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# Explaining the U-shape of the referral hiring pattern in a search model with heterogeneous workers

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## Abstract

This paper presents a search model with heterogeneous workers, social networks and endogenous search intensity. There are three job search channels available to the unemployed: costly formal applications and two costless informal channels - through family and professional networks. The gain from being employed is increasing in the productivity, so the lowest motivation for preparing formal applications is proved to be among the least productive worker types. We assume that professional contacts exhibit a strong degree of homophily, thus it is profitable for firms to direct their network search towards more productive incumbent employees. So the probability of a professional referral is increasing in the productivity of the worker, which mitigates the incentives to use the formal channel of search. Therefore, the model predicts that workers in the right (left) tail of the productivity distribution have the highest propensity of finding a job with a help of professional (family) contacts, whereas the formal channel of search is mostly utilized by workers in the middle range of the distribution. This explains the U-shaped referral hiring pattern in the model. Endogenous sorting of workers across channels also implies that professional (family) referrals are associated with wage premiums (penalties) compared to the formal channel of search.

**Keywords:** endogenous search intensity, family contacts, professional networks, U-shape, referral puzzle, wage premiums and penalties

**JEL Classification:** J23, J31, J38, J64

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# 1 Introduction

The purpose of this article is to explain a U-shape referral hiring pattern in a labour market matching model with heterogeneous workers, social networks and referrals. The U-shape relationship implies that referrals are mostly used by workers in the tails of the skill distribution, whereas all other workers in the middle are more likely to use a formal channel of job search. Brown et al. (2012) is the most recent study providing empirical evidence for this U-shape relationship in the United States. In particular, this study shows that referrals have a significantly greater impact on the overall probability of offer receipt for positions with lower education requirements. This is true for high school diplomas and associate's degrees. At the same time they report that referrals have a significantly larger positive impact on the probability of being interviewed for positions with a graduate rather than college degree requirement. This finding is also supported for the Netherlands in an earlier paper by Boxman et al. (1991), which was the first study to make this observation. To shed some light on this issue from a theoretical perspective we suggest to distinguish between the two types of social contacts – family and professional ties – and allow for the endogenous search intensity of workers and firms.

The ingredients of the model are as follows. Firms are homogeneous at the stage of a vacancy, but workers differ in their productivity which we also interpret as skill heterogeneity. There are two types of social contacts. Family contacts are exogenous in the model and serve as a residual method of search. In addition, every worker has a fixed number of professional contacts<sup>1</sup>. Ioannides and Datcher Loury (2004) report that acquired social contacts develop along dimensions such as race, ethnicity, religious affiliation and education. Therefore, in our model we assume a strong degree of network homophily along the productivity or the skill dimension. Thus, the job-finding rate through the network of professional contacts is skill-specific. In this setup, we distinguish between the three job search channels: formal applications to posted vacancies and two informal channels - through family and professional networks. Both informal channels of search are costless for workers, but preparing a formal application is associated with a positive effort cost. Moreover, through the endogenous group-specific advertising intensity firms can direct their network search towards particular groups of incumbent employees. This contrasts with the formal search channel, which is random and undirected.

There are two key predictions of the model which can be described in the following way:

- The model exhibits a strong U-shape referral hiring pattern: workers in the right (left) tail of the productivity distribution have the highest propensity of finding a job with a help of professional (family) contacts, whereas the formal channel of search is mostly utilized by workers in the middle range of the distribution;
- When the two types of social contacts are separated, family contacts are associated with wage penalties, whereas referrals from professional contacts are associated with wage premiums. The average effect of referrals on wages is ambiguous and depends on the relative proportions of high and low productivity workers in the population.

To the best of our knowledge there are no other studies that can generate these two predictions in a unified theoretical framework. First, we explain the mechanism which is generating the

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<sup>1</sup>The importance of relatives for job search is reported by Corak and Piraino (2011) and Kramarz and Skans (2014). For the role of former co-workers see Cingano and Rosolia (2012) and Glitz (2013).

U-shape. Every wage is an outcome of bargaining between the firm and the worker. Therefore, wages are increasing in the productivity, which is observable to the firm. With respect to the formal search channel, unemployed workers trade off the cost and the gain of effort, so the optimal search intensity is increasing in the productivity of the worker. Thus the least productive group of workers has a lowest job-finding rate associated with a formal channel. In addition, the probability of getting a job via professional referrals is also low for these workers due to a larger share of unemployed contacts in their networks. This latter finding is explained by the fact that the equilibrium unemployment rate is decreasing with a productivity of a worker. Low types are then relying on their family to find a job, which is a method of last resort in the model.

Another implication of bargaining is that firms' profits are increasing in the productivity of the hired applicant. As the network of professional contacts exhibits a strong degree of skill homophily, firms correctly anticipate a high productivity applicant if they approach an incumbent employee of the same type. Such a behavior of firms is based on the belief that people usually refer workers who are similar to themselves (Galenianos, 2014; Saloner, 1985; Montgomery, 1991). Since the advertising effort of firms is group-specific and endogenous, it is optimal for them to direct their search at incumbent workers of higher types. For unemployed workers this means that the job-finding rate through the network of professional contacts is increasing in the worker's type. So the formal channel of search is becoming less relevant for the more productive groups of applicants and is mostly utilized by workers in the middle range of the distribution. This explains the U-shape of the referral hiring pattern in the model. Specifically, in a benchmark calibration of the model, the fraction of workers relying on referrals falls down from 100% to 38% in the middle of the distribution and rises again to 67% for the most productive workers.

Next we analyze the effect of referrals on wages. As low productivity types are more likely to rely on family contacts, the equilibrium wage distribution of workers who used this channel is first order stochastically dominated by the distribution of workers who used a formal method. So the model predicts wage penalties associated with family contacts. In the benchmark calibration of the model the penalty is equal to 6%. High productivity workers are more likely to rely on professional contacts. Thus the equilibrium earnings distribution corresponding to this channel first order stochastically dominates the distribution of workers who used a formal method. Hence the network of professional contacts is associated with wage premiums. In the benchmark calibration of the model the premium is equal to 10%. Intuitively, wage penalties/premiums arise due to the self-selection of workers into a specific method of search and are robust to different shapes of the skill/productivity distribution. This is different for the average effect of referrals on wages. To illustrate this result we perform comparative statics analysis with respect to the shift parameter of the skill distribution. Then the average wage effect of referrals is negative (positive) in a labour market with a large fraction of low (high) types.

The second prediction of our model is empirically supported by Capellari and Tatsiramos (2013) for the United Kingdom and Meliciani and Radicchia (2011) for Italy. Specifically, Capellari and Tatsiramos (2013) report that high skilled workers with a better network quality of non-relatives experience wage premiums in the British labour market. In contrast, low skilled workers with a better network quality of relatives are more likely to experience a wage penalty associated with a referral. Likewise, empirical evidence presented in Meliciani and Radicchia

(2011) suggests that workers entering the labour market via professional contacts enjoy a wage bonus, whereas those recruited via referrals from family and close friends receive on average lower wages. Similar results are also reported by Sylos Labini (2004) for Italy and Antoninis (2006) for Egypt. Moreover, all these studies emphasize the importance of controlling for observable (and unobservable) worker characteristics when estimating the effect of a respective search channel, which is an implicit indicator of worker selection. For example, before controlling for worker and job characteristics, Antoninis (2006) reports that workers hired through a recommendation of a friend or a relative earn 21.6% less than an average worker. In contrast, recommendations from a former colleague are associated with a wage premium of 61%. However, both of these effects are strongly reduced in the magnitude once the skill requirement of the job is controlled for.

More empirical evidence in favour of our model is provided by Kramarz and Skans (2014) for Sweden and Kuzubas and Szabo (2014) for Indonesia. For example, the former study finds that parental networks matter more in the job search process for low educated youths even though there is a wage penalty in the first years of employment. Moreover, Kuzubas and Szabo (2014) report that in their sample low educated workers are more likely to find a job through family and close friends (52%) compared to college graduates (34%). In addition, Meliciani and Radicchia (2011) write that "people entering the labor market via relatives and friends contacts have lower levels of education, no specific competencies or training than the average and seem to be generally concentrated into lower occupational groups" (p.521). This is a direct empirical support for the selection mechanism described in our model.

Finally, we consider the overall effect of referrals on wages without separating into family and professional contacts. Our model predicts that the average effect is positive if the proportion of high productivity workers in the population is relatively high, otherwise it is negative. This finding may serve as an explanation of the mixed empirical evidence on the average wage effect of referrals. For example, Staiger (1990), Simon and Warner (1992) and Granovetter (1995) report that referrals are associated with wage premiums in the United States. The hypothesis of wage premiums is also supported by Margolis and Simonnet (2003) and Goos and Salomons (2007) for France and the United Kingdom. In contrast, Bentolila, Michelacci and Suarez (2010) report wage penalties in the United States and the European Union. This result is supported by Delattre and Sabatier (2007), Pistaferri (1999) as well as Addison and Portugal (2002) for France, Italy and Portugal respectively. This contradicting empirical evidence, which can be well described as a "referral puzzle", is summarized in Pelizzari (2010) who writes that "... in the European Union premiums and penalties to finding jobs through personal contacts are equally frequent and are of about the same size". When pooling data for 14 European countries, Pelizzari (2010) shows that referrals are associated with a wage penalty of 17.4% before controlling for worker characteristics. However, this effect is reduced to only 4.4% after controlling for observable worker traits and down to 2% when controlling for unobservables. Even though the penalty of 2% remains significant on average for the EU, it becomes insignificant for a number of countries. Again, this is an indicator for the strong selection of workers on observable characteristics.

Apart from empirics, our study is also related to a number of theoretical papers analyzing the role of social networks. Early economic studies on social contacts include Simon and Warner (1992), Montgomery (1991, 1992, 1994) and Mortensen and Vishwanath (1994). Both Simon and Warner (1992) and Montgomery (1991) emphasize that referrals reveal the quality of the match

to the employer and therefore should have a positive effect on wages. This result is similar to the positive wage effect of professional referrals in our model, however the possibility to rely on family contacts is not included in the early studies. Recent theoretical studies generating wage premiums associated with referrals include Kugler (2003), Ioannides and Soetevent (2006) and Galenianos (2014). Specifically, Kugler (2003) finds that the benefit of using referrals for firms is that they reduce the cost of monitoring, because workers can exert peer pressure on coworkers. As a result, firms relying on referrals find it cheaper to elicit effort by paying efficiency wages than firms using formal hiring methods. Ioannides and Soetevent (2006) support this result by showing that better connected workers experience lower unemployment rates and receive higher wages. This should be compared with our finding that more productive workers experience lower unemployment rates because they have a lower proportion of unemployed contacts in their network. Note that this result is different from Ioannides and Soetevent (2006) as all workers have the same fixed number of network contacts in our model. So it is the endogenous proportion of employed contacts that differs between the agents, whereas it is the total number of contacts which is different between workers in their study.

The group of papers that can generate wage penalties in a theoretical framework includes Bentolila et al. (2010) as well as Ponzio and Scoppa (2010). Ponzio and Scoppa (2010) argue that recruiters may favor low ability family ties over more talented applicants. This is the idea of favoritism in the recruiting process. Bentolila et al. (2010) find that social contacts can generate a mismatch between occupational choices and productive advantages of workers. This is particularly true for workers who failed to find a job in their occupation and followed a recommendation of a close family member. Horvath (2014) extends the mismatch result of Bentolila et al. (2010). As the probability that ties connect similar agents (homophily) increases, the mismatch level decreases in his model. Moreover, if this probability is sufficiently high, networks provide good matches at higher rate upon arrival than the formal market. Therefore, referrals can generate wage premiums if the homophily level in the society is high. Otherwise, if social contacts are strongly heterogeneous, the effect of referrals on wages may be negative due to a stronger mismatch between the skill of the worker and the type of the job.

The first idea that positive and negative effects of referrals are simultaneously valid for different types of contacts and can account for differences in the wage effects is due to Sylos Labini (2004) and Datcher Loury (2006) followed by Kuzubas and Szabo (2014). In a theoretical model confirmed by empirical evidence Sylos Labini (2004) shows that workers who find their jobs through professional referrals earn on average higher wages, whereas workers who are recommended by their relatives earn lower wages. Similarly Kuzubas and Szabo (2014) develop a theoretical model of a frictional labour market for Indonesia with two channels of search: inner networks (families) and outer networks defined as the ethnic language group. Using the inner network of relatives is costless for workers, which is also the same in our model, however there is a fixed cost of using the outer network. Thus it is mostly high skilled workers who pay this cost and use a large outer network. These results are similar in our model if the network of professional contacts is merged with potential employers into one large outer network. Nevertheless, our model is more specific as the formal channel is separated from professional contacts, which explains the U-shape referral pattern observed in developed economies.

Other theoretical papers which can explain wage premiums/penalties depending on the pa-



rameters of the labour market are Tumen (2013) and Zaharieva (2015). Tumen (2013) considers a population of workers heterogeneous with respect to the cost of maintaining connections. In his model well integrated workers with low costs have higher reservation wages and are able to bargain higher wages. Conversely, workers with higher costs accept wages below the market level. Zaharieva (2015) investigates the role of referrals in a matching model with on-the-job search. On the one hand, in her model better connected workers bargain higher wages for a given level of job-related productivity. This is the positive effect of outside opportunities on wages. On the other hand, employees rationally accept job offers from more productive employers and forward other offers to the unemployed contacts. Therefore, job offers transmitted through social contacts are biased in the direction of less productive employers. This selection mechanism can generate a negative effect of referrals on wages. To sum up, both papers by Tumen (2013) and Zaharieva (2015) can generate wage penalties or premiums in wages associated with social contacts, however, in each paper the mechanism is different from the present study.

The paper is organized as follows. Section 2 explains notation and the general economic environment. In section 3 we investigate the decisions of workers and firms and explain their choice of the search intensity. Section 4 illustrates our theoretical results by means of a numerical example, while section 5 includes a number of robustness checks for the model. Section 6 concludes the paper.

## 2 Labour market modeling framework

The labour market is characterized by the following properties. There is a continuum of infinitely lived risk neutral workers and firms discounting future at a common discount rate  $r$ . Firms are homogeneous, while workers have heterogeneous skills (schooling levels). Worker's skills are reflected in his/her productivity  $y_i$ ,  $i = 1..p$  which follows a discrete probability distribution  $f_i$ , such that  $\sum_{i=1}^p f_i = 1$ . Workers are perfectly informed about their productivity  $y_i$ , while firms with open vacancies are aware of the productivity distribution  $f_i$ ,  $\forall i = 1..p$ . At the same time firms learn the exact productivity of the worker upon the match, so there is no asymmetric information in the model. The highest productivity  $y_p$  is set to 1, while the lowest productivity  $y_1$  is equal to the unemployment benefit  $b$ .

Every worker can be either employed and producing output  $y_i$  or unemployed and searching for a job. Let  $u_i$  denote the mass of unemployed workers with productivity  $y_i$  and  $e_i$  – the mass of corresponding employees, so that  $e_i + u_i = f_i$ , since the total measure of workers is normalized to 1. There are three search channels in the labour market. First, unemployed workers can find a job by sending regular applications to open vacancies, this is the formal channel of job search with an endogenous job-finding rate  $\phi(s)$ . Variable  $s$  is the individual search effort of workers and may differ across agents belonging to different productivity groups, i.e.  $s_i$ . The formal channel of search is costly in terms of effort, since it requires preparing and sending job applications. However, a more intensive job search is associated with a higher probability of finding employment. Let  $C(s) = s^2/c$  denote the effort cost function, which is identical for all workers in the market.

Further, let all workers have an equal number of professional contacts  $n > 0$ . Employed workers provide referrals and transmit vacancy information to the unemployed members of

their network, this is the second channel of job search. To simplify the model we assume that professional contacts are only formed among workers with the same productivity level  $y_i$ . Therefore, the job-finding rate through the network of professional contacts is skill-specific and is denoted by  $\lambda_i$ . Empirical support for this assumption comes from the observation of strong homophily in social networks reported in Rivera, Soderstrom and Uzzi (2010). Finally,  $\lambda_0$  is a constant probability of hearing about a job from family members which is a third search channel in the model. In section 6 we endogenize  $\lambda_0$  as a form of robustness check for the model, however, it is constant in the rest of the paper.

Job referrals from professional contacts and family are the informal methods of search and are costless for workers. While there is strong agreement in the literature that getting help from family members is a costless method of search, it is less obvious for professional contacts. One explanation of this assumption is that in this paper we only focus on a group of colleagues and former coworkers of the agent which can be seen as a subgroup of all professional contacts. Empirical studies show that former coworkers are an important source of job-related information for the unemployed. For example, Cingano and Rosolia (2012) find that the median number of former coworkers in Italy is equal to 32. This number is higher in Germany and is equal to 43 according to Glitz (2013). Note that this type of professional contacts is created by means of daily job-related communication between the colleagues. Certainly, there are some costs of communication, however, even these costs are reduced due to the recent IT development (such as Facebook, LinkedIn and Xing) which makes it easier for workers to stay in touch with former colleagues. Therefore, in the model we assume that the cost of keeping professional contacts is negligibly small compared to the formal search channel and normalize it to zero. Yet another advantage of treating former colleagues as professional contacts is a strong degree of skill homophily between coworkers.

Firms are free to enter the labour market by opening new vacancies. Open vacancies are associated with a flow cost  $z$  on the side of the firm which can be understood as financial expenses for making vacancy information visible to the applicants. This includes posting vacancies in the newspapers, registering on the recruitment websites and participating in the job fairs. It may also include the cost of capital depreciation. Formal matching between unemployed workers and vacancies is random and discussed below. To model the process of network matching we extend the approach of Cahuc and Fontaine (2009) and assume that firms make a random draw from the pool of incumbent employees with an advertising intensity  $a$  per unit time. However, in our model the advertising intensity  $a$  is endogenous and can be specific to a given group of employees, i.e.  $a_i$ . Intuitively,  $a_i$  is an effort level with which the manager of an open vacancy is addressing an incumbent employee of type  $i$  to refer one of his/her contacts. This extension allows firms to direct their search more intensively towards the more productive group of workers. The advertising search intensity  $a$  is costly for firms with a cost function  $K(a) = a^2/k$ . Note that the advertising intensity  $a_i$  is chosen after the match with an employee and so the cost  $K(a_i)$  is unrelated to the cost  $K(a_j)$  for  $i \neq j$ . If the job position is filled with a worker, the firm obtains a flow profit  $y_i - w_i$ , where the wage  $w_i$  is bargained between the firm and the worker upon hiring. We use the Nash bargaining rule to determine wages. Every filled job can be destroyed for exogenous reasons at rate  $\delta$ .

Let  $m(x, v)$  denote a matching function between workers and firms, where  $v$  is the number of



open vacancies and  $x$  is the number of searching workers in efficiency units (either unemployed or employed, transmitting job offers to their unemployed contacts). Following the approach of Gautier et al. (2010) we assume that the matching technology is quadratic, that is  $m(x, v) = xv$ . This approach has been frequently used in the search literature, for example, Teulings and Gautier (2004) provide a number of explanations why this technology may be the most adequate assumption in a model with worker heterogeneity. The main reason is that this technology avoids congestion externalities between different worker types and jobs.

Consider matching between unemployed workers and open vacancies. The total number of searching unemployed workers weighted by their search intensity is given by  $x = \sum s_i u_i$ , so the number of contacts created through the formal method of search is  $v \sum s_i u_i$ . However, only proportion  $s_i u_i / \sum s_i u_i$  of these contacts are the matches between open vacancies and unemployed workers of type  $y_i$ . Therefore the number of matches between open vacancies and unemployed workers of type  $y_i$  is given by:

$$v \sum s_i u_i \cdot \frac{s_i u_i}{\sum s_i u_i} = v s_i u_i$$

This means that the job-finding rate through the formal channel of search is equal to  $\phi_i \equiv \phi(s_i) = v s_i u_i / u_i = v s_i$  and is increasing in the total number of vacancies  $v$  and the individual search intensity of unemployed workers  $s_i$ . In addition, from the perspective of firms, the probability of filling a job through the formal channel with a worker of type  $y_i$  is  $\phi_i u_i / v = s_i u_i$ .

Next consider matching between employed workers and open vacancies. The total number of employees in efficiency units is given by  $x = \sum a_i e_i$ , so the number of contacts between vacancies and employees with productivity  $y_i$  is equal to  $v \sum a_i e_i$ . However, only a fraction  $a_i e_i / \sum a_i e_i$  of these contacts are the matches between open vacancies and employees of type  $y_i$ . Every contacted employed worker transmits vacancy information to exactly one randomly chosen unemployed social contact out of a pool of  $n$  contacts. Here we assume that job information is only transmitted to the direct social links, so the job offer is lost if all  $n$  contacts are employed. The probability of being employed for an arbitrary worker of type  $y_i$  is equal to  $1 - \mu_i$ , where  $\mu_i \equiv u_i / f_i$  is the unemployment rate in a group of workers with productivity  $y_i$ . So the probability that all  $n$  contacts of the employee are also employed is equal to  $(1 - \mu_i)^n$ . This means that the number of matches between vacancies and unemployed workers of type  $y_i$  through the network of contacts is given by:

$$v \sum a_i e_i \cdot \frac{a_i e_i}{\sum a_i e_i} \cdot [1 - (1 - \mu_i)^n] = v a_i e_i [1 - (1 - \mu_i)^n]$$

where expression in the square bracket is the probability of having at least one unemployed contact out of  $n$ . The individual job-finding rate through the first informal search channel (professional contacts) is then equal to:

$$\lambda_i = v a_i \frac{e_i}{u_i} [1 - (1 - \mu_i)^n] = v a_i \frac{1 - \mu_i}{\mu_i} [1 - (1 - \mu_i)^n]$$

Note that  $\lambda_i$  is increasing in the number of vacancies  $v$  and the number of social contacts  $n$ . Moreover, a more intensive search by firms directed at workers of type  $y_i$ , that is a higher  $a_i$ , is raising the probability of finding a job for an unemployed worker of this type. From

the perspective of firms, the flow probability of filling a job with a professional contact of an incumbent employee of type  $y_i$  is equal to  $\lambda_i u_i / v = a_i e_i [1 - (1 - \mu_i)^n]$ .

### 3 Analysis of the model

#### 3.1 Workers and their choice of search effort

Let  $U_i$  ( $W_i$ ) denote the present value of being unemployed (employed) for the worker with productivity  $y_i, i = 1..p$ . The asset value of unemployed workers of type  $y_i$  is given by:

$$rU_i = b + (\lambda_0 + \lambda_i)(W_i - U_i) + v \max_s [s(W_i - U_i) - \frac{1}{c}s^2] \quad (1)$$

and reflects simultaneous availability of the three job search channels discussed above. The rent from employment is independent of the search channel and is denoted by  $R_i \equiv (W_i - U_i)$ . Workers choose costly effort  $s_i$  to maximize the present value of unemployment  $U_i$ , therefore the optimal level of search effort  $s_i$  obtains at the point where the marginal gain  $(W_i - U_i)$  is equal to the marginal cost  $C'(s)$ :

$$s_i = 0.5c(W_i - U_i) = 0.5cR_i$$

Therefore, the asset value of unemployment can be rewritten as:

$$rU_i = b + (\lambda_0 + \lambda_i)R_i + 0.25cvR_i^2 \quad (2)$$

Further, the asset value of employed workers of type  $y_i$  is given by:

$$rW_i = w_i - \delta(W_i - U_i) \quad (3)$$

and so the worker rent from employment is equal to the discounted net present value of earnings:  $R_i = (w_i - rU_i)/(r + \delta)$ . Combining this and equation (2) allows us to derive the optimal search effort  $s_i \equiv s(\lambda_i, w_i)$ . These results are summarized in Lemma 1:

**Lemma 1:** *Consider workers with productivity  $y_i$ . The optimal job-finding rate  $\phi(s_i) = vs_i$  through the formal channel of search is given by:*

$$vs_i = \sqrt{(r + \delta + \lambda_0 + \lambda_i)^2 + (w_i - b)cv} - (r + \delta + \lambda_0 + \lambda_i) \quad (4)$$

*The optimal search intensity  $s(\lambda_i, w_i)$  is increasing in the wage  $w_i$  but decreasing in  $\lambda_i$ , which is a job-finding rate through professional contacts. The optimal search intensity  $s(\lambda_i, w_i)$  is also decreasing in the number of vacancies  $v$ .*

**Proof:** Appendix I.

Lemma 1 shows that a higher wage  $w_i$  would motivate workers to exert more effort when applying for jobs. On the contrary, a higher job-finding rate through professional contacts  $\lambda_i$  improves the outside opportunities of workers, so the total rent from a job  $R_i$  is reduced. A lower rent then has a disincentive effect on the intensity of job search. In addition, there is a similar disincentive effect from a higher number of vacancies  $v$ , thus vacancies and effort are

substitutes in our setting.

### 3.2 Firms and the wage determination

From the perspective of firms, let  $J_i$  be the asset value of a job, filled with a worker of type  $y_i$ , and  $V$  be the present value of the open vacancy. We will come back to the determination of  $V$  in section 3.5. Once matched firms observe the productivity of the applicant, so  $J_i$  is given by:

$$rJ_i = y_i - w_i - \delta(J_i - V) \quad (5)$$

The equilibrium wages are determined by means of Nash bargaining with a disagreement-while-bargaining state  $U_i^D$  for type  $y_i$  worker and with  $\alpha \in (0, 1)$  being the workers' bargaining power, for example, as in Gautier (2002) and Hall and Milgrom (2008). This approach is close to the bargaining model with a risk of a negotiation breakdown by Binmore et al. (1986) and allows to simplify the model, while not influencing qualitatively the results. An unemployed worker gets a present value  $U_i^D$  during the disagreement time, while the employer obtains a present value  $V_i^D$ . We assume that during the time of negotiation neither the worker nor the firm continue searching for other partners. This is intuitive since there are no reasons for agents to exert costly search effort when they are already in the process of bargaining with a prospective partner. This means that neither the worker nor the firm pays the search cost during the period of negotiation, however, the worker still receives the unemployment benefit from the state. Thus,  $U_i^D$  and  $V_i^D$  can be written as:

$$rU_i^D = b + \delta(U_i - U_i^D) \quad rV_i^D = \delta(V - V_i^D)$$

These equations imply that vacancies have the same probability  $\delta$  of being destroyed during the bargaining as do existing jobs. Moreover, if the bargaining process breaks down for an exogenous reason, the worker becomes unemployed with a present value  $U_i$  and the position remains vacant with a present value  $V$ . The solution is the wage  $w_i$  maximizing the Nash objective function  $(W_i - U_i^D)^\alpha (J_i - V_i^D)^{1-\alpha}$  which can be written as:

$$\max_{w_i} \left( \frac{w_i + \delta U_i}{r + \delta} - \frac{b + \delta U_i}{r + \delta} \right)^\alpha \left( \frac{y_i - w_i + \delta V}{r + \delta} - \frac{\delta V}{r + \delta} \right)^{1-\alpha} \Rightarrow w_i = \alpha y_i + (1 - \alpha)b$$

This maximization problem shows that the wage is a weighted average between the unemployment benefit  $b$  and the productivity  $y_i$ . Therefore, wages are heterogeneous in the economy and resemble the productivity distribution in the population of workers. Let  $g_i$  denote the equilibrium distribution of wages, such that  $\sum g_i = 1$ . It is then given by:

$$g_i = \frac{e_i}{e} = \frac{f_i(1 - \mu_i)}{\sum f_i(1 - \mu_i)} = \frac{f_i(1 - \mu_i)}{1 - \sum f_i \mu_i}$$

where  $e = 1 - \sum f_i \mu_i$  is the equilibrium employment rate in the economy. Intuitively, if the employment rate of some worker group is smaller than the average, i.e.  $(1 - \mu_i) < (1 - \sum f_i \mu_i)$ , then this group is underrepresented in the earnings distribution compared to the initial productivity density  $f_i$ . The opposite holds when the employment rate of some worker group is larger than the average, so this group is overrepresented.

### 3.3 Type-specific unemployment rates

Consider workers with productivity  $y_i$ . The unemployment rate  $\mu_i = u_i/f_i$  can be found from the steady-state equation for unemployed workers. It can be written as:

$$0 = \dot{u}_i = \delta(f_i - u_i) - (\lambda_0 + \lambda_i + s_i v)u_i \quad (6)$$

and reflects the fact that the inflow into and the outflow out of unemployment are equalized in the steady state. Thus, the equilibrium unemployment rate  $\mu_i$  is equal to:

$$\mu_i = \frac{\delta}{\lambda_0 + s_i v + \lambda_i + \delta} = \frac{\delta}{\sqrt{(r + \delta + \lambda_0 + \lambda_i)^2 + \alpha(y_i - b)cv} - r} \Rightarrow \mu_i = \mu(\lambda_i, y_i) \quad (7)$$

Hence the equilibrium unemployment rate can be expressed as a function of the job-finding rate  $\lambda_i$  and the productivity  $y_i$ . Next, consider a partial relationship between  $\mu_i$  and  $\lambda_i$  for a fixed productivity  $y_i$ . A higher probability of finding a job through professional contacts (that is a higher  $\lambda_i$ ), has an indirect disincentive effect on the search intensity  $s(\lambda_i, y_i)$ . Consequently, a lower level of search effort through the formal channel raises the equilibrium unemployment rate  $\mu_i$ . This is an indirect effect which is operating through the outside opportunities of workers. At the same time a higher  $\lambda_i$  reduces the unemployment rate  $\mu_i$ . This is a direct effect since more unemployed workers find jobs by means of referrals. Equation (7) shows that the direct effect is dominating and describes a negative relationship between the unemployment rate  $\mu_i$  and the job-finding rate through professional contacts  $\lambda_i$ :

$$\frac{\partial \mu(\lambda_i, y_i)}{\partial \lambda_i} < 0 \quad \lim_{\lambda_i \rightarrow 0} \mu_i = \frac{\delta}{\sqrt{(r + \delta + \lambda_0)^2 + \alpha(y_i - b)cv} - r} \equiv \bar{\mu}_i > 0 \quad \lim_{\lambda_i \rightarrow \infty} \mu_i = 0$$

This is illustrated in figure 1, where  $\bar{\mu}_i$  denotes the upper limit of the unemployment rate  $\mu_i$  for a given fixed level of  $y_i$ . The corresponding curve is denoted by (UC).

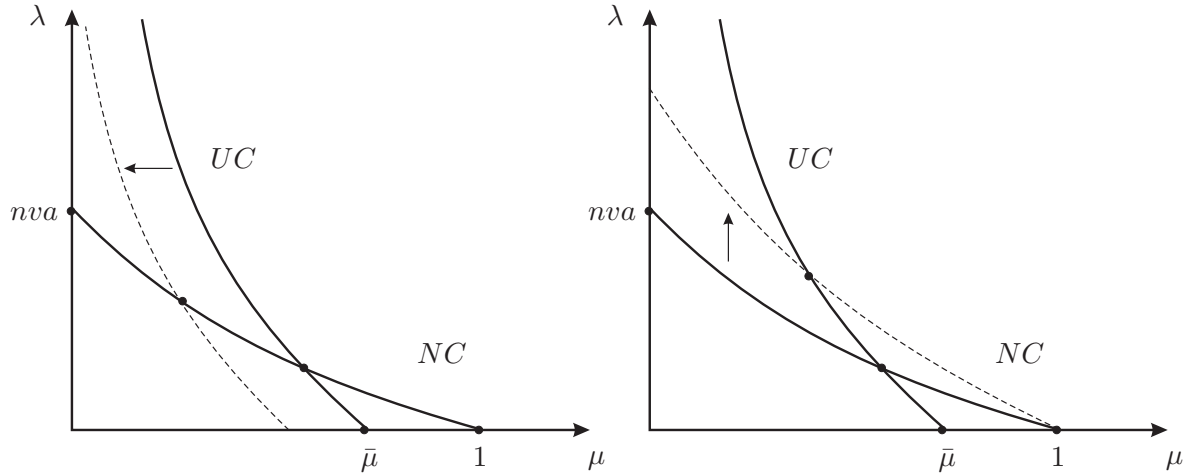


Figure 1: Intersection between  $\mu(\lambda, y)$  and  $\lambda(\mu, a)$  for a given advertising intensity  $a$  and a given productivity  $y$ . Left panel: changes in  $\lambda(\mu, a)$  and  $\mu(\lambda, y)$  given a positive shift in  $y$ . Right panel: changes in  $\lambda(\mu, a)$  and  $\mu(\lambda, y)$  given a positive shift in  $a$ .

Further, recall from section 2 that the job-finding rate by means of referrals  $\lambda_i$  depends on

the unemployment rate in the network  $\mu_i$ . In particular, it holds that:

$$\lambda_i = va_i \frac{1 - \mu_i}{\mu_i} [1 - (1 - \mu_i)^n] \quad \Rightarrow \quad \lambda_i = \lambda(\mu_i, a_i) \quad (8)$$

If more workers of a given type are employed (that is a lower  $\mu_i$ ) the possibilities for firms to communicate with this group of employees arise more frequently. And hence the contact rate between firms and unemployed workers of type  $y_i$  is increased. But on the other hand, a lower unemployment rate  $\mu_i$  implies a lower number of unemployed contacts in the network and therefore, a lower probability that the contacted employee will recommend someone for a job  $[1 - (1 - \mu_i)^n]$ . Lemma 2 shows that the indirect network effect is dominated by the direct effect of a higher contact rate between firms and unemployed workers and so equation (8) describes a negative relationship between variables  $\lambda_i$  and  $\mu_i$ . The corresponding curve is denoted by (NC).

**Lemma 2:** *For a given advertising intensity  $a_i$ , a lower unemployment rate  $\mu_i$  in a group of workers with productivity  $y_i$  implies a higher job-finding rate through the informal channel of search  $\lambda_i$ :*

$$\frac{\partial \lambda_i(\mu_i, a_i)}{\partial \mu_i} < 0 \quad \lim_{\mu_i \rightarrow 0} \lambda_i = nva_i \quad \lim_{\mu_i \rightarrow 1} \lambda_i = 0$$

**Proof:** Appendix II.

Based on the results from lemma 2, figure 1 shows that there is a unique intersection between the curves  $\mu(\lambda_i, y_i)$  and  $\lambda(\mu_i, a_i)$ . This implies that  $\mu_i$  is an implicit function of the productivity  $y_i$  and the advertising intensity  $a_i$ , formally:

$$\mu_i = \frac{\delta}{\sqrt{(r + \delta + \lambda_0 + \lambda(\mu_i, a_i))^2 + \alpha(y_i - b)cv - r}} \quad \Rightarrow \quad \mu_i = m(y_i, a_i)$$

To analyse the intuitive implications of this relationship consider workers with a higher productivity  $y_i$ . More productive workers expect to get a higher wage  $w_i$ , so the gain from finding a job is increasing in the productivity. This means that more able workers invest more effort in writing applications and preparing for a job interview. More intensive job search through the formal channel improves the job-finding rate  $vs(\lambda_i, w(y_i))$  and so the unemployment rate  $\mu(\lambda_i, y_i)$  is reduced for every value of  $\lambda_i$ . On figure 1 this is illustrated by the left-ward shift of the curve (UC) on the left panel. Since productivity does not enter directly into the job-finding rate through the network, there is no shift of the curve (NC). This means that the unemployment rate is unambiguously lower in more productive worker groups. Consequently a larger proportion of employees facilitates informal matching between open vacancies and unemployed workers and therefore the probability of finding a job by recommendation is increasing. These results are summarized in lemma 3:

**Lemma 3** *For a given advertising intensity  $a_i$ , the equilibrium unemployment rate  $\mu_i = m(y_i, a_i)$  is lower in more productive worker groups. Further, for every productivity group  $y_i$ , the equilibrium unemployment rate falls with a higher search effort by firms, formally:*

$$\frac{\partial m(y_i, a_i)}{\partial y_i} < 0 \quad \frac{\partial m(y_i, a_i)}{\partial a_i} < 0 \quad \lim_{a_i \rightarrow 0} m(y_i, a_i) = \lim_{\lambda_i \rightarrow 0} \mu(\lambda_i, y_i) = \bar{\mu}_i$$

**Proof:** Appendix III.

In addition, lemma 3 describes consequences of a higher search intensity by firms  $a_i$ . If firms exert more effort in contacting their employees, then the probability of finding a job by means of a referral is increased for every unemployment rate  $\mu_i$ . In figure 1 this is illustrated by the up-ward shift of the curve (NC) on the right panel. Since advertising intensity does not enter directly the unemployment equation, there is no shift of the curve (UC). This means that the job finding rate  $\lambda_i$  is unambiguously higher and the equilibrium unemployment rate is reduced.

### 3.4 Endogenous advertising rate for referral hiring

Let us now consider the optimal behavior of a firm with an open vacancy. Apart from formal applications the firm may also fill its vacancy through the informal channel of search. In particular, the firm should choose the optimal advertising intensity  $a_i$  for every worker type  $y_i$ . Intuitively, at rate  $a_i$  the firm is asking type- $y_i$  incumbent employees whether they can recommend a friend for the open vacancy. Similarly to the effort choice of the unemployed, there is a gain and a cost from advertising activity. The expected firm rent from contacting the incumbent employee of type  $y_i$  is equal to  $a(1 - (1 - \mu_i)^n)(J_i - V)$ , which is the probability that the job offer will be transmitted to the unemployed worker of this type times the present value of profits. This gives rise to the following maximization problem:

$$\max_a [a(1 - (1 - \mu_i)^n)(J_i - V) - \frac{1}{k}a^2]$$

The optimal  $a_i$  is, thus, given by:

$$a_i = 0.5k(1 - (1 - \mu_i)^n)(J_i - V) \quad \text{where} \quad J_i - V = \frac{(1 - \alpha)(y_i - b) - rV}{r + \delta} \quad (9)$$

This first order condition defines the level of advertising  $a_i$  as a function of  $\mu_i$  and  $y_i$ , that is  $a_i = a(\mu_i, y_i)$ . Therefore, for a given  $y_i$ , firms exert more advertising effort if they expect a higher proportion of unemployed workers in the network of the incumbent employee. In the following we consider the economy in the steady-state with a free-entry of firms, which means that  $V = 0$ . Figure 2 shows equilibrium for advertising effort and unemployment. Recall that  $\mu_i = m(y_i, a_i)$  slopes down in the space  $(\mu, a)$ : finding jobs becomes easier for unemployed workers if firms increase their advertising activities. Let this curve be denoted by (MA) (see figure 2). Equation (9) is the advertising curve and slopes up, let it be denoted by (AC). Group-specific equilibrium  $(\mu(y_i), a(y_i))$  is at the intersection of the two curves and it is unique.

Next compare the equilibrium vector of variables  $(\mu(y_i), a(y_i))$  across different productivity groups. On the one hand, more productive workers exert more effort in sending applications and preparing for the job interview, so their unemployment is lower for any advertising intensity  $a_i$ . On figure 2 this is illustrated by the inward shift of the curve (MA). On the other hand, for a given  $\mu_i$ , firms expect to earn higher profits from more productive network applicants, and so their advertising effort is higher when the firm is communicating with a more productive incumbent employee. This implies an upward shift of the advertising curve (AC) since firms' effort is increasing for every level of the unemployment rate  $\mu_i$ . Considering both changes as a combination shows that the equilibrium unemployment rate is lower in more productive worker



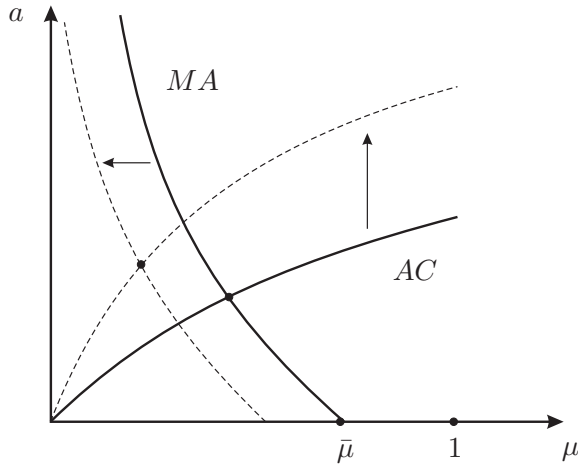


Figure 2: Determination of the type-specific unemployment rate  $\mu(a, y)$  with the endogenous advertising intensity of firms  $a(\mu, y)$ . Arrows indicate higher values of  $y$ .

groups. This result is described in proposition 1:

**Proposition 1:** (i) The group-specific equilibrium unemployment rate  $\mu_i$  is decreasing in the productivity  $y_i$ . (ii) The job-finding rate  $\lambda_i$  and the network advertising intensity  $a_i$  are both increasing in  $y_i$  if the elasticity of referral probability  $\rho(y_i) \equiv [1 - (1 - \mu(y_i))^n]$  with respect to the net productivity  $y_i - b$  is less than 1, formally:

$$-\frac{\partial \rho(y_i)}{\partial (y_i - b)} \cdot \frac{(y_i - b)}{\rho(y_i)} = -\frac{n(1 - \mu(y_i))^{n-1}}{1 - (1 - \mu(y_i))^n} \cdot \frac{\partial \mu(y_i)}{\partial y_i} \cdot (y_i - b) < 1 \quad (10)$$

**Proof:** Appendix IV.

Proposition 1 shows that there are two counteracting effects of  $y_i$  on the network advertising intensity  $a(y_i)$ . On the one hand, firms anticipate higher profits from more productive network applicants and direct their search towards worker groups with a higher  $y_i$ . But on the other hand, the equilibrium unemployment  $\mu(y_i)$  is decreasing in  $y_i$  which means that the average proportion of unemployed workers in the network is lower in less productive worker groups. From the perspective of firms this means a lower probability of referral hiring. Condition (10) then implies that the first direct effect of higher profits is dominating if the equilibrium unemployment rate is sufficiently inelastic.

Finally, the job-finding rate through professional contacts  $\lambda(y_i)$  can be now rewritten as:

$$\lambda(y_i) = \frac{a(y_i)v(1 - \mu(y_i))}{\mu(y_i)} \rho(y_i) = \frac{v(1 - \mu(y_i))}{\mu(y_i)} 0.5k\rho^2(y_i)J(y_i) \quad (11)$$

Recall that  $\lambda(y_i) = \lambda(\mu(y_i), a(y_i))$ . If the elasticity condition in proposition 1 is satisfied than more productive employees are more intensively approached by firms. So the probability of finding a job through professional contacts is increasing in the productivity. In addition, since the unemployment rate is decreasing in  $y_i$ , the probability that a randomly chosen employee is of type  $y_i$  is increasing in the productivity. Both of these factors imply that the probability of finding a job by recommendation is an increasing function of  $y_i$ , that is  $\partial \lambda(y_i) / \partial y_i > 0$  if  $\partial a(y_i) / \partial y_i > 0$ .

The primary purpose of this paper is to analyze which groups of workers are more likely to use family and professional contacts in the process of job search. To address this question we define the following new variables  $d_0(y_i)$  and  $d(y_i)$ . The former variable is an average proportion of workers with productivity  $y_i$  using family contacts in order to find a job. In contrast, the latter variable is an average proportion of workers using professional contacts. This means:

$$d_0(y_i) = \frac{\lambda_0}{\lambda_0 + \phi(y_i) + \lambda(y_i)} \quad d(y_i) = \frac{\lambda(y_i)}{\lambda_0 + \phi(y_i) + \lambda(y_i)}$$

The last possibility to find a job in the model is the formal channel of job search, so the average proportion of type  $y_i$  workers finding jobs by means of this channel can be found as  $1 - d_0(y_i) - d(y_i)$ . Which worker group is relying most on family contacts? To answer this question observe that:

$$d_0(y_i) = \frac{\lambda_0}{\lambda_0 + \phi(y_i) + \lambda(y_i)} = \frac{\lambda_0}{\sqrt{(r + \delta + \lambda_0 + \lambda(y_i))^2 + \alpha(y_i - b)cv} - r - \delta}$$

Therefore,  $d_0(y_i)$  is decreasing in  $y_i$  if the elasticity condition (10) is satisfied. On the one hand, more productive workers anticipate a larger present value of wages and exert more effort when preparing applications and, on the other hand, firms are searching more intensively for more productive applicants. Both of these factors imply that the proportion of workers finding jobs through family contacts is a decreasing function of  $y_i$ . In addition, observe that  $d_0(b) = 1$  (since  $\lambda(b) = 0$  and  $\phi(b) = 0$ ) which means that least productive workers rely exclusively on family contacts. Finally, it is not possible to predict in general whether variable  $d(y_i)$  is increasing or decreasing in  $y_i$ . We investigate this relationship numerically in section 4.

### 3.5 Wage distribution and the free-entry condition

The second purpose of our paper is to analyze the effect of referrals on wages. As mentioned in the earlier part of our paper the equilibrium wage distribution is given by  $g_i = e_i/e$  and shows the relative proportion of  $y_i$ -workers in the pool of employees. So the average wage in the economy can be found as  $\bar{w} = \sum g_i w_i$ . Next we find average wages conditional on the specific channel of search. To do so let  $\bar{w}^o$ ,  $\bar{w}^s$  and  $\bar{w}^n$  be the corresponding average wages conditional on the search method being the family, the formal application or the network of professional contacts. In addition, let  $h_i^o$ ,  $h_i^s$  and  $h_i^n$ ,  $\forall i = 1..p$ , be the respective wage distributions so that  $\sum h_i^o = 1$ ,  $\sum h_i^s = 1$  and  $\sum h_i^n = 1$ . For example,  $h_i^s$  is the equilibrium distribution of wages among employed workers who found a job by using the formal method of search. Each of these three distributions can be obtained as:

$$h_i^o = \frac{g_i d_0(y_i)}{\sum g_i d_0(y_i)} \quad h_i^s = \frac{g_i (1 - d_0(y_i) - d(y_i))}{1 - \sum g_i (d_0(y_i) + d(y_i))} \quad h_i^n = \frac{g_i d(y_i)}{\sum g_i d(y_i)} \quad \forall i = 1..p$$

Variable  $\sum g_i d_0(y_i)$  is the proportion of employees who found a job with a help of a family member. It is also the total measure of these workers since the total population size is normalized to 1. In a similar way,  $\sum g_i d(y_i)$  is the fraction of employees who found a job with a help of a professional contact. And the remaining part  $1 - \sum g_i (d_0(y_i) + d(y_i))$  is the proportion of workers who found a job through the formal method of search. Therefore, the three average

wages for each of the search channels can be found as:

$$\bar{w}^o = \sum w_i h_i^o \quad \bar{w}^s = \sum w_i h_i^s \quad \bar{w}^n = \sum w_i h_i^n$$

These equations allow us to compare the average wages  $\bar{w}^o$ ,  $\bar{w}^s$  and  $\bar{w}^n$  and to predict whether family and/or professional referrals are associated with a wage premium or a wage penalty relative to the formal method. Either of these results will depend on the self-selection of workers into the specific channels of search. For example, we expect that family contacts will be associated with a wage penalty as this search channel is the most prevalent among the groups of workers with low wages. Formally, one can show that family contacts are associated with a wage penalty if the distribution  $h_i^s$  first order stochastically dominates the distribution  $h_i^o$ :

$$\bar{w}^o = \sum_{i=1}^p w_i h_i^o = w_1 + \sum_{i=1}^{p-1} \Delta w_i (1 - H_i^o) \quad \text{and} \quad \bar{w}^s = \sum_{i=1}^p w_i h_i^s = w_1 + \sum_{i=1}^{p-1} \Delta w_i (1 - H_i^s)$$

so that  $\bar{w}^o < \bar{w}^s$  if  $H_i^s \leq H_i^o, \forall i = 1..p$

where  $\Delta w_i = w_{i+1} - w_i > 0$  since the wage is an increasing function of the productivity, and variables  $H_i^o$ ,  $H_i^s$  are the cumulative density functions so that  $H_i^o = \sum_{j=1}^i h_j^o$  and  $H_i^s = \sum_{j=1}^i h_j^s$ . The proof is presented in appendix V. In a similar way, define  $H_i^n = \sum_{j=1}^i h_j^n$  to be the cumulative density function of wages obtained with a help of professional contacts. This channel of search is then associated with a wage premium relative to the formal method, i.e  $\bar{w}^s < \bar{w}^n$ , if the distribution  $h_i^n$  first order stochastically dominates the distribution  $h_i^s$ , which is equivalent to  $H_i^n \leq H_i^s, \forall i = 1..p$ . Intuitively, this condition holds when firms rely on professional recommendations to match with high ability workers, which is the case in our model.

Finally, the last component of the model is the Bellman equation for an open vacancy with a present value denoted by  $V$ . Same as workers firms are simultaneously using each of the three search channels to fill an open vacancy. At rate  $\lambda_0 u_i/v$  the firm is matched with an unemployed worker of type  $y_i$  as a consequence of a family referral and at rate  $\phi(s_i)u_i/v = s_i u_i$  the firm is matched with a similar worker by means of a formal application. Note that a higher measure of unemployed workers with the productivity  $y_i$  and a more intensive job search  $s_i$  increase the probability of filling a vacancy with this type of worker. In addition, firms may contact one of the incumbent employees to ask for the referral. An applicant of type  $y_i$  is hired through this channel with a job-filling rate  $a_i u_i \rho_i$ , where we use notation  $\rho_i = \rho(y_i) = [1 - (1 - \mu(y_i))^n]$ . This latter term is the probability that the contacted employee will recommend an applicant for the open position. Thus, the value of an open vacancy is given by:

$$rV = -z + \frac{\lambda_0}{v} \sum u_i (J_i - V) + \sum s_i u_i (J_i - V) + \sum e_i \left( a_i \rho_i (J(y_i) - V) - a_i^2/k \right) \quad (12)$$

where  $z$  is the flow cost of filling a vacancy. Note also that the choice of the advertising intensity  $a_i \forall i = 1..p$  is compatible with the maximization of the present value of an open vacancy  $V$ . The free-entry condition of firms implies that  $V = 0$  in the steady-state equilibrium. Substituting the present value of profits  $J_i$  and the optimal advertising intensity  $a_i$  gives us the equilibrium

number of vacancies:

$$v = \lambda_0 \frac{1-\alpha}{r+\delta} \sum u_i(y_i - b) \left[ z - \frac{1-\alpha}{r+\delta} \sum s_i u_i(y_i - b) - 0.25k \frac{(1-\alpha)^2}{(r+\delta)^2} \sum e_i \rho_i^2 (y_i - b)^2 \right]^{-1}$$

This is the last equilibrium equation. So the equilibrium can be defined in the following way:

**Definition 1.** *Search equilibrium is a vector of variables  $(U_i, W_i, J_i, w_i, s_i, a_i, \mu_i)$ ,  $\forall i = 1..p$  as well as the number of vacancies  $v$  and the present value of an open vacancy  $V$ , satisfying the asset value equations for workers (1) and (3), for firms (5) and (12), the wage equations  $w_i = \alpha y_i + (1-\alpha)b$ , the optimal effort equations (4) and (9), the stationary unemployment conditions (7) and the free-entry condition  $V = 0$ .*

To analyze whether the equilibrium defined above is unique, consider first an economy without professional referrals, that is  $a_i = 0 \forall i = 1..p$ . The free-entry condition  $V = 0$  is then:

$$z = \sum \left( \frac{\lambda_0}{v} + s_i \right) \mu_i f_i J_i \quad \text{where} \quad \mu_i = \frac{\delta}{\sqrt{(r+\delta+\lambda_0)^2 + \alpha(y_i - b)cv - r}} \quad (13)$$

Note that a larger number of vacancies has a negative effect on the recruiting rate through families  $\lambda_0/v$  and on the individual search intensity  $s_i$  (see the result from lemma 1). On the one hand, firms compete stronger for applicants, and on the one hand, workers are demotivated and exert less effort in searching for jobs. At the same time, the unemployment rate  $\mu_i$  is reduced, which makes it even more difficult for firms to hire workers. So the right hand side of the free-entry condition (expected profits) is a decreasing function of  $v$ , whereas the left hand side is a fixed cost of hiring  $z$ . So there exists a unique equilibrium in this economy. This results are formalized in the following proposition:

**Proposition 2:** *Consider the labour market described in definition 1,*

- (a) *when professional networks are not utilized, i.e.  $a_i = 0 \forall i = 1..p$ , there exists a unique equilibrium, where the number of vacancies  $v$  is given by the free-entry condition (13).*
- (b) *when professional networks are utilized, there is a possibility of multiple equilibria.*

**Proof:** Appendix VI.

Part (b) of the above proposition indicates that professional networks may lead to the multiplicity of equilibria in our model. To see this consider a special case of the model with only one worker type ( $y > b$ ), where professional networks are the only channel of search. The free-entry condition ( $V = 0$ ) in this economy simplifies to yield:

$$z = 0.25k(1-\mu)(1-(1-\mu)^n)^2 J^2 \quad \text{where} \quad v = 0.5\delta(1-\mu)J/z$$

where the second equation comes from the equilibrium equation for unemployment  $\mu = \delta/(\delta+\lambda)$ . The term on the right hand side  $0.25k(1-\mu)(1-(1-\mu)^n)^2 J^2$  is the expected profit of firms. Note that it is equal to zero for  $\mu = 0$  and  $\mu = 1$  with an internal maximum for some intermediate value of  $\mu$ . Intuitively, it means that a larger number of unemployed agents raises the probability that a randomly contacted employee will recommend his/her contact for the job, so firm profits are increasing in  $\mu$  as long as  $\mu$  is relatively low. But when the number of unemployed workers

is increasing further, then there are fewer employees who can give a recommendation, which has a negative effect on profits. In the extreme case when  $\mu = 1$ , no one is employed and there is no hiring. Thus Appendix VI proves that in this economy there may be two equilibria with the same expected profits of firms; in the first one the unemployment rate is relatively low and there are many vacancies, in the second one the unemployment rate is relatively high and there are fewer vacancies.

Proposition 2 shows that our model is compatible with a multiplicity of equilibria. The first study highlighting the point that social networks in a frictional labour market may give rise to multiple equilibria is Cahuc and Fontaine (2009). However, our result is different from theirs as in their model there are two equilibria with and without networks whereas in our model professional networks are used in both equilibria. Finally, several equilibria may prevail even if all three search channels are used simultaneously, but we do not find it for realistic parameter values in the next section.

## 4 Numerical example

### 4.1 Search effort and the equilibrium unemployment curve

This section parameterizes the model to match the average labour market indicators in the OECD countries. We choose a unit period of time to be six months and set  $r = 0.01$  which corresponds to the annual discount rate of 2%. Further, we follow Shimer (2005) and set the unemployment benefit  $b$  equal to 0.4. Fontaine (2008) uses the value of 0.15 for the U.S. economy and 0.4 for the French economy. Gautier (2002) and Cahuc and Fontaine (2009) set  $b$  equal to 0.2. At the same time, Hall and Milgrom (2008) obtain a larger value of 0.71. Therefore, our choice of  $b$  is in the middle range of the typical values in the literature.

Given  $b = 0.4$ , the range of productivities in the model becomes  $[0.4..1]$ . Since education is one of the main components of the productivity, in the following we will interpret the least productive group of workers (with  $y_1 = 0.4$ ) as those who have completed compulsory education. At the same time the most productive group of workers with  $y_{25} = 1$  will be interpreted as those who have already completed a doctoral degree. The education difference between these two groups is equal to 12 years, including 4 years in the high school, 5 years in the college/university and 3 years for obtaining a doctoral degree. Given that in the model one period of time is equal to one semester, this gives us  $2 \cdot 12 + 1$  educational groups, so we set  $p = 25$ . The productivity gap between the two consequent worker groups is then equal to  $0.025^2$ .

Next, we take the value of the separation rate  $\delta = 0.15$  which corresponds to the average job duration of  $1/(2 \cdot 0.15) = 3.3$  years. Pissarides (2009) and Shimer (2005) choose the value of  $\delta$  equal to 0.1 with one unit of time being a quarter. This corresponds to the average job duration of  $1/(4 \cdot 0.1) = 2.5$  years. Hall and Milgrom (2008) choose the value of 3% per month, so the average job duration in their model is  $1/(12 \cdot 0.03) = 2.78$  years. Hobijn und Sahin (2009), however, when estimating the monthly job separation rate for OECD countries for the period

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<sup>2</sup>The choice of  $p$  is not essential for the model since neither of the group-specific variables depends on the total number of productivity groups. Nevertheless, with a large value of  $p$  the model can serve as an approximation of the market with a continuous productivity distribution and allows an intuitive interpretation of worker groups as those with different levels of educational attainment.

1968-2004 report to be at most 2.3% (Spain). So the average job duration is  $1/(12 * 0.023) = 3.6$  years. Therefore, our estimate falls within the standard bounds.

The job-finding rate through the channel of family contacts  $\lambda_0$  is chosen to be 0.3 and is defined by the unemployment rate of the least productive workers being equal to  $\mu(b) = \delta/(\delta + \lambda_0) = 0.33$ . Note that the productivity of these workers is equal to the unemployment benefit and so the gain from finding a job is zero, which means that sending formal job applications is too costly for this group of workers. At the same time,  $w(b) = b$  so firms obtain zero profits from hiring the least productive workers. Therefore, it is not profitable for firms to direct their search towards these employees. Hence, the only way for the unproductive workers to find a job is to rely on family contacts. For this reason the average unemployment duration of this group of workers is equal to 1.6 years and is the largest in the population.

We choose the number of workers' professional contacts equal to 50 as in Cahuc and Fontaine (2009), while Fontaine (2008) uses  $n = 40$  in a benchmark model of his paper. These numbers are in line with the empirical evidence, for example, in their recent study Cingano and Rosolia (2012) find that the median number of professional contacts in Italy is equal to 32. This number is higher in Germany and is equal to 43 according to Glitz (2013). The workers' bargaining power  $\alpha$  is set equal to 0.72 as in Shimer (2005). This means that the maximum wage in the economy is equal to  $0.72 \cdot 1 + (1 - 0.72) \cdot 0.4 = 0.832$ . With this parameter choice we can calculate the annual return to schooling which is implied by the model:  $\ln(0.83/0.4)/12 = 0.061$ , so the average salary increase for an additional year of education is equal to 6.1%. This value is in line with the parameters presented in Card (2001).

An important feature of our model is its invariance to the shape of the productivity distribution and the number of vacancies. Recall that the two key variables in the model  $d_0(y_i)$  and  $d(y_i)$  correspond to the relative fractions of workers finding jobs with a help of family and professional contacts. Thus these variables are defined in relative terms and are independent of the productivity distribution  $f_i$ . Moreover, the total number of vacancies only enters in the two multiplicative terms  $kv$  and  $cv$ , where  $k$  and  $c$  are the unobservable parameters of the two cost functions. To identify variables  $kv$  and  $cv$ , first, we define a median productivity group. According to the OECD report 2013 (table A1.1a), 47% of workers aged 25-64 in the US report the high school diploma to be the maximum educational achievement. Further, there is a small proportion of workers equal to 11% who did not complete their high school studies. Therefore, we can conclude that 58% of workers in the US have at most a high school diploma. In the model this corresponds to the productivity level  $y_8 = 0.575$  (8 semesters of schooling). So a median worker in the US has the level of schooling just below the high school. Based on this information we conjecture that a median worker in our model should have the productivity  $y_7 = 0.55$ .

In the second step, having defined a median worker group, we simultaneously choose variables  $kv$  and  $cv$  to target  $d_0(y_7) + d(y_7) = 0.4$  and  $\mu(y_7) = 0.087$ . The first of these conditions implies that 40% of workers in the median group find employment by means of referrals. This assumption is in line with the empirical observation that 30% to 60% of the employees in developed countries rely on social contacts in order to find a job (see Ioannides and Datcher Loury (2004) for an overview). The second condition implies that the unemployment rate in the median group of workers is equal to 8.7%. This number is the average unemployment rate in the United States in the recent years (BLS, 2009-2013). Our calibration strategy yields  $kv = 0.24$  and  $cv = 22.07$ .



Table 1 presents our calibration for the benchmark case.

Variable	Value	Explanation, source and target
$n$	50	Network size (Cahuc and Fontaine (2009))
$r$	0.01	Annual interest rate of 2%
$\lambda_0$	0.3	Unemployment of the least able worker=33%
$\delta$	0.15	Average job duration of 3.3 years
$\alpha$	0.72	Worker's bargaining power (Shimer (2005))
$b$	0.4	Unemployment benefit (Shimer (2005))
$p$	25	Number of productivity types
$cv$	22.07	Unemployment of the median worker=8.7%
$kv$	0.24	Referral hiring of the median worker=40%

Table 1: Values of the model parameters

Next we describe our results. Figure 3 (left panel) presents variables  $d_0(y_i)$ ,  $1 - d_0(y_i) - d(y_i)$  and  $d(y_i)$  for every worker group  $i = 1..25$ . These are the average proportions of workers finding employment by means of family contacts, formal applications and professional contacts respectively. As we proved in the theoretical part of the paper the average fraction of workers using family contacts to find a job,  $d_0(y_i)$ , is a decreasing curve and the lowest productivity group never finds jobs through channels other than family contacts. Therefore, the reliance on family contacts falls down from 100% for the least able workers to only 7% for the most productive group. Intuitively, even though family contacts become less important for more productive workers, our model does not exclude situations when talented employees are recommended and work for the same employer as their parents.

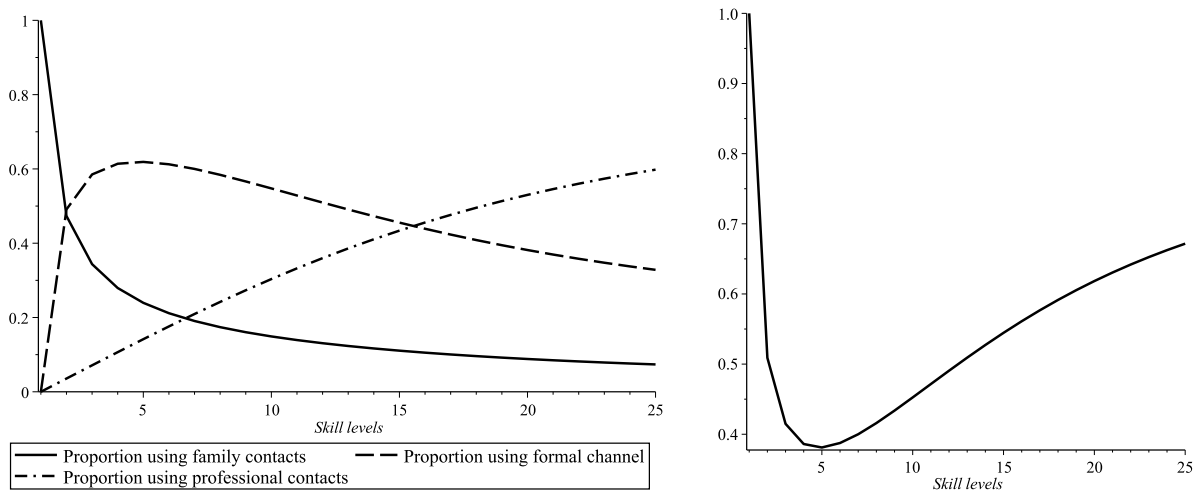


Figure 3: Left panel: The graph for average proportions of workers using family, professional contacts and the formal application in the job search process for different productivity levels. Right panel: The graph for the average proportions of workers using social contacts in the job search for different productivity levels (U-shape).

Now consider professional relations. Figure 4 (the right panel) shows that firms exert more advertising effort  $a(y_i)$  when targeting the more productive group of incumbent employees. This means that the elasticity condition in proposition 1 is satisfied and the positive effect of higher profits is dominating for firms. This in turn implies that the job finding rate  $\lambda(y_i)$  is an increasing function of productivity. On the one hand, even if firms contacted their incumbent employees

in a random and undirected manner they would be more likely to be in contact with a more productive worker as the equilibrium unemployment rate is decreasing with  $y_i$  (see the right panel of figure 4). On the other hand, it is profitable for firms to direct their search towards the more productive group of incumbent employees in the expectation of a good applicant. Therefore, both effects are reinforcing and amplifying each other and the network job-finding rate  $\lambda(y_i)$  is an increasing and a convex function of  $y_i$  (see the left panel of figure 4). Thus the average proportion of workers using professional contacts to find a job,  $d(y_i)$  is increasing from 0% for the least productive group up to 60% for the most productive group. Moreover, the network of professional contacts is a dominating channel of search for workers with a productivity above  $y_{15} = 0.75$ .

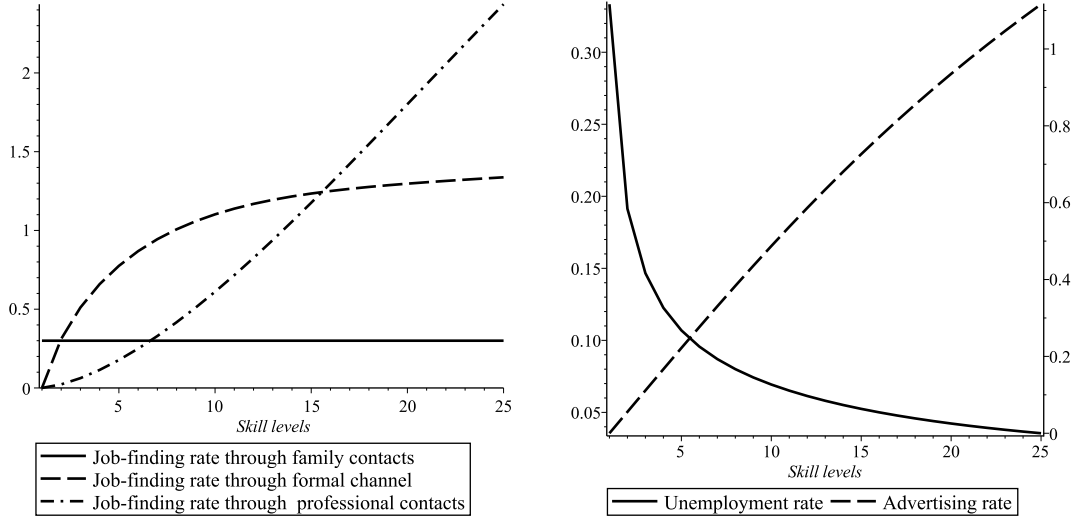


Figure 4: Left panel: The graphs for  $\lambda_0$ ,  $\phi(s(y_i))$  and for  $\lambda(y_i)$  for different productivity levels. Right panel: The optimal advertising rate  $a(y_i)$  by firms and the graph for the unemployment rate  $\mu(y_i)$  for different productivity levels.

Finally, consider the formal applications channel. Figure 3 (left panel) shows that the relative fraction of workers finding jobs through this channel,  $1 - d_0(y_i) - d(y_i)$  is increasing for productivities below  $y_5 = 0.5$  and decreasing thereafter. Intuitively, for the less able workers the probability of being referred for a job  $\lambda(y_i)$  is still relatively low, but the wage  $w_i$  is already sufficiently large to motivate these workers for preparing formal applications. However, as the productivity is increasing, workers' chances of being referred for a job are improving and the incentives to invest costly effort and time in preparing applications are mitigated. In line with this reasoning figure 4 (left panel) shows that the search effort  $s(w_i, \lambda_i)$  is an increasing but a concave function of  $y_i$  as the positive effect of a higher wage is partially neutralized by the negative effect of a higher  $\lambda_i$ . In addition, figure 3 (left panel) illustrates that the formal channel of search is dominating for workers in the middle range of productivities between  $y_2 = 0.425$  and  $y_{15} = 0.75$  reaching a maximum of 62% for workers with a productivity  $y_5 = 0.5$ .

To sum up, our model is able to jointly replicate a number of empirical observations. First, without separating social contacts into different types the model shows that the reliance on social contacts  $d_0(y_i) + d(y_i)$  has a distinct U-shape pattern falling down from 100% to 38% for workers with  $y_5 = 0.5$  and rising again to the level of 67% for the most productive workers (see figure 3

(right panel))<sup>3</sup>. Second, when the two types of contacts are separated, then family contacts are more likely to have a negative effect on wages since family referrals are strongly biased towards the left tail of the productivity distribution. In contrast, professional contacts are more likely to have a positive effect on wages since these contacts are biased towards the right tail of the distribution. However, the overall effect of referrals on wages is ambiguous and depends on the exact proportions of high and low types in the population. Therefore, we continue the analysis of wages by considering an explicit shape of the productivity distribution  $f_i$ .

## 4.2 Wage and productivity distributions

It is a well documented empirical fact (see Neal and Rosen (2000) and Mortensen (2003)) that a typical earnings distribution is hump-shaped and positively skewed with a mean value larger than the median. Therefore, it is often well approximated by the log-normal distribution. In our model the distribution  $f_i$  is discrete, so we use the Negative Binomial productivity distribution which is a discrete counterpart of the log-normal distribution. In particular, we rely on a special case of the density which is known as the Polya distribution. Given that this distribution has an infinite range we truncate it at  $i = 25$ . The productivity density  $f_i$  is then characterized by two parameters  $t$  and  $\pi$  and takes the form:

$$f_i = \frac{\tilde{f}_i}{\sum_{i=1}^{25} \tilde{f}_i} \quad \text{where} \quad \tilde{f}_i = \binom{i+t-2}{i-1} (1-\pi)^t \pi^{i-1} = \frac{\Gamma(i+t-1)}{(i-1)! \Gamma(t)} (1-\pi)^t \pi^{i-1}, \quad i = 1, 2, \dots$$

Negative Binomial a discrete probability distribution of the number of successes in a sequence of independent and identically distributed Bernoulli trials before a specified number of failures occurs. Here  $t$  is the specified number of failures and  $\pi$  is the probability of success. However, in the specific case of Polya distribution  $t$  is a real-valued parameter rather than an integer, which allows a more accurate approximation of the data. To identify parameters  $t$  and  $\pi$  we exploit the definition of the median worker having the productivity  $y_7 = 0.55$  and the fact that 58% of workers in the U.S. have at most a high school degree (corresponding to  $y_8 = 0.575$ ), mentioned above. Therefore, we set  $\sum_{i=1}^7 f_i = 0.5$  and  $\sum_{i=1}^8 f_i = 0.58$ .

Next, we consider the free-entry condition. To identify the cost of an open vacancy  $z$  we set the market tightness  $v/u$  equal to 1, where  $u = \sum_{i=1}^p \mu_i f_i$  is the equilibrium unemployment rate in the economy. This value coincides with the calibration of Shimer (2005) but it is larger than 0.72 chosen by Pissarides (2009) and derived from the Job Openings and Labor Turnover Survey (JOLTS). To defend our choice we argue that some vacancies are not captured by JOLTS, for example, some positions are filled without ever reporting a vacancy. In particular, these positions are very likely to be filled through social networks and, therefore, we consider the market tightness equal to 1 an appropriate choice for our model. The last set of parameters is presented in table 2:

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<sup>3</sup>As a form of robustness check, we have estimated our model with a fixed advertising intensity  $a_i = cst$ . This does not change our result that the unemployment rate is decreasing with a higher skill level and the job-finding rate through professional contacts is increasing. However, numerically this increase is rather small and dominated by the sharp rise in the search effort through the formal channel. So the frequency of workers finding jobs through networks is decreasing with the skill level which is not compatible with the U-shape hiring pattern. Thus, we find that for realistic parameter values, endogenous advertising intensity is necessary for the explanation of the right tail of the U-shape hiring curve.

Variable	Value	Explanation, source and target
$t$	2.98	58% have at most a high school degree ( $y_8 = 0.575$ ) (OECD (2013))
$\pi$	0.717	Productivity of a median worker $y_7 = 0.55$
$z$	0.390	Market tightness equal to 1 (Shimer (2005))

Table 2: Values of the model parameters

We get  $z = 0.39$  (flow cost of an open vacancy) in equilibrium, which is close to the value of 0.4 chosen for the formal search method by Cahuc and Fontaine (2009). Shimer (2005) has chosen the value of 0.213 for this parameter, while Fontaine (2008) uses the number 0.3. The choice of Pissarides (2009) is closer to our value, in his calibration the cost of an open vacancy is equal to 0.36. Another influential calibration is due to Hagedorn and Manovskii (2008). These authors find that the capital cost of an open vacancy is equal to 0.47 and the personnel cost is 0.11, which gives a value 0.58. Hence,  $z = 0.39$  is in the middle range of standard values.

The productivity density function  $f_i$  with  $t = 2.98$  and  $\pi = 0.717$  is illustrated by the solid curve on figure 5 (left panel). The equilibrium wage distribution  $g_i$ , defined in the theoretical part of the paper, is shown by the dashed curve on the same figure. The corresponding CDFs are shown on figure 5 (right panel). The wage distribution  $g_i$  first order stochastically dominates the productivity distribution  $f_i$ . This is because the unemployment rate is higher than the average among the less productive types and lower among the more productive. Both distributions are, however, very close to each other.

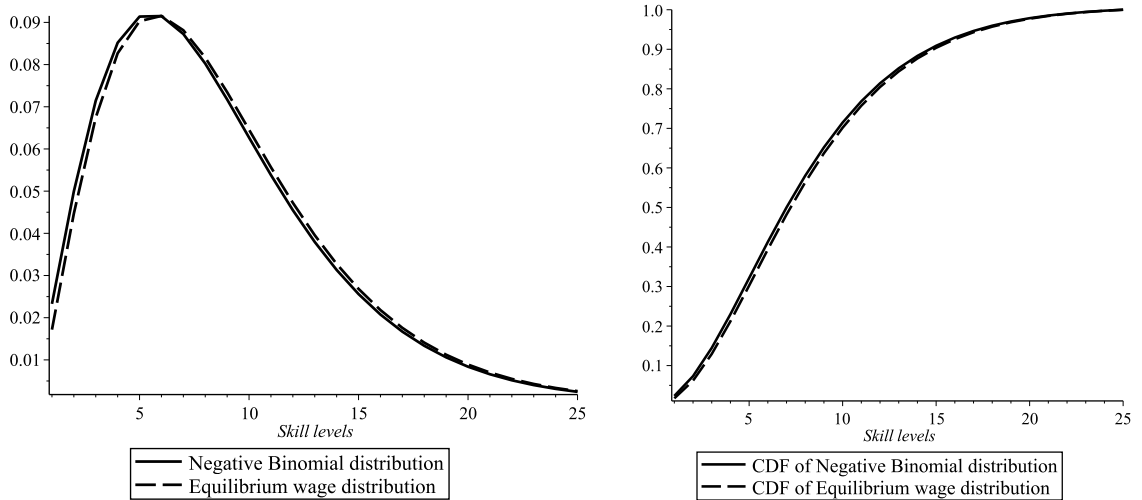


Figure 5: Left panel: The graphs for the probability mass functions of the Negative Binomial distribution and of the equilibrium wage distribution. Right panel: The graphs for the CDFs of the Negative Binomial distribution and of the equilibrium wage distribution.

The equilibrium wage/productivity distributions conditional on the job search channel  $h_i^o$  (black),  $h_i^s$  (red) and  $h_i^n$  (blue), as well as their corresponding CDFs are presented on figure 6. The wage distribution of workers finding jobs through their families,  $h_i^o$ , is first order stochastically dominated by the wage distribution of employees who used a formal application,  $h_i^s$ . The probability mass of the distribution  $h_i^o$  is mostly concentrated in the lower productivity range and so most of the employees in this group are the low productivity types with low wages. The average productivity of workers using the family channel is equal to 0.536 and the average

wage is  $\bar{w}^o = 0.498$ . In contrast, the average productivity of workers using the formal channel is equal to 0.580 and the average wage is  $\bar{w}^s = 0.530$ . Therefore, we conclude that the family search channel is associated with a *wage penalty* of 6% compared to the formal channel.

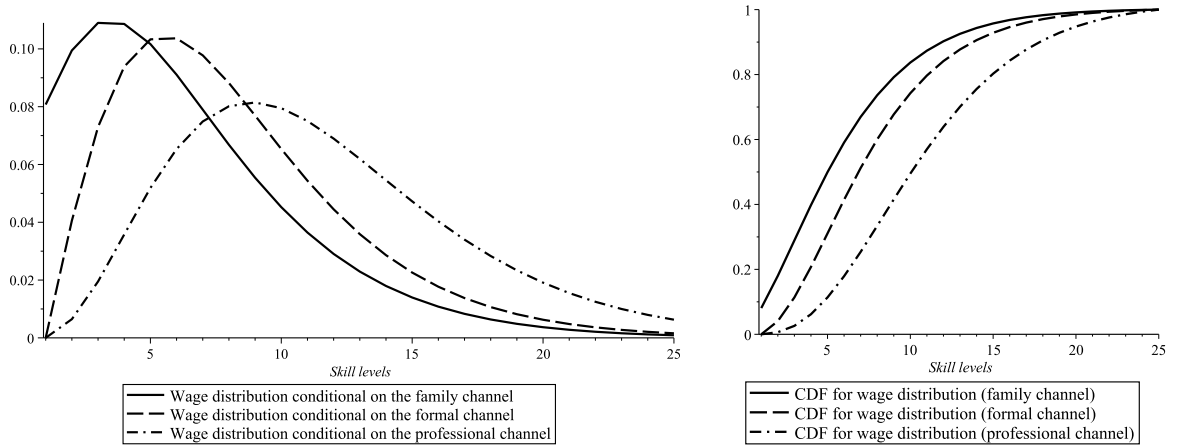


Figure 6: Left panel: The graphs for wage distribution functions  $h_i^o$ ,  $h_i^s$  and for  $h_i^n$  corresponding to different channels of job search. Right panel: The graphs for cumulative wage distribution functions  $H_i^o$ ,  $H_i^s$  and for  $H_i^n$ .

The second distribution,  $h_i^s$ , is in turn first order stochastically dominated by the wage distribution of workers who used professional contacts,  $h_i^n$ . Here the average productivity is equal to 0.656 and the average wage is  $\bar{w}^n = 0.584$  (see table 3). The probability mass of the distribution  $h_i^n$  is shifted to the right and so this density is mostly concentrated in the middle range of the productivities. This is due to the fact that the proportion of high productivity workers in the population is relatively low and almost all of them are employed. Finally, observe that finding a job with a help of professional contacts is associated with a *wage premium* of 10%. Therefore, we can rank  $\bar{w}^o < \bar{w}^s < \bar{w}^n$ , which is the second prediction of the paper: when the two types of contacts are separated, then family contacts are more likely to have a negative effect on wages, whereas professional contacts are more likely to have a positive effect. Capellari and Tatsiramos (2013) confirm this result with their empirical finding for the UK that high-skilled individuals, whose employed friends are non-relatives, earn higher wages and low-skilled individuals, whose employed friends are relatives, experience a wage penalty. Sylos Labini (2004) confirm this finding for Italy.

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.536	0.580	0.656	0.590
Average wage	0.498	0.530	0.584	0.536
Proportion of employees	0.213	0.541	0.247	1

Table 3: Model-generated statistics for the benchmark calibration

Based on table 3 we can also calculate the average wage of employees who found a job by means of referrals, i.e. both types of social contacts. Let this variable be denoted by  $\bar{w}^c$ :

$$\bar{w}^c = \frac{\bar{w}^o \sum g_i d_0(y_i) + \bar{w}^n \sum g_i d(y_i)}{\sum g_i d_0(y_i) + \sum g_i d(y_i)} = \frac{0.498 \cdot 0.213 + 0.584 \cdot 0.247}{0.213 + 0.247} = 0.544$$

$\bar{w}^c = 0.544$  is higher than  $\bar{w}^s = 0.530$ . Thus, in the benchmark case the positive effect of professional networks is dominating the negative effect of family contacts and job referrals are associated with a wage premium of 2.6%. However, this result is sensitive to the relative proportions of workers relying on family and professional relations. To elaborate on this point we perform comparative statics analysis with respect to parameter  $t$  which is a shift parameter of the distribution. Intuitively, a lower value of  $t$  corresponds to labour markets with a larger proportion of low skilled workers. In the first step, we find  $t^*$  for the neutral scenario when the effect of referrals on wages is zero. Other parameters remain unchanged. We get the value  $t^* = 2.21$ . Our results for the neutral scenario are presented in table 4. In the second step, we recognize that the effect of referrals should be negative for  $t < t^*$ . Therefore, we consider a wage penalty scenario as a symmetric case around  $t^*$ :  $t = 1.45 = 2.21 - (2.98 - 2.21)$ , subtracting from  $t^*$  the difference between the benchmark value of  $t = 2.98$  and  $t^*$ . Our results for the wage penalty scenario ( $t = 1.45$ ) are presented in table 5:

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.493	0.545	0.617	0.545
Average wage	0.467	0.504	0.556	0.504
Proportion of employees	0.270	0.538	0.193	1

Table 4: Model-generated statistics with  $t = 2.21$

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.454	0.511	0.575	0.498
Average wage	0.439	0.480	0.526	0.471
Proportion of employees	0.371	0.496	0.134	1

Table 5: Model-generated statistics with  $t = 1.45$

$f_i$  in the case of the neutral scenario ( $t = 2.21$ ) is close to our benchmark productivity distribution, although shifted to the left. Compared to the benchmark scenario, more workers are relying on their families in the job search process ( $0.270 > 0.231$ ) and less workers find jobs by means of professional contacts ( $0.193 < 0.247$ ). From table 4 we also see that family contacts have a negative effect of wages, whereas professional contacts are associated with a wage premium:  $\bar{w}^o = 0.467 < \bar{w}^s = 0.504 < \bar{w}^n = 0.556$ . Therefore, we conclude that these results are due to the self-selection of workers into channels and are robust to different specifications of the productivity distribution. The average referral wage for this case can be calculated from table 4:

$$\bar{w}^c = \frac{0.467 \cdot 0.270 + 0.556 \cdot 0.193}{0.270 + 0.193} = 0.504$$

This value is equal to  $\bar{w}^s$  and so the average effect of referrals on wages is equal to zero.

The scenario for the wage penalty ( $t = 1.45$ ) is associated with a further increase in the proportion of workers relying on families ( $0.371 > 0.270$ ) and a lower importance of professional contacts ( $0.134 < 0.193$ ). As before, table 5 confirms that family contacts are associated with a wage penalty, whereas professional networks with wage premium, since  $\bar{w}^o = 0.439 < \bar{w}^s =$



$0.480 < \bar{w}^n = 0.526$ . The average referral wage can be calculated from table 5:

$$\bar{w}^c = \frac{0.439 \cdot 0.371 + 0.526 \cdot 0.134}{0.371 + 0.134} = 0.462$$

This value is lower than  $0.480 = \bar{w}^s$  and so there is a wage penalty equal to 1.9%.

This section shows that the negative effect of family contacts and the positive effect of professional contacts are both robust to the exact specification of the productivity distribution in the population. However, the average effect of referrals on wages is sensitive to the specific distribution and can be positive or negative depending on the relative proportions of high and low productivity groups. Thus, our model provides an additional explanation for the ambiguous results reported in the empirical literature, which were summarized in the introduction.

## 5 Robustness checks

### 5.1 Family contacts

In this section we analyze whether the model is robust with respect to the modeling of the family search channel. There are potentially two ways for family members to help an individual find a job. First, employed family members may recommend their relatives for open vacancies, thereby connecting unemployed workers with job openings, which are not publicly advertised. This is the extensive margin of search, however, family recommendations are often associated with nepotism which is forbidden in many companies. Therefore, in this paper we follow a different approach by recognizing that employed family members can increase the speed at which the individual is sampling official job offers. This is the intensive margin of search effort. For simplicity suppose that every worker has exactly one family member, for example, a parent or a spouse. If this family member is employed, he/she continues searching for jobs in the formal way with a constant search intensity  $s_0$ . At rate  $\phi(s_0) = vs_0$  this family member is matched with an open vacancy and forwards this information to the unemployed relative. Thus we can extend the model, where the modified job-finding rate through the family channel,  $\lambda_0^i$ , is equal to the matching rate  $vs_0$  multiplied by the employment probability of the helping family member.

Further, the probability of being employed depends on the skill level of the helping member. In the benchmark model of the paper we assumed that the job-finding rate  $\lambda_0$  was constant across groups which can only be in the absence of skill homophily within families. However, this case is not completely satisfactory as there exists empirical evidence of positive correlation between parents and grown-up children and between spouses. For example, Hertz et al. (2007) report that the global average correlation between parent and child's schooling has held steady at about 0.4 for the past fifty years. In the United States this correlation is slightly above the average amounting to 0.46. Black et al. (2011) provide an overview of the literature on intergenerational mobility. Their analysis suggests an ordering with the lowest correlations in the North European countries, higher correlations in the UK and the highest correlation in the US. Their estimates of the intergenerational elasticity of earnings in the US are in the range between 0.5 and 0.6 which is higher than 0.3 for the UK<sup>4</sup>. Overall, this evidence suggests a positive correlation in the skill

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<sup>4</sup>Note that the correlation coefficient coincides with the elasticity parameter only if sons' and fathers' earnings have the same variance.

levels of family members. Moreover, this correlation is also positive among spouses. According to Smits (2003), for the US the odds ratio indicating the extent of educational homogamy among the higher educated has a value 8.5. This means that the odds of having a spouse with a high educational level is 8.5 times as high for a person who has also a high educational level than for somebody with a lower educational level. In order to account for the positive correlation of schooling levels we propose the following equation for  $\lambda_0^i$ :

$$\lambda_0^i = vs_0[\beta e_i + (1 - \beta)\bar{e}]$$

where  $0 \leq \beta \leq 1$  is a mixing parameter,  $e_i$  is the employment rate in the skill group  $i$  and  $\bar{e}$  is the employment rate in the median skill group ( $i = 7$ ). To understand this equation consider the two extreme cases. If  $\beta = 1$ , then the job-finding rate  $\lambda_0^i$  is equal to  $vs_0e_i$ , this is the case of strong homophily between family members. Note that in this case family members are situated in the same skill group  $i$  and so their employment rate is equal to  $e_i$ . In the opposite case, when  $\beta = 0$ , the job-finding rate is constant across groups,  $\lambda_0^i = vs_0\bar{e}$ , implying the absence of skill homophily. This is the benchmark case of the model, so we set  $vs_0\bar{e} = \lambda_0$ . Following the calibration above, the equilibrium employment rate of the median worker group ( $i = 7$ ) is equal to  $1 - 0.087 = 0.913$ , which gives us an estimate of the formal matching rate between firms and family members:  $\phi(s_0) = 0.33$ . This search intensity is relatively low given that the individual matching rate of unemployed workers is ranged between 0 and 1.33 for  $i = 1..25$ .

Intuitively, variable  $\beta$  can be seen as a fraction of type  $i$  workers with family members in the same group. Thus a larger value of  $\beta$  is associated with a stronger homophily of family members and a stronger correlation of skills within families. In order to find an estimate of  $\beta$  we set the correlation coefficient between family members equal to 0.46, which is the empirical estimate of Hertz et al. (2007). The correlation between skill levels of the family members can be obtained from the corresponding probability matrix, where the measure  $\beta f_i$  of type  $i$  workers are linked to family members in the same skill group. In contrast, a measure  $(1 - \beta)f_i$  of these workers are linked to family members with a median skill level  $y_7 = 0.55$ . In the special case  $\beta = 1$ , this matrix has zero entries off the diagonal as families are exclusively formed within the same skill group. Based on this probability matrix we find that a correlation coefficient of 0.46 corresponds to  $\beta = 0.225$ . For comparison, on figure 7 we also consider higher values of this parameter –  $\beta = 0.5$  and  $\beta = 0.75$  with the corresponding correlation coefficients equal to 0.69 and 0.86.

The left panel of figure 7 shows changes in the job-finding rate  $\lambda_0^i$  as we increase  $\beta$  (and thereby the correlation coefficient) from 0 to 1. For the purpose of illustration we only focus on low skilled workers  $i = 1..5$ , as changes for other groups are rather small. With a stronger homophily within the family, there is a higher probability that family members of low skilled workers are also low skilled. This makes their help in the search process less likely, thus the job-finding rate  $\lambda_0^i$  falls below  $\lambda_0 = 0.3$ . This drop is particularly pronounced for the least skilled group ( $i = 1$ ) as the job-finding rate falls down to 0.28 for the realistic scenario  $\beta = 0.225$  and down to 0.18 for the case of full homophily  $\beta = 1$ . This drop is smaller for the second group reaching the level of 0.26 for  $\beta = 1$ . The right panel of figure 7 shows changes in the unemployment rate for the low skilled workers. The unemployment rate of the least skilled rises from 0.33 to 0.35 for the realistic scenario  $\beta = 0.225$  and up to 0.46 for the case of full homophily.

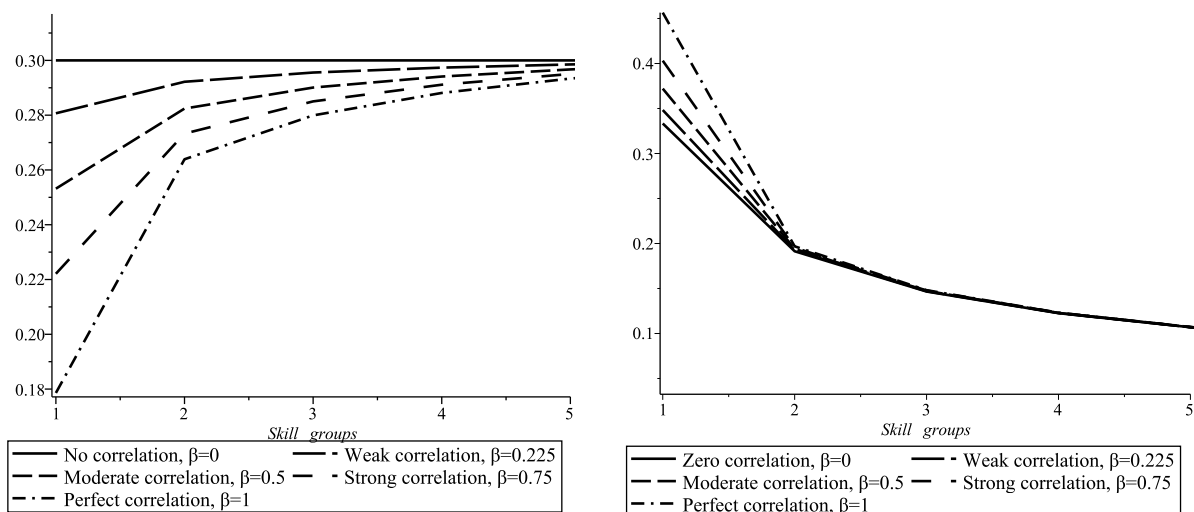


Figure 7: Left panel: Changes in the family job-finding rate for low skilled workers with the increase in  $\beta$  from 0 to 1. Right panel: Changes in the unemployment rate for the low skilled workers with the increase in  $\beta$  from 0 to 1.

At the same time, the change in unemployment is relatively small for all other groups. On the one hand, workers exert more individual effort in order to compensate for the lower help of the family, and on the other hand, more skilled groups are less dependent on families, so the overall effect of a stronger skill correlation on the unemployment rate is relatively small, with the exception of the first group.

Further, we have calculated average wages for each of the three search channels in the model for the extreme case of perfect skill correlation between family members (case  $\beta = 1$ ). They are given in table 6. In this scenario, unskilled workers are worse off as on average they are less likely to get help from their family members. This is reflected in the lower proportion of workers finding jobs through families compared to the benchmark scenario in table 3 ( $0.207 < 0.213$ ). As family contacts become less relevant, workers exert more effort in the formal channel, so there is a small increase in the proportion of workers finding jobs through the formal channel ( $0.545 > 0.541$ ). Despite these changes, the overall intuitive result of the model remains unchanged: family contacts are associated with wage penalties, whereas professional contacts are associated with wage premiums. Given that  $\beta = 1$  is an unrealistic hypothetical scenario, the changes are even smaller for the realistic case  $\beta = 0.225$ . Thus we conclude that our results are robust to the constant specification of  $\lambda_0$ .

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.541	0.579	0.656	0.590
Average wage	0.501	0.529	0.584	0.537
Proportion of employees	0.207	0.545	0.247	1

Table 6: Perfect skill correlation between family members

## 5.2 Hiring costs

As a final robustness check we present comparative statics results with respect to the hiring cost parameter  $z$ . This parameter primarily includes the costs of posting job ads in the media (newspapers, Internet, etc.). In the benchmark scenario, we have chosen  $z = 0.39$  to achieve a market tightness ratio ( $v/u$ ) equal to 1. The left panel of figure 8 shows changes in the vacancy rate  $v$  and in the average unemployment rate  $u$ . As the cost  $z$  is increasing from 0.2 to the benchmark case 0.39, firms post less vacancies and the average unemployment rate is increasing from 0.072 to the benchmark case 0.096. Thus the model captures the macroeconomic dynamics of the labour market as vacancies and unemployment are moving in the opposite directions. The right panel of figure 8 shows changes in the U-shape of the referral hiring pattern. If the cost parameter is decreasing, then firms have more vacancies which should be filled. This improves the formal job-finding rates  $\phi(s_i) = vs_i$  and the network matching rates  $\lambda_i$ . However, as the chances of finding jobs through professional contacts are improving (that is  $\lambda_i$  is increasing) workers optimally reduce their individual search effort  $s_i$ . Thus the initial rise in  $\phi(s_i)$  is moderated by the lower search intensity  $s_i$ . Overall, this implies that the ratio of workers finding jobs through professional contacts is increasing with a larger number of vacancies and the U-shape pattern becomes more pronounced. So the model predicts that networks are relatively more (less) utilized in the periods of expansions (recessions) compared to the formal search channel.

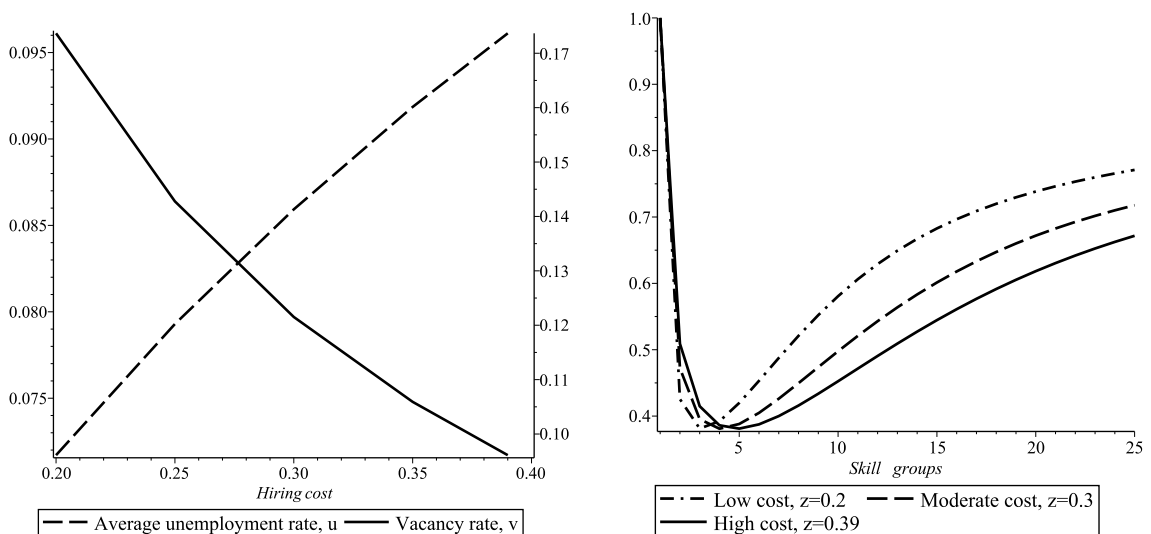


Figure 8: Left panel: Changes in the vacancy rate  $v$  and in the average unemployment rate  $u$  with the increase in vacancy cost  $z$ . Right panel: Changes in the U-shape of the referral hiring pattern for different vacancy costs  $z$ .

Investigating the link between vacancies and referral hiring is a relatively new research direction. To the best of our knowledge there are only two other studies dealing with this issue. First, in a theoretical model Horvath (2012) finds that in economic upturn the neighbors of an individual are more likely to be employed in high paying jobs and hence, it is more likely that the individual hears about a high paying job through them. This finding is intuitively similar to our result if a higher number of vacancies in our model is understood as an economic upturn in Horvath (2012). And second, Galeotti and Merlino (2014) find a U-shape relationship between the job-destruction rate and the network matching rate. This is an empirical finding which is in

line with their theoretical model. It means that improving economic conditions are associated with a higher utilization of networks in the beginning. But as economic conditions improve further, network matching becomes less relevant. The first part of the effect is compatible with our model, even though it does not predict a lower network matching rate at the pick of the economic expansion. Overall, we conclude that empirical evidence on this question is rather sparse and more empirical and theoretical work should be done in the future to investigate this issue in more details.

## 6 Conclusions

This paper develops a labour market matching model with a finite number of heterogeneous worker groups and three channels of job search: family contacts, formal applications and professional contacts. Moreover, the model relies on the assumption of network homophily meaning that workers connected in the network are all of the same productivity type. In this framework, we are able to generate a significant U-shape relationship between the frequency of referral hiring and the productivity/skill level of the worker.

Family contacts are exogenous in the model and serve as a residual method of search. Nevertheless, every worker may choose an endogenous search intensity, which can be interpreted as total effort and time invested in preparing job applications. It turns out that the gain from preparing applications is increasing in the worker type and so it is relatively costly for low productivity workers to rely on the formal channel. On the other hand, firms with open vacancies direct their network search towards more productive incumbent employees in the anticipation of higher profits. Therefore, the family channel of search is predominantly employed by workers in the left tail of the productivity distribution, whereas the network of professional contacts is largely used by more productive workers. These two mechanisms explain the U-shaped referral hiring pattern, which implies that the effect of professional (family) contacts on wages is positive (negative) with respect to the formal channel. This result is due to the endogenous sorting of workers across the channels and is robust to the exact shape of the productivity density. We believe these results shed some light on the contradicting empirical evidence and may serve as a further step in the explanation of the "referral puzzle", at least from a theoretical perspective.

Finally, our model may exhibit a multiplicity of equilibria if professional contacts are heavily utilized. For example, if there is only one type of workers and professional referrals are the only channel of search, then there exist two equilibria. In the first one there is high unemployment and a low number of vacancies, the opposite is true in the second equilibrium. Intuitively, when unemployment is high, there are many unemployed job seekers but very few employees who can give a recommendation. In the second equilibrium, the unemployment is low, so there are many employees who can give a recommendation but very few of their contacts are unemployed.

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## 8 Appendix

### Appendix I. Proof of Lemma 1:

The rent  $R_i$  can be obtained as a solution of the following quadratic equation:

$$0.25cvR_i^2 + (r + \delta + \lambda_0 + \lambda_i)R_i - (w_i - b) = 0$$

Since workers will only accept the job with  $R_i \geq 0$  it holds that:

$$R_i = \frac{2}{cv} [\sqrt{(r + \delta + \lambda_0 + \lambda(y_i))^2 + (w_i - b)cv} - (r + \delta + \lambda_0 + \lambda_i)]$$

therefore the optimal effort is given by  $s_i = 0.5cR_i$ , where  $R_i$  is increasing in the wage  $w_i$  but decreasing in  $\lambda_i$ :

$$\frac{\partial R_i}{\partial \lambda_i} = \frac{2}{cv} \left[ \sqrt{\frac{(r + \delta + \lambda_0 + \lambda_i)^2}{(r + \delta + \lambda_0 + \lambda_i)^2 + (w_i - b)cv}} - 1 \right] < 0$$

To reduce notation in the following let  $D_i \equiv r + \delta + \lambda_0 + \lambda_i$ . To prove that search effort  $s_i$  is a decreasing function of the number of vacancies  $v$ , differentiate it with respect to  $v$  to obtain:

$$\frac{\partial s_i}{\partial v} = \frac{1}{v^2} \left[ \frac{0.5(w_i - b)cv}{\sqrt{D_i^2 + (w_i - b)cv}} - \left( \sqrt{D_i^2 + (w_i - b)cv} - D_i \right) \right]$$

The function in the square bracket takes value zero at  $v = 0$ . It turns out that there are no other values of  $v$  delivering a zero to this function. To see this, differentiate expression in the square bracket to get:

$$-\frac{0.25(w_i - b)^2 c^2 v}{(D_i^2 + (w_i - b)cv)\sqrt{D_i^2 + (w_i - b)cv}} + \frac{0.5(w_i - b)c}{\sqrt{D_i^2 + (w_i - b)cv}} - \frac{0.5(w_i - b)c}{\sqrt{D_i^2 + (w_i - b)c}} < 0$$

Thus the function in the square bracket starts at zero and is downward sloping for any  $v > 0$ . However, this means that it is negative for any  $v > 0$ , so the derivative  $\partial s_i / \partial v$  is also negative for any  $v > 0$ .

### Appendix II. Proof of Lemma 2:

Differentiate  $\lambda_i$  with respect to  $\mu_i$  for a given fixed advertising intensity  $a_i$ :

$$\begin{aligned} \frac{\partial \lambda(\mu_i, a_i)}{\partial \mu_i} &= va_i \left[ -\frac{1}{\mu_i^2} [1 - (1 - \mu_i)^n] + \frac{1 - \mu_i}{\mu_i} n(1 - \mu_i)^{n-1} \right] \\ &= \frac{va_i}{\mu_i^2} \left[ -1 + (1 - \mu_i)^n + n\mu_i(1 - \mu_i)^n \right] \\ &= \frac{va_i}{\mu_i^2} [(1 - \mu_i)^n (n\mu_i + 1) - 1] \end{aligned}$$

Let  $\sigma(\mu)$  denote the first term in the square bracket (suppressing the subindex), i.e.  $\sigma(\mu) = (1 - \mu)^n (n\mu + 1)$ . Note that  $\sigma(0) = 1$  and  $\sigma(1) = 0$ . Moreover,  $\sigma(\mu)$  is a decreasing function of



$\mu$  for  $0 < \mu < 1$ :

$$\begin{aligned}\frac{\partial \sigma}{\partial \mu} &= -n(1-\mu)^{n-1}(n\mu+1-\mu+\mu) + (1-\mu)^n n \\ &= n[-(1-\mu)^n - (1-\mu)^{n-1}\mu(n+1) + (1-\mu)^n] = -n(1-\mu)^{n-1}\mu(n+1) < 0\end{aligned}$$

This proves that  $\sigma(\mu) = (1-\mu)^n(n\mu+1) < 1$  and, therefore,  $\partial\lambda(\mu_i, a_i)/\partial\mu_i < 0$  for  $0 < \mu < 1$ .

Next, applying the L'Hopital's rule one can show that:

$$\lim_{\mu_i \rightarrow 0} \lambda_i = va_i \frac{\lim_{\mu_i \rightarrow 0} (1 - (1 - \mu_i)^n)}{\lim_{\mu_i \rightarrow 0} \mu_i} = va_i \lim_{\mu_i \rightarrow 0} n(1 - \mu_i)^{n-1} = nva_i$$

and also  $\lim_{\mu_i \rightarrow 1} \lambda_i = 0$ . This completes the proof of lemma 2.

### Appendix III: Proof of Lemma 3

First, note that at the intersection between the curves  $\mu(\lambda, y)$  and  $\lambda(\mu, a)$  (see figure), the latter curve (NC) is flatter than the former curve (UC), this means:

$$0 > \frac{\partial\lambda(\mu, a)}{\partial\mu} > \left[ \frac{\partial\mu(\lambda, y)}{\partial\lambda} \right]^{-1} \Rightarrow 0 < \frac{\partial\lambda(\mu, a)}{\partial\mu} \cdot \frac{\partial\mu(\lambda, y)}{\partial\lambda} < 1$$

Taking a total derivative of  $\mu(\lambda, y)$  with respect to  $y$  yields the following:

$$d\mu = \frac{\partial\mu(\lambda, y)}{\partial\lambda} d\lambda + \frac{\partial\mu(\lambda, y)}{\partial y} dy \quad \text{and} \quad d\lambda = \frac{\partial\lambda(\mu, a)}{\partial\mu} d\mu$$

Therefore, we get the following result:

$$\frac{\partial m(y, a)}{\partial y} = \frac{d\mu}{dy} = \frac{\partial\mu(\lambda, y)}{\partial y} \left[ 1 - \frac{\partial\mu(\lambda, y)}{\partial\lambda} \cdot \frac{\partial\lambda(\mu, a)}{\partial\mu} \right]^{-1} < 0$$

since  $\partial\mu(\lambda, y)/\partial y < 0$  and expression in the square bracket is positive. Similarly, we can show that  $\partial m(y, a)/\partial a < 0$ .

### Appendix IV. Proof of Proposition 1:

(i) First, note the following results from before:

$$\frac{\partial m(y, a)}{\partial y} < 0 \quad \frac{\partial m(y, a)}{\partial a} < 0 \quad \frac{\partial a(m, y)}{\partial m} > 0 \quad \frac{\partial a(m, y)}{\partial y} > 0$$

Taking a total derivative of  $m(a, y)$  with respect to  $y$  yields the following:

$$\begin{aligned}dm &= \frac{\partial m(y, a)}{\partial y} dy + \frac{\partial m(y, a)}{\partial a} da \\ &= \frac{\partial m(y, a)}{\partial y} dy + \frac{\partial m(y, a)}{\partial a} \left[ \frac{\partial a(m, y)}{\partial m} dm + \frac{\partial a(m, y)}{\partial y} dy \right]\end{aligned}$$

$$dm \underbrace{\left[ 1 - \frac{\partial m(y, a)}{\partial a} \frac{\partial a(m, y)}{\partial m} \right]}_{>0} = \underbrace{\left[ \frac{\partial m(y, a)}{\partial y} + \frac{\partial m(y, a)}{\partial a} \frac{\partial a(m, y)}{\partial y} \right]}_{<0} dy \Rightarrow \frac{dm}{dy} < 0$$

(ii) Let  $\rho(y_i) \equiv [1 - (1 - \mu(y_i))^n]$  denote the probability of a referral, it then holds that  $a(y_i) =$

$0.5k\rho(y_i)J(y_i)$ , where  $J(y_i) = (1 - \alpha)(y_i - b)/(r + \delta)$  given the free-entry condition  $V = 0$ . Next differentiate  $a(y_i)$  with respect to  $y_i$  to obtain:

$$\frac{\partial a(y_i)}{\partial(y_i - b)} = \frac{\partial \rho(y_i)}{\partial(y_i - b)} J(y_i) + \rho(y_i) \frac{\partial J(y_i)}{\partial(y_i - b)} = \frac{\partial \rho(y_i)}{\partial(y_i - b)} \frac{(1 - \alpha)(y_i - b)}{r + \delta} + \rho(y_i) \frac{(1 - \alpha)}{r + \delta}$$

Therefore, 
$$\frac{\partial a(y_i)}{\partial y_i} = \frac{\partial a(y_i)}{\partial(y_i - b)} > 0 \quad \text{if} \quad \frac{\partial \rho(y_i)}{\partial(y_i - b)} \cdot \frac{(y_i - b)}{\rho(y_i)} > -1$$

**Appendix V.** Suppose the distribution  $h_i^s$  first order stochastically dominates the distribution  $h_i^o$ , then it holds  $H_i^s \leq H_i^o, \forall i = 1..p$ . The average wage  $\bar{w}^o$  can be written as:

$$\begin{aligned} \bar{w}^o &= \sum_{i=1}^p w_i h_i^o = w_1 h_1^o + w_2 h_2^o + w_3 h_3^o + \dots + w_{p-1} h_{p-1}^o + w_p h_p^o \\ &= w_1 (h_1^o + h_2^o + \dots + h_p^o) + (w_2 - w_1)(h_2^o + \dots + h_p^o) + \dots + (w_p - w_{p-1}) h_p^o \\ &= w_1 + \sum_{i=1}^{p-1} (w_{i+1} - w_i) \sum_{j=i+1}^p h_j^o = w_1 + \sum_{i=1}^{p-1} (w_{i+1} - w_i) \left(1 - \sum_{j=1}^i h_j^o\right) \\ &= w_1 + \sum_{i=1}^{p-1} (w_{i+1} - w_i) (1 - H_i^o) \end{aligned}$$

In a similar way, one can derive an equation for  $\bar{w}^s$ . So that  $\bar{w}^o < \bar{w}^s$  if  $H_i^s \leq H_i^o, \forall i = 1..p$ .

**Appendix VI:** Proof of proposition 2.

Consider the free-entry condition  $z = \sum (\frac{\lambda_0}{v} + s_i) \mu_i f_i J_i$ . In the absence of professional networks we get that  $D_i = r + \delta + \lambda_0$ . Next, applying the L'Hopital's rule one can show that:

$$\lim_{v \rightarrow 0} s_i = \frac{\lim_{v \rightarrow 0} \sqrt{D_i^2 + (w_i - b)cv} - D_i}{\lim_{v \rightarrow 0} v} = \lim_{v \rightarrow 0} \frac{0.5(w_i - b)c}{\sqrt{D_i^2 + (w_i - b)cv}} = \frac{0.5(w_i - b)c}{r + \delta + \lambda_0}$$

Similarly one can show that  $\lim_{v \rightarrow \infty} s_i = 0$ ,  $\lim_{v \rightarrow 0} \mu_i = \delta/(\delta + \lambda_0)$  and  $\lim_{v \rightarrow \infty} \mu_i = 0$ . Then we know that the right hand side of the free-entry condition is a decreasing function such that  $\lim_{v \rightarrow 0} (\frac{\lambda_0}{v} + s_i) \mu_i = \infty$  and  $\lim_{v \rightarrow \infty} (\frac{\lambda_0}{v} + s_i) \mu_i = 0$ . Thus there exists a unique intersection between the cost  $z$  on the left hand side and the expected profit on the right hand side.

To prove part (b), consider an economy with only one worker type ( $y > b$ ), where professional networks are the only channel of search. The free-entry condition ( $V = 0$ ) in this economy simplifies to yield  $z = 0.25k(1 - \mu)(1 - (1 - \mu)^n)^2 J^2$ . The first order derivative of function  $(1 - \mu)(1 - (1 - \mu)^n)$  is given by:

$$\frac{\partial(1 - \mu)(1 - (1 - \mu)^n)^2}{\partial \mu} = (1 - (1 - \mu)^n)[(1 - \mu)^n(1 + 2n) - 1]$$

Thus this function takes value zero at  $\mu = 0$ , it is then increasing to the unique maximum point at  $\mu = 1 - (1 + 2n)^{-1/n}$  and then falls down to zero for  $\mu = 1$ . So there exist two equilibria for sufficiently low value of the cost parameter  $z$ .

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