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Don't Kill the Goose that Lays the Golden Eggs: Strategic Delay in Project Completion

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Don’t Kill the Goose that Lays the Golden Eggs: Strategic Delay in Project Completion

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Abstract

It’s puzzling that most projects fail to complete within the predetermined timeframe given that timing considerations rank among the major goals in project management. We argue that when managers can extract private benefits from working on a project, project delay becomes optimal. We introduce a continuous-time framework for project management activities that incorporates this feature. A manager’s unobserved effort cumulatively increases the project’s success probability, but decreases the expected duration of the project and with it the expected flow of on-the-job benefits. A strict deadline limits incentives for effort delay, but also decreases the probability that the project will be terminated in due time. In this trade-off, the optimal deadline balances the increase in expected project value against the expected increase in project duration and costs. Because the manager does not want to “kill the golden goose” prematurely, he always prefers a stricter deadline compared to the principal. As a result, project completion is threatened by both effort provision over time and contractual agreements on time.

Keywords: Optimal deadline, Dynamic incentives, Strategic delay, Project completion

JEL-Codes: D82, M52
1 Introduction

Finishing projects on time is a basic prerequisite of doing business and is essential to a firm’s long-term success. However, many projects fail to complete within the predetermined timeframe imposing costly consequences on firms. Project delay ranks among the most common problems in product development, it captures projects from industrial manufacturing to construction; and in software engineering most projects are characterized by time overruns.¹ For existing theory, the inefficiency of delayed projects raises an explanatory question: Why are scheduling problems not properly taken into account if the associated losses should be foreseen? In this paper, we present a new explanation for this phenomenon that is based on the interaction of timing and incentives. Specifically, we argue that when managers use projects as a strategic instrument for maintaining their private benefits, project delay becomes optimal. To avoid project failure the principal must increase the manager’s time horizon for completing the project. However, increasing project duration increases incentives to postpone effort and produces costs of project delay.

In this context, our paper explores the inherent problem of providing incentives for agents to work effectively on projects that develop over time. Related to the seminal work of Aghion and Tirole (1997), managerial incentives to behave are often non-monetary. Instead, they refer to situations in which managers are motivated by the desire to keep their job because of the attached private benefits, including prestige, third-party-favors, or the gains from empire building. We examine the nature of the dynamic incentives in these settings and endogenize the optimal timing of contracts. Our model shows that on-the-job benefits cause managers to delay effort from early towards later stages of the project, putting project completion at risk. Moreover, the threat of losing the flow of private benefits prematurely induces managers to choose a stricter deadline than it is optimal from the principal’s point of view. The divergent interrelations between incompatible incentives and diverse timing preferences lead us to conclusions that are in line with the observed patterns emphasizing the role of organizational and managerial aspects in explaining project delay (van Genuchten (1991)).

To formally analyze the incentive effect of project deadlines, we introduce a principal-agent framework in continuous time. A principal hires a manager to carry out a project within a finite time horizon. The project’s success probability is stochastic and cumulatively increases with the manager’s effort. The firm does not observe the project’s progress and is only interested in seeing the project complete. The manager also derives utility from a successful project outcome, but additionally enjoys private benefits per period of tenure while working on the project. We will outline that this inconsistency in objectives incentivizes the manager to postpone effort, decreasing the probability of project success.

¹Worldwide data provided by The Economist (2009) shows that only 6% of senior executives and project managers confirm that their organization always delivers projects on time. Thereby, finishing projects on time and on budget is ranked as the most important goal in measuring project success. Similarly, a recent study on IT projects conducted by The Standish Group (2013) reports that 74% of such project are not finished by the predicted time.
Increasing the project deadline contains project failure - however, at the cost of project delay. Thus, for too short maturities, the probability that the project will be completed in due time is low. For too long maturities, the time average value of a successful project realization decreases. Consequently, under certain conditions, there is a positive, but finite optimal contract deadline.

We explore the impact of project deadlines on the incentives provided to managers and the resulting utility implications from a dynamic perspective. Specifically, the larger the manager’s tenure incentives relative to his success incentives, the stricter is the optimal deadline of the project. The intuition is that, if project success is comparatively valuable to the manager, then he should be given a large horizon to make sure the project will be completed in due time. Contrary, if the manager faces large incentives to postpone effort, then only a small horizon can discipline the manager and contain costly project delay. We investigate the boundaries of these findings by studying how the allocation of bargaining power in the negotiation process affects the choice of the optimal deadline. As a result, facing the additional threat of losing the flow of on-the-job benefits prematurely, the manager always prefers a stricter deadline compared to the principal. Our model shows that project failure not only follows from inefficiencies in project completion over time, but is also a consequence of timing considerations in contractual agreements.

**Literature.** Analyses of contract termination generally fall into two groups. The first approach endogenizes continuation and termination of projects. Previous work incorporates learning effects about a project’s fertility (Bergemann and Hege (1998), Simester and Zhang (2010), DeMarzo and Sannikov (2011), Kwon and Lippman (2011)), the incentives to divert cash flows for private benefits (DeMarzo, Fishman, He, and Wang (2012)), as well as exogenous shocks and project risk (Biais, Mariotti, Rochet, and Villeneuve (2010), Hoffmann and Pfeil (2010)). The main focus of our work is to analyze how limiting the time horizon through project deadlines relocates an agent’s dynamic incentives and with it also the stochastic probability and timing of project success. Thus, based on the seminal work of Holmström and Milgrom (1987), our model shares with Sannikov (2008) the existence of dynamic moral hazard and endogenous contract termination in continuous time framework. Specifically, Sannikov studies properties of optimal contracts when the past output path stochastically controls the agent’s continuation value, governing decisions about future employment.

In contrast, our work combines elements of endogenous project termination with models that incorporate timing considerations into contractual agreements. Thus, the other approach, the origin of which traces to Harris and Holmström (1987), focuses on employment duration as an element of optimal contracts. Related to the literature on job matching (see, for example, Jovanovic (1979)), Cantor (1988) analyzes how contract length influences a worker’s effort incentives and recontracting costs in the presence of career concerns. Guriev and Kvasov (2005) model trading, investment, and contracting decisions in continuous time and explore contract length as an incentive instrument. Methodologically, our paper is tied to Yang (2010) who studies team-related moral hazard in a continuous-time framework with finite horizon. The model investigates the optimal allocation of payments
between team members when their efforts decrease the probability of project failure over time. Similarly, in our model, the probability distribution governing the project’s success cumulatively increases with the manager’s efforts such that over time, firm and manager become more optimistic about eventual success of the project. However, Yang allows for a fixed (exogenous) time limit of the contract. In contrast, we lay open the incentive effect of project deadlines as a strategic (endogenous) instrument to punish effort delays.

Our paper most closely relates to the literature on optimal deadlines. Focusing on behavioral effects, Gutierrez and Kouvelis (1991) explore the implications of “Parkinson’s Law” which posits that “work expands as to fill the time available for its completion” (Parkinson, 1957) on the expected duration and optimal deadline of projects that involve multiple activities. O’Donoghue and Rabin (1999) and Herweg and Müller (2011) analyze the value of deadlines when agents procrastinate due to self-control problems. In contrast, Saez-Marti and Sjögren (2008) study the effect of deadlines when agents tends to be distracted from work and therefore front-load effort for precautionary reasons. With fully time-consistent preferences, Toxvaerd (2006) shows that the expected time for project completion mainly depends on the principal’s ability to commit to long-term contracts. Related to this work, Toxvaerd (2007) studies optimal project deadlines in the presence of adverse selection. The model of Bonatti and Hörner (2011) investigates the effect of deadlines in teams when both procrastination and free-riding effects are prevalent. Optimal deadline formation with team production is also analyzed by Campbell, Ederer, and Spinnewijn (2014) who address problems of free-riding and communication that lead to project delays. Lewis (2012) explores properties of optimal contracts in a dynamic theory of delegated search and characterizes compensation and performance deadlines for sequential discoveries. Our dynamic modeling approach is most closely related to Bonatti and Hörner (2014) who investigate effort, learning about ability, and compensation in a continuous-time framework with finite horizon. Endogenous deadlines are studied in terms of an agent’s optimal quitting decision on effort and wages when agents have career concerns. Abstracting from learning effects about ability, our model considers specific incentives provided by the desire to keep the private benefits associated with a job. More specifically, by investigating the dynamic interrelation of tenure versus success incentives, our model analyzes the optimal deadline as an incentive-based instrument to trade off the probability of a project’s success with its expected duration and costs. Overall, our model focuses on the efficient provision of incentives to workers and creates novel insights into the dynamics of optimal contracting for project management activities.

The paper continues in section 2 with the introduction of the analytical model. The analysis starts in section 3 where we explore the problem from the standpoint of the social optimum. Section 4 proceeds with the manager’s dynamic moral hazard problem for project activities. In section 5, we investigate the trade-off for the optimal deadline of a project from the manager’s and principal’s perspectives. Finally, section 6 summarizes the main results.
2 The Model

We study a dynamic principal-agent framework in continuous time. A firm (the principal) employs a manager to carry out a project. At heart of this model, the contract specifies an endogenous deadline $T \in [0, \bar{T}]$, where $\bar{T} > 0$ indicates the endogenous maximum lifetime of the project. The principal does not observe the project’s progress, but only the presence or absence of a success. Denote $\tilde{t}$ the unknown completion time of the project. The principal’s value of a successful realization of the project is $R > 0$ if the project is finished at $\tilde{t} \leq T$ or zero, otherwise. Principal and manager are risk neutral and have reservation utility 0. The manager is protected by limited liability implying that payments to the manager must be non-negative.

Related to Aghion and Tirole (1997), we assume that the manager does not respond to monetary incentives and receives a constant wage equal to his reservation utility of zero. Rather, while working on the project, he gets a private benefit of $b > 0$ per period of tenure. Private benefits might include the usual perks on the job, such as fringe benefits, perquisites, prestige, or gains from empire building. Thus, on the one hand, the longer the project continues, the larger are the aggregate flow benefits the manager can extract from the project. On the other hand, we allow for a fixed increase in the manager’s private benefits by $B > 0$ if the project is completed in due time. This is reasonable if we assume that private benefits are partly correlated with overall project profitability. Also, this interpretation is consistent with a more elaborate approach in which a successful manager can benefit from a good reputation or the possibility of signaling his ability.\footnote{Similarly, Dewatripont and Maskin (1995) distinguish between a good and a bad entrepreneur’s private benefits. Related to this approach, Stein (1997) assumes that private benefits are directly proportional to a project’s cash flow. More generally, Aghion and Tirole (1994) argue that even in the absence of explicit rewards, agents receive informal or non-contractual rewards from a successful innovation. Besides monetary ex-post gains in terms of salary increases or cash awards, such rewards also include reputational benefits and promotions.}

The manager works continuously on the project. His unobservable effort $e(t) \geq 0$ is costly, but cumulatively increases the probability of success. However, effort also decreases the expected duration of the project. If the project is completed at $\tilde{t} < T$, the manager remains employed until time $T$, but engages in a “rulebook slowdown” and enjoys no private benefit flow. Consequently, a subsequent contract can only be signed after expiration of the current contract. Consequently, the choice of the deadline represents a delay cost until both parties become available for a possible next project. That is, the later the project succeeds within the deadline of $T$, the larger are the aggregate private benefits of the manager.

The project’s absolute success rate (probability density of the success time) at time $s \in [0, T]$ is

$$f(s) = \alpha \int_{\tau=0}^{s} e(\tau) \, d\tau$$

\hfill (1)
The parameter $\alpha > 0$ measures the effectiveness of effort in making the innovation (e.g. manager’s experience, knowledge level, or technology-efficiency parameter). Hence, an effort $e(\tau)$ at any instant $\tau \in [0, T]$ increases the success rate by $\alpha e(\tau)$ for all future dates until $T$. Then, the probability of project completion before time $t \in [0, T]$, is given by the distribution function

$$Pr(\tilde{t} \leq t) = F(t) = \int_{s=0}^{t} f(s) \, ds = \int_{s=0}^{t} \int_{\tau=0}^{s} e(\tau) \, d\tau \, ds.$$  

(2)

Accordingly, $Pr(\tilde{t} > t) = 1 - Pr(\tilde{t} \leq t)$ is the probability that the project will not succeed until time $t \in [0, T]$. Thus, the manager will remain in employment with probability $1 - F(t)$,

$$Pr(\tilde{t} > t) = 1 - F(t) = 1 - \int_{s=0}^{t} f(s) \, ds = 1 - \alpha \int_{s=0}^{t} \int_{\tau=0}^{s} e(\tau) \, d\tau \, ds.$$  

(3)

The manager derives utility from staying at the project as well as from the success benefit, but has to bear the effort costs. The choice of efforts affects the manager directly through the effort costs $C(e(t))$, and indirectly through the probability function $F(t)$. The manager’s value of a breakthrough $B$ at any instant $t \in [0, T]$ is weighed with the density of success $f(t)$ at any time $t$. Integrating this from $t = 0$ to $T$ yields the expected revenue, $B \int_{t=0}^{T} f(t) \, dt$. On-the-job benefits are only collected on condition that success has not occurred. Consequently, the probability that the manager receives benefit $b$ at any time $t \in [0, T]$ is the probability of no success up to time $t$, or $1 - F(t)$. Integrated from $t = 0$ to $T$ yields the manager’s expected aggregate benefit flow, $b \int_{t=0}^{T} (1 - F(t)) \, dt$. To simplify analysis, we assume that the manager incurs effort costs over the entire horizon $t \in [0, T]$. With a quadratic cost structure of the form $C(e(t)) = c e(t)^2 / 2$, aggregate effort costs are given by $c \int_{t=0}^{T} e(t)^2 / 2 \, dt$. Finally, note that the terminal deadline influences the utility derived from periods beyond the termination of the actual project. Hence, to incorporate that the number of projects decreases with the duration of each contract, we apply the average utility approach (see Radner (1981), or Townsend (1982) for some early contributions). Then, the manager’s average expected utility from employment,

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3Focusing on the project’s expected effort costs, $\int_{t=0}^{T} (1 - F(t)) C(e(t)) \, dt$, would yield similar results with regard to the optimal effort $e^r$. Specifically, an effort at date $t$ would decrease the probability that costly effort is required in the future. Hence, under certain conditions, effort incentives would increase for all $t \in [0, T]$. However, the equilibrium effort would change only to a minor degree, but complicate the subsequent analysis substantially. Therefore, we abstract from epiphenomenal incentives in the model and assume that the manager bears effort costs during the entire duration of employment, $t \in [0, T]$. A possible economic interpretation is that in case of a premature termination of the project, the manager incurs a disutility from continued employment.

4Instead of applying the average utility approach, we could alternatively assume that both principal and agent discount their payoffs. Using the continuous discount factor $e^{-r t}$, with $r$ as the interest rate, we obtain the same result regarding equilibrium effort incentives than under the average utility approach. However, as we are not able to obtain closed form solutions for the optimal deadline under discounting, we employ the average utility criterion in this paper.
\[ E[U_M] = E[U]/T, \] is given by

\[
E[U] = \frac{1}{T} \left[ B \int_{t=0}^{T} f(t) \, dt + b \int_{t=0}^{T} (1 - F(t)) \, dt - \int_{t=0}^{T} C(e(t)) \, dt \right]
\[
= \frac{1}{T} \left[ \alpha B \int_{t=0}^{T} \int_{\tau=t}^{T} e(\tau) \, d\tau \, dt + b \int_{t=0}^{T} \left( 1 - \alpha \int_{s=0}^{t} \int_{\tau=s}^{T} e(\tau) \, d\tau \, ds \right) \, dt \right.
\]
\[
- \frac{1}{2} c \int_{t=0}^{T} e(t)^2 \, dt \left. \right] = \frac{1}{T} \left[ \alpha B \int_{t=0}^{T} \int_{\tau=t}^{T} e(\tau) \, d\tau \, dt + b \int_{t=0}^{T} \left( 1 - \alpha \int_{s=t}^{T} \int_{\tau=s}^{T} e(\tau) \, d\tau \, ds \right) \, dt \right.
\]
\[
- \frac{1}{2} c \int_{t=0}^{T} e(t)^2 \, dt \right]. \tag{4}
\]

The principal earns a payoff of \( R \) only upon completion of the project at \( \tilde{t} \leq T \). The manager’s effort is only valuable to influence the probability of a breakthrough. Thus, the expected revenue is \( R \int_{t=0}^{T} f(t) \, dt \). This yields the principal’s average expected utility,

\[ E[U_P] = E[U]/T, \]

\[
E[U_P] = \frac{1}{T} \left[ R \int_{t=0}^{T} f(t) \, dt \right] = \frac{1}{T} \left[ \alpha R \int_{t=0}^{T} \int_{\tau=t}^{T} e(\tau) \, d\tau \, dt \right]
\]
\[
= \frac{1}{T} \left[ \alpha R \int_{t=0}^{T} \int_{\tau=t}^{T} e(\tau) \, d\tau \, dt \right]. \tag{5}
\]

The timing of the model is as follows.

1. Employment: A firm hires a manager to work on a project for \( \min\{\tilde{t}, T\} \) periods.

2. Moral hazard: The manager’s costly effort \( e(t) \) increases the project’s success probability \( F(t) \), but decreases the expected project duration.

3. Payoffs:

   (a) Project succeeds at \( 0 < \tilde{t} \leq T \) with probability \( F(T) \):
   - Principal: project value \( R > 0 \)
   - Manager: success benefit \( B > 0 \), tenure benefits \( b > 0 \) at any \( t \in [0, \tilde{t}] \)

   (b) Project fails if \( \tilde{t} > T \) with probability \( 1 - F(T) \)
   - Principal: project value \( 0 \)
   - Manager: success benefit \( 0 \), tenure benefits \( b > 0 \) at any \( t \in [0, T] \)
3 First-Best Solution

In this section, we determine the optimal effort and the optimal project deadline when there exist no incentive problem but principal and manager jointly choose $e(t)$ and $T$ to maximize the expected net surplus of the agency. This surplus is the difference of the expected revenues over the expected costs and is equivalent to the sum of the average expected utilities of principal and manager, $E[\hat{U}_M] + E[\hat{U}_P]$,

$$\max_{e(t),T} E [\hat{U}] = \frac{1}{T} \left[ \alpha (B + R) \int_{t=0}^{T} \int_{\tau=t}^{T} e(\tau) \, d\tau \, dt 
+ b \int_{t=0}^{T} \left( 1 - \alpha \int_{s=t}^{T} \int_{\tau=s}^{T} e(\tau) \, d\tau \, ds \right) \, dt
- \frac{1}{2} c \int_{t=0}^{T} e(t)^2 \, dt \right].$$ (6)

To derive the first-best values, $e^{FB}(t)$ and $T^{FB}$, we proceed in two steps. In the first step, we maximize $E[\hat{U}]$ over $e(t)$ to obtain first-best effort depending on the deadline $T$, $e^{FB}(t,T)$. In the second step, we insert $e^{FB}(t,T)$ into $E[\hat{U}]$ and maximize over $T$.

Step 1: At each date $t \in [0,T]$, an effort $e(t)$ influences the aggregate utility $E[\hat{U}]$ in three ways. First, effort increases the probability of pocketing success benefits $B + R$ by $\alpha e(t)$ for the remaining $T - t$ dates of the project. Hence, the marginal benefit of effort at $t$ is given by $\alpha (B + R) (T - t)$. Second, effort decreases the probability of pocketing flow benefits $b$ for all future periods of the project. This renders the negative marginal effect of effort on the manager’s on-the-job benefits, $-\alpha b (T - t)^2/2$. Third, effort costs increase by $c e(t)^2/2$, yielding a negative marginal utility effect, $-c e(t)$. In equilibrium, the positive effect (success benefits) exactly offsets two negative effects (negative tenure effect and costs of effort). Maximizing $E[\hat{U}]$ point-wise w.r.t. $e(t)$ yields the following first-order condition:

$$\frac{1}{2} \alpha (T - t) (2 (B + R) - b (T - t)) - c e^{FB}(t,T) = 0.$$ (7)

Solving for the first-best effort leads to

$$e^{FB}(t,T) = \frac{\alpha (T - t) (2 (B + R) - b (T - t))}{2 c}. \quad (8)$$

Note that $F(t)$ is a probability implying that $0 \leq F(t) \leq 1$ for all $t \in [0,T]$. To ensure that $e^{FB}(t,T) \geq 0$ and hence $F(t) \geq 0$ for all $t \in [0,T]$ we require $T \leq 2 (B + R)/b$. Moreover, parameters satisfy

$$Pr(\tilde{t} \leq T) = F(T) = \alpha \int_{s=0}^{T} \int_{\tau=s}^{T} e^{FB}(\tau,T) \, d\tau \, ds \leq 1$$
$$= \alpha \int_{s=0}^{T} \int_{\tau=s}^{T} \frac{\alpha (T - \tau) (2 (B + R) - b (T - \tau))}{2 c} \, d\tau \, ds \leq 1. \quad (9)$$

From (9) follows an endogenous technical condition for the maximum deadline in the first-best solution, $T^{FB}$,

$$\alpha^2 \left( T^{FB} \right)^3 \frac{4 (B + R) - b T^{FB}}{24 \, c} = 1. \quad (10)$$

7
Step 2: Substituting \( e^{FB}(t, T) \) for \( e(t, T) \) into (6) leads to
\[
\max_T E \left[ \bar{U} \left( e^{FB}(t, T) \right) \right] = E \left[ \bar{U}_M \left( e^{FB}(t, T) \right) \right] + E \left[ \bar{U}_P \left( e^{FB}(t, T) \right) \right]
\]
\[
= \frac{\alpha^2 b T^3 (5 (B + R) - 2 b T)}{120 c} + b. \tag{11}
\]
Solving the first-order condition, \( d \left[ U \left( e^{FB}(t, T) \right) \right] /dT = 0 \), such that
\( d^2 \left[ U \left( e^{FB}(t, T) \right) \right] /dT^2 < 0 \), yields
\[
0 = \alpha^2 b \left( T^{FB} \right)^2 \left( 15 (B + R) - 8 b T^{FB} \right),
\]
\[
T^{FB} = \frac{15 (B + R)}{8 b}. \tag{12}
\]
We have \( T^{FB} < 2 (B + R)/b \). In addition, (10) requires that \( T^{FB} \leq \bar{T}^{FB} \), or, equivalently, that
\[
T^{FB} \leq 4 \sqrt{\frac{3 c}{17 \alpha^2 (B + R)}}. \tag{13}
\]
We derive the following proposition.

**Proposition 1** There is a threshold, \( T^{FB}_1 = 4 \sqrt{3 c/(17 \alpha^2 (B + R))} \), such that the first-best deadline \( T^{FB} \) only exists if \( T^{FB} \leq T^{FB}_1 \). Then, in the first-best solution,
\[
e^{FB}(t) = \frac{\alpha (T^{FB} - t) \left( 2 (B + R) - b (T^{FB} - t) \right)}{2 c}, \tag{14}
\]
\[
T^{FB} = \frac{15 (B + R)}{8 b}. \tag{15}
\]

Comparative statics:

1. \( \partial e^{FB}(t)/\partial t \biggr|_{\geq 0} \) for \( (B + R)/b \biggr|_{< 0} \) \( (T^{FB} - t) \),
2. \( \partial e^{FB}(t)/\partial R = \partial e^{FB}(t)/\partial B \geq 0 \), and \( \partial T^{FB}/\partial R = \partial T^{FB}/\partial B > 0 \),
3. \( \partial e^{FB}(t)/\partial b \leq 0 \), and \( \partial T^{FB}/\partial b < 0 \),
4. \( \partial e^{FB}(t)/\partial c \geq 0 \), and \( \partial T^{FB}/\partial c = 0 \),
5. \( \partial e^{FB}(t)/\partial c \leq 0 \), and \( \partial T^{FB}/\partial c = 0 \),
6. \( \partial e^{FB}(t)/\partial R = \partial e^{FB}(t)/\partial B = \partial e^{FB}(t)/\partial b = \partial e^{FB}(t)/\partial c \) if and only if \( t = T^{FB} \).
Proof: See the Appendix.

From the comparative statics it follows that an effort increasing end-to-end over \( t \) cannot be optimal. Rather, for \( (B + R)/b \geq T^{FB} \), the first-best effort \( e^{FB} \) is decreasing over \( t \). For \( (B + R)/b < T^{FB} \), \( e^{FB} \) is increasing up to \( t = T^{FB} - (B + R)/b \), and decreasing thereafter. Specifically, \( (B+R)/b \) is the total benefit if the project succeeds, \( B+R \), over the manager’s benefit \( b \) per period of tenure. We call it the success-to-tenure ratio. A higher \( b \) ceteris paribus leads to less effort: as tenure becomes more valuable the probability for project completion in early periods will be decreased. As a result, both first-best effort and first-best deadline are increasing in the success benefits \( B \) and \( R \), but decrease in the manager’s private tenure benefit \( b \). The endogenous threshold \( \bar{T}^{FB}_1 \) limits the project’s success probability to its feasible value. Consequently, \( \bar{T}^{FB}_1 \) increases with the cost factor \( c \), and decreases with the efficiency parameter \( \alpha \), and the success benefits \( B + R \). That is, the larger the effort \( e^{FB} \) (large \( \alpha \), \( B \), \( R \), small \( c \)), the faster is the increase in the project’s success probability, limiting the maximum feasible duration of the project.

4 Project Deadline and Dynamic Moral Hazard

What determines the manager’s optimal choice of effort \( e^*(t) \)? Conditional on arrival at date \( t \), an effort \( e(t) \) increases the probability of project termination and therewith the probability of pocketing success benefit \( B \) for the remaining \( T - t \) periods of the project. This renders the manager’s positive marginal effect of effort, \( \alpha B (T - t) \). However, investing effort is costly, and the more effort is exerted, the less likely on-the-job benefits \( b \) will be retained in the future. This indicates the two negative marginal effects: the negative tenure effect \( -\alpha b (T - t)^2/2 \), and the effort cost effect, \( -c e(t) \). Consequently, at each date \( t \in [0, T] \), the manager increases his effort as long as the benefits associated with this increase outweigh the resulting costs. Maximizing \( E[\hat{U}_M] \), as given in (4), point-wise with respect to \( e(t) \) yields

\[
0 = \frac{1}{2} \alpha (T - t) (2B - b(T - t)) - c e^*(t, T).
\]

This proves the following proposition.

**Proposition 2** The manager’s equilibrium effort is

\[
e^*(t, T) = \frac{\alpha (T - t) (2B - b(T - t))}{2c}
\]

for \( T \leq 2B/b \) such that \( F(t) > 0 \) holds for all \( t \in [0, T] \).

Figure 1 shows the manager’s equilibrium effort \( e^*(t, T) \) as a function of time when varying the project deadline \( T \). The right figure decomposes two incentive effects that channel the manager’s effort over time: success incentives \( \alpha B (T - t)/c \), and tenure incentives \( -\alpha b(T - t)^2/(2c) \). First, the larger the success benefit \( B \), the larger are the manager’s
Incentives to complete the project. As the success probability cumulatively increases with the manager’s effort, these incentives are most effective for large durations $T - t$. Second, the optimal effort decreases with the manager’s benefit flow $b$, as effort diminishes the expected duration of the project. Moreover, the strength of both effects decreases with the effort cost factor $c$, and increases with the efficiency parameter $\alpha$. Thus, for large $T - t$, the manager has a long horizon, thus he does not want to “kill the golden goose” at start. Consequently, negative incentives decrease over time, yielding concave tenure incentives in $T - t$. For $t = T$, both effects are zero and the manager exerts no effort at all.

Combined together, success incentives decrease the equilibrium effort over time, while tenure incentives delay effort from early to late periods of the manager’s employment. For loose deadlines (black curve), the manager has a long horizon to complete the project and success incentives are large. On the other hand, the manager faces large tenure incentives to withhold effort. This causes the manager’s optimal effort to be not monotone in the deadline $T$. Specifically, if the deadline is larger than the manager’s success-to-tenure ratio, $B/b < T$, then the equilibrium effort is single-peaked at the strictly positive value of $t = T - B/b$. For strict deadlines (gray curve), the manager’s maximum project duration is small and thus also the incentives to postpone effort. However, it proves to be more difficult to meet the deadline and we have small success incentives. Hence, if the deadline is weakly smaller than the manager’s success-to-tenure ratio, $B/b \geq T$, then the manager’s optimal effort decreases over time, $\partial e^*(t, T) / \partial t < 0$ for all $t \in [0, T]$.

Figure 2 compares the first-best effort, $e^{FB}(t, T)$, with the manager’s equilibrium effort, $e^*(t, T)$, over time. Note that the first-best effort $e^{FB}(t, T)$ reduces to the manager’s optimal effort $e^*(t, T)$ when $R = 0$. Hence, the first-best effort would correspond to the second-best solution if the manager would internalize the principal’s payoff from the project. Specifically, the larger the remaining duration of the project $T - t$, the larger is the difference between the manager’s success incentives, $\alpha B (T - t)/c$, and the joint success incentives of manager and principal, $\alpha (B + R) (T - t)/c$. For $t = T$, both efforts coincide and the optimal effort is zero, $e^{FB}(t, T) = e^*(t, T) = 0$. Hence, the more distant
Figure 2: Optimal effort $e(t, T)$ as a function of time $t$

The parameters are $\alpha = 0.01$, $b = 1$, $c = 20$, $B = 10$, as before, and $R = 10$. The continuous lines show the manager’s optimal effort $e^*(t, T)$ (cf. Figure 1), the dashed lines stand for the first-best effort $e^{FB}(t, T)$. As before, we have $T = 15$ for the black curve, and $T = 10$ for the gray curve.

As the deadline, the more the manager underinvests in effort, and the larger is then the difference in aggregate efforts, $\int_{t=0}^{T} (e^{FB}(t, T) - e^*(t, T)) \, dt$. Therefore, by disregarding the principal’s expected value of a success, the manager invests in effort both too little, and too late.

5 Optimal Deadline

What is the optimal deadline $T^*$ of the project? Initially, this involves the question of how the bargaining power in contract negotiation is allocated between principal and manager. While in many moral hazard agency models it is assumed that the principal is endowed with full bargaining, in reality, both parties enjoy at least some of the bargaining power. In general, given the incentives, any choice of contract length is consistent with maximizing the manager’s behavior (manager’s incentive compatibility constraint). Specifically, incentive compatibility implies that the choice of the optimal deadline $T^*$ is constrained by the manager’s equilibrium effort $e^*(t, T)$ as defined by (17). As $F(t)$ is a probability it follows that $0 \leq F(t) \leq 1$ for all $t \in [0, T]$. First, $T \leq 2B/b$ ensures that $e^*(t, T) \geq 0$ and hence $F(t) \geq 0$ for all $t \in [0, T]$. Second, parameters satisfy

$$Pr(\hat{t} \leq T) = F(T) = \alpha \int_{s=0}^{T} \int_{\tau=s}^{T} e^*(\tau, T) \, d\tau \, ds \leq 1$$

$$= \alpha \int_{s=0}^{T} \int_{\tau=s}^{T} \frac{\alpha (T - \tau)(2B - b(T - \tau))}{2c} \, d\tau \, ds \leq 1. \quad \text{(18)}$$

From (9) follows an endogenous technical condition for the maximum deadline in the second-best solution, $\bar{T}^{SB}$,

$$\frac{\alpha^2 (\bar{T}^{SB})^3}{24c} \frac{4B - b\bar{T}^{SB}}{2c} = 1. \quad \text{(19)}$$
Figure 3 shows manager’s and principal’s average expected utilities as dependent on the deadline $T$. The dashed lines give the optimal values at the two extremes, when the manager has full bargaining power (in the left picture, $T_M^* = 18.75$), and when the principal enjoys full bargaining power (in the right picture, $T_P^* = 20$). These boundaries will be analyzed more detailed, before considering the more general case of arbitrary allocations of bargaining power.

The parameters are $\alpha = 0.01$, $b = 1$, $c = 20$, $B = 10$, and $R = 10$, as before. The dashed lines give the optimal deadlines of manager ($T_M^*$) and principal ($T_P^*$), respectively.

5.1 Manager has full bargaining power

The manager maximizes his average expected utility $E[\bar{U}_M (e^*(t, T))]$ over $T$. Specifically, the manager balances the expected success benefit, $B \int_{t=0}^{T} f(t) \, dt$, and the expected flow benefits, $b \int_{t=0}^{T} (1 - F(t)) \, dt$, with the costs of effort, $\int_{t=0}^{T} C(e(t)) \, dt$. Incorporating the equilibrium effort $e^*(t, T)$ of Proposition 2 in (4), gives

\[
\max_T E[\bar{U}_M (e^*(\tau, T))] = \frac{1}{T} \left[ \alpha B \int_{t=0}^{T} \int_{\tau=t}^{T} e^*(\tau, T) \, d\tau \, dt \\
+ b \int_{t=0}^{T} \left( 1 - \alpha \int_{s=t}^{T} \int_{\tau=s}^{T} e^*(\tau, T) \, d\tau \, ds \right) \, dt \\
- \frac{1}{2} c \int_{t=0}^{T} e^*(t, T)^2 \, dt \right] \\
= \frac{\alpha^2 B T^2 (4B - b T)}{24c} - \left( \frac{\alpha^2 b T^3 (5B - b T)}{120c} - b \right) \\
- \frac{\alpha^2 T^2 (5B (4B - 3b T) + 3b^2 T^2)}{120c} \\
= \frac{\alpha^2 b T^3 (5B - 2b T)}{120c} + b. \tag{20}
\]
The manager’s average expected utility, $E[\bar{U}_M(e^*(t,T))]$, is influenced by three terms: The first positive term is the expected success benefit that depends on the probability of project completion. Even though this probability is strictly increasing with the deadline, the average value of the success benefit decreases with the employment duration, and the manager faces opportunity costs of project delay. The second positive term gives the expected on-the-job benefits. On the one hand, increasing the deadline induces the manager to delay effort in the first place. Initially, the probability of project completion decreases. On the other hand, aggregate efforts increase with the deadline and decrease the probability that the manager will pocket job-related benefits until the deadline. Thus, in contrast to the success benefit, average on-the-job benefits will only increase with the deadline if the incentives to delay effort are comparatively large. That is, if the manager’s success-to-tenure ratio, $B/b$, is sufficiently small. The third term is negative and reflects the manager’s costs of effort. Obviously, as the manager’s positive effort incentives are driven by the success benefit, average effort costs increase with the deadline if $B/b$ is sufficiently large.

The first-order condition, $dE[\bar{U}_M(e^*(t,T))] / dT = 0$, gives an implicit definition for the manager’s optimal deadline, $T^*_M$,

$$\alpha^2 b T^*_M^2 (15 B - 8 b T^*_M) = 0.$$  \hfill (21)

Equation (21) has three solutions for $T^*_M$. The second-order condition for an interior maximum, $d^2 E [\bar{U}_M(e^*(t,T))] / dT^2 < 0$, is

$$\alpha^2 b T^*_M (5 B - 4 b T^*_M) < 0$$  \hfill (22)

implying that the only admissible solution is given by

$$T^*_M = \frac{15 B}{8 b}. \hfill (23)$$

Hence, we have $T^*_M < 2 B/b$. Finally, (19) requires that $T^*_M \leq \bar{T}^{SB}$, or equivalently that

$$T^*_M \leq T^{SB}_1 = 4 \sqrt[3]{\frac{3 c}{17 \alpha^2 B}}.$$  \hfill (24)

This yields the following proposition.

**Proposition 3** Suppose the manager has full bargaining power at the time of contracting. Then, there is a threshold, $\bar{T}^{SB}_1 = 4 \sqrt[3]{3 c/(17\alpha^2 B)}$, such that an optimal deadline $T^*_M$ only exists if $T^*_M \leq \bar{T}^{SB}_1$. The optimal contract stipulates $T^*_M = 15 B/(8 b)$.

From Proposition 3 it follows that the optimal deadline $T^*_M$ increases with the manager’s success-to-tenure ratio, $B/b$. Why? If the project is highly valuable to the manager (large $B$), then he should choose a loose deadline to make sure it will be completed in due time. However, with a too loose deadline, the time average value of a successful outcome
decreases. Moreover, a strict deadline limits the probability of a premature termination of the project. This is especially acute if the manager’s on-the-job benefits are relatively large ($b$). Note that when the principal’s payoff from the project is zero, $R = 0$, not only first-best effort, but also the first-best deadline reduce to the manager’s optimum, $e^*(t, T) = e^{FB}(t, T)$ and $T^*_M = T^{FB}$. For $R > 0$, the manager always underinvests in effort, $e^*(t, T) < e^{FB}(t, T)$, and prefers a stricter deadline than in the first-best solution, $T^*_M < T^{FB}$. That is, the larger the principal’s payoff from the project, the more the manager’s equilibrium choice falls short the social optimum.

5.2 Principal has full bargaining power

The principal derives utility from a successful project outcome. Thus, his payoff is weighted with the probability density of the success time, $R \int_{t=0}^{T} f(t) \, dt$. Incorporating the equilibrium effort $e^*(t, T)$ of Proposition 2 in (5) yields

$$
\max_T E[\bar{U}_P (e^*(t, T))] = \frac{1}{T} \left[ \alpha R \int_{t=0}^{T} \int_{\tau=t}^{T} e^*(\tau, T) \, d\tau \, dt \right] = \frac{\alpha^2 R T^2 (4B - bT)}{24c}.
$$

From the principal’s point of view, for short maturities, the probability of project completion is small. For long maturities, the principal faces opportunity costs of project delay such that the average value of the ex post gains decreases. The first derivative with respect to $T$,

$$
\frac{dE[\bar{U}_P (e^*(t, T))]}{dT} = \frac{\alpha^2 R T (8B - 3bT)}{24c},
$$

is strictly positive for $0 < T \leq 2B/b$. Hence, the principal’s average expected utility increases with $T$ within the feasible parameter range. This implies that for $T \leq 2B/b$, the effect of an increasing probability of project completion always outweighs the effect of a decreasing average value of the ex post gains. Hence, the principal optimally chooses the maximum possible deadline of

$$
T^*_P = \frac{2B}{b}
$$

on condition that $T^*_P \leq \bar{T}^{SB}$, or equivalently that

$$
T^*_P \leq \sqrt{\frac{12c}{\alpha^2 B}}.
$$

This gives the following proposition.

**Proposition 4** Suppose the principal has full bargaining power at the time of contracting. Then, there is a threshold, $\bar{T}^{SB}_2 = \sqrt[3]{\frac{12c}{\alpha^2 B}}$, such that an optimal deadline $T^*_P$ only exists if $T^*_P \leq \bar{T}^{SB}_2$. The optimal contract is a corner solution which stipulates $T^*_P = 2B/b$. 

From the Propositions 3 and 4 it follows that the manager always prefers a stricter deadline compared to the principal, \( T_M^* < T_P^* \). The main intuition is that the principal only skims profits from employment if the project succeeds. That is, the manager’s on-the-job benefits affect the principal only indirectly through the manager’s choice of effort. Thereby, \( T_P^* \) is independent of the principal’s project payoff \( R \): the effect of \( R \) is not included in the manager’s optimal effort, \( e^*(t, T) \), and hence is not effective in creating incentives. Therefore, from a comparison of the Propositions 1 and 4 it follows that

\[
T_P^*(>) = T^{FB} \quad \text{for} \quad B^>(>) = 15 R. \tag{29}
\]

Hence, for a low (high) ratio of manager’s to principal’s success benefit \( B/R \), the principal’s optimal deadline is stricter (looser) than in the first-best solution, \( T_P^* < T^{FB} \). Note that (29) is independent of the manager’s flow benefit \( b \). The effect of \( b \) is completely captured by \( e^{FB}(t, T) \) and \( T^{FB} \), and also by \( e^*(t, T) \) and \( T_P^* \).

### 5.3 Arbitrary allocations of bargaining power

Now consider the more general case where manager and principal enjoy some of the bargaining power at contract negotiation. Specifically, let \( v \in [0,1] \) be the manager’s bargaining power at the time of contracting. We solve for the asymmetric Nash solution (Binmore, Rubinstein, and Wolinsky, 1986). The main idea is that the equilibrium of a bargaining game with alternating offers can be represented by the Nash bargaining solution. This definition is equivalent to maximizing the weighted product of the average expected individual utilities of manager and principal. Then, using the results of (20) and (25), we obtain

\[
\max_T E \left[ \bar{U}_M(e^*(t, T)) \right]^v E \left[ \bar{U}_P(e^*(t, T)) \right]^{1-v} = \left( \frac{\alpha^2 b T^3 (5 B - 2 b T)}{120 c} + b \right)^v \left( \frac{\alpha^2 R T^2 (4 B - b T)}{24 c} \right)^{1-v}. \tag{30}
\]

The optimal solution to \( T \) can be found through logarithmic transformation,

\[
\max_T v \ln E \left[ \bar{U}_M(e^*(t, T)) \right] + (1-v) \ln \left[ \bar{U}_P(e^*(t, T)) \right] = v \ln \left[ \frac{\alpha^2 b T^3 (5 B - 2 b T)}{120 c} + b \right] + (1-v) \ln \left[ \frac{\alpha^2 R T^2 (4 B - b T)}{24 c} \right]. \tag{31}
\]

The first-order condition, \( d(v \ln E[\bar{U}_M(e^*(t, T))] + (1 - v) \ln [\bar{U}_P(e^*(t, T))])/dT = 0 \), gives an implicit definition for the optimal deadline, \( T^*(v) \),

\[
0 = \alpha^2 T^*(v)^3 (20 (2 + v) B^2 - b T^*(v)) ((31 + 16 v) B - 2 (3 + v) b T^*(v)))
+ 120 (1 - v) c (8 B - 3 b T^*(v)) \tag{32}
\]
For $v = 1$ and $v = 0$, the optimal solutions are given by $T^*_M \leq \bar{T}^{SB}_1$ and $T^*_P \leq \bar{T}^{SB}_2$, respectively. Recall that the principal’s average expected utility increases with $T$ for $T \leq 2B/b$ such that the optimum is a corner solution, $T^*_P = 2B/b$. Hence, for $0 < v < 1$, we obtain $T^*(v) \in (T^*_M, T^*_P]$. Specifically, depending on the manager’s bargaining power $v$, we obtain a lower threshold for the optimal deadline, $\bar{T}^{SB}_3 \leq T^*(v)$, that decreases with $v$. Incorporating the endogenous technical condition of (19), which requires that $T^*(v) \leq \bar{T}^{SB}$, yields

$$\bar{T}^{SB}_3 = \frac{B \left( 131 - 84v - \sqrt{361 + 8(19 - 28v)v} \right)}{2(21 - 13v)b}.$$  \hspace{1cm} (33)

The second-order condition, $d^2 \frac{v \log E [\bar{U}_M (e^*(t,T))] + (1 - v) \log [\bar{U}_P (e^*(t,T))] }{dT^2} < 0$, holds for all $T^*(v) \in [T^*_M, T^*_P]$. We derive the following proposition.

**Proposition 5** In the Nash bargaining solution, there is a threshold,

$$\bar{T}^{SB}_3 = \frac{B \left( 131 - 84v - \sqrt{361 + 8(19 - 28v)v} \right)}{2(21 - 13v)b},$$  \hspace{1cm} (34)

such that an optimal deadline $T^*(v) \in [T^*_M, T^*_P]$, with $T^*_M \leq \bar{T}^{SB}_1$, $T^*_P \leq \bar{T}^{SB}_2$, only exists if $T^*(v) \geq \bar{T}^{SB}_3$. The optimal contract is characterized implicitly by

$$0 = \alpha^2 T^*(v)^3 \left( 20(2 + v)B^2 - bT^*(v) \left( (31 + 16v)B - 2(3 + v)bT^*(v) \right) \right) + 120(1-v)c \left( 8B - 3bT^*(v) \right).$$  \hspace{1cm} (35)

The corner solution at $T^*(v) = T^*_P = 2B/b$ occurs if optimal $T^*(v) \geq 2B/b$.

**Comparative statics:**

1. $\partial T^*(v)/\partial B > 0$,
2. $\partial T^*(v)/\partial R = 0$,
3. $\partial T^*(v)/\partial b < 0$,
4. $\partial T^*(v)/\partial \alpha \leq 0$,
5. $\partial T^*(v)/\partial c \geq 0$,
6. $\partial T^*(v)/\partial v < 0$,
7. $\partial T^*(v)/\partial \alpha = \partial T^*(v)/\partial c = 0$ if and only if either $v = 0$ or $v = 1$.

**Proof:** See the Appendix.

Proposition 5 shows that in the asymmetric Nash solution, the optimal deadline $T^*(v)$ takes values between the individually optimal solutions of manager and principal, $T^*_M$ and
As the principal’s true optimal deadline exceeds the boundary of the feasible region, $dE[U_P(e^*(t,T))]/dT > 0$ for $T \leq 2B/b$, we have that $T^*_P = 2B/2$. Consequently, the optimal deadline in the bargaining solution is $T^*(v) = T^*_P = 2B/2$ in case the algebraic term of $T^*(v)$ in (35) exceeds $2B/2$. Thereby, $T^*(v)$ increases with the manager’s success-to-tenure ratio, $B/b$, but is independent of the principal’s payoff $R$. Moreover, the optimal deadline decreases with the manager’s bargaining power $v$. Note that, for $0 < v < 1$, $T^*(v)$ increases with the cost factor $c$, and decreases with the efficiency parameter $\alpha$. The main intuition is that marginal effort costs $c$ decrease the equilibrium effort and with it the probability of project completion. Consequently, a larger horizon is required to complete the project. In contrast, $\alpha$ increases the productivity of effort in making the innovation, which results in a higher equilibrium effort and a higher probability of innovation. Therefore, an increase in $\alpha$ decreases the optimal deadline of the project.

In the context of the empirical literature, our results allow us to reinterpret the so-called Schumpeterian hypotheses (Schumpeter (1942)) with regard to the labor market. From a Schumpeterian viewpoint, the market power effect states a positive relationship between a firm’s market power and the supply of innovations. The firm size effect presumes that large firms will be more innovative than small ones (see Kamien and Schwartz (1982) and Cohen and Levin (1989), for an overview). In the argumentation of our model, imperfect competition in the labor market allows firms to determine deadlines as an incentive instrument to encourage innovation effort. As a result, greater labor market power at the firm level increases the optimal deadline of a project and with it the probability of its completion. Consequently, the presence of ex ante imperfect competition in the labor market should imply greater flows of innovations. Related to the concept of “creative destruction”, successful innovations increase a firm’s profits which may increase firm size and lead to market concentration in both product and labor markets. Thus, the extent to which labor markets are characterized by imperfect competition ex ante may account for ex post market power acquired by successful innovation. Therefore, not only innovation effort and project success should considered to be endogenous, but also their effect on market power and firm size. From the viewpoint of public policy, firms’ labor market power may be influenced by labor market regulation, for example with regard to trade unions. Also the implementation of governmental subsidy programs, like in the field of R&D projects, may affect market concentration and firm size. This raises the question about the desirability and the impact of market regulation with regard to both labor and product markets.

6 Conclusion

In this paper, we investigate the link between deadlines in project management and the incentives for project completion. We show that when managers can skim private benefits from working on a project, they are encouraged to postpone effort. In a trade-off between success and delay, the optimal project deadline balances the expected increase in project value with the increase in expected project duration and costs. As a result, the larger the
manager’s success incentives relative to his tenure incentives, the smaller is the delay of effort and the larger is then the optimal deadline of a project. As another result of the model, the optimal project deadline decreases with the manager’s bargaining power at contract negotiation. From the standpoint of the social optimum, managers both underinvest in effort and enforce too strict deadlines. It follows that project completion is threatened by both inefficiencies in effort provision over time and inefficiencies in contractual agreements on time.

Taking a broader view of the results, our model emphasizes the dynamics of project management activities and contributes to the debate on the optimal organizational design of projects. Even though project completion and deadline formation can be influenced by other unmodeled factors, such as technical requirements, the basic trade-off between effort incentives and completion time addressed in this paper accords with the widely observed patterns of projects that mostly fail to succeed within their initial timeframe. Thereby, our explanatory approach is in line with empirical evidence that emphasizes the role of organizational and managerial aspects in explaining project delay (van Genuchten (1991)). The results of our work derive testable predictions about the impact of environmental variables on the optimal deadline of a project and deliver new insights into the impact of labor market regulation on project success and the supply of innovations. Overall, our paper offers a new understanding of how the interaction of project timing and incentives influences the behavior of managers governed by the objective of not to “kill the golden goose” prematurely.

A Appendix

A.1 Proofs

Proof of Proposition 1 Substituting $e^{FB}(t, T)$ into (6) yields

$$\max_T E \left[ \bar{U} (e^{FB}(t, T)) \right] = E \left[ \bar{U}_M (e^{FB}(t, T)) \right] + E \left[ \bar{U}_P (e^{FB}(t, T)) \right]$$

$$= \frac{1}{T} \left[ \alpha (B + R) \int_{t=0}^{T} \int_{\tau=t}^{T} e^{FB}(\tau, T) \, d\tau \, dt \right.$$  

$$+ b \int_{t=0}^{T} \left( 1 - \alpha \int_{s=t}^{T} \int_{\tau=s}^{T} e^{FB}(\tau, T) \, d\tau \, ds \right) \, dt \right.$$  

$$- \frac{1}{2} c \int_{t=0}^{T} e^{FB}(t, T)^2 \, dt \right]$$

$$= \frac{b \alpha^2 T^3 \left( 5 (B + R) - 2 b T \right)}{120 c} + b,$$

which corresponds to the result of (11). From the first-order-condition,

$$\frac{dE \left[ \bar{U} (e^{FB}(t, T)) \right]}{dT} = \frac{\alpha^2 b \left( T^{FB} \right)^2 \left( 15 (B + R) - 8 b T^{FB} \right)}{120 c} = 0,$$  

(37)
we obtain three candidates for an optimum:

\[
T_{1}^{FB} = T_{2}^{FB} = 0,
\]

\[
T_{3}^{FB} = \frac{15 (B + R)}{8 b}.
\]

Only \( T_{3}^{FB} \) fulfills the second-order-condition,

\[
\frac{d^2 E [\bar{U} (e^{FB}(t,T))]}{dT^2} = \frac{\alpha^2 b T_{3}^{FB} (5 (B + R) - 4 b T_{3}^{FB})}{20 c} = -\frac{15 \alpha^2 (B + R)^2}{64 c} < 0. \quad (38)
\]

Thus,

\[
T^{FB} = T_{3}^{FB} = \frac{15 (B + R)}{8 b} \quad (39)
\]

if and only if \( T^{FB} \leq \bar{T}^{FB} \), hence if and only if (13) holds.

For the comparative statics with regard to the first-best effort,

\[
e^{FB}(t) = \frac{\alpha (T^{FB} - t) (2 (B + R) - b (T^{FB} - t))}{2 c}, \quad (40)
\]

we have

\[
\frac{\partial e^{FB}(t)}{\partial t} = -\frac{\alpha (B + R - b (T^{FB} - t))}{c}, \quad (41)
\]

\[
\frac{\partial e^{FB}(t)}{\partial B} = \frac{\partial e^{FB}(t)}{\partial R} = \frac{\alpha}{c} (T^{FB} - t) \geq 0, \quad (42)
\]

\[
\frac{\partial e^{FB}(t)}{\partial b} = -\frac{\alpha}{2 c} (t - T^{FB})^2 \leq 0, \quad (43)
\]

\[
\frac{\partial e^{FB}(t)}{\partial \alpha} = -\frac{\alpha}{2 c} (T^{FB} - t) \frac{2 c}{2 c^2} \geq 0, \quad (44)
\]

\[
\frac{\partial e^{FB}(t)}{\partial c} = -\frac{\alpha (T^{FB} - t) (2 (B + R) - b (T^{FB} - t))}{2 c^2} \leq 0, \quad (45)
\]

such that for \( t = T^{FB} \),

\[
\frac{\partial e^{FB}(t)}{\partial B} = \frac{\partial e^{FB}(t)}{\partial R} = \frac{\partial e^{FB}(t)}{\partial \alpha} = \frac{\partial e^{FB}(t)}{\partial c} = 0. \quad (46)
\]

With regard to \( T^{FB} \), we obtain

\[
\frac{\partial T^{FB}(t)}{\partial B} = \frac{\partial T^{FB}(t)}{\partial R} = \frac{15}{8 b} \geq 0, \quad (47)
\]

\[
\frac{\partial T^{FB}(t)}{\partial b} = -\frac{15 (B + R)}{8 b^2} < 0. \quad (48)
\]

\[\blacksquare\]
Proof of Proposition 5. To prove the comparative static results, we use the implicit function theorem. The first-order condition for a maximum requires that 
\[ \frac{d}{dT} \left( v \log E \left[ \hat{U}_M \left( e^*(t, T) \right) \right] + (1-v) \log \left[ \hat{U}_P \left( e^*(t, T) \right) \right] \right) /dT = 0. \]
Differentiating (31) with respect to \( T \), and equating to zero yields our implicit function, \( g(T^*(v), x) = 0 \). To simplify notation, we will omit the \( v \) in \( T^*(v) \) in the proof. We obtain
\[ g(T^*, x) = \frac{\alpha^2 T^{*3} V_1 + 120 (1 - v) c (8 B - 3 b T^*)}{T^* (4 B - b T^*)} (\alpha^2 T^{*3} (5 B - 2 b T^*) + 120 c) = 0, \]
where
\[ V_1 = 20 (2 + v) B^2 - b T^* ((31 + 16 v) B - 2 (3 + v) b T^*), \]
and \( x \) stands for one of the exogenous parameters \( B, R, b, \alpha, c, \) and \( v \), respectively. The expression in the numerator corresponds to (32) and (35). The comparative static results follow from \( \partial T^* / \partial x = - (\partial g(T^*, x) / \partial x) / (\partial g(T^*, x) / \partial T^*) \). The second-order condition for a maximum requires that the sign of the denominator is negative, \( \partial g(T^*, x) / \partial T^* = d^2 \left( v \log E \left[ \hat{U}_M \left( e^*(t, T) \right) \right] + (1-v) \log \left[ \hat{U}_P \left( e^*(t, T) \right) \right] \right) /dT^2 < 0, \)
\[ \frac{\partial g(T^*, x)}{\partial T^*} = \frac{V_2 - 14400 (1 - v) c^2 (16 B (2 B - b T^*) + 3 b^2 T^{*2})}{T^* (4 B - b T^*)^2 (\alpha^2 T^{*3} (5 B - 2 b T^*) + 120 c)^2} < 0, \]
where
\[ V_2 = -240 \alpha^2 c T^{*3} (4 B^2 (20 (2 - 5 v) B - 6 (6 - 19 v) b T^*) + b^2 T^{*2} (B (47 - 158 v) - 6 (1 - 3 v) b T^*)) - \alpha^4 T^{*6} (40 B^3 (10 (2 + v) B - 13 (2 + v) b T^*)) + 4 b^4 T^{*3} ((31 + 16 v) B + (3 + v) b T^*) + (523 + 288 v) b^2 B^2 T^{*2}). \]
Equation (52) is strictly negative for \( T^* \in [T_M^*, T_P^*] \). Now, consider the sign of the numerator, \( -\partial g(T^*, x) / \partial x \), for each exogenous parameter. If the first derivative is positive, \( \partial g(T^*, x) / \partial x > 0 \), then \( T^* \) is increasing with \( x \), \( \partial T^* / \partial x > 0 \). Contrary, if \( \partial g(T^*, x) / \partial x < 0 \), then we have \( \partial T^* / \partial x < 0 \). Taking derivatives, we obtain
\[ \frac{\partial g(T^*, B)}{\partial B} = \frac{2 \alpha^2 T^{*2} V_3 + 57600 (1 - v) b c^2}{(4 B - b T^*)^2 (\alpha^2 T^{*3} (5 B - 2 b T^*) + 120 c)^2} > 0, \]
where
\[ V_3 = 60 c \left( 40 B (6 v B + (1 - 4 v) b T^*) - (16 - 31 v) b^2 T^{*2} \right) + \alpha^2 b T^{*4} \left( 2 (5 B - 2 b T^*)^2 + 3 v (10 B^2 - b^2 T^{*2}) \right), \]
\[ \frac{\partial g(T^*, R)}{\partial R} = 0, \]
\[
\frac{\partial g(T^*, b)}{\partial b} = -\frac{2 \alpha^2 T^* v^3 V_4 + 57600 (1 - v) B c^2}{(4 B - b T^*)^2 (\alpha^2 T^* v^3 (5 B - 2 b T^*) + 120 c)^2} < 0,
\]
where
\[
V_4 = 480 c (5 + 11 v) B^2 - b T^* (2 (1 + 3 v) B - v b T^*) + \alpha^2 B T^* v^3 \left( 2 (5 B - 2 b T^*)^2 + 3 v (10 B^2 - b^2 T^*^2) \right),
\]
\[
\frac{\partial g(T^*, \alpha)}{\partial \alpha} = \frac{240 v \alpha c T^* v^2 (15 B - 8 b T^*)}{(\alpha^2 T^* v^3 (5 B - 2 b T^*) + 120 c)^2} \leq 0,
\]
\[
\frac{\partial g(T^*, c)}{\partial c} = -\frac{120 v \alpha^2 T^* v^2 (15 B - 8 b T^*)}{(\alpha^2 T^* v^3 (5 B - 2 b T^*) + 120 c)^2} \geq 0,
\]
\[
\frac{\partial g(T^*, v)}{\partial v} = \frac{2 \alpha^2 T^* v^3 \left( 2 B (5 B - 4 b T^*) + b^2 T^* v^2 \right) - 120 c (8 B - 3 b T^*)}{T^* (4 B - b T^*) (\alpha^2 T^* v^3 (5 B - 2 b T^*) + 120 c)} < 0.
\]
The results hold for all \( T^* \in [T^*_M, T^*_P] \). At the boundaries, \( v = 1 \) and \( v = 0 \), we have \( T^* = T^*_M \) and \( T^* = T^*_P \), respectively, such that
\[
\frac{\partial g(T^*, \alpha)}{\partial \alpha} = \frac{\partial g(T^*, c)}{\partial c} = 0.
\]

References


