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Beat the gun - protection against zero-profit imitation

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Beat the Gun – Protection against Zero-Cost Imitation

January 28, 2015

Abstract

The recent development of 3D printing raises the issue of how to protect manufacturing firms from product piracy. In this paper, we are interested in potential regulatory requirements to protect firms from falling victim of product piracy and associated quality choices. We employ a game-theoretic model of duopoly competition. One firm offers a high-quality product facing quality-related costs. An imitator views the product and produces an imitation using a low-cost production method as 3D printing. Our results indicate that copy protection by the high-quality firm yields a higher quality than under patent protection. However, the chosen quality level of the high-quality firm is highest in duopoly without protection. Optimal patent protection crucially depends on the underlying objective function. It is only socially optimal if the regulatory authority maximises GDP.

Keywords: Patents, Regulation, Product Imitation, Product Piracy, Vertical Product Differentiation, Price Competition, Market Entry, Welfare

JEL-Code: L13, L51, O31

1 Introduction

The recent development of 3D printing raises the issue of how to protect manufacturing firms from product piracy. 3D printing facilitates the production of three-dimensional objects from a digital model by additive manufacturing. It allows for building parts and devices with very complex geometries without tooling, fixtures and waste. Associated costs of production are reduced to the printer itself and required granulates. In contrast, the production of quality products, which are not subject to mass production and economies of scale, may face competitive disadvantage due to their traditional cost structure.

Production and the risk of imitation refers to a wide range of products, as 3D printing is universally applicable at different stages of manufacturing. Examples are products of art, fashion, gadgets, toys but also technological or medical products. An example that came up in the news recently was a gun. Made of several components, a gun was produced by 3D printing for the first time. And it is fully functional.

In this paper, we are interested in potential regulatory requirements to protect firms from falling victim of product piracy and the associated degree of product innovation. To do so, we employ a game-theoretic model of duopoly competition. One firm offers a high-quality product facing costs that are convex in quality and quality adjustment costs at later stages. Quality and associated costs can be interpreted as the result of research and development efforts in product innovation in the past. An imitator views the product and produces a clone of the same or less quality using a low-cost production method as 3D printing. Without loss of generality, we normalise the imitator's cost to zero. In order to avoid competition from the start, the high-quality firm can obtain a patent, which exclusively grants the right to offer a product of a certain quality for a specified time period. The imitator is restricted to design-around the high-quality product during the patent lifetime. After choosing the product qualities, firms engage in price competition. By the time the patent expires, both firms choose their qualities and prices anew. The patent's lifetime as

well as its height are determined by the regulatory authority prior to firms' interactions. We analyse whether the patent systems is beneficial given different welfare functions. In an extension of the model, we allow for investment in copy protection with linear cost by the high-quality firm, which slows down product imitation by others.

Our results indicate that optimal patent protection crucially depends on the underlying objective function. It is only socially optimal if the regulatory authority maximises GDP. Whenever the regulatory authority includes consumer surplus in its objective function, it is optimal to offer no patent protection at all. The imitator's profits are irrelevant in any case. If the regulatory authority has an interest in nurturing quality, it should abstain from patent protection: The chosen quality level of the high-quality firm is highest in duopoly without protection. Notably, copy protection by the high-quality firm yields a higher quality than under patent protection.

A patent will be granted for a specific amount of years, the patent lifetime. In earlier studies, protection was considered to be perfect during the patent lifetime (Nordhaus, 1969), creating a monopoly for the patentee. More recent papers also consider imperfect protection. Imperfect patent protection involves either a certain patent breadth (Gilbert and Shapiro, 1990), height (van Dijk, 1996) or width (Klemperer (1990)). Patent height imposes a minimum inventive step with respect to product quality or a restriction on the number of product attributes on competitors' products. Patent width refers to the range of applications, e.g. markets, for which a patent holds, a certain product space that will be covered by the patent. Instead, patent breadth can be interpreted as protection up to a certain level of product characteristics in a horizontal attribute space that are allowed to be imitated. This paper analyses imitations beyond patent height, which can be seen as designing-around the original high-quality product. Our patent protection is similar to van Dijk (1996). We extend this model as we allow for an adjustment in prices and qualities after the patent lifetime has expired.

Our analysis is related to papers that discuss conditions for optimal patent lifetime or patent breadth, such as Klemperer (1990), Gallini (1992) and Gilbert and Shapiro (1990). Thereby, the last two papers interpret the patent scope as the flow of profits, which the innovating firm earns during the patent lifetime. Klemperer (1990) instead defines patent width as how different product imitations have to be from the original product in terms of horizontal product differentiation. In contrast to these papers, we explore the impact of a vertically differentiated product imitation. In their 2007 article, Lambertini and Tedeschi also analyse patent protection in terms of patent lifetime and height in a duopoly setting. However, we explicitly examine whether the patent system is beneficial given different social preferences. Taking account of consumer surplus, for example, is of importance as patents impose deadweight losses on welfare due to restrictions on competition (Tirole, 1988). To do so, we explicitly model patent length and allow for quality adjustments in view of patent expiring date. This contrast the article by Lambertini and Tedeschi (2007), who refrain from modelling competition in the aftermath of patent protection.

As pointed out by earlier empirical studies, imitation costs are large (Mansfield, Schwartz, and Wagner, 1981). Papers supporting this finding, however, are based on traditional production technologies, decades before 3D printing was available. Our assumption that the imitator does not face any cost strongly rely on the innovative additive manufacturing process or comparable low-cost production methods.

In the literature on network economics, the topic of copy protection is also discussed, e.g., in Shy and Thisse (1999), Peitz and Waelbroeck (2003) and Belleflamme and Peitz (2010). A general finding is that firms may be better off refraining from copy protection. The driving force is that without copy protection, firms attract consumers, who do not only buy the product itself but also related complements. In contrast, products considered in this paper do not have any network externalities. Models on copying without network effects, are often applied to print media as journals and books or music (Novos and Waldman (1984), Johnson (1985) Liebowitz (1985) and Besen and Kirby (1989)). However, copy protection is only an exogenous

phenomenon instead of a strategically chosen instrument.

An application of patents to pharmaceutical markets by Kamien and Zang (1999) serves to illustrate how firms react facing the expiring of their patents. Kamien and Zang show that firms increase market shares by introducing generic substitutes for their products. This idea was carried forward by Regan (2008), Marjit, Kabiraj, and Dutta (2009) and Jost (2012). Although Kamien and Zang's patent system is different to ours, there is a common feature: The patentee is allowed to react to the patent expiring by either supplying a generic product (Kamien and Zang, 1999) or, in our case, by choosing the product quality anew.

Welfare losses are always connected to patents, as patents grant a reward to the patentee for disclosure of the product innovation. If the reward is considered to be too low, the patentee might decide to postpone the disclosure. This phenomenon was explored by Scotchmer and Green (1990) and Baker and Mezzetti (2005); or Shapiro (1985) and Mukherjee and Pennings (2004), who find that licensing is a possibility of dissemination. "The ease of inventing around" is considered to be one of the most important reasons not to patent (Cohen, Nelson, and Walsh, 2000). Indeed, the thread of product imitations is in place in our model. Our model implies an asymmetry with respect to quality-related cost and, therefore, competitors will always supply an imitation to the original product, as profits are always nonnegative. This is also a difference between the present analysis and previous ones, which analyse patent races or sequential product innovations, see e.g. Scotchmer (1991), Bessen and Maskin (2009).

Our model is in line with earlier papers on vertical differentiation, which show that the first firm entering the market chooses the higher quality as profits are higher compared to the lower quality alternative (Lehmann-Grube, 1997; Choi and Shin, 1992). This result is supported by various articles on market entry in vertical differentiated markets (Donnenfeld and Weber (1995), Lutz (1997), Hung and Schmitt (1992)). However, these articles do not take patents into account.

The following issues are of special interest for our analysis. First, we consider patent protection in an environment where market entry is never blockaded, as the imitator's quality cost are nil. Due to low-cost production technologies as 3D printing, this is a real-world risk that gains importance. Second, we specify the effects of design-arounds. We want to know which product qualities are supplied if a patent that is linked to quality is valid or close to its expiring date. Third, we take a closer look at who prefers patent protection and who not, which finds its expression in different formulations of the welfare function. And last, we allow for copy protection by the high-quality firm and its effect on quality choice in an extension of the model.

The paper is organised as follows. Section 2 describes the game-theoretic model. We analyse the equilibrium in Section 3 and optimal patent protection afterwards in Section 4. In an extension of the model, we consider copy protection by the high-quality firm in Section 5. Section 6 concludes with a discussion of the findings and implications for future research.

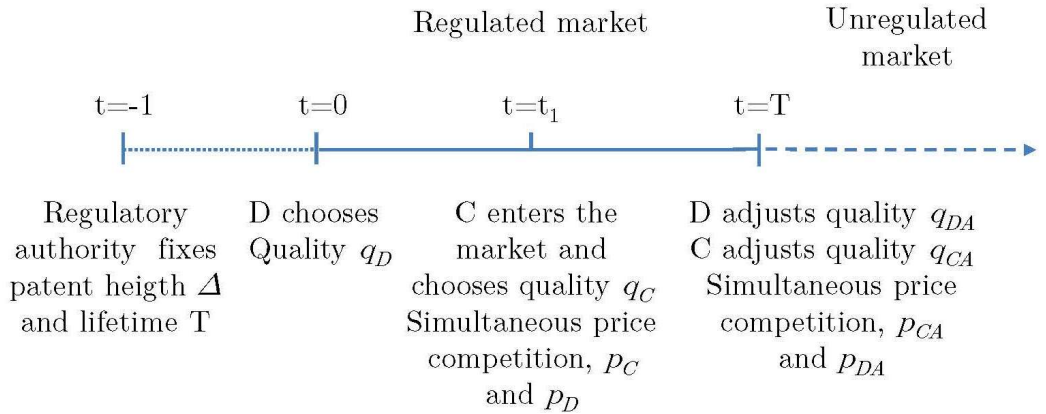
2 The Model

Consider a market for vertically differentiated products. Time is defined over an infinite continuous interval $[0, \infty)$. Firm D , the designer, is leading in the market and introduces a product with high quality q_D , for which it takes out a patent. During patent lifetime, the high-quality firm is protected against products of similar quality. Thus, imitator C is restricted to design around the high-quality product. Once patent lifetime has ended, firms compete in an unregulated duopoly and have the possibility to readjust qualities. The high-quality firm faces development costs that are quadratic in quality and quality adjustment cost at a later stage. The imitator has no costs.

The timing of the game is as follows. Prior to firms' interactions, at stage

$t = -1$, the regulatory authority determines patent protection resulting from patent expiring date T and patent height Δ . At stage $t = 0$, the designer D enters the market and offers a product of quality q_D , which implies quadratic quality-related costs. During time period $[0, t_1)$, the high-quality firm is the sole supplier in the market until imitator C recognises the high-quality product. At stage $t = t_1$, the imitator enters the market with a product of quality q_C . Due to patent protection, the imitator's product quality is restricted to $q_C \leq (1 - \Delta) q_D$. Simultaneous price competition takes place afterwards. At stage $t = T$, the patent expires and both firms readjust their product qualities. We assume that first the high-quality firm readjusts quality to q_{DA} and bears quality adjustment costs, before the imitator redetermines quality to q_{CA} . Imitator C is confined to react to D 's quality adjustment. These sequential quality choices reflect the strategic asymmetry between the high-quality firm's and the imitator's market entries at different point in times. Finally, simultaneous price competition takes place. Figure 1 illustrates the timing of the game.

Figure 1: Timing of the game



The different points in time determine different market constellations. Over the time span $[0, t_1)$, the high-quality firm D acts as a monopolist and offers products at price p_M . Over the time span $[t_1, T)$, both firms compete in a regulated duopoly by simultaneously choosing prices p_C and p_D . After the patent's expiring date over time period $[T, \infty)$, the duopoly is unregulated and both firms compete in prices

p_{CA} and p_{DA} . In the following, we refer to the regulated market when $t \in [0, T)$, where the patent protection is valid. Time period $t \in [T, \infty)$ denotes the unregulated market.

Consumers are assumed to differ in type θ , which is their marginal willingness-to-pay for quality. They are uniformly distributed on an interval so that $\theta \in [0, \theta_{max}]$. The indirect utility function of a consumer of type θ , who buys firm f 's product, $f = C, D$, of quality q_f at price p_f , is given by

$$u(f; \theta) = \theta q_f - p_f. \quad (1)$$

In the time span $[0, t_1)$, where the high-quality firm is the only supplier offering quality q_D at monopoly price p_M , θ_{0M} denotes the willingness-to-pay such that the utility from buying the high-quality firm's product is equal to the utility from buying nothing at all. If $0 < \theta_{0M} < \theta_{max}$, then θ_{0M} represents the marginal consumer who is indifferent between buying product D and nothing at all,

$$\theta_{0M} = \frac{p_M}{q_D}. \quad (2)$$

Only consumers whose willingness-to-pay is sufficiently high and greater than θ_{0M} buy exactly one unit of the good. Therefore, the market is not covered if $\theta_{0M} > 0$. From this, the monopolist's demand x_M can be calculated by

$$x_M = \theta_{max} - \theta_{0M}. \quad (3)$$

Analogously, in the time span $[t_1, T]$ where the imitator offers a lower quality q_C than D at price p_C , the marginal consumer who is indifferent between buying product C and nothing at all is given by

$$\theta_0 = \frac{p_C}{q_C}. \quad (4)$$

Moreover, the consumer whose utility from buying product C is equal to the utility from buying product D by θ_1 , $u(C, \theta_1) = u(D, \theta_1)$, has the willingness-to-pay

$$\theta_1 = \frac{p_D - p_C}{q_D - q_C}. \quad (5)$$

If $\theta_1 \leq 0$ ($\theta_1 \geq 1$), all consumers buy firm D 's (C 's) product. Whereas if $\theta_1 \in (0, 1)$, then θ_1 is the marginal consumer who is indifferent between buying from C and D . In this case, both firms have positive demand x_i with

$$x_C = \theta_1 - \theta_0 \quad (6)$$

$$x_D = \theta_{max} - \theta_1. \quad (7)$$

3 Equilibrium Analysis

We solve the game using backwards induction starting with the last period, which is the unregulated market. Only subgame perfect Nash-equilibria in pure strategies are taken into account.

3.1 Unregulated Market

From stage $t = T$ onwards, both firms compete in an unregulated duopoly. The discounted flows of profits henceforth are given by

$$\Pi_C(T, \infty) = \int_T^\infty x_{CA} p_{CA} e^{-rt} dt \quad (8)$$

$$\Pi_D(T, \infty) = \int_T^\infty x_{DA} p_{DA} e^{-rt} dt - \frac{c}{2} (q_{DA} - q_D)^2 e^{-rT}, \quad (9)$$

where x_{CA} (x_{DA}) is firm C 's (D 's) demand that can be derived analogously to equations (6) and (7). The term $\frac{c}{2} (q_{DA} - q_D)^2 e^{-rT}$ denotes the present value of the high-quality firm's quality adjustment costs that occur in period T from improving quality from q_D to q_{DA} . Cost parameter c is assumed to be positive, $c > 0$.

As firms sequentially readjust their product qualities at $t = T$ and then compete in prices for $t \geq T$, firms' optimal strategies are rather standard, see, for example, Choi and Shin (1992). Profit maximisation for given qualities yields equilibrium

prices

$$p_{CA}^* = \frac{q_{CA}(q_{DA} - q_{CA})}{4q_{DA} - q_{CA}} \theta_{\max} \quad (10)$$

$$p_{DA}^* = 2 \frac{q_{DA}(q_{DA} - q_{CA})}{4q_{DA} - q_{CA}} \theta_{\max}. \quad (11)$$

The imitator maximises its profits

$$\Pi_C(T, \infty) = \frac{q_{CA} q_{DA} (q_{DA} - q_{CA})}{(4q_{DA} - q_{CA})^2} \frac{\theta_{\max}^2}{r} e^{-rT}, \quad (12)$$

which yields the optimal product quality q_{CA}

$$q_{CA}^*(q_{DA}) = \frac{4}{7} q_{DA}. \quad (13)$$

Anticipating C 's best quality response, D maximises its discounted flow of profits

$$\begin{aligned} \Pi_D(T, \infty) &= 4 \frac{q_{DA}^2 (q_{DA} - q_{CA}^*(q_{DA}))}{(4q_{DA} - q_{CA}^*(q_{DA}))^2} \frac{\theta_{\max}^2}{r} e^{-rT} - \frac{c}{2} (q_{DA} - q_D)^2 e^{-rT} \\ &= \frac{7}{48} q_{DA} \frac{\theta_{\max}^2}{r} e^{-rT} - \frac{c}{2} (q_{DA} - q_D)^2 e^{-rT}. \end{aligned} \quad (14)$$

Hence, for a given initial quality q_D , the high-quality firm adjusts its product quality such that¹

$$q_{DA}^* = \frac{7}{48rc} \theta_{\max}^2 + q_D. \quad (15)$$

As result, the high-quality firm will always increase its quality, $q_{DA} > q_D$, as $\theta_{\max} > 0$, $r > 0$ and $c > 0$. In contrast, $c < 0$ implies that costs are refundable if the high-quality firm decreases its quality. We do not allow for this possibility but argue in contrast that quality costs of the initially chosen quality are sunk. Thus $q_{DA} > q_D$ and the high-quality firm will always increase its quality.

Firms' discounted equilibrium profits in the unregulated market then amount to

$$\Pi_C(T, \infty) = \frac{\theta_{\max}^2}{2304cr^2} (7\theta_{\max}^2 + 48crq_D) e^{-rT} \quad (16)$$

$$\Pi_D(T, \infty) = \frac{7\theta_{\max}^2}{4608cr^2} (7\theta_{\max}^2 + 96crq_D) e^{-rT}. \quad (17)$$

¹In case both firms simultaneously readjust product quality, the high-quality firm's optimal q'_{DA} is

$$q'_{DA} = \frac{7}{24rc} \theta_{\max}^2 + q_D.$$

The profits reflect the well-known high-quality advantage (see, e.g., Lehmann-Grube, 1997) as the high-quality firm's profits are larger than the imitator's profits. This result is due to the convex quality-related cost function of the high-quality firm and holds although the imitator has no costs.

3.2 Regulated Market

The imitator firm enters the market at $t = t_1$ with quality q_C . It observes the high-quality firm's product of quality q_D fixed in $t = 0$. The patent system requires that the imitator chooses a lower quality than the high-quality firm, $q_C \leq (1 - \Delta) q_D$. For given qualities, firms simultaneously set prices. Thus, in the regulated duopoly, firms maximise discounted profits with respect to prices

$$\Pi_i(t_1, T) = \int_{t_1}^T x_i p_i e^{-rt} dt, \quad (18)$$

for $i = C, D$, which yields the same optimal prices as in the unregulated market, see equations (10) and (11).

Given these equilibrium prices, the imitator chooses its optimal quality to maximize its discounted profits in the regulated market

$$\Pi_C(t_1, T) = \frac{q_{CA} q_{DA} (q_{DA} - q_{CA})}{(4q_{DA} - q_{CA})^2} \frac{\theta_{\max}^2}{r} (e^{-rT} - e^{-rt_1}) \quad (19)$$

given the constraint that quality is restricted by the patent system. Depending on the patent height Δ , but analogously to the analysis in the unregulated market, the optimal quality is

$$q_C^*(q_D) = \alpha q_D, \text{ with } \alpha = \min \left\{ \frac{4}{7}, 1 - \Delta \right\}. \quad (20)$$

The patent protection takes effect only if the regulatory authority chooses $\Delta > \frac{3}{7}$. For $\Delta < \frac{3}{7}$, the imitator will choose optimal quality $q_C = \frac{4}{7}$, which is unaffected by the patent. Thus, $\alpha \leq 4/7$.

3.3 Monopolistic Market

At $t = 0$, the high-quality firm anticipates the imitator's behaviour and maximises its discounted flow of profits during the monopoly, regulated $\Pi_D(t_1, T)$ and unregulated $\Pi_D(T, \infty)$ duopoly, which is given by

$$\Pi_D = \int_0^{t_1} x_M p_M dt + \Pi_D(t_1, T) + \Pi_D(T, \infty) - \frac{c}{2} q_D^2. \quad (21)$$

net of its quality cost $\frac{c}{2} q_D^2$ that occur in $t = 0$. Moreover, the high-quality firm's optimal price in the monopoly period is given by $p_M^*(q_D) = \frac{1}{2} q_D \theta_{\max}$. Using optimal price setting, we can rewrite D 's profits to

$$\begin{aligned} \Pi_D = & \left(\frac{1}{4} (1 - e^{-rt_1}) + \frac{4(1-\alpha)}{(4-\alpha)^2} (e^{-rt_1} - e^{-rT}) + \frac{7}{48} e^{-rT} \right) q_D \frac{\theta_{\max}^2}{r} \\ & + \frac{49}{4608} \frac{\theta_{\max}^4}{c r^2} e^{-rT} - \frac{c}{2} q_D^2. \end{aligned} \quad (22)$$

From this follows that the optimal quality q_D^* is given by

$$q_D^* = \frac{\theta_{\max}^2}{48(4-\alpha)^2 r c} (12(4-\alpha)^2 - 12\alpha(8+\alpha)e^{-rt_1} - (20+\alpha)(4-7\alpha)e^{-rT}). \quad (23)$$

Note that $q_D^* \leq \frac{\theta_{\max}^2}{4rc}$, and since $\alpha \leq 4/7$

$$\frac{\partial}{\partial T} q_D^* = \frac{(20+\alpha)(4-7\alpha)\theta_{\max}^2}{48(4-\alpha)^2 r c} e^{-rT} \geq 0 \quad (24)$$

and

$$\frac{\partial}{\partial \alpha} q_D^* = -\frac{4(2+\alpha)\theta_{\max}^2}{(4-\alpha)^3 c r} (e^{-rt_1} - e^{-rT}) \leq 0 \quad (25)$$

Proposition 1 *The high-quality firm D , whose high-quality product is protected by patent, increases quality the broader the patent protection, that is, the longer the patent lifetime T and the larger the patent height represented by a small α and a large Δ .*

The intuition of this finding is straightforward: As soon as the imitator is restricted in choosing its quality q_C , that is for $\Delta > \frac{3}{7}$, products in the regulated

market are more differentiated than the readjusted ones in the unregulated market. Since for more differentiated products price competition is less fierce, the high-quality firm can offer a higher price-quality ratio and, therefore, realises higher instantaneous profits in the regulated than in the unregulated duopoly. Moreover, since product differentiation is greater the higher the high-quality firm's product quality, a longer patent lifetime as well as a larger patent height increase the high-quality firm's marginal discounted returns from product development.

4 Patent Protection

Prior to the firms' decisions in $t = -1$, the regulatory authority determines patent protection T and Δ . It maximise social welfare SW , which can be calculated from the firms' discounted flow of profits Π_C and Π_D and the consumer surplus CS

$$SW = \beta (\gamma \Pi_C + \Pi_D) + (1 - \beta)CS. \quad (26)$$

Our social welfare function includes weights β and γ , where $\beta \in [0, 1]$ measures the degree with which the regulatory authority takes industry profits into account and $\gamma \in [0, 1]$ is the weight the authority attaches to the imitator's profits compared with the imitator's profits. See Appendix A for the calculation of social welfare.

The social welfare function is linear in β and γ . In the following, will therefore analyse the different β - γ constellations, which stand for different objective functions the regulatory authority might pursue. In particular, the regulatory authority might be interested in maximising solely consumer surplus. In an alternative consideration, the regulatory authority might want to maximise the gross domestic product (GDP), which is the value of production of the innovating and/or the imitating firm. This covers, e.g., a situation in which the imitating firm is located in a foreign country and the regulatory authority decides to consider only profits of the domestic high-quality firm. Finally, we analyse an Utilitarian welfare function, where all considered components have the same weights.

4.1 Consumer Surplus

Suppose that the regulatory authority maximises only consumer surplus, ignoring any firms' profits. Then $\beta = 0$, and the welfare function depends only on the consumer surplus $SW = CS$. In our model of vertical product differentiation, consumer surplus equals the discounted flow of indirect utility of all those consumers who buy either the high- or the low-quality product. For the regulated period, consumer surplus $CS(0, T)$ is the sum of consumer surpluses of the monopoly and the regulated duopoly over time

$$CS(0, T) = \int_0^{t_1} \int_{\theta_0^M}^{\theta_{\max}} (\theta q_D - p_M) d\theta \cdot e^{-rt} dt + \int_{t_1}^T \left(\int_{\theta_0}^{\theta_1} (\theta q_C - p_C) d\theta + \int_{\theta_1}^{\theta_{\max}} (\theta q_D - p_D) d\theta \right) \cdot e^{-rt} dt. \quad (27)$$

Analogously, consumer surplus for the unregulated duopoly $CS(T, \infty)$ is given by

$$CS(T, \infty) = \int_T^\infty \left(\int_{\theta'_0}^{\theta'_1} (\theta q'_C - p'_C) d\theta + \int_{\theta'_1}^{\theta_{\max}} (\theta q'_D - p'_D) d\theta \right) \cdot e^{-rt} dt \quad (28)$$

Summing up yields the welfare function

$$SW^{CS} = CS(0, T) + CS(T, \infty) \quad (29)$$

$$= \left(q_D + \frac{\alpha(28 - \alpha)}{(4 - \alpha)^2} q_D e^{-rt_1} + \left(\frac{49\theta_{\max}^2}{72cr} + \frac{(4 - 7\alpha)(16 - \alpha)}{3(4 - \alpha)^2} q_D \right) e^{-rT} \right) \frac{\theta_{\max}^2}{8r}.$$

Maximisation of the welfare function yields no interior solution. By comparing corner solutions with no patent protection $T = t_1$, $\Delta = 0$, which implies $\alpha = \frac{4}{7}$, and full patent protection $\Delta = \frac{4}{7}$ for $T \geq t_1$, $\alpha = 0$, we find that

Proposition 2 *If the regulatory authority is interested in maximising consumer surplus regardless of firms' profits, it is optimal to offer no patent protection, $\Delta = 0$.*

Proof: See Appendix B

Taking account of consumer preferences only, we are able to reproduce one of the most basic findings in economics: Consumer surplus gains from competition

because prices are lower compared to a situation where one firm has some degree of market power. Looking at it the other way around, the result suggests to not offer any patent protection. This is true although the high-quality firm chooses a lower quality the lower patent protection. In fact, price competition and associated low prices increase consumer surplus and overweight utility loss due to lower quality.

However, in our model we consider only quality-related costs corresponding to past R&D activities. Other R&D cost for investing in innovations, such as expenses for unsuccessful R&D projects, are not taken into account. Indeed, innovation is a risky process and many R&D projects fail or achieve only moderate success (Cooper, Edgett, and Kleinschmidt, 2004). To take costs of unsuccessful R&D projects into account, our model would require to regard fixed costs. By doing so, our welfare analysis might change, e.g. if fixed costs might be too high so that market entry is deterred resulting in lower consumer surplus. Such an analysis requires calibration of the respective parameters and goes beyond the purpose of this paper.

4.2 Gross Domestic Product

As the gross domestic product (GDP) comprises the market value of all final goods and services produced within the domestic country, we have to consider the high-quality firm's and the imitators geographical location. Under the assumption that the high-quality firm is a domestic firm, the GDP can either include the imitating firm's selling or not.

First, suppose that the imitator is located in a foreign country. If the regulatory authority wants to maximise GDP, it solely accounts for the market value of the high-quality firm's selling, which is given by D 's profits. Choosing weights $\beta = 1$ and $\gamma = 0$ implies the corresponding social welfare function, which is equal to D 's discounted flow of profits in equation (21)

$$SW^{GDP} = \Pi_D. \tag{30}$$

Second, suppose that the imitating firm is also domestic and the social welfare function also includes the imitator's profits, see equation (19). This equals weights $\beta = 1$ and $\gamma = 1$ and the social welfare is

$$SW^{GDP} = \Pi_C + \Pi_D. \quad (31)$$

The following proposition shows that the imitator's profits are irrelevant for the high-quality firm of the optimal patent system.

Proposition 3 *The regulatory authority maximises the GDP by choosing the most effective patent protection with patent lifetime $T^* \rightarrow \infty$ and patent height $\Delta^* = 1$. This result holds as long as the high-quality firm is a domestic firm, independent of whether the imitator firm is located in the domestic or a foreign country.*

Proof: See Appendix C

Maximum patent protection $T^* \rightarrow \infty$ and $\Delta^* = 1$ implies that the imitator is restricted to zero quality, $q_C = 1 - \Delta^* = 0$, forever and faces zero demand and profits. Therefore, optimal patent protection keeps the imitator out of the market and the domestic high-quality firm acts as a monopolist.

In case the imitator is a foreign firm, this results is not surprisingly since the high-quality firm's profits Π_D are larger if it is the sole supplier compared to D 's profits resulting from duopoly competition. In case the imitator firm is also a domestic firm, however, this result follows from the fact that overall duopoly profits are still less than monopoly profits with full patent protection.

Proposition 3 together with Proposition 2 implies that the optimal patent lifetime T as well as the optimal patent height Δ are both increasing in β :

$$\frac{\partial}{\partial \beta} T^* > 0 \text{ and } \frac{\partial}{\partial \beta} \Delta^* > 0. \quad (32)$$

If a larger weight β is attached to consumer surplus in the social welfare function, the patent protection is less restrictive. To put it differently, the regulatory authority

fixes a broader patent system, the more important industry profits are. This results follows immediately from a property of the social welfare: it is linear in β .

4.3 Utilitarian Welfare

Again, we distinguish two cases. If the imitator is located in a foreign country, the domestic regulatory authority will maximize a social welfare function that contains domestic welfare only. It can be obtained by fixing the weights² $\beta = 0.5$ and $\gamma = 0$. This yields social welfare as the sum of the two preceding cases represented by equations (29) and (30)

$$SW^{DW} = \Pi_D + CS. \quad (33)$$

Instead by choosing $\beta = 0.5$ and $\gamma = 1$, social welfare is given by an Utilitarian welfare function, where also the imitator's profits are taken into account and all components are considered equally³. This corresponds to the preceding cases as the sum of the two preceding cases represented by equations (29) and (31)

$$SW^U = \Pi_C + \Pi_D + CS. \quad (34)$$

Both with a foreign as well as a domestic imitator, the Utilitarian welfare function combines two cases that yield opposed results with respect to the optimal patent protection. Our results indicate that the positive effect on consumer surplus dominates and drives the results that it is optimal to grant no patent protection at all.

Proposition 4 *If the regulatory aims at maximising Utilitarian welfare, it is optimal to offer no patent protection at all. This result holds as long as the high-quality firm is a domestic firm, independent of whether the imitator firm is located in the domestic or a foreign country.*

²Henceforth, we use a monotonic transformation of the social welfare function for $\beta = 0.5$ so that $SW = 0.5(\gamma\Pi_C + \Pi_D) + 0.5 CS$ simplifies to $SW = \gamma\Pi_C + \Pi_D + CS$.

³With a domestic imitator firm, domestic welfare equals Utilitarian welfare. See Appendix D for calculations.

In both cases, the Utilitarian welfare function is decreasing in T . Therefore, there is no interior solution to the welfare maximisation problem, instead the corner solution with $T = t_1$, $\Delta = 0$ ($\alpha = \frac{4}{7}$), yields maximum welfare. Comparing this result to the maximisation of consumer surplus, we find that the inclusion of any kind of profit is irrelevant for the result.

5 Copy Protection

The previous analysis shows that depending on the social welfare function the optimal patent protection is not always in the high-quality firm's interests. In those situations the high-quality firm might have an incentive to investigate in its own private patent protection. We therefore extend our basic model and assume that the high-quality firm decides on an investment in copy protection, which slows down product imitation by others. In particular, investment in copy protection postpones the point in time t_1 , where the imitator enters the market, to a later point in time. Copy protection is costly, with τt_1 standing for linear copy protection costs. The timing of the extended game is as follows: At stage $t = -1$, the regulatory authority still determines the patent protection. At $t = 0$, the high-quality firm D enters the market, decides on quality and afterwards on copy protection. As before, the imitator enters the market in t_1 . In $t = T$, the patent expires.

Solving the game by backward induction, we build on our results from Section 3.1 for the unregulated market, $t \in [T, \infty)$. We can also adopt the imitators quality choice (20) in t_1 for the regulated market from the basic model. Taking account of these decisions yields the high-quality firm's profit, which equals profit (21) of the basic model minus copy protection cost τt_1

$$\Pi_D = \int_0^{t_1} (\theta_{max} - \theta_0^M) p_M dt + \Pi_D(t_1, T) + \Pi_D(T, \infty) - \frac{c}{2} q_D^2 - \tau t_1. \quad (21')$$

Optimal copy protection can be found by maximising profit (5) with respect to t_1 . Solving for t_1 yields copy protection that is negatively correlated with marginal copy

protection cost τ

$$t_1^*(q_D) = -\frac{1}{r} \left(\ln \frac{\tau 4 (4 - \alpha)^2}{\theta_{\max}^2 q_D \alpha (\alpha + 8)} \right).$$

Taking the subsequent choice on copy protection into account⁴, the high-quality firm maximises profits

$$\begin{aligned} \Pi_D = & \left(\frac{1}{4} - \frac{\tau (4 - \alpha)^2}{\theta_{\max}^2 q_D \alpha (\alpha + 8)} + \frac{4 (1 - \alpha)}{(4 - \alpha)^2} \left(\frac{\tau 4 (4 - \alpha)^2}{\theta_{\max}^2 q_D \alpha (\alpha + 8)} - e^{-rT} \right) + \frac{7}{48} e^{-rT} \right) q_D \frac{\theta_{\max}^2}{r} \\ & + \frac{49}{4608} \frac{\theta_{\max}^4}{c r^2} e^{-rT} - \frac{c}{2} q_D^2 \end{aligned} \quad (35)$$

and chooses optimal quality q_D^{**}

$$q_D^{**} = \left(\frac{1}{4} - \frac{(20 + \alpha)(4 - 7\alpha)}{48(4 - \alpha)^2} e^{-rT} \right) \frac{\theta_{\max}^2}{rc}. \quad (36)$$

Comparing the optimal quality of the high-quality firm in the basic model q_D^* (see equation (23)) with that of our extended version q_D^{**} , we find that

$$q_D^* = q_D^{**} - \frac{\tau}{rc} \frac{1}{q_D^{**}}. \quad (37)$$

Proposition 5 *When the high-quality firm has the possibility to invest in copy protection, it will do so as long as protection cost are not too high. Its optimal quality q_D^{**} is higher compared to the quality q_D^* without copy protection.*

Copy protection is a powerful tool for the high-quality firm. Consider those kind of goods to which copy protection as well as patent protection are applicable. If in this case, the regulatory authority does not supply any patent protection and copy protection costs are not too high⁵, the high-quality firm will decide to implement copy protection as it generates larger profits. However, empirical studies demonstrate, that the industry often fails in implementing copy protection with desired consequences, see e.g. Djekic and Loebbecke (2007) for software; Bauxmann et al. (2005) for the recording industry. Also strategic reasons for dropping copy protection (see Shy and Thisse, 1999) are not considered here.

⁴In order that $t_1 \geq 0$, we assume $\tau \leq \frac{8+\alpha}{4(4-\alpha)^2} q_D \alpha \theta_{\max}^2$.

⁵ $\tau < \frac{\alpha(8+\alpha)\theta^4}{192cr(4-\alpha)^4} (12(-4+\alpha)^2 - 6\alpha(8+\alpha)e^{-rt_1} + (20+\alpha)(-4+7\alpha)e^{-rT}) e^{-rt_1}$

6 Conclusions

This paper analyses potential regulatory requirements to protect firms from falling victim of product piracy and the associated degree of product innovation. We assume that one firm offers a high-quality product and an imitator firm reacts by supplying a clone of less quality. Specifically, we assume that the high-quality firm faces costs that are convex in quality and quality adjustment cost at later stages. In contrast, the imitator is supposed to use a low-cost production process, as 3D printing. Those cost are normalised to zero. The high-quality firm can obtain a patent, which exclusively grants the right to offer a product of a certain quality for a specified time period. After the patent's lifetime expired, both firms compete in an unregulated duopoly and qualities as well as prices are adjusted. The patent's lifetime as well as its height are determined by the regulatory authority.

Our results indicate that patent protection is not always socially optimal. Whether the regulatory authority should decide to offer patents on quality crucially depends on the formulation of the social welfare function. In most cases it is optimal to offer no patent protection at all. This is due to the fact that patent protection and consumer surplus are negatively correlated as consumers benefit from patent-free price competition. If contrarily the regulatory authority restricts its objective function to GDP, full patent protection (and monopoly arising thereby) is optimal. Thus, focusing on GDP solely neglects an important aspect: consumer surplus matters. Further, we find that the imitator's profits are irrelevant in the regulatory authority's objective. If the imitator offers a product of lower quality than the high-quality firm, it will also realise lower profits. The counterpart is known as the high-quality advantage of the designer firm (see, e.g., Lehmann-Grube, 1997). With a low absolute magnitude, those profits do not influence the results.

If the regulatory authority has an interest not in maximising the measurable objective but in nurturing quality, it should abstain from patent protection. Though the high-quality firm offers a product of higher quality the higher patent protection,

the absolute quality level is lower than without patent protection. Similar to patent protection, copy protection leads to lower quality than in the unregulated duopoly; but notably, copy protection by the high-quality firm yields a higher quality than under patent protection. This is an alternative way to protect against product imitations, which is only beneficial for the high-quality firm if costs are not too high.

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A Calculating Social Welfare

Social welfare is given by equation (26):

$$SW = \beta (\gamma \Pi_C + \Pi_D) + (1 - \beta) CS \quad (26)$$

Calculating the Consumer Surplus

The consumer surplus for the regulated market as defined in equation (27), can be calculated as follows

$$CS(0, T) = \frac{1}{8} q_D \theta_{\max}^2 + \frac{(5\alpha + 4)}{2(4 - \alpha)^2} q_D \theta_{\max}^2.$$

Consumer surplus for the unregulated duopoly as given by equation (28) yields

$$CS(T, \infty) = \frac{7}{576cr} (7\theta_{\max}^2 + 24cq_D) \theta_{\max}^2,$$

with q_D denoting D 's quality as given by equation (23). Consumer surplus (38) plus (A) yields social welfare SW^{CS} in equation (29).

Calculating Profits

Profits are given by

$$\begin{aligned} \Pi_C(t_1, T) &= \left(\frac{\alpha(1 - \alpha)}{(4 - \alpha)^2} q_D \theta_{\max}^2 \right) \cdot \frac{1}{r} (e^{-rt_1} - e^{-rT}) \\ \Pi_C(T, \infty) &= \left(\frac{1}{1152cr} (7\theta_{\max}^2 + 24cq_D) \theta_{\max}^2 \right) \cdot \frac{1}{r} e^{-rT} \\ \Pi_C &= \left(\frac{\alpha(1 - \alpha)}{(4 - \alpha)^2} q_D e^{-rt_1} + \left(\frac{(4 - 7\alpha)^2}{48(4 - \alpha)^2} q_D + \frac{7}{1152cr} \theta_{\max}^2 \right) e^{-rT} \right) \frac{1}{r} \theta_{\max}^2 \quad (38) \end{aligned}$$

$$\Pi_D = \left(1 - \frac{\alpha(8 + \alpha)}{(4 - \alpha)^2} e^{-rt_1} - \frac{(\alpha + 20)(4 - 7\alpha)}{12(4 - \alpha)^2} e^{-rT} \right) \frac{1}{4r} \theta_{\max}^2 q_D - \frac{c}{2} q_D^2 \quad (39)$$

Social welfare SW^{GDP} in equations (30) and (31) can be calculated from equations (38) and (39).

Social Welfare

The social welfare function can then be rewritten using

$$\begin{aligned} \beta (\gamma \Pi_C + \Pi_D) &= \beta \left(\frac{1}{4} q_D + \alpha \frac{4\gamma (1 - \alpha) - (8 + \alpha)}{4(4 - \alpha)^2} q_D e^{-rt_1} \right. \\ &\quad \left. + \left(\frac{7\gamma}{1152cr} \theta_{\max}^2 + \frac{(4 - 7\alpha)(\gamma(4 - 7\alpha) - (\alpha + 20))}{48(4 - \alpha)^2} q_D \right) e^{-rT} \right) \frac{1}{r} \theta_{\max}^2 \\ &\quad - \beta \frac{c}{2} q_D^2 \end{aligned}$$

and

$$\begin{aligned} (1 - \beta) CS &= (1 - \beta) \frac{\theta_{\max}^2}{8r} \\ &\quad \left(q_D + \frac{\alpha(28 - \alpha)}{(4 - \alpha)^2} q_D e^{-rt_1} + \left(\frac{49\theta_{\max}^2}{72cr} + \frac{(4 - 7\alpha)(16 - \alpha)}{3(4 - \alpha)^2} q_D \right) e^{-rT} \right) \end{aligned}$$

B Proof of Proposition 2

Using social welfare SW^{CS} , given by equation (29) and quality (23), welfare can be rewritten

$$\begin{aligned} SW^{CS} &= \frac{\theta_{\max}^4}{32cr^2} + \frac{\theta_{\max}^4}{1152cr^2(4 - \alpha)^4} \left(36 \alpha \left(\alpha(-28 + \alpha)(8 + \alpha) + (20 - 2\alpha)(-4 + \alpha)^2 e^{rt_1} \right) e^{-2rt_1} \right. \\ &\quad + \left((4 - 7\alpha)^2(-16 + \alpha)(20 + \alpha) + (-4 + \alpha)^2(2096 + \alpha(-1768 + 203\alpha)) e^{rT} \right) e^{-2rT} \\ &\quad + \left(12(4 + 5\alpha)(-80 + 232\alpha + 19\alpha^2) \right. \\ &\quad \left. \left. + (-84\alpha(8 + \alpha) - 3(20 + \alpha)(-4 + 7\alpha))(4 - \alpha)^2 \right) e^{-r(T+t_1)} \right) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial}{\partial T} SW^{CS} \frac{1152cr(4 - \alpha)^4}{\theta_{\max}^4} &= - \left(2(4 - 7\alpha)^2(-16 + \alpha)(20 + \alpha) e^{rt_1} \right. \\ &\quad - 3\alpha(-4 + 7\alpha)(-1072 + 5(-8 + \alpha)\alpha) e^{rT} \\ &\quad \left. + (4 - \alpha)^2(2096 + \alpha(-1768 + 203\alpha)) e^{r(T+t_1)} \right) e^{-r(2T+t_1)}. \end{aligned}$$

For $0 < \alpha < \frac{4}{7}$, the derivative above is always negative, $\frac{\partial}{\partial T} SW^{CS} < 0$. Thus, there is no interior solution. For the corner solutions full patent protection $\alpha = 0$ and no patent protection at all, $\alpha = \frac{4}{7}$ and $T = t_1$, social welfare simplifies to

$$\begin{aligned} CS^{full} &= \frac{\theta_{\max}^4}{1152cr^2} (36 - 20e^{-2rT} + 131e^{-rT}) \\ CS^{no} &= \frac{\theta_{\max}^4}{1152cr^2} (36 - 20e^{-2rt_1} + 131e^{-rt_1}). \end{aligned}$$

As $t > t_1$ and, thus, $e^{-rt_1} > e^{-rT}$, it is $CS^{no} > CS^{full}$. Hence no patent protection is optimal although a lower quality q_D results.

Q.E.D

C Proof of Proposition 3

$$\frac{\partial}{\partial T} SW^{GDP} = \frac{(4 - 7\alpha)(\alpha + 20)}{48(4 - \alpha)^2} e^{-rT} q_D \theta_{\max}^2 > 0$$

$$\text{as } \left(\frac{1}{4} - \frac{\alpha(8+\alpha)}{4(4-\alpha)^2} e^{-rt_1} - \frac{(4-7\alpha)(\alpha+20)}{48(4-\alpha)^2} e^{-rT} - \frac{cr}{\theta_{\max}^2} q_D \right) = 0.$$

$$\text{Hence, } \alpha = 0, \text{ as } \frac{\partial}{\partial \alpha} \left(-\frac{\alpha(8+\alpha)}{4(4-\alpha)^2} \right) = -4 \frac{\alpha+2}{(4-\alpha)^3} < 0 \text{ and } \frac{\partial}{\partial \alpha} q_D < 0$$

Q.E.D

D Proof of Proposition 4

The social welfare function (26) simplifies for $\beta = \frac{1}{2}$ to

$$\begin{aligned}
 SW = & \frac{\theta_{\max}^4}{4608cr^2(4-\alpha)^4} \left(144(4-\alpha)^4 \right. \\
 & + 144\alpha \left(2(1-\alpha)(1+2\gamma)(4-\alpha)^2 + \alpha(8+\alpha)(-10+\alpha+4(-1+\alpha)\gamma e^{rt_1}) \right) e^{-rt_1} \\
 & + 6\alpha(4-7\alpha) \left(-16(47+14\gamma) + \alpha(16+256\gamma+\alpha(7+22\gamma)) \right) e^{-r(T+t_1)} \\
 & + \frac{1}{2} \left((20+\alpha)(-44-8\gamma+\alpha(5+14\gamma))(4-7\alpha)^2 \right. \\
 & \quad \left. + 4(4-\alpha)^2(1616+208\gamma+7\alpha(-8(17+7\gamma)+\alpha(35+43\gamma)))e^{rT} \right) e^{-2rT} \Big) \\
 & \hspace{15em} (26')
 \end{aligned}$$

Hence,

$$\begin{aligned}
 \frac{\partial}{\partial T} SW \frac{4608cr(4-\alpha)^4}{\theta_{\max}^4} = & 6\alpha(-4+7\alpha) \left(16(47+14\gamma) - \alpha(16+256\gamma+\alpha(7+22\gamma)) \right) e^{-r(T+t_1)} \\
 & + 2(4-\alpha)^2 \left(-1616 - 208\gamma + 7\alpha(8(17+7\gamma) - \alpha(35+43\gamma)) \right) e^{-rT} \\
 & + (20+\alpha)(4-7\alpha)^2 \left(44 + 8\gamma - \alpha(5+14\gamma) \right) e^{-2rT}
 \end{aligned}$$

and because the second line of (40) is negative and has a greater value than the remaining lines, it is $\frac{\partial SW}{\partial T} < 0$ and no patent protection is optimal.

Q.E.D.

E Proof of Proposition 5

If $q_D = q_D(t_1)$ then for given q_D

$$\begin{aligned}
 \frac{\partial}{\partial t_1} \Pi_D &= q_D \frac{\alpha(\alpha+8)}{4(4-\alpha)^2} \theta_{\max}^2 e^{-rt_1} - \tau = 0 \\
 e^{-rt_1} &= \frac{\tau 4(4-\alpha)^2}{\theta_{\max}^2 q_D \alpha(\alpha+8)} \\
 t_1 &= -\frac{1}{r} \left(\ln \frac{\tau 4(4-\alpha)^2}{\theta_{\max}^2 q_D \alpha(\alpha+8)} \right)
 \end{aligned}$$

$$\begin{aligned}\Pi_D = & \left(\frac{1}{4} \left(1 - \frac{\tau 4 (4 - \alpha)^2}{\theta_{\max}^2 q_D \alpha (\alpha + 8)} \right) + \frac{4 (1 - \alpha)}{(4 - \alpha)^2} \left(\frac{\tau 4 (4 - \alpha)^2}{\theta_{\max}^2 q_D \alpha (\alpha + 8)} - e^{-rT} \right) \right. \\ & \left. + \frac{7}{48} e^{-rT} \right) q_D \frac{\theta_{\max}^2}{r} + \frac{49}{4608} \frac{\theta_{\max}^4}{c r^2} e^{-rT} - \frac{c}{2} q_D^2\end{aligned}\quad (40)$$

$$\frac{\partial}{\partial q_D} \Pi_D = \left(\frac{1}{4} - \frac{(\alpha + 20) (4 - 7\alpha)}{48 (4 - \alpha)^2} e^{-rT} \right) \frac{\theta_{\max}^2}{r} - c q_D \quad (41)$$

$$q_D = \left(\frac{1}{4} - \frac{(\alpha + 20) (4 - 7\alpha)}{48 (4 - \alpha)^2} e^{-rT} \right) \frac{\theta_{\max}^2}{c r} \quad (42)$$

$$q_D^* = \left(\frac{1}{4} - \frac{12\alpha (8 + \alpha)}{48 (4 - \alpha)^2} e^{-rt_1} - \frac{(20 + \alpha) (4 - 7\alpha)}{48 (4 - \alpha)^2} e^{-rT} \right) \frac{\theta_{\max}^2}{r c} \quad (43)$$

$$= q_D - \frac{\theta_{\max}^2}{r c} \frac{12\alpha (8 + \alpha)}{48 (4 - \alpha)^2} e^{-rt_1} \quad (44)$$