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Do age complementarities affect labour productivity? Evidence from German firm level data

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Do age complementarities affect labour productivity?

Evidence from German firm level data

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Abstract

In Germany as in many other European countries there will be a shift in the workforce's age structure in the next decades. The number of older workers will increase whereas the number of young and middle aged workers will decline. This paper provides evidence how demographic ageing affects labour productivity. It focuses on age complementarities between workers. Using a cross sectional linked employer-employee data set from 2012 translog cost functions are estimated. To maintain consistency with microeconomic theory, several parameter constraints are imposed. To control for the skill level of the workers a nested production structure is applied. This allows additionally to analyse age complementarities by skill level. Based on the estimated parameters pairwise elasticities of complementarity and factor price elasticities are calculated. The results indicate that workers that belong to different age groups are complementary factors. The complementarities especially arise between young and medium aged workers and are higher within groups of high skilled labour. Simulating how the expected shift in the age structure affects labour productivity indicates that the productivity of younger and middle aged workers will increase. In contrast, the productivity of older workers will decline, promoted by the complementary relationship between the different age groups.

JEL classification: C31, D24, J11, J31

Keywords: Age complementarities, demographic ageing, labour-labour substitution, linked employer-employee, translog

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1 Introduction

In Germany as in many other European countries there will be a shift in the age structure of the workforce in the next decades. According to a labour market projection commissioned by the Federal Ministry of Labour and Social Affairs the labour force aged 15 to 24 years declines from 2010 to 2030 by almost one million persons, and the labour force aged 25 to 54 years by almost five million. On the contrary, the labour supply of workers being between 55 and 74 years old increases by almost three million persons (BMAS, 2013). This shift towards older workers should put “downward pressure” on the productivity of workers with similar characteristics and “upward pressure” on the productivity of workers with different characteristics (Ottaviano & Peri, 2012: 158). Quantitatively, these effects depend on the degree of complementarity between the different groups. The larger the degree of complementarity between workers of a different age the larger are the effects of demographic ageing on the (relative) productivity of age groups.

Complementary relationships between workers who differ in age might exist since younger workers are different from older ones. Young workers tend to have more physical strength and agility, and are especially stronger in generating and recombining new knowledge. In contrast, older workers tend to have more experience, and abilities to grasp difficult situations as well as to use and apply existing skills (Horn & Cattell, 1967; Börsch-Supan & Weiss, 2007; Börsch-Supan, 2013).

Welch (1979) indeed finds that workers of different age are imperfect substitutes when he analyses the impact of the cohort size on earnings for the United States. This holds especially for high skilled labour. Focusing on manufacturing in the United States Grant and Hamermesh (1981) identify complementarities between young workers and different types of older workers as well, “though in no case is the complementary relationship very strong” (Grant & Hamermesh, 1981: 357). In their seminal work Card and Lemieux (2001) find that the cohort-specific relative supplies of college-equivalent labour is one driving force of a relative rise in college wage premium for younger workers in the United States, the United Kingdom, and Canada. They apply a nested CES framework with imperfect substitutability between workers with the same education but a different age. This framework is also used by Fitzenberger and Kohn (2006). Their findings for Germany confirm that employees of different age but same skill level are imperfect substitutes.

In addition, Arntz and Gregory (2014) detect for Germany that younger and older workers are complements in the knowledge production process. They analyze the effect of the workforce’s age structure on regional innovation by estimating flexible translog functions. One shortcoming of their study is that they do not analyse age complementarities within groups of workers with a comparable skill level as suggested by Card and Lemieux (2001), but only within the entire regional labour force.

Contrary to Arntz and Gregory (2014), I take the skill level of workers into account and focus on the complementary relationships between younger, middle aged and older workers within groups of low, medium, and high skilled labour. The results of Grant and Hamermesh (1981) indicate that the degree of complementarity between different pairs of labour varies. Moreover, Arntz and Gregory (2014) find that younger and older workers are complements in the production of knowledge, whereas the degree of complementarity between younger and middle aged and between middle aged and older workers is rather low. Flexible forms are needed to identify such differences. I therefore estimate translog functions in favour of CES-functions which base on “strong functional form assumptions” (Fitzenberger & Kohn, 2006: 24). Thereby I relax the assumption that the degree of complementarity is identical between different pairs of age groups within the same skill group which is made by e.g. Card and Lemieux (2001) and Fitzenberger and Kohn (2006).

As discussed by Hamermesh (1996: 64) microeconomic data is “likely to yield better estimates” of the parameters characterizing establishments’ labour demand than aggregate data. For this reason, I use a linked employer-employee data set and study age complementarities on the establishment level, i.e. at the workplace, and not on the aggregate or regional level. To maintain consistency with microeconomic theory I impose several parameter constraints when I estimate firm level translog cost functions. The parameter estimates are used to calculate elasticities of factor complementarities and factor price elasticities. Based on the factor price elasticities it is possible to simulate how demographic ageing affects wages if they are free to adjust (Grant & Hamermesh, 1981).

The remainder of this paper is structured as follows. First, I outline the theoretical framework. Second, the empirical specification and econometric issues are discussed. Third, the data is described, thereafter section four presents the estimation results and finally section five concludes.

2 Theoretical Framework

To identify complementarities between different types of labour, I apply a nested production framework which is in line with neoclassical production theory. This well established approach (see e.g. Card & Lemieux, 2001; Borjas, 2003; Fitzenberger & Kohn, 2006; Ottaviano & Peri, 2012; Haas et al., 2013) bases on the assumption that the demand for different types of labour is separable from the demand for other inputs (Hamermesh, 1996). As shown in Figure 1 the top level describes an establishment’s output Y as a function of technology A , capital C and labour L .

The labour aggregate L is distinguished according to the qualification of the workers (index: q). It is important to note, that L is not the pure sum of low, medium, and high skilled workers employed by the establishment, but compound labour input, that also depends on the degree of complementarity between differently skilled workers.

On the next level the skill specific labour input L_q itself is partitioned into groups that differ according to the age of the workers (index: $i \in \{y, m, o\}$): *younger workers* (y), *middle aged workers* (m), *older workers* (o). Estimates of this level provide information on age complementarities in the production process.

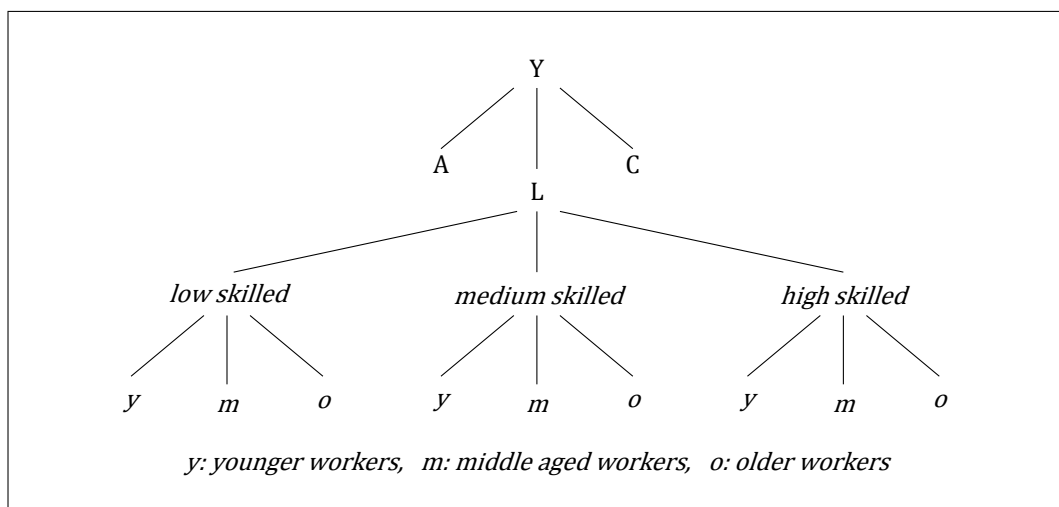


Figure 1: Nested production framework

Assuming factor prices are given to the establishments and that the establishments choose L_{iq} , i.e. the amount of labour with age i and skill level q , such that the costs to accumulate L_q units of compound labour with skill

level q are minimized. Then, the total costs of L_q are given by $C_q(\mathbf{w}_q, L_q)$, where \mathbf{w}_q is a vector containing the wages w_{iq} . I follow Christensen, Jorgenson, and Lau (1973) and approximate $\log(C_q(\mathbf{w}_q, L_q))$ by a function that is quadratic in the logarithms. Additionally, I assume that $\log(C_q(\mathbf{w}_q, L_q))$ is homogeneous in L_q and obtain (1).

$$\log(C_q(\mathbf{w}_q, L_q)) = \alpha_q + \sum_i \alpha_{iq} v_{iq} + \frac{1}{2} \sum_i \sum_j \beta_{ijq} v_{iq} v_{jq} + \gamma_q \ln L_q$$

with $v_{iq} := \log(w_{iq})$ and $l_q := \log(L_q)$ (1)

The assumption of homogeneity implies that the cost minimizing, relative composition of L_q with respect to the different age groups is independent of the level of L_q . Under this assumption its optimal composition is given by (2) which is a system of share equations that depend on the same parameters as (1).¹ S_{iq} is the value share of age group i within skill group q and the parameters β_{ijq} can be interpreted as share elasticities which indicate the skill level specific response of i 's value share to an increase of j 's wage (Jorgenson, 1986).

$$S_{iq}(\mathbf{w}_q) \equiv \frac{L_{iq}^* w_{iq}}{\sum_j L_{jq}^* w_{jq}} = \alpha_{iq} + \sum_j \beta_{ijq} v_{jq},$$
 (2)

Estimates of (2) enable to calculate Hick's Partial Elasticities of Complementarity c_{ij} (Hicks, 1970; Sato & Koizumi, 1973), and Factor Price Elasticities θ_{ij} (Sato & Koizumi, 1973) which provide information on age complementarities.²

In terms of a translog cost function c_{ij} is defined as (Hamermesh & Grant, 1979: 539):

$$c_{ij} = \frac{|\mathbf{G}_{ij}|}{|\tilde{\mathbf{G}}|},$$
 (3)

where $|\mathbf{G}_{ij}|$ is the cofactor of G_{ij} , $|\tilde{\mathbf{G}}|$ the determinant of the matrix

$$\tilde{\mathbf{G}} = \left[\begin{array}{c|ccc} 0 & G_1 & \dots & C_n \\ \hline G_1 & & & \\ \vdots & & & \\ G_n & & & \mathbf{G} \end{array} \right]$$

and $\tilde{\mathbf{G}}$ in turn a transformation of the bordered Hessian matrix of $C(\mathbf{w}, L)$ ³ with:

$$G_i = \frac{\partial C(\mathbf{w}, L)}{\partial w_i} \frac{w_i}{C(\mathbf{w}, L)} = S_i$$
 (4)

$$\mathbf{G} = \frac{\widehat{\mathbf{W}}[\nabla_{\mathbf{w}\mathbf{w}}^2 C(\mathbf{w}, L)]\widehat{\mathbf{W}}}{C(\mathbf{w}, L)} = \mathbf{B} - \widehat{\mathbf{S}} + \mathbf{S}\mathbf{S}^T.$$
 (5)

¹ (2) has been derived by taking the partial derivative of (1) with respect to v_{iq} and by applying Shephard's Lemma. See Appendix A.1 for a detailed description.

² Both elasticities can be calculated by skill level. For simplicity the index q is omitted in the description of the elasticities.

³ The bordered Hessian matrix and its derivation are given in Appendix A.2.

$\widehat{\mathbf{W}}$ is a diagonal matrix with the elements of the wage vector $\mathbf{w} \equiv [w_y, w_m, w_o]^T$ on the main diagonal: $\widehat{\mathbf{W}} \equiv \begin{bmatrix} w_y & 0 & 0 \\ 0 & w_m & 0 \\ 0 & 0 & w_o \end{bmatrix}$, \mathbf{B} a 3×3 matrix containing β_{ij} , and $\widehat{\mathbf{S}}$ a 3×3 diagonal matrix with the elements of the share

vector $\mathbf{s} \equiv [S_y, S_m, S_o]^T$ on the main diagonal: $\widehat{\mathbf{S}} \equiv \begin{bmatrix} S_y & 0 & 0 \\ 0 & S_m & 0 \\ 0 & 0 & S_o \end{bmatrix}$ (Diewert & Wales, 1987: 48).

Based on c_{ij} the factor price elasticity is given by (Sato & Koizumi, 1973):

$$\theta_{ij} = \frac{\partial \log(w_i)}{\partial \log(L_j)} = S_j c_{ij}. \quad (6)$$

If $c_{ij} > 0 (< 0)$ the age groups i and j are called quantity complements (substitutes) since an increase in L_{jq} leads to an increase (decline) in w_{iq} . θ_{ij} measures the percentage change in w_{iq} in response to a 1% increase in L_{jq} given wages are flexible (Grant & Hamermesh, 1981).

3 Empirical Specification and Econometric Issues

Since estimates of translog cost functions like (2) do not per se fulfill the properties of a microeconomic cost function, parameter constraints have to be imposed. To maintain consistency with Young's theorem of integrable functions symmetry constraints have to be fulfilled, i.e. $\beta_{ijq} = \beta_{jiq}$, $i \neq j$. In addition, $C_q(\mathbf{w}_q, L_q)$ has to be monotone as well as linear homogeneous and concave in w_{iq} (Jorgenson, 1986).

Monotonicity implies that the (predicted) cost shares have to be nonnegative. However, since $C_q(\mathbf{w}_q, L_q)$ is quadratic in the logarithms of the wages it is not possible to impose parameter constraints that ensure monotonicity (Jorgenson, 1986). Thus, whether this property is fulfilled has to be checked after the estimation.

The other two properties, linear homogeneity and concavity, however, can be imposed. Linear homogeneity requires that the parameters α_{iq} add up to unity and the parameters β_{ijq} add up to zero:

$$\sum_i \alpha_{iq} = 1, \quad \sum_i \beta_{ijq} = 0. \quad (7)$$

Concavity of $C_q(\mathbf{w}_q, L_q)$ implies that its Hessian matrix \mathbf{H}_q is negative semi-definite. Diewert and Wales (1987: 48) show that this is fulfilled, provided that $C(\mathbf{w}_q, L_{qf}) > 0$, if and only if the matrix \mathbf{G}_q (as defined in (5)) is negative semi-definite.

However, since \mathbf{H}_q and \mathbf{G}_q depend on the data, i.e. on the cost shares S_i , a different Hessian matrix and thus a different matrix \mathbf{G}_q is obtained for each observation. As described by Diewert and Wales (1987) and Ryan and Wales (2000), imposing concavity globally destroys the flexibility of the translog function and results in biased estimates. Therefore, Ryan and Wales (2000) suggest to impose concavity locally at a normalization point where the logarithm of each wage is zero instead of imposing it globally. In the normalization point S_{iq} equals α_{iq} such that parametric restrictions can be imposed. The ij th element of \mathbf{G}_q is then given by (Ryan & Wales, 2000: 255):

$$g_{ijq} = \beta_{ijq} - \alpha_{iq} \delta_{ij} + \alpha_{iq} \alpha_{jq} \quad (8)$$

with $\delta_{ij} = 1$ if $i = j$ and 0 otherwise.

Concavity of the cost function in the neighbourhood of the normalization point can be attained by setting $-\mathbf{G}_q = \mathbf{D}\mathbf{D}'_q$ (Ryan & Wales, 2000), where \mathbf{D} is a 3×3 lower triangular matrix.⁴ Solving (8) for β_{ijq} and replacing $-g_{ijq}$ by the ij th element of $\mathbf{D}\mathbf{D}'_q$ ⁵ yields non linear constraints that imposes concavity of $C_q(\mathbf{w}_q, L_q)$ in the neighbourhood of the normalization point (Ryan & Wales, 2000: 255):

$$\beta_{ijq} = -(DD')_{ijq} + \alpha_{iq}\delta_{ijq} - \alpha_{iq}\alpha_{jq} \quad (9)$$

Following Ryan and Wales (2000) I choose the sample mean as reference point.

If symmetry as well as local concavity are imposed and random disturbances added, the system of share equations (2) changes to (10). Since the parameters β_{ijq} are replaced by (9), the parameters d_{ijq} are estimated instead of β_{ijq} .⁶

$$\begin{aligned} S_{yqf} &= \alpha_{yq} + (-d_{yyq}d_{yyq} + \alpha_{yq} - \alpha_{yq}\alpha_{yq})v_{yqf} + (-d_{ymq}d_{yyq} - \alpha_{yq}\alpha_{mq})v_{mqf} + (-d_{yyq}d_{yoq} - \alpha_{yq}\alpha_{oq})v_{oqf} + \varepsilon_{yqf} \\ S_{mqf} &= \alpha_{mq} + (-d_{ymq}d_{yyq} - \alpha_{yq}\alpha_{mq})v_{yqf} + (-d_{ymq}d_{ymq} - d_{mmq}d_{mmq} + \alpha_{mq} - \alpha_{mq}\alpha_{mq})v_{mqf} \\ &\quad + (-d_{yoq}d_{ymq} - d_{moq}d_{mmq} - \alpha_{mq}\alpha_{oq})v_{oqf} + \varepsilon_{mqf} \\ S_{oqf} &= \alpha_{oq} + (-d_{yyq}d_{yoq} - \alpha_{yq}\alpha_{oq})v_{yqf} + (-d_{yoq}d_{ymq} - d_{moq}d_{mmq} - \alpha_{mq}\alpha_{oq})v_{mqf} + \\ &\quad (-d_{yoq}d_{yoq} - d_{moq}d_{moq} - d_{ooq}d_{ooq} + \alpha_{oq} - \alpha_{oq}\alpha_{oq})v_{oqf} + \varepsilon_{oqf}. \end{aligned} \quad (10)$$

The index combination qf allows to identify the units of observation, i.e. skill group q within establishment f . The indices y , m , and o refer to younger, middle aged and older workers, respectively that belong to qf . ε_{iqf} are unobservable random disturbances for the cost shares. As Jorgenson (1986) I assume that the expected value of all unobservable random disturbances is zero, that the error covariance matrix is the same for all units of observation, and that the random disturbances belonging to different observations are uncorrelated.

Since the cost shares S_{yqf} , S_{mqf} , and S_{oqf} sum up to unity for each observation qf , the sum of the error terms $\sum_i \varepsilon_{iqf}$ is equal to zero for all observations (Jorgenson, 1986). Hence, the error covariance matrix is singular. Greene (2012) therefore suggests to estimate just $n - 1$ share equations. Furthermore, he notes that computing maximum likelihood estimates of the parameters ensures invariance with respect to the choice of which share equation is dropped.⁷

As stressed for example by Grant and Hamermesh (1981) one only derives unbiased estimates of a cost function, or a system of share equations that base on factor prices, if they are exogenous. Hamermesh (1996) therefore argues that the cost function would ideally be embedded in a system of equations that also describes the labour supply reaction or workers' choices. As an alternative strictly exogenous variables might be used to generate instruments for the factor prices in the cost function. However, to find such variables is already a severe problem for studies that consider homogeneous labour. In the case of heterogeneous labour it is "insolvable" (Hamermesh, 1996: 70). Therefore, one has to decide whether to treat quantities as exogenous and to estimate a production function or to treat wages as exogenous and to estimate a cost function. Hamermesh (1996) argues that in case of microeconomic data it is reasonable to assume the latter since small units like establishments

⁴ The idea of employing the Cholesky factorization to impose concavity goes back to Lau (1978).

⁵ $\mathbf{D}\mathbf{D}' = \begin{bmatrix} d_{yy}^2 & d_{yy}d_{my} & d_{yy}d_{oy} \\ d_{yy}d_{my} & d_{my}^2 + d_{mm}^2 & d_{my}d_{oy} + d_{mm}d_{om} \\ d_{yy}d_{oy} & d_{my}d_{oy} + d_{mm}d_{om} & d_{oy}^2 + d_{om}^2 + d_{oo}^2 \end{bmatrix}$.

⁶ The adding-up constraints $\sum_i \alpha_i = 1$ and $\sum_i \beta_{ij} = 0$ are directly imposed when estimating the system.

⁷ I estimate (10) using Stata's iterative feasible generalized least squares estimator which is equivalent to ML estimation with multivariate normal disturbances and also invariant to the choice of the estimated share equations (see Stata's manual *nlsur - Estimation of nonlinear systems of equations*, page 9, <http://www.stata.com/manual.s13/rnl sur.pdf>).

face perfect elastic labour supply in the long run. As a robustness check I exclude all establishments with 50 employees or more to address that large establishments may have power to set wages.

4 Data

To estimate the parameters of (10) that are needed to calculate c_{ij} and θ_{ij} I use a data set that provides information on a 20 % random sample of all establishments located in Germany that employed at least one worker subject to social security contributions on June, 30th 2012. The public sector, temporary employment agencies and crop and animal production, hunting and related service activities are excluded. The remaining number of establishments amounts to 508,514. My raw data set bases on the IAB employment history (BeH, the German acronym of 'Beschäftigten-Historik') and contains all employment spells that were reported by the selected establishments in 2012, i.e. 9,124,309 individual spells.

The BeH bases on the integrated notification procedure for health, pension and unemployment insurance. Employers are required to submit notifications to the responsible social security agencies concerning all of their employees covered by social security as well as marginally employed at least once a year. The Federal Employment Agency collects and edits these notifications, which the IAB then subsequently integrates into the BeH. Thus, the BeH covers very reliable information on all employment spells as long as the workers are not exempt from social security contributions, like civil servants and self-employed persons (about 12 % of total employment in Germany).

Among others, each observation in the BeH provides information on the length of a specific employment spell and the paid gross wage sum during this time period. Due to the social security threshold the wage information is right-censored and in 2012 only reported up to €67,200 per year in West Germany and €57,600 per year in East Germany. To avoid biased estimates I impute the censored wages of employees subject to social security contributions with estimated wages.⁸

Based on the individual data from the BeH an establishment level data set is generated. To distinguish low, medium, and high skilled workers within the establishments the requirement level of the job is used, which is the second dimension of the German Classification of Occupations 2010 (KldB 2010). A detailed description can be found in Paulus and Matthes (2013). I define workers with an occupation that does not require a vocational training of at least two years as low skilled. Medium skilled workers are workers with an occupation that requires a vocational training of at least two years and workers with an occupation that requires at least a bachelor's degree, a qualification as master craftsman or technician or an equivalent technical school or college graduation are considered as high skilled.

I use the requirement level of the job in favor of the highest educational degree of a worker which is also available in the BeH to identify low skilled, medium skilled, and high skilled labour due to two reasons. First, the requirement level enables to assign the workers to the different skill groups according to the complexity of their actual task and not according to their formal level of qualification. Haas, Lucht, and Schanne (2013) show that workers with the same formal educational degree, but with different job requirements are imperfect substitutes. Second, 27% of the 9 million employment spells do not provide information on the educational degree of the worker, whereas the information on the requirement level is missing only in 2% of the spells.⁹

⁸ I follow Reichelt (2015) and apply an interval regression to estimate the true (daily) wages above the thresholds, see Appendix A.3.

⁹ If the requirement level is missing but the highest educational degree is not, then the later is used to assign the workers to one of the skill groups. This reduces the share of spells that cannot be assigned to one of the three groups to 0.07%. In 65.8% of the cases in

To which age group an employee belongs arises from her day of birth that is also given in the BeH. Employees younger than 30 years are considered as *younger workers*, workers aged 30 to 49 years as *middle aged workers* and workers that are 50 years old or older as *older workers*. 0.4% of the 9 million spells are not assigned to one of the age groups since the worker is implausible old (above 75 years). Figure 2 provides information on the distribution of the individual employment spells across the three age groups by skill level. 6% of the employment spells are not considered in the analysis, since I exclude apprentices from the analysis. Furthermore, employment spells of interns, and student trainees are only considered as long as the employees have at least a bachelors degree and have an occupation that requires a vocational training of at least two years or higher education.

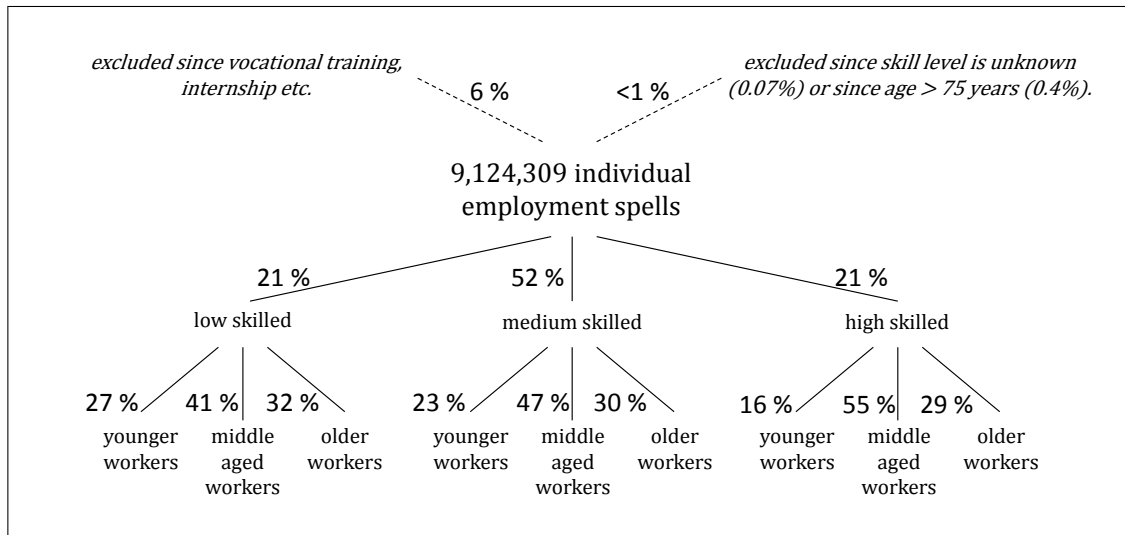


Figure 2: Employment spells by skill level and age of the employee

Aggregating the remaining individual employment spells by the skill level of the workers but not across establishments results in a data set providing information on the three skill groups within each establishment. 60% of these observations refer to groups of medium skilled workers, 22% to groups of low skilled workers and 18% to groups of high skilled workers. Skill groups with less than five employees are not considered in the analysis. The number of remaining observations amounts to 290,334. Each provides information on w_{iqf} and S_{iqf} , i.e. the wages and the cost shares of younger, middle aged, and older workers that belong to skill group q of establishment f . Summary statistics are provided by Table 1. Unfortunately, the BeH does not provide information on non-wage labour costs. Therefore, the calculated cost shares S_{iqf} refer to the wage costs only.

The wages of the different age-skill groups w_{iqf} , that enter (10) on the right hand side, are calculated on a daily basis. Hence, w_{iqf} indicates how much establishment f paid in 2012 on average per day to an employee with age i and skill level q . For the calculation of w_{iqf} only employment spells are used that refer to full-time employment. Moreover, only employment spells with a length of at least 8 days and a gross-daily wage above the limit of marginally employed are considered. In 40% of the cases it is not possible to calculate w_{iqf} since establishment f does not report employment spells of workers with age i and skill level q that fulfill the criteria above. In these cases $w_{iqr(f)}$ is used as an approximation of w_{iqf} , where $w_{iqr(f)}$ is the average gross daily wage of group iq in the region r in which establishment f is located. $w_{iqr(f)}$ can be interpreted as the daily wage that f would have to pay to employ one unit of type iq labour.

which information on the requirement level as well as on the highest educational degree is available the requirement level accords with the reported highest educational degree. In 16.6% the reported highest educational degree is higher and in 17.6% lower than the requirement level.

Table 1: Summary statistics

	Mean	SD	Min	Max
Groups of low skilled labour (N = 64405)				
Cost shares				
Younger labour	0.215	0.244	0.000	1.000
Middle aged labour	0.459	0.259	0.000	1.000
Older labour	0.326	0.266	0.000	1.000
ln(Average gross daily wages)				
Younger labour	3.976	0.227	2.558	5.621
Middle aged labour	4.155	0.295	2.565	6.268
Older labour	4.229	0.273	2.570	6.127
Groups of medium skilled labour (N = 173313)				
Cost shares				
Younger labour	0.205	0.213	0.000	1.000
Middle aged labour	0.501	0.232	0.000	1.000
Older labour	0.294	0.238	0.000	1.000
ln(Average gross daily wages)				
Younger labour	4.203	0.273	2.560	6.710
Middle aged labour	4.336	0.365	2.558	6.214
Older labour	4.424	0.351	2.559	6.460
Groups of high skilled labour (N = 52617)				
Cost shares				
Younger labour	0.121	0.178	0.000	1.000
Middle aged labour	0.561	0.237	0.000	1.000
Older labour	0.318	0.252	0.000	1.000
ln(Average gross daily wages)				
Younger labour	4.506	0.286	2.570	6.368
Middle aged labour	4.823	0.417	2.568	7.441
Older labour	4.960	0.408	2.567	6.959

5 Results

Table 2 provides estimates of (10). I start with a regression where I pool the observations that belong to different skill groups. Thus, I assume that the parameters and elasticities do not differ across q . The first column of Table 2 contains the estimated parameters of (10), inserting them into (9) and (7), i.e. concavity and the adding-up constraints, results in the actual parameters of the cost function. They are reported in the second column of Table 2. Based on these parameters the Hick's Partial Elasticities of Complementarity c_{ij} and the factor price elasticities θ_{ij} and θ_{ji} are calculated using (3) and (6), respectively.

The results indicate that all three age groups are complementary input factors. But the degree of complementarity differs. The highest degree is observed for young and middle aged workers. The factor price elasticity θ_{ym} indicates that an increase of the middle aged workforce by 1% leads to an increase of the wage of young workers by 0.5%. For comparison, the wage of older workers would rise only by 0.35% due to the 1% increase of the number of middle aged workers.

As discussed in section 3 the estimates can only be interpreted as causal effects if the wages are exogenous to the firms. As a robustness check I repeat the estimations with a reduced sample that only covers establishments with less than 50 employees. The additional results are summarized in Table A.2. The parameter estimates and calculated elasticities are 10% to 20% smaller than those reported in Table 2. However, the qualitative results are the same.

I can use the calculated own and cross factor price elasticities θ_{ij} to simulate how the demographic ageing might affect the productivity of the three age groups due to the expected shift in relative labour supply (see e.g. Grant & Hamermesh, 1981). It is assumed that relative wages are completely flexible. In the simulation I examine a scenario where the number of young workers decreases by 21%, and the number of middle aged workers by 16.5%. The number of older workers is assumed to increase by 3.9%.¹⁰ The simulation results are summarized in Table 3. Here again the predicted change is slightly smaller in absolute terms if establishments with 50 employees or more are excluded.

The simulated effects show that productivity of young workers will increase most (by 4% to 5%) due to the assumed changes in age structure. It is important to note that the own factor price elasticity of younger workers (θ_{yy}) would predict an increase of the wage rate by more than 13% (11% if larger firms are excluded) in response to the assumed decline of younger labour by 21%. However, the decline in the number of middle aged workers put downward pressure on the wages of younger workers due to the complementary relationship. The wage of young workers is therefore expected to rise only by 4% to 5%. Similarly, the decline in middle aged workers tend to increase their productivity. However, their wage rate increases only by about 3% since the number of younger workers declines as well.

For older workers the simulation predicts a strong decrease in productivity. Their wage is expected to decline by 8% to 9%. Predominantly this is caused by the decreasing number of younger and middle aged workers.

Relaxing the assumption that age complementarities are independent of the skill level, shows that they are often larger within groups of high skilled labour than within groups of low or medium skilled workers as indicated by Table 4.¹¹ This supports the findings of Welch (1979) who observes that the degree of complementarity increases by skill level. Welch (1979) argues that high skilled workers perform tasks which clearly require a

¹⁰ This scenario bases on the relative change of the population aged 15-29 years, 30-49 years, and 50-70 years that is expected by the German Federal Statistical Office until 2030 (DESTATIS, 2009). Changes in labour participation rates are not considered.

¹¹ The corresponding results that base on firms with less than 50 employees are summarized in Table A.3.

Table 2: Estimated parameters and calculated elasticities, average effects ($\alpha_{iq} = \alpha_i$, $\beta_{ijq} = \beta_{ij}$ is assumed)

i	j	α_i, d_{ij}	α_i, β_{ij}	c_{ij}	θ_{ij}	θ_{ji}
y		0.194*** (0.000)	0.194*** (0.000)			
m		0.503*** (0.000)	0.503*** (0.000)			
o			0.303*** (0.000)			
y	y	0.455*** (0.002)	-0.050*** (0.001)	-3.273*** (0.022)	-0.628*** (0.004)	-0.628*** (0.004)
m	m	0.528*** (0.001)	-0.066*** (0.001)	-0.801*** (0.004)	-0.403*** (0.002)	-0.403*** (0.002)
o	o		-0.137*** (0.001)	-1.394*** (0.006)	-0.426*** (0.002)	-0.426*** (0.002)
y	m	-0.192*** (0.002)	-0.010*** (0.001)	0.999*** (0.008)	0.502*** (0.004)	0.192*** (0.002)
y	o		0.061*** (0.001)	0.413*** (0.008)	0.126*** (0.002)	0.079*** (0.002)
m	o		0.076*** (0.001)	0.690*** (0.004)	0.211*** (0.001)	0.347*** (0.002)
N		290335				

Standard errors in parentheses, calculation based on Delta-method, significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The data is normalized at the sample mean. Monotonicity and concavity of the cost function are satisfied at all data points. The concavity constraint is locally imposed at the sample mean. The elasticities are calculated at mean predicted shares.

Table 3: Simulation results

	Percentage change in wage rate of ...		
	younger workers	middle aged workers	older workers
Wage effects based on estimates for the full sample (Table 2)	5.4 (0.06)	3.4 (0.02)	-9.0 (0.04)
Wage effects based on estimates for firms with less than 50 employees (Table A.2)	4.3 (0.05)	3.2 (0.02)	-8.4 (0.04)

Wage effects based on factor price elasticities (θ_{ij} , θ_{ji} in Table 2 and Table A.2, respectively) and the scenario: number of younger workers -21%, middle aged workers -16.5%, older workers: +3.9%. Wages are assumed to be completely flexible. Standard errors in parentheses, calculation based on Delta-method.

high formal education, but also extensive training after beginning to work. This promotes complementarities between younger and more experienced workers. Accordingly, the Hick's Partial Elasticities of Complementarity and factor price elasticities for low and medium skilled labour in Table 4 are always smaller than the corresponding elasticities reported in Table 2. There is one exception. For low skilled labour a – relative to the other skill levels – high degree of complementarity between younger and older workers is detected. One reasonable explanation is that jobs which require only little specific training are often connected with physical burden. Therefore, the presence of younger workers likely has a higher importance for the productivity of older workers in groups of low skilled workers than in other skill groups.

Within groups of high skilled labour younger and older workers are rather substitutes than complements as indicated by the results reported in Table 4. This finding contradicts the reasoning by Welch (1979) and supports the view of Fitzenberger and Kohn (2006). They argue that university graduates have in general (independent of their age) high competence to solve problems, which reduces complementarities.

Additionally, my results indicate that the productivity of young as well as of older high skilled increases with the number of middle aged high skilled workers. For young graduates the presence of middle aged professionals is likely to be of high importance since graduates who recently completed their education at the university lack experience to apply their knowledge. Especially middle aged workers might be important in this career phase since they have work experience and at the same time knowledge that is more up-to-date than that of older workers. For that reason, older workers with a job that has a very high degree of complexity likely benefit from the presence of middle aged workers as well.

Table 4: Elasticities by skill level

i	j	low skilled labour			medium skilled labour			high skilled labour		
		c_{ij}	θ_{ij}	θ_{ji}	c_{ij}	θ_{ij}	θ_{ji}	c_{ij}	θ_{ij}	θ_{ji}
y	y	-2.936*** (0.053)	-0.630*** (0.011)	-0.630*** (0.011)	-2.257*** (0.014)	-0.463*** (0.003)	-0.463*** (0.003)	-7.870*** (0.183)	-0.954*** (0.022)	-0.954*** (0.022)
m	m	-0.818*** (0.010)	-0.376*** (0.005)	-0.376*** (0.005)	-0.707*** (0.004)	-0.354*** (0.002)	-0.354*** (0.002)	-1.042*** (0.019)	-0.585*** (0.010)	-0.585*** (0.010)
o	o	-1.242*** (0.014)	-0.405*** (0.005)	-0.405*** (0.005)	-1.324*** (0.006)	-0.390*** (0.002)	-0.390*** (0.002)	-1.847*** (0.021)	-0.587*** (0.007)	-0.587*** (0.007)
y	m	0.890*** (0.019)	0.409*** (0.009)	0.191*** (0.004)	0.768*** (0.006)	0.385*** (0.003)	0.157*** (0.001)	1.892*** (0.053)	1.061*** (0.030)	0.229*** (0.006)
y	o	0.678*** (0.022)	0.221*** (0.007)	0.146*** (0.005)	0.265*** (0.006)	0.078*** (0.002)	0.054*** (0.001)	-0.337*** (0.043)	-0.107*** (0.014)	-0.041*** (0.005)
m	o	0.566*** (0.009)	0.185*** (0.003)	0.260*** (0.004)	0.669*** (0.004)	0.197*** (0.001)	0.335*** (0.002)	1.119*** (0.018)	0.355*** (0.006)	0.628*** (0.010)
N		64405			17313			52617		

Standard errors in parentheses, calculation based on Delta-method, significance levels: * p<0.05, ** p<0.01, *** p<0.001. The data is normalized at the respective sample mean. Monotonicity and concavity of the cost function are satisfied at more than 99% of the respective data points. The concavity constraint is locally imposed at the respective sample mean. The elasticities are calculated at mean predicted shares.

6 Conclusion

In this paper I analyse complementarities between younger, middle aged and older workers. This is of special interest since demographic ageing leads to a shift in the workforce's age structure in Germany as well as in many other European countries. Applying a nested and flexible production framework, enables me to analyse age complementarities within and by skill groups. Contrary to others, my analysis bases on a linked employer-employee data set. This allows to investigate age complementarities on establishment level, i.e. directly at the workplace.

The results indicate that workers who belong to different age groups are indeed complements and that the degree of complementarity differs across the considered age groups and by skill level. However, for each skill level the largest degree of complementarity is identified between younger and middle aged workers. Especially young high skilled workers benefit from the presence of middle aged high skilled colleagues. On the contrary, young high skilled workers and older high skilled workers are identified as substitutes rather than complementary groups of labour. It can be argued that young high skilled workers already have a high competence in general problem solving for which reason they do not benefit from the presence of older high skilled labour (Fitzenberger & Kohn, 2006). However, their productivity increases in the number of middle aged workers, likely, since the latter teach their younger colleagues how to apply their knowledge. This is necessary, because especially the complex tasks of high skilled workers needs in addition to the high formal education extensive training after beginning to work (Welch, 1979).

If my results are compared to those of Arntz and Gregory (2014) inconsistencies arise. Arntz and Gregory (2014) identify the largest degree of complementarity between young and older workers and no significant complementarities between young and middle aged workers. However, our results are not directly comparable due to several reasons. Arntz and Gregory (2014) do not analyse labour productivity in general, but focus on the generation of innovation, i.e. on the knowledge production process. In this special context the interaction of younger and older workers might be beneficial since workers with knowledge that is up-to-date and workers with extensive experience come together. However, one has to note that Arntz and Gregory (2014) do not analyse age complementarities within groups of workers that are comparable according to their skill level, but across all workers. This is likely to affect the results since the average educational level differs across cohorts. Additionally, Arntz and Gregory (2014) analyse age complementarities on the regional level and not on the firm level. Thus, they estimate how the age structure of the entire workforce within a region impact on regional innovation. Even though the regional context might be of importance for the generation of innovation it would be interesting to see whether their findings can be confirmed by a firm-level analysis. This should be taken into account in future research on age complementarities in the production process in general and on the generation of knowledge in particular.

Based on my results I simulate how the expected demographic ageing impacts on the wages of the different age groups. Due to the complementary relationship between older workers and the two other groups the expected decline in younger and middle aged workers put pressure on the wages of older workers. Under the assumption that wages are completely flexible one could expect a decline of about 8% to 9% until 2030. On the contrary, the wages of younger and middle aged workers are expected to rise by about 5% and 3%, respectively, since their relative shares decline.

A Appendix

A.1 Deriving the system of share equations

The partial derivative of $\log(C_q(\mathbf{w}_q, L_q))$ with respect to $\log(w_{iq})$ is

$$\frac{\partial \log(C_q(\mathbf{w}_q, L_q))}{\partial \log(w_{iq})} = \alpha_{iq} + \sum_j \beta_{ijq} v_{jq}. \quad (11)$$

$\frac{\partial \log(C_q(\mathbf{w}_q, L_q))}{\partial \log(w_{iq})}$ is the elasticity of $\log(C_q(\mathbf{w}_q, L_q))$ regarding w_{iq} . Thus:

$$\frac{\partial \log(C_q(\mathbf{w}_q, L_q))}{\partial \log(w_{iq})} = \frac{\frac{dC_q}{C_q}}{\frac{dw_{iq}}{w_{iq}}} = \frac{dC_q}{dw_{iq}} \frac{w_{iq}}{C_q}. \quad (12)$$

According to Shephard's Lemma (McFadden, 1978): $\frac{dC_q}{dw_{iq}} = L_{iq}^*$, where L_{iq}^* is the cost minimizing number of type iq workers given the price vector \mathbf{w}_q and L_q . Hence, the cost minimizing share S_{iq} of type iq workers in type q workers amounts to:

$$S_{iq} = \frac{L_{iq}^* w_{iq}}{\sum_j L_{jq}^* w_{jq}} = \alpha_{iq} + \sum_j \beta_{ijq} v_{jq}. \quad (13)$$

A.2 Deriving the bordered Hessian Matrix $\tilde{\mathbf{H}}$ of the cost function $C(\mathbf{w}, L)$

$$\tilde{\mathbf{H}} = \left[\begin{array}{c|ccc} 0 & C_1 & \dots & C_n \\ \hline C_1 & & & \\ \vdots & & \mathbf{H} & \\ C_n & & & \end{array} \right]$$

$$\mathbf{H} = \left[\begin{array}{ccc} C_{11} & \dots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \dots & C_{nn} \end{array} \right]$$

$$\begin{aligned} C(\mathbf{w}, L) &= \exp \left[\ln(a_0) + \sum_i \alpha_i \ln(w_i) + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln(w_i) \ln(w_j) + \gamma l \right] \\ C_i = \frac{\partial C(\mathbf{w}, L)}{\partial w_i} &= \exp[\cdot] \left(\alpha_i \frac{1}{w_i} + \sum_j \beta_{ij} \frac{1}{w_i} \ln(w_j) \right), \text{ with } \beta_{ij} = \beta_{ji} \\ &= \frac{C(\mathbf{w}, L)}{w_i} \left[\alpha_i + \sum_j \beta_{ij} \ln(w_j) \right] \\ &= \frac{C(\mathbf{w}, L)}{w_i} S_i (= L_i, \text{ Shephard's Lemma}) \end{aligned} \quad (14)$$

$$\begin{aligned} C_{ii} = \frac{\partial^2 C(\mathbf{w}, L)}{\partial w_i \partial w_i} &= \exp[\cdot] \left(\alpha_i \frac{1}{w_i} + \sum_j \beta_{ij} \frac{1}{w_i} \ln(w_j) \right) \left(\alpha_i \frac{1}{w_i} + \sum_j \beta_{ij} \frac{1}{w_i} \ln(w_j) \right) \\ &\quad + \exp[\cdot] \left(-\alpha_i \frac{1}{w_i} \frac{1}{w_i} + \left(-\sum_j \beta_{ij} \frac{1}{w_i} \frac{1}{w_i} \ln(w_j) + \beta_{ii} \frac{1}{w_i} \frac{1}{w_i} \right) \right) \\ &= C(\mathbf{w}, L) \left[\frac{1}{w_i} \left(\alpha_i + \sum_j \beta_{ij} \ln(w_j) \right) \frac{1}{w_i} \left(\alpha_i + \sum_j \beta_{ij} \ln(w_j) \right) + \frac{1}{w_i^2} \left(-\alpha_i - \sum_j \beta_{ij} \ln(w_j) + \beta_{ii} \right) \right] \\ &= \frac{C(\mathbf{w}, L)}{w_i^2} [S_i^2 - S_i + \beta_{ii}] \end{aligned} \quad (15)$$

$$\begin{aligned} C_{ij} = \frac{\partial^2 C(\mathbf{w}, L)}{\partial w_i \partial w_j} &= \exp[\cdot] \left(\alpha_i \frac{1}{w_i} + \sum_j \beta_{ij} \frac{1}{w_i} \ln(w_j) \right) \left(\alpha_j \frac{1}{w_j} + \sum_i \beta_{ij} \frac{1}{w_j} \ln(w_i) \right) + \exp[\cdot] \beta_{ij} \frac{1}{w_i w_j}, \quad i \neq j \\ &= C(\mathbf{w}, L) \frac{1}{w_i} \left(\alpha_i + \sum_j \beta_{ij} \ln(w_j) \right) \frac{1}{w_j} \left(\alpha_j + \sum_i \beta_{ij} \ln(w_i) \right) + C(\mathbf{w}, L) \beta_{ij} \frac{1}{w_i w_j} \\ &= \frac{C(\mathbf{w}, L)}{w_i w_j} \left[\left(\alpha_i + \sum_j \beta_{ij} \ln(w_j) \right) \left(\alpha_j + \sum_i \beta_{ij} \ln(w_i) \right) + \beta_{ij} \right] \\ &= \frac{C(\mathbf{w}, L)}{w_i w_j} [S_i S_j + \beta_{ij}] \end{aligned} \quad (16)$$

A.3 Imputation of censored wages

Since the wages in the BEH are only reported up to the social security threshold (2012: €67,200 per year in West Germany, €57,600 in East Germany), I have to estimate the true wages above the censoring limit. I use additional information from the BEH, i.e. the age, sex, and nationality of the workers as well as requirement level of the job, the employment status, the establishment size, the industry (2-digit level), the occupation (3-digit level), and the type of region the employees are working in.

Following Reichelt (2015) I apply interval regression to estimate the true wages above the threshold and add an error term, i.e. a random value from a normal distribution, to the estimates. Since the true wage has to be larger than the censoring limit, the random variable is drawn from a truncated distribution as suggested by Gartner (2005). Wages of certain groups like marginal employed, interns, and student trainees should not be affected by the social security threshold. They are therefore excluded from the interval regression, whose results are summarized in Table A.1. Figure A.1 depicts the wage distribution before and after imputation.

Table A.1: Results of an interval regression to impute right censored wages of employees liable to social security, dependant variable: $\ln(\text{Censored gross daily wage})$

Age	0.040***	(311.32)
Age squared	-0.000***	(-254.03)
Female	-0.189***	(-345.38)
German	0.074***	(96.50)
Requirement level according to KldB2010 (see Paulus & Matthes, 2013), reference: Unskilled or semi-skilled activities		
Specialist activities	0.204***	(283.20)
Complex specialist activities	0.408***	(409.92)
Highly complex activities	0.609***	(553.25)
Parttime	-0.527***	(-967.73)
Employment status, reference: Employees liable to social security without special characteristics (code:101)		
Employees in partial retirement (103)	0.118***	(73.02)
Causal worker (118)	0.466***	(94.22)
Employed pensioner (119)	-0.349***	(-104.63)
Seaman (140)	0.313***	(28.86)
Seaman in partial retirement (149)	0.706***	(12.59)
Establishment size: $\ln(\text{Number of employees})$	0.076***	(609.69)
Type of region, reference: Agglomerations in West Germany		
Urbanized region in West Germany	-0.038***	(-75.40)
Rural region in West Germany	-0.049***	(-60.32)
Agglomeration in East Germany	-0.127***	(-166.33)
Urbanized region in East Germany	-0.211***	(-240.32)
Rural region in East Germany	-0.227***	(-202.68)
Constant	2.423***	(74.32)
Total number of observations	6,173,784	
Censored observations	525,152	
Uncensored observations	5,648,632	

t-statistics in parentheses, significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Beside the explanatory variables reported in this table industry fixed effects (2-digit level) and occupation fixed effects (3-digit level) are considered.

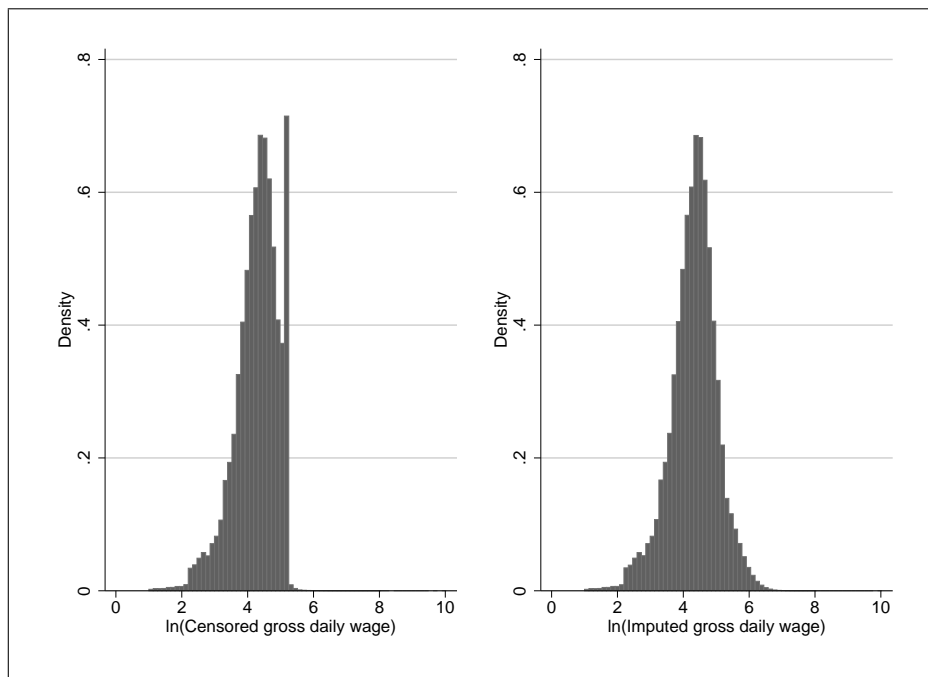


Figure A.1: Wage distribution before and after imputing gross daily wages of employees subject to social security contributions

A.4 Further results

Table A.2: Results based on a reduced sample: only establishments with less than 50 employees, average effect ($\alpha_{iq} = \alpha_i$, $\beta_{ijq} = \beta_{ij}$ is assumed)

i	j	α_i, d_{ij}	α_i, β_{ij}	c_{ij}	θ_{ij}	θ_{ji}
y		0.205*** (0.000)	0.205*** (0.000)			
m		0.502*** (0.001)	0.502*** (0.001)			
o			0.292*** (0.001)			
y	y	0.505*** (0.002)	-0.092*** (0.002)	-2.599*** (0.042)	-0.524*** (0.016)	-0.524*** (0.003)
m	m	0.543*** (0.001)	-0.093*** (0.002)	-0.730*** (0.007)	-0.367*** (0.002)	-0.367*** (0.002)
o	o		-0.169*** (0.002)	-1.339*** (0.006)	-0.396*** (0.002)	-0.396*** (0.002)
y	m	-0.220*** (0.002)	0.008*** (0.001)	0.854*** (0.007)	0.429*** (0.008)	0.172*** (0.001)
y	o		0.084*** (0.001)	0.322*** (0.007)	0.095*** (0.002)	0.065*** (0.001)
m	o		0.085*** (0.001)	0.658*** (0.004)	0.194*** (0.001)	0.331*** (0.002)
N		226290				

Standard errors in parentheses, calculation based on Delta-method, significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The data is normalized at the sample mean. Monotonicity and concavity of the cost function are satisfied at all data points. The concavity constraint is locally imposed at the sample mean. The elasticities are calculated at mean predicted shares.

Table A.3: Elasticities by skill level based on reduced sample: only establishments with less than 50 employees

i	j	low skilled labour			medium skilled labour			high skilled labour		
		c_{ij}	θ_{ij}	θ_{ji}	c_{ij}	θ_{ij}	θ_{ji}	c_{ij}	θ_{ij}	θ_{ji}
y	y	-2.320*** (0.044)	-0.524*** (0.010)	-0.524*** (0.010)	-2.081*** (0.013)	-0.434*** (0.003)	-0.434*** (0.003)	-5.203*** (0.113)	-0.730*** (0.016)	-0.730*** (0.016)
m	m	-0.706*** (0.009)	-0.329*** (0.004)	-0.329*** (0.004)	-0.679*** (0.004)	-0.340*** (0.002)	-0.340*** (0.002)	-0.869*** (0.016)	-0.488*** (0.009)	-0.488*** (0.009)
o	o	-1.151*** (0.014)	-0.356*** (0.004)	-0.356*** (0.004)	-1.298*** (0.006)	-0.377*** (0.002)	-0.377*** (0.002)	-1.711*** (0.021)	-0.509*** (0.006)	-0.509*** (0.006)
y	m	0.767*** (0.017)	0.357*** (0.008)	0.173*** (0.004)	0.723*** (0.006)	0.362*** (0.003)	0.151*** (0.001)	1.427*** (0.038)	0.802*** (0.021)	0.200*** (0.005)
y	o	0.540*** (0.019)	0.167*** (0.006)	0.122*** (0.004)	0.248*** (0.006)	0.071*** (0.002)	0.052*** (0.001)	-0.243*** (0.034)	-0.072*** (0.010)	-0.034*** (0.005)
m	o	0.502*** (0.009)	0.155*** (0.003)	0.234*** (0.004)	0.651*** (0.004)	0.189*** (0.001)	0.326*** (0.002)	0.967*** (0.016)	0.288*** (0.005)	0.543*** (0.009)
N		45302			14792			33696		

Standard errors in parentheses, calculation based on Delta-method, significance levels: * p<0.05, ** p<0.01, *** p<0.001. The data is normalized at the respective sample mean. Monotonicity and concavity of the cost function are satisfied at more than 99% of the respective data points. The concavity constraint is locally imposed at the respective sample mean. The elasticities are calculated at mean predicted shares.

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