formation, and hours worked.

The quantitative experiments reveal the following effects of demographic change comparing year 2010 to year 2050: 1) The skill premium declines by about 15 percentage points while the college educated share of the workforce increases by about 3 percentage points. 2) The interest rate falls by 1 percentage point while

the average wage increases by about 20% induced by a substitution of labor by capital and the aforementioned skill increase in the production. 3) The replacement rate falls massively by about 40 percentage points if the contribution rate to the public pension system is held fix.

Welfare effects of demographic change are substantial and vary between -3% and $+2\%$ of consumption in every period of lifetime depending on skill group and generation. All currently living generations lose. Despite the drop in the skill premium, demographic change benefits skilled over unskilled households. This is mainly due to the co-incident decline of the interest rate which makes borrowing for education less costly. While less able households benefit strongly from equilibrium effects arising from a higher college share in the workforce, more able households benefit rather from higher idiosyncratic human capital investments on the job.

As a secondary result, the quantitative experiments show that past skill-biased technological change will depress strongly the future skill premium by additional 15 percentage points due to an ongoing increase in the relative supply of college workers. This causes strong welfare losses for college households of up to 8% of consumption in every period of lifetime. Note that the prediction of a strongly declining earnings premium and the associated welfare consequences could be turned around in case of ongoing skill-biased technological change in the future. However, this paper remains agnostic with respect to the direction of future technological change in the sense that all future change in technology is assumed to be skill-neutral.

The investigation shows that it is important to consider the skill-composition of aggregate labor input in the context of aging economies and thereby extents the results of Ludwig, Schelkle, and Vogel (2012) by heterogeneity in formal skills. There are a couple of interesting extensions that future research may approach. First, in this model, a household trades off opportunity costs against pecuniary benefits when deciding on tertiary education. The latter does not involve any direct pecuniary costs, neither at the private nor at the public level. In reality, the German university system is mostly financed by public means. Aging economies are likely to fall short of financial means as the tax base erodes as Keuschnigg, Davoine, and Schuster (2013) argue. A link of tax and educational system would thus be an interesting extension. Furthermore, Winter (2014) states that both, parental transfers and borrowing constraints are important determinants of the college decision in the U.S. Kindermann (2014) argues that the former is also true for Germany. In a dynastic world, in which the ability of children is linked to the ability of their parents parental transfers might keep lower ability household out of college, as they would be disadvantaged along two dimensions. However, declining interest rates in the course of demographic change would then act as a counter-argument as they would harm rich and more able households more than the poor and less able. The evolution of interest rates and the capital structure

of the economy drives the college decision and is also important for the welfare consequences across skill groups. It is thus important to shed more light on the distributional consequences across skill groups in economic settings with an international capital market in the tradition of Krüger and Ludwig (2007). This is another task that I leave for future research.

A Theoretical Appendix

A.1 Recursive Household Problem

I hereafter define recursively the household problem. Households take returns as given and maximize their lifetime utility over the choice of consumption, hours supplied to the labor market and hours spent on skill accumulation. Note that the latter choice is restricted while a household is enrolled in college as described in section 3.3.

In order to derive the solution of the household problem I apply a transformation which assures that all variables are trend-stationary. This is consistent with the computational implementation of the solution of the household problem and does not alter results. Therefore, I divide variables which exhibit a trend growth arising from the exogenous technological progress, $\Upsilon_{t+1} = (1+g)\cdot\Upsilon_t$, by the technology level: $\tilde{k}^a_{t,j} := k^a_{t,j}/\Upsilon_t$, $\tilde{c}^a_{t,j} := c^a_{t,j}/\Upsilon_t$, $\tilde{r}^{\mathcal{S}}_t := r^{\mathcal{S}}_t/\Upsilon_t$, $\tilde{\mathcal{P}}_t(b^a_{t,j}) := \mathcal{P}_t(b^a_{t,j})/\Upsilon_t$, $\tilde{\vartheta}_t(b^a_{t,j}, r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j}) := \vartheta_t(b^a_{t,j}, r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j})/\Upsilon_t$.[33] Other variables are already trend-stationary and do not need to be transformed. In the following, I drop the indexes t and j for the sake of simplicity and indicate next period's variables by the symbol $'$, irrespective of whether they are only time dependent or age and time dependent.
Given the preferences of section 4.2 the de-trended household problem reads as:

$$
\begin{aligned}
v^a(\tilde{k}^a, h^a, b^a) = \max_{\tilde{c}^a, l^a, i^a, \tilde{k}^{a\prime}, h^{a\prime}, b^{a\prime}} \; & \{u(\tilde{c}^a, i^a, l^a) + \hat{\beta}\cdot v^{a\prime}(\tilde{k}^{a\prime}, h^{a\prime}, b^{a\prime})\} \\
\text{subject to} \\
\tilde{k}^{a\prime} &= \begin{cases} \frac{1}{1+g}\cdot\Big((1+r^K)\cdot\tilde{k}^a + (1-\tau)\cdot\tilde{r}^{\mathcal{S}}\cdot h^a\cdot l^a - \tilde{c}^a\Big) & if \quad j < j_r \\ \frac{1}{1+g}\cdot\Big((1+r^K)\cdot\tilde{k}^a + \tilde{\mathcal{P}}(b^a) - \tilde{c}^a\Big) & else \end{cases} \\
b^{a\prime} &= \begin{cases} \tilde{\vartheta}(b^a, \tilde{r}^{\mathcal{S}}\cdot h^a\cdot l^a) & if \quad j < j_r - 1 \\ b^a & else \end{cases} \\
h^{a\prime} &= \begin{cases} h_0 & if \quad j < j_w - 1 \;\wedge\; \mathcal{S} = \mathcal{C} \\ (1+\bar{h})\cdot h_0 & if \quad j = j_w - 1 \;\wedge\; \mathcal{S} = \mathcal{C} \\ \varphi^{\mathcal{S}}(h^a, i^a) & else \end{cases} \\
& h^a_0 = h_0 > 0,\; \tilde{k}^a_0 = \tilde{k}_0 = 0,\; b^a_0 = b_0 = 0 \\
& c^a, l^a, i^a \geq 0 \\
& i^a = \bar{i}^a \quad if \quad j < j_w - 1 \;\wedge\; \mathcal{S} = \mathcal{C}
\end{aligned}
\tag{18}
$$

[33] Note that the values of trend-stationary variables do not depend on the time period in a steady state of the economy in which the demographic age distribution of the economy is assumed to be fix over time.

where $\hat{\beta} := \beta \cdot \varsigma \cdot (1+g)^{1-\theta}$. Note that the transformation of the utility function in period t, age j follows $u(c, 1-i-l) = u(\tilde{c}, 1-i-l) \cdot \Upsilon^{1-\theta}$. That transformation also applies to the value function.

In the following, I derive the first order conditions (FOCs) of the de-trended household problem given initial conditions and the educational choice. Note that the constraint $i^a = 0$ never binds due to the production function of human capital. λ is the Lagrange multiplier associated with the constraint $l^a \geq 0$.

$$0 = u_{\tilde{c}} + \hat{\beta} \cdot v'_{\tilde{k}'} \cdot (-\frac{1}{1+g}) \quad if \quad j < J \tag{19}$$

$$0 = u_i + \hat{\beta} \cdot v'_{h'} \cdot \varphi_i^{\mathcal{S}} = 0 \quad if \quad j < j_r - 1 \wedge not(j < j_w \wedge \mathcal{S} = \mathcal{C}) \tag{20}$$

$$0 = u_l + \hat{\beta} \cdot \left(v'_{\tilde{k}'} \cdot \frac{1}{1+g} \cdot (1-\tau) \cdot \tilde{r}^{\mathcal{S}} \cdot h + v'_{b'} \cdot \tilde{\vartheta}_l \right) + \lambda \quad if \quad j < j_r \tag{21}$$

where $u_{\tilde{c}} := \partial u(\tilde{c}, i, l)/\partial \tilde{c}$, $u_i := \partial u(\tilde{c}, i, l)/\partial i$, $u_l := \partial u(\tilde{c}, i, l)/\partial l$, $\varphi_i^{\mathcal{S}} := \partial \varphi^{\mathcal{S}}(h, i)/\partial i$, $\tilde{\vartheta}_l := \partial \tilde{\vartheta}(b, \tilde{r}^{\mathcal{S}} \cdot h \cdot l)/\partial l$, $v'_{\tilde{k}'} = \partial v'(\tilde{k}', h', b')/\partial \tilde{k}'$, $v'_{h'} = \partial v'(\tilde{k}', h', b')/\partial h'$, and $v'_{b'} = \partial v'(\tilde{k}', h', b')/\partial b'$.

Next, I derive the first partial derivatives of the value function with respect to the state variables $\tilde{k}$, h, and b using the envelope theorem:

$$v_{\tilde{k}} = \begin{cases} \hat{\beta} \cdot v'_{\tilde{k}'} \cdot \frac{1}{1+g} \cdot (1 + r^K) & if \quad j < J \\ u_{\tilde{c}} \cdot (1 + r^K) & else \end{cases} \tag{22}$$

$$v_h = \begin{cases} \hat{\beta} \cdot \left(v'_{\tilde{k}'} \cdot \frac{1}{1+g} \cdot (1-\tau) \cdot \tilde{r}^{\mathcal{S}} \cdot l + v'_{h'} \cdot \varphi_h^{\mathcal{S}} + v'_{b'} \cdot \tilde{\vartheta}_h \right) & if \quad j < j_r \\ 0 & else \end{cases} \tag{23}$$

$$v_b = \begin{cases} \hat{\beta} \cdot v'_{b'} \cdot \tilde{\vartheta}_b & if \quad j < j_r \\ \hat{\beta} \cdot \left(v'_{\tilde{k}'} \cdot \frac{1}{1+g} \cdot \tilde{\mathcal{P}}_b + v'_{b'} \right) & if \quad j_r \leq j < J \\ u_{\tilde{c}} \cdot \tilde{\mathcal{P}}_b & else \end{cases} \tag{24}$$

where $\varphi_h^{\mathcal{S}} := \partial \varphi^{\mathcal{S}}(h, i)/\partial h$, $\tilde{\vartheta}_h := \partial \tilde{\vartheta}(b, \tilde{r}^{\mathcal{S}} \cdot h \cdot l)/\partial h$, $\tilde{\vartheta}_b := \partial \tilde{\vartheta}(b, \tilde{r}^{\mathcal{S}} \cdot h \cdot l)/\partial b$, and $\tilde{\mathcal{P}}_b := \partial \tilde{\mathcal{P}}(b)/\partial b$.

Using these equations yields the following FOCs:

$$u_{\tilde{c}} = \frac{1}{1+g} \cdot \hat{\beta} \cdot (1 + r^{K\prime}) \cdot u'_{\tilde{c}'} \quad if \quad j < J \tag{25}$$

$$-u_i = \hat{\beta} \cdot v'_{h'} \cdot \varphi_i^{\mathcal{S}} \quad if \quad j < j_r - 1 \wedge not(j < j_w \wedge \mathcal{S} = \mathcal{C}) \tag{26}$$

$$-u_l = \left((1-\tau) \cdot \tilde{r}^{\mathcal{S}} \cdot h + \frac{v'_{b'}}{v'_{\tilde{k}'}} \cdot (1+g) \cdot \tilde{\vartheta}_l \right) \cdot u_{\tilde{c}} + \lambda \quad if \quad j < j_r \tag{27}$$

A.2 Derivation of the Aggregate Resource Constraint

The individual budget constraints write as

$$k^a_{t+1,j+1} = (1 + r^K_t) \cdot k^a_{t,j} + (1 - \tau_t) \cdot r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j} + \mathcal{P}_t(b^a_{t,j}) - c^a_{t,j} \; \forall \; t, j.$$

I take the population weighted sum of the individual budget constraints in order to derive the aggregate resource constraint in period t:

$$\sum_{j=0}^{J} N_{t,j} \cdot \int k^a_{t+1,j+1} \cdot \zeta(a) \; da$$

$$= \sum_{j=0}^{J} N_{t,j} \cdot \int \left((1 + r^K_t) \cdot k^a_{t,j} + (1 - \tau_t) \cdot r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j} + \mathcal{P}_t(b^a_{t,j}) - c^a_{t,j} \right) \cdot \zeta(a) \; da$$

$\Leftrightarrow$

$$\sum_{j=0}^{J} N_{t,j} \cdot \int k^a_{t+1,j+1} \cdot \zeta(a) \; da$$

$$= \sum_{j=1}^{J} \varsigma_{t-1,j-1} \cdot N_{t-1,j-1} \cdot \int (1 + r^K_t) \cdot k^a_{t,j} \cdot \zeta(a) \; da + N_{t,0} \cdot \int (1 + r^K_t) \cdot k^a_{t,0} \cdot \zeta(a) \; da$$

$$+ \sum_{j=0}^{J} N_{t,j} \cdot \int (1 - \tau_t) \cdot r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j} \cdot \zeta(a) \; da + \sum_{j=0}^{J} N_{t,j} \cdot \int \mathcal{P}_t(b^a_{t,j}) \cdot \zeta(a) \; da$$

$$- \sum_{j=0}^{J} N_{t,j} \cdot \int c^a_{t,j} \cdot \zeta(a) \; da + \sum_{j=}^{J} (1 - \varsigma_{t-1,j-1}) \cdot N_{t-1,j-1} \cdot \int (1 + r^K_t) \cdot k^a_{t,j} \cdot \zeta(a) \; da$$

$$- \sum_{j=1}^{J} (1 - \varsigma_{t-1,j-1}) \cdot N_{t-1,j-1} \cdot \int (1 + r^K_t) \cdot k^a_{t,j} \cdot \zeta(a) \; da$$

$\Leftrightarrow$

$$\sum_{j=0}^{J} N_{t,j} \cdot \int k^a_{t+1,j+1} \cdot \zeta(a) \; da$$

$$= \sum_{j=1}^{J} N_{t-1,j-1} \cdot \int k^a_{t,j} \cdot \zeta(a) \; da + r^K_t \cdot \sum_{j=1}^{J} N_{t-1,j-1} \cdot \int k^a_{t,j} \cdot \zeta(a) \; da$$

$$+ \sum_{j=0}^{j_r - 1} N_{t,j} \cdot \int r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j} \cdot \zeta(a) \; da - \tau_t \cdot \sum_{j=0}^{j_r - 1} N_{t,j} \cdot \int r^{\mathcal{S}}_t \cdot h^a_{t,j} \cdot l^a_{t,j} \cdot \zeta(a) \; da$$

$$+ \sum_{j=j_r}^{J} N_{t,j} \cdot \int \mathcal{P}_t(b^a_{t,j}) \cdot \zeta(a) \; da - \sum_{j=0}^{J} N_{t,j} \cdot \int c^a_{t,j} \cdot \zeta(a) \; da$$

$$- \sum_{j=1}^{J} (1 - \varsigma_{t-1,j-1}) \cdot N_{t-1,j-1} \cdot \int (1 + r_t^K) \cdot k_{t,j}^a \cdot \zeta(a) \; da$$

Using the equilibrium conditions (8–10) as well as the following two conditions

- zero profits due to constant returns to scale production:

$$\begin{aligned} & \mathcal{F}_t(K_t, L_t^{\mathcal{H}}, L_t^{\mathcal{C}}) - (r_t^K + \delta^K) \cdot K_t - r_t^{\mathcal{H}} \cdot L_t^{\mathcal{H}} - r_t^{\mathcal{C}} \cdot L_t^{\mathcal{C}} = 0 \\ \Leftrightarrow \; & Y_t = (r_t^K + \delta^K) \cdot K_t + r_t^{\mathcal{H}} \cdot L_t^{\mathcal{H}} + r_t^{\mathcal{C}} \cdot L_t^{\mathcal{C}} \end{aligned}$$

- accumulation of the aggregate capital stock:

$$K_{t+1} = (1 - \delta^K) \cdot K_t + I_t^K$$

leads to the aggregate resource constraint holding in equilibrium:

$$\begin{aligned} K_{t+1} =& K_t + r_t^K \cdot K_t + r_t^{\mathcal{H}} \cdot L_t^{\mathcal{H}} + r_t^{\mathcal{C}} \cdot L_t^{\mathcal{C}} - C_t - G_t \\ \Leftrightarrow \quad Y_t =& C_t + I_t^K + G_t \end{aligned} \tag{28}$$

B Computational Appendix

B.1 Computational Implementation

The numerical solution is implemented in Fortran 90 using routines which are partly based on Press, Teukolsky, Vetterling, and Flannery (1996). The determination of the equilibrium path involves outer (aggregate model) and inner (household problem) loop iterations. Furthermore, a very outer loop serves for calibration purposes. At all stages, I apply an error tolerance level of at least $1 \cdot 10^{-4}$. Section B.2 contains details on the numerical solution of the household problem.

B.2 Solving the Household Problem

I solve the household problem for the policy functions $\{\tilde{c}_{t,j}^{a,\mathcal{S}}, i_{t,j}^{a,\mathcal{S}}, l_{t,j}^{a,\mathcal{S}}\}_{t,j,a,\mathcal{S}}$, i.e., de-trended consumption, hours spent on skill development, and hours supplied to the labor market for all combinations of ability type, schooling type, age, and time period. In addition, I solve for the optimal schooling decision at age $j = 0$ for all ability types in all time periods. In the following, I omit the superscripts a and $\mathcal{S}$ for convenience where applicable and indicate next period's variables by $'$ irrespective of whether it is age dependent, time dependent, or both.

I apply a backward shooting method using the equations of the household problem, its first order conditions, and the first derivatives of the value function as derived in section A.1.

1. Guess $(\tilde{k}_J, h_J, b_J)$.

2. Start at age $j = J$ where $v'_{\tilde{k}'} = v'_{h'} = v'_{b'} = v' = \tilde{k}' = 0$ $\tilde{k}$, h, and b are given , and the household chooses $l = i = 0$. Use (18) for determining $\tilde{c}$. Compute v from (18) and $v_{\tilde{k}}, v_h, v_b$ according to (22–24).

3. Go backwards in age for $j = J-1, J-2, ..., j_r$. Set $i = l = 0$. Given $u'_{\tilde{c}'}$ determine $\tilde{c}$ from (25), compute $\tilde{k}, h$, and b from (18). Compute $v_{\tilde{k}}, v_h, v_b$ according to (22–24) and v from (18).

4. Go backwards in age for $j = j_r - 1, j_r - 2, ..., 0$ and proceed as described below in order to determine $\tilde{c}$, i, and l. In the following cases, choices are restricted: At $j = j_r - 1$ set $i = 0$ as any time spent on skill development does not pay off in retirement. At $j = 0, ..., j_w - 1 \wedge \mathcal{S} = \mathcal{C}$, set $i = \bar{i}$ as the household is currently enrolled in college.

   (a) Determine $\tilde{c}, i, l$:

      i. Guess i.
      ii. Compute h from (18).
      iii. Compute $u_{\tilde{c}}$ from (25) and $u_l/u_{\tilde{c}}$ from (27).
      iv. Compute l from $u_{\tilde{c}}$ and $u_l/u_{\tilde{c}}$. If $l < 0$ set $l = 0$.
      v. Compute $\tilde{c}$ using the preference function $u(\cdot)$.
      vi. Compute $\hat{i}$ from (26).
      vii. If $\|\hat{i} - i\| > \epsilon$ go to step 4(a)i. and update the guess of i.[34]

   (b) Compute $v, \tilde{k}, h$, and b from equation (18) and $v_{\tilde{k}}, v_h, v_b$ according to (22–24).

5. If $\|(\tilde{k}_0, h_0, b_0) - (0, 1, 0)\| > \epsilon$ go to step 2 and update the guess of $(\tilde{k}_J, h_J, b_J)$.

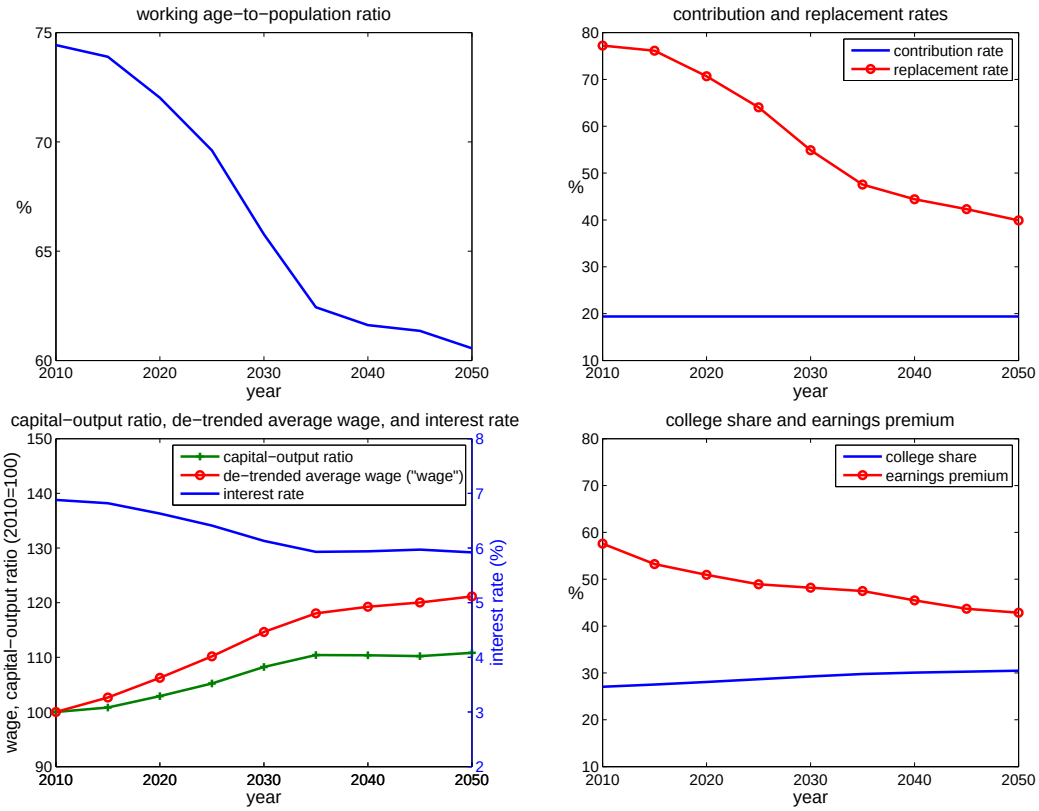
6. At the beginning of period $j = 0$, each household faces the decision on tertiary education. Given all policy rules resolved according to steps 2.–4., given its ability type, a, a household chooses formal schooling according to equation (2).

B.3 Aggregate Dynamics Beyond 2010 Without Past Skill-Biased Technological Change

Figure 8 shows the evolution of key aggregate measures in the main period of projection, from year 2010 to year 2050 in the auxiliary economy without past skill-biased technological change. The central findings can be summarized as follows: While 1) the replacement rate, 2) the interest rate, and 3) the earnings

[34] ϵ denotes the error tolerance level.

Figure 8: Aggregate Dynamics Beyond 2010 Without Past Skill-Biased Technological Change



Source: Auxiliary model without past skill-biased technological change in the main period of projection, 2010 to 2050: Selected aggregate measures. *Notes:* The replacement rate in the top right panel refers to the average ratio of the pension payment to a 65 year old household to the sum of net earnings (adjusted by wage inflation) over its working life. The average gross hourly wage in the bottom left panel is shown net of the trend growth arising from exogenous skill-neutral technological change in Υ_t over time. Capital-output ratio and de-trended average wage are normalized to 100 in 2010. The bottom right panel shows the average earnings premium of college graduates and the share of college graduates in the 25+ workforce in percent.

premium decline gradually over time, 4) the average wage level increases.

As an indicator for the development of overall economic inequality in the economy table 3 displays the change in the Gini coefficients of net total income and consumption from 2010 to 2050. The resulting numbers are similar to the

Table 3: Measures of Economic Inequality: The Change in Ginis from 2010 to 2050 Without Past Skill-Biased Technological Change

	net total income	consumption
$Gini(2050) - Gini(2010)$, baseline	+7.9	−3.6
$Gini(2050) - Gini(2010)$, fix demographics	−4.6	+2.2

Source: Auxiliary model without past skill-biased technological change. *Notes:* The numbers show percentage point changes. The top row displays the total change in Gini coefficients while the bottom row shows the corresponding values if the Gini coefficients in 2050 are recomputed using the age distribution of the population in year 2010. Net total income is the sum of capital income and net earnings, respectively pension income of a household.

results with past skill-biased technological change in the main text. However, the change in the income Gini shows to be about 2 to 3 percentage points more positive than in the case with past skill-biased technological case. This holds for both computed values (top and bottom row) and mirrors the development of the earnings premium which exhibits a smaller drop in the case without past skill-biased technological change as it is discussed in the main text.

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