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## Market Size Effects in New New Trade Theory

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# 1 Motivation

Policy makers and the public are concerned about the *relative size* of their economies. This is most clearly visible in the discussion about the increase of the relative weight of emerging countries.<sup>1</sup> New Trade Theory (Krugman, 1980; Helpman and Krugman, 1985) based on models with representative firms provides a theoretical rationale. With trade costs, product differentiation, increasing returns to scale at the firm-level, and imperfect competition, firms prefer to locate in the larger country, *ceteris paribus*, since this allows the majority of sales to be carried out without incurring transportation costs.

Market size affects equilibrium outcomes through different channels depending on modeling assumptions. In a single-sector model with homogeneous firms, “*the larger country, other things equal, will have the higher wage*” (Krugman, 1980, p. 954). Helpman and Krugman (1985) and much of the following literature study market size effects under the assumption that the wage is the same in both countries. The differentiated good sector featuring increasing returns to scale, monopolistic competition, and iceberg trade costs is accompanied by a linear outside sector producing a freely traded homogeneous good under perfect competition. Then, “*increasing-returns to scale industries, other things equal, [tend] to concentrate production near their larger markets*” (Helpman and Krugman, 1985, p. 197). This prediction is known as the *Home Market Effect* (HME).

We analyze the role of market size in a single-sector model with heterogeneous firms and selection á la Melitz (2003). We report the following analytical results. First, in the presence of trade costs, the larger country (or region) pays the higher wage, as in the single-sector Krugman (1980) model with homogeneous firms. Second, the larger country hosts an over-proportional share of firms. While this finding is similar to the prediction of Helpman and Krugman (1985), the mechanisms are different. In multi-sector models, the result is generated by endogenous labor supply to the differentiated good sector.<sup>2</sup> In the single-sector

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<sup>1</sup>In a special report, The Economist (Sept 24, 2011) argues “*The shift in economic power from West to East is accelerating ... The rich world will lose some of its privileges*”. It provides examples of policy makers’ obsession with “*grandeur and decline*” and China “*Becoming number one*”.

<sup>2</sup>In a model with a continuum of differentiated goods that differ in the elasticity of substitution, industry  $z$  is subject to a home market effect “if the large country’s share of varieties of  $z$  produced globally is greater than its

Melitz (2003) model with fixed labor supply, the result is generated by (domestic) selection. In the larger country, a larger fraction of firms remain purely domestic and therefore relatively smaller. This allows a larger number of them to exist in equilibrium, which, in turn, increases welfare per capita in the larger country due to the availability of a larger range of varieties. Importantly, while the mass of firms *attempting entry* is strictly proportional to the endowment size, the likelihood of a given firm to *successfully* cover its fixed costs is greater in the larger market.

Third, trade liberalization reduces the impact of country size differences on wages. It magnifies its effect on concentration of production, which is similar to the standard Home Market Magnification (HMME) effect. On total, trade liberalization results in convergence of real per capita income like in familiar models of comparative advantage. Moreover, concentration of production in the larger country is increasing in the degree of productivity dispersion, which constitutes a further HMME. Finally, the larger country has relatively fewer but larger exporters.

Our paper is related to a strand of the literature that studies the robustness of the HME. The HME appears robust to allowing for a non-CES demand structure (Ottaviano and van Ypersele, 2005), for oligopolistic rather than monopolistic market structure (Feenstra, Markusen and Rose, 2001), for many differentiated goods sectors (Hanson and Xiang, 2004), for the existence of multinational firms (Larch, 2007) or for more than two trading countries (Behrens, Lamorgese, Ottaviano and Tabuchi, 2009). Also, as shown by Helpman and Krugman (1985), one can have more than one factor of production if the production technology is homothetic.

The assumption of the linear outside sector is more critical. Davis (1998) introduces trade costs into the outside sector and shows that, when transport costs are identical for both types of goods, the HME vanishes. The HME re-emerges only if relative costs of trading differentiated goods are unusually high.

The usual modeling of the outside sector implies a perfectly elastic supply of labor to the increasing returns sector so that the higher attractiveness of the larger economy as a loca-

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share of world factor supplies" (Hanson and Xiang, 2005, p. 1111). Here, labor supply to a certain industry is also endogenous.

tion of production of differentiated goods is not offset by an increase in the wage rate. If the wage does adjust, the HME can be dampened or can even disappear (Head and Mayer, 2004). Crozet and Trionfetti (2008) have qualified this prediction.<sup>3</sup> They introduce Armington differentiation and trade costs into the outside sector, nesting Helpman and Krugman (1985) and Davis (1998). Their numerical results suggests that the HME survives but becomes non-linear.

The linear outside sector has also been used in models with heterogeneous firms.<sup>4</sup> Demidova and Rodriguez-Clare (2013) use a small economy Melitz (2003) model to show that eliminating the assumption of an outside sector reverses the result in Melitz and Ottaviano (2008) or Demidova (2008), where a country that unilaterally lowers trade costs experiences a decline in welfare. We show that the outside sector assumption can be replaced by a Melitz (2003) type mechanism by which heterogeneous firms select into different activities according to their productivity.

Our paper is related to Takahashi, Takatsuka, and Zeng (2013). They prove the existence of a home market effect in single-sector, homogeneous firms model with a second factor which is mobile across countries.<sup>5</sup> As in our analysis, the home market effects in wages and in firm shares go hand in hand. Their model predicts that the wage differential and the industrial location in the larger country evolve in an inverted U-pattern when freeness of trade rises. Relative real per capita income can be non-monotonous in trade costs. In contrast, we find monotonic relationships. When freeness of trade rises, the relative wage falls, the share of firms hosted by the larger country rises, and the real per capita gap declines.

Several papers analyze the role of market size in the presence of heterogeneous firms. In a two-sector Ricardian model with heterogeneous firms, without export selection, and where Foreign's sector 2 is a mirror image in technology of Home's sector 1 and vice versa, Okubo

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<sup>3</sup>In a multi-sector extension of the standard model, Krugman and Venables (1999) show that the HME continues to exist as long as there are at least some homogeneous goods sectors with zero transportation costs and some differentiated goods sectors with zero fixed costs.

<sup>4</sup>Other prominent papers that have used a linear, freely traded, perfectly competitive outside sector in a Melitz-type environment are Helpman, Melitz and Yeaple (2004), Grossman, Helpman and Szeidl (2006), Chor (2009), Baldwin and Okubo (2009), Baldwin and Forslid (2010), and Ossa (2011).

<sup>5</sup>Zeng and Uchikawa (2014) extend the analysis to a multi-country framework.

(2009) shows that the larger country pays the higher wage and the country with the larger expenditure share has a more than proportional number of exporters in a sector with a given technology. We analyze the market size effects on the relative mass of active firms in a single-sector model.

Melitz and Ottaviano (2008) analyze market size effects in an environment with an outside side sector and endogenous mark-ups. They find that the larger market exhibits larger and more productive firms as well as more product variety, lower prices, and lower mark-ups. Importantly, the size of a country's trading partner does not affect the productivity cut-off and hence product variety and welfare. In the Melitz (2003) model, in contrast, productivity cut-offs depend on relative country size. Moreover, firms in the larger country are on average less productive and charge higher prices.

Arkolakis, Ramondo and Rodriguez-Clare (2013) argue that the home market effect is a determinant of the location of innovation and production in an extension of the Melitz (2003) model where firms are allowed to separate innovation and production geographically. Our paper aims at developing an understanding of the market size effects in a standard Melitz (2003) model.

Our paper contributes to the literature on gains from trade with and without firm-level heterogeneity. Arkolakis, Costinot and Rodriguez-Clare (2012) discuss a general class of models encompassing the Melitz framework for asymmetric countries. They show that – conditional on the share of a country's spending on its own goods – heterogeneity and selection matter less for aggregate welfare than what has been hitherto believed. For our result, in contrast, heterogeneity is absolutely crucial. When a country commands a larger share of world population, its attractiveness as a production location increases. The increased demand for labor is accommodated along two margins: first, the relative wage of the country goes up; second, the average size of firms goes down. In a model with a linear outside sector, the first channel would not be present, and the HME is of maximum size. In a model with homogeneous firms, firm size cannot adjust, and the wage needs to rise until the number of firms is exactly proportional to the labor force. The more dispersed the productivity distribution, the stronger the link between average firm size and the number of firms, and, accordingly, the

more pronounced is the HME.

Melitz and Redding (2015) compare gains from trade predicted by a heterogeneous firm model to those predicted by a homogeneous firm model that is a special case with a degenerate productivity distribution. They find that due to the selection channel, gains from trade are larger in the heterogeneous than in the homogeneous firm model. They focus on gains from trade generated by trade liberalization with two symmetric countries. We explore the role of market size differences.

Moreover, our paper has implications for empirical work. Due to the prevalence of the HME in models of *increasing returns to scale* models, it has been used as a discriminating criterion to test for the validity of New Trade Theory (Davis and Weinstein, 1999, 2003).<sup>6</sup> Results are mixed. However, it is well possible that rejection of the HME is actually due to the empirical failure of the outside sector assumption rather than the more relevant increasing returns to scale feature.

The remainder of the paper is structured as follows. Chapter 2 describes the model. Chapter 3 proves our main results: the existence of the HME, and the magnification by lower trade costs or higher productivity dispersion. Chapter 4 contains a discussion of extensions. Chapter 5 concludes.

## 2 The Model

### 2.1 Basic environment

The model is the basic extension of Melitz (2003) to the case of two large asymmetric countries, indexed by  $i \in \{H, F\}$ . Each country is populated with  $L_i$  identical households. Labor is the only factor of production.<sup>7</sup> Importantly, each household *inelastically* supplies one unit of

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<sup>6</sup>A number of prominent empirical papers are Feenstra, Markusen, and Rose (2001), Head and Ries (2001), and Davis and Weinstein (2003), Hanson and Xiang (2004). In their survey, Head and Mayer (2004) conclude that “The evidence on HMEs accumulated in those papers is highly mixed”. More recently, Crozet and Trionfetti (2008) or Brülhart and Trionfetti (2009) find evidence in favor of the HME, while Pham, Lovely, and Mitra (2014) fail to find robust evidence.

<sup>7</sup>We devise the model as a single-factor framework. Multiple (non-traded) factors can be easily accommodated if one is willing to assume that variable and fixed inputs are in terms of a composite input, which combines

labor.<sup>8</sup> We will denote wages by  $w_i$ . The representative consumer has a standard Dixit-Stiglitz utility function defined over a continuum of differentiated varieties

$$U_i = \left[ \int_{z \in \Omega_i} q[z]^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where the measure of the set  $\Omega_i$  is the mass of available varieties,  $q[z]$  is the quantity of variety  $z$  consumed, and  $\sigma > 1$  is the elasticity of substitution.

Firms compete monopolistically in a *single* sector. After paying fixed setup costs  $w_i f^e$ , they obtain information about their productivity level  $\varphi$ . Productivities are sampled from a Pareto distribution whose c.d.f. is given by  $G_i[\varphi] = 1 - \varphi^{-\beta}$ , as in standard quantitative trade literature.<sup>9</sup> The shape parameter  $\beta$  is inversely related to productivity dispersion. Output is linear in  $\varphi$ . A firm in country  $i$  pays fixed market access costs  $w_i f_{ij}$  to serve consumers in country  $j$ . Selection implies that a firm does not necessarily serve one or both markets. Whenever advantageous, we use  $f_{ij} = f_{ji} = f^x$  and  $f_{ii} = f_{jj} = f^d$ . As usual, exporting involves symmetric iceberg trade costs  $\tau_{ij} = \tau_{ji} = \tau \geq 1$ , where  $\tau_{ii} = 1$ . Then,  $\tau_{ij} w_i / \varphi$  is the marginal cost of producing one unit of output in  $i$  and selling to  $j$ . In the following description of equilibrium conditions, we will be very brief since the model is very standard.

## 2.2 Equilibrium conditions

The first set of equilibrium conditions are *zero cutoff profit conditions*. They pin down the minimum productivity level  $\varphi_{ij}^*$  required for a firm in country  $i$  to make at least zero profits by selling in country  $j$ . Since we have two countries, there are *four* of those conditions:

$$\frac{E_j}{\sigma} \left( \frac{\rho P_j}{\tau_{ij} w_i} \varphi_{ij}^* \right)^{\sigma-1} = w_i f_{ij}, \quad i \in \{H, F\}, j \in \{H, F\}, \quad (2)$$

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different factor services in a constant returns to scale fashion. That composite input takes the role of labor in our analysis; all results stated in this paper would continue to hold.

<sup>8</sup>In the standard Krugman (1980) framework, Fujita, Krugman, and Venables (1999) consider a two-sector model with flexible elasticity of labor supply to the differentiated good sector. With perfectly elastic labor supply, the HME always appears, but if we approach the perfectly inelastic labor supply case, the HME will be reversed for some level of trade costs; see Head and Mayer (2004), p. 29f.

<sup>9</sup>Note that each variety  $z$  is produced by a single firm with productivity level  $\varphi$ . We henceforth index varieties by  $\varphi$ . CES preferences and Pareto jointly guarantee that trade flows obey gravity, see Costinot and Rodriguez-Clare (2015).

where  $\rho = (\sigma - 1)/\sigma \in [0, 1]$  is the inverse of the mark-up.<sup>10</sup> Since we assume balanced trade, aggregate expenditure  $E_j$  is equal to national income  $w_j L_j$ . The price index is given by

$$P_i^{1-\sigma} = \theta \sum_{j \in \{H, F\}} m_{ji} M_j \left( \frac{\rho \varphi_{ji}^*}{\tau_{ji} w_j} \right)^{\sigma-1}, i \in \{H, F\}, \quad (3)$$

where  $M_i$  denotes the (endogenous) mass of firms,  $m_{ij} = \left(1 - G[\varphi_{ij}^*]\right) / \left(1 - G[\varphi_i^e]\right)$  is the fraction of firms located in country  $i$  which serve market  $j$ . Note that  $\varphi_i^e$  denotes the productivity of the marginal firm (the least efficient operative firm).<sup>11</sup>  $\theta \equiv \beta / (\beta - (\sigma - 1))$  is a strictly positive constant.<sup>12</sup> The left hand side of (2) denotes profits of a firm with labor productivity  $\varphi_{ij}^*$ . They are proportional to aggregate profits  $E_j/\sigma$ . The right hand side denotes the value of fixed market entry costs. Firm-level profits increase in the foreign price level as the firm's competitive position there is improved; they decrease in  $w_i$  for the opposite reason. Hence, for a given demand level  $E_j P_j^{\sigma-1}$ , an increase in the wage rate  $w_i$  must be accompanied by stronger selection (an increase in the entry cutoff  $\varphi_{ij}^*$ ) in order to restore zero profits of the cutoff firm. For a given cost situation in country  $i$  (wage  $w_i$ ), also less productive firms can successfully enter country  $j$  if a rise in population (rise in  $L_j$ ) increases demand.

The second set of equilibrium conditions are *free entry conditions*. In each country, firms invest fixed setup costs until expected profits from entering  $(\theta - 1)w_i \sum_j m_{ij} f_{ij}$  are equal to entry costs discounted by the probability of successful entry  $p_i^{in} = 1 - G[\varphi_i^e]$  for  $i \in \{H, F\}$ ,  $j \in \{H, F\}$ . The *two* free entry conditions therefore are

$$(\theta - 1)p_i^{in} \sum_{j \in \{H, F\}} m_{ij} f_{ij} = f^e, i \in \{H, F\}. \quad (4)$$

Note that wages have dropped out from this condition. They nevertheless affect entry in general equilibrium. For a given probability of successful entry  $p_i^{in}$  and a given demand level

<sup>10</sup>The equilibrium conditions are derived in detail in (B.1) in the Appendix.

<sup>11</sup>Empirical evidence suggests that only the most productive firms export. The model reproduces this stylized fact if parameters are such that  $\varphi_{ij}^* > \varphi_{ii}^*$ . We refer to this situation as to the case of *conventional sorting*, whereby  $p_i^{in} = 1 - G[\varphi_{ii}^*]$ . Conventional sorting obtains if Home's share in the world labor endowment is not too large, i.e. if  $\lambda < \bar{\lambda} \equiv \lambda[\eta, \beta, \rho; f^x/f^d]$ , where  $\eta \equiv \tau^{-\beta} (f^x/f^d)^{1-\frac{\beta}{\sigma-1}} \in (0, 1]$  is a measure of the freeness of trade. We characterize  $\bar{\lambda}$  in Appendix A.1.4 and consider unconventional sorting in section 4.3.

<sup>12</sup>This is to ensure that the variance of the size distribution is finite.



$E_j P_j^{\sigma-1}$ , an increase in  $w_i$  comes along with decline in the export participation rate  $m_{ij}$ , which implies a fall in expected profits. Hence, an increase in the wage  $w_i$  must be accompanied by an increase in the probability of successful entry in order to restore zero profits in expectations.

Finally, there are *two labor market clearing conditions*. With the above equilibrium conditions and using the Pareto distribution, they simplify to

$$\frac{M_i \beta f^e}{p_i^{in} \rho} = L^i, i \in \{H, F\}. \quad (5)$$

If  $p_i^{in}$  were exogenous (or, as in Krugman (1980) equal to unity), labor supply and the mass of operative firms  $M_i$  would be proportional and there could not be a HME. Selection enables the existence of a HME.

Summarizing, we have four zero cutoff profit conditions (2), two free entry conditions (4) and two labor market clearing conditions (6) to pin down eight unknown endogenous variables of the model  $\{\varphi_{HH}^*, \varphi_{FF}^*, \varphi_{HF}^*, \varphi_{FH}^*; M_H, M_F; w_H, w_F\}$ . Knowledge of these equilibrium objects allows to determine  $m_{ij}$  and  $p_i^{in}$ . In the following, we use Foreign labor as the numeraire and denote  $w_H/w_F \equiv \omega$  as the relative wage.

### 2.3 Trade balance

Balanced trade is implicit in conditions (2), (4) and (6); it is implied by the representative agents in both countries each satisfying their respective budget constraints. Nonetheless, it is useful to make the *balanced trade* condition explicit. It can be written as

$$M_H \bar{r}_{HF} = M_F \bar{r}_{FH}, \quad (6)$$

where  $M_j \bar{r}_{ji}$  denotes aggregate sales of firms located in country  $j$  in market  $i$ . One can show that  $\bar{r}_{ij} = \sigma \theta w_i m_{ij} f_{ij}$ . Using the definition of  $m_{ij}$  expression along with equation (5), the trade balance condition (6) can be rewritten so that the relative wage  $\omega$  appears as a function of Home's share in the world labor endowment and the ratio of the two countries' export

productivity cutoffs:

$$\omega = \frac{1 - \lambda}{\lambda} \left( \frac{\varphi_{FH}^*}{\varphi_{HF}^*} \right)^{-\beta} \quad \text{with} \quad \lambda \equiv \frac{L_H}{L_H + L_F}. \quad (7)$$

So, the trade balance condition ensures that, for given relative country size, a shift in  $\omega$  will induce opposite movements in two countries' foreign market access threshold productivities  $\varphi_{FH}^*$  and  $\varphi_{HF}^*$ . Without loss of generality, we assume Home to be weakly larger than Foreign such that  $\lambda \geq 1/2$ .

## 2.4 Equilibrium

We characterize the equilibrium of the asymmetric Melitz model with the help of two separate equilibrium conditions in a diagram with  $p_H^{in}/p_F^{in} \equiv \chi$  on the y-axis and  $\omega$  on the x-axis. The two curves have opposite slopes and allow conducting comparative statics in an insightful and tractable manner. Equilibrium of the two-country asymmetric Melitz economy is given by the intersection of these curves.

**Lemma 1 (Equilibrium.)** *Suppose that Home is not too big such that in both countries only the most productive firms export, i.e.  $\lambda < \bar{\lambda} \equiv \lambda[\eta, \beta, \rho; f^x/f^d]$ . Then, there always exists a unique equilibrium at the intersection between a strictly downward-sloping convex **ex post profitability curve***

$$\chi \equiv \frac{p_H^{in}}{p_F^{in}} = \frac{1 - G[\varphi_{HH}^*]}{1 - G[\varphi_{FF}^*]} = \frac{\lambda}{1 - \lambda} \omega^{-\frac{2\beta - \rho}{\rho}}. \quad (8)$$

*and a strictly increasing convex (the latter under mild parameter restrictions) **ex ante profitability curve**:*

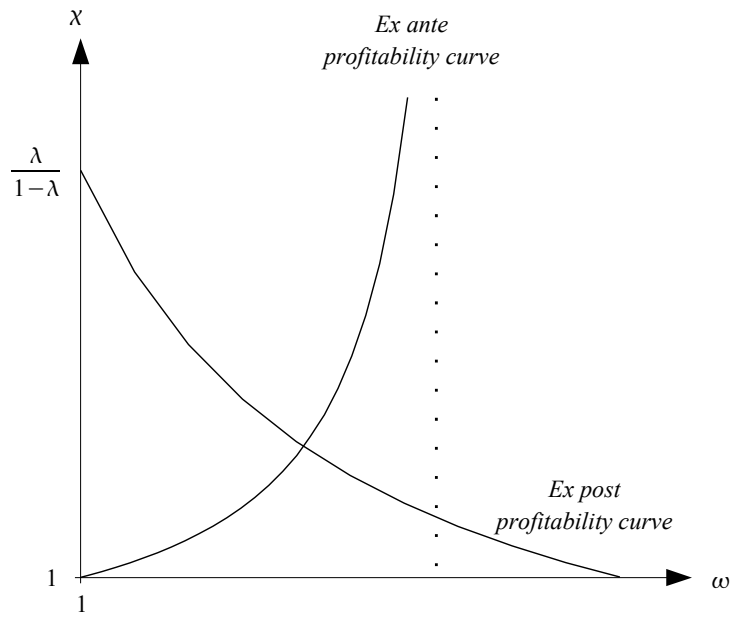
$$\chi = \frac{1 - \eta\omega^{-\frac{\beta}{\rho}}}{1 - \eta\omega^{\frac{\beta}{\rho}}}, \eta\omega^{\frac{\beta}{\rho}} \neq 1. \quad (9)$$

**Proof.** In Appendix A.1. ■

**Ex post profitability.** Under the assumption of conventional sorting, the “ex post profitability curve” (8) is obtained by combining all four zero cutoff profit conditions (2) and observing that balanced trade implies (7). At  $\omega = 1$ , we have  $\chi = \lambda/(1 - \lambda)$ . Moreover,  $\chi$  converges to

zero as  $\omega \rightarrow \infty$ . Figure 1 illustrates the locus. If  $w_i$  increases for a given demand level  $E_i P_i^{\sigma-1}$ , Home's relative labor costs go up, and a higher domestic entry cutoff is required in order to restore zero profits. The logic remains valid in general equilibrium. Hence, an increase in the relative wage  $\omega$  fosters the selection effect and is therefore associated to a fall in the relative probability of successful entry  $\chi$ . Equation (8) takes an *ex post* perspective in that it summarizes firm behavior after the resolution of uncertainty about productivity.

Figure 1: Existence and uniqueness of equilibrium



**Ex ante profitability.** Equation (9) constitutes a second relationship between relative entry probability  $\chi$  and relative wage  $\omega$ . The derivation starts from the free entry conditions (4). It makes use of balanced trade (7) and employs equations (2) to eliminate productivity cutoffs. It emerges from the point  $(1, 1)$  with a slope  $2\beta/(\rho(1-\eta))$ . Moreover, we have an asymptote at  $\omega = \eta^{-\rho/\beta} > 1$ . Figure 1 illustrates the locus. We refer to this schedule as to the “ex ante profitability curve”: if  $w_i$  increases, Home's exports become more expensive. For given Foreign demand  $E_j P_j^{\sigma-1}$ , this implies that Home's export participation rate declines, reducing expected profits. This disadvantage has to be compensated by a higher probability of successful entry in order to restore zero profits in expectations. The logic remains valid in general

equilibrium. Hence, the ex ante profitability curve illustrates a positive relationship the relative entry probability  $\chi$  and the relative wage rate  $\omega$ . It takes an *ex ante* perspective in that it relates to potential firms' decisions to sink setup costs and learn about their productivities. The curve is convex if  $\eta > \rho / (4\beta + \rho)$ ; a sufficient condition for this is  $\eta > 1/5$ .<sup>13</sup>

Note that Lemma 1 resembles the balance-of-payments condition that determines the wage rate in a single-sector model without selection if one equates equations (8) and (9) and considers the limiting case  $\beta \rightarrow \sigma - 1$ ; see equation (14) in Krugman (1980).

### 3 Market size effects in a single-sector Melitz model

We now use the graphical tool developed in the previous section to analyze how market size affects equilibrium outcomes and how trade costs interact with market size effects.

#### 3.1 Wage and selection effects

First, consider the effect of market size on wages. Figure 1 shows that with heterogeneous firms, this effect is coupled on a selection effect, resulting in the following proposition:

**Proposition 1** (*Wage and selection effects.*) *Suppose that conventional sorting holds. Then, the larger country pays the higher wage ( $\omega > 1$ ) and features a relatively lower selection effect ( $\chi > 1$ ).*

**Proof.** Immediate follows from the observations that the ex ante profitability curve emerges from (1,1) and is increasing with an asymptote at  $\omega = \eta^{-\rho/\beta}$ , while at  $\omega = 1$ , the downward-sloping ex post profitability curve results in  $\chi = \lambda / (1 - \lambda) > 1$  with an asymptote at  $\chi = 0$  when  $\omega$  goes to infinity. ■

The intuition for the result is simple: At given factor costs, firms find it more profitable to produce in the larger market as this minimizes payments of variable trade and market access costs. As in a single-sector model with homogeneous firms, this advantage is offset by a wage

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<sup>13</sup>With the realistic parameterization  $\rho = 0.74$  (i.e.  $\sigma = 3.8$ ), and  $\beta = 3.3$ , convexity requires that  $\eta > 0.053$ . That, in turn, requires an ad valorem tariff equivalent of 144 percent if  $f^x/f^d = 1$ , and even higher if  $f^x > f^d$ .

differential. Moreover, presence in the large country is particularly valuable for firms with intermediate productivity levels. Since they do not export, the higher wage in Home puts them at a competitive disadvantage in Home but not in Foreign. Hence, the selection effect is weaker in the larger country.

Note that in the single-sector Krugman (1980) model, there is no selection, and only the wage adjustment channel is active. Hence, the wage effect must be larger in models with homogeneous firms than in models with heterogeneous firms. Note that the welfare consequences in general differ across models. While a higher wage makes domestic varieties more expensive, thereby increasing the price index, the direct effect on labor income dominates such that real income increases. More domestic variety increases welfare as consumers have love for variety.

### 3.2 Home Market Effects

We now analyze the mapping between the share of firms located in the larger economy and its share in the world labor endowment. From equation (5) we know that the mass of active firms in each country is proportional to the labor force times the probability of successful entry.<sup>14</sup> This allows writing Home's share of firms as a function only of the relative entry probabilities  $\chi$  and of Home's labor share  $\lambda$

$$\phi \equiv \frac{M_H}{M_H + M_F} = \gamma\lambda \quad (10)$$

with

$$\gamma \equiv \frac{\chi}{1 + \lambda(\chi - 1)}. \quad (11)$$

Clearly,  $\gamma$  increases in  $\chi$  and falls in  $\lambda$ .

Before we proceed, we need a more precise definition of the phenomenon that we are interested in:

**Definition 1** *A weak (static) home market effect (weak HME) exists, if the share of firms*

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<sup>14</sup>Note that Home's share of firms that pay entry fixed costs is proportional to relative country size:  $M_H^e / (M_H^e + M_F^e) = \lambda$ .

located in Home is larger than its share in the world labor endowment, i.e., if

$$\phi > \lambda.$$

A **strong (dynamic) home market effect** (strong HME) exists, if an increase in Home's labor share yields a more than proportionate increase in Home's share of firms, i.e., if

$$\phi'(\lambda) > 1.$$

The weak form of the HME is the one usually discussed in the literature. The definition of the HME used by Helpman and Krugman (1985), Hanson and Xiang (2004) or Behrens, Lamorgese, Ottaviano, and Tabuchi (2009) coincide with it. In the standard case with a linear outside sector (with or without firm-level heterogeneity),<sup>15</sup>  $\gamma$  is equal to a constant  $\bar{\gamma}$  and so  $\phi = \bar{\gamma}\lambda$ . The weak HME materializes if and only if  $\bar{\gamma} > 1$ . Linearity of  $\phi$  in  $\lambda$  implies that the requirement for the strong HME is identical:  $\phi'(\lambda) = \bar{\gamma} > 1$ . Hence, in contrast to the single-sector model, in the presence of a linear outside sector, it is not interesting to distinguish a weak and a strong version of the HME.

Using Figure 1, it is very easy to show that the model exhibits a *weak HME*. Remembering  $\lambda > 1/2$ , equation (11) and Definition 1 imply that

$$\chi > 1 \Leftrightarrow \gamma > 1. \tag{12}$$

So, a weak HME exists if and only if the probability of successful entry,  $\chi$  is greater in the larger home economy than in Foreign, i.e.,  $p_H^{in}/p_F^{in} \equiv \chi > 1$ . Figure 1 establishes that this is indeed the case since  $\lambda/(1-\lambda) > 1$  so that all admissible realizations of  $\chi$  satisfy (12) and there is indeed a weak HME. Note the crucial role of firm-level heterogeneity: if all firms were identical, and thus, in a meaningful equilibrium,<sup>16</sup> all of them would find it worthwhile to produce, in both countries we would have  $p_i^{in} = 1$ , and hence  $\chi = 1$ . It follows that  $\gamma = 1$  and

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<sup>15</sup>See Appendix B.2 for the model with a linear outside sector.

<sup>16</sup>The parameter constellation could be such that no firm wants to operate ( $p_i^{in} = 0$ ). We exclude such shut-down equilibria.

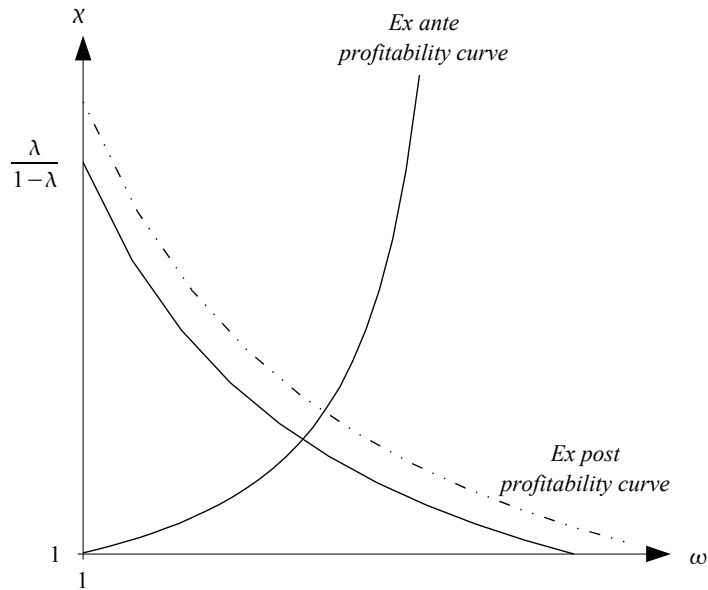
the relationship between  $\phi$  and  $\lambda$  would be one-to-one: there would not be a HME.

The *strong HME* is slightly more involved. It obtains when an increase in the labor share of a country leads to a more-than-proportionate increase in its share of firms. Denote by  $\epsilon_x$  the elasticity of some variable  $x$  with respect to  $\lambda$ . Then,

$$\epsilon_\phi = 1 + \epsilon_\gamma > 1 \Leftrightarrow \epsilon_\gamma > 0 \quad (13)$$

To verify the validity of the above condition, one needs to understand how  $\gamma$ , and hence  $\chi$ , depend on  $\lambda$ . This can be easily seen with the help of our figure, where the effect of an increase in Home's share of labor affects only the ex post profitability curve. It shifts upwards if  $\lambda$  increases; the shift is larger, the smaller  $\lambda$  is initially. Figure 2 illustrates this situation. Clearly, an increase in  $\lambda$  leads to a higher relative wage  $\omega$  and to a higher relative entry probability of Home.

Figure 2: Country size shock



So, as long as the conventional sorting condition holds,  $d\chi/d\lambda > 0$  and  $d^2\chi/d\lambda^2 < 0$ . Using (11), it is clear that around the symmetric equilibrium ( $\lambda = 1/2, \chi = 1$ ), the derivative of  $\gamma$  with respect to  $\lambda$  is given by  $(d\chi/d\lambda)/2$ . It follows that, around the symmetric equilibrium,  $\epsilon_\gamma > 0$  is positive and a strong HME exists. As  $\lambda$  grows away from symmetry, the positive

increments to  $\chi$  become smaller; moreover, a higher  $\lambda$  also has a direct negative effect on  $\gamma$ . It follows that  $\epsilon_\gamma > 0$  cannot hold for all  $\lambda$ . Let  $\bar{\lambda}$  denote the endowment share at which conventional sorting does no longer hold. It can be proved that the strong HME exists over an interval  $(1/2, \lambda^*)$  with the critical value  $\lambda^*$  bounded by  $\lambda^* < \bar{\lambda}$ .

We know from equation (11), that a change in the relative entry probability  $\chi$  translates into a change in the share of firms  $\phi$ . So, a shock on  $\lambda$  has both a “price effect” and a “quantity effect” (Head and Mayer, 2004). The higher relative wage of Home shifts the price distribution since it affects unit labor costs. It also affects the composition of productivities and the share of firms that do not find it worthwhile to operate (besides the obvious effect of increasing the number of firms that attempt entry). In the single-sector Krugman (1980) model, only the price effect emerges. Resources are fixed by full-employment and balanced trade conditions, where the latter ensures that the larger country pays the higher wage.

Note that the equilibrium relative entry probability  $\chi$  is concave in relative size  $\lambda$ . The reason is that (i) the ex ante profitability curve is concave in the relative wage, (ii) it is not shifted itself by a country size shock, and (iii) the relative wage is strictly increasing in the share of consumers.

These considerations allow stating the second key proposition of this paper:

**Proposition 2 (Home Market Effect).** *Suppose and that conventional sorting holds ( $\lambda < \bar{\lambda}$ ).*

*Then, the model exhibits*

*(a) a weak HME if Home’s endowment share lies in the interval  $(1/2, \bar{\lambda})$ , and*

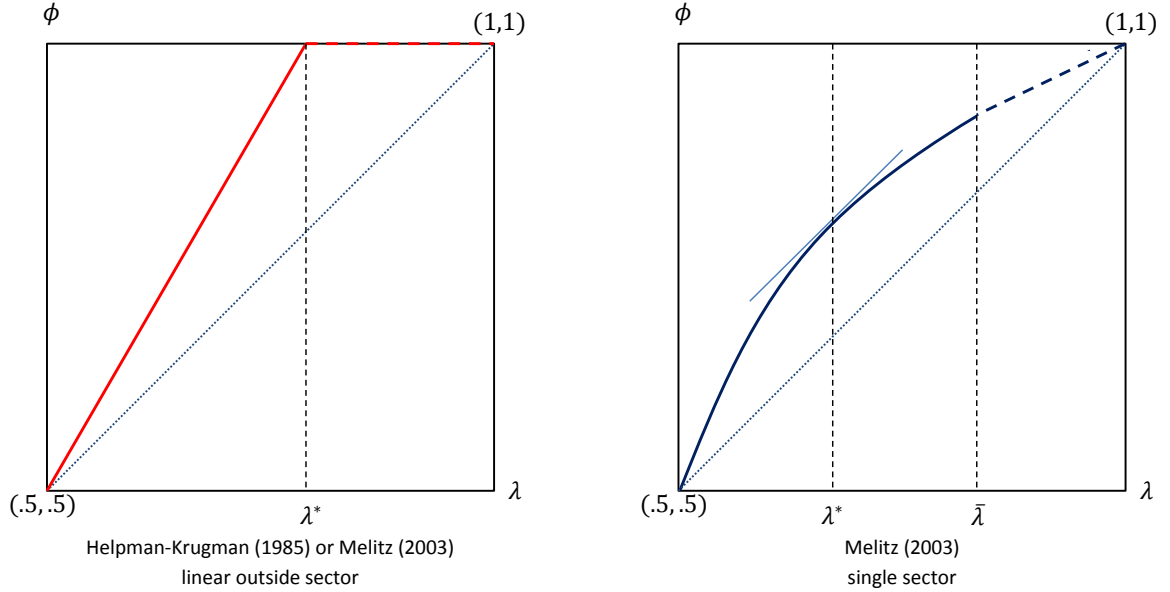
*(b) a strong HME if Home’s endowment share lies in the interval  $(1/2, \lambda^*)$ , where  $\lambda^* \leq \bar{\lambda}$ .*

**Proof.** In Appendix A.2. ■

It can be shown that the weak HME extends into the region of unconventional sorting; see the extension in section 4.3. Figure 3 anticipates this generalization. It graphically illustrates the strong and the weak HMEs in models with and without a linear outside sector. The diagram on the left hand is the standard illustration of the HME in the Helpman-Krugman (1985)



Figure 3: Home Market Effects with and without an outside sector

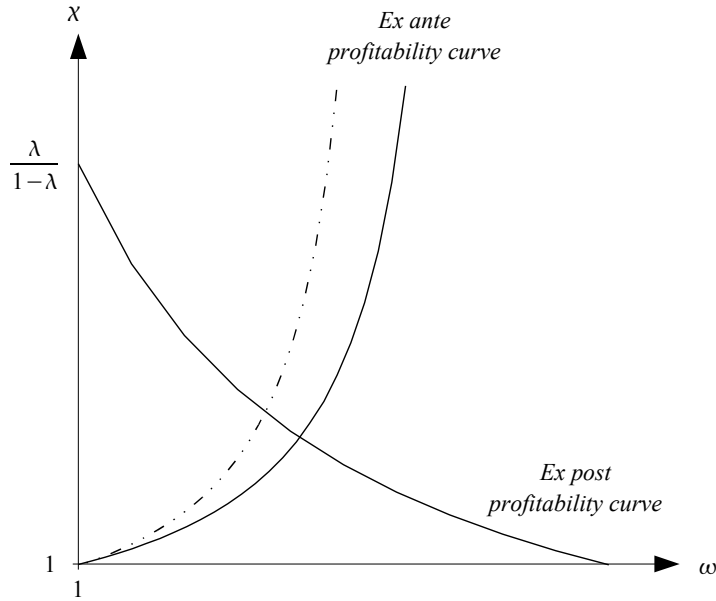


world. It also holds in the Melitz (2003) model with a linear outside sector.<sup>17</sup> In the interval  $\lambda < \lambda^*$ , there is a strong HME as the share of firms increases more than proportionately due to an increase in  $\lambda$ . When  $\lambda > \lambda^*$ , the large economy is fully agglomerated ( $\phi = 1$ ), so that the strong HME cannot hold anymore. Clearly, a weak HME exists over the entire interval. The diagram on the right-side illustrates the HMEs in the single-sector Melitz case. The functional relationship  $\phi(\lambda)$  clearly is increasing from  $(1/2, 1/2)$  to  $(1, 1)$ . It has a kink at  $\lambda = \bar{\lambda}$ , where conventional sorting no longer holds anymore. So, over the interval  $\lambda < \lambda^*$  the strong HME holds, while the weak HME obtains over the full interval. Moreover,  $\phi(\lambda)$  can be shown to be concave in the interval  $(1/2, \bar{\lambda})$  but is convex in the interval  $(\bar{\lambda}, 1)$ .

Finally, one can show that the existence of a linear outside sector in the Melitz (2003) model magnifies the HME relative to the case where the outside sector is absent. In particular, in the case with the outside sector, the slope of the locus  $\phi(\lambda)$  is equal to  $1 + 2\eta$ . In the single-sector case, it is equal to  $1 + \eta / (2 - \rho(1 - \eta) / \beta)$ ; see (A.3) in the Appendix for details. That the linear outside sector exaggerates the HME follows from the fact that  $1 - \eta < \beta / \rho$ .

<sup>17</sup>See Appendix B.2 for the model with a linear outside sector.

Figure 4: Trade liberalization



### 3.3 Home Market Magnification Effects

Next, we investigate how the strength of the HME is affected by the freeness of trade and by the extent of productivity dispersion. In the traditional Helpman-Krugman case with outside sector, lower variable trade costs make the HME more pronounced. They also make it more likely that the model degenerates to full agglomeration (where Home has all firms). This home market magnification effect also exists in our case; only it appears in a somewhat subtler form due to the absence of a linear outside sector. We also show that increased dispersion of productivity magnifies the HME.

**Trade liberalization.** Recall that freeness of trade is defined as

$\eta \equiv \tau^{-\beta} (f^x/f^d)^{1-\beta/(\sigma-1)}$ . It falls when variable trade costs  $\tau$  shrink and/or when foreign entry costs relative to domestic ones  $f^x/f^d$  fall. The precise origin of a change in  $\eta$  does not matter for the result. Conveniently,  $\eta$  only appears in the ex ante profitability curve but not in the ex post profitability crowding curve. Using Figure 1, the comparative statics with respect to  $\eta$  are therefore very easy. Figure 4 provides an illustration.

The intuition is the following. For a given wages and domestic entry cutoffs, higher free-

ness of trade increases the export participations rate and therefore expected profits in both countries. Free entry implies that the probability of successful entry has to go down in both countries. A symmetric drop in trade costs favors the smaller country, which is more open and now has easier access to the large market, which implies that the relative entry probability from the large country's perspective rises in response to symmetric trade liberalization. The ex post profitability locus is not affected since freeness of trade rises symmetrically. Hence, the equilibrium relative entry probability goes up, which translates into a larger HME since  $\gamma$  rises in  $\chi$  for given  $\lambda$ . Moreover, the equilibrium relative wage declines, so that higher freeness of trade leads to convergence of nominal wages.

The equilibrium relative entry probability  $\chi$  is also concave in the freeness of trade  $\eta$ . The reason is that (i) the ex post profitability curve is convex in the relative wage, (ii) it is not shifted itself by a freeness of trade shock, and (iii) the relative wage is strictly decreasing in the freeness of trade.

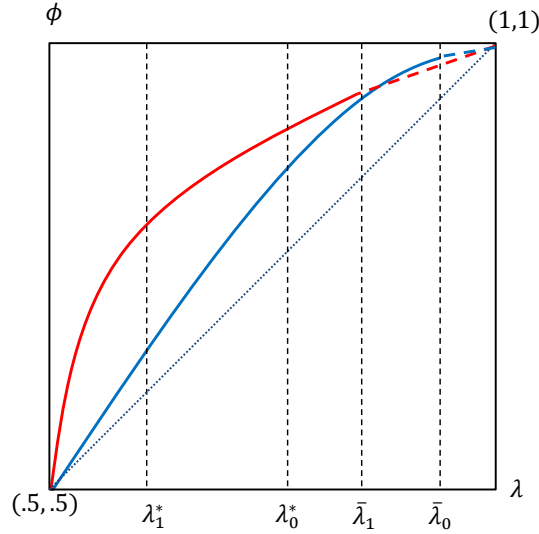
There are additional side effects. Higher  $\eta$  makes it more likely that the conventional sorting conditions fails to hold, so  $\bar{\lambda}$  falls and the kink in the  $\phi(\lambda)$  locus occurs earlier. Moreover, also the interval over which the strong HME can be observed shrinks since  $\lambda^*$  falls as well. Figure 5 provides an illustration in  $(\phi, \lambda)$  –space. As long as conventional sorting continues to hold, one can show that  $\gamma$  increases so that the HME becomes stronger.<sup>18</sup> As a corollary, there must be some interval over which also the strong HME becomes augmented.

**Productivity dispersion.** Next, we consider the comparative statics with respect to  $\beta$ , the shape parameter of the Pareto distribution, which is inversely related to the variance of the sampling distribution.  $\beta$  appears in both the ex ante and the ex post profitability curves. An increase in productivity dispersion (lower  $\beta$ ) rotates both curves upwards. Hence, higher productivity dispersion unambiguously leads to a higher relative entry probabilities  $\chi$ , while the effect on the wage rate depends on model parameters in a complicated fashion. A higher  $\chi$  translates into a higher  $\gamma$  so that the weak HME is magnified productivity dispersion goes up. The intuition for this result is that differences in domestic entry cutoffs due to size differen-

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<sup>18</sup>We provide a generalization of this result to the case of unconventional sorting in the extensions below.

Figure 5: Home market magnification due to trade liberalization



Blue line: Before trade liberalization. Red line: After trade liberalization

tials are magnified when the productivity dispersion is higher. In other words, selection is more important. It turns out to favor the larger country *ex ante* and *ex post*. So, for the emergence of a HME in the single-sector Melitz model, productivity dispersion is important.<sup>19</sup>

The following proposition summarizes this third key result of our analysis.

**Proposition 3 (Home Market Magnification Effects.)** *Suppose that conventional sorting holds.*

*Then, the weak home market effect is magnified by*

- (a) *trade liberalization and*
- (b) *in increase in productivity dispersion.*

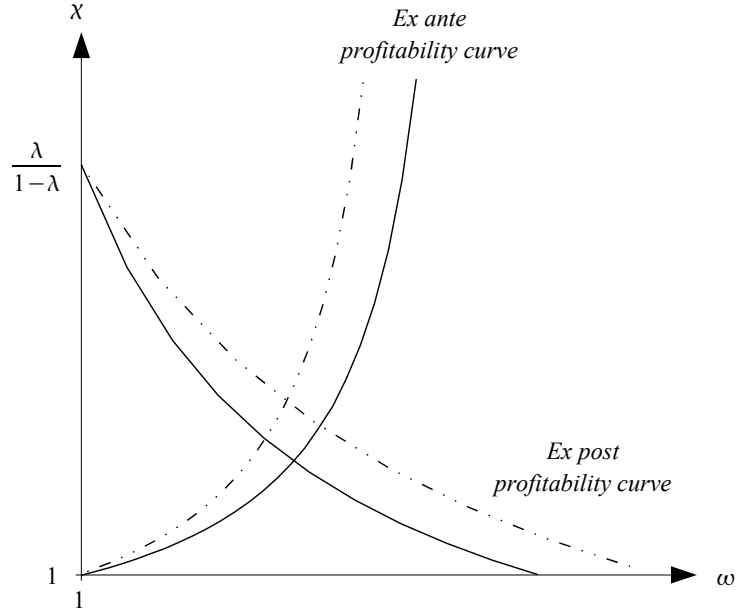
**Proof.** In Appendix A.4. ■

### 3.4 Real income

In this subsection we analyze the role of market size for real per capita income (welfare) and the effect of trade liberalization. The real wage is higher in the larger country. Trade liberalization leads to real per capita income convergence. This result is in sharp contrast to the

<sup>19</sup>This finding has important implications for empirical studies on the HME, such as Hanson and Xiang (2004). The model suggests that one important industry characteristic that shapes the size of the HME is the degree of productivity dispersion as captured by the shape parameter of the Pareto distribution.

Figure 6: Higher productivity dispersion



prediction of a model with a linear outside sector, where relative real per capita income is invariant to changes in trade costs.

Real per capita income is proportional to country size and to the domestic entry cutoff  $\varphi_{ii}$ . This observation follows from solving the domestic zero profit condition (1) for  $w_i/P_i$ , noting that aggregate expenditure is equal to labor income, i.e.  $E_i = w_i L_i$ . Hence, we can write *relative real per capita* income (the cross-country gap in living standards) as a function of the exogenous country size differential and a single endogenous variable that has played a key role in our analysis so far and whose properties are well understood, namely  $\chi$  :

$$W \equiv \frac{W_H}{W_F} = \left( \frac{\lambda}{1-\lambda} \right)^{\frac{1}{\sigma-1}} \chi^{-\frac{1}{\beta}}. \quad (14)$$

We know from Figure 1, that  $\chi < \lambda/(1-\lambda)$ . Together with the condition  $\beta > \sigma - 1$ , this ensures that  $W_H > W_F$ . So, the larger Home has the higher real per capita income.<sup>20</sup> For the same reason, it is easy to see that  $W_H/W_F$  increases with  $\lambda$  so that a more unequal distribution of the world labor endowment leads to more disparity in terms of welfare.

<sup>20</sup>This result extends to the case of unconventional sorting; see (A.7.3) in the Appendix.

Equation (14) is also useful to understand the fundamental drivers of international disparities. For given relative country size, an increase in the freeness of trade raises  $\chi$  and therefore lowers  $W_H/W_F$ . In other words, trade liberalization leads to convergence of real per capita income across countries. The intuition is that higher freeness of trade favors the more open country, which is the small country. In the model, convergence of real income is equivalent to factor price convergence.

Importantly, trade liberalization does not affect relative real per capita income in models that feature a linear outside sector.<sup>21</sup> The intuition for this result lies in the simple fact that fixing the wage rate also fixes mill prices in a CES environment. So, trade liberalization could affect  $W$  differently across countries, if cutoff productivity levels were differently affected by a change in  $\eta$ . This is, however, not possible when wages are insensitive to  $\lambda$  and  $\eta$ .

We have argued above that the presence of a linear outside sector exaggerates the importance of the HME. Interestingly, one can show that this does not imply that the cross-country welfare differential must be bigger, too. Quite the opposite is true. The welfare differential is larger in the absence of the outside sector.

**Proposition 4** (*per capita income and convergence.*) *Suppose that that conventional sorting holds. Then,*

- (a) *the larger economy exhibits the higher real per capita income, and*
- (b) *trade liberalization leads to real per capita income convergence across countries.*

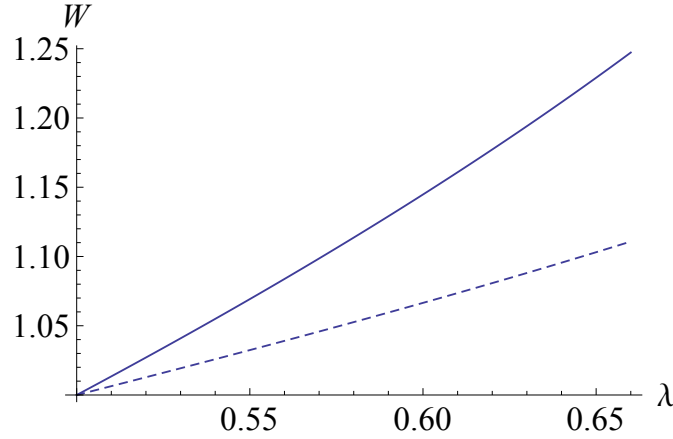
**Proof.** In the text. ■

In the extreme case where the freeness of trade  $\eta$  reaches its maximum (i.e. there are no variable trade costs,  $\tau = 1$ , and foreign market and domestic market access costs coincide,  $f^x/f^d = 1$ , such that  $\eta = 1$ ), wages are equalized across countries. Moreover, as there is no export selection, the price indices are the same in both countries as well. Hence, country size differences do not translate into real per capita income differences in the absence of export selection. Note that there is still domestic selection. With  $\eta = 1$ , the domestic zero profit conditions imply  $\chi = (\lambda/(1 - \lambda))^{\beta/(\sigma-1)} \geq 1$ .

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<sup>21</sup>See Appendix B.2 for a detailed description of this model.

Figure 7: The role of market size and trade costs for relative real per capita income



This figure shows relative real per capita income  $W = W_H/W_F$  as a function of Home's share of world factor supplies  $\lambda$ . We set  $\beta = 4.25$  and  $\sigma = 4$ . Freeness of trade is  $\eta = 0.1$  (solid line) and  $\eta = 0.9$  (dashed line).

In order to illustrate how country size differences translate into real per capita income differences and how this gap is affected by trade liberalization, we perform a simple numerical exercise with two countries. The calibration follows Melitz and Redding (2015). In line with respectively Bernard, Eaton, Jensen and Kortum (2003) and Simonovska and Waugh (2014a), we set the elasticity of substitution between varieties  $\sigma = 4$  and the Pareto shape parameter  $\beta = 4.25$ . The calibrated values for trade costs are  $\tau = 1.83$  and  $f^x/f^d = 0.545$  in order to match the observations that exports account for 14% of firm sales in U.S. manufacturing and that the export participation rate is 18% (Bernard, Jensen, Redding, and Schott (2007) when there are two symmetric countries. This calibration implies a freeness of trade  $\eta = 0.1$ .

Figure 7 plots relative real per capita income  $W \equiv W_H/W_F$  as a function of the share in world labor supply  $\lambda$ . The solid line represents a freeness of trade  $\eta = 0.1$ , in line with the calibration characterized above. When Home is 50% larger than Foreign ( $\lambda = 0.6$ ), the welfare gap is about 15%. The gap reduces to about 5% when freeness of trade increases to  $\eta = 0.9$ .

### 3.5 Margins of trade

The conventional Helpman and Krugman (1985) model has another important prediction, namely that the larger country should be a net exporter of the differentiated good and a net importer of the outside good. This prediction is less suited than the HME to empirically dis-

criminate models with increasing returns to scale at the firm level from models with based on comparative advantage since the latter can have similar predictions (Helpman, 1999; Hanson and Xiang, 2004). From a theoretical perspective, the trade pattern result is a direct corollary of the HME if the upper tier demand structure is homothetic. An increase in Home's labor force triggers an over-proportional expansion of the differentiated good sector, which requires that labor supply to and output of the homogeneous good sector shrink, whereas expenditures remain constant.

In a single-sector model, one cannot discuss sectoral trade pattern. However, it is easy to see that market size differences have implications for the margins of trade. Balanced trade implies that the value of exports and imports of a country are the same. The trade balance condition 6 can be rewritten as

$$\underbrace{\frac{m_{HF}M_H}{m_{FH}M_F}}_{\text{extensive}} \times \underbrace{\omega}_{\text{intensive}} = 1, \quad (15)$$

where we have used that  $\bar{r}_{ij} = \sigma\theta M_{ij} f_{ij}$  is the average value of sales of a firm located in  $i$  on market  $j$ . As the larger has the higher wage, equation (15) implies that the mass of exporters is smaller in the larger country. Given that the mass of firms in the larger country is larger, i.e.  $M_H > M_F$ , it must be true that the export participation rate is smaller in the larger country. We summarize this testable implication of the model as a corollary to proposition 1:

**Corollary 1** (*Margins of trade.*) *Exports of the larger country are dominated by the intensive margin; exports of the smaller country are dominated by the extensive margin.*

**Proof.** In the text. ■

Interestingly, the opposite result obtains if one assumes foreign market entry costs to be paid in the destination rather than the source country.



## 4 Extensions

The analysis in the previous sections relies on the assumption that productivities are Pareto-distributed. In section 4.1, we prove that the larger country pays a higher wage and hosts an over-proportional share of firms also under more general productivity distributions. Moreover, we show in section 4.2 that the weak HME not only materializes in endowments shares but also in demand shares. In section 4.3 we re-consider our results under unconventional sorting.

### 4.1 General productivity distribution

In this subsection, we generalize the argument to a situation where firms' productivity levels are sampled from a general productivity distribution. The functional forms of our ex post and ex ante productivity curves hinge on the assumption of the Pareto distribution. While it is difficult to derive results on the strong HME or on magnification effects, the following proposition characterizes the relative wage and establishes the weak HME for the case of conventional sorting with more general productivity distributions:

**Proposition 5 (General sampling distribution.)** *Suppose that productivity levels are sampled from a general productivity distribution and that conventional sorting holds. Then,*

- (a) *the larger country pays the higher wage, and*
- (b) *the economy exhibits a weak HME on the interval  $(1/2, \bar{\lambda})$ .*

**Proof.** In Appendix A.5. ■

In order to prove these results, we follow Demidova and Rodriguez-Clare (2013) and reduce the equilibrium conditions (2) - (6) to two equations in the relative wage  $\omega$  and Home's export cutoff  $\varphi_{HF}^*$ , which have opposite slopes. Relative endowments affect only one of these curves, which allows inference on the effect of a country size shock on the relative wage and the various cutoffs. Moreover, drawing on a generalized labor market clearing condition, we can derive our result on the home market effect.

## 4.2 HME in demand shares

Empirical work typically employs cross-industry data to study how shocks on *demand shares* affect production patterns. We do not pursue the straight-forward extension of our model to the multi-industry case, but characterize analytically the endogenous relationship between Home's share of firms ( $\phi$ ) and its share in world demand ( $\delta$ ).<sup>22</sup> Home's share in world demand (GDP) is given by

$$\delta \equiv \frac{w_H L_H}{w_H L_H + w_F L_F} = \frac{1}{1 + \frac{1-\lambda}{\lambda} \frac{1}{\omega}} \gg \lambda,$$

which is of course endogenous to the model. In the standard setup, with an identically parameterized linear outside sector in both countries, we would have  $\omega = 1$  and therefore  $\delta = \lambda$ . The HME in demand shares is then identical to the HME in endowment shares. In our case, this is of course different since  $\omega > 1$ . The demand share of the larger country is larger than its endowment share. Hence, whether Home's share of firms is larger than its share in world endowment is a priori unclear.

We define the (weak) home market effect in demand shares as an over-proportional relationship between the share of firms and the demand share, so that  $\delta > 1/2 \Rightarrow \phi > \delta$ . From equation (10), it is easy to see that the HME in demand shares requires  $\chi > \omega$ . Note that the condition for a HME in demand shares is stronger than the one for a HME in labor shares, which is  $\chi > 1$ .

One can show that a weak HME in demand shares arises if and only if  $\eta > \rho / (2\beta + \rho)$ ; a sufficient condition for this is  $\eta > 1/3$ .<sup>23</sup>

These results are summarized in the following proposition:

**Proposition 6 (Home Market Effect in demand shares.)** *Let  $\delta$  denote Home's share in GDP. Suppose that conventional sorting holds and that the freeness of trade is not too low ( $\eta > 1/3$ ). Then, the model exhibits a weak HME in demand shares ( $\phi > \delta$ ).*

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<sup>22</sup>To work with the share of production rather than the share of firms would require a multi-industry setup. In the single-sector framework that approach would lead to an identity ( $GDP = GDP$ ).

<sup>23</sup>With the simple (and very standard) calibration of section 3.4, the requirement for the weak HME is  $\eta > 0.1$ . This would represent a valorem tariff equivalent of 95 per cent ( $\tau = 1.946$ ). Hence, the condition is likely to be met in all reasonable circumstances.

**Proof.** In Appendix A.6. ■

The situation is more complicated if one takes differences in technologies into account. We model technology differences by differences in the lower bound of the Pareto productivity distribution, i.e. we allow for Ricardian-type differences in technology.

A weak HME obtains if the relative endowment of Home ( $L_H/L_F$ ) is large enough relative to  $B \equiv (b_H/b_F)^\beta$ , where  $b_i$  is the lower bound of the Pareto distribution in country  $i$ . The finding that the underlying cause for cross-country variation in demand shares matters for whether or not a HME exists, is important for empirical work. If a researcher runs a regression of  $\phi$  on  $\delta$ , it is important to control for some measure of  $B$  for example average TFP. Failing to do so may explain why the empirical literature has found mixed support for the (weak) HME so far.

### 4.3 Unconventional Sorting

It is a well documented stylized fact that only the most productive firms engage in exporting; see Bernard, Jensen, Redding, and Schott (2007). We have this situation “conventional sorting” and have so far assumed that it holds. In symmetric equilibrium,  $f^x > f^d$  is a sufficient condition to guarantee this sorting pattern.<sup>24</sup> Then, only the most productive firms will generate sufficiently large sales to overcome export fixed costs. However, when countries are asymmetric in size, the sorting pattern can reverse in the smaller country. The intuition for this lies in the fact that less productive firms would make negative profits on their small domestic market, for which fixed costs are high relative to revenue, while they can make profits on the large export market, where fixed costs are lower relative to revenue. When the sorting pattern reverses, the definition of the relative entry probability  $\chi$  needs to be adapted since  $p_F^{in}$  will now be given by  $1 - G[\varphi_{FH}^*]$ .

It is still true that the larger country commands the higher wage. Also equations (10) and

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<sup>24</sup>In the presence of variable trade costs, this requirement can be weakened.

(11) continue to hold. However, the *ex post profitability curve* now is given by

$$\chi = \frac{f^x/f^d}{\eta} \omega^{-\frac{\beta}{\rho}}. \quad (16)$$

It follows from the relative zero cutoff profit conditions for targeting the Home's market. Clearly, the MCC is strictly decreasing in  $\omega$ . In contrast to the case of conventional sorting, it is now independent of  $\lambda$  but depends on trade costs. The *ex ante profitability curve* (MPC) now reads

$$\chi = \frac{f^x/f^d}{1 - \frac{1-\lambda}{\lambda} \omega^{\frac{\beta-\rho}{\rho}} (\omega^{\beta/\rho} - \eta)}. \quad (17)$$

The ex ante profitability curve is strictly increasing in  $\omega$ , and now depends on  $\lambda$  as well as on trade costs. Similar to the case of conventional sorting, equilibrium can be determined by the intersection of the modified ex post and ex ante profitability curves. Evaluated at  $\omega = 1$ , the ex ante profitability curve yields  $\chi > 1$ . Hence, the weak HME also occurs under unconventional sorting.

Variable trade cost liberalization, in turn, shifts both curves downwards. Lowering relative export fixed costs additionally moves both curves proportionally. Hence, a higher freeness of trade lowers the relative probability, which dampens the home market effect. Higher  $\lambda$  also lowers the HME. This is a necessary consequence of the fact that  $\phi(1) = 1$ . We summarize these results in the following proposition:

**Proposition 7 (*Unconventional sorting*.)** *Assume that unconventional sorting holds. Then,*

- (a) *the larger country has a higher relative wage ( $\omega$ ) and higher relative participation probability ( $\chi$ ), so that a weak home market effect exists, and*
- (b) *trade liberalization reduces  $\chi$ , so that the (weak) home market effect is diminished.*

**Proof.** In Appendix A.7. ■

As a corollary, the model features a weak HME over all possible cross-country allocations of the world labor endowment. As in Crozet and Trionfetti (2008) the (weak) HME is non-linear. Their numerical exercise suggests that the HME is concave in  $\lambda$  for values of  $\lambda$  around the symmetric equilibrium and convex thereafter. In our setup, the HME is concave for  $\lambda < \bar{\lambda}$

and convex for  $\lambda > \bar{\lambda}$ .

## 5 Conclusion

This paper provides a tractable way to characterize a two-country single-sector asymmetric Melitz (2003) model for the purpose of conducting comparative statics. It does so without imposing a linear, perfectly competitive and frictionless outside sector, as the literature has usually chosen to do. The outside sector assumption has been criticized to be unrealistic and possibly important for aggregate results, such as welfare (Demidova and Rodriguez-Clare, 2013), or for the ability of the model to predict a Home Market Effect (HME), by which a large country attracts a more than proportionate share of producing firms (Davis, 1998).

The present analysis focuses on the HME, which has been used as a criterion to discriminate between trade models featuring increasing returns to scale and more conventional comparative advantage based setups. It shows that the unrealistic and potentially problematic outside sector assumption can be replaced by a Melitz-type selection mechanism, where only a fraction of heterogeneous firms sell to all markets. The resulting HME is non-linear, as the empirical analysis of Crozet and Trionfetti (2008) suggests. It is magnified by falling trade costs and by a higher degree of firm-level productivity dispersion. The HME translates into cross-country welfare differences. In contrast to the model with a linear outside sector, trade liberalization attenuates these cross-country differences and leads to real wage convergence. Firm-level heterogeneity is absolutely crucial for these results: in the Krugman (1980) single-sector model, no HME can arise.

The results presented in this paper are important for empirical work. First, since the outside-sector is not crucial for the existence of a HME, an empirical rejection of an over-proportionate relation between a country's share of firms and its share of endowments is indeed evidence against increasing returns and not against the linear outside sector. Second, empirical tests that fail to control for the level of technology may wrongly reject the existence of an endowment-driven HME. Finally, the prediction on the margins of trade can be used to empirically discriminate models where export fixed costs are paid in the source country from

models where these costs are paid in the destination country.

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## A Proofs of Lemmas and Propositions

### A.1 Proof of Lemma 1 (Equilibrium)

We first derive the ex post and ex ante profitability curves under the assumption that conventional sorting holds. We then derive the critical share of Home's labor endowment in world labor endowment  $\bar{\lambda}$ .

#### A.1.1 Derivation of ex post profitability curve

In order to derive the ex post profitability curve, we use the **zero cutoff profit conditions** in relative terms and the balanced trade condition. Taking  $F$  as the target market and using the two associated zero cutoff profit conditions

$$\left(\frac{\varphi_{HF}^*}{\varphi_{FF}^*}\right)^{\sigma-1} = \tau^{\sigma-1} \omega^\sigma \frac{f^x}{f^d}. \quad (18)$$

Taking  $H$  as the target market and dividing the two associated zero cutoff profit conditions

$$\left(\frac{\varphi_{FH}^*}{\varphi_{HH}^*}\right)^{\sigma-1} = \tau^{\sigma-1} \omega^{-\sigma} \frac{f^x}{f^d} \quad (19)$$

Using equations (18) and (19) together with the trade balance condition (7), we obtain

$$\chi = \frac{\lambda}{1-\lambda} \omega^{-\frac{2\beta-\rho}{\rho}},$$

where  $\chi \equiv (\varphi_{FF}^*/\varphi_{HH}^*)^\beta$  denotes Home's relative entry probability.

#### A.1.2 Derivation of ex ante profitability curve

In order to derive the ex ante profitability curve, we use the **free entry conditions** in relative form along with the zero cutoff profit conditions and the balanced trade condition.

In relative form, the free entry conditions read

$$\chi = \frac{f^d + \left(\frac{\varphi_{FF}^*}{\varphi_{FH}^*}\right)^\beta f^x}{f^d + \left(\frac{\varphi_{HH}^*}{\varphi_{HF}^*}\right)^\beta f^x} \quad (20)$$

Using the zero cutoff profit conditions and the trade balance condition to eliminate  $\varphi^*$  terms, we obtain

$$\frac{\varphi_{FF}^*}{\varphi_{FH}^*} = \tau^{-1} \omega^{\frac{\rho-\beta}{\rho\beta}} \left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{\beta}} \left(\frac{f^x}{f^d}\right)^{\frac{1}{1-\sigma}} \quad (21)$$

from (7) and (18) and

$$\frac{\varphi_{HH}^*}{\varphi_{HF}^*} = \tau^{-1} \left(\frac{\lambda}{1-\lambda}\right)^{-\frac{1}{\beta}} \omega^{\frac{\beta-\rho}{\beta\rho}} \left(\frac{f^x}{f^d}\right)^{\frac{1}{1-\sigma}}$$

from (7) and (19). Substituting out these expressions, we get

$$\chi = \frac{1 + \omega^{-\frac{\beta-\rho}{\rho}} \left(\frac{\lambda}{1-\lambda}\right) \tau^{-\beta} \left(\frac{f^x}{f^d}\right)^{-\frac{1-\sigma+\beta}{\sigma-1}}}{1 + \omega^{\frac{\beta-\rho}{\rho}} \left(\frac{1-\lambda}{\lambda}\right) \tau^{-\beta} \left(\frac{f^x}{f^d}\right)^{-\frac{1-\sigma+\beta}{\sigma-1}}}.$$

Using  $\eta \equiv \tau^{-\beta} \left(\frac{f^x}{f^d}\right)^{-\frac{\beta-(\sigma-1)}{\sigma-1}}$  and the ex post profitability curve to substitute out  $\frac{\lambda}{1-\lambda} \omega^{-\frac{2\beta-\rho}{\rho}}$  in the numerator and denominator, we obtain

$$\chi = \frac{1 - \eta \omega^{-\frac{\beta}{\rho}}}{1 - \eta \omega^{\frac{\beta}{\rho}}}.$$

### A.1.3 Characteristics of ex post and ex ante profitability curves

**Ex post profitability curve.** The ex post profitability curve implies a downward-sloping and convex relationship between  $\chi$  and  $\omega$  as

$$\frac{\partial \chi}{\partial \omega} < 0 \text{ and } \frac{\partial^2 \chi}{\partial \omega^2} > 0.$$

Evaluated at  $\omega = 1$ , the ex post profitability curve takes the value  $\chi = \frac{\lambda}{1-\lambda} \geq 1$ .  $\chi$  is bounded from below by 0.

**Ex ante profitability curve.** Evaluated at  $\omega = 1$ , we have  $\chi = 1$ . Given that  $\chi$  is restricted to positive values, we have  $\omega < \eta^{-\frac{\rho}{\beta}}$ . The ex ante profitability curve implies an upward-sloping relationship between  $\chi$  and  $\omega$

$$\frac{\partial \chi}{\partial \omega} = \frac{\beta \eta}{\rho} \frac{\chi \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}}{\omega \left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)} > 0.$$

Convexity of the ex ante profitability curve potential curve requires that the freeness of trade is not too small

$$\eta > \frac{\rho}{4\beta + \rho}.$$

In order to see this, we compute

$$\frac{\partial^2 \chi}{\partial \omega^2} = \frac{\partial \chi}{\partial \omega} \frac{\omega^{-1}}{\chi \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}} \left( 2 \frac{\partial \chi}{\partial \omega} \omega^{\frac{\beta+\rho}{\rho}} + \frac{\beta - \rho}{\rho} \chi \omega^{\frac{\beta}{\rho}} - \frac{\beta + \rho}{\rho} \omega^{-\frac{\beta}{\rho}} \right).$$

The sign of the second derivative is the sign of

$$h[\omega] = (\beta + \rho) \left( \eta \chi \omega^{\frac{2\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} + \eta \right) + 2\beta \eta + (\beta - \rho) \chi \omega^{\frac{\beta}{\rho}}.$$

Evaluating in symmetric equilibrium ( $\omega = \chi = 1$ ), we obtain

$$2(4\beta \eta + \rho \eta - \rho) > 0 \Leftrightarrow \eta > \frac{\rho}{4\beta + \rho}.$$

Moreover, it is easy to check that  $h[\omega]$  is strictly increasing in  $\omega$ . The reason is that  $\chi$  is increasing in  $\omega$ . Moreover,  $\beta > \rho$ .

#### A.1.4 Derivation of conventional sorting cutoff $\bar{\lambda}$

Cutoff level  $\bar{\lambda}$  up to which Foreign first serves domestic and then export market is implicitly defined by  $\varphi_{FF}^* = \varphi_{FH}^*$

$$\varphi_{FF}^* = \varphi_{FH}^* \Leftrightarrow 1 = \tau^{-1} \bar{\omega}^{\frac{\rho-\beta}{\rho\beta}} \left( \frac{\bar{\lambda}}{1-\bar{\lambda}} \right)^{\frac{1}{\beta}} \left( \frac{f^x}{f^d} \right)^{\frac{1}{1-\sigma}} \quad (22)$$

Ex post and ex ante profitability curves imply

$$\frac{\bar{\lambda}}{1-\bar{\lambda}} = \frac{\eta - \bar{\omega}^{\beta/\rho}}{\eta \bar{\omega}^{\beta/\rho} - 1} \bar{\omega}^{(\beta-\rho)/\rho}. \quad (23)$$

Using this expression to substitute out  $\frac{\bar{\lambda}}{1-\bar{\lambda}}$  from equation (22) and solving for  $\bar{\omega}$ , we obtain

$$\bar{\omega}^{\frac{\beta}{\rho}} = \frac{\eta \tau^{-\beta} \left(\frac{f^x}{f^d}\right)^{-\frac{\beta}{\sigma-1}} + 1}{\tau^{-\beta} \left(\frac{f^x}{f^d}\right)^{-\frac{\beta}{\sigma-1}} + \eta}.$$

Using the definition of  $\eta$ , we can write  $\bar{\omega}^{\frac{\beta}{\rho}}$  as

$$\bar{\omega}^{\frac{\beta}{\rho}} = \frac{\eta + \frac{1}{\eta} \frac{f^x}{f^d}}{1 + \frac{f^x}{f^d}}.$$

This expression can be used to back out  $\bar{\lambda}$  from equation (23).

## A.2 Proof of Proposition 2 (Home Market Effect)

### A.2.1 Weak HME

In symmetric equilibrium ( $\lambda = 1/2$ ), we have  $\chi = \omega = \gamma = 1$ . In response to a labor share shock, the ex post profitability curve shifts upwards

$$\frac{\partial \chi}{\partial \lambda} = \frac{\omega^{-\frac{2\beta-\rho}{\rho}}}{(1-\lambda)^2} > 0.$$

The ex ante profitability curve is unaffected. Then, in equilibrium  $\chi > 1$  for  $\lambda > 1/2$ .  $\gamma$  is increasing in  $\chi$

$$\frac{\partial \gamma}{\partial \chi} = \frac{1-\lambda}{[1+\lambda(\chi-1)]^2} \geq 0,$$

where the equality occurs for  $\lambda = 1$ . Hence, for  $\lambda > 1/2$ , we have  $\gamma > 1$ , which is the definition of the weak HME.

## A.2.2 Strong HME

Remember that

$$\phi = \gamma\lambda, \gamma = \frac{\chi}{1 + \lambda\chi - \lambda}$$

with

$$\frac{\partial\gamma}{\partial\lambda} = \frac{\frac{\partial\chi}{\partial\lambda} [1 + \lambda(\chi - 1)] - \chi \left( \chi + \lambda \frac{\partial\chi}{\partial\lambda} - 1 \right)}{[1 + \lambda(\chi - 1)]^2}$$

Then

$$\begin{aligned} \epsilon_\gamma &\equiv \frac{\lambda \frac{\partial\gamma}{\partial\lambda}}{\gamma} = \frac{\frac{\partial\chi}{\partial\lambda} \lambda [1 + \lambda(\chi - 1)] - \lambda\chi \left( \chi + \lambda \frac{\partial\chi}{\partial\lambda} - 1 \right)}{1 + \lambda(\chi - 1)} \frac{1}{\chi} \\ &= \epsilon_\chi (1 - \phi) - \phi \left( \frac{\chi - 1}{\chi} \right), \end{aligned}$$

where  $\epsilon_\chi \equiv \frac{\partial\chi}{\partial\lambda} \frac{\lambda}{\chi}$ . We have  $\epsilon_\gamma > 0$  if

$$\epsilon_\chi > \frac{\phi}{1 - \phi} \left( \frac{\chi - 1}{\chi} \right).$$

An alternative way to write this is

$$\epsilon_\chi > \frac{\frac{\chi\lambda}{1 + \lambda\chi - \lambda}}{\frac{1 - \lambda}{1 + \lambda\chi - \lambda}} \left( \frac{\chi - 1}{\chi} \right) = \frac{\lambda}{1 - \lambda} (\chi - 1) \Leftrightarrow \frac{\epsilon_\chi}{\chi - 1} > \frac{\lambda}{1 - \lambda}.$$

The conjecture is that we can identify a downward- and an upward-sloping curve such that the left hand side and the right hand side are equal for some unique  $\lambda^* \in (1/2, 1)$ .

Using  $\epsilon_\chi = \frac{\partial\chi}{\partial\lambda} \frac{\lambda}{\chi} = \frac{\partial\chi}{\partial\omega} \frac{\omega}{\chi} \frac{\partial\omega}{\partial\lambda} \frac{\lambda}{\omega}$ , we can rewrite the inequality as

$$\frac{\frac{\partial\chi}{\partial\omega} \frac{\omega}{\chi}}{\chi - 1} > \frac{\lambda}{1 - \lambda} \frac{1}{\frac{\partial\omega}{\partial\lambda} \frac{\lambda}{\omega}}.$$

We prove that the left hand side is downward-sloping in  $\omega$  and therefore downward-sloping in  $\lambda$ , whereas the right hand side is increasing in  $\lambda$ .

**The  $\frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} / (\chi - 1)$  locus.** Recall that

$$\frac{\partial \chi}{\partial \omega} = \frac{\eta \beta \left( \omega^{-\frac{\beta}{\rho}} - \eta \right) + \left( \omega^{\frac{\beta}{\rho}} - \eta \right)}{\rho \left( 1 - \eta \omega^{\frac{\beta}{\rho}} \right)^2} > 0.$$

The elasticity of  $\chi$  in  $\omega$  is then

$$\frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} = \frac{\eta \beta \left( \omega^{-\frac{\beta}{\rho}} - \eta \right) + \left( \omega^{\frac{\beta}{\rho}} - \eta \right)}{\rho \left( 1 - \eta \omega^{\frac{\beta}{\rho}} \right) \left( 1 - \eta \omega^{-\frac{\beta}{\rho}} \right)},$$

where

$$\Leftrightarrow \frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} = \epsilon_{\chi} / \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)$$

Moreover,

$$\chi - 1 = \eta \frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{1 - \eta \omega^{\frac{\beta}{\rho}}} > 0,$$

and therefore

$$\frac{\frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi}}{\chi - 1} = \frac{\beta \left( \omega^{-\frac{\beta}{\rho}} - \eta \right) + \left( \omega^{\frac{\beta}{\rho}} - \eta \right)}{\rho \left( 1 - \eta \omega^{-\frac{\beta}{\rho}} \right) \left( \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right)}$$

**Slope of  $\frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} / (\chi - 1)$  in  $\omega$ .** We conjecture that  $\partial \left[ \frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} / (\chi - 1) \right] / \partial \omega < 0$ . To check this, define

$$\begin{aligned} f[\omega] &= \omega^{-\frac{\beta}{\rho}} + \omega^{\frac{\beta}{\rho}} - 2\eta > 0 \\ g[\omega] &= \left( 1 - \eta \omega^{-\frac{\beta}{\rho}} \right) \left( \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right) \\ &= \omega^{\frac{\beta}{\rho}} - \eta - \omega^{-\frac{\beta}{\rho}} + \eta \omega^{-\frac{2\beta}{\rho}} > 0 \end{aligned}$$

We have to show that

$$f'[\omega] g[\omega] - f[\omega] g'[\omega] < 0,$$



where

$$f'[\omega] = -\frac{\beta}{\rho}\omega^{-\frac{\beta}{\rho}-1} + \frac{\beta}{\rho}\omega^{\frac{\beta}{\rho}-1} = \frac{\beta}{\rho} \left( \frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\omega} \right) > 0$$

$$g'[\omega] = \frac{\beta}{\rho}\omega^{\frac{\beta}{\rho}-1} + \frac{\beta}{\rho}\omega^{-\frac{\beta}{\rho}-1} - \frac{2\beta}{\rho}\eta\omega^{-\frac{2\beta}{\rho}-1} = \frac{\beta}{\rho} \left( \frac{\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta\omega^{-\frac{2\beta}{\rho}}}{\omega} \right) > 0.$$

The last inequality follows from  $\omega < \eta^{-\frac{\rho}{\beta}}$

$$\begin{aligned} & \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta\omega^{-\frac{2\beta}{\rho}} \\ & > \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \left( \eta^{-\frac{\rho}{\beta}} \right)^{-\frac{2\beta}{\rho}} \\ & = \left( \omega^{\frac{\beta}{\rho}} - \eta^3 \right) + \left( \omega^{-\frac{\beta}{\rho}} - \eta^3 \right) \end{aligned}$$

We can rewrite the necessary condition as

$$\frac{\beta}{\rho} \frac{1}{\omega} \left( \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right)^2 \left( 1 - \eta\omega^{-\frac{\beta}{\rho}} \right) < \frac{\beta}{\rho} \frac{1}{\omega} \left( \omega^{-\frac{\beta}{\rho}} + \omega^{\frac{\beta}{\rho}} - 2\eta \right) \left( \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta\omega^{-\frac{2\beta}{\rho}} \right).$$

Since  $1 - \eta\omega^{-\frac{\beta}{\rho}} < 1$  and  $\omega^{-\frac{2\beta}{\rho}} < 1$ , a sufficient condition is

$$\left( \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right)^2 < \left( \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \right)^2 \iff 0 < \omega^{\frac{\beta}{\rho}} - \eta.$$

**The  $\frac{\lambda}{1-\lambda} \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)^{-1}$  locus.** The trade balance condition implies

$$TB[\lambda, \omega] \equiv \frac{\eta\omega^{\frac{\beta}{\rho}} - 1}{\eta - \omega^{\frac{\beta}{\rho}}} \omega^{-\frac{\beta-\rho}{\rho}} - \frac{1-\lambda}{\lambda} = 0.$$

By the implicit function theorem, we have

$$\frac{\partial \omega}{\partial \lambda} = - \frac{\partial TB}{\partial \lambda} / \frac{\partial TB}{\partial \omega},$$

where

$$\frac{\partial TB}{\partial \lambda} = \frac{1}{\lambda^2}$$

and

$$\frac{\partial TB}{\partial \omega} = -\omega^{-\frac{\beta}{\rho}} \left( \frac{\beta}{\rho} \frac{1 - \eta^2}{\left(\eta - \omega^{\frac{\beta}{\rho}}\right)^2} \omega^{\frac{\beta}{\rho}} + \frac{\beta - \rho}{\rho} \frac{\eta \omega^{\frac{\beta}{\rho}} - 1}{\eta - \omega^{\frac{\beta}{\rho}}} \right) < 0.$$

Then,

$$\frac{\lambda}{1 - \lambda} \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)^{-1} = -\lambda \frac{\lambda}{1 - \lambda} \omega^{-\frac{\beta - \rho}{\rho}} \left( \frac{\beta}{\rho} \frac{1 - \eta^2}{\left(\eta - \omega^{\frac{\beta}{\rho}}\right)^2} \omega^{\frac{\beta}{\rho}} + \frac{\beta - \rho}{\rho} \frac{\eta \omega^{\frac{\beta}{\rho}} - 1}{\eta - \omega^{\frac{\beta}{\rho}}} \right).$$

Using the trade balance condition to substitute out  $\lambda/(1 - \lambda)$  on the right hand side, we obtain

$$\begin{aligned} \frac{\lambda}{1 - \lambda} \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)^{-1} &= -\lambda \left( \frac{\beta}{\rho} \frac{1 - \eta^2}{\left(\eta - \omega^{\frac{\beta}{\rho}}\right) \left(\eta \omega^{\frac{\beta}{\rho}} - 1\right)} \omega^{\frac{\beta}{\rho}} + \frac{\beta - \rho}{\rho} \right) \\ &= \lambda \left( \frac{\beta}{\rho} \frac{1 - \eta^2}{\eta^2 + 1 - \eta \left(\omega^{-\frac{\beta}{\rho}} + \omega^{\frac{\beta}{\rho}}\right)} + \frac{\beta - \rho}{\rho} \right). \end{aligned}$$

**Slope of the  $\frac{\lambda}{1 - \lambda} \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)^{-1}$  locus.** It is easy to check that  $\frac{\lambda}{1 - \lambda} \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)^{-1}$  rises in  $\lambda$ , given that  $\omega$  increases in  $\lambda$ . Ignoring

Hence, the downwards-sloping  $\frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} / (\chi - 1)$  locus and the upward-sloping  $\frac{\lambda}{1 - \lambda} \left( \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega} \right)^{-1}$  locus determine a unique  $\lambda^*$  such that for  $\lambda < \lambda^*$  a strong HME occurs.

### A.3 Slope of $\phi(\lambda)$ in symmetric equilibrium

**Preliminaries.** It is easy to check that  $\partial \phi / \partial \lambda = \gamma + \lambda \partial \gamma / \partial \lambda$ , where the first term is the direct effect of a country size shock on  $\lambda$ . The second term represents the indirect effect due to adjustments in the relative entry probability. We have already shown that

$$\frac{\partial \gamma}{\partial \lambda} = \frac{\gamma}{\lambda} \left[ \epsilon_{\chi} (1 - \phi) - \phi \left( \frac{\chi - 1}{\chi} \right) \right],$$

where  $\epsilon_\chi$  denotes the elasticity of  $\chi$  in  $\lambda$ . Applying the chain rule, we can rewrite  $\epsilon_\chi$  as  $\epsilon_\chi = \frac{\partial \chi}{\partial \omega} \frac{\omega}{\chi} \frac{\partial \omega}{\partial \lambda} \frac{\lambda}{\omega}$ . At symmetric equilibrium, we have  $\phi = \lambda = 1/2$  and  $\omega = \chi = \gamma = 1$ . Hence, we have

$$\frac{\partial \chi}{\partial \omega} = \frac{2\eta\beta}{\rho(1-\eta)} \text{ and } \frac{\partial \omega}{\partial \lambda} = \frac{1}{\lambda^2} \frac{\rho}{2\frac{\beta}{1-\eta} - \rho}.$$

Moreover, we have

$$\epsilon_\chi = \frac{\partial \chi}{\partial \lambda} = 2\eta \left(1 - \frac{\rho(1-\eta)}{2\beta}\right)^{-1}.$$

**Home market effect.** At symmetric equilibrium, the slope of the  $\phi$ -curve is given by

$$\frac{\partial \phi}{\partial \lambda} = 1 + \eta \left(1 - \frac{\rho(1-\eta)}{2\beta}\right)^{-1},$$

which is strictly smaller than the slope of  $\phi$  in  $\lambda$  for the case with the outside sector, which is equal to  $1 + 2\eta$  (see Appendix B.2) since  $\rho(1-\eta)/\beta < 1$ .

**Home market magnification effect.** At the symmetric equilibrium, we have  $\frac{\partial^2 \phi}{\partial \lambda \partial \eta} = \frac{\partial \gamma}{\partial \eta} + \lambda \frac{\partial^2 \gamma}{\partial \lambda \partial \eta}$ , where  $\frac{\partial \gamma}{\partial \eta} = \frac{\partial \gamma}{\partial \chi} \frac{\partial \chi}{\partial \eta} > 0$  and  $\frac{\partial^2 \gamma}{\partial \lambda \partial \eta} = 2 \left(1 - \frac{\rho}{2\beta}\right) \left(1 - \frac{\rho(1-\eta)}{2\beta}\right)^{-2}$ . In contrast, the model with an outside sector implies  $\partial^2 \phi / \partial \lambda \partial \eta = 2$ .

## A.4 Proof of Proposition 3 (Home Market Magnification Effects)

### A.4.1 Trade liberalization

Evaluated at  $\omega = 1$ , the ex ante profitability curve takes the value  $\chi = 1$ , which does not depend on  $\eta$ . The ex ante profitability rotates upwards in response to a freeness of trade shock since

$$\frac{\partial \chi}{\partial \eta} = \frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)^2} > 0.$$

The locus of the ex post profitability curve is unaffected. Then,  $\chi$  has to increase, which raises  $\gamma$  and therefore magnifies the HME. This proves part (a) of proposition (3). The equilibrium value of  $\omega$  declines in response to a freeness of trade shock.

#### A.4.2 Productivity dispersion

Evaluated at  $\omega = 1$ , the ex post profitability curve yields  $\chi = \lambda/(1 - \lambda)$ , which does not depend on  $\beta$ . Moreover, we have

$$\frac{\partial \chi}{\partial \beta} = -\frac{2\lambda}{\rho(1 - \lambda)} \omega^{-\frac{2\beta - \rho}{\rho}} \ln[\omega] < 0$$

since  $\omega > 1$ .

Evaluated at  $\omega = 1$ , the ex ante profitability curve yields  $\chi = 1$ , which does not depend on  $\beta$ . Moreover, we have

$$\frac{\partial \chi}{\partial \beta} = \frac{\eta \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta}{\rho \left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)^2} \ln[\omega] + \frac{\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\left(1 - \eta \omega^{\frac{\beta}{\rho}}\right)^2} \frac{\partial \eta}{\partial \beta}$$

where the indirect effect through adjustment in the relative wage is non-negative since  $\omega^{\frac{\beta}{\rho}} - \eta > 0$ ,  $\omega^{-\frac{\beta}{\rho}} - \eta > 0$ , and  $\omega \geq 1$ . The direct effect through changes in the freeness of trade is negative since

$$\frac{\partial \eta}{\partial \beta} = -\eta \left( \ln[\tau] + \frac{1}{\sigma - 1} \ln \left[ \frac{f^x}{f^d} \right] \right) < 0$$

as  $\tau \geq 1$  and  $f^x/f^d > 1$ .

The sign of  $\partial \chi / \partial \beta$  is then given by the sign of

$$\frac{1}{\sigma - 1} \left( \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \right) \ln[\omega] - \left( \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right) \left( \frac{1}{\sigma - 1} \ln \left[ \frac{f^x}{f^d} \right] + \ln[\tau] \right)$$

Hence,  $\partial \chi / \partial \beta < 0$  requires

$$\left( \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta \right) \ln[\omega] < \left( \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}} \right) \left( \ln \left[ \frac{f^x}{f^d} \right] + (\sigma - 1) \ln[\tau] \right),$$

Note that the expression on the left hand side and on the right hand side intersect at  $\omega = 1$ . We conjecture that for  $\omega > 1$ , the expression on the right hand side rises faster than the expression on the left hand side. Let

$$\begin{aligned}
f[\omega] &\equiv \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta, \\
g[\omega] &\equiv \ln[\omega], \\
h[\omega] &\equiv \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right) \left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma - 1) \ln[\tau]\right).
\end{aligned}$$

Then,

$$\begin{aligned}
f'[\omega] &= \frac{\beta \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\rho \omega} > 0, \\
g'[\omega] &= \frac{1}{\omega} > 0, \\
h'[\omega] &= \frac{\beta \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}}{\rho \omega} \left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma - 1) \ln[\tau]\right).
\end{aligned}$$

The slope of the left hand side is given by

$$f'[\omega] g[\omega] + f[\omega] g'[\omega] = \frac{\beta \omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}}{\rho \omega} \ln[\omega] + \frac{\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta}{\omega}.$$

Hence, we require

$$\frac{\beta}{\rho} \left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}\right) \left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma - 1) \ln[\tau]\right) > \frac{\beta}{\rho} \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right) \ln[\omega] + \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta.$$

Noting that  $\ln[\omega] < \rho \ln[\tau] + \frac{\beta - (\sigma - 1)}{\beta \sigma} \ln\left[\frac{f^x}{f^d}\right]$ , a sufficient condition reads

$$\begin{aligned}
&\frac{\beta}{\rho} \left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}\right) \left(\ln\left[\frac{f^x}{f^d}\right] + (\sigma - 1) \ln[\tau]\right) \\
> &\frac{\beta}{\rho} \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right) \left(\rho \ln[\tau] + \frac{\beta - (\sigma - 1)}{\beta \sigma} \ln\left[\frac{f^x}{f^d}\right]\right) + \omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}} - 2\eta.
\end{aligned}$$

Rearranging terms, we get

$$\begin{aligned}
-2\eta < &\left[\frac{\beta \sigma - \sigma - 1}{\sigma - 1} \left(\omega^{\frac{\beta}{\rho}} + \omega^{-\frac{\beta}{\rho}}\right) - \frac{\beta - (\sigma - 1)}{\sigma - 1} \left(\omega^{\frac{\beta}{\rho}} - \omega^{-\frac{\beta}{\rho}}\right)\right] \ln\left[\frac{f^x}{f^d}\right] \\
&+ \left[(\sigma - 1) \omega^{\frac{\beta}{\rho}} + (\sigma + 1) \omega^{-\frac{\beta}{\rho}}\right] \beta \ln[\tau].
\end{aligned}$$

Collecting terms, we obtain

$$\left[ \beta \omega^{\frac{\beta}{\rho}} + (\sigma + 1) \frac{\beta - \frac{2}{\sigma+1}(\sigma - 1)}{\sigma - 1} \omega^{-\frac{\beta}{\rho}} \right] \ln \left[ \frac{f^x}{fd} \right] + \left[ (\sigma - 1) \omega^{\frac{\beta}{\rho}} + (\sigma + 1) \omega^{-\frac{\beta}{\rho}} \right] \beta \ln [\tau] > -2\eta,$$

where the inequality holds since we have only positive terms on the left hand side (remember that  $\sigma > 1$  and  $\beta > \sigma - 1$ ) and a negative term on the right side.

Hence,  $\chi$  rises in response to an drop in  $\beta$  (higher dispersion), which raises  $\gamma$  and magnifies the HME. This proves part (b) of proposition 3.

### A.5 Proof of Proposition 5 (General sampling distribution)

As proposed by Demidova and Rodriguez-Clare (2013), we reduce the model's equilibrium conditions (2), (4) and (6) to a system of two equations in  $\omega$  and  $\varphi_{HF}^*$ . The first curve draws on zero cutoff profit and free entry conditions and is independent of the productivity distribution. The relative cutoff profit conditions for targeting Foreign is given by

$$\varphi_{HF}^* = \tau \left( \frac{f^x}{fd} \right)^{\frac{1}{\sigma-1}} \omega^{\frac{1}{\rho}} \varphi_{FF}^*. \quad (24)$$

Foreign's domestic entry cutoff  $\varphi_{FF}^*$  is a function of its export cutoff  $\varphi_{FH}^*$  by free entry. Moreover, using the relative cutoff profit conditions for entry into Home,  $\varphi_{FH}^*$  can be expressed as a function of the relative  $\omega$  and  $\varphi_{HH}^*$ . The latter is a function of  $\varphi_{HF}^*$  by free entry. Hence, our first equilibrium condition constitutes an upward sloping and concave relationship between the relative wage and Home's export cutoff. It is important to note that the locus of this curve is not affected by country size.

Under a general productivity distribution, the balanced trade condition can be rewritten as

$$\sigma (\varphi_{HH}^*)^{-\rho} (\psi_H + 1) = \frac{\lambda}{1 - \lambda} \tau^\rho \left( \frac{f^x}{fd} \right)^{\frac{1}{\sigma}} (\varphi_{FH}^*)^{-\rho} (\psi_F + 1), \quad (25)$$

where

$$\psi_i = \frac{fd}{f^x} \left( \frac{\varphi_{ij}^*}{\varphi_{ii}^*} \right)^{\sigma-1} \frac{\int_{\varphi_{ii}^*} \varphi^{\sigma-1} dG_i[\varphi]}{\int_{\varphi_{ij}^*} \varphi^{\sigma-1} dG_i[\varphi]}.$$

The left hand side of equation (25) is independent of  $\omega$ . Moreover, it rises in  $\varphi_{HF}^*$ . The right

hand side can be expressed as a function of  $\varphi_{FH}^*$ .  $\varphi_{FH}^*$  has to rise in  $\varphi_{HF}^*$ . By free entry,  $\varphi_{HH}^*$  falls. Relative entry in Home reads

$$\varphi_{FH}^* = \tau \left( \frac{f^x}{f^d} \right)^{\frac{1}{\sigma-1}} \omega^{-\frac{1}{\rho}} \varphi_{HH}^*. \quad (26)$$

Since  $\varphi_{FH}^*$  rises and  $\varphi_{HH}^*$  falls,  $\omega$  must fall to restore equilibrium. Hence, trade balance establishes a negative relationship between Home's export cutoff and the relative wage.

Consider a country size shock. For a given relative wage, the right hand side of the trade balance curve must be larger, which can only come about by an increase in  $\varphi_{HF}^*$ . Hence, a country size shock shifts the trade balance locus upwards. We also conclude that the larger country pays a higher wage, which proves part (a) of the proposition.

Using the free entry conditions, we can write the ratio of active firms as

$$\frac{M_H}{M_F} = \frac{\lambda}{1-\lambda} \frac{f^e/p_F^{in} + f^d + f^x m_{FH}}{f^e/p_H^{in} + f^d + f^x m_{HF}}. \quad (27)$$

We have argued above that a country size shock raises Home's export cutoff  $\varphi_{HF}^*$ . By free entry, Home's domestic entry cutoff falls. Hence, the denominator in equation (27) falls. It follows from equation (26) that Foreign's export cutoff  $\varphi_{FH}^*$  falls since  $\varphi_{HH}^*$  falls and  $\omega$  rises. By free entry,  $\varphi_{FF}^*$  rises. Then, the numerator of the above equation rises, which implies that the effect of the country size shock on the relative mass of firms is magnified.

It is easy to check that  $M_H/M_F = \phi/(1-\phi) > \lambda/(1-\lambda)$  directly translates into  $\phi > \lambda$ , which constitutes a weak home market effect.

## A.6 Proof of Proposition 6 (Home Market Effect in demand shares)

We first augment the model by differences in the lower bound of the productivity distribution  $b_i$ . This leaves the zero cutoff profit conditions unchanged, but affects various equations of the model. The relative entry probability is given by

$$\chi \equiv \frac{1 - G_H[\varphi_{HH}^*]}{1 - G_F[\varphi_{FF}^*]} = B \left( \frac{\varphi_{FF}^*}{\varphi_{HH}^*} \right)^\beta,$$

where  $B \equiv (b_H/b_F)^\beta$ . The export participation rate now reads

$$m_{ij} = \left(\frac{b_i}{b_j}\right)^\beta \left(\frac{\varphi_{ii}}{\varphi_{ij}}\right)^\beta.$$

The balanced trade condition becomes

$$\omega B \frac{\lambda}{1-\lambda} = \left(\frac{\varphi_{FH}^*}{\varphi_{HF}^*}\right)^{-\beta}.$$

This has an important implication for the ex post profitability curve, which now reads

$$\chi = B^2 \frac{\lambda}{1-\lambda} \omega^{-\frac{2\beta-\rho}{\rho}}.$$

Relative free entry reads

$$\chi = \frac{f^d + \left(\frac{\varphi_{FF}^*}{\varphi_{FH}^*}\right)^\beta f^x}{f^d + \left(\frac{\varphi_{HH}^*}{\varphi_{HF}^*}\right)^\beta f^x},$$

where the  $b_i$  disappeared because it is part of  $\chi$ .

Using the zero cutoff profit conditions and balanced trade, we obtain

$$\chi = \frac{1 + \omega^{-\frac{\beta-\rho}{\rho}} B \frac{\lambda}{1-\lambda} \eta}{1 + \omega^{\frac{\beta-\rho}{\rho}} B^{-1} \left(\frac{\lambda}{1-\lambda}\right)^{-1} \eta}.$$

Using the ex post profitability curve to substitute out  $B^2 \frac{\lambda}{1-\lambda} \omega^{-\frac{2\beta-\rho}{\rho}}$ , we obtain the ex ante profitability curve

$$\chi = B \frac{1 - B \omega^{-\frac{\beta}{\rho}} \eta}{B - \omega^{\frac{\beta}{\rho}} \eta}.$$

The ex post profitability curve is downward sloping in  $\omega$ . Evaluated at  $\omega = 1$ , we have  $\chi = B^2 \frac{\lambda}{1-\lambda}$ , which is the upper bound for  $\chi$ . The ex ante profitability curve is upward sloping since

$$\frac{\partial \chi}{\partial \omega} = B \frac{\beta \eta \omega^{-\frac{\beta+\rho}{\rho}} \left( B^2 + \omega^{\frac{2\beta}{\rho}} - 2B \eta \omega^{\frac{\beta}{\rho}} \right)}{\rho \left( B - \eta \omega^{\frac{\beta}{\rho}} \right)^2} > 0,$$



where the inequality follows from rewriting the terms in brackets in the nominator as

$$B^2 - 2B\omega^{\frac{\beta}{\rho}} + \omega^{\frac{2\beta}{\rho}} + 2B\omega^{\frac{\beta}{\rho}}(1 - \eta) = \left(B - \omega^{\frac{\beta}{\rho}}\right)^2 + 2B\omega^{\frac{\beta}{\rho}}(1 - \eta) > 0.$$

Note that the ex ante profitability curve features an asymptote at  $\omega = \left(\frac{B}{\eta}\right)^{\frac{\rho}{\beta}}$ . We consider the range  $\omega \in [1, \left(\frac{B}{\eta}\right)^{\frac{\rho}{\beta}})$ . In the supported range, the denominator is always positive. The sign of the numerator is unclear as  $B > 1, \eta < 1, \omega > 1$ .

Home's share of firms is given by

$$\phi = \left(1 + \frac{M_F}{M_H}\right)^{-1} = \left(1 + \frac{b_F^\beta L_F (\varphi_{FF}^*)^{-\beta}}{b_H^\beta L_H (\varphi_{HH}^*)^{-\beta}}\right)^{-1} = \left(1 + \frac{1 - \lambda}{\lambda} \frac{1}{\chi}\right)^{-1}$$

while Home's share of demand is

$$\left(1 + \frac{1 - \lambda}{\lambda} \frac{1}{\omega}\right)^{-1}.$$

A weak HME in demand shares occurs if  $\chi > \omega$ .

A necessary condition for  $\chi > \omega$  is that the value of the market potential curve exceeds the value of  $\chi = B^2 \frac{\lambda}{1 - \lambda} \omega^{-\frac{2\beta - \rho}{\rho}}$  for  $\chi = \omega$ , which is given by

$$\tilde{\omega} = B^2 \frac{\lambda}{1 - \lambda} \tilde{\omega}^{-\frac{2\beta - \rho}{\rho}} \Leftrightarrow \tilde{\omega}^{\frac{2\beta}{\rho}} = B^2 \frac{\lambda}{1 - \lambda} \Leftrightarrow \tilde{\omega} = B^{\frac{\rho}{\beta}} \left(\frac{\lambda}{1 - \lambda}\right)^{\frac{\rho}{2\beta}}.$$

Hence, we require

$$\chi[\tilde{\omega}] > \tilde{\omega} \Leftrightarrow \chi[\tilde{\omega}] = B \frac{1 - B\tilde{\omega}^{-\frac{\beta}{\rho}} \eta}{B - \tilde{\omega}^{\frac{\beta}{\rho}} \eta} > \tilde{\omega}.$$

By rearranging terms, we have

$$B^2 \left( \left(\frac{\lambda}{1 - \lambda}\right)^{\frac{\beta + \rho}{2\beta}} \eta^{-1} B^{\frac{\rho}{\beta}} - \left(\frac{\lambda}{1 - \lambda}\right)^{\frac{1}{2}} \eta^{-1} + 1 \right) < \tilde{\omega}^{\frac{2\beta + \rho}{\rho}}.$$

Inserting the expression for  $\tilde{\omega}$  and solving for  $\eta$ , we obtain

$$\eta > \ell^{\frac{1}{2}} \frac{B^{\frac{\rho}{\beta}} \ell^{\frac{\rho}{2\beta}} - 1}{B^{\frac{\rho}{\beta}} \ell^{\frac{2\beta + \rho}{2\beta}} - 1},$$

where  $\ell \equiv \lambda / (1 - \lambda)$  with  $\ell \geq 1$ .

In the presence of symmetric country sizes, i.e.  $\ell$ , the necessary condition for a weak HME in demand shares reduces to

$$\eta > 1.$$

This condition can never hold. To the contrary, we always have  $\eta < 1$ . Hence, if countries differ only in the lower productivity bound, the richer country hosts an under-proportional share of firms. This proves part (b) of our proposition.

We now prove that the weak HME in demand shares arises under a mild condition if countries only differ in their country size. With  $B = 1$ , the necessary condition reads

$$\eta > \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1}.$$

If  $\ell$  increases, the numerator increases, but the denominator increases faster, such that the term on the right hand side declines. Then, a sufficient condition for a weak HME in demand shares is

$$\eta > \lim_{\ell \rightarrow 1} \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1}.$$

Employing l'Hospital's rule, we find

$$\lim_{\ell \rightarrow 1} \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1} = \lim_{\ell \rightarrow 1} \frac{\frac{\rho}{2\beta} \ell^{-\frac{2\beta-\rho}{2\beta}}}{\frac{2\beta+\rho}{2\beta} \ell^{\frac{\rho}{2\beta}}} = \frac{\rho}{2\beta + \rho}.$$

## A.7 Proof of Proposition 7 (Unconventional sorting)

### A.7.1 Weak HME

The relative entry probability  $\chi$  is given by

$$\chi = \frac{1 - G[\varphi_{HH}^*]}{1 - G[\varphi_{FH}^*]} = \left( \frac{\varphi_{FH}^*}{\varphi_{HH}^*} \right)^\beta.$$

The two zero cutoff profit conditions from targeting Home imply

$$\chi = \tau^\beta \left( \frac{f^x}{f^d} \right)^{\frac{\beta}{\sigma-1}} \omega^{-\frac{\beta}{\rho}} = \frac{f^x}{f^d} \eta^{-1} \omega^{-\frac{\beta}{\rho}}, \quad (28)$$

which constitutes the ex post profitability curve.

Equation (23) can be rewritten as

$$\frac{\lambda}{1-\lambda} = \omega^{\frac{\beta-\rho}{\rho}} \frac{\omega^{\beta/\rho} - \eta}{1 - \eta \omega^{\beta/\rho}}.$$

We use equation (28) to substitute out  $\omega^{\frac{\beta}{\rho}}$  in the denominator and solve for  $\chi$ , which yields a ex ante profitability curve

$$\chi = \frac{f^x}{f^d} \frac{1}{1 - \frac{1-\lambda}{\lambda} \omega^{\frac{\beta-\rho}{\rho}} (\omega^{\beta/\rho} - \eta)}$$

The ex ante profitability curve is strictly increasing in  $\omega$ .

We have  $\chi > 1$  since

$$1 - \frac{1-\lambda}{\lambda} \omega^{\frac{\beta-\rho}{\rho}} (\omega^{\beta/\rho} - \eta) < 1.$$

Together with equation (11), this proves the existence of a (weak) HME under unconventional sorting.

### A.7.2 Home Market Magnification Effect

Lower variable trade cost shift the down both the ex post and ex ante profitability curves. Lower export fixed cost additionally shift both curves proportionally. Hence, a higher freeness of trade lowers the relative entry probability under unconventional sorting, which implies that the home market effect is dampened under unconventional sorting.

### A.7.3 Welfare differential

Using equations (19) and (21), we can rewrite relative welfare as

$$\frac{W_H}{W_F} = \left( \frac{\lambda}{1-\lambda} \right)^{\frac{\beta-(\sigma-1)}{\beta(\sigma-1)}} \omega^{\frac{2\beta-\rho}{\beta\rho}} > 1.$$

This is a general expression which holds under conventional and unconventional sorting.  $\omega$  is increasing in  $\lambda$  also under unconventional sorting. The inequality follows from  $\lambda > 1/2$ ,  $\omega > 1$ ,  $\beta > (\sigma - 1)$ , and  $2\beta > \rho$ . While under conventional sorting an increase in the freeness of trade reduces  $\omega$  and therefore the welfare differential, it is unclear whether trade liberalization leads to real per capita income convergence under unconventional sorting because the effect on  $\omega$  is ambiguous.

## B Details to Derivations (not intended for publication)

### B.1 Derivation of equilibrium conditions

**Zero cutoff profit conditions.** Demand for any variety is given by

$$q[z] = R_i P_i^{\sigma-1} p[z]^{-\sigma},$$

where the price index to (1) is given by  $P_i^{1-\sigma} = \int_{z \in \Omega_i} p[z]^{1-\sigma} dz$  and  $R_i$  denotes aggregate expenditure.<sup>25</sup> Given the demand function, the price charged at the factory gate is  $w_i / (\rho\varphi)$ .

Then, operating profits of a firm from country  $i$  on market  $j$  are

$$\pi[\varphi] = R_j P_j^{\sigma-1} \left( \frac{\rho\varphi}{\tau_{ij} w_i} \right)^{\sigma-1} / \sigma - w_i f_{ij}.$$

The zero cutoff profit conditions follow from noting that  $\pi[\varphi_{ij}^*] = 0$ .

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<sup>25</sup>Note that each variety  $z$  is produced by a single firm with productivity level  $\varphi$ . We henceforth index varieties by  $\varphi$ .

**Price index.** Using the zero cutoff profit condition, we can write the price level  $P_i$  as

$$\begin{aligned}
P_i^{1-\sigma} &= \sum_{j \in \{H,F\}} \int_{\varphi_{ji}^*}^{\infty} \left( \frac{\tau_{ji} w_j}{\rho \varphi} \right)^{1-\sigma} M_j m_{ji} \frac{dG[\varphi]}{1 - G[\varphi_{ji}^*]} \\
&= \sum_{j \in \{H,F\}} \left( \frac{\tau_{ji} w_j}{\rho} \right)^{1-\sigma} M_j m_{ji} \theta (\varphi_{ji}^*)^{\sigma-1} \\
&= \theta \sum_{j \in \{H,F\}} m_{ji} M_j \left( \frac{\rho \varphi_{ji}^*}{\tau_{ji} w_j} \right)^{\sigma-1},
\end{aligned}$$

where  $\theta \equiv \beta / (\beta - \sigma + 1)$  is a positive constant.

**Free entry condition.** Using optimal demand and the zero cutoff profit condition, we obtain the following expression for expected profits of a firm in country  $i$  from entering

$$\begin{aligned}
\bar{\pi}_i &= \sum_{j \in \{H,F\}} \int_{\varphi_{ij}^*}^{\infty} \pi_{ij}[\varphi] \frac{dG[\varphi]}{1 - G[\varphi_{ij}^*]} \\
&= \sum_{j \in \{H,F\}} m_{ij} \left( \theta \frac{R_j P_j^{\sigma-1}}{\sigma} \left( \frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} (\varphi_{ij}^*)^{\sigma-1} - w_i f_{ij} \right) \\
&= \sum_{j \in \{H,F\}} m_{ij} \left( \theta R_j P_j^{\sigma-1} \left( \frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} R_j^{-1} P_j^{1-\sigma} \left( \frac{\tau_{ij} w_i}{\rho} \right)^{\sigma-1} w_i f_{ij} - w_i f_{ij} \right),
\end{aligned}$$

which reduces to the expression in the text.

**Labor market clearing condition.** Labor market clearing is given by

$$L_i = M_i^e f^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^*}^{\infty} \frac{\tau_{ij} q_{ij}[\varphi]}{\varphi} \frac{dG[\varphi]}{1 - G[\varphi_{ij}^*]} = M_i \theta \sigma \sum_j m_{ij} f_{ij},$$

where the second equality follows from inserting  $M_i^e = M_i / p_i^{in}$ , using the free entry condition to substitute out  $f^e$ , and using the zero cutoff profit conditions to substitute out the cutoff productivity levels. The formula in the text follows from using the free entry condition to substitute out  $\sum_j m_{ij} f_{ij}$  and noting that  $\theta \sigma / (\theta - 1) = \beta / \rho$ .

**Trade balance condition.** In analogy to expected profits, we can write expected revenues of a firm in country  $i$  from selling to country  $j$  as

$$\bar{r}_{ij} = \int_{\varphi_{ij}^*}^{\infty} r_{ij}[\varphi] \frac{dG[\varphi]}{1 - G[\varphi_{ii}^*]} = \sigma \theta w_i m_{ij} f_{ij}.$$

Using this expression, the labor market clearing condition, the definition of  $m_{ij}$  and exploiting symmetry of fixed cost, we obtain the balanced trade condition (7).

## B.2 Melitz (2003) with outside sector

In this appendix, we derive equilibrium in the presence of an outside sector. We show that the economy exhibits a weak and a strong home market effect. As in Helpman and Krugman (1995), the home market effect is linear in  $\lambda$ . Moreover, we discuss welfare implications.

**Basic environment.** The model is augmented by a homogeneous good produced under constant returns to scale and perfect competition. Utility takes the Cobb-Douglas form, where  $\mu$  denotes the share of expenditure spent on differentiated varieties. The outside good is freely tradable. Hence, wages are equalized and henceforth normalized to unity. Welfare per worker is given by the inverse of the *aggregate* price index. Using  $P_i$  to denote the price index of the differentiated goods sector and defining  $\tilde{\mu} \equiv (1 - \mu)^{1-\mu} \mu^\mu$ , we can write welfare per worker as

$$W_i = \tilde{\mu} / P_i^\mu.$$

**Equilibrium.** The free entry conditions are unaffected. In the *zero cutoff profit conditions*, however, wages drop. In relative form, the zero cutoff profit condition becomes

$$\varphi_{ji}^* = \tau \left( \frac{f^x}{f^d} \right)^{\frac{1}{\sigma-1}} \varphi_{ii}^*.$$

Hence, we can substitute out export cutoffs from the *free entry conditions*. We obtain two equations in two unknowns, which can be used to solve for the domestic entry cutoffs as

$$(\varphi_{ii}^*)^\beta = (\theta - 1) \frac{(f^d)^\beta}{f^e} (1 + \eta).$$

It is important to note that the cutoff productivity levels do not depend on country size. Hence, they are symmetric across countries irrespectively of the country size distribution.

*Labor market clearing implies*

$$\xi_i L_i = M_i^e f^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^*} \frac{\tau_{ij} q_{ij} [\varphi]}{\varphi} \frac{dG [\varphi]}{1 - G [\varphi_{ij}^*]} = M_i \theta \sum_j m_{ij} f_{ij},$$

where  $\xi_i$  denotes the fraction of workers employed in the differentiated good sector.

Using the free entry condition, we obtain

$$M_i = \frac{\xi_i L_i}{\bar{r}_i};$$

where  $\bar{r}_i \equiv \sum_j \bar{r}_{ij} = \frac{\beta f^e}{\rho} (\varphi_{ii}^*)^\beta$ . Expected revenues  $\bar{r}_{ij}$  are given by

$$\bar{r}_{ij} = \int_{\varphi_{ij}^*} r_{ij} [\varphi] \frac{dG [\varphi]}{1 - G [\varphi_{ij}^*]} = \theta \sigma m_{ij} f_{ij}.$$

Due to the symmetry of the cutoffs, we henceforth suppress the subscripts of the revenue terms. Note that  $\bar{r}^x / \bar{r}^d$  reduces to  $\eta$ .

Balanced trade is given by

$$M_i \bar{r}^x = M_j \bar{r}^x + (1 - \mu) L_i - (1 - \xi_i) L_i$$

where the term on the left hand side represents country  $i$ 's exports of the differentiated good. The first term on the right hand side represents  $i$ 's imports of the differentiated good. The remaining terms reflect  $i$ 's imports of the homogeneous good (spending on the homogeneous good minus value of domestic homogeneous good production).

Substituting out  $M_i$  and  $\bar{r}_{ij}$  from balanced trade and using  $\bar{r}_i = \bar{r}_j$ , we obtain

$$\xi_i = \mu \frac{\bar{r}}{\bar{r}^d} - \xi_j \frac{L_j \bar{r}^x}{L_i \bar{r}^d}, i \in \{H, F\}.$$

Solving the system of two equations in  $\xi_i$  and  $\xi_j$ , we obtain

$$\xi_i = \mu \frac{\frac{\bar{r}}{\bar{r}^d} - \frac{L_j \bar{r}^x}{L_i \bar{r}^d} \frac{\bar{r}}{\bar{r}^d}}{1 - \left(\frac{\bar{r}^x}{\bar{r}^d}\right)^2}, i \in \{H, F\},$$

Moreover, we have

$$\xi_H > \xi_F \Leftrightarrow \lambda > 1/2.$$

Hence, the smaller country is a net exporter of the homogeneous good.

**Home market effect.** The labor clearing conditions implies that Home's share of firms active in the differentiated good sector can be written as

$$\phi = \frac{1}{\lambda + (1 - \lambda) \frac{\xi_F}{\xi_H}} \lambda.$$

Note that entry cutoffs have dropped due to their symmetry. We have seen above that  $\xi_H > \xi_F$ , which constitutes a weak home market effect.

We can rewrite  $\phi$  as

$$\phi = \frac{1}{1 + \frac{1-\lambda}{\lambda} \frac{\bar{r} - \frac{\lambda}{1-\lambda} \bar{r}^x}{\bar{r} - \frac{1-\lambda}{\lambda} \bar{r}^x}} = \frac{\lambda \bar{r} - (1 - \lambda) \bar{r}^x}{\bar{r}^d}$$

with

$$\frac{\partial \phi}{\partial \lambda} = 1 + 2 \frac{\bar{r}^x}{\bar{r}^d} = 1 + 2\eta > 1$$

The following observations stand out. First, the weak HME is linear in  $\lambda$ . Second, there exists a strong HME. Third, the HME is magnified by a reduction in trade barriers (HMME).

**Real per capita income.** Using the domestic zero cutoff profit condition, we can rewrite welfare per worker as

$$W_i = \tilde{\mu} \left( \frac{\beta L_i}{\sigma f^d} \right)^{\frac{\mu}{\sigma-1}} (\rho \varphi_{ii}^*)^\mu,$$



where  $\tilde{\mu} \equiv (1 - \mu)^{1-\mu} \mu^\mu$  is a constant. The following observations stand out. First, a shock on the relative country size  $\lambda$  affects both countries symmetrically. The reason that domestic entry cutoffs are independent of the country size distribution. Second, an increase in freeness of trade rises the domestic entry cutoff and therefore increases welfare per worker in both countries. Third, an increase in the freeness of trade leaves relative welfare per worker unaffected.