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# Safe, or not safe? Covered Bonds and Bank Fragility

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## Abstract

We explore the relationship between asset encumbrance and bank funding in the context of covered bonds – a form of collateralized debt. Covered bond issuance influences the incidence of bank runs by unsecured creditors and, in turn, conditions in the unsecured funding market influence the bank’s choice of asset encumbrance. The more reliant is the bank on secured finance and the more it encumbers assets on its balance sheet, the more concentrated are losses on unsecured creditors and the more fragile the bank. But as more stable long-term debt is added to the funding mix, the greater is the expected value of bank equity. We solve for the optimal choice of asset encumbrance and wholesale funding. Our model sheds light on how losses of confidence in funding markets, macroeconomic shocks, and improved crisis resolution frameworks affect the extent of balance sheet collateralization.

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**Keywords:** Asset encumbrance, Bank funding, Covered bonds, Global games, Rollover risk.

**JEL classifications:** D82, G01, G21.

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## 1. Introduction

Following the global financial crisis of 2007/9, banks have increasingly relied on secured wholesale lending markets to finance their activities (CGFS, 2013). But the form of that collateralized funding has varied substantially across jurisdictions. While in the United States banks have typically accessed short-term secured funding through repo markets, in Europe the focus has been on long-term secured funding in the form of covered bonds. Indeed, the moribund state of private-label securitization markets since the crisis has prompted some commentators and policymakers to suggest covered bonds as a means of revitalizing mortgage finance in the United States (Paulson (2009); Financial Times, 2012).

Covered bonds are secured senior debt issued by banks and have a long history in Europe, stretching back to the 18th century.<sup>1</sup> They are secured by *encumbering*, or ring-fencing, a pool of high quality assets – typically mortgages or public sector loans – on the issuing bank’s balance sheet. These ring-fenced assets are placed within a bankruptcy-remote special vehicle called the *cover pool*. Since the cover pool is on balance sheet, regular capital requirements apply, in contrast to off-balance sheet asset-backed securities.

In the event of financial distress, covered bondholders have a preferential claim over the cover pool. Bankruptcy remoteness ensures that covered bondholders can always access their collateral. Critically, the cover pool is *dynamic* – banks must replenish non-performing assets with performing ones of equivalent value and quality over the life of the bond to maintain the requisite collateralization.<sup>2</sup>

These institutional features make covered bonds a safe asset for investors and a cheap funding source for banks. While covered bonds have yet to be defaulted on, the dynamic ring-

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<sup>1</sup>See Schwarcz (2011) for a description of the legal aspects of covered bonds. Packer et al. (2007) provide an overview of the covered bond market prior to the crisis. Rixtel and Gasperini (2013) survey bank funding trends in the euro area in the wake of the global financial crisis.

<sup>2</sup>Covered bondholders are also protected by *dual recourse*. If the value of the cover pool is insufficient to meet obligations, covered bondholders have a claim of the shortfall on unencumbered assets, where this claim is equal in seniority to other creditors. In what follows, we do not model this feature of covered bonds.

fence asymmetrically transfers credit risks onto, and erodes the seniority of, other at holders such as unsecured creditors. This suggests a two-way interaction between secured and unsecured funding markets. Covered bond issuance influences the incidence of bank runs by unsecured creditors and, in turn, conditions in the unsecured funding market influence the choice of asset encumbrance and covered bonds by the bank.

Despite their longevity as a financial instrument, there is surprisingly little formal analysis of covered bonds. This paper fills that gap by developing a general equilibrium model of bank funding and asset encumbrance in which covered bonds assume center stage. Our approach steps outside the Modigliani and Miller (1958) framework by assuming market segmentation between investors in secured and unsecured funding markets, and makes use of global games techniques (Morris and Shin (2003); Rochet and Vives (2004); Goldstein and Pauzner, 2005). The ex-post rollover decisions of unsecured bank creditors reflect a coordination problem that is characterized by a unique equilibrium at which a wholesale debt run occurs. Critically, we link the ex-post debt run to the bank's ex-ante choice of funding. Our analysis accounts for both the costs and benefits of asset encumbrance as we derive a unique joint equilibrium in secured and unsecured funding markets.

We emphasize two effects. The first, which we dub a **risk concentration effect**, highlights that the more reliant is the bank on covered bonds, the greater the asset encumbrance, the larger the impact of credit losses particularly on unsecured wholesale debt holders, and the greater the incidence of a bank run. The second, which we call the **bank funding effect**, acts in the opposite direction. The more assets are encumbered and the more covered bonds are issued to finance profitable investment, the greater the expected value of bank equity. This lowers the potential for a run by unsecured creditors. Absent bailouts and deposit insurance, a bank must take both these effects into account when choosing the optimal level of asset encumbrance and the amount of covered bonds to issue.

The comparative static results of our model shed light on the way in which secured and unsecured bank funding markets interact. A loss of confidence in unsecured funding

markets, in the form of heightened strategic complementarity amongst market participants, leads to more frequent ex-post runs and bankruptcy. To compensate for the heightened risk, wholesale investors seek a higher face value of unsecured debt. Moreover, to avoid bankruptcy, the banker ex-ante pledges more resources to unsecured wholesale investors and thereby reduces the level of asset encumbrance. In sum, the level of asset encumbrance decreases, while the effect on promised returns in the unsecured market are ambiguous.

The risk of a sharp decline in house prices is often cited as a key financial stability risk to banking systems relying heavily on covered bonds. We therefore consider how an exogenous shock to the profitability of mortgage assets influences asset encumbrance and unsecured funding. In our model, unsecured wholesale investors react to a fall in profitability by requiring a higher face value of unsecured debt. In turn, the banker maintains more unencumbered assets, reducing the overall level of asset encumbrance.

We also analyze the effect that institutional reforms, such as improved bank crisis resolution processes, that lower the costs of premature liquidation can have on the ex-ante optimal mix of secured and unsecured funding. In the event of bankruptcy, and unencumbered assets are liquidated, the proceeds to unsecured wholesale investors is higher. Thus, ex-ante, these investors require lower promised returns. This, in turn, permits the banker to reduce the stock of unencumbered assets and thus encumber more asset.

The recent theoretical literature on banking crises has largely focused on how shocks to haircuts – excess collateral per unit of funding – propagate and amplify financial stress.<sup>3</sup> But it has yet to fully explore the interplay between secured and unsecured bank funding in the context of the move towards greater collateralization of balance sheets. Two recent papers that attempt to do so are Gai et al. (2013) and Matta and Perotti (2014).<sup>4</sup> Both use global game techniques to focus on the implications of secured repo funding for bank

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<sup>3</sup>See, for example, Brunnermeier and Pedersen (2009), Adrian and Shin (2010), Geanakoplos (2010), Gorton and Metrick (2012), and Gai et al. (2011).

<sup>4</sup>Eisenbach et al. (2014) also study similar issues in a setup where rollover risk is exogenous.

fragility, an issue that has greater resonance for the UK and US banking systems. Our paper is the first to attempt a comparable analysis of covered bonds, the dominant form of secured funding in continental European banking systems.<sup>5</sup>

Gai et al. (2013) analyze how the liquidity and solvency risk of a bank changes with the composition of funding. They distinguish between short- and long-term funding and show how “dashes for collateral” by short-term secured creditors can occur. When secured debt is short-term, the bank’s liquidity risk becomes more sensitive to collateral values since, in the event of a run, expected losses are covered by the liquidation of pledged collateral. The interim resources available to unsecured creditors varies with pledged collateral and pledged collateral, in turn, varies with its resale value.

Matta and Perotti (2014) adopt a similar perspective in their study of the interaction between secured repo funding and unsecured lending. But, unlike Gai et al. (2013) who focus exclusively on the mechanics of the ex post run, Matta and Perotti also model the ex ante funding mix. In this respect, their analysis is closely related to ours, although the contractual setting generating safety is different. We explicitly model the institutional features of covered bonds and dynamic ring fencing of the cover pool on the bank’s balance sheet means that asset encumbrance is explicit. By contrast, in Matta and Perotti (2014), the bank securitizes the (safe) part of the project return and pledges it as collateral to secured lenders. Asset encumbrance is, therefore, implicit and sensitive to collateral resale values.

The paper proceeds as follows. In Section 2, we set out the model structure before describing in section 3 the ex post rollover decision of unsecured creditors and the ex ante optimal choice of asset encumbrance and funding by the bank. Section 4 presents our comparative static results, and a final section concludes with a discussion of avenues for further work. All proofs are relegated to the appendices.

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<sup>5</sup>The existing literature on covered bonds is empirical. Carbo-Valverde et al. (2012) examine the extent to which covered bonds can substitute for mortgage-backed securities. And Prokopczuk et al. (2013) study how market liquidity and asset quality influences covered bond pricing.

## 2. Model

Our model builds on Rochet and Vives (2004). There are three dates,  $t = 0, 1, 2$ , and the economy is populated by risk-neutral agents. There are two groups of wholesale investors – short-term investors of unit mass and long-term investors of large mass  $\delta \in (0, \infty)$ . Investors receive a unit endowment at  $t = 0$  that can either be consumed immediately or placed with a financial intermediary (‘the banker’) that has access to profitable opportunities. Reflecting their time preferences, short-term investors can consume at each date,  $U = C_0 + C_1 + C_2$ , while long-term investors can only consume at the initial or final date,  $U = C_0 + C_2$ . The banker consumes at the final date,  $U = C_2$ .

A unit investment placed with the banker at  $t = 0$  by short-term investors can be withdrawn at either the interim or final date. The endogenous face value of unsecured debt raised,  $D \in [1, \infty)$ , is independent of the withdrawal date.<sup>6</sup> The funds are used by the banker to acquire high-quality assets (such as mortgages) that yield a gross return,  $R > 1$ , if held until the final date. Premature liquidation yields a fraction  $\psi \in (0, \frac{1}{R})$  of the final-date return, perhaps because of the cost of fire sales (Shleifer and Vishny, 1992) or the loss of relationship specific knowledge when asset ownership changes (Diamond and Rajan, 2001).

The banker can attract additional funding from long-term investors at  $t = 0$  through issuing covered bond  $CB_0 \geq 0$ . The covered bond is secured by publicly ring-fencing a fraction  $\alpha \in [0, 1]$  of bank assets and placing them in a bankruptcy-remote vehicle, the cover pool, on the bank’s balance sheet. These ring-fenced (or encumbered) assets are valued mark-to-market and we denote the face value of the cover pool at  $t = 2$  by  $CB \equiv \psi R \alpha$ , which reflects the over-collateralization associated with secured debt. The banker invests the proceeds from issuing covered bonds in additional high-quality assets. Table 1 summarizes the

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<sup>6</sup>Demandable debt arises endogenously in the presence of idiosyncratic liquidity risk (Diamond and Dybvig, 1983) or a demand for absolutely safe claims (Gennaioli et al. (2013); Ahnert and Perotti, 2014). Furthermore, demandable debt may also prevent moral hazard of the banker (Calomiris and Kahn (1991); Diamond and Rajan, 2001). In what follows, we assume that the banker is able to invest in high quality assets only.

bank's balance sheet at  $t = 0$ .

(cover pool)	$\alpha$	$CB_0$
(unencumbered assets)	$(1 - \alpha) + CB_0$	1

Table 1: Balance sheet of the bank at  $t = 0$

A defining feature of covered bonds is the dynamic replenishment of the cover pool in the event of adverse shocks to the balance sheet. Covered bonds require the banker to maintain the value of the cover pool at all dates, by replacing non-performing assets in the cover pool with performing unencumbered assets. Thus, the replenishment protects (secured) covered bond holders, but it disadvantages unsecured debt holders since the entire shock is concentrated on them. We suppose that the balance sheet suffers a shock,  $S \in \mathbb{R}$ , at the final date, which the banker observes at the interim date. The shock is drawn from a continuous probability distribution function  $f(S)$ , and cumulative distribution function  $F(S)$ . The shock is bounded above by the value of assets on the balance sheet,  $S \leq R[1 + CB_0] \equiv \hat{S}$ . We assume  $f' \leq 0$ , whereby small shocks are more likely than large shocks. Table 2 illustrates the effect of a small shock  $S > 0$  on the balance sheet at  $t = 2$  in the event that short-term investors roll over their unsecured funds, where  $E$  is the value of equity.

(cover pool)	$R\alpha$	$CB$
(unencumbered assets)	$R[(1 - \alpha) + CB_0] - S$	$D$
		$E$

Table 2: Balance sheet at  $t = 2$  (after a small shock and rollover of unsecured debt)

Bankruptcy occurs when the value of unencumbered assets is insufficient to repay unsecured debt holders. At this point, the bank is closed and each of these debt holders is assumed to receive an equal share of liquidated unencumbered assets at the interim date:

$$\min \left\{ D, \psi \left( R[(1 - \alpha) + CB_0] - S \right) \right\},$$

while (secured) covered bond holders receive the proceeds of the liquidated cover pool. Under limited liability, the equity value of the bank is zero in the event of bankruptcy. Otherwise,



equity value is the value of investment net of debt payments, that is

$$E \equiv \max \left\{ 0, R(1 + CB_0) - S - CB - D \right\}.$$

The banker maximizes the expected value of bank equity.

Wholesale investors are assumed to delegate the decision to roll over their funds at  $t = 1$  to a group of professional fund managers indexed by  $i \in [0, 1]$ . If a proportion of managers  $\ell \in [0, 1]$  refuses to roll over, the banker must liquidate the amount  $\ell D / \psi > \ell D$  to serve withdrawals. Thus, bankruptcy occurs whenever

$$R \left[ (1 - \alpha) + CB_0 \right] - S - \frac{\ell D}{\psi} < (1 - \ell)D, \quad (1)$$

where  $R \left[ (1 - \alpha) + CB_0 \right] - S$  is the value of unencumbered assets and the banker must serve  $(1 - \ell)D$  of withdrawals at the final date.

The simultaneous rollover decision of fund managers is governed by their compensation. In the case of bankruptcy, a manager's relative compensation from rolling over is negative,  $-C < 0$ . Otherwise, the relative compensation from rolling over is a benefit,  $B > 0$ . We define  $\gamma \equiv \frac{C}{B+C} \in (0, 1)$  to be a measure of the conservatism of managers. This specification ensures global strategic complementarity in the rollover decisions of fund managers.<sup>7</sup>

Figure 1 illustrates the dominant strategies of fund managers when the shock  $S$  is common knowledge. If all unsecured debt is rolled over,  $\ell = 0$ , bankruptcy occurs when the shock is larger than an *upper bound*, namely,

$$\bar{S} \equiv R[(1 - \alpha) + CB_0] - D, \quad (2)$$

and it is a dominant strategy for fund managers not to roll over whenever  $S > \bar{S}$ . Likewise,

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<sup>7</sup>Goldstein and Pauzner (2005) analyze a bank-run game with one-sided strategic complementarity.

when  $\ell = 1$ , no unsecured debt is rolled over and bankruptcy is avoided whenever the shock is smaller than a *lower bound*,

$$\underline{S} \equiv R[(1 - \alpha) + CB_0] - \frac{D}{\psi} < \bar{S}. \quad (3)$$

In this case, it is a dominant strategy for fund managers to roll over for  $S < \underline{S}$ . The upper and lower dominance regions depicted in Figure 1 are always well defined. That is, for all funding choices at  $t = 0$ ,  $\underline{S} > -\infty$  and  $\bar{S} \leq R(1 + CB_0) - 1 < \hat{S}$ .

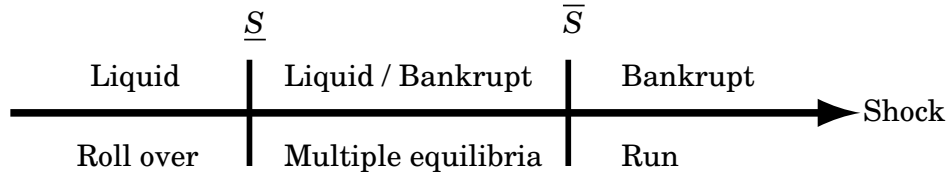


Figure 1: Tripartite classification of the shock

In order to sidestep issues of multiple equilibria that stem from assuming that there is complete information about the shock, we suppose that each fund manager  $i$  receives a noisy private signal at the interim date:

$$x_i \equiv S + \epsilon_i, \quad (4)$$

where idiosyncratic noise  $\epsilon_i$  is drawn from a continuous distribution  $G$  with support over the interval  $[-\epsilon, \epsilon]$ , where  $\epsilon > 0$ . Idiosyncratic noise is independent of the shock and independently and identically distributed across fund managers. Table 3 summarizes the timeline.

<b>Initial date</b> ( $t = 0$ )	<b>Interim date</b> ( $t = 1$ )	<b>Final date</b> ( $t = 2$ )
1. Unsecured debt issuance	1. Banker observes credit shock	1. Banker repays secured debt
2. Investment	2. Dynamic replenishment	2. Banker repays unsecured debt
3. Asset encumbrance	3. Private signals about shock	3. Banker consumes equity
4. Secured debt issuance	4. Unsecured debt withdrawals	
5. Additional investment	5. (Bankruptcy)	

Table 3: Timeline of events.

### 3. Equilibrium

We solve the model backwards, analyzing first the decisions of fund managers to roll over unsecured debt at the interim date. Next, we turn to the optimal choice of secured (covered bond) and unsecured funding at the initial date. Thus, funds managers treat as given the composition of funding, namely the amount of secured covered bond funding  $CB_0 \in [0, \delta]$ , the extent of asset encumbrance  $\alpha \in [0, 1]$ , and the cost of unsecured debt  $D \in [1, \infty)$ .

**Proposition 1. Unsecured debt rollover subgame.** *If private noise vanishes, there exists a unique Bayesian equilibrium in each subgame. It is characterized by a bankruptcy threshold,  $S^*$ , and a signal threshold  $x^*$ . Fund manager  $i$  rolls over unsecured debt if and only if  $x_i < x^*$  and bankruptcy occurs if and only if  $S > S^* = R[(1 - \alpha) + CB_0] - \kappa D \in (\underline{S}, \bar{S})$ , where  $\kappa \equiv 1 + \gamma \left( \frac{1}{\psi} - 1 \right) \in \left( 1, \frac{1}{\psi} \right)$  and  $x^* \rightarrow S^*$  for  $\epsilon \rightarrow 0$ .*

*The bankruptcy threshold decreases in the level of asset encumbrance and the cost of unsecured debt, but increases in the amount of secured funding attracted:*

$$\frac{\partial S^*}{\partial \alpha} = -R < 0, \quad \frac{\partial S^*}{\partial CB_0} = R > 0, \quad \frac{\partial S^*}{\partial D} = -\kappa < 0. \quad (5)$$

**Proof.** See Appendix A. ■

Greater asset encumbrance reduces the amount of unencumbered assets available to meet the demand from unsecured debt holders, which increases the probability of default on unsecured debt. This induces fund managers to withdraw funding for a larger range of shocks. In contrast, an increase in the amount of secured funding received from long-term investors increases the amount of unencumbered assets, so fund managers withdraw for a smaller range of shocks. Moreover, more expensive unsecured funding implies a smaller range of shocks for which all fund managers are repaid in full, which induces them to withdraw funds for a larger range of shocks.

We next consider the secured funding market. The banker chooses the level of asset encumbrance and the level of covered bond issuance at the initial date to maximize the expected value of equity, subject to the participation constraint of long-term investors, taking the cost of unsecured funding as given. First, we derive the expected value of the banker's equity. For a small shock,  $S < S^*$ , equity is  $E(S) = R(1 + CB_0) - S - CB - D$ , while bankruptcy occurs for larger shock,  $S > S^*$ . Thus, the expected value of bank equity is

$$\pi \equiv \int_{-\infty}^{S^*} E(S) dF(S) = F(S^*)[R(1 + CB_0) - CB - D] - \int_{-\infty}^{S^*} S dF(S). \quad (6)$$

Second, we identify the maximum amount of secured funding that the banker can attract for any given level of asset encumbrance  $\alpha$ . This amount,  $CB_0^*(\alpha)$ , is derived from the participation constraint of long-term investors who invest in covered bonds:

$$1 \leq F(\tilde{S}) \left( \frac{CB}{CB_0} \right) + \int_{\tilde{S}}^{\hat{S}} \psi \frac{R(1 + CB_0) - S}{CB_0} dF(S). \quad (7)$$

The outside option of long-term investors is to consume their endowment at  $t = 0$ . Investing in covered bonds buys a claim to the fraction  $1/CB_0$  of the face value  $CB$ , backed by the covered pool. If there is no bankruptcy,  $S \leq S^*$ , then long-term investors are repaid in full. In the event of bankruptcy,  $S > S^*$ , the implications for long-term investors depend on the shock size. For sufficiently small shocks,  $S \leq \tilde{S} \equiv R[1 - \alpha + CB_0]$ , bankruptcy remoteness of the cover pool ensures full repayment. But larger shocks,  $\tilde{S} < S \leq \hat{S}$ , entail partial default even on secured covered bonds. Figure 2 shows the effective seniority of debt claims implied by dynamic the replenishment of the cover pool.

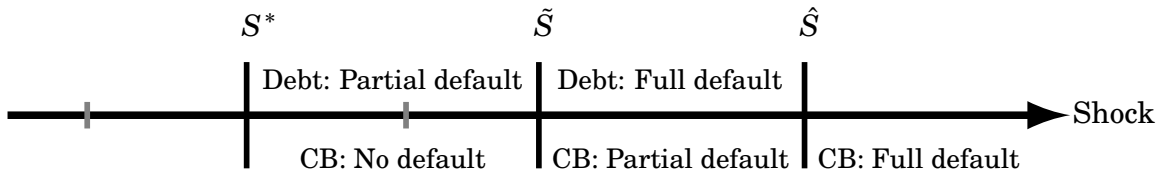


Figure 2: Classification of default regions. The **grey** lines indicate the upper and lower dominance regions.

**Proposition 2. Secured funding.** *Provided there are sufficiently many long-term investors,  $\delta \geq \underline{\delta} \equiv CB_0^*(1)$ , there exists a unique amount of covered bond funding  $CB_0^*(\alpha) \in [0, \delta]$  implicitly defined by:*

$$CB_0^*(\alpha) = F(\tilde{S}) CB + \psi \int_{\tilde{S}}^{\hat{S}} (\hat{S} - S) dF(S) \quad (8)$$

*for any given amount of asset encumbrance,  $\alpha \in [0, 1]$ . Moreover, greater asset encumbrance raises more secured funding from long-term investors via covered bonds:*

$$\frac{dCB_0^*(\alpha)}{d\alpha} \equiv \frac{F(\tilde{S})}{F(\tilde{S}) + \frac{1}{\psi R} - 1} \in (0, 1), \quad (9)$$

*but with diminishing returns to scale,*

$$\frac{d^2CB_0^*(\alpha)}{d\alpha^2} = \frac{\psi R^2 f(\tilde{S}) \left( \frac{dCB_0^*(\alpha)}{d\alpha} - 1 \right) (1 - \psi R)}{[1 - \psi R (1 - F(\tilde{S}))]^2} < 0. \quad (10)$$

**Proof.** See Appendix B. ■

Proposition 2 highlights a **bank funding channel**. Encumbering more assets allows the banker to issue more covered bonds. As more secured funding is attracted, the banker expands its balance sheet via additional investment, which increases the expected equity value. Henceforth, we assume that long-term investors wealth is abundant,  $\delta \geq \underline{\delta}$ . But there are diminishing returns to scale from encumbering assets. Greater encumbrance increases the range of credit shocks over which secured debt holders can suffer a loss. Partial default on covered bonds occurs for shocks  $S \in (\tilde{S}, \hat{S})$  as depicted in Figure 2. These losses offset some of the beneficial effect of balance sheet expansion and results in diminishing returns.

Issuing secured debt also has the effect of asymmetrically shifting credit risk onto unsecured debt holders. Greater asset encumbrance concentrates the credit shock onto unsecured debt, which leads to higher instances of unsecured debt runs. This **risk concentration channel** mitigates the indirect beneficial effects of the bank funding channel on the bankruptcy threshold. Corollary 1 summarizes.

**Corollary 1.** *Encumbering more assets reduces the bankruptcy threshold  $S^*(\alpha, CB_0^*(\alpha))$ :*

$$\frac{dS^*}{d\alpha} = \frac{\partial S^*}{\partial \alpha} + \frac{\partial S^*}{\partial CB_0^*} \frac{dCB_0^*}{d\alpha} = R \left( \frac{dCB_0^*}{d\alpha} - 1 \right) < 0. \quad (11)$$

In our model, the banker internalizes both the bank funding and risk concentration channels when encumbering assets:

$$\alpha^*(D) \equiv \arg \max_{\alpha \in [0,1]} \pi(\alpha, CB_0^*(\alpha); D) \quad (12)$$

Figure 3 shows how the banker's expected equity value  $\pi(\cdot)$  changes with asset encumbrance in the case of an exponential shock distribution. The objective function is strictly concave and has a global maximum at the optimal interior level of asset encumbrance. Proposition 3 provides sufficient conditions for an interior level of asset encumbrance.

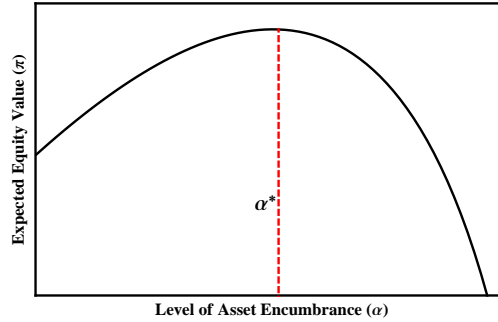


Figure 3: Expected equity value as a function of the level of asset encumbrance. Additional parameters were  $R = 3$ ,  $\psi = 0.2$ ,  $k = 0.025$ , and  $D = 1.1$ , where the shock was exponentially distributed with intensity  $\lambda = 1.1$ .

**Proposition 3. Optimal asset encumbrance.** *If asset profitability is high,  $R \geq \underline{R}$ , there exists a unique choice of asset encumbrance,  $\alpha^*(D) \in (0,1)$ , and a corresponding amount of secured funding,  $CB_0^*(\alpha^*(D))$ . The choice of asset encumbrance is implicit defined by:*

$$F(S^*(\alpha^*)) \left( \frac{dCB_0^*(\alpha^*)}{d\alpha} - \psi \right) + f(S^*(\alpha^*)) [(\kappa - 1)D + \alpha^*(1 - \psi)R] \left( \frac{dCB_0^*(\alpha^*)}{d\alpha} - 1 \right) = 0. \quad (13)$$

*Thus, more expensive unsecured funding induces the banker to encumber fewer assets:*

$$\frac{d\alpha^*(D)}{dD} < 0. \quad (14)$$

**Proof.** See Appendix C, where we define the lower bound  $\underline{R}$ . ■

When asset profitability is high, the marginal amount of secured funding attracted from encumbering assets exceeds the liquidation value,  $\psi$ . Hence, the banker finds it optimal to encumber some assets,  $\alpha^*(D) > 0$ . Our earlier assumption of  $\psi R < 1$  limits the amount of balance sheet encumbrance,  $\alpha^*(D) < 1$ . The optimal level of asset encumbrance, and thus the volume of secured funding, is affected by the cost of unsecured funding. More expensive unsecured debt (a higher face value  $D$ ) reduces the bankruptcy threshold for any level of encumbrance. As a result, unsecured debt runs and bankruptcy occur more frequently. To avoid these adverse outcomes, where all equity value is wiped out, the banker encumbers fewer assets in order to meet withdrawals of unsecured debt. We label this precautionary motive the **equity preservation effect**.

We now turn to the supply of unsecured funding. Competitive short-term investors break even in expectation. As shown in Figure 2, for a small shock,  $S < S^*$ , there is no unsecured debt run,  $l^* = 0$ , and short-term investors are repaid in full, receiving  $D$ . However, for a larger shock,  $S^* < S < \tilde{S}$ , a debt run occurs,  $l^* = 1$ , that leads to bankruptcy and partial default on unsecured debt. There is full default for large shocks,  $S > \tilde{S}$ . Thus, the face value of unsecured debt demanded by short-term investors for any given level of asset encumbrance,  $D^*(\alpha)$ , is given by the solution to the fixed point equation

$$1 = F(S^*)D + \psi \int_{S^*}^{\tilde{S}} [\tilde{S} - S] dF(S). \quad (15)$$

**Proposition 4. Unsecured funding.** *Suppose short-term investors find it optimal to lend when all assets are encumbered and the face value of unsecured debt is equal to the value*

of the unencumbered assets. Then, there exists a unique face value of unsecured debt for any given level of asset encumbrance,  $D^*(\alpha)$ , which is implicitly defined by equation (15). Furthermore, greater asset encumbrance raises the equilibrium face value of unsecured debt,

$$\frac{dD^*(\alpha)}{d\alpha} > 0. \quad (16)$$

**Proof.** See Appendix D, where we state the technical condition sufficient for existence. ■

Corollary 1 states that the total marginal effect of asset encumbrance on the bankruptcy threshold is negative. Thus, for a given distribution of shocks, there will be more instances of unsecured debt runs leading to bankruptcy. In these instances, the banker defaults on unsecured debt obligations and instead pays short-term investors an equal share of the liquidated unencumbered assets. To counter these adverse outcomes, short-term investors require a higher face value of unsecured debt in order to break even.

**Proposition 5. Equilibrium in secured and unsecured funding markets.** *There exists a unique joint equilibrium in both funding markets. It is characterized by a face value of unsecured debt  $D^* \equiv D^*(\alpha^*)$  and a choice of asset encumbrance,  $\alpha^* \equiv \alpha^*(D^*)$ , and the corresponding volume of secured funding (covered bonds),  $CB_0^*$ .*

**Proof.** See Appendix E. ■

Figure 4 depicts the unique joint equilibrium,  $(D^*, \alpha^*)$  between unsecured and secured (covered bond) funding markets and. The equity preservation effect renders the banker's optimal level of asset encumbrance downward sloping in the cost of unsecured debt. However, since greater asset encumbrance leads to more runs, unsecured debt holders require a higher compensation in order to lend to the banker. The unique equilibrium is given by the intersection of the two schedules.



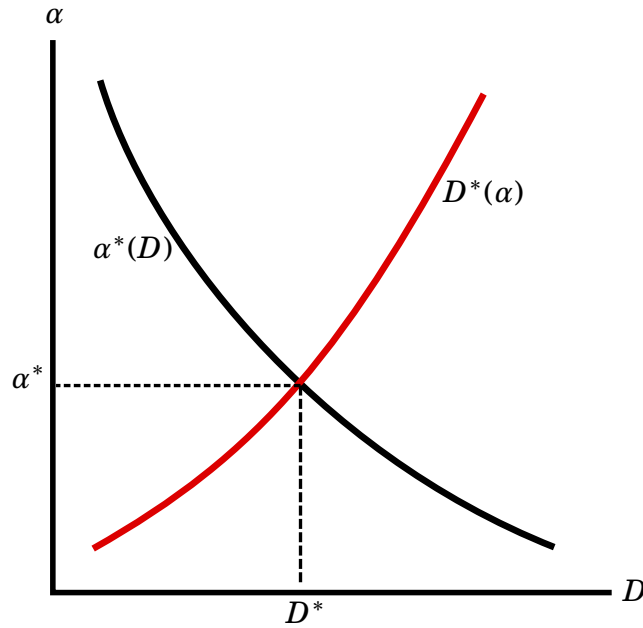


Figure 4: Joint equilibrium in secured (covered bond) and unsecured funding markets.

#### 4. Comparative Statics

We next analyse how changes in exogenous parameters affect the equilibrium amount of asset encumbrance, and thus the issuance of secured covered bond funding, and the cost of unsecured funding. Specifically, we consider premature liquidation value ( $\psi$ ) that affects the degree of strategic complementarity amongst market participants in the unsecured wholesale funding market, the conservatism of these market participants ( $\gamma$ ), and the profitability of assets underlying the issuance of covered bonds ( $R$ ).

**Proposition 6. Investor conservatism.** *If fund managers are more conservative (higher  $\gamma$ ), they have lower incentives to roll over unsecured funding, which results in a lower level of asset encumbrance but an ambiguous effect on the face value of unsecured debt.*

**Proof.** See Appendix F. ■

Figures 5 illustrate the effect of an increase in investor conservatism  $\gamma$ . As a result, greater incentives of wholesale funding market participants not to roll over unsecured debt results in a higher probability of bankruptcy and default on unsecured debt. To compensate

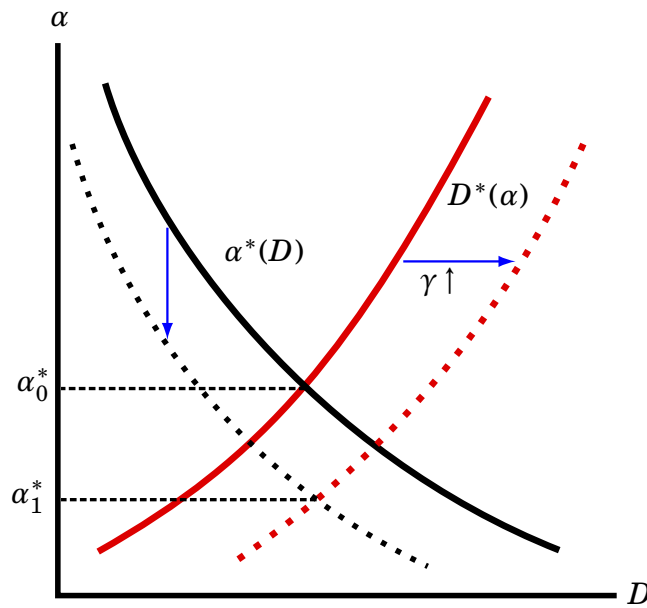


Figure 5: A marginal increase in the conservativeness of wholesale fund managers  $\gamma$

for these adverse outcomes, short-term investors require a higher face value of debt to break even in expectation, which shifts the schedule  $D^*(\alpha)$  outwards. On the other hand, the banker also responds to greater conservatism of wholesale fund managers by reducing the amount of asset encumbrance. This effect is akin to the equity preservation effect and shifts the schedule  $\alpha^*(D)$  downwards. With more unencumbered assets, default on wholesale funding occurs less frequently. As such, short-term investors require a lower face value of unsecured debt in order to break even. In sum, the overall effect of higher conservatism is fewer asset encumbrance but an ambiguous effect on the face value of unsecured debt.

**Proposition 7. Investment profitability.** *Greater profitability of (mortgage) assets results in a higher level of asset encumbrance and an ambiguous effect on the cost of unsecured funding.*

**Proof.** See Appendix G. ■

Figure 6 shows the effect of an increase in the profitability of mortgage assets  $R$ . Since unencumbered assets are more profitable, short-term investors require a lower face value

of unsecured debt, for any given level of asset encumbrance  $\alpha$ . As a result, the schedule  $D^*(\alpha)$  shifts inwards. At the same time, more profitable mortgage assets implies that the banker wishes to encumber more assets, for any given cost of unsecured debt. Therefore, the schedule  $\alpha^*(D)$  shifts upwards. In sum, the overall effect of a higher profitability of mortgage assets is greater encumbrance of assets but an ambiguous effect on the cost of unsecured funding. Intuitively, there are fewer but more profitable unencumbered assets available to short-term investors.

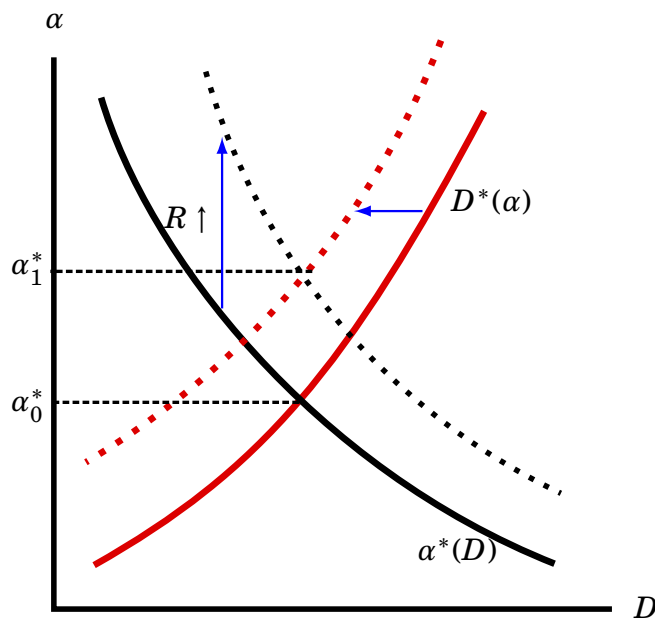


Figure 6: A marginal increase in the profitability of mortgage assets  $R$

**Proposition 8. Liquidation value.** *A larger liquidation value of assets (higher  $\psi$ ) reduces the degree of strategic complementarity among wholesale fund managers. Thus, more asset are encumbered to issue secured debt, but effect on the cost of unsecured debt is ambiguous.*

**Proof.** See Appendix H. ■

Figure 7 illustrate the effect of an increase in the liquidation value  $\psi$ . In the event of bankruptcy, unsecured debt holders receive more from the liquidated assets. Thus, for

any given level of asset encumbrance  $\alpha$ , they require a lower face value of unsecured debt to break even, which shifts the schedule  $D^*(\alpha)$  inwards. At the same time, as  $\psi$  increases, the degree of strategic complementarity among wholesale fund managers is lowered, which reduces the range of credit shocks over which bankruptcy occurs. Thus, the banker can raise more funds from encumbering assets such that, for any given cost of unsecured debt, the optimal level of asset encumbrance is higher. This shifts the schedule  $\alpha^*(D)$  upwards. In sum, the overall effect of a higher liquidation value is more asset encumbrance but an ambiguous effect on the cost of unsecured debt. Intuitively, there are fewer but more liquid unencumbered assets available to short-term investors.

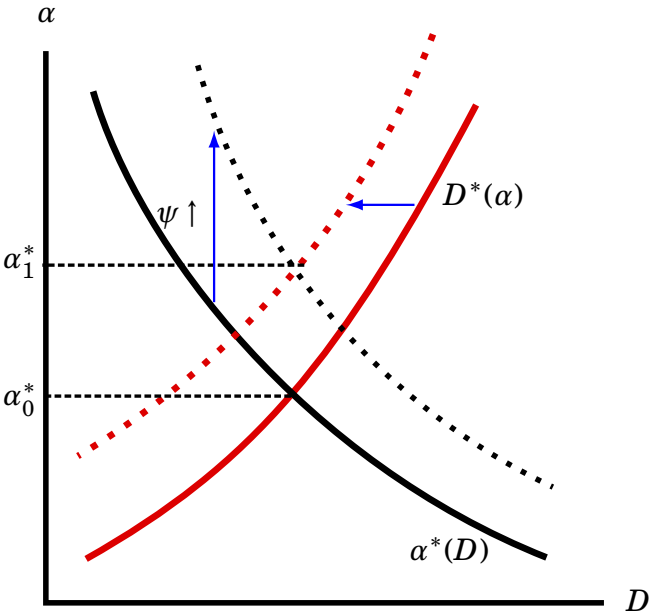


Figure 7: A marginal increase in the liquidation value  $\psi$

## 5. Conclusion

In this paper, we have explored some implications of the increased collateralization of bank balance sheets. In so doing, we present the first formal model of covered bonds and examine how the covered bond funding market interacts with the unsecured funding market.

Our analysis is strictly positive. We decompose the influence of covered bonds into two distinct balance sheet effects. First, covered bond issuance asymmetrically shifts the risk of bank distress onto unsecured creditors (risk concentration effect). And second, the greater the asset encumbrance, the greater is the expected value of bank equity (bank funding effect). Absent bailouts and deposit insurance, the bank fully internalizes these channels when choosing its funding mix and the level of asset encumbrance. The comparative static results suggest how balance sheet collateralization is influenced by losses of confidence in unsecured credit markets, macroeconomic shocks to asset profitability, and the efficacy of crisis resolution measures.

Although we eschew a normative analysis, our results suggest that there may be grounds to be cautious about the quest for safety by financial intermediaries in the wake of the global financial crisis. While a micro-prudential perspective suggests that increased long-term secured funding seems desirable, since it increases funding diversity and lowers run risk, a macro-prudential perspective suggests reasons to be less sanguine. High encumbrance levels reduce the assets available to unsecured creditors, leading to higher costs of borrowing from such funding sources and heightening incentives to finance on secured terms. And, as assets are increasingly encumbered, the financial system as a whole may become riskier as it is more susceptible to procyclical swings in the underlying value of collateral assets (Haldane, 2012).

We conclude by identifying several avenues for further work. First, our model makes the strong assumption that the banker always invests funds in high quality activities. Since

Calomiris and Kahn (1991), a sizeable literature has developed that explores the trade-off between ex-ante moral hazard and the disciplining effect of a run by unsecured creditors. Extending our analysis to allow for the possibility of moral hazard by the banker would enable a proper study of how asset encumbrance and the optimal funding mix affects this trade-off.

Second, our approach assumes a monopolistic banker in the market for covered bonds. Under other market structures, the amount of covered bond funding raised by encumbering assets may be lower, although we expect that our main insights on asset encumbrance and bank fragility should remain unchanged. Encumbering assets allows banks to raise covered bond funding that can be invested in profitable projects. Since competition for covered bond funding attenuates the bank funding effect, the banker is likely to encumber fewer assets.

Third, covered bonds are typically backed by a heterogeneous mixture high quality assets, typically mortgages and public debt. Following a shock to the balance sheet, the dynamic replenishment of the cover pool can alter the risk profile of unencumbered assets, possibly substantially. If low risk assets are swapped into the cover pool first, then the concentration of risk onto unsecured wholesale debt holders could be exacerbated. Asset heterogeneity may therefore amplify the mechanisms described in our paper.

Finally, some policy implications of our model merit further investigation. For instance, requiring banks to hold liquidity cushions (either voluntarily or involuntarily) will influence the joint equilibrium in the two funding markets. Also, a number of countries, notably Australia, Canada, and New Zealand have introduced formal limits on asset encumbrance levels and established covered bond legislation to make clear the rights of secured and unsecured creditors in the event of financial distress. There is a suggestion that a limit on asset encumbrance might usefully curb negative externalities – such as fire sales and systemic risk – that may follow bankruptcy of a financial institution. Further analysis on the implications of covered bonds and asset encumbrance for systemic risk and crisis resolution is an important area for future work.

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## Appendix A. Proof of Proposition 1

In each rollover subgame, it is sufficient to establish the existence of a unique Bayesian equilibrium in threshold strategies for the limit of vanishing private noise,  $\epsilon \rightarrow 0$ .<sup>8</sup> Each fund manager  $i$  uses a threshold strategy, whereby unsecured debt is rolled over if and only if the private signal suggests that the credit shock is small,  $x_i < x^*$ . Hence, for a given realization  $S \in [\underline{S}, \bar{S}]$ , the proportion of fund managers who do not roll over debt is:

$$\ell(S, x^*) = \text{Prob}(x_i > x^* | S) = \text{Prob}(\epsilon_i > x^* - S) = 1 - G(x^* - S).$$

A critical mass condition states that bankruptcy occurs when the credit shock reaches a threshold  $S^*$ , where the proportion of unsecured debt not rolled over is evaluated at  $S^*$ :

$$R \left[ (1 - \alpha) + CB_0 \right] - S^* - \ell(S^*, x^*) \frac{D}{\psi} = \left( 1 - \ell(S^*, x^*) \right) D \quad (\text{A.1})$$

The posterior distribution of the credit shock conditional on the private signal is derived using Bayes' rule. The indifference condition states that the fund manager who receives the critical signal  $x_i = x^*$  is indifferent between rolling over and not rolling over unsecured debt:

$$\gamma = \text{Pr}(S < S^* | x_i = x^*). \quad (\text{A.2})$$

Using the definition of the private signal  $x_j = S + \epsilon_j$  of the indifferent fund manager, we can state the conditional probability as follows:

$$1 - \gamma = \text{Pr}(S \geq S^* | x_i = x^*) = \text{Pr}(S \geq S^* | x_i = x^* = S + \epsilon_j) \quad (\text{A.3})$$

$$= \text{Pr}(x^* - \epsilon_j \geq S^*) = \text{Pr}(\epsilon_j \leq x^* - S^*) \quad (\text{A.4})$$

$$= G(x^* - S^*) \quad (\text{A.5})$$

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<sup>8</sup>Morris and Shin (2003) show that only threshold strategies survive the iterated deletion of strictly dominated strategies. See also Frankel et al. (2003).

Thus, the indifference condition implies that  $x^* - S^* = G^{-1}(1 - \gamma)$ . Inserting the indifference condition into  $\ell(S^*, x^*)$ , the proportion of fund managers who do not roll over when the credit shock is at the critical level  $S^*$  is perceived by the indifferent fund manager to be:

$$\ell(S^*, x_i = x^*) = 1 - G(x^* - S^*) = 1 - G(G^{-1}(1 - \gamma)) = \gamma. \quad (\text{A.6})$$

Therefore, the bankruptcy threshold is  $S^* = R[(1 - \alpha) + CB_0] - \kappa D$ . If private noise vanishes, the signal threshold also converges to this value. The partial derivatives of the bankruptcy threshold  $S^*$  follow immediately.  $\square$

## Appendix B. Proof of Proposition 2

The derivative of the banker's expected value of equity with respect to secured funding raised is:

$$\frac{\partial \pi}{\partial CB_0} = RF(S^*) + Rf(S^*)[(\kappa - 1)D + (1 - \psi)\alpha R] > 0 \quad (\text{B.1})$$

for all feasible values of  $\alpha$ ,  $CB_0$ , and  $D$ . Therefore, the banker raises as much secured funding as possible.

The participation constraint of long-term investors can be written as:

$$CB_0 \leq F(\tilde{S})CB + \int_{\tilde{S}}^{\hat{S}} g(S) dF(S), \quad (\text{B.2})$$

where  $g(S) = \psi[R(1 + CB_0) - S]$ , which implies that  $g(\tilde{S}) = CB$  and  $g(\hat{S}) = 0$ . Taking the derivatives with respect to the amount of secured funding raised from long-term investors, the left-hand side has a unit slope, while the right-hand side's slope is (by Leibniz rule):

$$\begin{aligned} \frac{dRHS}{dCB_0} &= f(\tilde{S})CB \frac{d\tilde{S}}{dCB_0} - g(\tilde{S})f(\tilde{S}) \frac{d\tilde{S}}{dCB_0} - g(\hat{S})f(\hat{S}) \frac{d\hat{S}}{dCB_0} + \psi R(F(\hat{S}) - F(\tilde{S})) \\ &= \psi R(1 - F(\tilde{S})) \geq 0 \end{aligned} \quad (\text{B.3})$$

since  $F(\hat{S}) = 1$ . Moreover, since  $\psi R < 1$ , we have that  $\frac{dRHS}{dCB_0} < 1$ . It thus follows that, as secured funding  $CB_0$  increases, the left-hand side of the participation constraint increases faster than the right-hand side. Hence, if long-term investors are sufficiently abundant, there exists a unique solution  $CB_0^*(\alpha)$  given by the binding participation constraint:

$$CB_0^*(\alpha) = F\left(R[(1-\alpha) + CB_0^*(\alpha)]\right) \alpha \psi R + \psi \int_{R[(1-\alpha) + CB_0^*(\alpha)]}^{R[1 + CB_0^*(\alpha)]} \left(R(1 + CB_0^*(\alpha)) - S\right) dF(S), \quad (\text{B.4})$$

Naturally, we have that  $CB_0^*(0) = 0$ . Finally, observing that the right-hand side of the participation constraint increases in the level of asset encumbrance  $\alpha$

$$\frac{RHS}{d\alpha} = \psi R F(\tilde{S}) > 0, \quad (\text{B.5})$$

it follows that greater asset encumbrance leads to the banker raising more secured funding:

$$\frac{dCB_0^*(\alpha)}{d\alpha} \equiv \frac{F\left(R[(1-\alpha) + CB_0^*(\alpha)]\right)}{\frac{1}{\psi R} - 1 + F\left(R[(1-\alpha) + CB_0^*(\alpha)]\right)} > 0. \quad (\text{B.6})$$

For long-term investors to be sufficiently abundant, we assume that  $\delta \geq \underline{\delta} \equiv CB_0^*(1)$ , where the lower bound is implicitly and uniquely defined by:

$$\underline{\delta} = \psi R F(R\underline{\delta}) + \psi \int_{R\underline{\delta}}^{R[1 + \underline{\delta}]} \left(R(1 + \underline{\delta}) - S\right) dF(S). \quad (\text{B.7})$$

One can show that  $\underline{\delta} < 1$ .

### Appendix C. Proof of Proposition 3

Taking the risk concentration channel and the bank funding channel into account, and the cost of unsecured funding as given, the banker chooses the level of asset encumbrance

$\alpha \in [0, 1]$  to maximize the expected value of equity. This yields the first-order condition

$$IA \equiv F(S^*) \left( \frac{dCB_0^*(\alpha)}{d\alpha} - \psi \right) + f(S^*)[(\kappa - 1)D + \alpha^*(1 - \psi)R] \left( \frac{dCB_0^*(\alpha)}{d\alpha} - 1 \right) = 0, \quad (\text{C.1})$$

where  $IA$  is the implicit function that defines  $\alpha^*$  for any given  $D$ .

An interior solution  $\alpha^*(D) \in (0, 1)$  requires that  $\frac{dCB_0^*}{d\alpha} > \psi$ . This inequality is toughest to hold for  $\alpha = 1$ , which yields the sufficient condition  $R \geq \underline{R}$ , where the lower bound on returns is implicitly defined by  $F(\underline{R}\delta) = \frac{1 - \underline{R}\psi}{\underline{R}(1 - \psi)} \in (0, 1)$ . We state for future reference:

$$\frac{dCB_0^*}{d\alpha} - \psi = \frac{(1 - \psi)F(\tilde{S}) - \psi \left( \frac{1}{\psi R} - 1 \right)}{F(\tilde{S}) + \frac{1}{\psi R} - 1} \quad (\text{C.2})$$

$$1 - \frac{dCB_0^*}{d\alpha} = \frac{\frac{1}{\psi R} - 1}{F(\tilde{S}) + \frac{1}{\psi R} - 1} \quad (\text{C.3})$$

Concentrating on the interior solution, we show that this is a maximum of the banker's expected equity value. The second derivative of the equity value with respect to  $\alpha$  is:

$$\begin{aligned} \frac{1}{R} \frac{dIA}{d\alpha} &= \frac{d^2CB_0^*}{d\alpha^2} \{F(S^*) + f(S^*)[\alpha^*R(1 - \psi) + D(\kappa - 1)]\} \\ &+ \left( \frac{dCB_0^*}{d\alpha} - 1 \right) \left\{ f(S^*) \left[ \frac{dS^*}{d\alpha} + R(1 - \psi) \right] + f'(S^*) \frac{dS^*}{d\alpha} [\alpha^*R(1 - \psi) + D(\kappa - 1)] \right\} \\ &+ \frac{dS^*}{d\alpha} f(S^*)(1 - \psi). \end{aligned}$$

The third term is strictly negative since  $dS^*/d\alpha < 0$ . For the second term, since  $dCB_0^*/d\alpha < 1$ , it too is negative if the expression in the parenthesis is positive. Since  $f' \leq 0$ , a sufficient condition for this is  $\frac{dS^*}{d\alpha} + R(1 - \psi) > 0$  or, equivalently,  $\frac{dCB_0^*}{d\alpha} - \psi > 0$ . This condition, however, is guaranteed by the sufficient condition on the credit shock distribution for an interior  $\alpha^* \in (0, 1)$  solution (see above). Finally, the amount of covered bond funding raised as more assets are encumbered satisfied decreasing returns to scale, the banker's expected equity value function is strictly concave, and the choice  $\alpha^*$  is the global maximum.

A higher face value of unsecured debt affects the first-order condition according to:

$$\frac{dIA}{dD} = f(S^*(\alpha^*)) \frac{dS^*(\alpha^*)}{dD} \left( \frac{dCB_0^*(\alpha^*)}{d\alpha} - \psi \right) + f(S^*(\alpha^*)) (\kappa - 1) \left( \frac{dCB_0^*(\alpha^*)}{d\alpha} - 1 \right) \quad (\text{C.4})$$

$$+ f'(S^*(\alpha^*)) \frac{dS^*(\alpha^*)}{dD} [(\kappa - 1)D + \alpha^*(1 - \psi)R] \left( \frac{dCB_0^*(\alpha^*)}{d\alpha} - 1 \right) < 0, \quad (\text{C.5})$$

where  $dS^*(\alpha^*)/dD = -\kappa < 0$ .

From the Implicit Function Theorem it follows that the derivative of the optimal level of encumbrance with respect to the face value of unsecured debt is

$$\frac{d\alpha^*}{dD} = - \frac{\frac{dIA}{dD}}{\frac{dIA}{d\alpha}} < 0. \quad (\text{C.6})$$

#### Appendix D. Proof of Proposition 4

The equilibrium face value of unsecured debt,  $D^*(\alpha)$ , is defined by the implicit function:

$$ID \equiv F(S^*)D + \psi \int_{S^*}^{\tilde{S}} [\tilde{S} - S] dF(S) - 1 = 0.$$

First, observe that the value of the unsecured debt claim decreases in the level of asset encumbrance,

$$\frac{dID}{d\alpha} = D[1 - \kappa\psi]f(S^*) \frac{dS^*}{d\alpha} - \psi R \left( 1 - \frac{dCB_0^*(\alpha)}{d\alpha} \right) \int_{S^*}^{\tilde{S}} dF(S) < 0. \quad (\text{D.1})$$

Intuitively, more asset encumbrance reduces both the pool of unencumbered assets and the range of credit shocks for which unsecured debt holders are repaid in full, so the overall effect on the value of the unsecured debt claim is negative.

Second, we show that the right-hand falls short of the left-hand side at the lower bound  $D = 1$  for all values of  $\alpha$ . Since this is toughest to hold for  $\alpha = 0$ , we need to show that

$1 > ID(D = 1, \alpha = 0)$ . This condition always holds:

$$1 > ID(D = 1, \alpha = 0) = F(R - \kappa) + \psi \int_{R - \kappa}^R (R - S) dF(S) \quad (D.2)$$

$$\leq F(R - \kappa) + \psi \int_{R - \kappa}^R R dF(S) \quad (D.3)$$

$$= (1 - \psi R)F(R - \kappa) + \psi R F(R) < 1. \quad (D.4)$$

Thus,  $1 > ID(D = 1, \alpha)$  for all  $\alpha \in [0, 1]$ .

Third, the right-hand side is strictly concave in the face value of unsecured debt,

$$\frac{d^2 ID}{dD^2} = -\kappa f(S^*)[2 - \kappa\psi] + \kappa^2[1 - \kappa\psi]Df'(S^*) < 0, \quad (D.5)$$

since  $f'(S) \leq 0$ . Therefore, the existence and uniqueness of  $D^*(\alpha)$  for all  $\alpha \in [0, 1]$  is implied by  $ID(D = \tilde{S}, \alpha) > 1$  for all  $\alpha \in [0, 1]$ , since at most all unencumbered assets can be promised.

Fourth, since the value of the unsecured debt claim decreases in asset encumbrance, we have:

$$ID(D = \tilde{S}) \geq ID(D = \tilde{S}, \alpha = 1) = R\underline{\delta}F(R\underline{\delta}(1 - \kappa)) + \psi \int_{R\underline{\delta}(1 - \kappa)}^{R\underline{\delta}} (R\underline{\delta} - S) dF(S). \quad (D.6)$$

Hence,  $R\underline{\delta}F(R\underline{\delta}(1 - \kappa)) + \psi \int_{R\underline{\delta}(1 - \kappa)}^{R\underline{\delta}} (R\underline{\delta} - S) dF(S) > 1$  is a sufficient condition for the existence of a unique  $D^*(\alpha)$  for any given level of asset encumbrance.

It follows that, at  $D^*(\alpha)$ , the first derivative of the value of the unsecured debt claim with respect to the face value of unsecured debt is positive,

$$\left. \frac{dID}{dD} \right|_{D^*(\alpha)} > 0. \quad (D.7)$$

By the implicit function theorem, we have that more asset encumbrance leads to a

larger face value of unsecured debt for short-term investors to break even,

$$\left. \frac{dD^*(\alpha)}{d\alpha} \right|_{D^*(\alpha)} > 0. \quad (\text{D.8})$$

## Appendix E. Proof of Proposition 5

Existence and uniqueness is guaranteed by two boundary conditions because of the positive slope of the break-even condition of short-term investors ( $\left. \frac{dD^*(\alpha)}{d\alpha} \right|_{D^*(\alpha)} > 0$ ) and the negative slope of the optimal encumbrance choice of the banker ( $d\alpha^*/dD < 0$ ). First,  $\alpha^*(D = 1) < 1$ , which follows directly from Proposition 3. Second,  $D^*(\alpha = 0) < R$ . We show that this condition is always satisfied.  $D^*(\alpha = 0)$  is implicitly defined by

$$1 = F(R - \kappa D)D + \psi \int_{R - \kappa D}^R [R - S]dF(S) \quad (\text{E.1})$$

$$< F(R - \kappa D)D + \psi \int_{R - \kappa D}^R R dF(S) \quad (\text{E.2})$$

$$= \psi R F(R - \kappa D)(D - \psi R). \quad (\text{E.3})$$

Since the right-hand side, evaluates at  $D^*(\alpha)$ , increases in  $D$ , then  $ID$  is largest for  $D = R$ . Therefore, an upper bound on the value of the RHS is

$$\overline{ID} = (1 - \psi)RF(R(1 - \kappa)) + \psi R < R \quad (\text{E.4})$$

Thus,  $D^*(\alpha = 0) < R$ . This completes the short proof.

TA: checked until here.



## Appendix F. Proof of Proposition 6

We turn to the influence of investor conservatism  $\gamma$ , and thus the strategic complementarity among fund managers, on the two curves  $IA$  and  $ID$ . First, for the effect on the curve  $ID$ , which determines  $D^*(\alpha)$ , a marginal increase in the conservativeness ratio,  $\gamma$  reduces the required face value of unsecured debt:

$$\begin{aligned}
 \frac{dID}{d\gamma} &= f(S^*) \frac{dS^*}{d\gamma} D - \psi (\tilde{S} - S^*) f(S^*) \frac{dS^*}{d\gamma} \\
 &= f(S^*) \frac{dS^*}{d\gamma} (1 - \psi) (1 - \gamma) D \\
 &= -f(S^*) (1 - \gamma) \left( \frac{1}{\psi} - 1 \right) (1 - \psi) D^2 < 0.
 \end{aligned} \tag{F.1}$$

From the implicit function theorem it thus follows that

$$\left. \frac{dD^*}{d\gamma} \right|_{D^*(\alpha)} = - \left. \frac{dID/d\gamma}{dID/dD^*} \right|_{D^*(\alpha)} > 0, \tag{F.2}$$

where we use our earlier result that  $dID/dD^* > 0$  in equilibrium. Consequently, for any given level as asset encumbrance, as fund managers become more conservative, unsecured debt runs become more frequent, and bankruptcy, and thus costly liquidation of unencumbered assets, occurs more often. To ensure the continued participation of short-term investors, a higher face value of debt,  $D$ , is required for them to break even in expectation.

For the influence of  $\gamma$  on the banker's optimal choice of asset encumbrance, for a given face value of unsecured debt  $D$ , we have:

$$\begin{aligned}
 \frac{dIA}{d\gamma} &= \left( \frac{dCB_0^*(\alpha)}{d\alpha} - 1 \right) \left[ f(S^*) \frac{dS^*}{d\gamma} + f'(S^*) \frac{dS^*}{d\gamma} [(\kappa - 1)D + (1 - \psi)R] + f(S^*) D \left( \frac{1}{\psi} - 1 \right) \right] \\
 &\quad + (1 - \psi) f(S^*) \frac{dS^*}{d\gamma} \\
 &= \left( \frac{dCB_0^*(\alpha)}{d\alpha} - 1 \right) f'(S^*) \frac{dS^*}{d\gamma} [(\kappa - 1)D + (1 - \psi)R] + (1 - \psi) f(S^*) \frac{dS^*}{d\gamma} < 0,
 \end{aligned} \tag{F.3}$$

since  $dS^*/d\gamma < 0$ . Consequently, from the implicit function theorem we obtain that

$$\left. \frac{d\alpha^*}{d\gamma} \right|_{\alpha^*(D)} = - \left. \frac{dIA/d\gamma}{dIA/d\alpha} \right|_{\alpha^*(D)} < 0, \quad (\text{F.4})$$

since  $dIA/d\alpha < 0$  as demonstrated in the proof of Proposition 3. Thus, as the conservatism of fund managers increases, for any given value  $D$ , the banker responds by decreasing the fraction of assets encumbered.

Taking both results together yields the stated affect on  $\alpha^*$  and, in general, an ambiguous effect on  $D^*$ .

## Appendix G. Proof of Proposition 7

Finally, we consider the influence of mortgage asset profitability  $R$  on the two curves  $IA$  and  $ID$ . First, we need to understand the effect of changes in  $R$  on the equilibrium amount of covered bond funding raised for any given level of asset encumbrance,  $CB_0^*(\alpha)$ , which is defined by the implicit function  $ICB$ :

$$F(\tilde{S})CB + \psi \int_{\tilde{S}}^{\hat{S}} [\hat{S} - S] dF(S) - CB_0^*. \quad (\text{G.1})$$

From the proof of Proposition 2, we know that

$$\frac{dICB}{dCB_0^*} = \psi R (1 - F(\tilde{S})) - 1 < 0. \quad (\text{G.2})$$

Furthermore, we have that

$$\frac{dICB}{dR} = \psi [F(\tilde{S})\alpha + (1 - F(\tilde{S})) (1 + CB_0^*)]. \quad (\text{G.3})$$

Thus, we obtain that

$$\frac{dCB_0^*}{dR} = \frac{\psi [F(\tilde{S})\alpha + (1 - F(\tilde{S})) (1 + CB_0^*)]}{1 - \psi R (1 - F(\tilde{S}))} > 0, \quad (\text{G.4})$$

for all  $\alpha$ . As a result,  $\frac{dS^*}{dR} = 1 - \alpha + CB_0^* + R \frac{dCB_0^*}{dR}$ , so:

$$\frac{dS^*}{dR} = 1 - \alpha + CB_0^* + \frac{\alpha F(\tilde{S}) + (1 + CB_0^*)(1 - F(\tilde{S}))}{F(\tilde{S}) + \frac{1}{\psi R} - 1} > 0. \quad (\text{G.5})$$

This result allows us to calculate the effect of changes in  $R$  on the curve  $ID$ . We have:

$$\frac{dID}{dR} = f(S^*) \frac{dS^*}{dR} D^* (1 - \psi)(1 - \gamma) + \psi \frac{d\tilde{S}}{dR} [F(\tilde{S}) - F(S^*)] > 0. \quad (\text{G.6})$$

Also recall from the proof of Proposition 4 that  $\frac{dID}{dD} \Big|_{D^*(\alpha)} > 0$ . Thus, a marginal increase in asset profitability reduces the required face value of unsecured debt:

$$\frac{dD^*}{dR} \Big|_{D^*(\alpha)} = - \frac{dID/dR}{dID/dD^*} \Big|_{D^*(\alpha)} < 0. \quad (\text{G.7})$$

We now turn to the effect of changes in asset profitability on the banker's choice of asset encumbrance. First, we establish that

$$\frac{d^2 CB_0^*}{d\alpha dR} = \frac{d^2 CB_0^*}{dR d\alpha} = \frac{\frac{F(\tilde{S})}{\psi R^2} + f(\tilde{S}) \left( \frac{1}{\psi R} - 1 \right) \frac{d\tilde{S}}{dR}}{\left( F(\tilde{S}) + \frac{1}{\psi R} - 1 \right)^2} > 0. \quad (\text{G.8})$$

From this, we obtain that the effect of an increase in  $R$  on the IA is

$$\begin{aligned} \frac{dIA}{dR} &= F(S^*) \frac{d^2 CB_0^*}{dR d\alpha} + f'(S^*) \frac{dS^*}{dR} [(\kappa - 1)D + \alpha^*(1 - \psi)R] \left( \frac{dCB_0^*}{d\alpha} - 1 \right) \\ &\quad + f(S^*) \left[ \frac{dS^*}{dR} \left( \frac{dCB_0^*}{d\alpha} - \psi \right) + (1 - \psi)\alpha^* \left( \frac{dCB_0^*}{d\alpha} - 1 \right) + [(\kappa - 1)D + \alpha^*(1 - \psi)R] \frac{d^2 CB_0^*}{dR d\alpha} \right] \end{aligned} \quad (\text{G.9})$$

The terms that multiply  $f'(S^*)$  are non-negative. Inserting  $F(S^{**})$  from  $IA = 0$ , we note

that all terms are positive but those that multiply  $\alpha^*(1-\psi)f(S^*)$ . Therefore, a sufficient condition for the entire expression to be strictly positive is that the terms that multiply  $\alpha^*(1-\psi)f(S^*)$  are non-negative:

$$R(1-\psi)\frac{d^2CB_0^*}{d\alpha dR} \geq \left(1 - \frac{dCB_0^*}{d\alpha}\right)\left(\frac{dCB_0^*}{d\alpha} - \psi\right). \quad (\text{G.10})$$

Since  $f(\tilde{S})\left(\frac{1}{\psi R} - 1\right)\frac{d\tilde{S}}{dR} > 0$ , a sufficient condition is  $(1-\psi)F(\tilde{S}) + \psi\left(\frac{1}{\psi R} - 1\right)^2 \geq 0$  which always holds. As a result,  $\frac{dIA}{dR} > 0$ .

Taking both results together yields the stated affect on  $\alpha^*$  and, in general, an ambiguous effect on  $D^*$ .

## Appendix H. Proof of Proposition 8

We turn to the influence of the liquidation value  $\psi$  on the two curves,  $IA$  and  $ID$ , and thus the equilibrium outcome. First, for the effect on the curve  $ID$ , which determines  $D^*(\alpha)$ , a marginal increase in  $\psi$  has a positive impact on the required face value of unsecured debt:

$$\frac{dID}{d\psi} = Df(S^*)\frac{dS^*}{d\psi} + \psi \int_{S^*}^{\tilde{S}} (\tilde{S} - S^*) dF(S) - \psi \kappa Df(S^*)\frac{dS^*}{d\psi} + \int_{S^*}^{\tilde{S}} f(S)R \frac{dCB_0^*}{d\psi} dS > 0. \quad (\text{H.1})$$

As the liquidation value increases, short-term investors receive more in the event of bankruptcy and liquidation of the unencumbered assets. As a result, a lower face value of unsecured debt is required to break even in expectation, which we obtain by the implicit function theorem:

$$\frac{dD^*(\alpha)}{d\psi} = -\frac{dID/d\psi}{dID/dD^*} < 0, \quad (\text{H.2})$$

for each  $\alpha$ . Thus, a marginal increase in  $\psi$  leads to an inward shift in the short-term investors' schedule for the face value of unsecured debt.

Second, the effect of a marginal increase in  $\psi$  on the curve  $IA$ , which determines the

banker's optimal choice of asset encumbrance, is:

$$\begin{aligned}
\frac{dIA}{d\psi} &= -F(S^{**}) \\
&+ f(S^{**}) \frac{dS^*}{d\psi} \Big|_{\alpha=\alpha^*} \left( \frac{dCB_0^*}{d\alpha} \Big|_{\alpha=\alpha^*} - \psi \right) \\
&+ \frac{d^2CB_0^*}{d\alpha d\psi} \Big|_{\alpha=\alpha^*} (F(S^{**}) + f(S^{**}) [\alpha^* R(1-\psi) + D(\kappa-1)]) \\
&+ \left( \frac{dCB_0^*}{d\alpha} \Big|_{\alpha=\alpha^*} - 1 \right) f'(S^{**}) \frac{dS^*}{d\alpha} \Big|_{\alpha=\alpha^*} [\alpha^* R(1-\psi) + D(\kappa-1)] \\
&+ \left( \frac{dCB_0^*}{d\alpha} \Big|_{\alpha=\alpha^*} - 1 \right) f(S^{**}) \left[ -\frac{\gamma D}{\psi^2} - \alpha^* R \right]. \tag{H.3}
\end{aligned}$$

Each of the expressions on the second to fifth lines above are positive, while the term on the first line is negative.

Next, from the first-order condition,  $IA(\alpha^*, D) = 0$ , we can solve for

$$F(S^{**}) = \underbrace{\left( \frac{1 - \frac{dCB_0^*}{d\alpha} \Big|_{\alpha=\alpha^*}}{\frac{dCB_0^*}{d\alpha} \Big|_{\alpha=\alpha^*} - \psi} \right)}_{\beta > 0} f(S^{**}) [\alpha^* R(1-\psi) + D(\kappa-1)]. \tag{H.4}$$

If we substitute  $F(S^{**})$  into equation (H.3), and group the terms on the first and third lines, we obtain that  $dIA/d\psi > 0$  whenever

$$f(S^{**}) [\alpha^* R(1-\psi) + D(\kappa-1)] \left( -\beta + \frac{d^2CB_0^*}{d\alpha d\psi} \Big|_{\alpha=\alpha^*} \times [\beta + 1] \right) \geq 0. \tag{H.5}$$

Since  $f(S^{**}) \geq 0$  and  $[\alpha^* R(1-\psi) + D(\kappa-1)] > 0$ , it follows that the expression in the parenthesis must be non-negative. This condition can be simplified to

$$(1-\psi) \frac{d^2CB_0^*}{d\alpha d\psi} \Big|_{\alpha=\alpha^*} + \frac{dCB_0^*}{d\alpha} \Big|_{\alpha=\alpha^*} \geq 1. \tag{H.6}$$

The cross derivative of  $CB_0^*(\alpha^*)$  with respect to  $\alpha$  and  $\psi$  is

$$\begin{aligned} \left. \frac{d^2 CB_0^*}{d\alpha d\psi} \right|_{\alpha=\alpha^*} &= \frac{\left(\frac{1}{\psi R} - 1\right) f(\tilde{S}(\alpha^*)) \left. \frac{d\tilde{S}}{d\psi} \right|_{\alpha=\alpha^*} + \frac{F(\tilde{S}(\alpha^*))}{\psi^2 R}}{\left(\frac{1}{\psi R} - 1 + F(\tilde{S}(\alpha^*))\right)^2} \\ &= \underbrace{\frac{\left(\frac{1}{\psi R} - 1\right) f(\tilde{S}(\alpha^*)) \left. \frac{d\tilde{S}}{d\psi} \right|_{\alpha=\alpha^*}}{\left(\frac{1}{\psi R} - 1 + F(\tilde{S}(\alpha^*))\right)^2}}_{\xi > 0} + \underbrace{\frac{\frac{1}{\psi^2 R}}{\frac{1}{\psi R} - 1 + F(\tilde{S}(\alpha^*))}}_{\Xi > 1/\psi} \left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*}, \end{aligned} \quad (\text{H.7})$$

which is greater than  $dCB_0^*/d\alpha|_{\alpha=\alpha^*}$ . Substituting the above expression into our sufficient condition, it now reduces to

$$(1 - \psi) \left[ \xi + \Xi \left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*} \right] + \left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*} > 1. \quad (\text{H.8})$$

The left hand side of the above condition simplifies to

$$\begin{aligned} &(1 - \psi)\xi + [1 + \Xi(1 - \psi)] \left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*} \\ &\geq (1 - \psi)\xi + \frac{1}{\psi} \left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*}. \end{aligned}$$

Multiplying throughout by  $\psi$ , the sufficient condition yields

$$\psi(1 - \psi)\xi + \left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*} > \psi, \quad (\text{H.9})$$

which on reorganizing yields

$$\left. \frac{dCB_0^*}{d\alpha} \right|_{\alpha=\alpha^*} - \psi > -\psi(1 - \psi)\xi, \quad (\text{H.10})$$

which is always true since the left-hand side is always positive in equilibrium. In sum,

$$\frac{d\alpha^*(D)}{d\psi} > 0, \quad (\text{H.11})$$

for each  $D$ . Thus, a marginal increase in  $\psi$  leads to an outward shift of the optimal asset encumbrance schedule of the banker.