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Extending the Ramsey Equation further: Discounting under Mutually Utility Independent and Recursive Preferences

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Extending the Ramsey Equation further: Discounting under Mutually Utility Independent and Recursive Preferences

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PRELIMINARY VERSION
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Abstract

I revisit the consumption discount rate for a novel combination of standard assumptions. To disentangle risk and time preferences, I consider a decision maker with recursive preferences à la Kreps and Porteus (1978). Moreover I assume that preferences are mutually utility independent in the sense of Koopmans (1960). In an infinite horizon setting with independent growth risk and constant elasticity of substitution, the consumption discount rate is diminished by a previously unrecognised horizon effect. This effect may be significant if the rate of pure time preference is moderately small.

Keywords: discounting, intertemporal decision making, uncertain growth, risk aversion, recursive utility, Kreps-Porteus preferences, Risk-Sensitive preferences, utility independence, horizon, climate change

JEL codes: H43, D81, D90, Q54

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1 Introduction

Standard approaches to discounting under certainty originate from the most popular model of intertemporal choice: the discounted utility model as introduced by Samuelson (1937) and axiomatised by Koopmans (1960).¹ This model yields the well known Ramsey Equation which aggregates the determinants of the consumption discount rate - impatience and a wealth effect - in an intuitive manner. The predominance of the Ramsey Equation as an organising principle for discounting sure benefits was recently confirmed by a panel of leading experts on intergenerational discounting (Arrow et al. 2012).

A crucial assumption which is built into the discounted utility model - and thus into the standard approach to discounting under certainty - is preference independence. Preferences over the consumption of one generation are (mutually) preference independent if they are independent of the consumption levels of generations living in the past and in the future. This assumption largely simplifies the preference representation. In a deterministic setting, Koopmans (1960) showed that preference independence constitutes the key axiom for the existence of an additively separable intertemporal utility function.

Recent contributions in the discounting literature emphasise the role of risk and risk aversion. Gollier (2002a, 2002b) motivates an Extended Ramsey Equation which incorporates discounting for reasons of precaution in the presence of growth risk. The additional effect on a one period (instantaneous) discount rate is marginal, however. This insignificance of the growth risk is partly due to an immanent drawback of the additive expected utility framework from which the Extended Ramsey Equation originates. In this framework it is not possible to disentangle risk aversion from the intertemporal elasticity of substitution (IES). In the additive expected utility model,

¹See Frederick et al. (2002) for a comprehensive discussion of the discounted utility model's historical origins.

a meaningful degree of risk aversion goes along with an unrealistically small IES. Gollier (2002a), Hector (2013) and Traeger (2011, 2014) approach this deficit by employing recursive preferences of the Kreps-Porteus type (Kreps and Porteus 1978) to identify the consumption discount rate and its determinants. As the degree of risk aversion can be varied independently of the IES in the Kreps-Porteus framework, it is possible to account for higher degrees of risk aversion. Higher degrees of risk aversion then imply a pronounced effect of growth risk on the consumption discount rate.

Utility independence - the equivalent of preference independence in a risky world - is either implicitly assumed or dismissed without discussion in the cited literature on discounting under growth risk. In Gollier's (2002a, 2002b) Extended Ramsey Equation, utility independence goes along with the time additive structure of the expected utility function. Kreps-Porteus recursive preferences, in contrast, are not utility independent by default. In particular, the Epstein-Zin (Epstein and Zin 1989) parameterisation of Kreps-Porteus recursive preferences does not represent utility independent preferences. Without discussing the abandonment of the utility independence assumption, Traeger (2011, 2014) employs the Epstein-Zin (EZ) parameterisation to derive an Extended Ramsey Equation for EZ preferences.

The focus of the analysis at hand is on the discount rate of a decision maker whose preferences are Kreps-Porteus recursive as well as (mutually) utility independent. The Kreps-Porteus framework is chosen for its flexibility with respect to the disentanglement of risk aversion and the IES. Utility independence is postulated as it is a broadly accepted assumption for intertemporal social welfare considerations, such as those underlying the Ramsey Equation and the Extended Ramsey Equation. In a first instance, I show that utility independence restricts a Kreps-Porteus recursive decision maker's preferences to a very specific parametric form, namely to the constant absolute risk aversion form of Hansen and Sargent's (1995) Risk-Sensitive (RS) preferences. Coming from a decision maker with RS preferences, I analyse

the instantaneous consumption discount rate in an infinite horizon setting. This is done under the standard assumptions of independently distributed growth risk and constant elasticity of substitution.

I find that the discount rate of the considered decision maker is subject to an effect that is not present in previous approaches to discounting under risk. This effect, which is denoted as the ‘horizon effect’, may diminish the discount rate to a significant extent. The horizon effect is a function of the length of the time horizon *after* the period of discount, the degree of temporal risk aversion, the variance of growth risk, and the rate of pure time preference. The dependence on the time horizon after the period of discount discloses that the standard practice of cutting off decision problems after a given number of periods is problematic. In particular, Kreps-Porteus recursive frameworks which consider only two-period decision problems for simplification, exclude the horizon effect by construction. In infinite horizon settings like the one considered in this analysis, the role of temporal risk aversion, the role of risk itself, and the role of the rate of pure time preference on the consumption discount rate are amplified in comparison to their roles in the Ramsey Equation, the Extended Ramsey Equation and the Extended Ramsey Equation for EZ preferences (henceforth: the Ramsey Equation and its (previous) extensions). This point is illustrated through an analytical solution for the consumption discount rate of a decision maker with RS preferences. I refer to this analytical solution as the Extended Ramsey Equation for Risk-Sensitive preferences.

To avoid confusion, note that Gollier (2002a, 2002b) also refers to an effect on the discount rate that is connected to the time horizon. This effect is different from what I have in mind, however. In Gollier’s contributions, ‘horizon’ refers to the time horizon between the present period and the period to which the discount rate applies. Here, ‘horizon’ refers to the time horizon after the period for which one discounts. Closer to my understanding of the horizon effect is Traeger (2011). He also points to the fact

that the ‘planning horizon’ after the period for which one discounts may affect the discount rate. While Traeger is aware of the existence of the (planning) horizon effect in a very general Kreps-Porteus recursive setting, he does not study it in detail. In the contrary, as I discuss below, he eliminates the effect by employing an Epstein-Zin parameterisation of Kreps-Porteus preferences with homogeneous felicity.

In section 2 I describe the notion of preference and utility independence. I present the terminology and formal definitions in the static context of multiattribute utility theory to familiarise the reader with these concepts. In section 3, I introduce the preferences of the decision maker under consideration. I develop a definition of (mutual) utility independence for preferences over temporal lotteries which is then imposed on a Kreps-Porteus recursive decision maker. I show that the specified preferences are of the Risk-Sensitive type. In section 4, I examine the consumption discount rate of a decision maker with Risk-Sensitive preferences in two steps. First, I prove the existence and the direction of the horizon effect. Second, I derive an Extended Ramsey Equation for RS preferences, discuss its relation to the Ramsey equation and its previous extensions, and emphasise the special role of the rate of pure time preference. Section 5 concludes and suggests future research.

2 Background: Utility independence

Assumptions of preference or utility independence are standard in the context of utility functions $U(x_1, x_2, \dots, x_n)$ that aggregate the felicity from different attributes. The representation of preferences over multiple attributes is largely simplified if preferences over a specific attribute (or over lotteries on an attribute) are independent of common levels of other attributes. If mutual preference or utility independence holds, preferences over deterministic attributes and preferences that satisfy the axioms of expected utility theory can be represented through a utility function that is decomposable into smaller units: $U(x_1, x_2, \dots, x_n) = f(u_1(x_1), u_2(x_2), \dots, u_n(x_n))$. In

particular, an (expected) utility function over n attributes can be decomposed into an additive or multiplicative form if preferences satisfy mutual preference or utility independence.²

In this section, I describe the notion of mutual preference and utility independence in the (mostly) static context of multiattribute utility theory (MAUT). The purpose of this description is to familiarise the reader with the basic idea behind these independence concepts. This familiarity will help the understanding of the next section, in which I adjust the definition of mutual utility independence to the temporal and recursive setting of my analysis.

The main reference for independence concepts in the deterministic or expected utility context of MAUT is Keeney and Raiffa's (1976) volume on Decisions with Multiple Objectives. For comprehensive surveys on various independence assumptions, their implications in MAUT and the relevant literature see Farquhar (1977) and Yilmaz (1978). The definitions of conditional preferences, preference independence and utility independence below are as in Farquhar.

2.1 Terminology and conditional preferences

Consider a decision maker with preferences \succeq on a set of possible outcomes X , which contains n different attribute sets X_i with $i = 1, 2, \dots, n$. The set of possible outcomes is the Cartesian product of the attribute sets: $X = X_1 \times X_2 \times \dots \times X_n$. An element $x_i \in X_i$ is a specific level of attribute i . A specific outcome $x \in X$ is written as the n -tuple $x = (x_1, x_2, \dots, x_n)$. In risky situations, the decision maker's preferences \succeq are defined over the set P of lotteries on X . An element $p \in P$ is a lottery that assigns probabilities l^ω , with $\omega = 1, \dots, N$ and $\sum_{\omega=1}^N l^\omega = 1$, $l^\omega > 0 \forall \omega$, to specific outcomes $x^\omega \in X$, such that $p = \sum_{\omega=1}^N l^\omega x^\omega$.

For the definitions of preference and utility independence below it will be useful to

²Keeney and Raiffa (1976).

partition the attribute space and introduce conditional preference relations. The attribute space $i = 1, 2, \dots, n$ can be partitioned into the nonempty sets I and \bar{I} such that $X = X_1 \times X_2 \times \dots \times X_n$ can be expressed as $X = X_I \times X_{\bar{I}}$. The set of lotteries on X_I is then denoted as P_I , and $p_I \in P_I$ denotes a specific marginal distribution of p on X_I . A conditional preference relation is a preference relation that is defined over lotteries in one set, while holding the outcome in a different set fixed. Given a fixed outcome in $X_{\bar{I}}$, an unconditional preference relation \succeq on P can be expressed as a conditional preference relation $\succeq_{x_{\bar{I}}}$ on P_I . That is, rather than defining \succeq over lotteries $(p_I, x_{\bar{I}}), (p'_I, x_{\bar{I}}) \in P$ with marginal probabilities $p_I, p'_I \in P_I$ on X_I and probability 1 for the outcome $x_{\bar{I}} \in X_{\bar{I}}$, we can define $\succeq_{x_{\bar{I}}}$ over the marginals $p_I, p'_I \in P_I$. The conditional preference relation $\succeq_{x_{\bar{I}}}$ thus restricts the unconditional preference relation \succeq to those $p \in P$ that assign probability 1 to $x_{\bar{I}}$. Formally

$$p_I \succeq_{x_{\bar{I}}} p'_I \text{ if and only if } (p_I, x_{\bar{I}}) \succeq (p'_I, x_{\bar{I}}) \quad \forall p_I, p'_I \text{ in } P_I.$$

2.2 Mutual preference and utility independence

Preferences over outcomes in one attribute set may or may not depend on the specific levels of the remaining attributes. If the preference order over levels in the attribute set X_I is independent of the outcome in a different attribute set $X_{\bar{I}}$, we say that X_I is preference independent of $X_{\bar{I}}$. Formally, preference independence (PI) can be defined as follows:

Definition 1 (*Preference independence*)

X_I is preference independent of $X_{\bar{I}}$ if and only if $\succeq_{x_{\bar{I}}} = \succeq_{x'_{\bar{I}}}$ on $X_I \quad \forall x_{\bar{I}}, x'_{\bar{I}} \in X_{\bar{I}}$.

Note that preference independence is not a symmetric condition: Given that X_I is preference independent of $X_{\bar{I}}$ we cannot infer that $X_{\bar{I}}$ is preference independent of X_I and vice versa. A symmetric condition, namely mutual preference independence

(MPI), is, however, easily constructed:

Definition 2 (*Mutual preference independence*)

X_I and $X_{\bar{I}}$ are mutually preference independent if and only if X_I is preference independent of $X_{\bar{I}}$ and $X_{\bar{I}}$ is preference independent of X_I .

Preferences \succeq over X which satisfy MPI on the whole domain (i.e. each subset $X_I \in X$ is PI of its complement $X_{\bar{I}} \in X$) are representable by an additive utility function (Keeney and Raiffa 1976, theorem 3.6).

Preference independence restricts preferences that are defined over a set of deterministic attributes. The analogue for preferences defined over lotteries is utility independence. If the preference order over lotteries in P_I on X_I is independent of outcomes in $X_{\bar{I}}$, we say that X_I is utility independent of $X_{\bar{I}}$. The definition of utility independence (UI) mirrors that of preference independence. The difference is only in the set over which preferences are defined:

Definition 3 (*Utility independence*)

X_I is utility independent of $X_{\bar{I}}$ if and only if $\succeq_{x_{\bar{I}}} = \succeq_{x'_{\bar{I}}}$ on $P_I \forall x_{\bar{I}}, x'_{\bar{I}} \in X_{\bar{I}}$.

If utility independence holds, then all conditional preference relations $\succeq_{x_{\bar{I}}}$ on P_I preserve the same order among all $p_I \in P_I$. This includes degenerate lotteries that assign probability 1 to specific levels in the attribute set X_I . Thus, whenever X_I is utility independent of $X_{\bar{I}}$, it must also be true that X_I is preference independent of $X_{\bar{I}}$. The converse is not generally true.

Just like preference independence, utility independence is not a symmetric condition: Given that X_I is utility independent of $X_{\bar{I}}$ we cannot infer that $X_{\bar{I}}$ is utility independent of X_I and vice versa. The symmetric condition is called mutual utility independence (MUI):

Definition 4 (*Mutual utility independence*)

X_I and $X_{\bar{I}}$ are mutually utility independent if and only if X_I is utility independent of $X_{\bar{I}}$ and $X_{\bar{I}}$ is utility independent of X_I .

Preferences \succeq over P which satisfy MUI on the whole domain (i.e. each subset $X_I \in X$ is UI of its complement $X_{\bar{I}} \in X$) and which comply with von Neumann and Morgenstern's expected utility axioms are representable by an additive or multiplicative utility function (Keeney and Raiffa 1976, theorem 6.1). A standard additive expected utility function is mutually utility independent on the whole domain.

2.3 Temporal context

Consider the aggregation of an infinite number of attributes x_t with $t = 1, 2, 3, \dots$ which differ with respect to the period at which they occur. An intertemporal social welfare function constitutes such an aggregation. A distinctive feature of this aggregation is the temporal order of the attributes. Due to this order, assumptions of utility independence can be given a temporal interpretation. If utility independence is geared towards the past, one speaks of history independence, if it is geared towards the future, one calls it future independence.

To be more specific, define by $X = X_1 \times X_2 \times X_3 \dots$ the space of possible consumption paths over an infinite horizon. History independence of preferences over lotteries P_t on an attribute set $X_t \in X$ requires that X_t is utility independent of each $X_\tau \in X$ with $\tau < t$. Likewise, future independence of preferences over lotteries P_t on X_t requires that X_t is utility independent of each X_τ with $\tau > t$. If for some t, τ with $t < \tau$, it holds that preferences over P_t on X_t are future independent and those over P_τ on X_τ are history independent, then it must also be true that X_t and X_τ are mutually utility independent. If preferences over each P_t on $X_t \in X$ are both future and history independent, then each pair $X_t, X_\tau \in X$ with $t, \tau = 1, 2, \dots, \infty$ and $t \neq \tau$

is mutually utility independent. We then simply say that preferences over P on X are mutually utility independent on the whole domain.

Preferences which are history and future independent, which satisfy von Neumann and Morgenstern's expected utility axioms, and which are defined over lotteries on intertemporal consumption paths, are representable by an additive or multiplicative intertemporal utility function (Meyer 1976, theorem 9.2). Correspondingly, preferences represented by the standard additive (intertemporal) expected utility function are mutually utility independent on the entire domain.

In a temporal but deterministic context, Koopmans (1960) proved that several axioms, among them a crucial assumption on period independence, warrants the existence of the additive discounted utility model.³ In the following I extend Koopmans' requirement for independence of preferences defined over deterministic consumption paths to a larger domain, namely to the domain of temporal lotteries.

3 Preferences: Recursive and MUI

Intergenerational decision making involves allocating resources across many different generations. These generations differ with respect to their consumption level as well as with respect to the degree of consumption risk to which they are exposed. A decision maker who optimises intertemporal welfare evaluates the consumption and risk levels according to his preferences, in particular according to his intertemporal elasticity of substitution (IES) and his degree of risk aversion. These two preference characteristics are entangled in the standard model of intertemporal choice under risk, namely in the additive expected utility model. To model the preferences of an intertemporal decision maker in a more flexible manner, I resort to the recursive

³More specifically, Koopmans (1960) showed that stationary, time-consistent, period independent preferences over infinite deterministic consumption paths are represented by $U(x) = \sum_{t=1}^{\infty} \beta^{t-1} u(x_t)$. Under addition of a continuity axiom, he showed that the utility discount factor (rate of pure time preference) must be such that $0 < \beta < 1$.

utility representation of Kreps and Porteus (1978). A Kreps-Porteus (KP) recursive preference representation enables the disentanglement of a decision makers' degree of risk aversion from the IES.

KP recursive preferences are defined over objects called temporal lotteries. The definitions of mutual preference and utility independence above, however, concern preferences that are defined over deterministic attributes or over lotteries on attribute sets. The results of Keeney and Raiffa (1976), Meyer (1976) and Koopmans (1960) on the decomposition of a utility function when preferences satisfy definitions 1, 2, 3, or 4, are therefore not directly applicable in the context considered here.

To study how mutual utility independence restricts a Kreps-Porteus recursive preference representation, mutual utility independence for preferences over temporal lotteries must be defined. To this end, denote the set of temporal lotteries by D and write a specific temporal lottery as $(x_1, \tilde{x}_2, \tilde{x}_3, \dots) \in D$. A temporal lottery consists of a certain attribute x_1 for the initial period and (potentially) uncertain attributes \tilde{x}_t for $t > 1$. Note that the set of degenerate temporal lotteries (deterministic consumption paths) X^∞ is a subset of D .⁴ Mutual utility independence for preferences over temporal lotteries can be defined in the following way:

Definition 5 (*MUI for preferences over temporal lotteries*)

Preferences \succeq over the set of temporal lotteries D are mutually utility independent if

$$\begin{aligned} (x_1, \tilde{x}_2, \dots, \tilde{x}_{t-1}, x_t, \tilde{x}_{t+1} \dots) &\succeq (x'_1, \tilde{x}'_2, \dots, \tilde{x}'_{t-1}, x_t, \tilde{x}'_{t+1} \dots) \\ &\Downarrow \\ (x_1, \tilde{x}_2, \dots, \tilde{x}_{t-1}, x'_t, \tilde{x}_{t+1} \dots) &\succeq (x'_1, \tilde{x}'_2, \dots, \tilde{x}'_{t-1}, x'_t, \tilde{x}'_{t+1} \dots) \end{aligned}$$

$$\forall x_t, x'_t \in X_t.$$

⁴For a more comprehensive discussion of temporal lotteries see Kreps and Porteus (1978), Epstein and Zin (1989), or Bommier and Le Grand (2014).

Definition 5 is now imposed on Kreps-Porteus recursive preferences. Denote by \succeq^D a preference relation over the set of temporal lotteries D . Suppose \succeq^D is KP recursive and let $U^D : D \rightarrow \mathbb{R}$ represent such preferences. Since U^D represents KP recursive preferences, it must satisfy the recursion

$$U^D(x_1, m) = W(x_1, E_m[U^D]), \quad (1)$$

where $E_m[\cdot]$ is the expectation with respect to the probability measure m on D .⁵

Suppose in addition that the considered preference relation \succeq^D satisfies mutual utility independence according to definition 5. Note that the assumption of MUI on D implies MUI on the subdomain $X^\infty \subseteq D$ as well. Given MUI of \succeq^D on D , the form of U^D can be narrowed down in two steps.

First, I restrict the form of U^D such that it represents only preferences that are MUI on the subdomain $X^\infty \subseteq D$. To this end, I use Koopmans' (1960) representation result for period independent preferences. His definition of period independence accords to my definition of mutual utility independence.⁶ Koopmans shows that a preference relation \succeq^X over X^∞ which satisfies continuity, sensitivity, stationarity and mutual utility independence can be represented by an additive discounted utility function $U^X : X^\infty \rightarrow \mathbb{R}$:

$$U^X(x_1, x_2 \dots) = u(x_1) + \beta U^X(x_2, x_3 \dots). \quad (2)$$

Note that U^D and U^X represent the same (mutually utility independent) preferences on X^∞ . Since U^D and U^X represent the same ordinal preferences, there exists some increasing ϕ such that $U^D = \phi(U^X)$ (Kihlstrom and Mirman 1974). Denoting by $W^D(x, y)$ and $W^X(x, y)$ the aggregators of U^D and U^X and using $U^D = \phi(U^X)$, we

⁵See e.g. Bommier and Le Grand (2014).

⁶Meyer (1976) shows that the combination of Koopmans' postulates on stationarity and period independence (Koopmans' postulate 3') induce complete pairwise preferential independence. Complete pairwise preferential independence corresponds to our definition of mutual utility independence on the domain X^∞ (see section 3.6.3 in Keeney and Raiffa 1976).

can write $W^D(x, y) = \phi(W^X(x, \phi^{-1}(y)))$ and thus⁷

$$U^D(x_1, m) = \phi(u(x_1) + \beta\phi^{-1}(E_m U^D(x_2, m))). \quad (3)$$

Equation (3) restricts the form of U^D such that it represents only preferences which satisfy MUI on X^∞ .

Second, I restrict the form of U^D further such that it represents only preferences that are MUI on the entire domain D . To this end, one needs to restrict ϕ in such a way that U^D represents preferences with constant absolute risk aversion. I show this in appendix 7.1. The implications for a renormalised ordinal KP recursive utility function $U = \phi^{-1}(U^D)$ are summarised in theorem 1.

Theorem 1 (*Representation of KP recursive preferences that satisfy MUI*)

Consider a decision maker with preferences that satisfy the Kreps-Porteus recursion (equation 1). Suppose that these preferences are mutually utility independent over the set of temporal lotteries D (definition 5). Such preferences can be represented by a utility function $U : D \rightarrow \mathbb{R}$ of the following form:

$$U(x_1, m) = u(x_1) - \frac{\beta}{k} \ln(E_m \exp(-kU(x_2, m))). \quad (4)$$

Equation (4) is the constant absolute risk aversion form of Hansen and Sargent's (1995) Risk-Sensitive (RS) preferences. The parameter k measures the decision maker's degree of temporal risk aversion. We say that the decision maker is temporally risk averse if $k > 0$ and temporally risk loving if $k < 0$. Temporal risk aversion can be understood as aversion towards risk on continuation utility, which is given by $U(x_2, m)$ in equation (4).

⁷Let $W^X(x, y) = u(x) + \beta y$ such that $W^D(x, y) = \phi(u(x) + \beta\phi^{-1}(y))$. For $U^D(x_1, m) = W^D(x_1, E_m[U^D])$ this yields equation (3).

For $k = 0$, equation (4) nests the additive (intertemporal) expected utility function

$$U(x_1, m) = u(x_1) + \beta E_m U(x_2, m). \quad (5)$$

A decision maker with $k = 0$, i.e. an additive expected utility decision maker, is called temporally risk neutral. Such a decision maker is neutral towards risk on continuation utility. Aversion towards risk on consumption x_t is solely governed by the curvature of the felicity function $u(x_t)$, which simultaneously defines the intertemporal elasticity of substitution.

The implications of the mutual utility independence assumption become obvious if one considers the case of independently distributed risk on the attributes \tilde{x}_t . If the attributes are statistically independent, the assumption of mutual utility independence on KP recursive preferences implies the additive separability of the respective utility function. In particular, if preferences are represented by equation (4) and risk on consumption \tilde{x}_t is independently distributed, the utility function can be written as⁸

$$U(x_1, \hat{x}_2, \dots) = u(x_1) + \beta \sum_{t=2}^{\infty} \beta^{t-2} u(\hat{x}_t), \quad (6)$$

where \hat{x}_t is certainty equivalent consumption in t . For the RS decision maker under consideration, \hat{x}_t is derived from $u(\hat{x}_t) = -\frac{1}{k} \ln(E_{t-1} \exp(-ku(\tilde{x}_t)))$. If lotteries on \tilde{x}_t are degenerate (i.e. if consumption is deterministic), then (6) is equivalent to Koopmans' (1960) additive discounted utility function, i.e. equation (2).

I am not the first to connect assumptions of mutual utility independence and Kreps-Porteus recursive preferences. Bommier and Le Grand (2014) remark that their Kreps-Porteus recursive preference specification under scrutiny, namely Risk-Sensitive preferences, satisfies mutual utility independence. Above I approached the issue from a different angle, however. I showed more formally that Kreps-Porteus recursive pref-

⁸See appendix 7.2.

erences which satisfy mutual utility independence must be of the Risk-Sensitive type.⁹

4 Implications for discounting

I showed above that Kreps-Porteus recursive preferences which satisfy mutual utility independence are restricted to a specific parametric form, namely to that of Risk-Sensitive preferences. In this section I analyse the instantaneous consumption discount rate of a decision maker with such preferences.

An instantaneous consumption discount rate $DR_{1,2}$ compares the effects on intertemporal utility $U(x)$ when consumption in the first and in the second period are marginally changed:

$$DR_{1,2} = -\ln \frac{\partial U(x)/\partial x_2}{\partial U(x)/\partial x_1}. \quad (7)$$

I show below that the consumption discount rate of a decision maker with RS preferences is subject to an effect which is not present in the well known Ramsey Equation and its extensions. This effect is denoted as the ‘horizon effect’.

4.1 Defining the horizon effect

A horizon effect is present whenever the consumption discount rate is affected by circumstances that realise only after the period for which one discounts. For the instantaneous discount rate $DR_{1,2}$ (equation 7) this is the case if the value of period 2 consumption relative to that of consumption in period 1 is subject to circumstance in periods $t \geq 3$.

To formalise the horizon effect, I compare the instantaneous consumption discount rate in two situations, A and B . In both situations I consider a decision maker whose

⁹Related to but quite different from my approach is Traeger (2012). In a finite (rolling) horizon framework with Kreps-Porteus recursive preferences, he derives a constant absolute risk aversion parameterisation of Kreps-Porteus recursive preferences (Risk-Sensitive preferences) from an assumption denoted as ‘coinciding last outcome independence’.

preferences are defined over a temporally infinite domain.

In situation *A*, the decision maker faces a world that consists of an infinite number of existing generations. An existing generation t consumes x_t and derives felicity $u(x_t)$ from this consumption. The instantaneous consumption discount rate that applies to this situation is denoted $DR_{1,2}^{\bar{T}=\infty}$, where \bar{T} is the ‘last’ period in which a generation exists. In this situation, the instantaneous consumption discount rate of a KP recursive decision maker is derived as

$$DR_{1,2}^{\bar{T}=\infty} = -\ln \beta - \ln \frac{E_1 \left[\frac{\phi'(U_2)}{\phi'(\phi^{-1}(E_1[\phi(U_2)]))} u'(\tilde{x}_2) \right]}{u'(x_1)} \quad (8)$$

with $U_t = u(\tilde{x}_t) + \beta \phi^{-1}(E_t[\phi(U_{t+1})]) \quad \forall t = 2, 3, \dots$

Note that equation (8) may be subject to circumstances that apply to periods $t \geq 3$ since the continuation utility U_2 is a function of these values.

In situation *B*, generations in $t \geq 3$ do not exist.¹⁰ A generation t that does not exist is assigned zero felicity: $u(-) = 0$.¹¹ The respective discount rate is denoted $DR_{1,2}^{\bar{T}=2}$. In this situation, the instantaneous consumption discount rate of a KP recursive decision maker is written as

$$DR_{1,2}^{\bar{T}=2} = -\ln \beta - \ln \frac{E_1 \left[\frac{\phi'(u(\tilde{x}_2))}{\phi'(\phi^{-1}(E_1[\phi(u(\tilde{x}_2))])} u'(\tilde{x}_2) \right]}{u'(x_1)}. \quad (9)$$

Note that equation (9) is independent of values in periods $t \geq 3$.

In both equations, β is the utility discount factor. The term $(-\ln \beta)$ therefore constitutes the rate of pure time preference.

¹⁰Alternatively we could assume that generations $t \geq 3$ in situation *B* do exist and have consumption \tilde{x}_t which is not correlated with consumption in period 2. The implied discounting function would be the same as in the case where we assume that generations in $t \geq 3$ do not exist.

¹¹The ‘-’ stands for the consumption level of a non-existent generation. Note that this is different from just assuming that an existing generation has zero consumption. For an enlarged discussion of this point see Bommier (2013). He considers preferences that are defined over a finite lifetime, but with an infinite number of possibilities for the length of this lifetime.

Given the description of situations A and B , I formally define the horizon effect in the following way:

Definition 6 (*Horizon effect*)

An instantaneous consumption discount rate $DR_{1,2}^{\bar{T}=\infty}$ (equation 7) is subject to a horizon effect whenever

$$DR_{1,2}^{\bar{T}=\infty} \neq DR_{1,2}^{\bar{T}=2}.$$

The comparison of equations (8) and (9) in light of definition 6 reveals that the discount rate of a KP recursive decision maker is subject to a horizon effect whenever

$$E_1 \left[\frac{\phi'(U_2)}{\phi'(\phi^{-1}(E_1[\phi(U_2)]))} u'(\tilde{x}_2) \right] \neq E_1 \left[\frac{\phi'(u(\tilde{x}_2))}{\phi'(\phi^{-1}(E_1[\phi(u(\tilde{x}_2)]))} u'(\tilde{x}_2) \right]. \quad (10)$$

The fractions on both sides of this inequality adjust the statistical probability of a given state of the world for risk aversion with respect to risk on the continuation utility. The continuation utility is U_2 in situation A and $u(\tilde{x}_2)$ in situation B . I refer to these fractions as ‘risk aversion adjustment factors’. The product of the risk aversion adjustment factor and the statistical probability of a given state of the world is called a ‘risk aversion adjusted probability’.¹²

Equation (10) clarifies that a horizon effect may exist whenever the risk aversion adjusted probabilities of a given state of the world are not equivalent for $DR_{1,2}^{\bar{T}=\infty}$ and $DR_{1,2}^{\bar{T}=2}$. Note that the adjustment factors of a decision maker with additive expected utility preferences are 1 in each state of the world since $\phi(\cdot)$ is linear in this case. Hence, the risk aversion adjusted probabilities of such a decision maker are equal to the statistical probabilities and equation (10) holds with equality. It follows that the discount rate of an additive expected utility decision maker is never subject to a horizon effect.

¹²A formal definition of risk aversion adjusted probabilities is provided in appendix 7.4.

4.2 The discount rate of a RS decision maker

In theorem 1 I stated that the preferences of a KP recursive and mutually utility independent decision maker are representable by the Risk-Sensitive utility function, as specified in equation (4). The discount rate of a RS decision maker is thus restricted to a parametric form with $\phi(z) = -\exp(-kz)$.

For the discount rate $DR_{1,2}^{\bar{T}=\infty}$ of situation *A* above (equation 8), this implies the form

$$DR_{1,2}^{\bar{T}=\infty} = -\ln \beta - \ln \frac{E_1 \left[\frac{\exp(-kU_2)}{E_1[\exp(-kU_2)]} u'(\tilde{x}_2) \right]}{u'(x_1)} \quad (11)$$

with $U_t = u(\tilde{x}_t) - \frac{\beta}{k} \ln(E_t[\exp(-kU_{t+1})]) \quad \forall t = 2, 3, \dots$

The discount rate that corresponds to situation *B*, namely $DR_{1,2}^{\bar{T}=2}$ as specified in (9), is written as

$$DR_{1,2}^{\bar{T}=2} = -\ln \beta - \ln \frac{E_1 \left[\frac{\exp(-ku(\tilde{x}_2))}{E_1[\exp(-ku(\tilde{x}_2))]} u'(\tilde{x}_2) \right]}{u'(x_1)}. \quad (12)$$

Equations (11) and (12) define instantaneous consumption discount rates for a decision maker with KP recursive mutually utility independent (equivalently: Risk-Sensitive) preferences. Under (11), the decision maker faces a world that consists of an infinite number of generations. Under (12), the decision maker only accounts for the first two generations. Each generation $t \geq 2$ that is taken into account in (11) and (12) has possibly uncertain consumption \tilde{x}_t . Equation (11) is the subject under scrutiny in the remaining analysis, equation (12) serves as a benchmark to determine the existence, the direction, and the size of the horizon effect.

Before I go to the main analysis, let me point to a number of conditions under which the existence of a horizon effect acting on (11) can be excluded. $DR_{1,2}^{\bar{T}=\infty}$ is free from a horizon effect ($DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2}$) if $\beta = 0$, if $u(x_t)$ is linear, if there is no risk in period 2, if $k = 0$, or if the risk on \tilde{x}_t is independently distributed. I show and

discuss this in appendix 7.3. In the next section I assume that these assumptions are not met, hence a horizon effect may exist.

4.3 Existence and direction of the horizon effect

I examine the instantaneous consumption discount rate $DR_{1,2}^{\bar{T}=\infty}$ of a temporally risk averse Risk-Sensitive decision maker under a set of assumptions that are standard in the discounting literature. In particular, I assume that the decision maker has at least some valuation for generations living in $t \geq 2$ ($\beta > 0$), that the felicity function is concave and characterised by constant elasticity of substitution (CES), and that consumption *growth* is risky and independently distributed. Note that the riskiness of consumption growth implies that risk on consumption itself cannot be independently distributed.

The discount rate of the decision maker under consideration is specified in (11) with $k > 0$ and $0 < \beta < 1$.¹³ The consumption growth rate $g_t = \frac{x_t}{x_{t-1}} - 1 > -1 \forall t$, is subject to independently distributed risk. Generation t obtains CES felicity from $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ with $\rho < 1$, where $IES = \frac{1}{1-\rho}$ is the intertemporal elasticity of substitution. Given this setting, I examine how the discount rate depends on the horizon in $t \geq 3$. In particular, I prove in appendix 7.4 that the horizon effect reduces the discount rate $DR_{1,2}^{\bar{T}=\infty}$ relative to $DR_{1,2}^{\bar{T}=2}$. This finding is formalised in proposition 1:

Proposition 1 (*Existence and direction of the horizon effect*)

Consider the discount rate of a RS decision maker (equation 11). Assume $k > 0$, $0 < \beta < 1$ and $\tilde{g}_t > -1 \forall t \geq 2$. The horizon effect exists and reduces the discount rate (i.e. $DR_{1,2}^{\bar{T}=\infty} < DR_{1,2}^{\bar{T}=2}$) for either of the two following specifications:

1. $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ ($\rho < 1$, $IES > 0$)

¹³Note that the proof of proposition 1 does not depend on assuming $\beta < 1$. Yet this standard assumption will be convenient for the existence of a limit in the analytical solution of the next section.

where \tilde{g}_2 is risky and g_t is deterministic $\forall t \geq 3$

2. $u(x_t) = \ln x_t$ ($\rho = 0$, $IES = 1$)

where \tilde{g}_t is risky and independently distributed $\forall t \geq 2$

Note that statement 1 in proposition 1 still holds if one employs the more common CES felicity function $u(x_t) = \frac{x_t^\rho}{\rho}$. This is because the discount rate of a RS decision maker is invariant towards the addition of the constant $-\frac{1}{\rho}$ to felicity. Note furthermore that proposition 1 also holds if one substitutes $DR_{1,2}^{\bar{T}=\infty}$ by $DR_{1,2}^{\bar{T}}$ with $\bar{T} = 3, 4, \dots, \infty$. That is, the findings on the existence and the direction of the horizon effect are not restricted to an infinite horizon setting, but hold for any discount rate that is examined in a setting with $\bar{T} \geq 3$. This is obvious from the proof of proposition 1 in appendix 7.4, which does not depend on assuming $\bar{T} = \infty$ but merely presumes that $\bar{T} \geq 3$.

Proposition 1 states that the instantaneous consumption discount rate of a RS decision maker in a standard discounting setting depends on the horizon after the period for which one discounts, i.e. on the horizon after period 2. The standard practise of cutting off the horizon after the period of discount, i.e. looking at $DR_{1,2}^{\bar{T}=2}$ rather than $DR_{1,2}^{\bar{T}=\infty}$ as in Gollier (2002a) and Hector (2013), is therefore problematic in the context considered here. How problematic it is depends on the size of the horizon effect, which I elaborate on in the next section.

On first sight, the existence of the horizon effect may seem to be at odds with the assumption of mutual utility independence: Imposing mutual utility independence on preferences - and hence imposing history and future independence - leads to a discount rate that explicitly depends on the future through the horizon effect. On closer inspection, this result is not surprising. To see this, recall that the combination of MUI and KP recursivity, i.e. assuming RS preferences, implies the additive separability of the decision maker's utility function if risk on consumption \tilde{x}_t is independently distributed. A discount rate that is derived from such an additively separable utility

function is not subject to a horizon effect. The independence of preferences over risk in period 2 from the consumption levels in $t \neq 2$, *together* with the statistical independence of consumption risk, imply the absence of a horizon effect. The horizon effect enters the stage only as we give up the statistical independence of consumption risk and instead assume independently distributed risk on growth. Risk on consumption growth in period 2 (whether independently distributed or not) goes along with correlated risk on consumption at each $t \geq 2$. Hence, risk in period 2 does not only affect the riskiness of period 2 consumption, but also affects the riskiness of consumption in $t = 3, 4, \dots$ and thereby leads to risk on (continuation) utility U_3 . The more periods are aggregated in U_3 , i.e. the longer the horizon is, the bigger is this risk on continuation utility in absolute terms. A Risk-Sensitive decision maker is averse towards risk on continuation utility and thus adjusts the discount rate in accordance with the size of this risk. This is eventually reflected in the horizon effect.

The technicalities behind the horizon effect can be sketched by a simple example. Suppose for simplicity that $\bar{T} = 3$ is the last period in which a generation exists. Only second period consumption growth \tilde{g}_2 is risky. Consumption growth in the third period, g_3 , is deterministic and thus independent of the risk in period 2. Period 3 consumption itself is not independent of period 2 consumption: both are functions of \tilde{g}_2 . The consumption levels in periods 2 and 3 are given by $\tilde{x}_2 = (1 + \tilde{g}_2)x_1$ and $\tilde{x}_3 = (1 + \tilde{g}_2)(1 + g_3)x_1$. The discount rate of the RS decision maker can then be written as

$$DR_{1,2}^{\bar{T}=3} = -\ln \beta - \ln \frac{E_1 \left[\frac{\exp(-ku(\tilde{x}_2)) \exp(-k\beta u(\tilde{x}_3))}{E_1[\exp(-ku(\tilde{x}_2)) \exp(-k\beta u(\tilde{x}_3))]} u'(\tilde{x}_2) \right]}{u'(x_1)}.$$

The exponential that contains (continuation) utility $u(\tilde{x}_3)$ cannot be taken out of the expectation operator in the adjustment factor since \tilde{x}_3 , like \tilde{x}_2 , is conditional on period 2 information. Hence, period 3 values do not cancel out. The risk aversion adjustment factor is thus a function of the horizon after period 2 and $DR_{1,2}^{\bar{T}=3}$ is

consequently subject to a horizon effect.

4.4 Analytical solution

I restrict the setting of the last section further to derive an analytical solution for the instantaneous consumption discount rate. This allows for a comparison with the Ramsey Equation and its extensions. It also provides insights on the magnitude of the horizon effect and its interrelation with the rate of pure time preference.

In line with the standard in the discounting literature, I assume that growth rates are not only independently but also normally distributed at each point in time, i.e. $\tilde{g}_t \sim N(\mu_t, \sigma_t^2)$. Under this additional assumption and with $u(x_t) = \ln x_t$ ($IES = 1$), it is possible to derive an analytical solution for the instantaneous consumption discount rate of a RS decision maker (equation 11).

For a general horizon \bar{T} , I show in appendix 7.5 that the analytical solution is

$$DR_{1,2}^{\bar{T}} = -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k - \frac{\sigma_2^2}{2} 2k\beta \sum_{\tau=3}^{\bar{T}} \beta^{\tau-3}. \quad (13)$$

As $\bar{T} \rightarrow \infty$, the geometric series in the last term of equation (13) approaches the limit $\frac{1}{1-\beta}$. The analytical solution for the discount rate that corresponds to situation *A* is thus

$$DR_{1,2}^{\bar{T}=\infty} = -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k - \frac{\sigma_2^2}{2} 2k \frac{\beta}{1-\beta}. \quad (14)$$

For $\bar{T} = 2$, i.e. in situation *B* in which the horizon is cut off after the period to which the discount rate applies, the sum in (13) is zero. The analytical solution for the discount rate that corresponds to situation *B* is thus

$$DR_{1,2}^{\bar{T}=2} = -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k. \quad (15)$$

The difference between equations (14) and (15) constitutes the horizon effect that

acts on $DR_{1,2}^{\bar{T}=\infty}$. The horizon effect drives a wedge between the discount rate in an infinite horizon setting and that in a 2-period setting:

$$DR_{1,2}^{\bar{T}=\infty} - DR_{1,2}^{\bar{T}=2} = -\frac{\sigma_2^2}{2} 2k \frac{\beta}{1-\beta}. \quad (16)$$

Equation (16) confirms statement 2 of proposition 1 for a setting with $\tilde{g}_t \sim N(\mu_t, \sigma_t^2)$. Since $k > 0$ and $0 < \beta < 1$, the horizon effect reduces $DR_{1,2}^{\bar{T}=\infty}$ relative to $DR_{1,2}^{\bar{T}=2}$ as long as second period growth is risky ($\sigma_2^2 \neq 0$).

4.5 Comparison to the literature

To connect to the Ramsey Equation and its previous extensions, I rewrite equation (14) in terms of the utility discount rate $\delta = -\ln \beta$. I refer to this analytical solution as the Extended Ramsey Equation for RS preferences.

Definition 7 (*Extended Ramsey Equation for RS preferences*)

Given independently and normally distributed risk on \tilde{g}_t and $u(x_t) = \ln x_t$, the instantaneous consumption discount rate of a RS decision maker (equation 11) is written as

$$DR_{1,2}^{\bar{T}=\infty} = \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k - \frac{\sigma_2^2}{2} 2k \frac{1}{\delta} \quad (17)$$

which is denoted ‘Extended Ramsey Equation for RS preferences’.

The Ramsey Equation (Ramsey 1928) constitutes the most widely accepted organising principle for deterministic consumption discounting in an intergenerational context (Arrow et al. 2012). The Ramsey equation is written as

$$DR_{1,2}^{RE} = \delta + (1 - \rho) \mu_2, \quad (18)$$

where $(1 - \rho) = 1/IES$ and $\mu_2 = g_2$ since growth is deterministic. The first term in equation (18) is the rate of pure time preference. It discounts second period

felicity according to the decision maker's degree of impatience or empathic distance. The second term is called the wealth effect. This effect accounts for consumption discounting due to differences in the consumption levels of the first and the second generation. An increase in period 2 consumption is less valuable than an increase in present consumption if the second generation is richer than the present generation ($g_2 > 0$), and if the decision maker is averse towards such consumption inequalities ($IES > 0$).

The first two terms of the Extended Ramsey Equation for RS preferences (equation 17) correspond to the $IES = 1$ specification of the Ramsey Equation (equation 18). The last three terms in (17) are nil in the deterministic additive discounted utility environment of the Ramsey Equation ($\sigma_2^2 = 0$, $k = 0$).

The Extended Ramsey Equation (Gollier 2002a, 2002b) extends the Ramsey Equation to a world with normally distributed risk on the consumption growth rate \tilde{g}_2 . It is written as

$$DR_{1,2}^{EEE} = \delta + (1 - \rho) \mu_2 - \frac{\sigma_2^2}{2} (1 - \rho)^2. \quad (19)$$

The third term in (19) reduces the consumption discount rate according to the decision maker's aversion towards second period risk. Note that risk aversion is measured by the inverse of the IES in this setting, i.e. by the factor $(1 - \rho)$ in the last term of (19). Since risk aversion cannot be too high in a setting where risk aversion and the IES are entangled, the last term is small for moderate sizes of σ_2^2 .

The first three terms of the Extended Ramsey Equation for RS preferences (17) correspond to the $IES = 1$ specification of equation (19). Since the Extended Ramsey Equation yields the discount rate of a decision maker with additive expected utility preferences ($k = 0$), the last two terms of (17) are not present in (19).

The Extended Ramsey Equation for Epstein-Zin preferences (Traeger 2011, 2014) yields the consumption discount rate of a KP recursive decision maker under an EZ

parameterisation:¹⁴

$$DR_{1,2}^{EZ} = \delta + (1 - \rho)\mu_2 - \frac{\sigma_2^2}{2}(1 - \rho)^2 - \frac{\sigma_2^2}{2}RIRA|1 - (1 - \rho)^2| \text{ if } \rho \neq 0 \quad (20)$$

$$DR_{1,2}^{EZ} = \delta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2}(-2\alpha) \quad \text{if } \rho = 0 \quad (21)$$

The respective decision maker is characterised by CES felicity $u(x_t) = \frac{x_t^\rho}{\rho}$ and faces independently and normally distributed growth risk. The decision maker's degree of relative temporal risk aversion is measured by $RIRA$, which is a function of ρ and of Arrow Pratt risk aversion $(1 - \alpha)$. For a given IES , a temporally risk averse decision maker ($RIRA > 0$, $\alpha < \rho$) is more risk averse than an additive expected utility decision maker. The discount rate of such a temporally risk averse decision maker is thus smaller than the discount rate that results from Gollier's Extended Ramsey Equation, which is obtained for $RIRA = 0$ ($\rho = \alpha$).

The first four terms of the Extended Ramsey Equation for RS preferences (17) resemble the $\rho = 0$ ($IES = 1$) specification of the Extended Ramsey Equation for EZ preferences (equation 21). The Extended Ramsey Equation for EZ preferences, however, is not subject to a horizon effect. This is true regardless of the number of periods taken into account in the underlying decision problem. In fact, the planning horizon \bar{T} of the setting in which Traeger (2011) derives equation (20) is finite but exceeds the period to which the discount rate applies (here: period 2). My calculations in appendix 7.6 confirm the absence of the horizon effect in the discounting function of an EZ decision maker.

These comparisons with the Ramsey Equation and its previous extensions highlight the novelty of the horizon effect. The fifth term in (17), which constitutes the horizon effect, is unique to the consumption discount rate $DR_{1,2}^{\bar{T}=\infty}$ of a decision maker

¹⁴See Traeger (2011) for a derivation of this equation in a multiperiod setting or Traeger (2014) for a derivation in a two period setting. Note that Traeger refers to this equation as the 'consumption discount rate in the isoelastic setting with intertemporal risk aversion' rather than as the 'Extended Ramsey Equation for EZ preferences'.

with Risk-Sensitive preferences. Since the rate of pure time preference δ is usually considered to be small, the horizon effect may be quite significant, even for moderate degrees of temporal risk aversion. I enlarge upon this point in the next section.

Before I close this section, let me point to an apparent inconsistency that stands out when we compare the Extended Ramsey Equations for RS and EZ preferences with $IES = 1$. To see this apparent inconsistency, note that the Risk-Sensitive and the Epstein-Zin specification of Kreps-Porteus recursive preferences are equivalent if $u(x_t) = \ln x_t$, i.e. if $IES = 1$. One would thus expect to find the same instantaneous consumption discount rate for both specifications in this special case. What I find here, instead, is that the Extended Ramsey Equation for RS preferences is subject to a horizon effect, whereas the Extended Ramsey Equation for EZ preferences is free from a horizon effect.

The cause of this apparent inconsistency is that a homogeneous CES felicity function, $u(x_t) = \frac{x_t^\rho}{\rho}$, is employed for the derivation of (20) and (21). The homogeneity of this function eliminates the horizon effect in the EZ case, as is evident from the calculations in appendix 7.6. A logarithmic felicity function in the contrary, which is often treated as the limit of $u(x_t) = \frac{x_t^\rho}{\rho}$ when $IES = 1$, is not homogeneous. In fact, $u(x_t) = \ln x_t$ is not the limit of the homogeneous CES function $u(x_t) = \frac{x_t^\rho}{\rho}$, but rather the limit of the non-homogeneous CES function $u(x_t) = \frac{x_t^\rho - 1}{\rho}$. These two specifications are often used interchangeably since the addition of the constant $-\frac{1}{\rho}$ to $\frac{x_t^\rho}{\rho}$ does not change preferences over x_t . What the addition of this constant does, however, is to eliminate the homogeneity of $u(x_t)$. Without this homogeneity, there may exist a horizon effect, even for a decision maker with EZ preferences.

4.6 The role of the rate of pure time preference

Much of the disagreement on the adequate size of the consumption discount rate stems from different views on the proper value of the rate of pure time preference

$\delta = -\ln \beta$. On one side of the debate are the prescriptionists who argue for $\delta = 0$ in deterministic contexts, since in their opinion, there is no ethical justification to value future generations less than current generations. An argument for $\delta > 0$ which is often accepted by this group is to discount felicity due to a positive probability for the extinction of future generations. The low rate of pure time preference applied in the Stern Review (Stern 2007), $\delta = 0.001$, reflects this attitude. On the other side of the debate are the proponents of the descriptive approach, who require the consumption discount rate to reflect market interest rates. The rate of pure time preference estimated from financial market data is generally higher than that accepted by the prescriptionists. Nordhaus (2008), e.g., employs a rather high rate of pure time preference of $\delta = 0.015$.

In the absence of a horizon effect, the connection between the rate of pure time preference and the consumption discount rate is a one to one relationship. Increasing δ augments the consumption discount rate by the same amount. In this case, the task of the rate of pure time preference is solely to discount the felicity of the generation to which the consumption discount rate applies. In the Ramsey Equation and its previous extensions, all of which are not subject to a horizon effect, δ takes on this single role.

If the consumption discount rate is subject to a horizon effect as in the Extended Ramsey Equation for RS preferences, the role of the rate of pure time preference is twofold. As in the Ramsey Equation and its previous extensions, δ accounts for discounting second period felicity. This role is assumed by the first term on the right hand side of equation (17). Yet δ also appears in the term which defines the horizon effect, namely in the last term on the right hand side of equation (17). The impact of δ in this role is such that the absolute magnitude of the horizon effect decreases as δ increases. A smaller absolute magnitude of the horizon effect then implies a bigger discount rate, since the horizon effect impacts $DR_{1,2}^{\bar{T}=\infty}$ negatively. Corollary

1 summarises this twofold role of the rate of pure time preference in the Extended Ramsey Equation for RS preferences.

Corollary 1 *(A twofold role of the rate of pure time preference)*

Consider the Extended Ramsey Equation for Risk-Sensitive preferences (equation 17).

The rate of pure time preference affects $DR_{1,2}^{\bar{T}=\infty}$ positively through two distinct terms:

term 1 of (17): the bigger δ is, the more is period 2 felicity discounted

term 5 of (17): the bigger δ is, the smaller is the absolute value of the horizon effect

The significance of the rate of pure time preference in determining the size of the horizon effect can easily be demonstrated. Compare the size of the horizon effect for Stern's parameter value to that of Nordhaus. For Stern's $\delta = 0.001$, the fifth term in equation (17) is $-\frac{\sigma_2^2}{2}2k \cdot 1000$. For Nordhaus' $\delta = 0.015$, the horizon effect takes on the value $-\frac{\sigma_2^2}{2}2k \cdot 67$. The horizon effect is thus 15 times bigger (in absolute terms) under Stern's value for the rate of pure time preference.

It is evident from this example that the rate of pure time preference plays an important role in determining the size of the horizon effect. Whether the horizon effect itself plays an important role in determining the size of the instantaneous consumption discount rate however, also depends on the values of σ_2^2 and k .

Kocherlakota (1996) estimates the standard deviation of the consumption growth rate from US time series data to be $\sigma = 3.6\%$. Gollier (2002a) and Traeger (2011, 2014) use this estimate (or a rounded 4%) in the Extended Ramsey Equation and the Extended Ramsey Equation for EZ preferences. With $\sigma_2 = 0.036$, the horizon effect in equation (17) takes on the value $-1.3k$ for $\delta = 0.001$ and $-0.09k$ for $\delta = 0.015$.

Choosing an adequate value for k is problematic. The value of k is not only highly relevant in determining the size of the horizon effect, but also largely unexplored. Plugging in 'best guesses' for the value of k may illustrate the significance of the horizon effect in determining the consumption discount rate. Approximating a reasonable

range for the value of k , however, requires a thorough discussion which extends the scope of the present analysis. I defer this discussion to future research.

5 Conclusion

I examined the instantaneous consumption discount rate of a decision maker whose preferences are Kreps-Porteus recursive and mutually utility independent. In a first instance, I showed that such preferences are restricted to the Risk-Sensitive preference specification of Hansen and Sargent (1995). I then went on to analyse the discount rate of a decision maker with Risk-Sensitive preferences. The analysis was conducted in a setting with constant elasticity of substitution and independently distributed risk on consumption growth. I showed that this discount rate may be subject to a horizon effect whenever the horizon of the decision maker's intertemporal utility function extends the period to which the discount rate applies. To compare to the Ramsey Equation, the Extended Ramsey Equation, and the Extended Ramsey Equation for Epstein-Zin preferences, I derived an analytical solution for the discount rate under consideration. To this end, I restricted the setting further such that the *IES* equals one and consumption growth risk is normally distributed. The resulting discounting function was denoted as the Extended Ramsey Equation for Risk-Sensitive preferences. On the basis of this analytical solution, I highlighted the twofold role which the rate of pure time preference takes on in a discounting equation that is subject to a horizon effect.

The technicalities that lead to the horizon effect are straightforward. The horizon effect is a direct implication of a number of assumptions which, taken individually, are either standard or are considered to be suitable for intergenerational discounting in my analysis. These assumptions are risk on consumption growth, an infinite horizon, Kreps-Porteus recursivity, and mutual utility independence. Assuming (independently distributed) risk on growth is common in the discounting literature and

more in line with reality than an assumption of independently distributed risk on consumption itself. Postulating an infinite horizon is less arbitrary and more general than cutting off the horizon at some period. Employing the Kreps-Porteus recursive framework rather than the additive expected utility model allows for the disentanglement of the degree of risk aversion from the *IES*, and thus for a more flexible parameterisation of the decision maker's preferences.

Determining the appropriateness of mutual utility independence in the context of intergenerational discounting requires closer examination. Albeit no such examination exists in the literature, MUI prevails as an assumption on preferences in the most popular discounting equations, namely in the Ramsey Equation and the Extended Ramsey Equation. These equations are derived from intertemporally additive utility functions (the discounted utility model and the additive expected utility model) which build on an implicit or explicit MUI assumption. Critics of the discounted utility model sometimes argue that MUI is too restrictive and fails to comply with preference reversals or habit formation as empirically observed in the preferences of individuals.¹⁵ However, this criticism is geared towards the preferences of an individual over his lifetime consumption, rather than towards the preferences of a decision maker over the consumption of several generations. Existing criticism regarding the MUI assumption does therefore not apply in the context of the present analysis.

If the prevalence of MUI in the most popular discounting equations says anything about its validity in intergenerational discounting, one can conclude that it is an appropriate assumption. Furthermore, a first intuition suggests that MUI is an attractive assumption from a normative point of view - especially in the context of intergenerational decision making. Mutual utility independence prevents that preferences that concern one generation are conditioned on the wellbeing of other generations. MUI may therefore be considered to be more egalitarian than assuming some form of dependence.

¹⁵See e.g. Fredrick et al. (2002) and Kleindorfer et al. (1993) who enlarge upon this criticism.

I conclude with two suggestions for future research. First, the validity of the MUI assumption in the context of intergenerational decision making deserves further study from normative economics and moral philosophy. I showed that imposing MUI on KP recursive preferences has severe implications for the determinants of the discount rate. Second, a thorough discussion on the size of temporal risk aversion in the Risk-Sensitive framework is needed. Given a range of reasonable values of temporal risk aversion, more illuminating conclusions on the size of the horizon effect could be drawn. Thus, more precise statements on the significance of the horizon effect in determining the consumption discount rate could be made.

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7 Appendix

7.1 Proof of theorem 1

In this proof I show that KP recursive preferences which satisfy mutual utility independence are representable by (4). First, I denote by ${}_3x = (x_3, x_4, x_5, \dots)$, ${}_3x' = (x'_3, x'_4, x'_5, \dots)$ two specific deterministic consumption paths (outcomes) in ${}_3X = X_3 \times X_4 \times X_5 \times \dots$. Using this notation, I consider the specific temporal lotteries $(x_1, p_2, {}_3x), (x_1, p'_2, {}_3x), (x'_1, p_2, {}_3x'), (x'_1, p'_2, {}_3x') \in D$ where x_1 and x'_1 are two specific levels in X_1 and $p_2, p'_2 \in P_2$ are two specific lotteries over X_2 .

Second, I consider a decision maker with KP recursive preferences that are defined on D and satisfy mutual utility independence. Denote these preferences as \succeq^D . By the definition of mutual utility independence for temporal lotteries (definition 5) it must be true that

$$(x_1, p_2, {}_3x) \succeq^D (x_1, p'_2, {}_3x) \iff (x'_1, p_2, {}_3x') \succeq^D (x'_1, p'_2, {}_3x').$$

Employing the notion of conditional preferences we can equivalently write

$$\succeq_{x_1, {}_3x}^D = \succeq_{x'_1, {}_3x'}^D \text{ on } p_2, p'_2 \in P_2, \text{ for (all) } (x_1, {}_3x), (x'_1, {}_3x') \in X_1 \times {}_3X. \quad (22)$$

Now let U^D represent \succeq^D and consider $U^D(x_1, m) = W^D(x_1, E_m[U^D])$. Given the temporal lottery $(x_1, p_2, {}_3x)$, we write $U^D(x_1, p_2, {}_3x) = W^D(x_1, E_m[U^D(x_2, {}_3x)])$ where $U^D(x_2, {}_3x) = \phi(U^X(x_2, {}_3x))$ and $U^X(x_2, {}_3x) = u(x_2) + \beta U^X({}_3x)$. Now let $v(p_2) = E_m \phi(u(x_2) + \beta U^X({}_3x))$ represent $\succeq_{x_1, {}_3x}^D$. By (22) (i.e. by MUI), $v(p_2)$ represents $\succeq_{x_1, {}_3x}^D \forall {}_3x \in {}_3X$ (and $\forall x_1 \in X_1$). Put differently, a certainty equivalent \hat{x}_2 (which could be derived from $v(p_2)$), which makes a decision maker with $\succeq_{x_1, {}_3x}^D$ indifferent to receiving the lottery p_2 , is independent of the specific level of ${}_3x$. This just means that a decision maker with preferences $\succeq_{x_1, {}_3x}^D$ is constantly absolute risk

averse, which in turn implies $\phi(z) = -\exp(-kz)$. Using $\phi(z) = -\exp(-kz)$ in (3) and renormalising by $U = \phi^{-1}(U^D)$ yields (4).

7.2 RS preferences for independently distributed \tilde{x}_t

Suppose preferences are represented by (4) and \tilde{x}_t for $t > 1$ is risky and independently distributed. Plugging continuation utilities $U(x_2, m)$, $U(x_3, m), \dots$ into the initial utility function $U(x_1, m)$ yields

$$U(x_1, m) = u(x_1) - \frac{\beta}{k} \ln \left(E_m \left[\exp(-ku(\tilde{x}_2)) \left(E_m \left[\exp(-ku(\tilde{x}_3)) (E_m \exp(-k(\dots)))^\beta \right]^\beta \right) \right] \right).$$

Since \tilde{x}_t is independently distributed, the last equation can be written as

$$U(x_1, m) = u(x_1) - \frac{\beta}{k} \ln E_m \exp(-ku(\tilde{x}_2)) - \frac{\beta^2}{k} \ln (E_m \exp(-ku(\tilde{x}_3))) - \frac{\beta^3}{k} \ln (E_m \exp(-k(\dots))).$$

Now note that the terms $-\frac{1}{k} \ln (E_m \exp(-ku(\tilde{x}_t)))$ can be substituted for by $u(\hat{x}_t)$ since they determine certainty equivalent consumption \hat{x}_t . Thus we can further simplify the last equation and write

$$U(x_1, m) = u(x_1) + \beta u(\hat{x}_2) + \beta^2 u(\hat{x}_3) + \dots = u(x_1) + \beta \sum_{t=2}^{\infty} \beta^{t-2} u(\hat{x}_t).$$

7.3 Absence of the horizon effect

In this section I discuss conditions under which the instantaneous consumption discount rate of a decision maker with Risk-Sensitive preferences (equation 11) is not subject to a horizon effect. Although these conditions are fairly obvious, I evolve on them to facilitate the general understanding of the horizon effect.

The requirement for the absence of the horizon effect is that $DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2}$. This requirement is met under the following specifications.

1) $\beta = 0$

If $\beta = 0$, the utility function of a RS decision maker (equation 4) is $U_1 = u(x_1)$, independently of the length of the horizon \bar{T} taken into account. The respective decision maker has no valuation for generations living in $t \geq 2$ and therefore applies an infinite discount rate to period 2 consumption values. This is true irrespective of the existence of generations in periods $t \geq 3$. Hence the discount rates $DR_{1,2}^{\bar{T}=\infty}$ and $DR_{1,2}^{\bar{T}=2}$ are equivalent and there is no horizon effect.

2) $u(x_t)$ linear

If $u(x_t)$ is linear, $u'(x_t)$ is a constant and thus independent of x_t (which may or may not be risky). Thus one can write $u'(x_1) = u'(\tilde{x}_2) = c$ which reduces (4) to

$$DR_{1,2}^{\bar{T}=\infty} = -\ln \beta - \ln E_1 \left[\frac{\exp(-kU_2)}{E_1[\exp(-kU_2)]} \right] = -\ln \beta.$$

This is equivalent to $DR_{1,2}^{\bar{T}=2}$ under linear $u(x_t)$. Hence $DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2}$.

Intuitively, the absence of the horizon effect is explained by the absence of (risk aversion adjusted) probabilities. Since risk on $u'(\tilde{x}_2)$ plays no role if $u(x_t)$ is linear, there is no role for probabilities or risk aversion adjusted probabilities. The horizon of the decision problem - which enters the discounting equation through the adjustment factor - has therefore no effect on the discount rate.

3) no risk in period 2

If there is no risk in period 2 (but potentially in periods $t > 2$), the risk aversion adjustment factor in equation (4) can be written as

$$\frac{\exp(-kU_2)}{E_1[\exp(-kU_2)]} = \frac{\exp(-ku(x_2)) \cdot \exp(\beta \ln(E_2[\exp(-kU_3)]))}{\exp(-ku(x_2)) \cdot \exp(\beta \ln(E_2[\exp(-kU_3)]))} = 1.$$

The expectation operator E_1 can be neglected since x_2 is certain and since the uncertain continuation utility U_3 is transformed into a certainty equivalent by the expectation operator E_2 . Without the expectation operator E_1 , the numerator and the denominator cancel each other out. The discount rate is then simply

$$DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2} = -\ln \beta - \ln \left[\frac{u'(x_2)}{u'(x_1)} \right].$$

The intuition is as in the case where $u(x_t)$ is linear. If there is no risk on $u'(x_2)$, then there is no role for risk aversion adjustment factors and thus no channel through which the horizon $t \geq 3$ could enter the discounting function.

4) $k = 0$

If $k = 0$, the risk aversion adjustment factor $\frac{\exp(-kz)}{E_1[\exp(-kz)]}$ equals one. Thus,

$$DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2} = -\ln \beta - \ln E_1 \left[\frac{u'(\tilde{x}_2)}{u'(x_1)} \right].$$

The intuitive explanation is that $k = 0$ restricts $DR_{1,2}^{\bar{T}=\infty}$ to the discount rate of a decision maker who is temporally risk neutral, i.e. a decision maker whose preferences are representable by an additive expected utility function. A discount rate that is derived from an additive utility function only depends on values of the present period and values of the period that is discounted. This means that the instantaneous discount rate $DR_{1,2}^{\bar{T}=\infty}$ is independent of values in periods $t \geq 3$, and thus not subject to a horizon effect.

5) risk on \tilde{x}_t independently distributed

If risk on \tilde{x}_t is independently distributed, then the discounting equation (11) of the Risk-Sensitive decision maker can be written as

$$DR_{1,2}^{\bar{T}=\infty} = \delta - \ln E_1 \left[\frac{\exp(-ku(\tilde{x}_2)) \cdot \exp(\beta \ln(E_2[\exp(-kU_3)]))}{E_1[\exp(-ku(\tilde{x}_2))] \cdot \exp(\beta \ln(E_2[\exp(-kU_3)]))} \frac{u'(\tilde{x}_2)}{u'(x_1)} \right].$$

The exponential function containing U_3 was taken out of the expectation operator E_1 since the risk contained in U_3 is independent of period 2 information. As this exponential appears in the numerator as well as in the denominator, it cancels out and we get $DR_{1,2}^{\bar{T}=\infty} = DR_{1,2}^{\bar{T}=2}$.

The absence of a horizon effect for independently distributed \tilde{x}_t is a direct consequence of the mutual utility independence of the decision maker. I already showed in equation (6) of section 3 that a RS decision maker who faces independently distributed \tilde{x}_t has an additive utility function. As with $k = 0$, the additivity of the decision maker's utility function implies the absence of a horizon effect.

7.4 Proof of proposition 1

In this section I proof that the horizon effect diminishes the discount rate under the conditions stated in proposition 1. The proof employs the notions of comonotonicity and countercomonotonicity which are defined as follows.

Definition 8 (*Strict comonotonicity and strict countercomonotonicity*).

Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations of a single random variable \tilde{x} :

$$(Z_1, Z_2) = (g_1(\tilde{x}), g_2(\tilde{x})).$$

If g_1 and g_2 are strictly increasing in \tilde{x} , then Z_1 and Z_2 are called comonotonic.

If g_1 is strictly increasing and g_2 is strictly decreasing in \tilde{x} , or vice versa, then Z_1 and Z_2 are called countercomonotonic.

Furthermore, the proof uses a lemma that I refer to as the risk aversion adjusted covariance inequality. Before stating the lemma, I define formally what I mean by a risk aversion adjusted probability, a risk aversion adjusted expectation operator and

a risk aversion adjusted covariance. A prolonged discussion of these concepts can be found in Hector (2013).

A risk aversion adjusted probability twists the statistical probability of a given state of the world $\omega = 1, \dots, N$ to account for temporal risk aversion with respect to continuation utility. In particular, I define the risk aversion adjusted probability of state ω as

$$\pi^\omega = l^\omega \frac{\phi'(U_2)}{\phi'(\phi^{-1}(E_1[\phi(U_2)]))},$$

where l^ω is the statistical probability of state ω and the fraction is the risk aversion adjustment factor as mentioned in section 4.1. Note that $0 \leq \pi^\omega \leq 1 \forall \omega$ and $\sum_{\omega=1}^N \pi^\omega = 1$, which allows for the interpretation of π^ω as a probability.

A risk aversion adjusted expectation operator for a random variable \tilde{x} and some function $g(\tilde{x})$ is then defined as

$$E^\pi [g(\tilde{x})] = \sum_{\omega=1}^N \pi^\omega g(x^\omega).$$

This expectation operator employs risk aversion adjusted probabilities in the place of statistical probabilities.

Finally, a risk aversion adjusted covariance between two random variables or functions $g_1(\tilde{x}_1)$ and $g_2(\tilde{x}_2)$ is a covariance which is constructed from risk aversion adjusted expectation operators:

$$cov^\pi [g_1(\tilde{x}_1), g_2(\tilde{x}_2)] = E^\pi [g_1(\tilde{x}_1) g_2(\tilde{x}_2)] - E^\pi [g_1(\tilde{x}_1)] E^\pi [g_2(\tilde{x}_2)].$$

We are now ready to state a lemma on the risk aversion adjusted covariance inequality. A proof of this lemma, which is a close analogue to theorem 43 in Hardy et al. (1934), is contained in Hector (2013).

Lemma 1 (*Risk aversion adjusted covariance inequality*).

Consider two random variables Z_1 and Z_2 that are strictly monotonic transformations of a single random variable \tilde{x} . If Z_1 and Z_2 are strictly comonotonic, then

$$\text{cov}^\pi [Z_1, Z_2] > 0.$$

The inequality is reversed if Z_1 and Z_2 are strictly countercomonotonic.

Let us now turn to the actual proof of proposition 1. The decision maker under consideration has mutually utility independent KP recursive preferences. His instantaneous discount rate for a setting with horizon $\bar{T} > 2$ is thus given by (11). Assume that $k > 0$, $0 < \beta < 1$ and $g_t > -1 \forall t \geq 2$.

To prove statement 1 of proposition 1, assume furthermore that felicity is given by $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ with $\rho < 1$ and that only second period consumption growth \tilde{g}_2 is risky. The consumption growth rate in $t \geq 3$ is deterministic.

To prove statement 2 of proposition 1, assume that felicity is given by $u(x_t) = \ln(x_t)$ and that consumption growth \tilde{g}_t in $t \geq 2$ is risky and independently distributed.

proof of statement 1

Suppose $u(x_t) = \frac{x_t^\rho - 1}{\rho}$ with $\rho < 1$ and only period 2 growth is uncertain. Continuation utility U_2 can be rewritten in a simple manner since all risk resolves in period 2:

$$U_2 = u(\tilde{x}_2) + \sum_{t=3}^{\bar{T}} \beta^{t-2} u(\tilde{x}_t). \quad (23)$$

With $\tilde{x}_2 = (1 + \tilde{g}_2)x_1$, felicity in $t \geq 3$ can be written as

$$u(\tilde{x}_t) = u\left(\tilde{x}_2 \prod_{\tau=3}^t (1 + g_\tau)\right) = \frac{\tilde{x}_2^\rho}{\rho} \left[\prod_{\tau=3}^t (1 + g_\tau) \right]^\rho - \frac{1}{\rho}. \quad (24)$$

Plugging the felicity function (equation 24) into the continuation utility U_2 (equation 23) and (23) into the discounting function (equation 11), $DR_{1,2}^{\bar{T}}$ is given by

$$DR_{1,2}^{\bar{T}} = -\ln \beta - \ln E_1 \left[\frac{\exp \left(-ku(\tilde{x}_2) - \frac{k}{\rho} \tilde{x}_2^\rho h(\bar{T}) \right) u'(\tilde{x}_2)}{E_1 \left[\exp \left(-ku(\tilde{x}_2) - \frac{k}{\rho} \tilde{x}_2^\rho h(\bar{T}) \right) \right] u'(x_1)} \right] \quad (25)$$

where $h(\bar{T}) = \sum_{t=3}^{\bar{T}} \left(\beta^{t-2} \left[\prod_{\tau=3}^t (1 + g_\tau) \right]^\rho \right)$.

In the next step I study how $DR_{1,2}^{\bar{T}}$ changes as the horizon \bar{T} changes. To this end I would need to examine the derivative of $h(\bar{T})$ with respect to \bar{T} . However, as the domain of $h(\bar{T})$ is discrete, $h'(\bar{T})$ does not exist. Thus I define a function $\hat{h}(\bar{T}) : \mathbb{R}^+ \rightarrow \mathbb{R}$ with $\hat{h}(\bar{T}) = h(\bar{T}) \forall \bar{T} \in \mathbb{N}$. The function $\hat{h}(\bar{T})$ is assumed to constitute a smooth interpolation between the discrete points defined by $h(\bar{T})$ at all $\bar{T} \in \mathbb{N}$. I then examine the derivative of $\hat{h}(\bar{T})$ rather than that of $h(\bar{T})$. Since $\hat{h}(\bar{T})$ is strictly increasing, its derivative $\hat{h}'(\bar{T})$ is positive.

Substituting all $h(\bar{T})$ by $\hat{h}(\bar{T})$ and taking the derivative of (25) with respect to \bar{T} yields

$$\frac{\partial DR_{1,2}^{\bar{T}}}{\partial \bar{T}} = \frac{E_1 \left[f(\tilde{g}_2) u'(\tilde{x}_2) \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right]}{E_1 \left[f(\tilde{g}_2) u'(\tilde{x}_2) \right]} - \frac{E_1 \left[f(\tilde{g}_2) \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right]}{E_1 \left[f(\tilde{g}_2) \right]} \quad (26)$$

where $f(\tilde{g}_2) = \exp \left(-ku(\tilde{x}_2) - \frac{k}{\rho} \tilde{x}_2^\rho \hat{h}(\bar{T}) \right)$.

Thus

$$\frac{\partial DR_{1,2}^{\bar{T}}}{\partial \bar{T}} \gtrless 0$$

whenever

$$\frac{E_1 \left[f(\tilde{g}_2) u'(\tilde{x}_2) \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right]}{E_1 \left[f(\tilde{g}_2) u'(\tilde{x}_2) \right]} - \frac{E_1 \left[f(\tilde{g}_2) \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right]}{E_1 \left[f(\tilde{g}_2) \right]} \gtrless 0.$$

After multiplying the last equation with $E_1 [f(\tilde{g}_2) u'(\tilde{x}_2)] / E_1 [f(\tilde{g}_2)]$ we can write

$$E_1 \left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} u'(\tilde{x}_2) \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right] - E_1 \left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} u'(\tilde{x}_2) \right] E_1 \left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right] \underset{\leq}{\geq} 0. \quad (27)$$

Now note that $\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} = \frac{\exp(-ku(\tilde{x}_2) - \frac{k}{\rho} \tilde{x}_2^\rho \hat{h}(\bar{T}))}{E_1[\exp(-ku(\tilde{x}_2) - \frac{k}{\rho} \tilde{x}_2^\rho \hat{h}(\bar{T}))]} = \frac{\exp(-kU_2)}{E_1[\exp(-kU_2)]}$ is the risk aversion adjustment factor as mentioned earlier. We can thus rewrite (27) in terms of a risk aversion adjusted expectation operator E_1^π :

$$E_1^\pi \left[u'(\tilde{x}_2) \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right) \right] - E_1^\pi [u'(\tilde{x}_2)] E_1^\pi \left[\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right] \underset{\leq}{\geq} 0. \quad (28)$$

Equation (28) is the risk aversion adjusted covariance between $u'(\tilde{x}_2)$ and $\left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right)$, both of which are functions of the single random variable \tilde{g}_2 . By lemma 1, the sign of the risk aversion adjusted covariance can be determined from the comonotonicity characteristics of $u'(\tilde{x}_2)$ and $\left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right)$. The comonotonicity characteristics in turn are determined by the derivatives of $u'(\tilde{x}_2)$ and $\left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right)$ with respect to the random variable \tilde{g}_2 . Here we have

$$\begin{aligned} \frac{\partial u'(\tilde{x}_2)}{\partial \tilde{g}_2} &= u''(\tilde{x}_2) x_1 < 0 \\ \frac{\partial \left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right)}{\partial \tilde{g}_2} &= k \tilde{x}_2^{-\rho} \hat{h}'(\bar{T}) x_1 > 0, \end{aligned}$$

Hence, $u'(\tilde{x}_2)$ and $\left(\frac{k}{\rho} \tilde{x}_2^\rho \hat{h}'(\bar{T}) \right)$ are countercomonotonic by definition 8. By lemma 1, countercomonotonicity implies a negative risk aversion adjusted covariance (equation 28), which in turn implies $\frac{\partial DR_{1,2}^{\bar{T}}}{\partial \bar{T}} < 0$.

proof of statement 2

Suppose $u(x_t) = \ln x_t$ and consumption growth \tilde{g}_t in $t \geq 2$ is uncertain and indepen-

dently distributed. Starting with the continuation utility in \bar{T} , I plug $U_{\bar{T}}$ into $U_{\bar{T}-1}$, $U_{\bar{T}-1}$ into $U_{\bar{T}-2}$ and so on until I arrive in period $t = 2$:

$$U_2 = \left(\sum_{\tau=2}^{\bar{T}} \beta^{\tau-2} \right) \ln(\tilde{x}_2) - \frac{1}{k} q(\tilde{g}_\tau) \quad (29)$$

$$\text{where } q(\tilde{g}_\tau) = \sum_{\tau=3}^{\bar{T}} \left(\beta^{\tau-2} \ln E_{\tau-1} \left[(1 + \tilde{g}_\tau)^{-k \sum_{\tau'}^{\bar{T}} \beta^{\bar{T}-\tau'}} \right] \right).$$

Equation (29) exposes the additive separability of U_2 into a first term which collects \tilde{g}_2 (note that $\tilde{x}_2 = (1 + \tilde{g}_2) x_1$) and a second term, namely $(-k^{-1} q(\tilde{g}_\tau))$, which collects \tilde{g}_t for $t \geq 3$. The latter term is independent of risk that reveals in period 2. Hence, upon plugging the continuation utility (29) into discounting equation (11), all terms containing $q(\tilde{g}_\tau)$ can be taken out of the expectation operator E_1 and subsequently cancel out. The instantaneous discount rate for a horizon \bar{T} is thus

$$DR_{1,2}^{\bar{T}} = -\ln \beta - \ln E_1 \left[\frac{\exp(-kh(\bar{T}) \ln(\tilde{x}_2)) u'(\tilde{x}_2)}{E_1[\exp(-kh(\bar{T}) \ln(\tilde{x}_2))] u'(x_1)} \right] \quad (30)$$

$$\text{where } h(\bar{T}) = \sum_{\tau=2}^{\bar{T}} \beta^{\tau-2}.$$

Equation (30) depends on the length of the horizon \bar{T} through $h(\bar{T})$.

The direction of this dependency is studied by taking the derivative of $DR_{1,2}^{\bar{T}}$ with respect to \bar{T} . As in the proof of statement 1, we substitute $h(\bar{T})$ by its continuous analogue $\hat{h}(\bar{T})$. Then,

$$\frac{\partial DR_{1,2}^{\bar{T}=\infty}}{\partial \bar{T}} = \frac{E_1 \left[f(\tilde{g}_2) u'(\tilde{x}_2) \left(k \hat{h}'(\bar{T}) \ln(\tilde{x}_2) \right) \right]}{E_1 [f(\tilde{g}_2) u'(\tilde{x}_2)]} - \frac{E_1 \left[f(\tilde{g}_2) \left(k \hat{h}'(\bar{T}) \ln(\tilde{x}_2) \right) \right]}{E_1 [f(\tilde{g}_2)]}$$

$$\text{where } f(\tilde{g}_2) = \exp \left(-k \hat{h}(\bar{T}) \ln(\tilde{x}_2) \right).$$

The direction of the inequality $\frac{\partial DR_{1,2}^{\bar{T}=\infty}}{\partial \bar{T}} \gtrless 0$ is then equivalent to the direction of the inequality

$$E_1 \left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} u'(\tilde{x}_2) \left(k \hat{h}'(\bar{T}) \ln(\tilde{x}_2) \right) \right] - E_1 \left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} u'(\tilde{x}_2) \right] E_1 \left[\frac{f(\tilde{g}_2)}{E_1[f(\tilde{g}_2)]} \left(k \hat{h}'(\bar{T}) \ln(\tilde{x}_2) \right) \right] \gtrless 0,$$

which, as in the precedent proof, can be stated as a risk aversion adjusted covariance:

$$E_1^\pi \left[u'(\tilde{x}_2) \left(k \hat{h}'(\bar{T}) \ln(\tilde{x}_2) \right) \right] - E_1^\pi [u'(\tilde{x}_2)] E_1^\pi \left[k \hat{h}'(\bar{T}) \ln(\tilde{x}_2) \right] \gtrless 0. \quad (31)$$

Since

$$\begin{aligned} \frac{\partial u'(\tilde{x}_2)}{\partial \tilde{g}_2} &= u''(\tilde{x}_2) x_1 < 0 \\ \frac{\partial (k \hat{h}'(\bar{T}) \ln(\tilde{x}_2))}{\partial \tilde{g}_2} &= k \hat{h}'(\bar{T}) \frac{1}{(1 + \tilde{g}_2)} > 0, \end{aligned}$$

$u'(\tilde{x}_2)$ and $(k \hat{h}'(\bar{T}) \ln(\tilde{x}_2))$ are countercomonotonic according to definition 8. By lemma 1 it is then implied that equation (31) is negative and hence $\frac{\partial DR_{1,2}^{\bar{T}}}{\partial \bar{T}} < 0$.

7.5 Analytical solution of the discount rate

Consider a RS decision maker with discount rate (11). Assume consumption growth \tilde{g}_t is risky and independently distributed in all $t \geq 2$. Furthermore assume that $u(x_t) = \ln x_t$ and $\tilde{g}_t > -1 \forall t \geq 2$. In the proof of statement 2 of proposition 1, I already showed that the instantaneous discount in this setting can be written as

$$DR_{1,2}^{\bar{T}} = -\ln \beta - \ln E_1 \left[\frac{\exp(-kh(\bar{T}) \ln(\tilde{x}_2))}{E_1[\exp(-kh(\bar{T}) \ln(\tilde{x}_2))]} \frac{u'(\tilde{x}_2)}{u'(x_1)} \right]$$

where $h(\bar{T}) = \sum_{\tau=2}^{\bar{T}} \beta^{\tau-2}$.

With $\tilde{x}_2 = (1 + \tilde{g}_2) x_1$, $u'(x_t) = x_t^{-1}$, and after some rearrangements and after using $\ln(1 + \tilde{g}_2) \approx \tilde{g}_2$ (for \tilde{g}_2 small), the last equation can be written as

$$DR_{1,2}^{\bar{T}} = -\ln \beta - \ln E_1 \left[\frac{\exp \left((-kh(\bar{T}) - 1) \tilde{g}_2 \right)}{E_1 \left[\exp \left(-kh(\bar{T}) \tilde{g}_2 \right) \right]} \right].$$

Now assume in addition that $\tilde{g}_t \sim N(\mu_t, \sigma_t^2) \forall t \geq 2$. The moment generating function $M_{\tilde{y}}(a) \equiv E[\exp(a\tilde{y})]$ of a normally distributed random variable $\tilde{y} \sim N(\mu, \sigma^2)$ is $M_{\tilde{y}}(a) = \exp\left(a\mu + \frac{\sigma^2}{2}a^2\right)$. Using the moment generating function of \tilde{g}_2 in the last equation, we can write

$$\begin{aligned} DR_{1,2}^{\bar{T}} &= -\ln \beta - \ln \frac{\exp \left((-kh(\bar{T}) - 1) \mu_2 + \frac{\sigma_2^2}{2} (-kh(\bar{T}) - 1)^2 \right)}{\exp \left(-kh(\bar{T}) \mu_2 + \frac{\sigma_2^2}{2} (kh(\bar{T}))^2 \right)} \\ &= -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2kh(\bar{T}) \end{aligned}$$

or equivalently

$$DR_{1,2}^{\bar{T}} = -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k - \frac{\sigma_2^2}{2} 2k\beta \sum_{\tau=3}^{\bar{T}} \beta^{\tau-3}. \quad (32)$$

For $\bar{T} = \infty$, we can substitute the sum in (32) by the limit of a geometric series, $\lim_{\bar{T} \rightarrow \infty} \sum_{t=0}^{\bar{T}} \beta^t = \frac{1}{1-\beta}$, and thus write

$$DR_{1,2}^{\bar{T}=\infty} = -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k - \frac{\sigma_2^2}{2} 2k \frac{\beta}{1-\beta}.$$

If $\bar{T} = 2$, the sum in equation (32) is zero, and the analytical solution for $DR_{1,2}^{\bar{T}=2}$ is thus

$$DR_{1,2}^{\bar{T}=2} = -\ln \beta + \mu_2 - \frac{\sigma_2^2}{2} - \frac{\sigma_2^2}{2} 2k.$$

7.6 No horizon effect for EZ preferences

Epstein-Zin preferences with homogeneous CES felicity $u(x_t) = \frac{x_t^\rho}{\rho}$ are representable by a KP recursive utility function of the form

$$U_t = \frac{x_t^\rho}{\rho} + \beta \left(E_t U_{t+1}^\alpha \right)^\frac{\rho}{\alpha}. \quad (33)$$

This is the KP recursive EZ preference representation employed by Traeger (2011, 2014) from which (20) and (21) can be obtained. The instantaneous discount rate of a decision maker with preferences as in (33) is written as

$$DR_{1,2}^{\bar{T}>2} = -\ln \beta - \ln \frac{E_1 \left[\frac{U_2^{\frac{\alpha}{\rho}-1} u'(\tilde{x}_2)}{\left(E_1 \left[U_2^{\frac{\alpha}{\rho}} \right] \right)^{1-\frac{\rho}{\alpha}}} \right]}{u'(x_1)}, \quad (34)$$

where uncertain consumption \tilde{x}_t can be written as $\tilde{x}_t = (1 + \tilde{g}_2) x_1 \frac{\tilde{x}_t}{(1+\tilde{g}_2)x_1}$. Exploiting the homogeneity of $u(\tilde{x}_t)$ we can write $u(\tilde{x}_t) = \rho \frac{((1+\tilde{g}_2)x_1)^\rho}{\rho} u\left(\frac{\tilde{x}_t}{(1+\tilde{g}_2)x_1}\right) = \rho u(\tilde{x}_2) u\left(\frac{\tilde{x}_t}{\tilde{x}_2}\right)$, where the argument of the latter felicity function is statistically independent of \tilde{g}_2 since risk on growth is independently distributed. Starting with the $t = \bar{T}$ (terminal period) specification of (33), we can solve recursively for continuation utility U_2 . In each recursion, the independent distribution of \tilde{g}_t enables us to factor the term $\rho u(\tilde{x}_2)$ out:

$$\begin{aligned} U_{\bar{T}} &= \rho u(\tilde{x}_2) u(\tilde{x}_{\bar{T}}/\tilde{x}_2) \\ U_{\bar{T}-1} &= \rho u(\tilde{x}_2) \left(u(\tilde{x}_{\bar{T}-1}/\tilde{x}_2) + \beta (E_{\bar{T}-1} [u(\tilde{x}_{\bar{T}}/\tilde{x}_2)]^\frac{\alpha}{\rho})^\frac{\rho}{\alpha} \right) \\ U_{\bar{T}-2} &= \rho u(\tilde{x}_2) \left(u(\tilde{x}_{\bar{T}-2}/\tilde{x}_2) + \beta (E_{\bar{T}-2} [u(\tilde{x}_{\bar{T}-1}/\tilde{x}_2) + \beta (E_{\bar{T}-1} [u(\tilde{x}_{\bar{T}}/\tilde{x}_2)]^\frac{\alpha}{\rho})^\frac{\rho}{\alpha}]^\frac{\alpha}{\rho})^\frac{\rho}{\alpha} \right) \\ U_{\bar{T}-3} &= \rho u(\tilde{x}_2) (\dots). \end{aligned}$$

Finally we arrive at

$$U_2 = \rho u(\tilde{x}_2) h(\bar{T}), \quad (35)$$

where $h(\bar{T})$ is a function that depends on the horizon \bar{T} as well as on the (uncertain) growth rates \tilde{g}_t with $t > 2$. Note that the term $h(\bar{T})$ is independent of risk on period 2 growth, \tilde{g}_2 . Plugging (35) into the discounting equation of the EZ decision maker (equation 34) yields

$$DR_{1,2}^{\bar{T}>2} = -\ln \beta - \ln \frac{E_1 \left[\frac{u(\tilde{x}_2)^{\frac{\alpha}{\rho}-1} h(\bar{T})^{\frac{\alpha}{\rho}-1}}{\left(E_1 \left[u(\tilde{x}_2)^{\frac{\alpha}{\rho}} h(\bar{T})^{\frac{\alpha}{\rho}} \right] \right)^{1-\frac{\rho}{\alpha}}} u'(\tilde{x}_2) \right]}{u'(x_1)}.$$

Since the risk contained in $h(\bar{T})$ is independent of the risk in period 2, $h(\bar{T})$ can be taken out of the expectation operator and thus cancels out. We are left with

$$DR_{1,2}^{\bar{T}>2} = -\ln \beta - \ln \frac{E_1 \left[\frac{u(\tilde{x}_2)^{\frac{\alpha}{\rho}-1} u'(\tilde{x}_2)}{\left(E_1 \left[u(\tilde{x}_2)^{\frac{\alpha}{\rho}} \right] \right)^{1-\frac{\rho}{\alpha}}} \right]}{u'(x_1)}, \quad (36)$$

which is independent of the horizon after $t = 2$, hence $DR_{1,2}^{\bar{T}>2} = DR_{1,2}^{\bar{T}=2}$. I have thus shown that the instantaneous discount rate $DR_{1,2}^{\bar{T}>2}$ of the KP recursive EZ decision maker with homogeneous felicity $u(\tilde{x}_t)$ and independently distributed growth risk is not subject to a horizon effect.