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Forecasting Euro Area Recessions in real-time with a mixed-frequency Bayesian VAR

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Abstract
In this paper I use the predictive distribution of the back-, now- and forecasts obtained with a mixed-frequency Bayesian VAR (MF-BVAR) to provide a real-time assessment of the probability of a recession in the euro area for the period from 2003 until 2013. Using a dataset that consists of 135 monthly data vintages and covers 11 soft and hard monthly indicators as well as quarterly real GDP, I show that the MF-BVAR is able to capture current economic conditions extremely well. For both recession periods in the sample, the Great Recession of 2008/2009 and the European debt crisis 2011/2013, the MF-BVAR real-time recession probabilities soar right at the onset of the pending slump of GDP growth. By contrast a BVAR estimated on quarterly data detects both recessions with a substantial delay. While typically non-linear discrete-choice or regime switching models have to be used to predict rare events such as recessions, my results indicate that the MF-BVAR can not only compete with other nowcasting approaches in terms of the accuracy of point forecasts, but also reliably detect rare events through the corresponding predictive distribution which is easily available as a by-product of the estimation procedure.

Keywords: Nowcasting, Forecasting Recessions, Mixed-frequency data, Bayesian VAR
JEL-Codes: C53, E32, E37
1 Introduction

Reliable forecasts of macroeconomic activity are highly important for the decision making process of economic policy makers. However, the problems associated with real-time data availability such as for example mixed data frequencies, the irregular and varying publication lags of various macroeconomic indicators and data revisions pose huge challenges to practical forecasters (see Giannone et al. (2008) for a detailed discussion of these issues).

Recently, the success of different econometric forecasting methods in providing a reliable assessment of the prevailing economic conditions in terms of point forecasts, while coping with the outlined difficulties, has been demonstrated. These methods include bridge equation models (see e.g. Baffigi et al. (2004) and ECB (2008)), MIDAS-models (see e.g. Kuzin et al. (2011) and Schumacher (2014)) and factor models (see e.g. Schumacher and Breitung (2008), Rünstler et al. (2009) and Banbura and Rnstler (2011)) as well as combinations of the aforementioned methods (see e.g. Angelini et al. (2011) and Marcellino and Schumacher (2010)). The relative accuracy of these methods has been studied for example in Foroni and Marcellino (2014) and Schwarzmüller (2015).

However, in addition to precise point forecasts a reliable and timely prediction of business cycle turning points and in particular of recessions can be extremely useful for the design of appropriate economic policy. After all, the effectiveness of monetary and fiscal policy measures may depend on the current phase of the business cycle (see Auerbach and Gorodnichenko (2012) for empirical evidence for fiscal policy and Lo and Piger (2005) for monetary policy). Yet, rare events such as recessions are typically very hard to capture for linear forecasting models like those mentioned above. Hence, for this purpose researchers often turn to non-linear models such as discrete-choice or regime switching models (for recent applications see Nyberg (2010), Bellgo and Ferrara (2009) and Pauwels and Vasnev (2014) for discrete choice models and Chauvet and Piger (2008), Nalewaik (2012) and Camacho et al. (2014) for regime switching models).

In this paper I demonstrate that the mixed-frequency Bayesian vector autoregression (MF-BVAR) proposed by Schorfheide and Song (2015) is capable of meeting both of the above outlined forecasting needs. The model is estimated on mixed-frequency real-time data with ragged edges and provides not only extremely precise point forecasts but can also reliably detect recessions in real-time through the corresponding predictive distribution which is easily available as a by-product of the estimation procedure. Of course, this advantage is in principle inherent to all forecasting models estimated with Bayesian techniques. For example, Österholm (2012) uses the predictive distribution of a quarterly BVAR to estimate the probability of a recession in the US in the third and fourth quarter of 2008. However, due to the fact that the MF-BVAR can exploit information in a much timelier manner by incorporating high-frequency indicators into the estimation process the resulting real-time recession probabilities are considerably more precise than those obtained with models estimated on quarterly data only.

Within the empirical application in this paper I use the predictive distribution of the MF-BVAR to provide a real-time assessment of the recession probabilities in the euro area for the period from 2003 until 2013. Using a real-time dataset that consists of 135 monthly data vin-
tages and covers 11 soft and hard monthly indicators as well as quarterly real GDP, I show that the MF-BVAR is able to capture current economic conditions extremely well. For both recession periods in the sample, the Great Recession of 2008/2009 and the European debt crisis 2011/2013, the MF-BVAR real-time recession probabilities soar right at the onset of the pending slump of GDP growth. By contrast a BVAR estimated on quarterly data indicates both recessions with a substantial delay (which is in line with the findings in Österholm (2012) for the US).

Previous papers have focused mainly on predicting recessions with financial variables. In this context, especially the predictive power of the yield spread, i.e. the difference between the long- and short-term interest rate, has been emphasized (see among many others Estrella and Mishkin (1998), Chauvet and Potter (2002), Wright (2006), Nyberg (2010) and Clements and Harvey (2011) for the US as well as Duarte et al. (2005), Moneta (2005) and Chionis et al. (2010) for the euro area). However, as pointed out by Fornari and Lemke (2010) due to the determined expansionary response of monetary policy in 2008 the term spread rose sharply which indicated a very short recession. Thus, in light of the actual course of the Great Recession of 2008/2009 the usefulness of the yield spread seems to have deteriorated considerably.

Most of the papers using financial variables to detect recessions estimate some version of a discrete choice model such as static or dynamic probit or logit models. Here a binary indicator variable, which is related to an unobservable latent business cycle indicator, is regressed on a set of explanatory variables. Recently this approach has been combined with the vector autoregressive model in order to allow a better assessment of the underlying dynamics of the system as well as to be able to provide iterative multi-step forecasts. For example, Dueker (2005) proposes a Qual VAR which incorporates binary information into the VAR framework and finds that the values of the recession probabilities provided by this model are closely in line with the actual US recession periods as declared by the NBER business cycle dating committee. Fornari and Lemke (2010) estimate a Prob VAR which lacks some of the dynamics captured by the Qual VAR but is considerably easier to estimate. They find that for the US and Germany the model can reliably signal slowdowns of economic activity in the US.

However, while the Prob VAR, the Qual VAR as well as all the other discrete choice models have been proven to be quite successful in detecting recessions ex post, these models are of limited usefulness when it comes to the prediction of recessions in real-time. This is because, as pointed out by Dueker (2005), in a real-time setting these models would require the inclusion of real-time data on whether the economy was currently believed to be in a recession or not, i.e. a real-time value for the binary indicator variable. This data might not be available or of very limited reliability which constitutes a serious drawback of this class of models for real-time applications.

1The underlying theoretical argument can be summarized as follows: Since the long-term interest rate reflects expectations about the values of future short-term interest rates, the spread reflects agent’s believes on the future stance of monetary policy. While tighter monetary policy typically slows down economic activity and the term spread flattens there is a strong historical relationsship between the yield spread and the odds of a recession.

2Moreover these models can solely be used to detect business cycle turning point while the MF-BVAR produces reliable point and density now- and forecasts as well.
Real-time forecasting of recessions is of course an incredibly challenging task (see Hamilton (2011) for a discussion of the underlying issues and an overview of recent attempts). However, a few related papers have been successful in forecasting recessions in a real-time setting. For example, Chauvet and Piger (2008) estimate a monthly dynamic factor Markov-switching model for the US, while Schreiber (2014) estimates a large monthly VAR with automated coefficient restrictions following the general-to-specific method for the US and Germany. However, since both of these approaches can not deal with mixed data frequencies they have to rely on monthly industrial production as a proxy of overall economic activity. Aastveit et al. (2014) solve the mixed-frequency issue by applying the non-parametric Bry-Boschan rule to detect recessions to a bridge equation model nowcast and compare the accuracy of the therewith obtained real-time recession probabilities to those obtained with an autoregressive Markov-switching model for Norwegian GDP. However, according to their results these models are not able to forecast business cycle turning points in a timely manner in real time. Bellgo and Ferrara (2009) use a broad set of financial market data and estimate time-varying probit models with parameters evolving according to a Markov-chain as well as time-varying factor augmented probit models. They find that these models would have been very useful to provide an early signal of the Great Recession in the euro area. Finally, Camacho et al. (2014) estimate an extension of the dynamic Markov-switching factor model which is capable of dealing with mixed-frequency data and ragged edges to compute euro area GDP growth now- and forecasts as well as to detect euro area recessions in real-time. Their non-linear model captures not only comovements across various economic indicators through a common business cycle factor but also regime shifts. This allows for significant improvements in the timeliness with which they can detect economic downturns. Similar to the MF-BAVR, the dynamic Markov-switching factor model solves the issue of missing values at the end of a data vintage due to varying publication lags through a time-varying state space representation that is estimated using a Kalman Filter that simply skips missing values. However, one clear advantage of the MF-BVAR over the dynamic Markov-switching factor model is the fact that it accounts for the uncertainty related to the model point forecasts in form of the predictive distribution which is easily available as a by-product of the estimation procedure and which can be used for inference on future paths of the target variable. On this account, the associated MF-BVAR recession probabilities are deterministic functions of the target variable, i.e. euro area GDP growth, and not of an binary indicator variable as is the case for most other approaches.

The remainder of this paper is structured as follows. In section 2 I describe the Bayesian mixed-frequency VAR and outline the estimation approach. In section 3 I describe the dataset used for the empirical application, while in section 4 I outline the forecasting approach and explain how the real-time recession probabilities are obtained. The results are presented in section 5. Finally, in section 6 I conclude.
2 The Bayesian Mixed-Frequency VAR

Consider the following high-frequency VAR(p)

\[ X_t = C + A_1 X_{t-1} + ... + A_p X_{t-p} + \epsilon_t, \]

where the \((nx1)\) vector \(X_t = (x_{1,t}, x_{n,m,t}, \ldots, x_{m,1,t}, \ldots, x_{m,n,t})'\) contains \(n^m\) observable high-frequency (e.g. monthly) indicators and \(n^q\) unobservable high-frequency variables, which are published at a lower frequency (e.g. quarterly) only. Following Banbura et al. (2010) I include the variables in log-levels rather than growth rates to not lose information that might possibly be contained in the trends. \(p\) denotes the number of lags included in the estimation, \(C\) is a \((nx1)\) vector of constants, \(A_1, \ldots, A_p\) are \((nxn)\) parameter matrices and \(\epsilon_t\) is a \((nx1)\) vector of independently identically distributed white noise error terms with zero mean and covariance matrix \(\Sigma\).

The high-frequency model outlined in equation (1) can be rewritten in state-space form. For that define a \((npx1)\) vector of states as \(Z_t = (X_t', X_{t-p+1}')'\). Then, the transition equation reads

\[ Z_t = D + FZ_{t-1} + \nu_t. \]

The first \(n\) rows of the \((npx1)\) vector \(D\) contain the vector of constants \(C\) while the remaining entries of \(D\) equal zero. The \((nxnp)\) matrix \(F\) is given as

\[
F = \begin{pmatrix}
A_1 & A_2 & \ldots & A_p \\
I_n & 0 & \ldots & 0 \\
0 & I_n & \ldots & 0 \\
0 & 0 & \ldots & I_n
\end{pmatrix}
\]

where \(I_n\) denotes the identity matrix of dimension \(n\). \(\nu_t\) is a \((npx1)\) vector of independently identically distributed white noise error terms with zero mean and a covariance matrix \(\Psi\) which contains the matrix \(\Sigma\) in the upper-left submatrix and is zero otherwise.

Following Schorfheide and Song (2015) I define a \((nx1)\) vector of observations as \(Y_t = (y_{1,t}, \ldots, y_{n,m,t}, y_{1,t}, \ldots, y_{m,n,t})'\) and set up the measurement equation as

\[ Y_t = H_t Z_t \]

with

\[ H_t = S_t \Lambda. \]

To account for the fact that the low-frequency variables \(y_{1,t}, \ldots, y_{m,n,t}\) are not observable in every period and that some of the high-frequency variables might be missing at the current edge due to publication lags, the form of matrix \(H_t\) is time-varying. In particular, \(H_t\) can be decomposed into a \((nxn)\) diagonal time-varying selection matrix \(S_t\) and the \((nxnp)\) aggregation matrix \(\Lambda\).
which is given as

\[
\Lambda = \begin{pmatrix}
1 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & & & \ddots & 0 \\
\lambda
\end{pmatrix}
\]  

(6)

The \((n^q \times np)\) submatrix \(\lambda\) aggregates the unobservable high-frequency variables \(x^q\) into their low-frequency counterparts \(\tilde{y}^q\), i.e. \(\tilde{y}^q = \lambda x^q\).

Of course, the variables in \(\tilde{y}^q\) are included in the measurement equation (4) only for periods \(t^*_q\) in which they are observable. Thus, for example, if the VAR(p) outlined in equation (1) is specified at monthly frequency and the low frequency variables \(y^q\) are published on a quarterly basis then \(\tilde{y}^q\) would be included in the measurement equation (4) only in every third month, i.e. \(t^*_q = (3, 6, 9, \ldots)\). In practice, this is governed through the selection matrix \(S_t\).

The same principle applies for the variables in \(y^m\) at the current edge. In particular, let \(T_b\) denote the last period for which the data panel is balanced. Then the selection matrix \(S_t\) is given as

\[
S_t = \begin{pmatrix}
\Theta^m & \ldots & 0 \\
0 & \ldots & \Theta^q
\end{pmatrix}
\]  

(7)

where the diagonal elements of the diagonal \((n^m \times n^m)\) submatrix \(\Theta^m\) are given as

\[
\theta^m_{ii,t} = \begin{cases}
1 & \text{for } t \leq T_b \cup t > T_b \cap t = t^*_m \\
0 & \text{otherwise}
\end{cases}
\]  

(8)

where \(t^*_m\) denotes periods at the current data edge in which a particular high-frequency indicator is available. Similarly, the diagonal element of the diagonal \((n^q \times n^q)\) submatrix \(\Theta^q\) are

\[
\theta^q_{ii,t} = \begin{cases}
1 & \text{for } t \leq T_b \cap t = t^*_q \\
0 & \text{otherwise.}
\end{cases}
\]  

(9)

2.1 Estimation

The state space mixed-frequency VAR outlined in equations (2) - (9) is estimated with Bayesian techniques. This involves the estimation of the marginal posterior distributions of the unknown VAR parameters \(A_1, \ldots, A_p, C\) and \(\Sigma\) as well as the estimation of the unknown state vector \(Z_t\). I implement a Gibbs sampler which proceeds according to the following steps.

Step 1: Setting the priors and derivation of initial states For the unknown parameters of the VAR \(A_1, \ldots, A_p, C\) and \(\Sigma\), I implement a version of the Normal inverse Wishart prior (see e.g. Kadiyala and Karlsson, 1997) that retains the main principles of the widely used Minnesota prior (see Litterman, 1986). This prior implies that the VAR coefficients \(A_1, \ldots, A_p\) are assumed to be a priori independently and normally distributed, while with respect to the constant \(C\) the prior is assumed to be diffuse. The residual covariance matrix \(\Sigma\) is assumed to a priori follow
an Inverse Wishart distribution with scale matrix $\tilde{S}$ and degrees of freedom $\bar{\alpha}$.

One of the main principles of the Minnesota prior is to center each equation of the VAR around a random walk with drift. Thus the prior mean for the VAR coefficients $A_1, ..., A_p$ is specified as:

$$E[(A_{\ell})_{ij}] = \begin{cases} 
1 & \text{for } i = j, \ell = 1 \\
0 & \text{otherwise}
\end{cases}$$

(10)

Moreover, this prior also incorporates the belief that more recent lags of a variable should provide more reliable information for the estimation than less recent lags. The zero coefficient prior on more recent lags is therefore not imposed as tightly as on less recent lags which is captured by specifying the prior variance as

$$Var[(A_{\ell})_{ij}] = \begin{cases} 
\frac{\lambda^2}{\tilde{\sigma}_i^2} & \text{for } i = j \\
0 & \text{otherwise}
\end{cases}$$

(11)

where $\ell = 1, ..., p$ is the lag length, $\lambda$ is a hyperparameter governing the importance of the prior beliefs relative to the data and $\sigma_i/\sigma_j$ is a scale parameter adjusting the prior for the different scale and variability of the data. For the implementation, $\sigma_i$ is set equal to the standard deviation of the residuals of a simple univariate autoregression for each variable over a presample $t = T^-,...,0$. Following Schorfheide and Song (2015) I augment the prior outlined above to constrain the sum of coefficients of the VAR (see e.g. Sims and Zha, 1998) as well as to incorporate the belief that the variables in the VAR follow a common stochastic trend.

I implement the prior outlined above using dummy variables $Y^D$ and $X^D$, which are given as

$$Y^D = \begin{pmatrix}
\frac{\text{diag}(\sigma_1...\sigma_n)}{\lambda} \\
0_{n(p-1)\times n} \\
\text{diag}(\sigma_1...\sigma_n) \\
0_{1\times n} \\
\text{diag}(\mu_1...\mu_n)\gamma \\
\eta\mu_1...\eta\mu_n
\end{pmatrix},
X^D = \begin{pmatrix}
\frac{\text{diag}(1...p)\otimes\text{diag}(\sigma_1...\sigma_n)}{A} \\
0_{n\times np} \\
0_{1\times np} \\
P \otimes \frac{\text{diag}(\mu_1...\mu_n)\gamma}{c} \\
(P' \otimes \text{diag}(\mu_1...\mu_n)\eta)' \\
\eta
\end{pmatrix},$$

(12)

where $P$ is a $(1\times p)$ matrix of ones, $c$ is a very small number reflecting the diffuse prior for the constant $C, \mu_1, ..., \mu_n$ are the variable means over the presample $t = T^-,...,0$ and $\gamma$ and $\eta$ govern the tightness of the sum of coefficients constraint and the common stochastic trend prior, respectively. With these dummy variables the moments of the prior distributions for the parameters $A_1, ..., A_p, C$ and the residual covariance matrix $\Sigma$ can be computed as $E[(A_1, ..., A_p, C)] = \bar{\Sigma} = (X^D X^D)^{-1} X^D Y^D, Var[(A_1, ..., A_p, C)] = V = (X^D X^D)^{-1}, \bar{\Sigma} = (Y^D - X^D \bar{A})'(Y^D - X^D \bar{A})$ and $\bar{\alpha} = T^D - n(p - 1) - 1$, where $T^D$ is the number of rows of $Y^D$.

Given these prior moments I sample the state vector $Z_0$ conditional on the presample $t = T^-, ..., 0$ by applying the Kalman filter to the state space system (2) - (9). I chose a presample that ranges from April 1991 until December 1992. The initial value of the state vector $Z_{T^-}$ is obtained using actual data for the high-frequency variables, while for the low-frequency variables I follow Schorfheide and Song (2015) and assume that the unobserved high-frequency
variables are equal to the observed low-frequency averages.

**Step 2: Sampling of the VAR coefficients**  Conditional on the most recent draw of the state vector $Z_i^{t-1}$ the parameters $A^i_1, ..., A^i_p, C^i$ and $\Sigma^i$ are drawn from their respective posterior distributions\(^3\). With the Normal inverse Wishart prior the VAR coefficients $(A^i_1, ..., A^i_p, C^i)$ follow a multivariate t-distribution with mean $\bar{A}$, covariance matrix $\bar{S} \otimes \bar{V}$ and degrees of freedom $\bar{\alpha}$, while $\Sigma \sim IW(\bar{S}, \bar{\alpha})$. The respective moments of these distributions can be computed using the dummy variables outlined in equation (\(12\)). In particular, augment the transition equation (\(2\)) with the dummy variables to obtain $Y^* = [(Z_i^{t-1})', Y^D]'$ and $X^* = [(Z_i^{t-1})', X^D]'$. Then $\bar{A} = (X^* X^*)^{-1} X^* Y^*$, $\bar{V} = (X^* X^*)^{-1}$, $\bar{S} = (Y^* - X^* \bar{A})'(Y^* - X^* \bar{A})$ and $\bar{\alpha} = T^* - np + 1$, where $T^*$ is the number of rows of $Y^*$.

**Step 3: Sampling of the states**  Given the most recent draws of $(A^i_1, ..., A^i_p, C^i)$ and $\Sigma^i$ I sample the state vector $Z_i^t$ by applying the Kalman filter and the Carter-Kohn algorithm to the state space system (\(2\)) - (\(9\)).

Steps 2 and 3 are repeated 10000 times. The draws of $A_1, ..., A_p, C, \Sigma$ and $Z_t$ obtained during last 7500 iterations approximate the respective marginal distributions.

### 3 Data

For the empirical application in this paper I use a dataset that consists of eleven monthly variables for the euro area, namely harmonized consumer prices, industrial production excluding construction, unemployment, new passenger car registrations, money supply (M1 and M3), the Economic sentiment indicator as well as indices for the stock markets, the oil price and confidence in the industry and retail sales sectors. Moreover, a quarterly series for real euro area GDP is included. All series were obtained from the real time database of the Statistical Data Warehouse of the European Central Bank\(^4\) in December 2014. The dataset consists of 135 monthly data vintages\(^5\) from October 2002 until December 2014 all of which start in January 1991. Each of these data vintages provides a historic snapshot of the data as it was available at the time. This implies that the dataset perfectly reflects the publication lag of each variable with respect to the reference date, i.e. the date the snapshot was taken, as well as changes in the data flow over time due to recent improvements in the timeliness of various indicators. All series are seasonally adjusted and natural logarithms are taken for all variables not expressed in rates.

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\(^3\)Here $i$ denotes the $i^{th}$ iteration of the Gibbs Sampler.

\(^4\)A detailed description of the database, the variables included as well as the treatment of issues such as data revisions, changing variable definitions and the changing composition of the euro over time can be found in Giannone et al. (2010).

\(^5\)The very few unavailable vintages were replaced with the most recent ones.
4 Short-term Forecasting Approach

For euro area GDP, which is published at quarterly frequency, the first official release is available only in the third month after the end of the respective quarter\(^6\). This implies that not only is current quarter GDP unknown throughout the quarter but also last quarter GDP is not available, at least in the first and second month of a quarter. Thus, in order to assess the real-time recession probabilities in a certain quarter an estimate of current quarter GDP (referred to as the nowcast) and in some cases also of last quarter GDP (referred to as the backcast) has to be computed. Moreover, the GDP now- and backcasts have to account for the different (and changing) publication lags of the various monthly indicators throughout the quarter under consideration. In this application I compute three GDP now- and backcasts per quarter using the data vintages available in the first, second and third month of the quarter, respectively. These nowcasts (backcasts) are given as \(\hat{y}_{tq} \mid \Omega_j\) with \(j = 1, 2, 3\) (\(j = 1, 2\)), where \(\hat{y}_{tq}\) is the nowcast for GDP growth in quarter \(t\) and \(\Omega_j\) denotes the information set available in the \(j^{th}\) month of the quarter for which the nowcast is computed. Besides, I also compute up to \(h^q\)-quarter ahead forecasts for GDP growth for each information set denoted as \(\hat{y}_{tq+h^q} \mid \Omega_j\).

4.1 Forecasting GDP growth

The quarterly back-, now- and forecast \(\hat{y}_{tq+h^*}\) for \(h^* = -1, 0, 1, ..., h^q\) are computed from the monthly forecasts for unobservable monthly GDP obtained with the mixed-frequency BVAR outlined in section 2. In particular, the model is estimated as described above using the data vintage available at month \(T\) with information up to month \(T\). For each of the 7500 retained Gibbs draws of the VAR coefficients \(A_i, A_{ip}, C_i, \Sigma_i\) and the vector of states \(Z_i\), the shock vectors \(e_{T+h}\) are drawn from \(e_{T+h} \sim (0, \Sigma_i)\) and equation (1) is iterated forward to compute forecasts for the monthly observable and unobservable variables \(\hat{X}_{T+h}\) with \(h = 1, ..., h^m\). Taking into account the information set implied by \(T\) the forecasts for unobservable monthly GDP are transformed into their quarterly counterparts based on equation (4) which are then used to compute the implied quarterly GDP growth rates \(\hat{y}_{tq+h^*}\). The final now- and forecasts are obtained by computing the average over all 7500 thereby obtained estimates. Note that the set of 7500 \(\hat{y}_{tq+h^*}\) approximates the predictive distribution of the back-, now- and forecasts which will be used to compute the real-time recession probabilities.

4.2 Assessing real-time recession probabilities

Österholm (2012) defines the probability of a recession as the probability that the current and the following quarter both display negative GDP growth, i.e. \(Pr(\hat{y}_{tq} \mid \Omega_j < 0 \land \hat{y}_{tq+1} \mid \Omega_j < 0)\), a definition that is widely used. However, in this real-time application this definition is not perfectly suitable since it does not account properly for the publication lag of euro area GDP growth. Notably in the first two months of a given quarter a real-time recession probability as-

\(^6\)To be precise the first flash estimate of euro area GDP growth is published about 45 days after the end of the respective quarter. Since the monthly data vintages used in this application are snapshots of the available data taken within the first half of each month, I assume that the data on GDP growth is updated only in every third month.
assessment should take into account that no estimate of last quarter GDP growth is available yet. Therefore, I will define the real-time probability of a recession as $Pr(\hat{y}_{t-1}^{q} \mid \Omega^{j} < 0 \cap \hat{y}_{t}^{q} \mid \Omega^{j} < 0)$ for $j = 1, 2$ and $Pr(\hat{y}_{t}^{q} \mid \Omega^{3} < 0 \cap \hat{y}_{t+1}^{q} \mid \Omega^{3} < 0)$ in every third month of a given quarter. Thus for month $T$ the real-time recession probability $\psi_T$ can be obtained by evaluating the joint cumulative predictive distribution function of the two closest GDP growth estimates at $(0, 0)$. In practice, this is simply done by counting the number of paths of $\hat{y}_{t+h}^{q}$ that fulfill the above defined criterion and divide their number by the total number of Gibbs draws.

4.3 Forecast Evaluation

The real-time recession probabilities as well as the accuracy of the density and point forecasts obtained with the mixed-frequency Bayesian VAR (MF-BVAR) are evaluated against a quarterly Bayesian VAR (QF-BVAR) and a simple quarterly univariate autoregression (AR). The QF-BVAR is estimated for the same data as the MF-BVAR but aggregated to quarterly frequency. The estimation strategy for the models is corresponds to that of the MF-BVAR outlined above.

All models are estimated on a rolling window of 120 observations starting in October 2002. The GDP back-, now- and one quarter ahead forecasts obtained with the tree different models are evaluated for the period ranging from 2003 Q1 until 2013 Q4. Thus for each information set $\Omega^j$ I have $T^* = 44$ point forecasts available for the evaluation. The accuracy of these point forecasts is assessed by means of mean squared forecast errors which is given as

$$MSFE^j_h = \frac{1}{T^*} \sum_{t=1}^{T^*} (y_{t+h}^{q} - \hat{y}_{t+h}^{q} \mid \Omega^j)^2.$$ (13)

In principle, $y_{t+h}^{q}$ denotes realized period $t^q + h$ euro area GDP growth. However, in the literature there is disagreement on whether $y_{t+h}^{q}$ should be the first release for period $t^q + h$ or rather a revised final estimate. While the former has an advantage in capturing the real-time environment a forecaster is facing, the latter can be seen as a more precise assessment of the fundamental value of $y_{t+h}^{q}$. In this application, I will focus on the latter concept, however the results using first releases of euro area GDP growth are very similar and are available upon request.

Further, I will evaluate the predictive distributions of the MF-BVAR and the QF-BVAR in terms of sharpness and calibration. To this end I will compute the continuously ranked probability score (CRPS) which is given as the average over the evaluation sample of the period $CRPS$ which is given by

$$CRPS^j_h = E_P[Y - y_{t+h}^{q}] - \frac{1}{2} E_P[Y - Y'],$$ (14)

where $E_P$ denotes the expectations operator for the predictive cumulated distribution of the MF-BAVR and the QF-BVAR, respectively and $Y$ and $Y'$ are random draws of this distribution (see e.g. Gneiting et al. (2007)).
5 Results

5.1 Evaluation of GDP growth point and density forecasts

In this section the results of the forecast evaluation of the MF-BVAR and the QF-BVAR for the period ranging from 2003 Q1 until 2013 Q4 are reported. In order to assess the accuracy of the point now- and forecasts obtained with both models, table 2 shows the models’ mean squared forecast errors relative to those of a simple univariate quarterly autoregressive benchmark model (AR) for the different information sets (IS 1, ..., 6) available for the computation of the forecasts over the different horizons. In particular, IS 1 denotes the first month in the last quarter, IS 2 the second month and so on, while IS 4 – 6 denote the three months of the current quarter. Note that since the AR benchmark and the QF-BVAR do not incorporate within-quarter information, the data available to these models is updated only every third month. Hence, at IC 3 – 5 the same level of predictive information is included into the estimation.

According to the entries both models clearly outperform the AR benchmark in most cases. Only in the third month of the current quarter (IS 6) the advantage diminishes which is of course not surprising given that at this point in time the flash estimate of last quarter GDP growth is available for all models. Further, while the multivariate QF-BVAR apparently provides more precise forecasts than the univariate benchmark model, the entries in table 2 also reveal that the monthly within quarter information that the MF-BVAR can incorporate improves the models’ forecast accuracy considerably. Particularly in the first two months of the current quarter (IS 4 and 5) the MF-BVAR outperforms the QF-BVAR by far. For the one quarter ahead forecast the difference between the two models’ MSFEs is less pronounced, especially at the longest horizon (IS 1).

Table 1: RMSFE.

<table>
<thead>
<tr>
<th>IS</th>
<th>h^* = 1</th>
<th>h^* = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>MF-BVAR</td>
<td>0.68</td>
<td>0.31</td>
</tr>
<tr>
<td>QF-BVAR</td>
<td>0.67</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Notes: The entries denote mean squared forecast errors for euro area GDP growth relative to an AR benchmark evaluated for the sample ranging from 2003Q1 until 2013Q4. h^* = 1 refers to the one quarter ahead forecast, while h^* = 0 denotes the nowcast. IS denotes the respective information set, i.e. the monthly data availability when computing the fore- and nowcast of a respective quarter.

In terms of density forecasts the MF-BVAR generally outperforms the QF-BVAR as well, especially at within quarter information sets, but to a lesser extend. This can be seen from table 2 where the CRPS for both models is shown. While for the largest and the smallest forecast horizons (IS 1 and 6) the density forecasts of both models seem to be equally accurate, for all other horizons the MF-BVAR’s CRPS is smaller. This indicates that the MF-BVAR predictive cumulated distribution resembles more closely the realized cumulative distribution of euro
area GDP growth over the evaluation sample.

Table 2: CRPS.

<table>
<thead>
<tr>
<th>IS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h^*=1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MF-BVAR</td>
<td>0.43</td>
<td>0.30</td>
<td>0.35</td>
<td>0.26</td>
<td>0.26</td>
<td>0.34</td>
</tr>
<tr>
<td>QF-BVAR</td>
<td>0.43</td>
<td>0.43</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Notes: The entries denote the CRPS of the predictive distributions of the MF-BVAR and the QF-BVAR for the sample ranging from 2003Q1 until 2013Q4. $h^*=1$ refers to the one quarter ahead forecast, while $h^*=0$ denotes the nowcast. IS denotes the respective information set, i.e. the monthly data availability when computing the fore- and nowcast of a respective quarter.

This picture is confirmed when we look at plot of the predictive distributions of the models. From figure 1 it can clearly be seen that the uncertainty of the MF-BVAR density forecasts diminishes considerably, i.e. the 90% confidence bands become much narrower, the smaller the forecast horizon. For the third month of the current quarter (IS 6) realized euro area GDP growth lies mostly within the bands of the MF-BVAR predictive distribution, even for the Great Recession period. By contrast, the predictive distribution of the QF-BVAR shown in figure 2 is a lot smoother compared to that of the MF-BVAR. However, eventhough the QF-BVAR predictive distribution seems to display slightly less forecast uncertainty realized euro area GDP growth lies outside the 90% confidence bands considerably more often.

5.2 MF-BVAR real-time recession probabilites

The CEPR business cycle dating committee is responsible of officially identifying business cycle turning points for the euro area. However, this always happens with a substantial delay since the committee seems to prioritize correctness over timeliness. For example, the euro area business cycle peak that occurred in the first quarter of 2008 (in January as dated by the CEPR committee) was announced only on the 31st March 2009. Similarly, the trough in the second quarter of 2009 (in April according to the CEPR committee) as well as the peak in the third quarter of 2011 were announced with more than 12 months delay. Recently the CEPR business cycle dating committee has abandoned its practice to also announce the month of the business cycle turning point. While other authors have attempted to establish a monthly business cycle chronology for the euro area (see Anas et al. (2008) and Billio et al. (2012) for recent examples covering also the Great Recession of 2008/2009), their assessment of for example the Great Recession, which they date from September 2008 until July 2009, differs slightly from that of the CEPR committee.

7The probability integral transform (PIT) of the predictive distributions of both models (plotted in figures 4 and 5 in the appendix) indicate that the MF-BVAR forecast densities are slightly worse calibrated that those of the QF-BVAR. However, with only 44 quarters for the evaluation an assessment of the empirical validity of both models seems not very reliable.
Figure 1: MF-BVAR predictive density for euro area GDP growth
Figure 2: QF-BVAR predictive density for euro area GDP growth
As outlined in section 4 the real-time recession probabilities for the euro area are computed using the predictive distribution of the MF-BVAR and the QF-BVAR. Figure 3 plots the models’ recession probabilities computed for each month over the period ranging from 2007Q1 until 2013Q4 thus covering both recessions in the evaluation sample as dated by the CEPR business cycle dating committee\(^8\) (shaded areas). It can be seen very clearly that the MF-BVAR recession probabilities (solid line) are much timelier than those obtained with the QF-BVAR (dashed line).

For the Great Recession the MF-BVAR real-time recession probabilities start rising steadily from 5% in July 2008 to 96% in January 2009 and decrease thereafter until September 2009 where they reach a level of less than 10%. Given that the business cycle peak prior to the great slump in euro area GDP growth was announced by the CEPR only in March 2009 the MF-BVAR would have been able to deliver a considerably faster assessment of the prevailing economic situation. In this context the usefulness of within quarter high-frequency information becomes apparent again. The QF-BVAR signals the ongoing recession only from March 2009 onwards, while the respective real-time recession probabilities reach a level below 10% only in December 2010 signaling the end of the recession.

These results indicate that the MF-BAR can clearly compete with other approaches to de-

\(^8\)Given the most recent data vintage available in December 2014, euro area GDP growth was negative from 2008 Q2 until 2009 Q2 and from Q4 2011 until Q1 2013. While for the Great Recession and the beginning of the European debt crisis this coincides with the assessment of the CEPR committee, it might be too early to evaluate the model based recession probabilities at the current data edge.
tect euro area recessions in real time. For example, Bellgo and Ferrara (2009) report that with a recession detection criterion similar to the one used here, they would have been able to identify the Great Recession in November 2008. Hamilton (2011) uses the approach proposed by Camacho et al. (2014) with daily data and finds that the corresponding odds of a recession in the euro area would have started to increase sharply at the end of July 2008 and remained high until the end of April 2009. Thus while the MF-BVAR and the dynamic Markov-switching factor model perform equally well at signalling the onset of the Great Recession, the latter model seems to have captured the trough of the Great Recession better. However, in order to provide a reliable comparison of the performance of both models they would of course have to be estimated with the same data.

For the European debt crisis the MF-BVAR real-time recession probabilities have started to increase sharply in June 2011, while those of the QF-BVAR rose only from December 2011 onwards and to a much more gradual extent never exceeding 40%. Since September 2013 the MF-BVAR has been signaling very low recession odds. By contrast the QF-BVAR real-time recession probabilities are still elevated. Given that euro area GDP growth was negative from the fourth quarter of 2011 until the first quarter 2013, the MF-BVAR was clearly able to capture the onset of the recession much earlier. However, with respect to the end of the European debt crisis it might be too early to draw a conclusion. After all, the CEPR has not yet declared an end to the recession despite recent positive GDP growth rates.
6 Conclusion

In this paper I use the predictive distribution of the back-, now- and forecasts obtained with a mixed-frequency Bayesian VAR (MF-BVAR) to provide a real-time assessment of the probability of a recession in the euro area for the period from 2003 until 2013. Using a real-time dataset that consists of 135 monthly data vintages and covers 11 soft and hard monthly indicators as well as quarterly real GDP, I show that the MF-BVAR is able to capture current economic conditions in the euro area extremely well.

Not only does the MF-BVAR clearly outperform a BVAR estimated on quarterly data for all information sets and over all forecast horizons in terms of point forecasts. For both recession periods in the sample, the Great Recession of 2008/2009 and the European debt crisis 2011/2013, the MF-BVAR real-time recession probabilities soar right at the onset of the pending slump of GDP growth. By contrast the quarterly BVAR detects both recessions only with a substantial delay.

Typically non-linear discrete-choice or regime switching models are used to predict rare events such as recessions. However, in order to truly detect recessions in a timely manner these models would have to incorporate real-time data on what phase of the business cycle the economy was currently believed to be in into the estimation process. Since the official dating of business cycle turning points takes place only with a substantial delay models relying on discrete business cycle indicators seem not well suited for real-time applications.

By contrast the MF-BVAR is estimated on high-frequency macroeconomic indicators which allows for a very timely and precise signaling of the current stance of economic activity. Moreover, the predictive distribution of the model forecasts which is easily available as a by-product of the estimation procedure not only fully accounts for the uncertainty related to the model’s predictions but it can also be used to reliably spot rare events in real-time.

While this application focuses on euro area GDP growth and the detection of recessions in real time, the approach could easily be extended to other issues. For example, the predictive distributions of the MF-BVAR for inflation could be used to assess deflationary risks. Further research could also focus on developing clear criteria to translate the model based real-time recession probabilities into binary yes/no signals. However, the corresponding cut-off probabilities would need to take into account the loss function for the underlying problem at hand.

Another natural extension would be to specify the model in a frequency higher than monthly and incorporate a lot more financial variables thus allowing for an even timelier assessment of economic conditions. Beyond that, the MF-BVAR is in principle also suited to conduct conditional forecasting exercises in real-time or to analyze the impulse response functions of the model based real-time recession probabilities to economic shocks or different policy measures.
References


19
Appendix
Figure 4: MF-BVAR Probability Integral Transform
Figure 5: QF-BVAR Probability Integral Transform