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# Bias-corrected estimation in mildly explosive autoregressions\*

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## Abstract

This paper provides a comprehensive Monte Carlo comparison of different finite-sample bias-correction methods for autoregressive processes. We consider situations where the process is either mildly explosive or has a unit root. The case of highly persistent stationary is also studied. We compare the empirical performance of the plain OLS estimator with an OLS and a Cauchy estimator based on recursive demeaning, as well as an estimator based on second differencing. In addition, we consider three different approaches for bias-correction for the OLS estimator: (i) bootstrap, (ii) jackknife and (iii) indirect inference. The estimators are evaluated in terms of bias and root mean squared errors (RMSE) in a variety of practically relevant settings. Our findings suggest that the indirect inference method clearly performs best in terms of RMSE for all considered orders of integration. If bias-correction abilities are solely considered, the jackknife works best for stationary and unit root processes. For the explosive case, the bootstrap and the indirect inference can be recommended. As an empirical application, we study Asian stock market overvaluation during bubbles and emphasize the importance of bias-correction for explosive series.

**Key words:** Autoregression · Explosive behavior · Bias-correction · Bubbles · Stock market valuation

**JEL classification:** C13, C22, G12

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# 1 Introduction

Measuring and estimating the persistence of macroeconomic and financial time series is a long standing issue in econometrics. The most common framework for assessing the persistence is the autoregressive model. But, a major practical problem is the inherent bias of the conventional OLS estimator. Its bias increases amongst two dimensions: a small sample size and a true autoregressive parameter in the vicinity of unity are disadvantageous. Given a relatively small sample size, it is a complicated task to estimate the persistence if the process is (i) either stationary, but highly persistent, (ii) exhibits a unit root or (iii) is mildly explosive. As we argue, these situations are likely to occur in practice.

In finance and economics, it is a well established fact that most time series are characterized by high persistence or even stochastic trends, see e.g. Nelson and Plosser (1982) and Schotman and van Dijk (1991). Another important empirical issue is the instability of parameters, which is often observed and documented (see e.g. Stock and Watson, 1996). During the past decade, a literature on structural changes in persistence emerged, see e.g. Chong (2001), Kim (2000), Harvey et al. (2006) and Leybourne et al. (2007) amongst many others. In order to cope with potential time-variation in the parameters, users often apply the popular rolling window technique. Under these empirically relevant circumstances, the issue of unbiased and efficient estimation of persistence becomes particularly important: Typically, a relatively small window size is chosen in order to capture time-variation.<sup>1</sup> If a bubble or a crisis occurs in this particular window, some financial and economic time series are likely to exhibit explosive behaviour. Leading examples for time series with at least local explosive roots are stock prices (as caused by bubbles, see Diba and Grossman, 1988), price-dividend and price-earnings ratios (as caused by a dominant regime of chartist traders, see Lof, 2012), house and oil prices (due to speculation, see Homm and Breitung, 2012, Clark and Coggin, 2011 and Shi and Arora, 2012), hyperinflation (due to a collapse of a country's monetary system, see Casella, 1989), exchange rates (due to speculation van Norden, 1996 and Pavlidis et al., 2012) and the US Debt/GDP ratio (due to unsustainable fiscal policies, see Yoon, 2011) amongst others.

The complicated estimation of autoregressive processes in finite-samples sparked a fruitful area of research. Kendall (1954), Shaman and Stine (1988), Tjøstheim and Paulsen (1983), Tanaka (1984) and Abadir (1993) provide analytic derivations of asymptotic expansions which can be used for bias-correction, see also Abadir (1995) for the context of unit root testing. Approximately median-unbiased estimation is proposed in e.g. Andrews (1993), Andrews and Chen (1994) and Roy and Fuller (2001). Restricted maximum likelihood estimation is considered in Cheang and Reinsel (2000). Bootstrap-based bias-correction procedures have been suggested by e.g. Kilian (1998), Hansen (1999) and Kim (2003). Indirect inference has been put forward in MacKinnon and Smith (1998) and Gouriéroux et al. (2000). Jackknifing based on Efron (1979) is recently studied in Chambers (2013). Importantly, we note that the main body of the literature

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<sup>1</sup>Small samples may also arise from a low frequency of recorded data or newly released indexes. In another situation the researcher might want to split the sample into several parts (due to possible structural breaks) in order to compare pre- and post break periods.

focuses on stationary autoregressive models and on the unit root case while the case of (mildly) explosive behaviour has received less attention.

This work compares the OLS estimator with several approaches to tackle the bias problem. A very simple way of mitigating the bias is to use recursive demeaning see So and Shin (1999b). Standard demeaning leads to a correlation between the regressors (lags of the series) and the error term. Moreover, one can also use the Cauchy estimator (see So and Shin (1999a)) which is also builds upon a recursive mean adjustment. It is approximately median-unbiased even for unit root and explosive processes. This property makes it more attractive in comparison to the analytic median-bias correction proposed in Roy and Fuller (2001) which rules out explosiveness by construction. We also consider a recently proposed estimator based on second differencing by Chen and Kuo (2013). Moreover, we study three different principles of bias-correction applied to the OLS estimator. Amongst these are the bootstrap technique by Kim (2003), the jackknife approach by Chambers (2013) and the indirect inference method by Phillips et al. (2011), who propose a technique for autoregressive processes, based on the work of MacKinnon and Smith (1998) and Gouriéroux et al. (2000). Indirect inference estimators to correct the small sample bias have a long tradition, e.g. see Gouriéroux et al. (1993) and Smith (1993). In a recent contribution, Gouriéroux et al. (2010) extend this principle to dynamic panel data models. The indirect inference estimator allows for explosiveness in addition to highly persistent and unit root behavior, see also Phillips (2012) for a recent contribution on its limit theory. The finite-sample properties of recently suggested techniques are not fully explored and a comprehensive comparison has not been conducted yet.

In our Monte Carlo study, we consider small sample sizes, normal and fat-tailed innovations, ARCH disturbances and misspecification of the autoregressive lag structure. Furthermore, we also study the case where a linear deterministic trend is included in the autoregressive model. We evaluate the performance of the estimators by means of bias and root mean squared errors (RMSE). Our results allow several practical recommendations. For instance, we find that the indirect inference estimator performs very well in terms of RMSE, irrespective of whether the series is generated by a stationary process, has a unit root or follows an explosive path. The recommendations for bias-correction (without considering the variance of estimators) are more diverse. We distinguish situations where the practitioner (i) can either rule out stationarity or explosiveness a priori or (ii) aims at using a robust method with balanced performance across the levels of persistence. Regarding (i), the jackknife is highly recommendable in absence of explosiveness, while the bootstrap and indirect inference perform very well for explosive series. Interestingly, these two methods rank second and third for stationary and unit root series which makes their use advisable in case (ii). Thus, if explosiveness can not be ruled out a priori, bootstrap and indirect inference are the best robust choices in terms of bias-reduction (given our set of possible estimators). It is worthwhile to emphasize that the indirect inference estimator has less computational cost than the bootstrap and that it is the clear winner in terms of RMSE.

We provide an empirical application to Asian stock markets during short periods of explosiveness

due to the possible presence of bubbles. The results highlight the importance of bias-correction. We discuss the implications of different estimates for the growth rate of the bubble in terms of implied overvaluation of the stock markets.

The paper is organized as follows. Section 2 describes the different estimators and bias-correction techniques. Our simulation results are presented in Section 3. The empirical application is located in Section 4. Conclusions are drawn in Section 5. Further simulation results and all codes are available from the authors upon request.

## 2 Finite-sample bias corrections

### 2.1 Bias of the OLS estimator

Point of departure is the inherent bias of the OLS estimator in autoregressive models. In order to illustrate the problem, we simulate the empirical performance of the OLS estimator. We focus on finite samples and the possibility of mild explosiveness in a simple autoregressive framework:

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t, \quad (1)$$

where  $\varepsilon_t$  is assumed to be a martingale difference sequence. We consider the cases of stationarity and unit roots, i.e.  $|\rho| < 1$  and  $\rho = 1$ , and the case where  $\rho$  satisfies  $\rho = 1 + c/k_T$ , with  $c > 0$  and  $k_T$  being a sequence tending to infinity such that  $k_T = o(T)$  as  $T \rightarrow \infty$ . In the latter case, the autoregressive parameter is local-to-unity in the sense that  $\rho \rightarrow 1$  as  $T \rightarrow \infty$ . For finite  $T$  (as considered in this work),  $\rho$  deviates moderately from unity. Asymptotic theory for this case is developed in Phillips and Magdalinos (2007).

Figure 1 shows the AR(1) case as in equation (1) for two different sample sizes of  $T = 25$  and  $T = 50$ . The true autoregressive parameter  $\rho$  (on the  $x$ -axis) ranges from 0.6 to 1.2 which measures the persistence of the process. For simplicity, we set  $\mu = 0$ ,  $y_0 = 0$  and  $\varepsilon_t \stackrel{iid}{\sim} N(0, 1)$ . The bias  $B(\hat{\rho}, \rho) = \hat{\rho} - \rho$  is given on the  $y$ -axis, while the true parameter  $\rho$  takes values from 0.6 to 1.2 on the  $x$ -axis. The bias of the *plain* OLS estimator is given as a solid line, while other estimators (OLS estimator with recursive demeaning, Cauchy estimator and an OLS estimator based on second differencing of  $y_t$ ) are discussed below.

— FIGURE 1 ABOUT HERE —

The results confirms earlier theoretical findings that the OLS bias depends on the true value of the autoregressive parameter. The smaller the sample size, the more severe is the downward bias. The vicinity of unity is the region where the bias is strongest. Furthermore, it can be seen that the bias reduces for explosive processes and approaches zero at some point, but that the estimation of mildly explosive processes is still heavily biased.<sup>2</sup> For other estimators, we observe

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<sup>2</sup>It shall be noted that the problem still persists for higher-order autoregressive models. Further simulation results are available upon request.

a much better performance for stationary and unit root models. For instance, the estimator based on second differencing reduces the bias by a considerable amount. Moreover, the Cauchy estimator also performs well and notably better than the recursive OLS estimator. For the explosive case, we have already some interesting preliminary findings: The bias function  $B(\hat{\rho}, \rho)$  is asymmetric with respect to  $\rho$  around unity: the OLS bias vanishes faster for explosive models. After a certain threshold value of  $\rho$ , the OLS bias disappears. Clearly, this threshold value depends on the sample size  $T$ . Alternative estimators (recursive OLS and Cauchy) are reducing the bias for moderate deviations from unity and are thus preferable to the plain OLS estimator. But, for stronger deviations, the choice becomes less clear as the bias function switches its sign for alternative estimators yielding to an upward bias. For values of  $\rho$ , for which  $B(\hat{\rho}, \rho) \approx 0$ , the OLS estimator is preferable in terms of bias. It shall be noted that the second differencing estimator appears to be quite risky in the sense that it may lead to a dramatic upward bias once the time series reaches a certain level of persistence.

Our extensive simulation study in Section 3 takes several important dimensions of bias reduction into account. Beside a broad comparison of estimators and bias-correction techniques (as discussed in the following subsections), we consider the uncertainty of not knowing whether the series is in fact stationary, unit root non-stationary or explosive. In the latter case, it obviously plays an important role *how* explosive the series is. As this is ultimately unknown, it is desirable to use some *robust* estimator in practice unless one can rule out certain cases by assumption.<sup>3</sup> Moreover, we shall take the price of bias reduction in terms of possibly inflated variances into account by additionally comparing root mean squared errors. We proceed by presenting three alternative estimators (as shown in Figure 1). Thereafter, we turn our attention to three bias-correction techniques for the plain OLS estimator. First, a bootstrap method by Kim (2003) is discussed, followed by a simple Jackknife approach advocated by Chambers and Kyriacou (2013) and finally, we describe the indirect inference approach used in Phillips et al. (2011).

## 2.2 Alternative estimators

### 2.2.1 Recursive mean adjustment

An alternative way to handle the deterministic component  $\mu$  is to consider recursive mean adjustment, see So and Shin (1999b). OLS estimation of the autoregressive parameters is then applied to the series  $\bar{y}_t = t^{-1} \sum_{s=1}^t y_s$ . The ordinary mean adjustment scheme leads to a correlation of the regressors with the error term which is a source of the bias. By applying a recursive mean adjustment, this problem is mitigated. Simulation results in So and Shin (1999b) suggest that the bias and the mean squared error can be improved substantially.<sup>4</sup> The resulting estimator is

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<sup>3</sup>One example would be stock prices, where stationarity in the  $I(0)$  sense would be very unlikely. For such series, one may focus on the subset  $\rho \geq 1$ . Of course, there are many time series for which such pre-exclusion of stationarity is less easily achieved.

<sup>4</sup>Recursive mean adjustment has also been considered in the field of cointegration, see for instance Born and Demetrescu (2014).

labeled as  $\hat{\rho}^R$ :

$$\hat{\rho}^R = \frac{\sum_{t=2}^T (y_t - \bar{y}_{t-1})(y_{t-1} - \bar{y}_{t-1})}{\sum_{t=2}^T (y_{t-1} - \bar{y}_{t-1})^2}. \quad (2)$$

The plain OLS estimator  $\hat{\rho}$  only differs from  $\hat{\rho}^R$  by the fact that the OLS estimator uses  $\bar{y}_T$  instead of  $\bar{y}_{t-1}$ .

### 2.2.2 Cauchy estimator

The Cauchy estimator, see So and Shin (1999a), build upon  $\hat{\rho}^R$  as it uses recursive mean adjustment as well. The estimator for an AR(1) process is given by

$$\hat{\rho}^C = \frac{\sum_{t=2}^T (y_t - \bar{y}_{t-1}) \text{sign}(y_{t-1} - \bar{y}_{t-1})}{\sum_{t=2}^T |y_{t-1} - \bar{y}_{t-1}|} \quad (3)$$

where  $\text{sign}(x_t) = 1$  if  $x_t \geq 0$  and  $\text{sign}(x_t) = -1$  if  $x_t < 0$ . This estimator can be interpreted as an instrumental variable estimator where  $\text{sign}(y_{t-1} - \bar{y}_{t-1})$  serves as the instrument. Moreover, it can also be seen as a weighted LS estimator with weights  $w_t = |y_{t-1} - \bar{y}_{t-1}|^{-1}$ . This interpretation is based on the fact that  $\text{var}(y_t | y_{t-1}, \dots) = \sigma w_t^{-1}$ . The Cauchy estimator  $\hat{\rho}^C$  exhibits some nice properties. It is approximately median-unbiased for all values of  $\rho$  whereas the OLS estimator is median-biased.<sup>5</sup> Another important feature is that the estimator is applicable for explosive autoregressive processes, where  $\rho > 1$ . This is in contrast to the approximately median-unbiased Roy and Fuller (2001) estimator, for example.

### 2.2.3 Second difference-based estimator

In a recent contribution, Chen and Kuo (2013), suggest an estimator for the autoregressive parameter in AR(1) processes which is based on the second difference of  $y_t$ . Their OLS estimator is based on the following transformed regression model

$$2\Delta_2 y_t = \rho \Delta_2 y_{t-1} + \zeta_t,$$

where  $\Delta_2 = (1 - L^2)$  with  $L$  denoting the lag operator. The error term  $\zeta_t$  is given as  $\zeta_t = 2\Delta_2 y_t - \rho \Delta_2 y_{t-1}$ . Importantly, the regressor  $\Delta_2 y_{t-1}$  is uncorrelated with the error  $\zeta_t$ . The estimator for  $\rho$  is obtained by applying OLS to the transformed regression model. The resulting estimator reads

$$\hat{\rho}^{SD} = \frac{2 \sum_{t=2}^T \Delta_2 y_{t-1} \Delta_2 y_t}{\sum_{t=2}^T \Delta_2 y_{t-1}^2}.$$

The second difference-based estimator  $\hat{\rho}^{SD}$  exhibits the asymptotic property that  $\sqrt{T}(\hat{\rho}^{SD} - \rho)$  is distributed as a Normal random variable with zero mean and a variance of two for  $-1 < \rho \leq 1$ . As the authors argue, the estimator is inconsistent for explosive processes as  $\Delta_2 y_t$  is non-stationary, but it maintains its properties for mildly explosive processes for which  $\rho^{2T}/\sqrt{T} \rightarrow 0$  holds. In fact, this only allows for very mild deviations in the direction of explosiveness. Interestingly, the

<sup>5</sup>An estimator  $\tilde{\rho}$  for  $\rho$  is said to be median-unbiased if  $P(\tilde{\rho} \geq \rho) \geq 1/2$  and  $P(\tilde{\rho} \leq \rho) \geq 1/2$ .

simulation results in Chen and Kuo (2013) exclude the mildly explosive case. We note that mean adjustment does not play a role for this estimator as the constant deterministic term already vanishes after taking the first difference.

## 2.3 Bias-correction techniques

### 2.3.1 Bootstrap bias-corrected estimator

The first bias-correction technique we consider is the bootstrap. Kim (2003) proposes the use of the bootstrap for a correction of the OLS bias which is analogous to the procedure by Kilian (1998). This method involves the generation of a large number of pseudo-data sets using the estimated coefficients and re-sampled residuals. Pseudo-data sets shall resemble the dependence structure that is present in the original data set. The bias of the OLS estimator can be estimated as follows: Estimate the model via OLS and obtain the estimates  $\hat{\theta} = (\hat{\mu}, \hat{\rho})'$ . Generate a pseudo-data set  $\{y_t^b\}_{t=1}^T$  based on these estimates according to

$$y_t^b = \hat{\mu} + \hat{\rho}y_{t-1}^b + u_t^b ,$$

where  $u_t^b$  is a random draw with replacement from the OLS residuals  $\{\hat{u}_t\}_{t=1}^T$ .  $B$  sets of pseudo-data are generated. Each pseudo-data set gives a bootstrap parameter estimate  $\hat{\theta}^b = (\hat{\mu}^b, \hat{\rho}^b)'$  by estimating the model  $y_t^b = \mu + \rho y_{t-1}^b + v_t$ ,  $b = 1, \dots, B$ . We obtain the sequence  $\{\hat{\theta}^b\}_{b=1}^B$  and the average bias of  $\hat{\theta}^b$  is estimated as  $\tilde{\theta} - \hat{\theta}$ , where  $\tilde{\theta}$  is the sample average of  $\{\hat{\theta}^b\}_{b=1}^B$ , i.e.

$$\tilde{\theta} \equiv \frac{1}{B} \sum_{b=1}^B \hat{\theta}^b .$$

Using this bootstrap-based estimator for the bias, a bias-correction for  $\hat{\theta}$  can be directly obtained via

$$\hat{\theta}^B = \hat{\theta} - (\tilde{\theta} - \hat{\theta}) = 2\hat{\theta} - \tilde{\theta} .$$

This estimator computes the OLS estimation bias for a process with parameter values  $\hat{\theta}$  and uses this bias as approximation for the true bias of  $\hat{\theta}$ . For further details regarding this estimator, the interested reader is referred to Kim (2003). The validity of the bootstrap procedure for the explosive case (with finite error variance) is given in Basawa et al. (1989). Datta (1995) shows that the bootstrap also works for heavy-tailed innovations. The unit root case is treated in Basawa et al. (1991), Datta (1996) and extended in Inoue and Kilian (2002) to general deterministic terms and higher-order autoregressive models.<sup>6</sup>

### 2.3.2 Jackknife bias-corrected estimators

In general, Bao and Ullah (2007) show that the expected value of the OLS estimator  $\hat{\theta} = (\hat{\mu}, \hat{\rho})$  has the form

$$E(\hat{\theta}) = \theta + \frac{a}{T} + O(T^{-2}) .$$

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<sup>6</sup>See also Berkowitz and Kilian (2000) for a survey on bootstrap techniques for time series.



Shaman and Stine (1988) show that  $a = -(1 + 3\rho)$  for  $\hat{\rho}$ . If the full sample  $Y$  is divided into  $m$  sub-samples  $Y_j$  of same length  $l$ ,  $j = 1, \dots, m$ , and  $\hat{\theta}^j$  is the OLS estimate for  $\theta$  in sub-sample  $Y_j$ , then the jackknife statistic

$$\hat{\theta}^J = \left(\frac{T}{T-l}\right)\hat{\theta} - \left(\frac{l}{T-l}\right)\tilde{\theta}$$

where  $\tilde{\theta} = \frac{1}{m} \sum_{j=1}^m \hat{\theta}^j$  satisfies  $E(\hat{\theta}^J) = \theta + O(T^{-2})$  and is thus able to reduce the bias. Chambers (2013) proposes and compares various jackknife techniques to reduce the small sample bias. In this work we focus on one of the methods in the comparison of Chambers (2013): the non-overlapping sub-samples jackknife. This estimator has good bias-correction properties without the considerable increase of the RMSE of higher-order jackknife estimators. The time series is splitted in  $m$  non-overlapping sub-samples,

$$Y_j = (y_{[(j-1)T/m+1]}, \dots, y_{[jT/m]})', \quad j = 1, \dots, m.$$

In the following, we work with  $m = 2$  sub-samples, because the procedure with this particular choice of  $m$  has the best bias-correction properties according to Chambers (2013) (Table 1). This simplifies the jackknife statistic to

$$\hat{\theta}^J = 2\hat{\theta} - \tilde{\theta}.$$

The intuition behind this approach is almost the same as in the bootstrap approach of Kim (2003). The average bias in the sub-samples is higher because of the smaller sample size and therefore a bias-reduction is induced. The difference to the bootstrap procedure is that the average bias is calculated on sub-samples of the true process and not on pseudo-data.<sup>7</sup> It should be noted that the introduced jackknife procedure is only valid as long as the process is stationary, see Chambers (2013). The unit root case is tackled in Chambers and Kyriacou (2013). To our best knowledge, the (mildly) explosive case has not been under consideration so far.

### 2.3.3 Indirect inference bias-corrected estimator

We now turn to a simulation-based estimator relying on the concept of indirect inference. The following exposition draws heavily from Phillips et al. (2011). The basic idea of this simulation-based estimator is to consider initially the OLS estimator labeled as  $\hat{\rho}$ . Consider a set of simulated series with AR(1) coefficient equal to some  $\rho$ , i.e.  $\{y_t^h(\rho)\}_{h=1}^H$ ,  $h = 1, 2, \dots, H$ .  $H$  denotes the total number of available simulation paths.<sup>8</sup> For each single  $h \in 1, 2, \dots, H$ , we obtain an OLS estimate denoted as  $\hat{\rho}^h(\rho)$ . The indirect inference estimator (which belongs to the class of

<sup>7</sup>We have also experimented with a higher-order non-overlapping sub-samples jackknife estimator, i.e. J(2,3) in the notation of Chambers (2013). Our results show that a further bias-reduction (in comparison to the simple jackknife estimator) can be achieved, but that the increase in variance is substantial. Moreover, the simple jackknife already delivers superb bias-correction. For this reason, we focus on the simple version in the following.

<sup>8</sup>In order to generate  $\{y_t^h(\rho)\}_{h=1}^H$ , we assume normal errors in the following. Unreported simulation results suggest that this assumption is not crucial. The performance is nearly unaffected even under skewed and fat-tailed distributions. Results are available upon request from the authors.

extremum estimators) is given by

$$\hat{\rho}_H^{II} = \arg \min_{\rho \in \Theta} \left\| \hat{\rho} - \frac{1}{H} \sum_{h=1}^H \hat{\rho}^h(\rho) \right\| ,$$

where  $\Theta$  is a compact parameter space and  $\|\cdot\|$  is a distance metric. For  $H \rightarrow \infty$  one obtains

$$\hat{\rho}^{II} = \arg \min_{\rho \in \Theta} \|\hat{\rho} - q(\rho)\| ,$$

where  $q(\rho) = E(\hat{\rho}^h(\rho))$  is the so-called binding function. Given invertibility of  $q$ , the indirect inference estimator results as

$$\hat{\rho}^{II} = q^{-1}(\hat{\rho}) .$$

So the idea of this estimator is to have a grid of possible true values for  $\rho$  and the corresponding average OLS estimates  $H^{-1} \sum_{h=1}^H \hat{\rho}^h(\rho)$ . The estimate  $\hat{\rho}$  is compared to the average OLS estimates.  $\hat{\rho}^{II}$  is now the value which leads to the average OLS estimate with the minimal distance to  $\hat{\rho}$ . The finite-sample bias-correction stems from the simulation of  $q(\rho)$ . Precision is naturally expected to be increased with rising  $H$ , although it can be computationally costly. Nonetheless, the binding function has to be simulated only once and can thus be applied afterwards without any further simulation or re-sampling. This is a fundamental difference to the bootstrap approach. Furthermore, the indirect inference estimator is applicable even for mildly explosive processes, see PWY.

### 3 Finite-sample properties

#### 3.1 Monte Carlo setup

In this section we investigate the properties of various bias-correction methods via Monte Carlo simulation. The foci of this analysis are the bias-reduction and the RMSE of these estimators. The simulation setup is as follows: We consider autoregressive models of the structure

$$y_t = \mu + \rho y_{t-1} + \beta \Delta y_{t-1} + \varepsilon_t$$

with  $\varepsilon_t \sim N(0, \sigma^2)$ .<sup>9</sup> The autoregressive parameter  $\rho$  measures the persistence of  $y_t$  and takes values  $\rho = \{0.85, 0.9, 0.95, 0.99, 1, 1.01, 1.02, 1.03, 1.04, 1.05\}$ . The considered samples sizes are  $T = \{25, 50\}$ . The degree of explosiveness is mild in our design and corresponds to typical values in financial applications. The intercept  $\mu$  is set equal to zero for simplicity.<sup>10</sup> Regarding the initial condition we consider  $y_0 = \varepsilon_0 / \sqrt{1 - \rho^2}$  for stationary AR models, i.e. when  $\rho < 1$ ;

<sup>9</sup>Additional results for stable distributed errors with fat tails and heteroscedastic errors (via a GARCH(1,1) structure) are available upon request. In order to save space, these results are not reported. The main insight from the results for these settings is that the performance of estimators is typically unaffected.

<sup>10</sup>We have also studied the case where a linear deterministic time trend enters the estimation equation, i.e.

$$y_t = \mu + \delta t + \rho y_{t-1} + \varepsilon_t .$$

The additional estimation uncertainty arising from the inclusion of a linear time trend manifests itself in a rising bias of the OLS estimator. All procedures are found to perform worse compared to the case where the linear trend is omitted.

otherwise we set  $y_0 = \sqrt{T}\sigma\alpha$ . For simplicity, we set  $\sigma^2 = 1$  and  $\alpha = 1$ , yielding  $y_0 = \sqrt{T}$ . For ease of exposition and comparison, we set  $\beta = 0$  and focus on first-order models.<sup>11</sup>

The number of Monte Carlo repetitions is set to 20,000 for each single experiment. The number of bootstrap repetitions is set equal to 499. The binding function for the indirect inference estimator is simulated with  $\rho = \{0.60, 0.61, \dots, 1.20\}$  and  $\beta = \{-0.90, -0.89, \dots, 0.90\}$ . The number of simulation paths  $H$  equals 20,000 in the AR(1) case and  $H = 1,000$  for AR(2) models. In an unreported comparison between different values for  $H$ , we find that there are only marginal changes in the results as long as  $H \geq 100$ . This means that the indirect inference procedure can be applied at low computational costs with negligible loss of precision.

### 3.2 Bias

We present simulation results for the bias multiplied with 100 (for increased visibility) in Figures 2 for  $T = 25$  and in Figure 3 for  $T = 50$ , respectively.

— FIGURES 2–3 ABOUT HERE —

We start by discussing the results for the smaller sample size and comment on differences to the case of  $T = 50$  later on. The downward bias of the OLS estimator is strong for all considered values of  $\rho$ . The worst performance is obtained for values of  $\rho$  in the vicinity of unity. For explosive processes, the bias is less severe, but still of major relevance. Even for  $\rho = 1.05$ , the bias is of such a magnitude that the OLS estimator does not exceed unity on average. This means that even the process is explosive, the OLS estimator has an average bias of approximately  $-0.1$  and will thus yield average point estimates around 0.95. The performance of the OLS estimator with recursive demeaning is more promising over whole range of  $\rho$ , but it is worthwhile to use the Cauchy estimator instead. This estimator is still downward biased. But, it indicates explosiveness at least for  $\rho = 1.05$  where the simulated bias equals  $-0.039$ . The bootstrap approach performs substantially better than the Cauchy estimator. For rather explosive series, the bias function switches its sign leading to a small upward bias for  $\rho \geq 1.04$ . For stationary series ( $\rho < 1$ ), the bootstrap estimator performs similar to the second differencing estimator. The latter has some additional advantages for very persistent stationary and for unit root processes. For explosive series, it shows a somewhat risky behaviour: after a certain threshold (here: 1.02), the estimator turns to be severely upward distorted. This is not entirely surprising given the restrictive condition  $\rho^{2T}/\sqrt{T} \rightarrow 0$ . We conclude that the second difference estimator is performing reasonably well for stationary and unit root processes, and also for very mildly explosive cases. But, for typical values of  $\rho$  in financial and economic applications, it seems that the estimator can not be recommended. Somewhat similar, the jackknife approach performs extremely well for stationary and unit root cases. In fact, it offers superb bias-correction abilities at nearly no

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<sup>11</sup>In unreported simulations, we also compared the estimators for the case where  $\beta \neq 0$ . In case of correct specification, the exact order of the autoregressive model does alter our main findings. Over-fitting of the autoregressive model is not harmful, while under-fitting turns out to be an important issue. Therefore, the lag length shall be carried out on the basis of liberal selection procedures like the AIC.

extra computational cost. For explosive series, however, results are less favorable. Finally, the indirect inference estimator appears to offer a well balanced performance over the whole range of  $\rho$ .<sup>12</sup> In many cases, the indirect inference estimator ranks second or third with a small bias.

For  $T = 50$ , all estimators perform better as more information is available. Again, the jackknife approach leads to the best performance among all approaches for values  $\rho \leq 1$ . It has some problems for  $\rho > 1$ . The indirect inference estimator and the bootstrap approach work also very well, especially for explosive series. It is interesting to note that for  $\rho = 1.05$ , the bias of the plain OLS estimator is very close to zero. In this case other estimators are less effective (except of the bootstrap): they typically lead to some minor upward distortion.

### 3.3 Root mean squared error

Another useful measure for evaluating the small-sample performance of competing estimators is the root mean squared error (RMSE). Both components, bias and variance, contribute to this measure. By considering the RMSE, we are able to judge the price in terms of an increased variance for reducing the bias in relative terms. The results for the root mean squared errors (multiplied with  $T$ ) are shown in Figures 4 for  $T = 25$  and in Figure 5 for  $T = 50$ , respectively.

— FIGURES 4–5 ABOUT HERE —

Again, we start by interpreting the results for  $T = 25$ . Some clear cut results can be obtained. The best performing method (with clear advantages over its competitors) is indirect inference. Our second recommendation would be the bootstrap, while the second difference estimator is dominated by all others even for stationary and unit root processes. The reason being its relatively large variance. The plain OLS estimator also performs better than the jackknife for all values of  $\rho$ . It is interesting to note that the recursively demeaned OLS estimator outperforms the Cauchy estimator, especially for less persistent series. Both techniques are on equal footing for explosive series. The bootstrap has advantages over the recursively demeaned OLS estimator for a broad range of values around unity. In cases of more extreme persistence (either stationary or explosive), it is as good as the former estimator. The indirect inference estimator is the clear winner in this competition as it offers reasonable bias-correction together with a relatively low variance. We have seen that bias correction can be improved, but only at high costs. Taking both issues simultaneously into account, we recommend to use  $\hat{\rho}^{II}$  in practice for small sample sizes of  $T = 25$ .

Regarding the results for  $T = 50$ , we first note that the indirect inference estimator is still our recommended procedure. However, the differences to its competitors are less visible for rather explosive series with  $\rho \geq 1.03$ . For stationary autoregressive series with moderate persistence the

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<sup>12</sup>In additional unreported simulations, we have extended the range of values for  $\rho$  in order to check whether the bias function for the jackknife, the bootstrap and the indirect inference principles lead to increasing upward biases as  $\rho$  increases as well. The results indicate that this is not the case. For larger values of  $\rho$ , the bias functions tend to zero.

bootstrap performs similarly well. The recursively demeaned OLS estimator has still advantages over the Cauchy estimator. For large values of  $\rho$ , we find that the OLS estimator performing much better than for less persistent series. The main reason being that the bias vanishes and that its variance is relatively low as well. The second difference estimator is excluded from Figure 5 due to its bad performance.

## 4 Empirical application

As an illustration of the different procedures, we study two Asian stock indices. We consider the Shanghai 50 and the Hang Seng 45 indexes during periods of accumulated growth and thus possible explosiveness. For this reason, we decide to exclude the second difference-based estimator. The data is kindly provided by the authors of the study by Homm and Breitung (2012). The sample periods were chosen according to their breakpoint estimation results for the start-date of the explosive regimes. The end-dates are estimated similarly as suggested in Breitung and Kruse (2013). The resulting samples sizes are  $T = 24$  and  $T = 56$ , respectively. These sample sizes match well with our simulation setup for  $T = 25$  and  $T = 50$ . In the following, we apply the estimators to (i) the *levels* and (ii) the *logs* of the price series. In practical applications as the one by PWY, authors often use both ways for comparison.<sup>13</sup>

— FIGURE 6 ABOUT HERE —

To begin with, we do not observe substantial differences between the results for the levels and logs of prices. We therefore focus on the results for the logged series. Table 1 presents the estimation results and contains sample information like size and time span. In all cases, the OLS estimator produces the smallest point estimates indicating a downward bias. Results for the bias-corrected estimators may differ remarkably.

For the shorter series of the Shanghai 50 index, we find estimates in the range of [1.010 – 1.193]. The minimal value is attached to the plain OLS estimator, while the maximal value belongs to the bootstrap estimator. The recommendable procedures for the explosive case are the indirect inference estimator  $\hat{\rho}^{II}$  and the bootstrap-based estimator  $\hat{\rho}^B$ . While the former one suggest a growth rate of 10.5% per month, the latter leads to a substantially larger estimate of 19.3%. For the longer series there we find much less discrepancy between the outcomes of the two methods. For the Hang Seng 45 index we find less variation in the estimation results (excluding the second difference estimator).

These two series exemplify the use of (and need for) bias-correction techniques. As there is a strong discrepancy between bias-corrected and plain estimates, the need for an adjustment is clear. Following the discussion in PWY (p.217-218), we would like to judge the importance of bias-correction also from an economic viewpoint. Let us make a couple of standard assumptions

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<sup>13</sup>The lag length for the autoregressive specification is selected via AIC with a maximal number of lags equal to  $[4(T/100)^{1/4}]$ . For both series, a first-order model is selected.

Estimator	Shanghai 50	Hang Seng 45
$\widehat{\rho}$	1.010	1.013
$\widehat{\rho}^R$	1.061	1.066
$\widehat{\rho}^C$	1.098	1.062
$\widehat{\rho}^B$	1.193	1.048
$\widehat{\rho}^J$	1.137	1.026
$\widehat{\rho}^{II}$	1.105	1.056
$T$	24	56
Start	Dec 2005	Mar 2003
End	Nov 2007	Oct 2007

Table 1: Empirical estimation results for log stock price indices.

and suppose that the no-arbitrage condition

$$P_t = \frac{1}{1+R} E_t(P_{t+1} + D_{t+1})$$

holds.  $P_t$  is the real stock price,  $D_t$  is the real dividend and  $R$  is a positive and constant discount rate. A log-linear approximation leads the solution  $p_t = p_t^f + b_t$ , where the log of the stock price  $p_t$  is composed of a fundamental price  $p_t^f$  and a bubble component  $b_t$ . The fundamental value  $p_t^f$  is determined by expected future dividends only. The bubble is a submartingale given by  $b_t = (1+g)b_{t-1} + \varepsilon_t$  with  $\varepsilon_t$  being a martingale difference sequence. Obtaining an estimate for  $\rho$  larger than one implies that the log stock price grows at a rate of  $(\widehat{\rho} - 1)\%$  per month. Assuming that the log dividends are non-explosive,  $b_t$  is also explosive and grows with rate  $\widehat{g} \geq (\widehat{\rho} - 1)\%$ . Now, the importance of estimating  $\rho$  precisely stems from the following facts. Given an initial overvaluation of the stock market of  $a_0 > 1$ , i.e.  $P_0 = a_0 P_0^f$ , the bubble starts at  $b_0 = \log(P_0/P_0^f) = \log(a_0) > 0$ . After  $t$  periods, the expected level of the bubble component is  $b_t = \widehat{\rho}^t \cdot \log(a_0)$ . This leads to an estimate of overvaluation in the *level* of the stock market index  $P_t$  given by  $\widehat{a}_t \equiv \exp(b_t) = \exp(\widehat{\rho}^t \cdot \log(a_0))$ .

A calculation where the initial overvaluation is mild and equals five percent ( $a_0 = 1.05$ ), yields estimated overvaluations of (based on the indirect inference estimator) of  $\widehat{a}_T^{II} = (70.9\%, 180.6\%)'$  for the Shanghai 50 and the Hang Seng 45, respectively. Using the plain OLS estimator leads to  $\widehat{a}_T = (6.4\%, 10.6\%)'$  instead. Obviously, these differences are economically significant. The Shanghai 50 index peaked at 6092 on October 16, 2007 and stabilized around 3000 basis points during 2009 and 2010. By considering such a value as being close to the unobserved fundamental value, this would imply a rough estimate of overvaluation equal to approximately 103% which is in line with the previous calculations. Similarly, the Hang Seng 45 index reached its peak at the value of 31638 on October 30, 2007 and stabilized around 13500 basis points during December 2008 – March 2009. The implied overvaluation equals approximately 134%. We conclude that the estimates of the growth rates seem to be reasonable.

## 5 Conclusions

This paper compares different bias-correction techniques for autoregressive processes. Among these are the a bootstrap-based estimator, an indirect inference estimator and a jackknife estimator. In addition, we study the finite-sample performance of an estimator based on second differencing, a recursively demeaned OLS estimator and the related Cauchy estimator. We thus compare established techniques to newly proposed procedures in a comprehensive way. In particular, we focus on situations where the sample size is relatively small and data is highly persistent, exhibits a unit root or is even mildly explosive. When the popular rolling window framework is applied for assessing the possibly time-varying persistence of a time series, sample sizes are typically small. Moreover, it is reasonable to expect that time series undergo changes in persistence during different regimes and episodes. These changes can be either driven by episodes of speculation (leading to temporary bubbles) or policy induced (typically leading to a reduction in persistence). Therefore, we study an empirically relevant situation and provide practical recommendations for future applications.

A large-scale simulation study of bias and root mean squared errors of estimators reveals the following results: The substantial bias of the OLS estimator can be remarkably reduced across the whole range of considered autoregressive parameter values. The most promising approaches are the indirect inference estimator, the bootstrap approach and the jackknife estimator. The indirect inference estimator provides excellent bias-correction in various settings (i.e. heavy-tailed errors, GARCH errors, linear trend and misspecified autoregression) together with a reasonably low variance, while the jackknife estimator performs often best in terms of bias-correction, but has a clearly larger variance rendering this estimator less recommendable in terms of RMSE. The bootstrap approach often performs as a second best choice.

As an empirical application, we consider two Asian stock markets indexes during boom periods. Our results highlight the need for bias-correction and we discuss the results in light of implied stock market over-valuations. It turns out that the lower bound for bubble growth rates can be estimated reasonably by applying the indirect inference principle.

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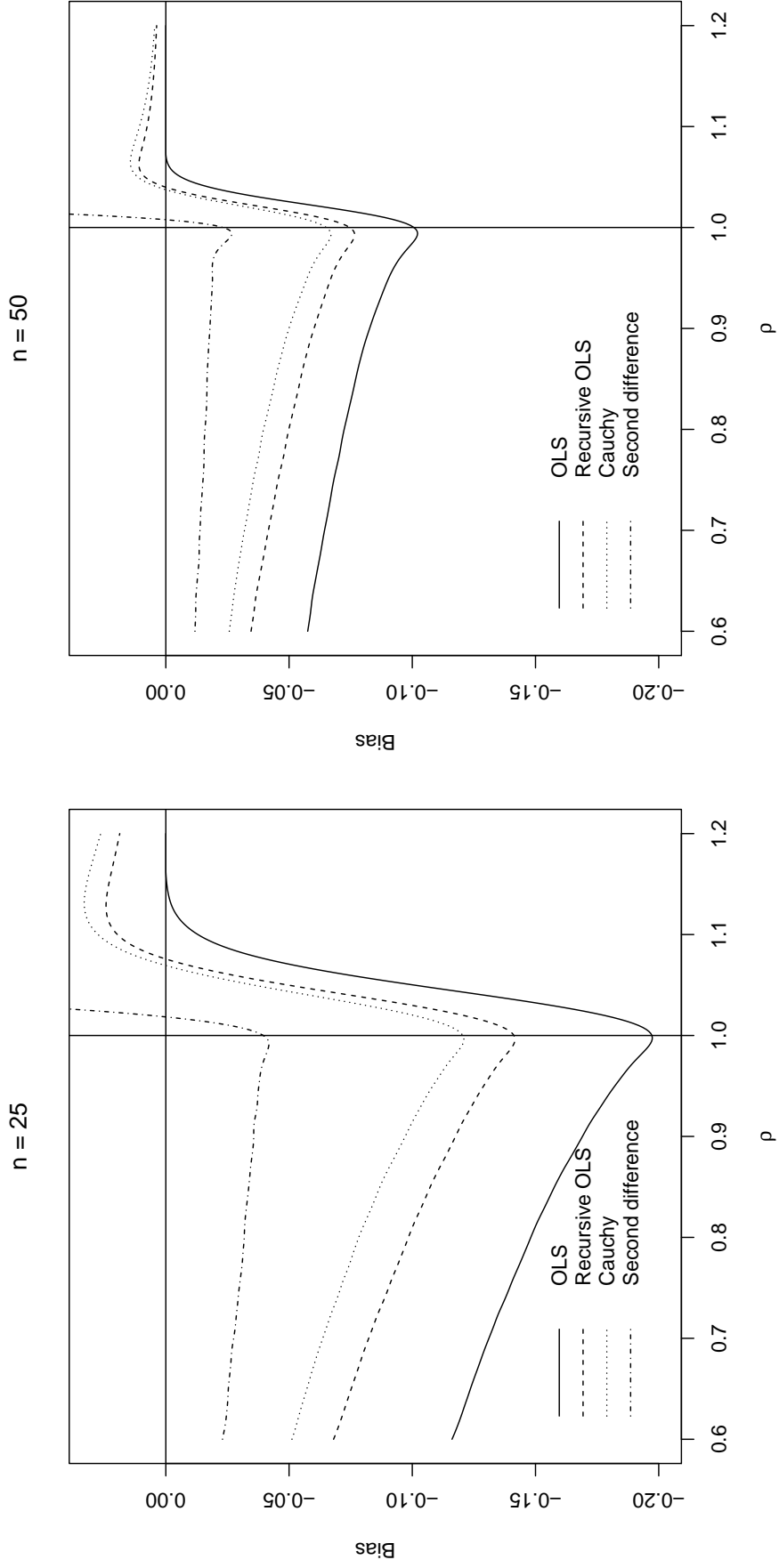


Figure 1: Estimation bias for different estimators for different values of  $\rho$  in AR(1) processes.

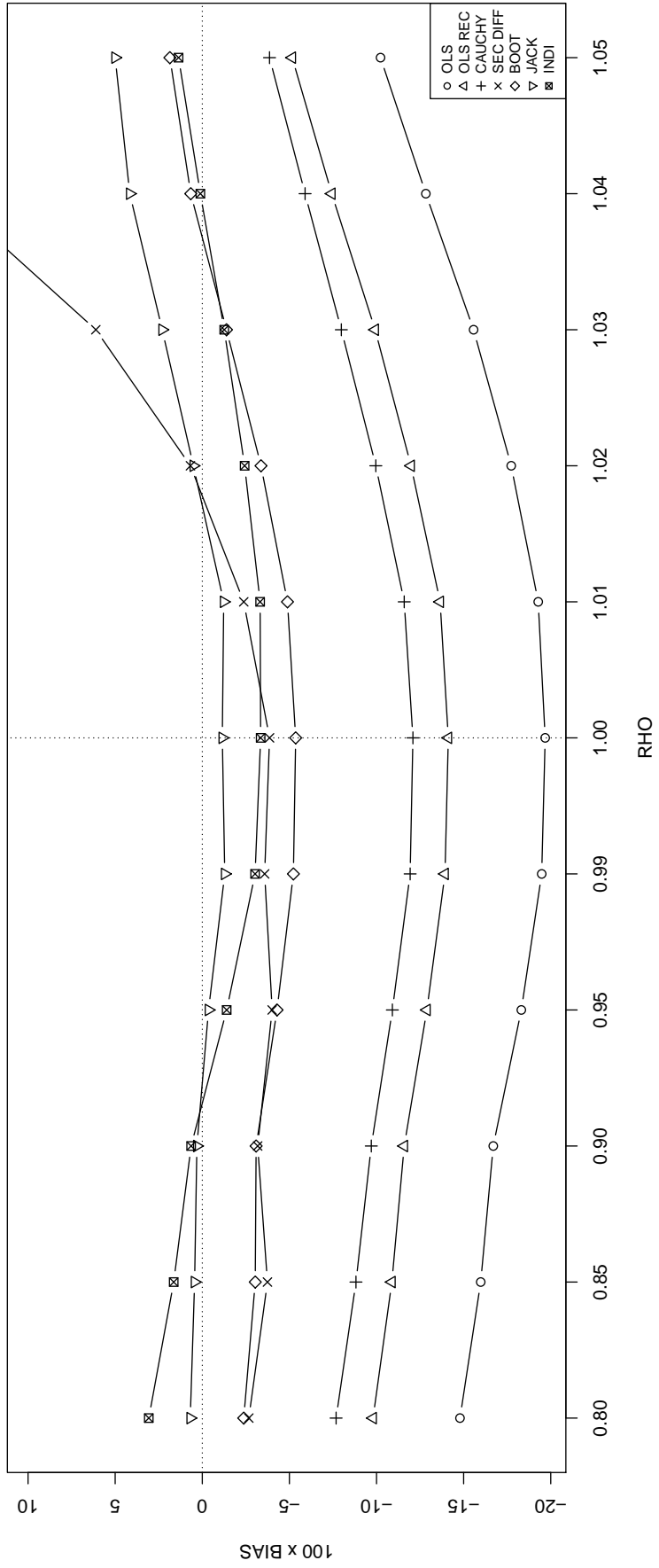


Figure 2:  $100 \times$  Bias for  $T = 25$  for estimators:  $\hat{\rho}$  (OLS),  $\hat{\rho}^R$  (OLS REC),  $\hat{\rho}^C$  (CAUCHY),  $\hat{\rho}^{SD}$  (SEC DIFF),  $\hat{\rho}^B$  (BOOT),  $\hat{\rho}^J$  (JACK) and  $\hat{\rho}^{INDI}$  (INDI).

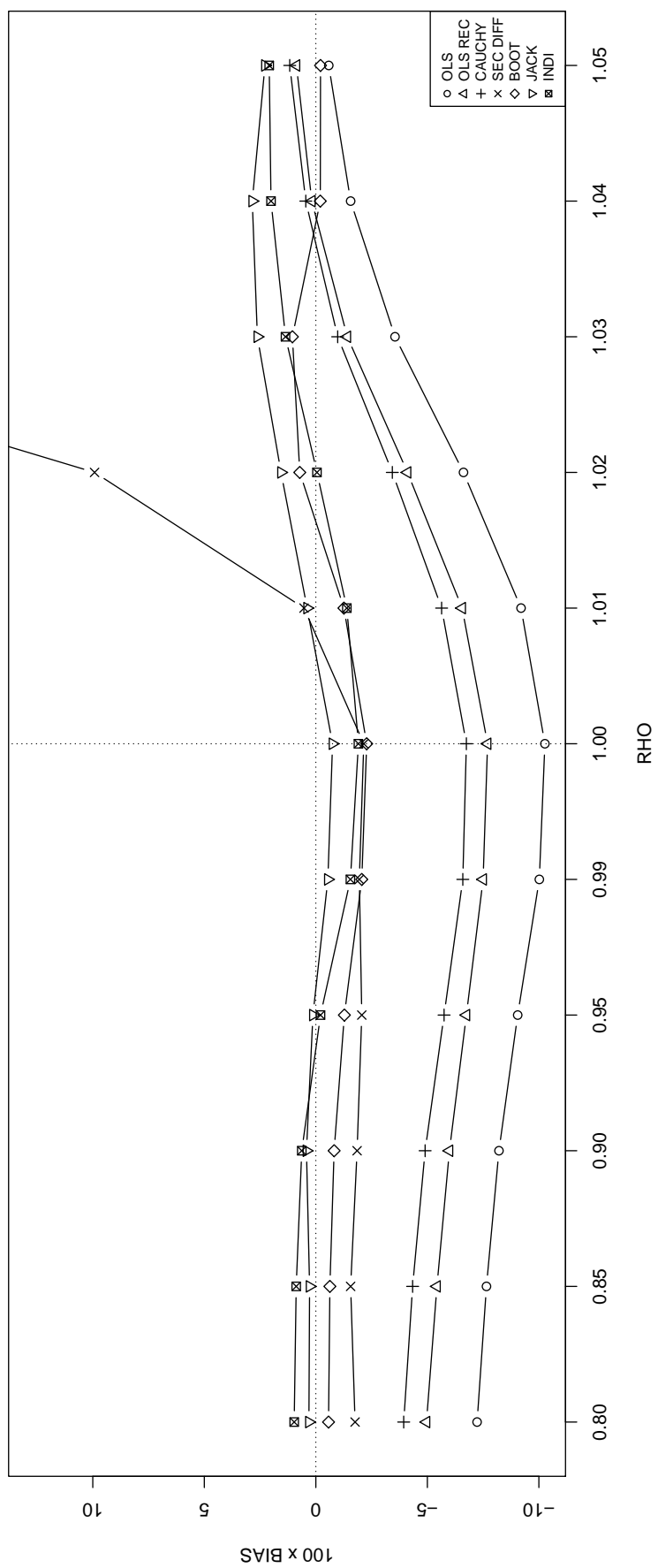


Figure 3:  $100 \times$  Bias for  $T = 50$  for estimators:  $\hat{\rho}$  (OLS),  $\hat{\rho}^R$  (OLS REC),  $\hat{\rho}^C$  (CAUCHY),  $\hat{\rho}^{SD}$  (SEC DIFF),  $\hat{\rho}^B$  (BOOT),  $\hat{\rho}^J$  (JACK) and  $\hat{\rho}^{INDI}$  (INDI).

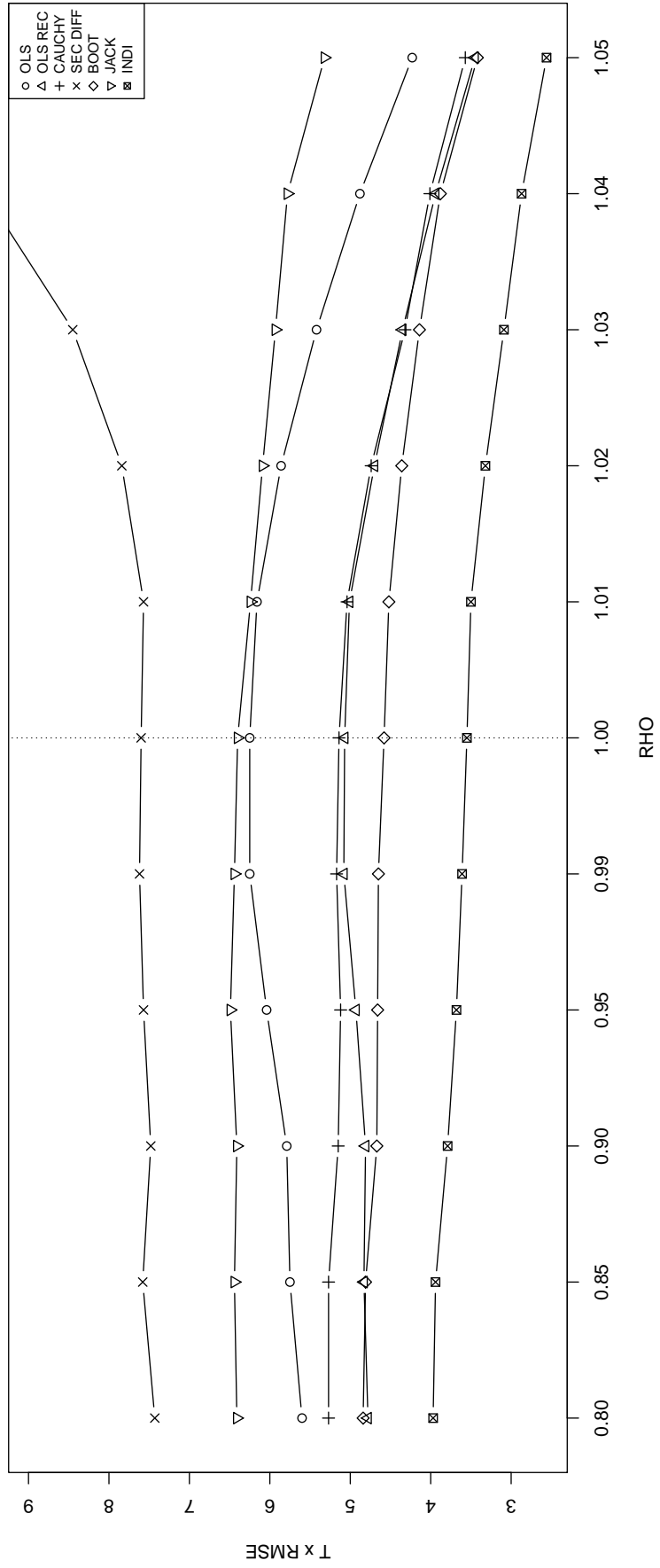


Figure 4:  $T \times \text{RMSE}$  for  $T = 25$  for estimators:  $\hat{\rho}$  (OLS),  $\hat{\rho}^R$  (OLS REC),  $\hat{\rho}^C$  (CAUCHY),  $\hat{\rho}^{SD}$  (SEC DIFF),  $\hat{\rho}^B$  (BOOT),  $\hat{\rho}^J$  (JACK) and  $\hat{\rho}^{INDI}$  (INDI).

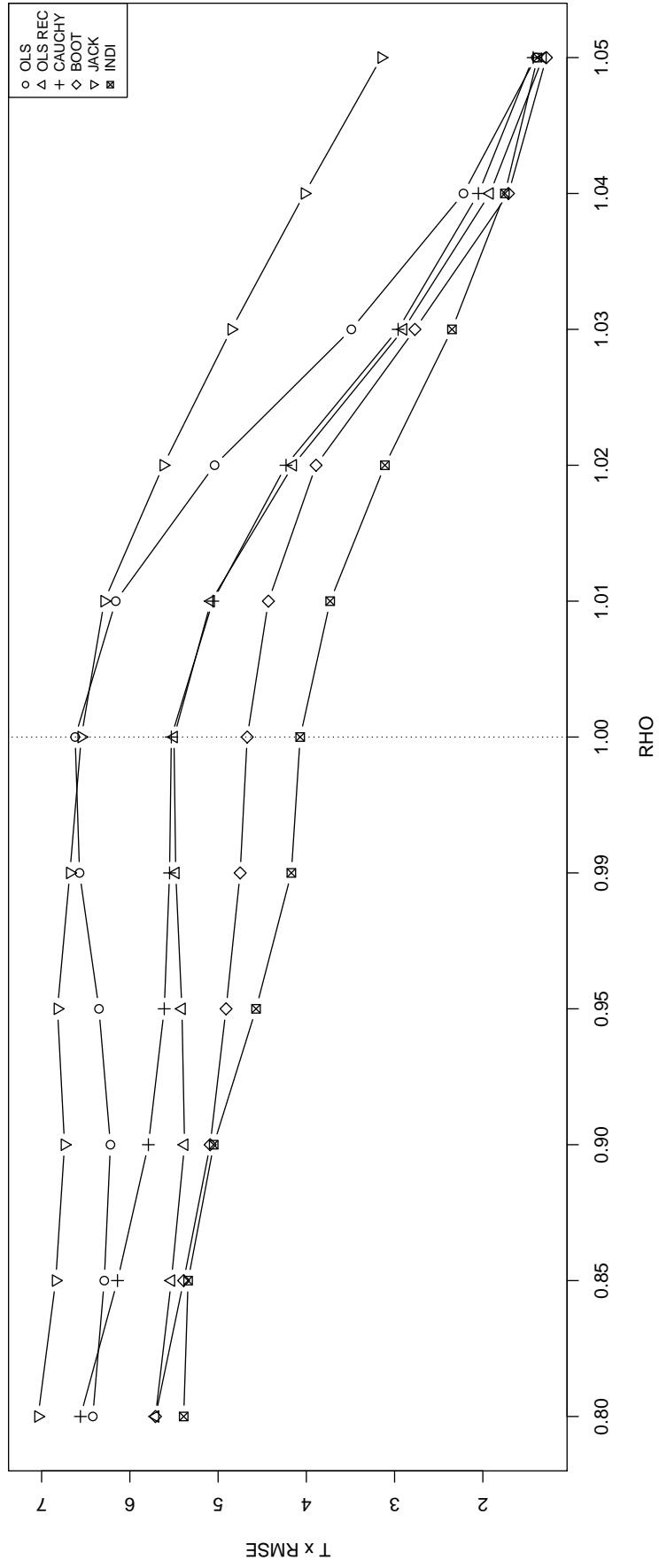


Figure 5:  $T \times \text{RMSE}$  for  $T = 50$  for estimators:  $\hat{\rho}$  (OLS),  $\hat{\rho}^R$  (OLS REC),  $\hat{\rho}^C$  (CAUCHY),  $\hat{\rho}^B$  (BOOT),  $\hat{\rho}^J$  (JACK) and  $\hat{\rho}^{SD}$  estimator (SEC DIFF) is excluded.

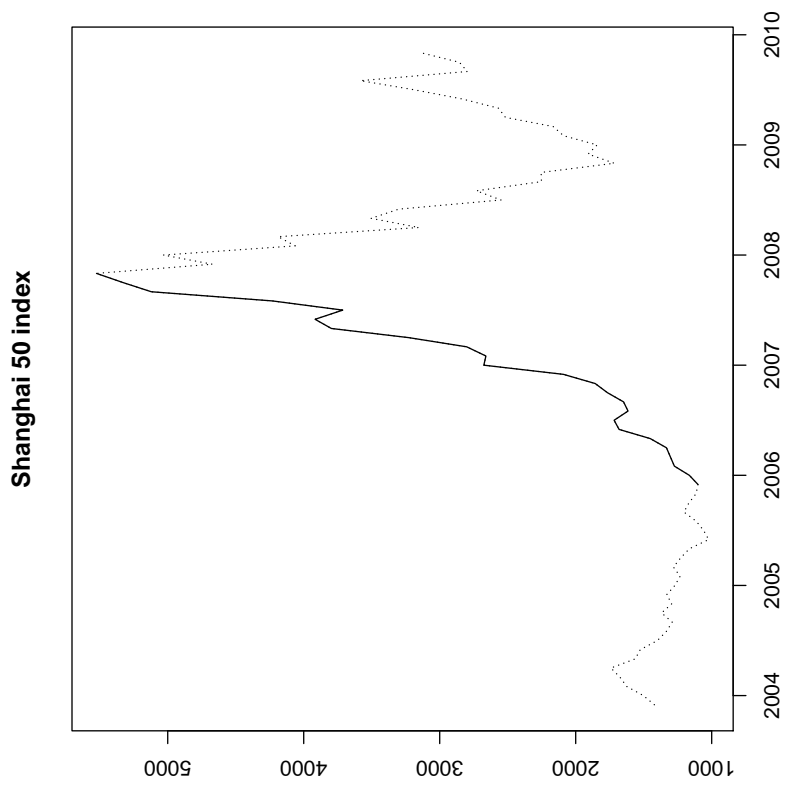
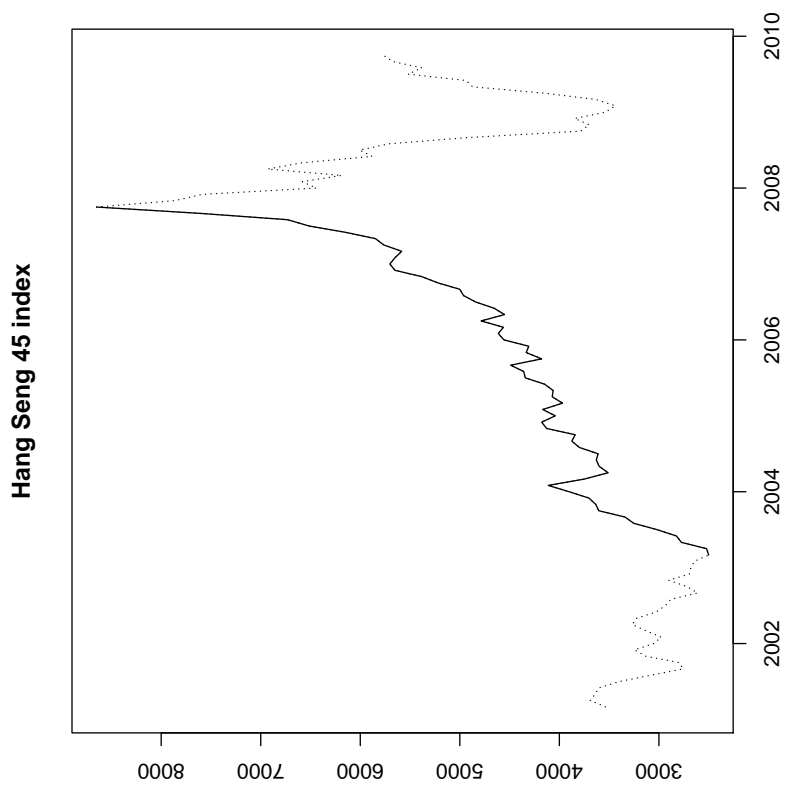


Figure 6: Two Asian stock indexes in *real levels*. Solid lines highlight the sample used for estimation.