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## Product Scope and Endogenous Fluctuations

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# Product Scope and Endogenous Fluctuations\*

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## Abstract

Recent empirical evidence suggests that product creation is procyclical and it occurs largely within existing firms. Motivated by these findings, the current paper investigates the role of intra-firm product scope choice in a general equilibrium economy with oligopolistic producers. We show that the multi-product nature of firms makes the economy significantly more susceptible to sunspot equilibria. The estimated indeterminate model generates artificial business cycles that closely resemble empirically observed fluctuations.

*Keywords:* Indeterminacy, sunspot equilibria, multi-product firms, business cycles, Bayesian estimation.

*JEL Classification:* E32.

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# 1 Introduction

This paper explores a model of business cycles in which product creation and firm dynamics generate *soi-disant* sunspot equilibria which ultimately drive movements in the economy's real output. It builds on a growing body of empirical work that suggests that a large portion of firms are multi-product producers. Bernard, Redding and Schott (2010), for example, report that close to half of US manufacturing firms produce in multiple 5-digit SIC industries. The importance of this finding becomes apparent once noticing that these firms account for about 90 percent of total sales. Broda and Weinstein (2010) arrive at similar conclusions. In particular, they document that over 90 percent of product creation and destruction occurs within firms (i.e. as firms adjust their product scopes). This alludes that the contribution to aggregate output from product scope variations is at least as important as that from net business formation.

The current paper picks up on these observations by laying out an artificial economy that generates procyclical product creation within firms, while also giving rise to endogenous business cycles. Specifically, we investigate the roles of net product creation and net business formation in a general equilibrium economy with oligopolistic intermediate goods firms. Endogenous net product creation (in particular via changes in firms' product scopes) creates sunspot equilibria at very realistic parametric situations. To demonstrate this, we estimate the indeterminate model and show that a combination of both belief shocks (i.e. sunspots) and fundamental shocks generates artificial business cycles that resemble empirically observed fluctuations.

Indeterminacy arises in the economy because net business formation and firms' product scope choice affect labor demand. The two effects lead to efficiency gains; phrased alternatively, net product creation gives rise to an endogenously shifting efficiency wedge. Furthermore, the oligopolistic market structure leads to countercyclical markups that act as an additional shifter of production possibilities – as a consequence, the wage-hours locus becomes upwardly sloping. Intuitively, sunspots come into effect as follows. Assume that people feel more optimistic about the future path of income: a wealth effect that causes a rise in the demands for consumption and leisure. Labor supply shifts inwards along an upwardly sloping wage-hours locus, thereby raising employment and output, and subsequently allowing the initial expect-

tations of higher incomes to become self-fulfilling.<sup>1</sup>

Our artificial economy parallels Feenstra and Ma (2009) and Minniti and Turino (2013) who introduce multi-product firms into general equilibrium. While also studying business cycles, however, Minniti and Turino (2013) consider fundamental disturbances only.<sup>2</sup> Relating to endogenous fluctuations, Jaimovich (2007) demonstrates how procyclical net business formation can lead to indeterminacy via the generation of countercyclical markups. Pavlov and Weder (2012) investigate the role of variety effects in generating sunspot equilibria. Both of these papers feature mono-product firms and hence do not consider firms' product scope choices. Furthermore, while most of the indeterminacy literature simulates calibrated models by sunspot shocks only, we use Bayesian methods to estimate the indeterminate model with both sunspots and fundamental disturbances to preferences and technology. By and large, we follow estimation approaches put forward by Farmer, Khramov, and Nicolò (2014) and Lubik and Schorfheide (2004).<sup>3</sup> Our findings suggest that about a third of U.S. output fluctuations are related to sunspots events.

The remainder of this paper evolves as follows. Section 2 lays out the model. Section 3 analyzes the local dynamics. Variable capital utilization is added to the economy in Section 4. The indeterminate model is estimated and simulated in Section 5. Section 6 concludes.

## 2 Model

The economy consists of intermediate good firms who are large relative to the size of the market and are able to choose how many products to produce. These goods are differentiated and hence bring about market power for these firms. The commodities are bought by competitive firms that weld them together into the final good that can be consumed or, by adding it to the capital stock, invested. People own the two factors of production and rent out their respective services on competitive markets.

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<sup>1</sup>See Benhabib and Farmer (1994).

<sup>2</sup>Moreover, we separate the elasticity of substitution parameters from the variety effects (a.k.a. taste for variety or increasing returns to specialization) in the production of final goods, which makes the theoretical mechanisms in our paper far more transparent.

<sup>3</sup>See Farmer and Guo (1995) for an early attempt to estimate a sunspot model.

## 2.1 Final goods

Final output,  $Y_t$ , is produced under perfect competition using the range of intermediate inputs supplied by  $M_t$  multi-product firms indexed  $i$ . Each firm supplies  $N_t(i)$  varieties of goods. Accordingly, the final good is constructed via two nested CES aggregators. The first encompasses the varieties from an individual firm  $i$  that, when put together, compose

$$Y_t(i) = N_t(i)^{1+\tau} \left( \frac{1}{N_t(i)} \int_0^{N_t(i)} y_t(i, j)^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \quad \tau > 0, \gamma > 1. \quad (1)$$

Here  $y_t(i, j)$  is the amount of the unique intermediate good  $j$  produced by firm  $i$ . Parameters  $\tau$  and  $\gamma$  stand for the intra-firm variety effect and the elasticity of substitution between goods, respectively. The firm-composite goods are then stacked together to yield the final output

$$Y_t = M_t^{1+\omega} \left( \frac{1}{M_t} \int_0^{M_t} Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad \omega \geq 0, \theta > 1. \quad (2)$$

Here,  $\omega$  is the inter-firm variety effect and  $\theta$  is the elasticity of substitution between the firms' composite goods. Variety effects are separated from the elasticity of substitution as there is no *a priori* reason for a strong link between them.<sup>4</sup> Moreover, the separation allows us to clearly distinguish the variety effect and its impacts from that of imperfect competition. As we will see later, the intra-firm variety effect is crucial for firms to produce more than a single product. Feenstra and Ma (2009) develop a related framework in which they assume  $\theta = \gamma$ . However, Broda and Weinstein's (2010) work suggests that these parameters are not equal, accordingly we will also calibrate the model following their findings.

The profit maximization problem yields

$$y_t(i, j) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \left( \frac{p_t(i, j)}{P_t(i)} \right)^{-\gamma} M_t^{\omega(\theta-1)-1} N_t(i)^{\tau(\gamma-1)-1} Y_t \quad (3)$$

where

$$P_t(i) = N_t(i)^{\frac{1}{\gamma-1}-\tau} \left( \int_0^{N_t(i)} p_t(i, j)^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} \quad (4)$$

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<sup>4</sup>Benassy (1996).

is the price index for firm  $i$ 's goods and the aggregate price index satisfies

$$P_t = M_t^{\frac{1}{\theta-1}-\omega} \left( \int_0^{M_t} P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (5)$$

In words, the demand for each variety depends negatively on its price, positively on the aggregate price index  $P_t$ , and positively (negatively) on the firm price index  $P_t(i)$  if  $\gamma > \theta$  ( $\gamma < \theta$ ).

## 2.2 Intermediate good firms

Each intermediate firm chooses how many different products it brings to the market and at what price it sells them. These tasks are solved in two stages. In the first, product scopes are decided. During the second stage, firms set their pricing rules by acting as Bertrand competitors in the product market.<sup>5</sup> Each period, the number of active firms is determined by a zero profit condition.

Intermediate goods are produced using capital,  $k_t(i, j)$ , and labor,  $h_t(i, j)$ , that are supplied on perfectly competitive factor markets. The production technology is Cobb-Douglas and involves two fixed costs. The variety-level fixed cost,  $\phi$ , applies once a variety is added to the production line. It restricts the amount of varieties a firm will produce and at the same time implies that it is only profitable to produce multiple products if the intra-firm variety effect is operating. The firm-level fixed cost,  $\phi_f$ , provides economies of scope. It determines the number of active firms via a zero-profit condition. Hence, a firm's output is given by

$$\int_0^{N_t(i)} y_t(i, j) dj = \int_0^{N_t(i)} [k_t(i, j)^\alpha h_t(i, j)^{1-\alpha} - \phi] dj - \phi_f \quad \phi > 0, \phi_f > 0. \quad (6)$$

Each firm sets prices to maximize profits

$$\pi_t(i) = \int_0^{N_t(i)} p_t(i, j) y_t(i, j) - w_t h_t(i, j) - r_t k_t(i, j) dj \quad (7)$$

where  $w_t$  and  $r_t$  are the labor and capital rental rates. Following Yang and Heijdra (1993), intermediate good firms are large enough to take the

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<sup>5</sup>This is a subgame perfect Nash equilibrium concept.

aggregate price index into consideration when making their pricing decision.<sup>6</sup> Appendix A.2 shows that a firm charges the same price,  $p_t(i)$ , for all of its varieties. Then, the optimal markup,  $\mu_t(i) = p_t(i)/mc_t$  becomes

$$\mu_t(i) = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1}$$

where  $mc_t$  is the marginal cost of producing an additional variety, and  $\epsilon_t(i)$  is firm  $i$ 's market share:

$$\epsilon_t(i) \equiv \frac{P_t(i)Y_t(i)}{P_t Y_t} = \frac{N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta}}{\int_0^{M_t} N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta} di}.$$

which increases in the number of goods  $N_t(i)$ . This highlights the importance of the intra-firm variety effect,  $\tau$ . Without it, the market share would not depend on the product scope. Profits would be decreasing in  $N_t(i)$  because of the variety-level fixed cost  $\phi$  and hence, firms would only produce a single product.

Firms determine their optimal number of products by maximizing profits with respect to  $N_t(i)$  by taking into account the effect on its own and other firms' pricing decisions (see Appendix A.3). The first-order condition is

$$\theta P_t Y_t \left( \frac{p_t(i) - mc_t}{p_t(i)} \right)^2 \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + Y_t \epsilon_t(i) \left( \frac{p_t(i) - mc_t}{p_t(i)} \right) \frac{\partial P_t}{\partial N_t(i)} = mc_t \phi \quad (8)$$

which can be understood as follows. The first term on the left-hand side corresponds to the presence of the intra-firm variety effect: introducing a new product increases the firm's market share and its profits. The second term stands for the impact of product scope on the aggregate price index. Specifically, a higher product scope reduces the aggregate price index,  $\partial P_t / \partial N_t(i) < 0$ , which from (3) leads to a lower demand for firm  $i$ 's products. The right-hand side of (8) represents the cost of producing one additional variety.

### 2.3 Symmetric equilibrium

In the symmetric equilibrium, each firm produces the same number of varieties,  $N_t(i) = N_t$ , charges the same price,  $p_t(i) = p_t$ , and has the same

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<sup>6</sup>Appendix A.4 shows that, at least in the present framework, under monopolistic competition, product scope and markups are constant over the business cycle.

market share  $\epsilon_t(i) = 1/M_t$ . Let us designate the final good to be the numeraire,  $P_t = 1$ , and therefore from (4) and (5), the price of a variety is determined by the two variety effects:

$$p_t = N_t^\tau M_t^\omega.$$

Using the above, (1) and (2), output per variety is

$$y_t = \frac{Y_t}{p_t N_t M_t}. \quad (9)$$

The markup simplifies to

$$\mu_t = \frac{\theta(M_t - 1)}{\theta(M_t - 1) - M_t} \quad (10)$$

which decreases with firm entry. It is this mechanism that renders the markup countercyclical. Furthermore, an increase in the firm's product scope raises its own price and reduces the prices of other firms: to lower price competition, firms under-expand their product scopes in comparison to the case of monopolistic competition where such strategic linkages are absent. The extent of this under-expansion can be seen by substituting  $\partial\epsilon_t(i)/\partial N_t(i)$  and  $\partial P_t/\partial N_t(i)$  into (8) and rearranging:

$$y_t(\mu_t - 1)\tau(\theta - 1) \left[ \frac{(M_t - 1)(\theta + (1 - \theta)M_t)}{\theta(M_t - 1) + M_t^2(1 - \theta)} - \frac{1}{M_t(\theta - 1)} \right] = \phi.$$

The term in the square brackets is less than one and is increasing in  $M_t$ : the strategic effect of the product scope decision becomes less important as the number of firms increases and this gives an incentive to introduce new varieties. When  $M_t$  becomes very large this term approaches unity and the markup converges to its monopolistic competition level of  $\theta/(\theta - 1)$ . Intuitively, as the number of firms grows, the impact on the market share of adding an additional variety becomes smaller, which has then a less impact on the price of the variety. Further rearrangement yields the product scope

$$N_t = \frac{\tau Y_t}{\phi p_t} \left[ \frac{(\theta - 1)(M_t - 1)}{\theta(1 - M_t) + M_t^2(\theta - 1)} + \frac{1}{M_t[M_t(1 - \theta) + \theta]} \right].$$

Using (6), (9) and the zero profit condition determines  $M_t$  as

$$M_t = \frac{(\mu_t - 1)K_t^\alpha H_t^{1-\alpha}}{\mu_t(N_t\phi + \phi_f)} \quad (11)$$



where  $K_t = M_t N_t k_t$  and  $H_t = M_t N_t h_t$ . To obtain aggregate output, we substitute (6) in (9), and use (11) to simplify:

$$Y_t = \frac{p_t}{\mu_t} K_t^\alpha H_t^{1-\alpha} \quad (12)$$

where  $p_t/\mu_t$  is an endogenous efficiency wedge that arises in the absence of changes to fundamentals. Finally, the equilibrium real wage and rental rate are given by

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} \quad \text{and} \quad r_t = \alpha \frac{Y_t}{K_t}.$$

## 2.4 People

There is a nonatomic measure-one space of agents. We assume that the individuals' preferences depend on consumption and leisure and that they can be represented by a utility function of the form

$$U = \int_0^\infty e^{-\rho t} u(C_t, H_t) dt \quad \rho > 0.$$

Here,  $\rho$  denotes the subjective rate of time preference and period utility,  $u(., .)$ , is separable in consumption,  $C_t$ , and hours worked,  $H_t$ . It takes on the functional form

$$u(C_t, H_t) = \ln C_t - v \frac{H_t^{1+\chi}}{1+\chi} \quad v > 0, \chi \geq 0$$

where  $\chi$  is the inverse of the Frisch labor supply elasticity. Logarithmic utility is the only additive-separable form consistent with balanced growth. The agents own the capital stock and sell labor as well as capital services. Any generated profits,  $\Pi_t$ , flow back to them. Let  $X_t$  denote investment, then the period budget is constrained by

$$w_t H_t + r_t K_t + \Pi_t \geq X_t + C_t$$

where investment is added to the capital stock such that:

$$\dot{K}_t = X_t - \delta K_t \quad 0 < \delta < 1.$$

Time derivatives are denoted by dots and  $\delta$  stands for the constant rate of physical depreciation of the capital stock. The solution to the maximization problem entails

$$v H_t^\chi = \frac{w_t}{C_t} \quad (13)$$

and

$$\frac{\dot{C}_t}{C_t} = r_t - \delta - \rho. \quad (14)$$

Equation (13) describes the agents' leisure-consumption trade-off, while (14) is the intertemporal Euler equation. In addition the transversality condition must hold.

### 3 Dynamics

This section analyzes the local dynamical properties of various versions of the artificial economy. To do so, we log-linearize the equilibrium conditions and arrange the dynamical system to

$$\begin{bmatrix} \dot{K}_t/K_t \\ \dot{C}_t/C_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}.$$

Hatted variables denote percent deviations from their steady-state values and  $\mathbf{J}$  is the  $2 \times 2$  Jacobian matrix of partial derivatives. Note that  $C_t$  is a non-predetermined variable and that  $K_t$  is predetermined. Indeterminacy means that the number of stable eigenvalues of  $\mathbf{J}$  exceeds the number of predetermined variables. In the present model, for indeterminacy both roots of  $\mathbf{J}$  must be negative, i.e.  $\text{Det}\mathbf{J} > 0 > \text{Tr}\mathbf{J}$ . For numerical explorations, we calibrate standard parameters at a quarterly frequency as  $\alpha = 0.3$ ,  $\rho = 0.01$ ,  $\delta = 0.025$  and  $\chi = 0$  which is set in line with most studies of indeterminacy to make a comparison straightforward.

#### 3.1 Mono-product model

To better illustrate the contribution of the firms' product scope decisions on indeterminacy, we first consider the case of mono-product firms. Figure 1 presents the stability zones, assuming that the variety effect depends on the elasticity of substitution between intermediate goods:  $\omega = 1/(\theta - 1)$ . The figure indicates that we prohibit situations where  $\theta < \mu/(\mu - 1)$  to rule out  $M < 0$ . As can be seen, the minimum steady state markup allowing for indeterminacy is  $\mu = 1/(1 - \alpha) = 1.429$ , which implies a variety effect at  $1/(\theta - 1) = 0.429$ . This exactly corresponds to the result reported in Pavlov and Weder (2012) for a mono-product model with monopolistic competition.

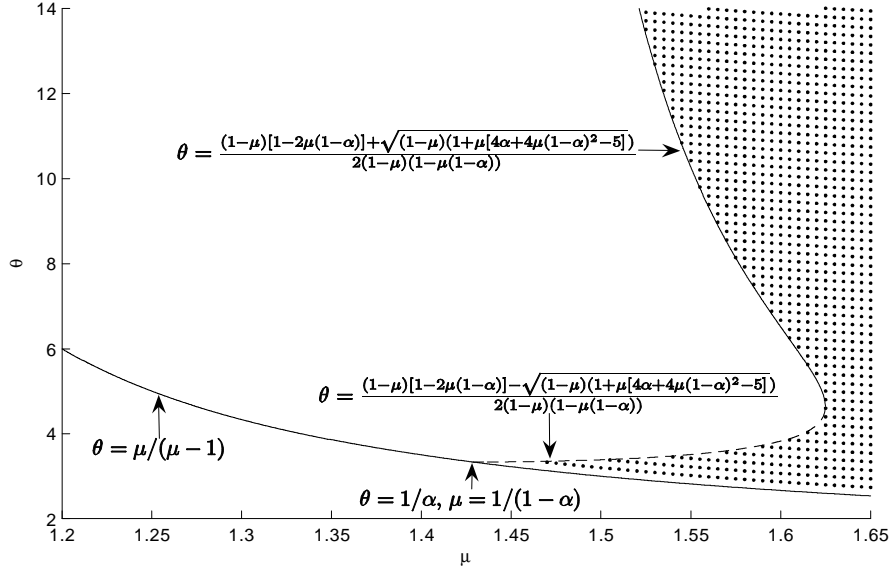


Figure 1: Mono-product model.

Why is this the case? Note that from (10), the steady state number of firms is

$$M = 1 + \frac{\mu}{\mu(\theta - 1) - \theta}.$$

Now, as  $\theta$  approaches  $\mu/(\mu - 1)$ , the number of firms approaches infinity: the markup and local dynamics converge to the case of monopolistic competition. This implies via (10) that the minimum  $\mu$  needed for generating indeterminacy is not lower under oligopolistic competition. On the other hand, Figure 1 also shows that the required variety effect drops considerably with higher values of  $\theta$ . This is because greater substitutability between differentiated goods (and hence a lower variety effect) and/or a higher steady state markup imply a lower number of firms and a more elastic markup over the business cycle. Therefore, the dashed stability line in the figure is upwardly sloping because the lower variety effect (via higher  $\theta$ ) needs to be offset by a higher markup elasticity (via higher  $\mu$ ). Yet, the line eventually becomes downwardly sloping because the gain from the higher markup elasticity starts to dominate the influence of the lower variety effect on the endogenous efficiency wedge as goods become closer substitutes.

## 3.2 Multi-product model

Figure 2 presents the numerical indeterminacy region for the multi-product model.<sup>7</sup> We set  $\omega = \tau = 1/(\theta-1) = 1/(\gamma-1)$  to allow for multi-product firms. Once again, the model converges to the one with monopolistic competition along the  $\theta = \mu/(\mu - 1)$  line. This is because the equality implies that both the markups and the product scopes are constant over the business cycle (see Appendix A.4). Under oligopolistic competition, however, the entry of new competitors reduces existing firms' market shares and encourages them to expand their product scopes. This additional channel of product creation reduces the minimum steady state markup, for example, for elasticities of substitution at  $\theta = \gamma = 14$ , a markup of  $\mu = 1.3$  is enough for indeterminacy.<sup>8</sup> At this point, the variety effect is only  $\omega = \tau = 0.077$  compared to the required size of 0.429 under monopolistic competition. Phrased alternatively, the complementarity feature of oligopolistic markets and endogenous product choice makes sunspot equilibria much easier to obtain.<sup>9</sup>

## 4 Capital utilization

The last section has demonstrated that when firms are able to choose their product scopes the possibility of sunspot equilibria increases. Next, it is shown that levels of market power can be reduced even further by augmenting the multi-product model by variable capital utilization. Each intermediate good firm  $i$  now operates the production technology

$$\int_0^{N_t(i)} y_t(i, j) dj = \int_0^{N_t(i)} [U_t^\alpha k_t(i, j)^\alpha h_t(i, j)^{1-\alpha} - \phi] dj - \phi_f$$

where  $U_t$  stands for the utilization rate of capital set by its owners. Capital accumulation follows

$$\dot{K}_t = X_t - \delta_t K_t = X_t - \frac{1}{\varrho} U_t^\varrho K_t \quad \varrho > 1.$$

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<sup>7</sup>We report numerical results since analytical expressions became simply too incommensurable.

<sup>8</sup>It can be shown that for very high values of  $\theta$ , the markup required for indeterminacy is as low as 1.05, albeit this appears in only a small parametric region.

<sup>9</sup>Figure A1 shows that the removal of the inter-firm variety effect retains the plausibility of indeterminacy in the multi-product model.

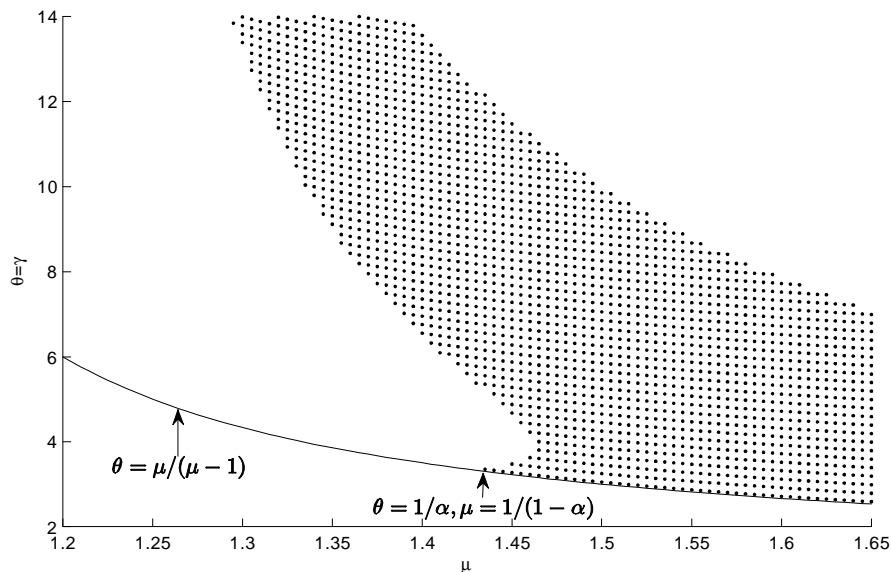


Figure 2: Multi-product model,  $\theta = \gamma$ .

In the symmetric equilibrium, the aggregate production function is

$$Y_t = \frac{p_t}{\mu_t} (U_t K_t)^\alpha H_t^{1-\alpha}$$

and the optimal rate of capital utilization entails

$$r_t = U_t^{\varrho-1}.$$

The calibration remains as in the previous section and  $\varrho = (\rho + \delta)/\delta = 1.4$  follows from steady state first-order conditions.<sup>10</sup> Then, Figure 3 demonstrates how the introduction of variable capital utilization significantly reduces the level of market power and the elasticities of substitution that are required for indeterminacy. In particular, the minimum steady state markup falls below 1.1. This occurs because, like lower markups and higher product variety, higher utilization increases the demand for labor.

Figure 4 allows  $\gamma \neq \theta$ . Estimates of the level of markups in the U.S. in value added data range from 1.2 to 1.4 and so our choice of  $\mu = 1.3$  lies in

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<sup>10</sup>See Wen (1998).

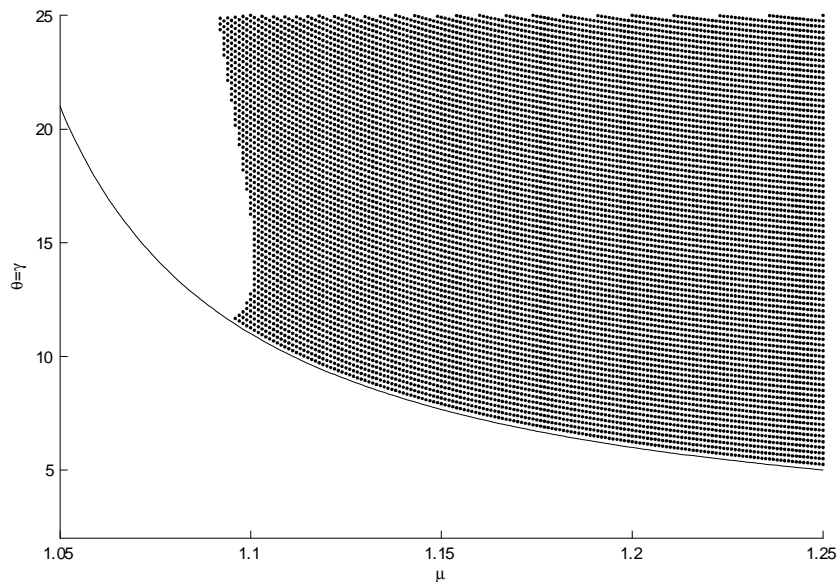


Figure 3: Multi-product model with variable capital utilization,  $\theta = \gamma$ .

the middle of these numbers (see Jaimovich, 2007). Again, the figure's line indicated by  $\theta = \mu/(\mu - 1)$  guarantees a strictly positive number of firms,  $M$ . Sunspots now become a very realistic scenario in the multi-product economy. Broda and Weinstein's (2010) estimation suggests that  $\theta = 7.5$  and  $\gamma = 11.5$  are plausible values for the two elasticities of substitution. Clearly, this  $\gamma - \theta$  combination entails indeterminacy.

Moreover, given the above combination of  $\gamma$  and  $\theta$  values, for a positive  $M$  the steady state markup must be  $\mu = 1.154 = 7.5/(7.5 - 1)$  or higher. Yet, at that value, the economy is in a sunspot equilibrium.<sup>11</sup> We conclude that sunspot equilibria are well in line with what could be considered a empirically reasonable calibration.

To further gain understanding about the effect of sunspots, the impulse responses of various variables are plotted in Figure 5 - on impact, the sunspot shock moves output 1 percent above its steady state. The calibration of the discrete-time version of the economy involves  $\alpha = 0.3$ ,  $\delta = 0.025$ ,  $\chi = 0$ , a discount factor at  $\beta \equiv (1 + \rho)^{-1} = 0.99$  and Broda and Weinstein's (2010)

<sup>11</sup>Changing the steady state markup leaves the sunspots zone basically unchanged while the  $\theta = \mu/(\mu - 1)$  line shifts up or down. See Figure A2.

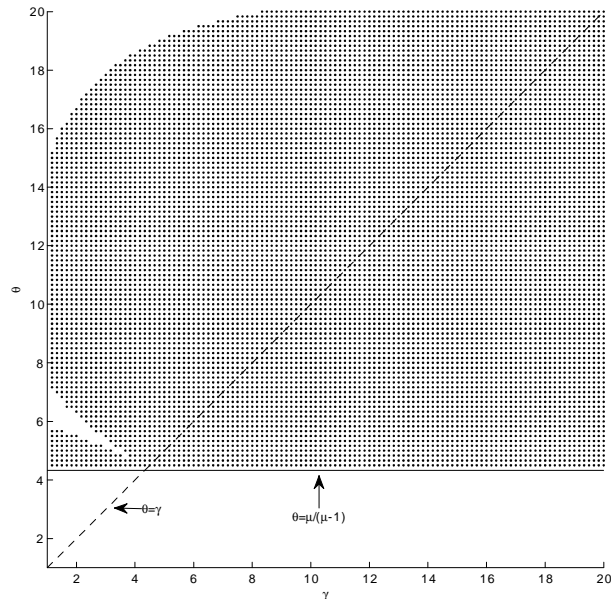


Figure 4: Multi-product model with variable utilization,  $\mu = 1.3$ .

suggestion that  $\theta = 7.5$  and  $\gamma = 11.5$ . Additionally, the steady state markup is set equal to 1.3 (the next section contains justification for this assumption). The impulse response functions reveal that both net product creation and net business formation positively comove with output, with the former being more volatile than the latter. It can also be seen that output per variety is countercyclical. This is due to the cannibalization effect: an introduction of a new variety reduces the demand for existing varieties. The markup fluctuates countercyclically. These combined effects lead to an upwardly sloped wage-hours locus which gives way to the self-fulfilling sunspots mechanism outlined earlier.

## 5 Estimation and simulations

We have shown that intra-firm product creation can generate indeterminacy under very plausible situations. Although this can be considered as progress, it would be rendered void if the model is unable to replicate the basic business cycle facts. This is done next by using U.S. quarterly data to estimate the

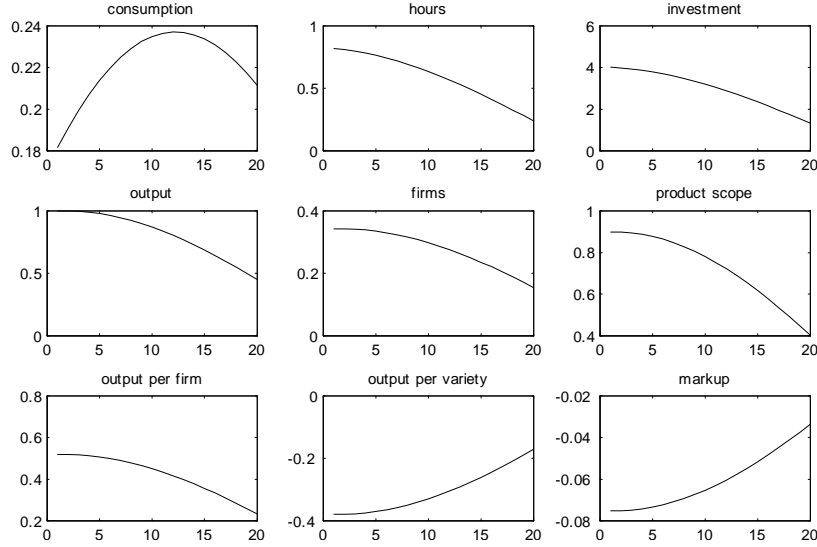


Figure 5: Impulse responses to a sunspot shock (percent deviations from the steady state).

indeterminate model and then comparing simulation results with a set of moments that characterize U.S. aggregate fluctuations (see Appendix A.5 for the data sources).

## 5.1 The model

The model employed here is a discrete time economy with capital utilization – parametric sunspot zones are roughly identical to the continuous time variant of the artificial economy. We furthermore add fundamental aggregate supply and demand shocks to the economy. The first source of fundamental uncertainty, labor augmenting technological progress,  $A_t$ , affects all firms equally and implies that aggregate output is given by a version of (12):

$$Y_t = \frac{p_t}{\mu_t} (U_t K_t)^\alpha (A_t H_t)^{1-\alpha}.$$

It is non-stationary and follows the process

$$\ln A_t = \ln A_{t-1} + \ln g_t$$



where

$$\ln g_t = (1 - \psi_A) \ln g + \psi_A \ln g_{t-1} + \varepsilon_t^A, \quad 0 \leq \psi_A < 1.$$

Here  $\ln g$  is the average growth rate and  $\varepsilon_t^A$  is an i.i.d. disturbance with variance  $\sigma_A^2$ .<sup>12</sup> The second fundamental disturbance is a preference shock to the agent's utility of consumption – a stand-in for aggregate demand shocks. Period utility now takes the form

$$u(C_t, H_t) = \ln(C_t - \Delta_t) - v \frac{H_t^{1+\chi}}{1+\chi}$$

where a positive shock to  $\Delta_t$  increases the marginal utility of consumption that leads to an urge to consume as in Baxter and King (1992) or Weder (2006). It follows the process

$$\ln \Delta_t = \psi_\Delta \ln \Delta_{t-1} + \varepsilon_t^\Delta, \quad 0 \leq \psi_\Delta < 1$$

with the shock variance  $\sigma_\Delta^2$ . In the model, this shock drives the economy's labor wedge, i.e. the gap between the marginal rate of consumption-leisure substitution and the marginal product of labor. Hence, our estimation will allow a much wider interpretation than mere shocks to preferences – a more agnostic reading would include changes to monetary policy for example.

It is now well known that under indeterminacy, the economy's response to shocks is not uniquely determined and that sunspots propagate fundamental disturbances (see Lubik and Schorfheide, 2003 and 2004). We follow Farmer, Khramov, and Nicolò (2014) in dealing with such loose expectation errors. Specifically, we reclassify the expectation error to output,  $\eta_t^Y$ , as a new exogenous shock:<sup>13</sup>

$$\hat{Y}_t = E_{t-1} \hat{Y}_t + \eta_t^Y.$$

Understanding that fundamental shocks have an effect on output on impact, we go a step further by breaking down the expectation error into fundamental and non-fundamental components:

$$\eta_t^Y = \Omega_A \varepsilon_t^A + \Omega_\Delta \varepsilon_t^\Delta + \varepsilon_t^s$$

where the parameters  $\Omega_A$  and  $\Omega_\Delta$  determine the effect of technology and preferences shocks on output and  $\varepsilon_t^s$  is an i.i.d. sunspot shock that is independent of fundamentals with variance  $\sigma_s^2$ .

<sup>12</sup>Since  $A_t$  displays a stochastic trend, the model is then detrended. For example, detrended output is given by  $\tilde{Y}_t = Y_t/A_t$  and  $\hat{Y}_t = \ln \tilde{Y}_t - \ln \tilde{Y}$ , where  $\tilde{Y}$  is the steady state value.

<sup>13</sup>Our results are robust to the choice of expectation error.

## 5.2 Estimation

The model is estimated via Bayesian methods using the quarterly real per capita growth rates of output, consumption, investment and the logarithm of per capita hours worked from 1948:I-2012:IV as observables.<sup>14</sup> The measurement equation is thus

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln X_t - \ln X_{t-1} \\ \ln H_t - \ln H \end{bmatrix} = \begin{bmatrix} \hat{Y}_t - \hat{Y}_{t-1} + \hat{g}_t \\ \hat{C}_t - \hat{C}_{t-1} + \hat{g}_t \\ \hat{X}_t - \hat{X}_{t-1} + \hat{g}_t \\ \hat{H}_t \end{bmatrix} + \begin{bmatrix} \ln g \\ \ln g \\ \ln g \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m.e.} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where  $\varepsilon_t^{m.e.}$  is a measurement error restricted to account for not more than ten percent of output growth.

The parameters that are calibrated remain the same as in the previous sections:  $\alpha = 0.3$ ,  $\delta = 0.025$ ,  $\chi = 0$ ,  $\beta = 0.99$ ,  $\theta = 7.5$ , and  $\gamma = 11.5$ . Furthermore, the quarterly growth rate of per capita real GDP implies that the growth rate of labor augmenting technological progress is  $\ln g = 0.0046$ .

The remaining parameters are estimated using the stochastic artificial economy in log-linear form. These parameters are the steady state markup,  $\mu$ , the parameters that portray the stochastic processes, i.e.  $\psi_A$ ,  $\psi_\Delta$ ,  $\sigma_s$ ,  $\sigma_A$ ,  $\sigma_\Delta$ ,  $\Omega_A$ ,  $\Omega_\Delta$  and a measurement error  $\sigma^{m.e.}$ . We follow Christiano, Trabandt, and Walentin (2011) by using endogenous priors to prevent overly high estimated model variances. Table 1 presents the initial prior and posterior distributions for the estimated parameters. We assume a gamma distribution for  $\mu$  with a lower limit of 1.154 to keep the steady state number of firms strictly positive, i.e.  $M > 0$ . The mean is centered around the middle of value-added markup estimates for the U.S. (see Jaimovich, 2007). A wide uniform distribution is employed for the expectation error parameters  $\Omega_A$  and  $\Omega_\Delta$ . The other parameters follow quite standard calibrations, hence, we refrain from expounding on these. We use the Metropolis-Hastings algorithm to obtain 500,000 draws from the posterior mean and adjust the scale in the jumping distribution to achieve a 30 percent acceptance rate.

As can be seen from the table, all estimated parameters are relatively precise as revealed by the percentiles. The estimated markup is well inside the empirically plausible range. High persistence is found for preference

<sup>14</sup>Clearly, we would have liked to include data on the number of firms and the product scope. However, no (long) time series are available for these variables.

shocks while the persistence of the shock to the growth rate of technology is close to zero. The signs of  $\Omega_A$  and  $\Omega_\Delta$  are as expected since detrended output also falls (rises) in response to permanent technology (demand) shocks in the determinate version of this economy as well as in a plain-vanilla RBC model.<sup>15</sup>

Prior distribution for model parameters					Posterior distribution	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
$\mu$	[1.154, $+\infty$ ]	Gamma	1.3	0.05	1.334	[1.319, 1.348]
$\psi_A$	[0, 1)	Beta	0.5	0.2	0.024	[0.010, 0.037]
$\psi_\Delta$	[0, 1)	Beta	0.5	0.2	0.982	[0.977, 0.987]
$\sigma_s$	$R^+$	Inverse Gamma	0.1	Inf	0.692	[0.657, 0.727]
$\sigma_A$	$R^+$	Inverse Gamma	0.1	Inf	0.789	[0.751, 0.825]
$\sigma_\Delta$	$R^+$	Inverse Gamma	0.1	Inf	0.527	[0.505, 0.549]
$\sigma^{m.e.}$	[0, 0.35]	Uniform	0.175	0.101	0.350	[0.349, 0.350]
$\Omega_A$	[-3, 3]	Uniform	0	1.732	-0.414	[-0.502, -0.325]
$\Omega_\Delta$	[-3, 3]	Uniform	0	1.732	1.028	[0.909, 1.145]

Inf implies two degrees of freedom for the inverse gamma distribution. Standard deviations are in percent terms.

Table 2 shows that the model fits the data well. The table presents the second moments of the U.S. data and of the estimated artificial economy. The model slightly overpredicts the variance of the growth rates but does a better job at matching the variances of the Hodrick Prescott (HP) filtered series. The relative volatilities as well as the co-movements of the main macroeconomic variables line up with data. Furthermore, as can be seen by the autocorrelation functions (ACF), the rich internal propagation mechanism of the indeterminate model produces persistence in the growth rates without having to rely on various real frictions used in the literature.

<sup>15</sup>While detrended output falls in response to permanent technology shocks, Figure A3 demonstrates that output per capita rises above its trend.

Business Cycle Dynamics						
	Data			Model		
$x$	$\sigma_x$	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF	$\sigma_x$	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	0.98	1	0.37	1.18	1	0.21
$\ln(C_t/C_{t-1})$	0.57	0.52	0.21	0.89	0.73	0.03
$\ln(X_t/X_{t-1})$	2.43	0.67	0.54	3.34	0.76	0.34
$\ln(H_t/H_{t-1})$	0.93	0.74	0.64	0.85	0.79	0.27
		$\rho(x, Y)$			$\rho(x, Y)$	
$Y_t$	1.69	1	0.85	1.44	1	0.80
$C_t$	0.89	0.78	0.82	0.87	0.74	0.73
$X_t$	4.99	0.79	0.89	4.71	0.90	0.82
$H_t$	1.98	0.88	0.90	1.14	0.99	0.80

$\sigma_Y$  denotes the standard deviation of output and  $\rho(x, Y)$  is the correlation of variable  $x$  and output. The last four variables have been HP filtered.

The relative contribution of each of the three shocks to output, consumption, investment and hours worked is displayed via a variance decomposition (Table 3). When considering growth rates, the decomposition suggests that output fluctuations are caused by an about equal split between the three disturbances. Investment appears to be mainly driven by sunspots and movements in consumption are largely caused by demand and technology shocks. The importance of sunspots remains largely unchanged when the series are HP filtered. However, the role of technology shocks slightly diminishes as demand shocks now explain most of consumption fluctuations.

Variance Decomposition				
	$\varepsilon_t^s$	$\varepsilon_t^A$	$\varepsilon_t^\Delta$	$\varepsilon_t^{m.e.}$
$\ln(Y_t/Y_{t-1})$	39.86	22.39	28.91	8.85
$\ln(C_t/C_{t-1})$	2.59	46.90	50.51	0
$\ln(X_t/X_{t-1})$	73.47	12.29	14.24	0
$\ln(H_t/H_{t-1})$	51.68	11.41	36.91	0
$Y_t$	48.14	21.79	30.07	0
$C_t$	4.96	11.31	83.72	0
$X_t$	67.11	20.27	12.62	0
$H_t$	52.19	13.68	34.13	0

### 5.3 Robustness

To demonstrate the robustness of the above insights, we next put forward two alternative models. Model 2 picks the forecast error on consumption

(instead of output) as the exogenous sunspot shock,  $\eta_t^C$ . As before, it is split into fundamental and non-fundamental components. In Model 3 we follow the approach of Farmer, Khramov, and Nicolò (2014). Here, the sunspot shock is simply the forecast error, i.e.  $\eta_t^Y = \varepsilon_t^s$ , with variance  $\sigma_\eta^2$ . Intuitively, since output is forward looking, this expectation error should be correlated with fundamental shocks. Yet, it is also a sunspot shock as it can cause movements in economic activity without any shifts to fundamentals. Assuming a uniform distribution, we thus estimate the correlations between  $\eta_t^Y$  and the fundamental shocks,  $\varepsilon_t^A$  and  $\varepsilon_t^\Delta$ . The priors for the other parameters are kept the same as in the baseline model. As can be seen in Table 4, and this echoes the findings of Farmer, Khramov, and Nicolò (2014), our estimation results are robust to the choice and formation of the expectation error.<sup>16</sup>

Name	Model 2: $\eta_t^C$		Model 3: $\eta_t^Y = \varepsilon_t^s$	
	Mean	90% Interval	Mean	90% Interval
$\mu$	1.333	[1.319,1.347]	1.333	[1.319,1.347]
$\psi_A$	0.024	[0.011,0.037]	0.024	[0.011,0.038]
$\psi_\Delta$	0.982	[0.977,0.987]	0.982	[0.977,0.987]
$\sigma_s$	0.126	[0.113,0.139]	-	-
$\sigma_A$	0.788	[0.751,0.825]	0.789	[0.752,0.825]
$\sigma_\Delta$	0.527	[0.506,0.548]	0.527	[0.505,0.548]
$\sigma^{m.e.}$	0.350	[0.349,0.350]	0.350	[0.349,350]
$\Omega_A$	-0.234	[-0.254,-0.214]	-	-
$\Omega_\Delta$	1.187	[1.165,1.208]	-	-
$\sigma_\eta$	-	-	0.938	[0.898,0.977]
$\rho_{A,\eta}$	-	-	-0.347	[-0.428,-0.268]
$\rho_{\Delta,\eta}$	-	-	0.575	[0.519,632]

## 6 Conclusion

Previous studies have shown that procyclical product creation via entry and exit of mono-product firms can be an important source of sunspot equilibria. Yet, recent empirical evidence suggests that product creation occurs largely within existing firms. Motivated by these findings, the current paper investigates the role of intra-firm product scope adjustments in a general equilibrium economy with oligopolistic producers. It shows that the multi-product nature of firms makes the economy significantly more susceptible

<sup>16</sup>The model fit is virtually identical to Table 2 and is not presented to conserve space.

to sunspot equilibria. The estimated indeterminate model driven by both belief and fundamental disturbances generates artificial business cycles that closely resemble empirically observed fluctuations. Our study elucidates that sunspots cause a non-negligible portion of the U.S. business cycle.

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## A Appendix

### A.1 Price elasticity of demand

This Appendix derives the demand elasticities of an intermediate good with respect to changes in its own price and the price of other goods produced by the same firm. Taking logs of (3) we obtain

$$\begin{aligned} \ln y_t(i, j) &= -\gamma \ln p_t(i, j) - (\theta - \gamma) \ln P_t(i) + \theta \ln P_t + \ln Y_t \\ &\quad + [\tau(\gamma - 1) - 1] \ln N_t(i) + [\omega(\theta - 1) - 1] \ln M_t. \end{aligned}$$

From (4)

$$\frac{\partial \ln P_t(i)}{\partial \ln p_t(i, j)} = \left( \frac{p_t(i, j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}.$$

Then from (5)

$$\frac{\partial \ln P_t}{\partial \ln p_t(i, j)} = \left( \frac{p_t(i, j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1}.$$

Then the price elasticity of demand is

$$\begin{aligned} \frac{\partial \ln y_t(i, k)}{\partial \ln p_t(i, j)} &= \underbrace{-\gamma}_{\text{absent for } k \neq j} - (\theta - \gamma) \left( \frac{p_t(i, j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \quad (\text{A.1}) \\ &+ \theta \left( \frac{p_t(i, j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1}. \end{aligned}$$

Note that under monopolistic competition, firms are too small to influence the aggregate price index,  $P_t$ , and hence the last term in (A.1) would be absent.

## A.2 Markups

This Appendix derives the optimal markups of intermediate good firms. Firm  $i$  maximizes profit (7) subject to the constraint (6):

$$\begin{aligned} \mathcal{L} &= \int_0^{N_t(i)} p_t(i, j) y_t(i, j) - w_t h_t(i, j) - r_t k_t(i, j) dj \\ &+ \Lambda_t \left( \int_0^{N_t(i)} [z_t k_t(i, j)^\alpha h_t(i, j)^{1-\alpha} - \phi] dj - \phi_f - \int_0^{N_t(i)} y_t(i, j) dj \right). \end{aligned}$$

Optimality gives

$$\frac{\partial \mathcal{L}}{\partial p_t(i, j)} = y_t(i, j) + \int_0^{N_t(i)} [p_t(i, j) - \Lambda_t] \frac{\partial y_t(i, j)}{\partial p_t(i, j)} dj = 0 \quad (\text{A.2})$$

$$\frac{\partial \mathcal{L}}{\partial h_t(i, j)} = -w_t + \Lambda_t (1 - \alpha) z_t k_t(i, j)^\alpha h_t(i, j)^{-\alpha} = 0 \quad (\text{A.3})$$

$$\frac{\partial \mathcal{L}}{\partial k_t(i, j)} = -r_t + \Lambda_t \alpha z_t k_t(i, j)^{\alpha-1} h_t(i, j)^{1-\alpha} = 0. \quad (\text{A.4})$$



The Lagrange multiplier,  $\Lambda_t$ , is obtained by combining (A.3) and (A.4) and amounts to the marginal cost,  $mc_t$ , of producing one more variety:

$$mc_t \equiv \Lambda_t = \frac{w_t^{1-\alpha} r_t^\alpha}{z_t(1-\alpha)^{1-\alpha} \alpha^\alpha}.$$

Hence, the costs of production are

$$\int_0^{N_t(i)} w_t h_t(i, j) + r_t k_t(i, j) dj = mc_t \left( \int_0^{N_t(i)} [y_t(i, j) + \phi] dj + \phi_f \right)$$

and profits are

$$\pi_t(i) = \int_0^{N_t(i)} y_t(i, j) [p_t(i, j) - mc_t] dj - mc_t [N_t(i)\phi + \phi_f]. \quad (\text{A.5})$$

Substituting (A.1) into (A.2) and some algebra yields

$$\begin{aligned} y_t(i, j) - \gamma \frac{y_t(i, j)}{p_t(i, j)} [p_t(i, j) - mc_t] &= \int_0^{N_t(i)} \frac{y_t(i, k)}{p_t(i, j)} [p_t(i, k) - mc_t] dk \\ &\times \left( \frac{p_t(i, j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left[ \theta - \gamma + \theta \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \right]. \end{aligned}$$

Substituting (3) for  $y_t(i, j)$ , the above equation simplifies to

$$\begin{aligned} P_t Y_t \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \left[ 1 - \gamma \frac{p_t(i, j) - mc_t}{p_t(i, j)} \right] &= \\ \int_0^{N_t(i)} y_t(i, k) [p_t(i, k) - mc_t] dk \left[ \theta - \gamma - \theta \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \right]. \end{aligned}$$

As the second part of this equation is the same for all  $j \in [0, N_t(i)]$ , this implies that firm  $i$  will charge the same price for all of its varieties. Hence,  $p_t(i, j) = p_t(i, k) = p_t(i)$  and the equation simplifies to

$$\begin{aligned} 1 - \gamma \frac{p_t(i) - mc_t}{p_t(i)} &= \quad (\text{A.6}) \\ N_t(i)^{\tau(\gamma-1)} \left( \frac{p_t(i)}{P_t(i)} \right)^{1-\gamma} \frac{p_t(i) - mc_t}{p_t(i)} \left[ \theta - \gamma - \theta \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \right]. \end{aligned}$$

To solve for firm  $i$ 's markup, first note from (4) that  $P_t(i) = N_t(i)^{-\tau} p_t(i)$ . Then using this together with (1), (3) and (5), we can express firm  $i$ 's market share,  $\epsilon_t(i) \equiv P_t(i)Y_t(i)/(P_t Y_t)$ , as

$$\epsilon_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} = \frac{N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta}}{\int_0^{M_t} N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta} di}. \quad (\text{A.7})$$

As long as  $\tau > 0$ , the price index  $P_t(i)$  is decreasing in  $N_t(i)$ , and so increasing the product scope increases the firm's market share. Finally, the markup,  $\mu_t(i) \equiv p_t(i)/mc_t$ , can be found by rearranging (A.6):

$$\mu_t(i) = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1}. \quad (\text{A.8})$$

### A.3 Product scope

This Appendix derives the firms' optimal product scope. Substituting (3) into (A.5), then using (4) and (A.7), we rewrite profits as

$$\pi_t(i) = \left( \frac{p_t(i) - mc_t}{p_t(i)} \right) P_t Y_t \epsilon_t(i) - mc_t [N_t(i)\phi + \phi_f].$$

Firm  $i$  takes the number of firms and their product scopes as given and maximizes its profits with respect to  $N_t(i)$  by taking account the effect of its product scope decision on its own and all other producers' pricing decisions. The first-order condition is

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \theta P_t Y_t \left( \frac{p_t(i) - mc_t}{p_t(i)} \right)^2 \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + Y_t \epsilon_t(i) \left( \frac{p_t(i) - mc_t}{p_t(i)} \right) \frac{\partial P_t}{\partial N_t(i)} - mc_t \phi = 0. \quad (\text{A.9})$$

We now calculate  $\partial \epsilon_t(i)/\partial N_t(i)$  and  $\partial P_t/\partial N_t(i)$  and then substitute in (A.9) to obtain firm  $i$ 's product scope. Differentiating (A.7) with respect to  $N_t(i)$  yields

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1) \frac{\epsilon_t(i)}{N_t(i)} - (\theta - 1)\epsilon_t(i) \left[ \frac{1}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} - \frac{1}{P_t} \frac{\partial P_t}{\partial N_t(i)} \right]. \quad (\text{A.10})$$

Note that the second term on the right hand side of (A.10) would not be present in the case of monopolistic competition. As we will see,  $\partial p_t(i)/\partial N_t(i)$  and  $\partial P_t/\partial N_t(i)$  are positive and negative, respectively; implying that firms

contract their product scopes compared to the case of monopolistic competition. We rewrite the aggregate price index (5) as

$$P_t = M_t^{\frac{1}{\theta-1}-\omega} \left( \int_0^{M_t} N_t(k)^{-\tau(1-\theta)} p_t(k)^{1-\theta} dk \right)^{\frac{1}{1-\theta}}.$$

Then, after some algebra  $\partial P_t / \partial N_t(i)$  can be expressed as

$$\frac{\partial P_t}{\partial N_t(i)} = P_t^\theta M_t^{\omega(\theta-1)-1} \left[ \int_0^{M_t} N_t(k)^{-\tau(1-\theta)} p_t(k)^{-\theta} \frac{\partial p_t(k)}{\partial N_t(i)} dk - \tau N_t(i)^{-\tau(1-\theta)-1} p_t(i)^{1-\theta} \right]. \quad (\text{A.11})$$

We now show that the first term in the square brackets is equal to zero. From (A.8)

$$\frac{p_t(k)}{p_t(k) - mc_t} = \theta - \theta \epsilon_t(k).$$

Then

$$\int_0^{M_t} \frac{p_t(k)}{p_t(k) - mc_t} dk = \theta M_t - \theta.$$

Differentiating with respect to  $N_t(i)$  gives

$$\int_0^{M_t} -\frac{mc_t}{[p_t(k) - mc_t]^2} \frac{\partial p_t(k)}{\partial N_t(i)} dk = 0$$

which under symmetry collapses to

$$(M_t - 1) \frac{\partial p_t(k)}{\partial N_t(i)} + \frac{\partial p_t(i)}{\partial N_t(i)} = 0.$$

Replacing  $\partial p_t(k) / \partial N_t(i)$  in (A.11) with  $-[\partial p_t(i) / \partial N_t(i)] / (M_t - 1)$  and assuming symmetry, the first term in the square brackets drops out and some rearrangement yields

$$\frac{\partial P_t}{\partial N_t(i)} = -\tau P_t \frac{\epsilon_t(i)}{N_t(i)}.$$

An increase in the product scope therefore reduces the aggregate price index. Inserting this result in (A.10) gives

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1) \frac{\epsilon_t(i)}{N_t(i)} [1 - \epsilon_t(i)] - (\theta - 1) \frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)}. \quad (\text{A.12})$$

The next step is to compute  $\partial p_t(i)/\partial N_t(i)$ . From (A.8) we obtain

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{\theta m c_t}{[1 - \theta + \theta \epsilon_t(i)]^2} \frac{\partial \epsilon_t(i)}{\partial N_t(i)}.$$

Then using this in (A.12) and some simplification yields

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1) \frac{\epsilon_t(i)}{N_t(i)} \frac{[1 - \epsilon_t(i)]^2 (\theta [1 - \epsilon_t(i)] - 1)}{\theta (1 - \epsilon_t(i)) [1 - \epsilon_t(i)] - 1}.$$

Here,  $\partial \epsilon_t(i)/\partial N_t(i) > 0$  and hence  $\partial p_t(i)/\partial N_t(i) > 0$ . Inserting  $\partial \epsilon_t(i)/\partial N_t(i)$  and  $\partial P_t/\partial N_t(i)$  into (A.9), assuming symmetry where  $\epsilon_t(i) = \epsilon_t = 1/M_t$ , and some rearrangement gives

$$N_t = \frac{\tau P_t Y_t}{p_t \phi} \left[ \frac{(\theta - 1)(M_t - 1)}{\theta(1 - M_t) + M_t^2(\theta - 1)} + \frac{1}{M_t[M_t(1 - \theta) + \theta]} \right].$$

#### A.4 Monopolistic competition

This Appendix shows that under monopolistic competition, markups and the product scope are constant over the business cycle. Moreover, this implies that the local dynamics and conditions for indeterminacy are identical to the mono-product model described in Pavlov and Weder (2012).

The procedure is similar to that used in the previous appendices. When firms are too small to influence the aggregate price index,  $P_t$ , the price elasticity of demand becomes

$$\frac{\partial \ln y_t(i, k)}{\partial \ln p_t(i, j)} = \underbrace{-\gamma}_{\text{absent for } k \neq j} - (\theta - \gamma) \left( \frac{p_t(i, j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}.$$

Substituting it in (A.2) and some algebra gives a constant markup

$$\mu_t(i) = \frac{p_t(i)}{m c_t} = \frac{\theta}{\theta - 1}.$$

In determining the product scope, differentiating profit by  $N_t(i)$  leads to the first order condition

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \frac{P_t Y_t}{\theta} \frac{\partial \epsilon_t(i)}{\partial N_t(i)} - m c_t \phi = 0$$

where

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1) \frac{\epsilon_t(i)}{N_t(i)}$$

and hence

$$N_t = \frac{\tau P_t Y_t}{\phi p_t M_t}.$$

Substituting  $N_t$  in firm  $i$ 's profits,  $\pi_t(i) = 0$ , and solving for  $M_t$  gives

$$M_t = \frac{P_t Y_t}{p_t \phi_f} \left[ \frac{1}{\theta - 1} - \tau \right].$$

Finally, combining the last two equations yields:

$$N_t = \tau \frac{\phi_f}{\phi} \left[ \frac{1}{\theta - 1} - \tau \right]^{-1}.$$

Since the markup and the product scope are constant over the business cycle, the linearized model is identical to the constant markup mono-product model presented in Pavlov and Weder (2012).

## A.5 Data Sources

This Appendix details the source and construction of the U.S. data used in Section 5. All data is quarterly and for the period 1948:I-2012:IV.

1. Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2009) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Private Domestic Investment, Fixed Investment, Residential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Nonfarm Business Hours. Index 2009=100, seasonally adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

8. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

9. GDP Deflator = (2)/(1).

10. Real Per Capita Consumption,  $C_t = [(3) + (4)]/(9)/(8)$ .

11. Real Per Capita Investment,  $X_t = [(5) + (6)]/(9)/(8)$ .

12. Real Per Capita Output,  $Y_t = (1)/(8)$ .

13. Per Capita Hours Worked,  $H_t = (7)/(8)$ .

## A.6 Extra Figures

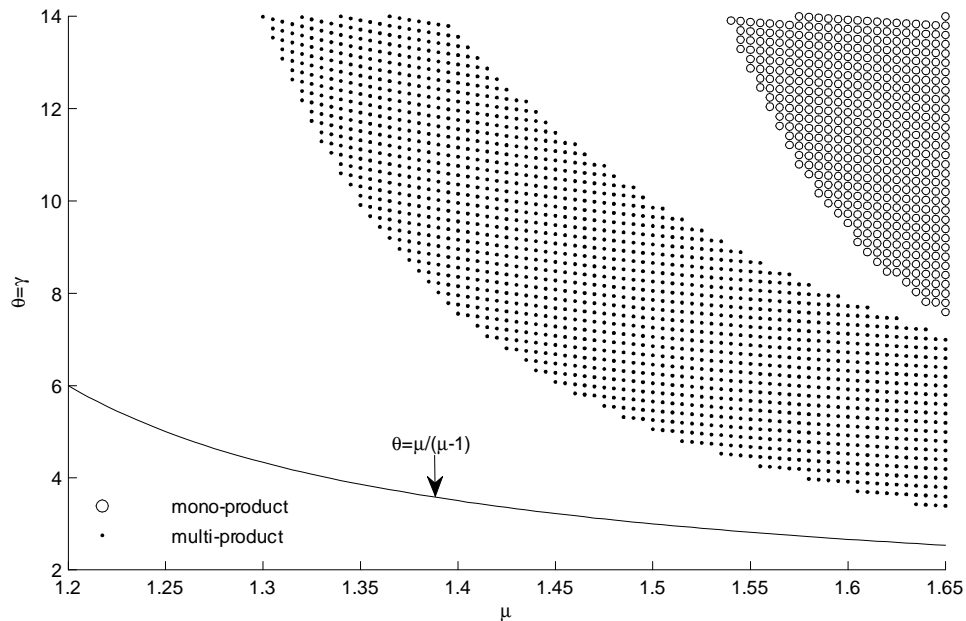


Figure A1: Multi and mono-product models with  $\omega = 0$ .

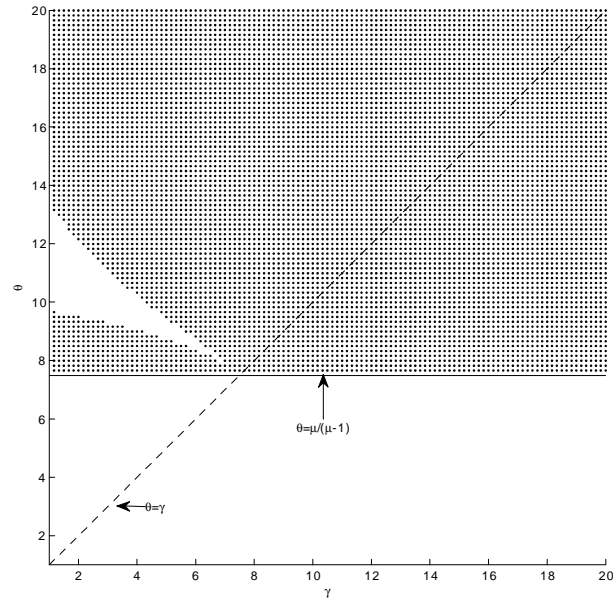


Figure A2: Multi-product model with variable capital utilization,  $\mu = 1.154$ .

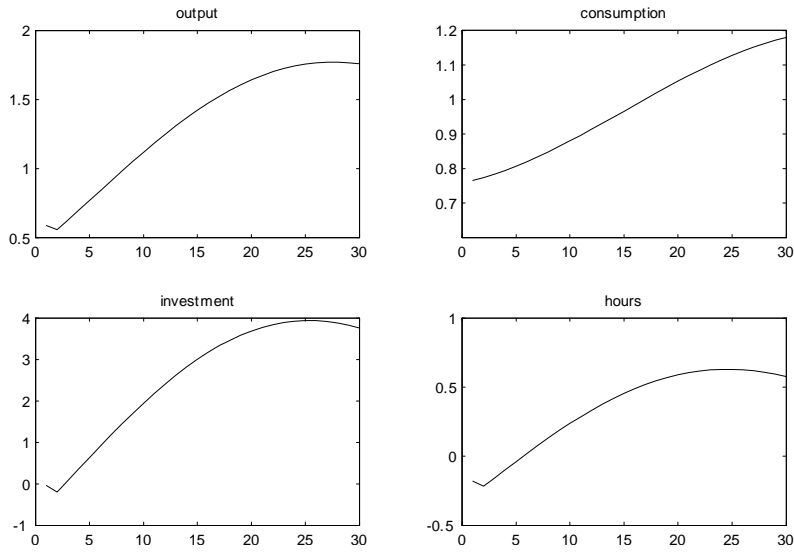


Figure A3: Impulse responses to a permanent technology shock (percent deviations from the balanced growth path).