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Uncertainty shocks and non-fundamental debt crises: An ambiguity approach


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Uncertainty shocks and non-fundamental debt crises: An ambiguity approach

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- Preliminary -

Abstract

This paper analyses empirically and theoretically the effects of uncertainty shocks on sovereign default risk. It describes a novel mechanism for non-fundamental debt crises induced by uncertainty shocks that are defined as time-varying levels of ambiguity surrounding the macroeconomic fundamental of the economy. A business cycle model with strategic sovereign default is augmented with ambiguity averse investors with multiple-priors utility. I find that uncertainty shocks increase the risk of default as perceived by worst case investors’ beliefs. Sovereign and private sector interest rates rise due to a spillover channel that unfolds through the domestic banking sector. A crisis zone is characterised where worst case investors’ beliefs lead to non-fundamental debt crises. The model’s predictions are shown to be in line with impulse responses obtained from a VAR analysis for a panel of four Euro area countries. Specifically, the dichotomy of sovereign debt pricing in the core and periphery countries can partly be rationalised by accounting for ambiguity premia.

Keywords: Uncertainty shocks; Non-fundamental crises; Multiple-priors utility; Ambiguity aversion; Interbank market

JEL Classifications: D81; E32; E44; F34
1 Introduction

In a recent debate, policy makers and academics discussed potential ‘mispricing’ or ‘over-pricing’ of debt in the course of the European sovereign debt crisis.\(^1\) Based on the concept of multiplicity of equilibria in sovereign debt markets, it has been argued that the problem of indeterminacy may give rise to ‘bad’ equilibria which lead to self-fulfilling debt crises and eventually imposes dead weight losses due to outright sovereign default.\(^2\) This view seems to be backed by a growing strand of empirical literature which finds that yields on public debt are partly unexplained by macroeconomic fundamentals (Aizenman et al. 2013, Beirne & Fratzscher 2013, D’Agostino & Ehrmann 2013).

This paper empirically and theoretically analyses the effects of uncertainty shocks on the market pricing of sovereign debt. It describes a novel mechanism for non-fundamental debt crises induced by uncertainty shocks that are defined as time-varying levels of ambiguity surrounding the macroeconomic fundamental of the economy. The main finding of the paper is that pricing decisions in government debt markets that have been referred to as ‘over-pricing’ can partly be rationalised if one accounts for ambiguity premia charged by investors that exhibit a strict preference for knowing probabilities.

In the first part of the paper, I estimate a structural vector autoregressive (VAR) model for a panel of four Euro area countries, namely France, Germany, Italy and Spain, that includes an uncertainty measure to explain sovereign default risk. I use monthly data over the time period 2008 to 2014 which exhibits elevated levels and high volatility of uncertainty. The empirical findings can be summarized in two main observations. First, there is an overall significant positive effect of uncertainty on sovereign default risk that lasts up to one year for countries in the Euro area periphery, while the effect is not significant for core countries. Second, a shock on macroeconomic uncertainty pushes up bank lending rates. The empirical results imply that uncertainty shocks matter for sovereign debt pricing and that these shock might be propagated through the banking sector. However, the cross-country responses are not symmetric. Further, strong identification assumptions in the empirical model limit conclusions about the causal relationship between fundamental uncertainty and default risk.

In order to investigate the interaction between uncertainty shocks and sovereign credit risk, a real business cycle model with strategic sovereign default is augmented with ambiguity averse investors with multiple-priors utility. The economic fundamental in the economy is uncertain in a sense that agents do not know the exact underlying probabilistic model of the evolution of the aggregate productivity state (Knight 1921). Limited confidence in the own forecasting technology and the availability of less tangible information lead investors to form a set of prior beliefs about the conditional mean expectation on future productivity. In line with multiple-priors utility (Gilboa & Schmeidler 1989, Epstein & Wang 1994), investors select the worst case prior and

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1 See IMF (2012), European Commission (2012), Corsetti et al. (2013) and Grauwe & Ji (2013), among others. The notion of ‘over-pricing’ refers to yields on sovereign debt as used e.g. by Aizenman et al. (2013).

2 Calvo (1988) is a classic reference for multiplicity of equilibria in sovereign debt markets which might lead to a Pareto-inferior equilibrium.
act as if there is just one belief in the set.

Investors purchase government securities which are subject to endogenous default risk due to limited commitment (Eaton & Gersovitz 1981). In line with the quantitative literature on optimal sovereign default, the government inherits the previously accumulated stock of public debt. If the government decides to service the debt, it is able to issue new bonds and roll-over existing debt. The macro model features a bank-sovereign nexus with heterogeneous banks as in Engler & Groe Steffen (2014). Banks use government bonds as collateral on the domestic interbank market to obtain wholesale funding. Non-financial firms need working capital loans provided by banks in order to produce. Outright sovereign default involves an ex post penalty through a freeze on the interbank market which triggers a credit crunch. Further, adverse shocks are amplified in this setting. When the quality of collateral falls in response to increased levels of sovereign risk, the risk premium for secured debt on the inter-bank market rises and makes the allocation of financial resources in the economy less efficient, leading to a further decline in output.

Following Ilut & Schneider (2014), uncertainty shocks stochastically increase or tighten the degree of ambiguity surrounding the macroeconomic fundamental each period. One possible interpretation is the time-varying degree of macroeconomic forecast dispersion. A natural advantage of this procedure is that the degree of ambiguity can be disciplined by the data. In a dynamic setting with default expectations that depend on the future evolution of the fundamental state of the economy, uncertainty shocks translate into ambiguous pay-offs from holding government debt. As a result, investors ask for higher yields in the form of an ambiguity premium to roll-over the existing stock of government debt which is labelled as the pricing channel of uncertainty shocks.

Shocks to ambiguity can be an important source of economic fluctuations in this setting. However, the model propagation mechanism is distinct from Ilut & Schneider (2014). Uncertainty shocks amplify frictions in the domestic interbank market. At elevated levels of fundamental ambiguity, investors’ worst case beliefs lead via the pricing channel to a loss in the quality of collateral used for secured lending on the interbank market. Lower bond prices induce a higher interest rate on wholesale loans that render borrowing on the interbank market less attractive. As a result, the arising spillover channel of uncertainty shocks to the financial sector lowers available credit to non-financial firms in the current period which dampens aggregate production.

The model is used to characterize the conditions for non-fundamental roll-over crises in sovereign debt markets. In analogy to Cole & Kehoe (2000), uncertainty shocks give rise to a crisis zone where the repayment decision of the government is determined by the realisation of aggregate ambiguity. However, in contrast to Cole & Kehoe (2000) who assume that an exogenous realisation of a sun-spot variable leads to a refusal to roll-over public debt, the emergence of a debt crisis is here derived endogenously from investors’ preferences that interact with the level of ambiguity surrounding the macroeconomic fundamental and limited commitment on the side of the government. Within the crisis zone, the joint effect of the pricing and spillover channel may render the option of outright default relatively more attractive than servicing the debt, even though the macroeconomic fundamental might also support debt repayment. Thereby, worst case beliefs of investors induce a roll-over debt crisis.
The findings of the model are related to a number of crisis models in the literature with multiplicity of equilibria, most of them following the seminal bank-run model of Diamond & Dybvig (1983) in which asymmetric information gives rise to a coordination problem among investors. Implications for sovereign debt markets are studied in Calvo (1988), Alesina et al. (1990) and Giavazzi & Pagano (1990). Morris & Shin (2004) and Corsetti et al. (2006) apply global games methods to solve the arising equilibrium selection problem. Corsetti et al. (2013) analyse the special case of indeterminacy of a New Keynesian model at the zero lower bound without explicitly modelling a default decision. They find that spillovers from sovereign risk exacerbate the problem of indeterminacy, concluding that this makes an economy more prone to self-fulfilling beliefs. Roch & Uhlig (2014) and Kirsch & Rühmkorf (2013) analyse bailout policies in the presence of sun-spot induced roll-over crises, analogous to Cole & Kehoe (2000). This paper departs from this strand of literature since the derived equilibrium is always unique as the pricing schedule of government debt represents a fixed-point to which the agents’ expectations converge. Instead, uncertainty shocks affect investors’ beliefs directly due to the adoption of multiple-priors utility.

The modelling of preferences with ambiguity aversion is closely related to the concept of multiplier preferences from the robust control theory (Hansen et al. 1999, Hansen & Sargent 2001). Costa (2009) and Pouzo & Presno (2013) apply multiplier preferences to an optimal sovereign default model with endowment shocks. They find, similar to the present study, that concerns about model misspecification generate higher yields on sovereign debt. However, the concept of multiplier preferences is not able to accommodate the analysis of uncertainty shocks. Since uncertainty premia arise from the degree of robustness required by investors specified by a penalty function, probability distortions are endogenously related to the fundamental state and not independent orthogonal structural shocks. Further, the model presented in this paper provides an endogenous propagation mechanism of uncertainty shocks on aggregate production which is not present in the previously analysed endowment settings.

Multiple-priors utility have been applied to a range of topics such as asset pricing (Epstein & Wang 1994) and business cycle analysis (Ilut & Schneider 2014). In this paper, a novel propagation mechanism of uncertainty shocks on the business cycle is laid out which arises from the interaction of limited commitment of the government on one side with investors’ multiple-priors preferences on the other side. As has been shown previously, ambiguity aversion has first-order effects on asset pricing in contrast to risk aversion which has been studied by Lizarazo (2013) in sovereign debt markets. Thereby, the pricing of government debt does not depend on the wealth level of investors or the correlation of investors’ income with sovereign default risk.

Finally, Durdu et al. (2013) analyse how news affect the pricing of sovereign debt. They show that news shocks shift and reshape the probability density function for the productivity shock. As news shocks comprise an informative signal, their effect is symmetric in a sense that good news imply higher bond prices while bad news lead to lower bond prices. Uncertainty shocks cannot generate such symmetric responses, as

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3 A detailed survey of the literature is provided by Guidolin & Rinaldi (2010).
4 Notwithstanding, risk shocks can have first-order implications for the business cycle, as shown by Justiniano & Primiceri (2008), Fernández-Villaverde et al. (2011), and Christiano et al. (2014).
only worst case beliefs are considered by ambiguity averse agents.\textsuperscript{5}

The paper is structured as follows. Section 2 contains the empirical SVAR analysis of uncertainty shocks in the Euro area. The theoretical model of optimal sovereign default in the presence of ambiguity averse investors and uncertainty shocks is presented in Section 3. A discussion of simulation results and the propagation mechanisms follows in Section 4. The final section concludes.

2 Empirical analysis

2.1 Determinants of sovereign credit risk and the role of uncertainty

What is the empirical relationship between uncertainty about macroeconomic fundamentals and sovereign credit risk? While there exists a fast growing strand of empirical literature since the start of the global financial crisis in 2008 that investigates the pricing of sovereign debt in advanced economies, the role of fundamental uncertainty has been studied only to a very limited extent. This section briefly summarizes the major empirical findings on the determinants of sovereign credit risk and outlines the conceptual framework for the following empirical analysis.

In how far can sovereign credit risk be explained by macroeconomic fundamentals? Laubach (2009) finds a positive effect of the government budget deficit and the debt-to-GDP ratio on long-run US Treasury yields using a 30-year sample. Borgy et al. (2011) estimate an affine term structure model for a panel of eight EMU countries. They also underline the importance of the fiscal position as a major fundamental variable of a country that matters for the country yield spread. At the same time, Aizenman et al. (2013) find in a cross country panel that a part of the premium on euro area periphery debt during the crisis, especially in 2010, cannot be explained by measures of fiscal space or other macro fundamentals. Haan et al. (2014) show that the explanatory power of macro fundamentals and the degree of ‘mispricing’ in empirical work is very sensitive to the modelling assumptions. Evaluating different empirical specifications, the finding of ‘over-pricing’ is most robust in the case of Greece, Portugal and Ireland, but less so for Italy or Spain.

In how far are country interest rates affected by common factors? There is a lot of empirical evidence that the Lehman crash of 2008 gave globally rise to a higher risk perception with significant spillovers to sovereign debt markets. On the basis of a cross-country principal component analysis, Longstaff et al. (2011) point to the dominance of a global market factor relative to country-specific fundamentals. Hagen et al. (2011) show that coefficients to fiscal imbalances have increased after the the Lehman shock and explain this with a shift in general risk aversion. Also, Bernoth & Erdogan (2012) find evidence for time-varying coefficients in European sovereign

\textsuperscript{5}The literature on expectation driven business cycles encompasses several settings where news affect agents’ decisions, e.g. Jaimovich & Rebelo (2009), Beaudry & Portier (2006, 2007), Schmitt-Groh\textemdash & Uribe (2012), Barsky & Sims (2012), Blanchard et al. (2013), and a survey by Krusell & McKay (2010).
debt markets. D’Agostino & Ehrmann (2013) confirm this finding in a panel of G7 countries. They conclude that time variation in risk perception lead to over-pricing of risk in European peripheral countries during the crisis period which is interpreted as re-denomination risk stemming from the possibility of a collapse of the European monetary union. However, all these studies miss to include a measure of fundamental uncertainty which increased in response to the Lehman crash in 2008 (Bloom 2009) to explain sovereign credit risk.

There are several complementary explanations for an increase and intensified co-movement in CDS rates across countries since the onset of the global financial crisis. First, contagion has been identified as a source for regional spillovers. Favero & Missale (2012) find evidence for contagion in Europe in response to an increase in global risk aversion. Beirne & Fratzscher (2013) distinguish between ‘wake-up call’-, regional-

5Berndt & Obreja (2010) demonstrate that there have been similar consequences in European corporate CDS markets.
and pure 'herding' contagion. They find only a minor role for pure contagion during the European sovereign debt crisis and stress that there has been an 'under-pricing' of risk in the run-up to the crisis. Second, there is the concept of systemic sovereign risk investigated by Ang & Longstaff (2013) who compare the degree of common credit risk between European sovereigns and US states. They find that the extend of systemic sovereign risk is substantially lower in the US than in Europe, and that financial market variables are an important source for these differences. This indicates to a third source for higher sovereign credit risk in Europe, the existence of a bank-sovereign nexus. Dieckmann & Plank (2012) find that financial sector variables matter to explain CDS spreads in all advanced countries due to implicit or explicit bailouts. This is particularly the case in Europe, as was previously shown by Ejsing & Lemke (2011). Fratzscher & Rieth (2015) confirm the interconnectedness between sovereign and bank sector risk in a time-series approach, while showing that monetary policy intervention achieved to moderately ease the adverse feedback loop. Fourth, liquidity of CDS markets is a relevant factor when it comes to pricing decisions, particularly during crisis episodes (Brunnermeier & Pedersen 2009, Calice et al. 2013, Schumacher 2014).

The conceptual approach chosen here to explain sovereign credit risk in the Euro area builds on the hypothesis that part of the unexplained share in CDS premia in European sovereign debt markets during the crisis period might be due to shocks to the confidence in macroeconomic fundamentals. Bloom (2009) documents that uncertainty is quite volatile at a lower frequency, related to political or economic shocks like wars or economic crises. He uses stock market volatility as a proxy for economic uncertainty. While second-moment realised risk shocks and their effect on asset pricing and the business cycle have been investigated previously in the literature (Justiniano & Primiceri 2008, Fernández-Villaverde et al. 2011, Christiano et al. 2014), I take a different approach by building on a related strand of papers which investigates the role of Knightian uncertainty for the business cycle, as e.g. in Ilut & Schneider (2014). In their model, agents do not have a full probabilistic assessment of the future growth path but suffer from model uncertainty. This seems to be a plausible assumption given that professional forecasters often disagree in their assessment about future macroeconomic fundamentals. Figure 1 shows the variance of point forecasts for GDP growth rates at a certain time horizon among the survey of professional forecasters conducted by the ECB. As forecast dispersion seems to increase during crises, I expect a positive correlation between economic uncertainty and sovereign default risk.

2.2 Model specification and data

How important were uncertainty shocks during the European sovereign debt crisis? This section presents a time-series analysis to the dynamic behaviour of credit risk in response to uncertainty shocks in the Euro area over the period 2008 to 2014. The empirical model follows a structural VAR with additional exogenous variables (VAR-X). The choice of variables is guided by three objectives. First, the model should contain determinants for sovereign default risk and aggregate production as I am

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7See Bowles et al. (2007) and Dovern et al. (2012) for a detailed discussion of forecast dispersion measures and economic uncertainty contained in the survey.
interested in the response to these variables to uncertainty shocks. Second, in order to account for severe disruptions in financial markets in the sample period, a measure is included that addresses exogenous financial shocks. Third, given the relatively short sample period, a parsimonious setting with few variables is preferred.

The considered empirical model has \( n = 5 \) endogenous variables which include a measure of the general government debt ratio, uncertainty, industrial production, sovereign credit risk, and the spread of the bank lending rate over the money market rate, summarised in the vector \( y_t \). Further, I include \( m = 4 \) exogenous variables which are the real US interest rate, the spread of Moody’s corporate bond yields rated Baa over those rated Aaa, the spread of the unsecured European money market rate over the overnight-index swap rate (OIS), and the real effective exchange rate, summarized in the \( x_t \) vector.

\[
y_t = \begin{bmatrix}
    \text{debt/gdp}_t \\
    \text{epu}_t \\
    \text{cds}_t \\
    \beta\hat{p}_t \\
    \hat{r}_t
\end{bmatrix}, \quad x_t = \begin{bmatrix}
    \hat{R}^{\text{ns}}_t \\
    \text{crs}_t \\
    \text{mm}_t \\
    \text{reer}_t
\end{bmatrix}
\]

The model is estimated separately for four member countries of the Euro area: France, Germany, Italy and Spain. The country panel is chosen due to data availability. The frequency of observations is monthly, starting in January 2008 and ending in December 2014. The system takes the form

\[
y_t = c + B(L)y_t + \Gamma(L)x_t + \epsilon_t, \quad (1)
\]

with matrices of polynomials in the lag operator \( L \), \( B(L) = B_0 L^0 + \ldots + B_p L^p \) and \( \Gamma(L) = \Gamma_0 L^0 + \ldots + \Gamma_q L^q \). Further, note that \( c \) is a \( n \times 1 \) vector of constants, \( B_i \) are \( n \times n \) matrices and \( \Gamma_i \) are matrices of dimension \( n \times m \).

In the system, endogenous variables depend on contemporaneous and past values of endogenous and exogenous variables. Identification of structural shocks is obtained through impact restrictions by the ordering of endogenous variables in the stacked vector \( y_t \). Specifically, I assume that the impact matrix \( B_0 \) is lower triangular such that variables ordered at lower ranks are assumed to not have a contemporaneous effect on the higher ranked variables (Sims 1980, 1986). In the specification considered, a lag length of order one is chosen for endogenous and exogenous variables.

As an important fundamental macro variable to determine sovereign credit risk, the debt-to-GDP ratio at a quarterly frequency enters the set of endogenous variables first. Next, the uncertainty measure considered is the Economic Policy Uncertainty (EPU) index. The index is based on the frequency of newspaper articles related to

\footnote{For the identification of the model, define \( \Psi(L) = \Psi_0 + \Psi_1 L + \ldots = [I - B(L)]^{-1} \) with \( \Psi_0 = I \), i.e. an infinite polynomial on the lag operator. This allows to obtain the VMA-X representation of the model \( (8) \) as \( y_t = \Psi(1)c + \Psi(L)\Gamma(L)x_t + \Psi(L)\epsilon_t \). The structural shocks \( \epsilon_t \) can then be estimated from the equation \( \mu + \Delta(L)x_t + C(L)\epsilon_t = \Psi(1)c + \Psi(L)\Gamma(L)x_t + \Psi(L)\epsilon_t \), where \( \epsilon_t \sim N(0, I) \) such that parameters can be estimated by OLS.}

\footnote{Lag lengths have been validated using various statistical criteria, such as the Akaike Information Criterion (AIC), the likelihood ratio test, and the Bayesian Information Criterion (BIC).}
economic policy uncertainty and conducted in Europe on a national level for Germany, France, Italy, Spain and the United Kingdom (Baker et al. 2013).\textsuperscript{10} An augmented Dickey-Fuller test (ADF) declines the presence of a unit-root within the sample period. I use credit default swaps with a maturity of five years as a measure for sovereign default risk.\textsuperscript{11} Although the ADF cannot reject the null of a unit root, CDS rates enter the system in levels for two reasons. First, on theoretical grounds it can be argued that interest rates do not follow a non-stationary process over a sufficiently long time period. Second, the level information is the objective for the underlying analysis where I am interested in the intensity of sovereign default risk. Contrary, real industrial production is seasonally adjusted and enters the model in first differences. Finally, the spread of the bank lending rate is constructed as follows: I subtract the 3-month Eonia rate from the national bank lending rate to non-financial firms of loans with a volume of more than one million euros and a maturity up to one year. Similar to developments in CDS markets, the ADF test cannot reject the null of a unit-root. As this is again strongly related to the sample period, the bank lending spread enter the empirical model in levels.

As exogenous variables, I construct the real US interest rate by subtracting the average inflation rate in the previous four months from the three month interest rate on US Treasury bills. Further, I include a typical credit risk measure by constructing the spread of yields on US corporate bonds with a Moody’s rating of Baa over those with a Aaa-rating. Both indicators are meant to capture exogenous global shocks in the sample period which have been identified in previous studies as changes in general risk aversion. The European money market spread is formed as the difference between the three month Euribor and the overnight index swap (OIS) and should control for tensions in interbank markets as a source for spillovers between banks and sovereigns. Finally, the real effective exchange rate comprises changes in the competitiveness of a country which might affect default risk in the medium term.\textsuperscript{12}

2.3 Uncertainty shocks, sovereign credit risk, and spillovers

The dynamic effects of uncertainty shocks identified in the empirical model are analysed next. Figure 2 presents the impulse responses of a one standard deviation increase in the uncertainty measure on sovereign default risk. There is a significant and long lasting response of default risk in Spain and Italy which lasts up to one year. On the contrary, there is practically no response of sovereign credit risk in the cases of Germany and France. This clear separation of Euro area member countries in a core an a periphery is remarkable. It highlights that there is an underlying relationship between economic uncertainty and sovereign default risk which affects the country groups asymmetrically. While financial markets anticipate an increase in sovereign default risk in response to economic policy uncertainty, there is no such increase in the core countries.

What is the underlying propagation channel between uncertainty and sovereign

\textsuperscript{10}Wisniewski & Lambe (2014) also evaluate the impact of EPU on sovereign default risk for a broader panel of countries outside the Euro area.

\textsuperscript{11}See Amato (2005) for an in-depth discussion of CDS rates as proxy for default risk.

\textsuperscript{12}Details regarding data sources can be found in the Appendix.
Figure 2: Effect of uncertainty shock on sovereign credit risk (IRFs)

![Graph showing the effect of uncertainty shock on sovereign credit risk for Germany, France, Spain, and Italy.](image)

Note: Orthogonalized impulse response functions. Dashed lines are 95 percent confidence intervals. The impulse is a one standard deviation increase in uncertainty (EPU index), the response the CDS rate (in percentage points).

Credit risk in the Euro area periphery? In the context of the European sovereign debt crisis, the mutual feedback effect between financial sector risk and sovereign credit risk has repeatedly been identified as a major candidate for shock amplification (Dieckmann & Plank 2012, Fratzscher & Rieth 2015). In order to evaluate the role of uncertainty shocks in the context of the hypothesis of a downward spiral in the presence of a bank-sovereign nexus, I consider the effect of a uncertainty shock on the bank lending rate. If the bank-sovereign nexus is strong, there should be an increase in the bank lending rate in response to uncertainty.

As expected, there is a positive reaction of private sector interest rates to an one-off increase in economic uncertainty, as shown in Figure 3. Although the separation of the country sample in core and periphery is not as strong, the amplitude of the impulse response is significantly higher in Italy and Spain in the medium term compared to Germany and France.

The empirical evidence presented here does not provide a conclusive answer to the role of uncertainty shocks in determining sovereign default risk. The impulse responses provide only suggestive evidence for the propagation of uncertainty shocks in the course of the European sovereign debt crisis. The problem of endogeneity and causality remains and limited observations pose challenges to a robust empirical identification strategy. I therefore present in the next section a theoretical model that investigates...
the effects of uncertainty shocks in a macroeconomic framework. The model is able to rationalise the different responses in the core and periphery of the Euro area to uncertainty shocks and pins down a possible propagation mechanism.

3 Theoretical model

3.1 Overview

The point of departure is a RBC model of a small open economy with a benevolent government. In line with the literature on strategic sovereign default (Eaton & Gersovitz 1981), the government cannot commit to service its obligations from previous debt issuances but takes an optimal default decision each period. I follow Engler & Große Steffen (2014) in modelling endogenous output costs of default due to disruptions in the domestic interbank market where government bonds serve as collateral.

The innovation in this paper is that there is ambiguity about the future aggregate productivity state. International investors have preferences that exhibit ambiguity aversion. This leads to pricing decisions different to the case of subjective expected utility maximization (SEU). Following the recursive multiple-priors model, ambiguity averse agents form a worst case belief which pins down their optimal inter-temporal
decisions when confronted with ambiguity about the true data generating process of aggregate productivity.

Additional to a benevolent government and a central bank, the domestic economy is populated by a representative household, a unit mass of non-financial firms and heterogeneous banks. Time is infinite and discrete $t = (0, 1, \ldots)$. Endogenous states, i.e. total government debt ($B_t$) and domestic debt held by the domestic banking sector ($B^D_t$), are given from period $t - 1$ decisions. Exogenous states are stochastic and given by aggregate total factor productivity ($z_t$) and the degree of ambiguity about the future fundamental state of the economy ($a_t$). After the government has learned about the aggregate state $s \in (B_t, B^D_t, z_t, a_t)$ at the beginning of each period when shocks realise, it decides to default ($\delta_t = 1$) or repay ($\delta_t = 0$) its debt. In case of repayment, non-financial firms produce a tradeable final good. They receive working capital loans from heterogeneous banks which re-allocate financial resources on the domestic interbank market. Borrowing in the interbank market is collateralised with government debt and is affected by changes in sovereign risk perceptions.

If the government defaults on its debt, the economy falls into financial autarky with an exogenous probability of re-accessing capital markets. Since public debt cannot be issued in autarky states, there is no collateral available. Interbank intermediation breaks down and the economy suffers an endogenous credit crunch. An overview of the sequence of events is presented in the Appendix, Figure 9.

3.2 Households

A representative household derives utility from consumption ($c_t$) and leisure ($1 - l_t$). It provides labour ($l_t$) to non-financial firms and receives a wage ($w_t$) in return. The household owns non-financial firms and banks in the economy and receives profits ($\Pi_t$) and dividend payments ($D_t$) in a lump-sum transfer at the end of each period. The inter-temporal savings decision to smooth consumption is done by the benevolent government through transfer payments ($T_t$). The household maximizes life-time utility subject to a budget constraint

$$\max_{\{c_t, l_t\}} E_t \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t),$$

$$s.t. \quad c_t = w_t l_t + \Pi_t + D_t + T_t,$$

where $E_t$ denotes the rational expectations operator and the utility function $u(\cdot)$ satisfies the inada conditions. Optimal labour supply is determined from the combined first order conditions for consumption and labour:

$$- \frac{u_t(c_t, 1 - l_t)}{u_c(c_t, 1 - l_t)} = w_t$$

3.3 Non-financial firms and technological change

The non-financial sector produces a tradeable final good $Y_t$ with a Cobb-Douglas production function with variable labour input $l_t$ and a constant capital stock $K$, thus
\[ Y_t = e^{z_t} F(l_t, K) \] with \( z_t \) denoting aggregate total factor productivity. Within-period working capital loans \( \kappa_t \) are required to finance production input factors up-front. These loans pay an interest rate \( r^e_t > 0 \). Let \( \eta \) denote the share of the wage bill which needs to be financed before production starts. Then, the firm’s profit maximisation problem reads

\[
\max_{\{l_t, \kappa_t\}} e^{z_t} F(l_t, K) - w_t l_t - \kappa_t r^e_t, \tag{4}
\]

s.t. \( \kappa_t \geq \eta w_t l_t. \tag{5} \)

Optimal labour demand is pinned down by the condition

\[ F_t = w_t (1 + \eta r^e_t) \tag{6} \]

Technological change is stationary, stochastic and persistent. I closely follow Ilut & Schneider (2014) and introduce ambiguity about the true data generating process that drives aggregate productivity into the set-up. The true law of motion for productivity is described by

\[ z_t = \rho_z z_{t-1} + u_t + \mu^*_t, \tag{7} \]

where \( u_t \) is a mean zero iid shock with variance \( \sigma^2_u \). Further, let \( \mu^*_t \) denote a deterministic sequence of changes in productivity. It is assumed that its long-run empirical distribution converges to an iid normal distribution with mean zero and variance \( \sigma^2_z - \sigma^2_u > 0 \) which is independent of stochastic shocks \( u_t \).

Since only aggregate productivity \( z_t \) is observable, there is no probabilistic assessment for the deterministic component \( \mu^*_t \). An econometrician would find by observing the sequence of productivity realisations \( z^t = (z_1, \ldots, z_t) \) that innovations to \( z_t \) are iid normal with mean zero and variance \( \sigma^2_z \). Therefore, there is no possibility of learning since the processes \( \mu_t \) and \( u_t \) cannot be disentangled.\(^{13}\)

As a result, agents are not only confronted with ambiguity about the future realisation of productivity which they infer from iterating equation (7) one period forward. Since the law of motion (7) covers a hole class of different DGPs, there is also ambiguity about the underlying probabilistic model. This requires additional structural assumptions on individual preferences for the formation of expectations as will be explained in detail in the optimisation problems of each class of agents in the subsequent sections.\(^{14}\)

### 3.4 Banks

The modelling of the heterogeneous banking sector follows the setting in Engler & Große Steffen (2014). Let there be an infinite amount of banks of measure one in the domestic economy. Banks are endowed with previously accumulated household deposits \( N \). Further, they carry over government bonds \( B^{D}_{t-1} \) from period \( t-1 \) decisions.

\(^ {13} \)See Epstein & Schneider (2007) for a formalisation of learning under ambiguity.

\(^ {14} \)In particular in Section 3.6 on the preferences of ambiguity averse investors which is deferred after the discussion of the government problem for a better comprehensibleness.
There are two types of banks, \( \tau \in \{p, u\} \). At an exogenous probability \( \pi^p \), a bank turns into a productive bank \((\tau = p)\), which is matched with a non-financial firm. At the complement probability \( \pi^u = 1 - \pi^p \), a bank is of the unproductive type \((\tau = u)\). These banks are not matched, but offer their financial resources as interbank loans \((M_t)\) on the domestic interbank market to type \( p \) banks at the interest rate \( r^M_t \).

Productive banks use deposits \( N^p \) and interbank loans to extend working capital loans to non-financial firms. Two frictions are assumed to be present in the domestic interbank market. First, type \( p \) banks borrowing wholesale funds want to self-insure against potential roll-over risk given the inherent maturity mismatch. Banks build excess reserves which are a constant fraction of interbank loans, \( R_t = M_t/\phi \), with \( \phi > 1 \).

Second, interbank loans are collateralised with government debt. Unproductive banks who lend in the interbank market thereby lower counterparty risk in the market for wholesale funding. The collateral constraint takes the form

\[
M_t \leq \frac{(1 - \delta_t)B^{D,p}_t}{\chi},
\]

where \( \chi \) denotes a constant haircut parameter. The term \((1 - \delta_t)\) alludes to the riskiness of government debt. In case the government decides to default, government debt loses its collateral value. We apply a short-cut and calibrate a risk-premium on collateralised debt with risky underlying assets. Specifically, let \( \Psi(q_t) \) denote a risk premium on secured interbank debt that is assumed to be decreasing in the price of government bonds \( q_t \) (Barro 1976).

Each bank maximizes a discounted infinite stream of dividend payments to the representative household, using the bank discount factor \( \beta^b \),

\[
\max E_t \left[ \sum_{j=0}^{\infty} \beta^b D^r_{t+j} \right].
\]

Additional to interbank loans, working capital and excess reserves, banks of both types may also invest in the deposit facility of a central bank. Central bank deposits \( R^{d,\tau}_t \) are remunerated at a constant exogenous rate \( r^R \).

Domestic banks form expectations under the paradigm of subjective expected utility (SEU), hence they are assumed to be not ambiguity averse. Banks treat the evolution of TFP from the perspective of an econometrician, i.e. as an AR(1) mean zero process, \( z_{t+1} = \rho z_t + \varepsilon_t^z \) with \( \text{iid} \) innovations \( \varepsilon_t^z \sim \mathcal{N}(0, \sigma^2_z) \).

Having discussed the available investment opportunities and the objective function of banks \( \tau = \{p, u\} \), we next derive the efficiency conditions for optimal intra-temporal loan decisions and inter-temporal collateral holdings. We start with a representative

\footnotetext[15]{This can be interpreted according to domestic banks having more confidence in their own forecasting model such that they are less inclined to consider less tangible information in contrast to foreign investors.}
productive bank. The maximization problem for a type \( p \) banks is

\[
W^p(s(t)) = \max_{\{B_{t+1}^D,M_t,R_t^p,R_{t+1}^p\}} D^p_t(s(t)) + \beta^p E_t [W(s(t+1))],
\]

subject to a flow of funds constraint, and a non-negativity constraint for central bank deposits and dividend payments

\[
N^p + M_t = \kappa_t + R_t^p, \quad (10)
\]

\[
R_t^p, D_t^p \geq 0. \quad (11)
\]

The term \( s(t) \) is a short cut representation of the state \( s = (B_t, B_t^D, z_t, a_t) \) at period \( t \). Dividend payments of type \( p \) banks amount to

\[
D_t^p = (1 - \delta_t) B_t^D + (1 + r_t^M) M_t + (1 + R_t) R_t^p - (1 - \delta_t) \tilde{q}_t B_{t+1}^D - N^p, \quad (12)
\]

with \( R_t^p = R_t^e + R_t^d \). \quad (13)

Writing down the corresponding maximization problem of type \( u \) banks, one obtains

\[
W^u(s(t)) = \max_{\{B_{t+1}^D,M_t,R_t^u,R_{t+1}^u\}} D^u_t(s(t)) + \beta^u E_t [W(s(t+1))]
\]

subject to the flow of funds constraint and non-negativity requirement on central bank deposits and dividend payments

\[
N^u = M_t + R_t^{d,u}, \quad (14)
\]

\[
R_t^{d,u}, D_t^u \geq 0. \quad (15)
\]

Type \( u \) banks’ dividend payments amount to

\[
D_t^u = (1 - \delta_t) B_t^D + (1 + r_t^M) M_t + (1 + R_t) R_t^u - (1 - \delta_t) \tilde{q}_t B_{t+1}^D - N^u \quad (16)
\]

The continuation value for both types of banks is identical, since the probabilities of types in the consecutive period are independently and identically distributed. Therefore, we obtain

\[
E_t [W(s(t+1))] = E_t [\pi^p W^p(s(t+1)) + \pi^u W^u(s(t+1))]. \quad (17)
\]

The efficiency conditions for financial intermediation in the domestic banking sector can be derived from the respective Lagrangian to the optimization problems of banks \( \tau = \{p, u\} \). For productive banks, we obtain

\[
r_t^\kappa = R_t^R + \mu_t^p, \quad (18)
\]

\[
r_t^\kappa = r_t^M + \frac{r_t^\kappa - r_t^R}{\phi} + \lambda_t \chi, \quad (19)
\]

\[
\tilde{q}_t = \beta^b E_t [W_{BD}(s(t+1))] + \mu_t^{D,p}. \quad (20)
\]
For unproductive banks, one gets

$$r_t^M = r^R + \Psi(q_t) + \mu^u, \quad (21)$$

$$\tilde{q}_t = \beta^b E_t [W_{BP}(s(t + 1))] + \mu_t^{D,u}. \quad (22)$$

With the envelope condition for (17), the pricing equation for collateral assets for both types of banks reads

$$\tilde{q}_t = \beta^b (1 - \pi_t^\delta) + \pi^p \beta^b E_t (\lambda_{t+1}) + \mu_t^D + \mu_t^{B,P}. \quad (23)$$

The equilibrium quantities of collateral are found using the no-arbitrage condition

$$\tilde{q}_t - q_t = 0 \text{ where } q_t \text{ is the price of government bonds at international capital markets as explained below. Using (23), this condition can be re-written as}$$

$$\beta^b (1 - \pi_t^\delta) + \pi^p \beta^b E_t (\lambda_{t+1}) + \mu_t^D + \mu_t^{B,P} - q_t = 0. \quad (24)$$

### 3.5 Government problem

The benevolent government maximizes the infinite life-time utility of the representative household. To do so, it has two decision variables. There is the binary decision to default $\delta_t \in \{0, 1\}$. In case of debt repayment, the government also takes a borrowing decision, $B_{t+1}$. Formally, the government objective function reads,

$$V_t(s(t)) = \max_{\{\delta_t\}} \{V_t^{nd}, V_t^d\}, \quad (25)$$

where $V_t^{nd}, V_t^d$ denote the values under repayment and default, respectively.

The value under repayment is given by

$$V_t^{nd}(s(t)) = \max_{\{c_t, B_{t+1}\}} \{U(c_t, 1 - l_t)$$

$$+ \beta E_t [V_{t+1}(s(t + 1))])\}, \quad (26)$$

subject to the aggregate resource constraint in the economy

$$c_t = e^{zt} K^\alpha l_t^{1-\alpha} + (B_t + B^D_t) - q_t (B_{t+1} + B^D_{t+1})$$

$$= Y_t - B^*_t + q_t B^*_{t+1} \quad (27)$$

and the set of partial equilibrium conditions under repayment.

Further, let the value under default be given by

$$V_t^d(s(t)) = \max_{\{c_t\}} \{U(c_t, 1 - l_t)$$

$$+ \beta E_t [(\theta V_t^{nd}(0, 0, z_{t+1}, a_{t+1}) + (1 - \theta)V_t^d(0, 0, z_{t+1}, a_{t+1})])\} \quad (28)$$

subject to the set of partial equilibrium conditions without interbank trading and the resource constraint under autarky:

$$c_t = Y_t \quad (29)$$
Conditional on default, the government writes off the entire stock of government debt. The economy falls into financial autarky with a probability to return to international and domestic capital markets. With the complement probability \((1 - \theta)\) it will remain in the autarky state in the consecutive period.

The government’s decisions characterise a repayment set \(\Gamma^{nd}\) and a default set \(\Gamma^d\) within the state space:

\[
\Gamma^{nd}_t(B_t, B^D_t) = \{ s^e = (z_t, a_t) \in s : V^{nd}_t(s(t)) \geq V^d_t(s(t)) \}
\]

\[
\Gamma^d_t(B_t, B^D_t) = \{ s^e = (z_t, a_t) \in s : V^d_t(s(t)) > V^{nd}_t(s(t)) \}
\]

The default probability is defined as the conditional cumulative probability density over the productivity and the ambiguity state in period \(t + 1\) from the default set \(\Gamma^d\), given the future endogenous state \((B_{t+1}, B^{D}_{t+1})\):

\[
\pi^{d}_t(B_{t+1}, B^{D}_{t+1}, z_t, a_t) = \int^z \int^a f(z_{t+1}, z_t, a_{t+1}, a_t) dz_{t+1} da_{t+1} \quad (30)
\]

### 3.6 Ambiguity averse international investors and uncertainty shocks

International investors are modelled in line with the literature on optimal sovereign default with the exception that I assume that they are ambiguity averse. Confronted with ambiguity about the true value of the deterministic component of aggregate productivity \(\mu^*_t\), investors gather less tangible information as e.g. newspaper articles in order to arrive at an assessment of the deterministic component of aggregate productivity. The degree of ambiguity after considering all information is captured by a numeric value \(a_t\) and summarized in a belief set \(\mathcal{P}_t\) which collects multiple priors on the conditional mean of \(\mu^*_t\). This belief set is assumed to be symmetrically centred around zero, \(\mathcal{P}_t = [-|a_t|, |a_t|]\), thus the DGP of productivity from equation (7) is constrained by investors’ beliefs to

\[
z_{t+1} = \rho z_t + \mu_t + u_{t+1}, \quad \text{with } \mu_t \in [-|a_t|, |a_t|] \quad (31)
\]

The set of prior beliefs is affected by uncertainty shocks. Specifically, the boundaries of the belief set are widened when ambiguity increases. An increase in \(|a_t|\) can thereby be interpreted as a loss of confidence in the own forecasting model in response to either a deterioration of the quality of intangible information or an increase in forecast dispersion.\(^{16}\) The level of uncertainty follows an exogenous AR(1) process which is known to agents and takes the form

\[
a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \varepsilon_a^a, \quad (32)
\]

where \(\bar{a} \geq 0\) denotes the unconditional mean, \(\varepsilon^a_t\) is an iid disturbance with variance

\(^{16}\)Note that the model does not structurally explain how \(a_t\) is determined. Instead, I follow Ilut & Schneider (2014) and use available data in order to discipline the evolution of uncertainty according to its law of motion (32).
\( \sigma_u^2 \) which is uncorrelated with \( u_t \) from (31).

Each period, investors are required to choose a specific conditional mean \( \mu_t^p \) from the set of priors in order to form expectations and forecast productivity according to the law of motion of \( z_t \) from equation (31). I follow Gilboa & Schmeidler (1989) and Epstein & Schneider (2003) and adopt maxmin preferences in recursive notation which lead investors to select a worst case prior.\(^{17}\) Specifically, the international investor minimizes the expected continuation utility under ambiguity subject to the constraint that the prior \( \mu_t^p \) must lie in the period \( t \) belief set.

Applying the maxmin preference structure to the optimization problem of a risk-neutral representative international investor who is confronted with sovereign default risk yields the following optimization problem:

\[
\max_{\{B_{t+1}\}} \Pi_t = -q_tB_{t+1}^* + \min_{\{\mu_t^p \in \mathcal{P}_t\}} E_t^p \left( \frac{1 - \delta_{t+1}}{1 + rf} \right) B_{t+1}^*
\] \hspace{1cm} (33)

The minimization step in (33) requires that the expectations operator with multiple priors \( E_t^p \) is evaluated under the prior which yields the lowest value to the investor.

**Proposition 1.** Let \( p_0 \) denote the worst case belief from the perspective of an ambiguity averse international investor. Then, the prior from the set \( \mathcal{P}_t \) that minimizes expected utility has mean \( \mu_{t0} = -a_t \).

**Proof.** See Appendix.\(^{18}\)

As a result, the international investor acts as if there is just one belief in the set \( \mathcal{P}_t \). The investor uses equation (31) to form the forecasting rule for aggregate productivity

\[
E_t^p(z_{t+1}) = \rho z_t - a_t,
\] \hspace{1cm} (34)

where \( \mu_t = -a_t \) follows from the worst case belief.\(^{19}\) The pricing condition for government debt with multiple priors yields

\[
q_t(B_{t+1}, B_{t+1}^D, z_t, a_t) = \min_{\{\mu^p \in [-a,a]\}} E_t^p \left( \frac{1 - \delta_{t+1}}{1 + rf} \right),
\] \hspace{1cm} (35)

where the expected default probability \( E_t^p(\delta_{t+1}) = \pi_t^s(B_{t+1}, B_{t+1}^D, z_t, a_t) \) is given by (30).

\(^{17}\)The adoption of worst case beliefs, formalized by the maxmin representation of expected utility, can be derived from the axioms of uncertainty aversion and certainty independence, see Epstein (1999) and Epstein & Schneider (2003). The recursive formulation also allows for dynamic consistency of preferences.

\(^{18}\)It can be shown that the default probability of the sovereign is monotonically decreasing in productivity, \( \partial \delta_t / \partial z_t < 0 \). This is a necessary and sufficient condition such that one can conclude that the minimizing prior is indeed the lowest element in the belief set, \( \mu_t = -a_t \), as this minimizes the expected pay-off to the investor.

\(^{19}\)See the Appendix for a detailed description of the formation of a one period ahead conditional forecast in a discrete state space using numerical methods.
4 Simulation results

4.1 Calibration

The calibration of the model does not aim to quantify the effects of uncertainty shocks in the Euro area. The main reason is it is hard to find a suitable counterpart to the ambiguity process in the data. Instead, I choose plausible parameter values that have been identified in the literature to analyse the implications of uncertainty shocks in models of optimal sovereign default. Most parameters are taken from the calibration in Engler & Große Steffen (2014) who use Spanish data for the quantitative model fit (Table 2). A detailed description and discussion of calibration targets and estimated values can be found there, too.

<table>
<thead>
<tr>
<th></th>
<th>Mode</th>
<th>.05 percentile</th>
<th>.95 percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambiguity persistence</td>
<td>$\rho_a$</td>
<td>0.887</td>
<td>0.873</td>
</tr>
<tr>
<td>Level ambiguity</td>
<td>$n$</td>
<td>0.955</td>
<td>0.983</td>
</tr>
<tr>
<td>Ambiguity</td>
<td>$\sigma_n$</td>
<td>0.134</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Notes: Estimates reproduced from Table 1. in Ilut & Schneider (2014).

Novel are the parameters that govern the dynamic process of uncertainty. I follow Ilut & Schneider (2014) in their calibration approach. The level of ambiguity is bounded due to a model consistency criterion. Agents with ambiguity aversion are expected to associate not more uncertainty to the true data generating process than there is observable volatility in past aggregate productivity realisations. Since the process for $\mu_t^*$ is assumed to converge to an iid normal sequence with mean zero and variance $\sigma_z^2 - \sigma_a^2$, the mean level of uncertainty $\bar{a}$ is bounded from above. Ilut & Schneider (2014) re-parametrize the ambiguity process as $\bar{a} = n \sigma_z$ and estimate the numerical values for $(\rho_a, n, \sigma_n)$ using Bayesian techniques in a log-linear approximation to a New Keynesian model. The re-parametrisation allows to impose that the model consistency criteria are respected by an adequate choice of prior distributions in the estimation step. I use the posterior distribution of the parameter estimates by Ilut & Schneider (2014) on $n$ and $\sigma_n$ in order to calibrate the ambiguity process $(\rho_a, \bar{a}, \sigma_a)$, see Table 1.

4.2 Pricing and spillover effect of uncertainty shocks

In order to gauge the effect of uncertainty shocks, let’s consider the model with uncertainty shocks as in the baseline calibration. Elevated levels of uncertainty translate through investors’ preferences monotonically into lower bond prices at a given debt level. Figure 4 illustrates the effect of uncertainty on bond prices for a fixed level of aggregate productivity. It confirms the typical pricing schedule for government borrowing in quantitative default models (Arellano 2008). The more a government

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$^{20}$E.g. to achieve a performance of forecasts in which the worst case belief is the best forecast at least 5 percent of the times, one needs $\bar{a} < 2\sigma_z$. 

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Table 2: Baseline calibration

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean level of uncertainty</td>
<td>$\bar{a}$</td>
<td>0.0162</td>
<td>a)</td>
</tr>
<tr>
<td>Variance of uncertainty</td>
<td>$\sigma_a$</td>
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<td>a)</td>
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<tr>
<td>Persistence of uncertainty</td>
<td>$\rho_a$</td>
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<td>a)</td>
</tr>
<tr>
<td>Household’s discount factor</td>
<td>$\beta$</td>
<td>0.92</td>
<td>b)</td>
</tr>
<tr>
<td>Capital stock</td>
<td>$K$</td>
<td>11.92</td>
<td>b)</td>
</tr>
<tr>
<td>Working capital requirement</td>
<td>$\eta$</td>
<td>1.074</td>
<td>b)</td>
</tr>
<tr>
<td>Liquid liabilities</td>
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<td>b)</td>
</tr>
<tr>
<td>Efficiency parameter $\mathcal{P}$</td>
<td>$\phi$</td>
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<td>b)</td>
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<tr>
<td>Probability for lending bank</td>
<td>$\pi^p$</td>
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<td>b)</td>
</tr>
<tr>
<td>Collateral requirement</td>
<td>$\chi$</td>
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<td>b)</td>
</tr>
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<td>Capital share of output</td>
<td>$\alpha$</td>
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<td>c)</td>
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<td>Risk aversion</td>
<td>$\gamma$</td>
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<td>d)</td>
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<tr>
<td>Wage elasticity of labor supply</td>
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<td>d)</td>
</tr>
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<td>Riskfree rate</td>
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<td>d)</td>
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<td>Bank’s discount factor</td>
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<td>b)</td>
</tr>
<tr>
<td>Probability of reentry</td>
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<td>Richmond &amp; Dias (2009)</td>
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<tr>
<td>Persistence of TFP shock</td>
<td>$\rho$</td>
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<td>Mendoza &amp; Yue (2012)</td>
</tr>
<tr>
<td>Std.dev. of TFP shock</td>
<td>$\sigma_x$</td>
<td>0.017</td>
<td>Mendoza &amp; Yue (2012)</td>
</tr>
<tr>
<td>Rate on central bank reserves</td>
<td>$r^R$</td>
<td>0.0077</td>
<td>b)</td>
</tr>
<tr>
<td>Cost function collateral</td>
<td>$\psi$</td>
<td>0.245</td>
<td>OLS estimate</td>
</tr>
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</table>

Notes: (a) Ilut & Schneider (2014); (b) Engler & Große Steffen (2014); (c) Smets & Wouters (2003); (d) Mendoza & Yue (2012).

borrows today, the lower is the price at which a bond can be sold in primary markets, since the probability of repayment shrinks with higher debt levels. An increase in the level of uncertainty $a_t$ shifts the bond price schedule inwards. This has two effects. First, to roll-over the amount of public debt accumulated in previous periods, the government has to pay a higher interest rate. This pricing channel of uncertainty makes the option of servicing debt relatively less valuable today, as the government anticipates the higher burden of debt repayment in the future.

Second, lower bond prices affect equilibrium allocations through the sovereign risk channel on the interbank market. The spillover channel of uncertainty shocks is illustrated in Figure 5. As the outlook for future debt repayment deteriorates given higher levels of ambiguity, risk premia on collateralised wholesale funding rise such that the interbank rate increases (Panel 5a). The sharp decline in interbank intermediation in response to higher funding costs propagates into lower loan supply through matched banks (Panels 5b,5c). Due to the working capital requirement (5), cash-in-advance constrained non-financial firms can hire labour only up to the point where wage bills can be financed through bank lending. In equilibrium, bank lending rates increase in response to uncertainty shocks to align supply and demand on the domestic credit market. Non-financial firms demand less labour at a given wage rate since the wedge between the marginal product of labour and the wage rate in equation (6) increases by $w_t \eta r^f_t$. As a result, shocks to the level of ambiguity are a source of macroeconomic
fluctuations in aggregate output.

The propagation mechanism of uncertainty on sovereign default risk is linked to the endogenous default penalty. As ambiguity on future TFP realisations dampens financial intermediation today, it simultaneously lowers the costs implied by a freeze on the domestic interbank market. As it is less beneficial to service outstanding debt, the spillover channel pushes up sovereign default risk.

### 4.3 Non-fundamental roll-over crises

This section characterises under which conditions uncertainty might induce a non-fundamental roll-over crisis in sovereign debt markets. The previous section laid out how the pricing and spillover channel of uncertainty shocks lower the value in the repayment case, $V^{nd}(s(t))$. Figure 6 illustrates the case for which the joint effect of ambiguity leads to non-fundamental default. Note that the value of repayment is a strictly increasing function in the absolute level of ambiguity, $|a_t|$, as belief sets $P_t$ are symmetric in the realisation $a_t$. According to the composition of repayment today from equation (25), the spillover channel affects the contemporaneous utility in the repayment regime, while the pricing channel lowers the continuation value.

The default value $V^d(s(t))$ is equally a decreasing function in $|a_t|$. Intuitively, the continuation value conditional on re-accessing financial markets is higher if the level of ambiguity is lower, such that the component $V^d_{t+1}(0, 0, z_{t+1}, a_{t+1})$ is affected by current realisations of $a_t$ due to the persistence in ambiguity, $\rho_a > 0$.

The sensitivity in the contemporaneous realisation of ambiguity is significantly higher for repayment ($V^{nd}$) than for defaulting ($V^d$). Figure 6 depicts the case where both values intersect for a given economic fundamental $z_t$. While the government decides to service the debt at low levels of ambiguity, it defaults for higher levels given that the default condition $V^d > V^{nd}$ is satisfied. I define non-fundamental default in the model as follows:
Figure 5: Spillover channel of uncertainty shocks

(a) Interbank rate

(b) Interbank loans

(c) Credit

Note: Variation of bond price schedules and allocations on the interbank market along current uncertainty realisations $a_t$. Productivity is normalised at $e^z = 1$.

Definition. (Non-fundamental default) A non-fundamental default is given in case when there is optimal default under the ambiguity-restricted price schedule at non-zero levels of uncertainty (i), whereas the government services the debt in case there is zero uncertainty (ii). Technically, the conditions for a non-fundamental default in the model are:

(i) $s^f = (z_t, a_t \leq 0) \in s : V^d_t(B_t, B^D_t, z_t, a_t) > V^{nd}_t(B_t, B^D_t, z_t, a_t)$,

(ii) $\land s^f = (z_t, a_t = 0) \in s : V^d_t(B_t, B^D_t, z_t, 0) \leq V^{nd}_t(B_t, B^D_t, z_t, 0)$.

Allowing for ambiguity aversion leads to a structural interpretation of an investors’ run in the presence of multiple equilibria as discussed in Cole & Kehoe (2000). Typically, models with strategic complementaries for investors’ behaviour as in the seminal bank-run model by Diamond & Dybvig (1983) require an exogenous sunspot event to determine which equilibrium materialises, the run or the roll-over equilibrium. One

21 Note that there is still a difference between a model with SEU-investors and $a_t = 0$.

22 See also Obstfeld (1986), ?.
avenue to solve the problem of equilibrium selection is provided by the literature on global games (Corsetti et al. 2006, Morris & Shin 2006). Ambiguity averse investors’ preferences present an alternative to sunspot equilibria and global games. The problem of multiplicity of equilibria is not present here, although the multiple prior model alludes to the fact that different outcomes might be possible. However, maxmin preferences among investors and the fact that the level of ambiguity $a_t$ is public information aligns default expectations of different agents in the model economy as investors act as if there is only one belief in their set of possible priors. As a result, the equilibrium is always unique and determined.

Figure 7 illustrates the default set conditional on the level of uncertainty along the productivity dimension (Panel 7a), and the domestic debt dimension (Panel 7b). The dark shaded area indicate a default for sure at any level of uncertainty. The grey shaded area is the repayment set, again no matter what level of uncertainty materialises. In contrast, the yellow coloured area in between the default and the repayment sets is the region where default is pinned down by uncertainty: A high realisation of $a_t$ might eventually trigger outright default, while lower levels of uncertainty make debt repayment the optimal policy choice (cf. Figure 6). Following Cole & Kehoe (2000), we denote this area as the crisis zone. If the economy is in the crisis zone, a non-fundamental default might occur in a sense that worst case beliefs of international investors are able to induce outright sovereign default on public debt, conditional on sufficiently elevated levels of ambiguity regarding the fundamental state.

It is the presence of a crisis zone in a model of strategic sovereign default augmented with ambiguity averse investors which allows to rationalise the empirical impulse responses obtained in section 2. A country which is in or close to the crisis zone as defined in the theoretical model will exhibit a much stronger response in sovereign credit risk and bank lending rates to a one-off increase in the level of uncertainty than a country which is far outside the crisis zone. The development of sovereign yields in the Euro area can therefore partly be explained by distinguishing between countries
4.4 Business cycle implications

This section analyses the quantitative implications of uncertainty shocks for business cycle fluctuations. Using a production economy with financial frictions as in Engler & Große Steffen (2014), it is possible to treat time-varying degrees of ambiguity about the fundamental state of the economy as a source for structural shocks that are endogenously propagated into equilibrium allocations. This dimension of the analysis is absent in related studies as Costa (2009) and Pouzo & Presno (2013) who look at robust control preferences in endowment economies.

I simulate the model 1500 times over a time period of 864 quarters. I subtract the first one hundred observations as a burn-in period. The remaining 191 years of simulated data represent the time span for which historical default frequencies are available (Sturzenegger & Zettelmeyer 2007). Further, 48 periods prior to a default are isolated and used for the computation of statistical moments produced by the model. Table 3 presents the moment obtained by averaging over all numerical simulations. The first column presents the results of the baseline model. For a comparative static analysis, columns (2) and (3) present a regime with higher uncertainty and lower uncertainty, respectively, each calibrated according to the 0.05 and 0.95 percentiles of the posterior estimates for parameters \( (n, \sigma_n) \) from Table 1.

The results show that higher mean level of ambiguity \( \bar{a} \) lowers the probability of default. This result is due to two effect which both lead to an overall reduction in the frequency of default with higher mean level of ambiguity. First, the marginal benefit from borrowing in international capital markets shrinks in the mean level of ambiguity, as investors charge additional ambiguity premia. As shown in Figure 8, this leads to an
Table 3: Simulation results

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) High uncertainty</th>
<th>(3) High volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of default</td>
<td>0.023</td>
<td>0.001</td>
<td>0.018</td>
</tr>
<tr>
<td>Non-fundamental defaults</td>
<td>0.02254</td>
<td>0.00044</td>
<td>0.01763</td>
</tr>
<tr>
<td>Sovereign risk premium</td>
<td>0.3821</td>
<td>0.2371</td>
<td>0.3787</td>
</tr>
<tr>
<td>Debt-to-GDP ratio</td>
<td>14.8817</td>
<td>15.2994</td>
<td>15.9852</td>
</tr>
<tr>
<td>Dom. debt share</td>
<td>69.1603</td>
<td>74.9044</td>
<td>69.92</td>
</tr>
<tr>
<td>Credit premium</td>
<td>0.7747</td>
<td>0.5562</td>
<td>0.7079</td>
</tr>
</tbody>
</table>

Correlations with output
- Lending: 0.969, 0.971, 0.973
- Sovereign premium: 0.076, -0.012, 0.151
- Credit premium: 0.098, 0.262, 0.239

Correlations with sovereign premium
- Credit premium: 0.246, 0.325, 0.164
- Lending: 0.018, -0.081, 0.117

overall reduction in the debt-to-GDP ratio. Further, domestic banks are not ambiguity averse. Thus, a higher share of total government debt is held by domestic banks when the mean level of ambiguity increases according to the no-arbitrage condition.23 Both effects taken together lead to a monotonic reduction in the frequency of default with the mean level of ambiguity.

5 Conclusion

This paper analyses empirically and theoretically the effects of uncertainty shocks on sovereign default risk and financial intermediation. The main finding is that the presence of ambiguity averse investors helps to rationalise the dichotomy in sovereign debt pricing in the course of the European sovereign debt crisis.

In a first step, a VAR analysis is conducted for a panel of four Euro area member countries. I find that sovereign credit risk increases in response to economic uncertainty in Spain and Italy, while there is no significant response in the core countries Germany and France. Further, bank lending rates have the tendency to react stronger in the periphery than in the core countries.

This dichotomy in the Euro area is further analysed in a macro model with strategic debt repayment and ambiguity averse international investors. Investors form worst case beliefs in line with the multiple-priors model of Gilboa & Schmeidler (1989). In a setting with strategic sovereign default, this leads to a pessimistic evaluation of the repayment probability of sovereign bonds. As a result, investors charge a higher interest rate to roll-over the existing stock of public debt when the level of ambiguity surrounding the macroeconomic fundamental is high.

23This is a feature that has recently been discussed under the notion of re-patriation of sovereign debt, see e.g. Bruttì & Sauré (2013).
The arising pricing channel of uncertainty shocks makes debt repayment less beneficial, as higher amounts of debt need to be repaid in the near future. Further, a spillover channel arises through secured wholesale lending on the domestic interbank market where government bonds serve as collateral. A reduction in interbank intermediation gives rise to an increase in private sector interest rates. The endogenous default penalty, consisting of a credit crunch if interbank intermediation breaks down, is lowered and gives rise to even higher sovereign default risk.

The model provides a propagation mechanism for uncertainty shocks which is distinct from Ilut & Schneider (2014). Financial market outcomes, business cycle fluctuations and endogenous sovereign default risk are jointly affected by time-varying degrees of ambiguity on the future macroeconomic fundamental. Similar to Cole & Kehoe (2000), a crisis zone emerges where outright sovereign default can occur even if macroeconomic fundamentals are sound enough to sustain debt repayment at lower levels of ambiguity.

The model’s predictions are broadly in line with the empirical VAR analysis. Specifically, the possibility of non-fundamental roll-over crises is able to rationalise the dichotomy in impulse responses in the Euro area. The periphery countries seem to be in or close to the crisis zone, as there is a significant response in sovereign default risk to economic uncertainty. The propagation via domestic interest rates also seems to be broadly consistent with empirical impulse responses. On the contrary, core countries are distant enough to the crisis zone such that an increase in fundamental uncertainty does not affect the judgement of ambiguity averse investors on sovereign default risk.


Knight, F. H. (1921), Risk, uncertainty, and profit, Hart Schaffner & Marx and Houghton Mifflin Co., Boston and MA.


A Timing of events

Figure 9: Timing
B Economic policy uncertainty index

Figure 10: Economic policy uncertainty index

Note: The economic policy uncertainty index (EPU) measures frequency of appearances of the word ‘uncertain’ or ‘uncertainty’ in newspaper articles related to economic content, see Baker et al. (2013) for further details. The measure for the Euro area in panel (a) is the average over Germany, France, Italy and Spain. The series are obtained via Thomson Reuters Datastream.