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Debt consolidation with long-term debt


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Debt consolidation with long-term debt

Alexander Scheer*

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Abstract

The Great Recession has sent debt levels to a post-WWII high for several advanced economies, reviving the discussion of fiscal consolidation. This paper assesses the macroeconomic implications of tax-based versus spending-based consolidation within the framework of a New Keynesian model with long term government debt. Three results stand out: First, tax-based consolidations are inflationary whereas spending-based ones are deflationary. Second, the net benefits of inflation increase in the average maturity of outstanding debt: inflation revalues debt more efficiently, while distortions due to price dispersion remain unaffected - the maturity effect. Third, as a result, tax-based consolidations can become superior to spending cuts if the average maturity is high enough. Quantitatively, the threshold is two years for US data in 2013. The previous mechanism illustrates the importance of inflation in the consolidation process, even if raising its target rate is considered not to be an option.

Keywords: Consolidation, Long term debt, Monetary policy, Fiscal policy, Inflation, New-Keynesian models

JEL-Codes: H63, E63

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1 Introduction

The Great Recession has sent debt levels to a post-WWII high for several advanced economies. As figure 1 and table 1 illustrate, debt to GDP ratios have increased from 2007 to 2013 by roughly 37 percentage points for a selection of 19 OECD countries. This increase was driven by a sharp reduction in GDP coupled with debt-financed discretionary fiscal stimulus and financial sector support, see IMF (2011). These elevated debt to GDP ratios have revived discussions on optimal debt levels and possible ways to reduce them. Although the literature does not agree on a specific level of debt (see the next section), it seems that the current ratios are not considered as optimal. A natural question arising is how to reduce that debt in the least distortionary way - that is when to reduce it and which instruments to use. Most of the literature focuses on the latter\(^1\) by studying a New Keynesian Model with one period debt. However, table 2 shows that the average maturity for most advanced economies is at least 4 years, usually 6.2 years (without Greece and UK) while for Greece\(^2\) it is 16 years and 14.7 for the UK.

In this paper I analyze the macroeconomic implications of permanently reducing the debt to GDP ratio within the framework of a New Keynesian Model with long term government debt. My contribution is to assess how the relative attractiveness of tax- vs. spending-based consolidation depends on the average maturity of outstanding debt. First, both fiscal adjustments have an effect on the inflation rate. Increasing the labor tax rate is inflationary as household will bargain for higher pretax wages which firms will partly accommodate by raising their prices, see Eggertsson (2011).\(^3\) Reducing public demand has a dampening effect on inflation as firms will lower prices to attract private demand. Second, the higher the average maturity of nominal debt the lower its real value for a given inflation rate, see for instance Aizenman and Marion (2011).\(^4\) Third, as a result, tax-hikes can become less disruptive than spending-cuts if the average maturity is high enough: the inflationary (deflationary) effect of tax-based (spending-based) consolidation reduces (increases) the real value of debt and thus the necessary fiscal adjustment needed.\(^5\)

\(^1\)There is yet little known about the optimal time for reducing debt levels, but the literature on fiscal stimulus and austerity in times of aggregate distress coupled with zero lower bound problems can be instructive.

\(^2\)The increase in maturity for Greece was due to the rescue packages. In 2011 the average maturity was 7 years.

\(^3\)An alternative tax instrument is the consumption tax rate (VAT). Feldstein (2002) has argued how credible VAT increases affect inflation expectations.

\(^4\)To be more precise, the reduction in the real value of government debt depends on the persistence of the (unexpected) inflation rate. I.e. if there is a one time only shock that leads to a price increase only in this one period than the real value of debt is c.p. similar for one period debt compared to a longer maturity debt.

\(^5\)Note that distortions from inflation are independent of the maturity, see Fischer and Modigliani (1978) or Ambler (2007)
Table 1: Debt between 2007 - 2013. Notes: Debt changes are in percentage points. Average is unweighted and for this subgroup only. Source: IMF (2013). General government gross debt; downloaded on 25.08.2014.

<table>
<thead>
<tr>
<th>Country</th>
<th>Debt Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greece</td>
<td>+67</td>
</tr>
<tr>
<td>Iceland</td>
<td>+62</td>
</tr>
<tr>
<td>Ireland</td>
<td>+98</td>
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<tr>
<td>Japan</td>
<td>+60</td>
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<td>Portugal</td>
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<td>Spain</td>
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<td>UK</td>
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<td>US</td>
<td>+41</td>
</tr>
<tr>
<td>ø</td>
<td>+61</td>
</tr>
</tbody>
</table>

Figure 1: Evolution of debt to GDP ratios. Source: International Monetary Fund, World Economic Outlook Database, October 2013, IMF (2013). General government net lending/borrowing; Percent of GDP; downloaded on 25.08.2014.
In order to analyze how the maturity of outstanding debt affects the relative attractiveness of fiscal consolidations I set up a New Keynesian Model with long term debt modeled à la Krause and Moyen (2013) and a target ratio of debt to GDP that is reduced permanently by 10%-points within 10 years. Fiscal policy is captured by simple feedback rules that increase (decrease) the tax rate (government spending) if the actual debt to GDP ratio is above its target rate.

First, I assess possibly positive welfare gains in the long run since lower debt levels imply more free resources to allocate for higher spending or lower tax rates. The welfare equivalent consumption variation (CV) is positive which indicates that households are better off with a lower debt level.

Second, for the transition towards the new steady state I calibrate the model to match US data in 2013. To compare the relative desirability of each debt reduction tool, I use two measures, the “fiscal sacrifice ratio” (FSR) that quantifies the output drop for a given debt reduction and the overall CV incorporating the transitional dynamics. The FSR is positive for both consolidation schemes, which shows that transitions are in general costly in terms of output, but the costs are a bit lower for the tax-based scenario. The CV is positive for tax hikes but negative for spending cuts which indicates that households prefer a tax-based
These results change if I consider only short term debt since spending cuts become much more preferable than tax hikes. The FSR is between 2 to 5 times lower when public expenditures are adjusted and the CV is 10 times smaller than for tax-hikes, although it is still negative. For intermediate values of maturities the CV is a monotonically increasing (decreasing) when consolidation is accomplished by tax (spending) adjustments. The FSR decreases for tax hikes the higher the maturity but stays relatively constant for spending cuts.

The present paper is closely related to Coenen et al. (2008) and Forni et al. (2010). The former use a two-country open-economy model of the euro area to evaluate the macroeconomic consequences of various fiscal consolidation schemes. They find positive long-run effects on output and consumption combined with considerable short-run adjustment costs and possibly distributional effects. The latter has a more detailed description of the public sector and shows that a 10 percentage point reduction of the debt to GDP ratio obtained by reducing expenditure and taxes can be welfare improving. Erceg and Lindé (2013) use a medium scaled two-country DSGE model to compare the effects of tax- vs. expenditure-based fiscal consolidation with different degrees of monetary policy accommodation. With an independent central bank, government spending cuts are less costly in reducing public debt than tax hikes since the latter reduces potential output through its distortionary nature, whereas spending cuts can be partly accommodated by a cut in the policy rate that crowds-in private demand. Hall and Sargent (2011) document that in the US after WWII, most of the debt was reduced by steady positive GDP growth rates. They use a detailed accounting scheme to assess the contribution of growth, primary surpluses and real interest rates on the debt level. As growth is not a direct policy option (at least in the short run) I focus only on changes of primary surpluses. I also do not consider direct default nor to inflate debt away as both instruments might entail tremendous costs. However, as the present analysis shows, even if raising the inflation target is not a direct policy tool it still matters whether fiscal adjustments are inflationary or deflationary.

The paper proceeds by a small de-tour of optimal debt levels followed by a description of the model and the solution technique. Section 4 examines the long-run benefits of a lower debt level and section 5 presents the short term dynamics. While section 6 offers some robustness...

If, on the other hand, only spending cuts are available, as is the case if prevailing tax rates are already revenue maximizing, the transition towards a lower debt level would be welfare detrimental. If, on top, the amount of debt consolidation increases, spending cuts become even more costly. That might explain partly the reluctance of some highly indebted countries to reduce their debt ratio. There is some evidence in Trabandt and Uhlig (2011) that, for instance, Italy is relatively close to the revenue maximizing labor tax rate. At the same time, Italy has not managed to reduce their debt level compared to other periphery countries.

See for instance Barro and Gordon (1983) or, more recently, Roubini (2011) on why inflation is neither desirable nor likely to reduce debt.
results, the final section concludes.

2 Optimal debt levels

There is quite an elaborate literature on optimal debt levels which in general shows debt to be either indeterminate, positive or negative. Barro (1979) shows in a simple framework that it is optimal to keep marginal tax rates constant to reduce distortions and that debt entails a unit root which soaks up part of the financing need. Aiyagari et al. (2002) formalized that approach in a Ramsey model, however, they find that debt optimally is negative to reduce distortions from taxes. In Aiyagari and McGrattan (1998) government debt increases the liquidity of agents in an incomplete markets setup and increases consumption smoothing and thus overall welfare. Von von Weizsaecker (2011) argues that government debt is a warranty, not a threat, for price stability as it raises the natural rate of interest which would have been negative due to demography. A number of researchers have brought attention towards possibly adverse effects of too much debt for the economy. First, higher debt levels might be harmful for growth as Reinhart and Rogoff (2010, 2013) have documented an inverse relationship between government debt and growth for higher levels of debt. Second, high debt levels might give rise to the existence of a “crisis zone”, in which the probability to default is determined by beliefs of the agents, as in Cole and Kehoe (2000) or Conesa and Kehoe (2012). This provides an incentive for the government to reduce it outstanding liabilities to exit that zone. Third, high debt levels may lead to inflation as shown by Sargent and Wallace (1981), Woodford (1995), Cochrane (1999) or Sims (2013). In those models inflation rises in equilibrium to reduce the real amount of government debt if the fiscal authority is constrained to adjust its real primary surpluses and thus does not provide the necessary fiscal backup.

3 Model set up

In this section I first describe the structural model before I continue to explain the solution method and the parameterization.

I use a closed economy New Keynesian Model with the extension of long term bonds as in Krause and Moyen (2013) augmented by fiscal policy rules. There are three agents in the economy: households that maximize their life time utility, firms that maximize profits and a government authority that sets distortionary labor tax rates and the level of public expenditures in order to keep the actual debt level close to some target rate. The household derives utility from consumption of a private and public good and from leisure. The asset

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*Reinhart et al. (2012) and Panizza and Presbitero (2013) provide a comprehensive survey of empirical research on the existence and significance of thresholds and the causality of the negative relationship.
market consists of a one period risk-free bond and a second market where long term bonds can be traded. An important feature of the latter debt market is that any long term bond matures stochastically. All households supply their labor services in a competitive labor market. On the production side there are two types of firms. The monopolistic competitive firms hire labor to produce intermediate goods and sell the goods to the final-good firm. They face nominal rigidities à la Calvo (1983) when setting their optimal price. The final-good firm uses the intermediate goods in a constant-elasticity-of-substitution (CES) production function to produce an aggregate good à la Dixit and Stiglitz (1977) that is sold to the households in a perfectly competitive market. The monetary authority follows a standard Taylor rule that reacts on deviations from inflation.

3.1 Long term bonds

A central innovation compared to previous studies is the use of an extended maturity structure for long term bonds where I follow Krause and Moyen (2013). Each unit of this outstanding debt pays an interest rate $i^L_t$ and matures next period with probability $\gamma$ in which case it also pays back the principal. With probability $1 - \gamma$ the bond survives until the next period. It is easily shown that the average maturity is thus captured by $\frac{1}{\gamma}$. The long term average interest rate $i^L_t$ will be a weighted sum of previously set long term interest rates on newly issued long term debt $i_{L,n}^t$. As the household holds a representative portfolio of long term bonds, a fraction $\gamma$ matures each period. Therefore, $\gamma$ determines not only the average maturity but also the amount of bonds maturing every period, see the discussion below. Every period the household can buy a newly issued long term nominal bond denoted by $B_{L,n}^t$. The interest rate on this bond, $i^L_{t,n}$, is going to be priced according to a no arbitrage condition stemming from the households first order conditions.

Since every period a fraction $\gamma$ matures the stock of long term bonds evolves as

$$B_{L}^t = (1 - \gamma)B_{L}^{t-1} + B_{L,n}^t$$

(1)

The average interest expenses of the portfolio $i^L_t B_{L}^t$ can be written recursively as well, namely

$$i^L_t B_{L}^t = (1 - \gamma)i^L_{t-1} B_{L,n}^{t-1} + i^L_{t,n} B_{L,n}^n$$

(2)

The advantage of that modeling approach relative to alternatives is that the steady state tax rate is independent of the maturity whereas it would depend for instance when using

\footnote{For a complete description see Krause and Moyen (2013)}
3.2 Households

Households maximize their life time utility given by

$$E_0 \sum_{t=0}^{\infty} \beta t \left\{ \frac{C_1^{1-\sigma}}{1-\sigma} + \frac{G_1^{1-\sigma_g}}{1-\sigma_g} - \frac{\chi_1^{N_1+\phi}}{1+\phi} \right\}$$

subject to

$$P_t C_t + B_t + B_t^{L,n} \leq P_t W_t N_t (1-\tau_t) + (1 + i_{t-1}) B_{t-1} + (\gamma + i_{t-1}^{L}) B_{t-1}^{L} + P_t D_t$$

They earn after tax wage income, the returns from the short and long term bonds and dividends from firm ownerships and use its income for private consumption and to buy new short and long term bonds. Denote with $\lambda_t$ the Lagrange multiplier attached to the budget constraint while $\mu_t$ is the multiplier of the average interest payments for the representative portfolio (equation 2) after the amount of newly issued long term bonds from equation 1 have been substituted in.\(^{11}\) The representative household maximizes its life time utility by choosing $C_t, B_t, B_t^{L}, i_t^{L}$ and $N_t$. Note that the interest rate on newly issued long term debt $i_t^{L,n}$ is taken as given, similar to the short term interest rate. However, the average interest rate $i_t^{L}$ depends on the composition of newly issued and outstanding bonds and can thus be chosen indirectly by the household.

The first order conditions for the short term bond holdings yield the familiar Euler equation:

$$\lambda_t = C_t^{-\sigma}$$

$$C_t^{-\sigma} = \beta E_t \left\{ \frac{1 + i_t}{1 + \pi_{t+1}} C_{t+1}^{-\sigma} \right\}$$

One can show that the optimality conditions for the long term bond have to satisfy

$$1 = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{1 + \pi_{t+1}} \left[ 1 + i_t^{L,n} - \mu_{t+1}(1-\gamma) (i_t^{L,n} - i_t^{L,n}) \right] \right\}$$

while $\mu_t$ is the Lagrange multiplier attached to the interest payments which evolves according to

$$\mu_t = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{1 + \pi_{t+1}} (1 + (1-\gamma) \mu_{t+1}) \right\}.$$

---

\(^{10}\)Alternative specifications can be found in Faraglia et al. (2013), Chatterjee and Eyigungor (2012) or Hatchondo and Martinez (2009). In Woodford (2001) any bond $b_t$ is bought at $q_t$ and lasts forever with an exponentially decaying coupon payment of factor $\rho$. In steady state the price of the bond $q_t$ depends on the maturity and thus affects the steady state tax rate.

\(^{11}\)To arrive at the expression one has to scale $\mu_t$ by $\lambda_t$. 
Note that in steady state the multiplier is \( \mu = \frac{1}{i + \gamma} \) which is the pricing function for a one-period bond if \( \gamma = 1 \) and a consol if \( \gamma = 0 \). Therefore, one can interpret \( \mu \) as the price of the stochastic bond. As can be seen from equation 6 the price is higher than for short-term debt.

In case of \( \gamma = 1 \) equation 5 implies \( i_{t+1}^{L,n} = i_t \) and the second Euler equation collapses to the first one. The two Euler equations 4 and 5 constitute the no arbitrage condition for investing in the short and long term bond. The right hand sight of 5 is the expected payoff of the long term debt valued by the stochastic discount factor. It consists of two parts, the first, \( 1 + i_t^{L,n} \) is the return if the bond would mature next period. The second, \( -\mu_{t+1}(1 - \gamma) (i_{t+1}^{L,n} - i_t^{L,n}) \), can be interpreted as the capital loss (gain) that arises from a rise (fall) in the newly issued long term rate. The no arbitrage condition implies that once the household expects a rise of the interest rate for long term bonds, i.e. \( i_{t+1}^{L,n} > i_t^{L,n} \), he asks for a premium with a higher interest rate \( i_t^{L,n} \) to compensate the investment in a long term bond today as it ties resources for several periods. The household thus takes into account the direct return plus the opportunity costs of having resources fixed in a long term contract. The remaining FOC yields the labor supply

\[
W_t(1 - \tau_t) = \chi_n N_t^\phi C_t^\sigma. \tag{7}
\]

### 3.3 Firms

The final good firm uses intermediate goods from the monopolistic competitive firm and produces a final good with a CES production function. Its demand for each intermediate good \( j \) is given by

\[
y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{- \epsilon} y_t^d
\]

while \( y_t^d \) is the household demand for a final good.

Each intermediate good firm produces its good \( y_t(j) \) according to \( y_t(j) = A_t N_t(j) \) while \( N_t(j) \) is the amount of labor and \( A_t \) aggregate technology. As the production function exhibits constant returns to scale marginal costs are independent of the level of production and equal to

\[
mc_t = \frac{W_t}{A_t}. \tag{8}
\]

Each firm sets a profit maximizing prize subject to Calvo (1983) nominal friction. The FOC of the firm can be cast into the following recursive forms:

\[
g_t^1 = \lambda_t mc_t y_t^d + \beta \theta E_t \left\{ g_{t+1}^1 \right\} \tag{9}
\]

\[
g_t^2 = \lambda_t y_t^d + \beta \theta E_t \left\{ g_{t+1}^2 \right\} \tag{10}
\]
while the optimal price is equal to

\[
\frac{P^*_t}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{q_t^1}{g_t^1}.
\]

The price index evolves according to

\[
1 = \theta(1 + \pi_t)^{t-1} + (1 - \theta) \left( \frac{P^*_t}{P_t} \right)^{1-\epsilon}.
\]

### 3.4 Government

Fiscal policy is captured by simple feedback rules that increase (decrease) the tax rate (government spending) if the actual debt to GDP ratio is above some target ratio \(\eta^B_t\). The latter will be reduced exogenously to a lower value \(\eta^{new}_B < \eta^{old}_B\) which summarizes the desire to reduce debt levels permanently. Furthermore, it seems plausible that policymakers plan to reduce the target ratio gradually to avoid potentially large adverse consequences on output.

To capture this gradualism I follow Coenen et al. (2008) and use the following law of motion:

\[
\eta^B_t = (1 - \rho_b)\eta^{new}_B + \rho_b \eta^{t-1}_B
\]

where \(\rho_b\) is chosen such that the debt to GDP target converges to its new level of \(\eta^{new}_B\) after approximately 40 quarters. The government budget constraint is given by

\[
B_t + B^{L,n}_t + P_t \tau_t W_t N_t = P_t G_t + (1 + i_{t-1})B_{t-1} + (\gamma + i^L_t)B^{L}_{t-1}
\]

which states that the government finances its public expenditures and interest payments with labor taxes or the issuance of new debt.

#### 3.4.1 Tax consolidation

If consolidation is achieved by increases in the tax rate, the fiscal feedback rule is given by

\[
\tau_t - \tau^{new} = \phi_\tau \left( \frac{B^L_t + B_t}{P_t} - 4Y^{new} \eta^B_t \right).
\]

Note that \(Y^{new}\) is the steady state output level after consolidation has taken place such that in steady state \(\tau = \tau^{new}\) is consistent with a debt ratio of \(\eta^{new}_B\). The parameter \(\phi_\tau\) captures the pace of adjustment. The larger its value the closer the actual debt ratio to the target.

As will be explained in detail in the next section, the new steady state implies a different optimal amount of public expenditures. This enhances transparency with respect to the instruments used as the initial and the end steady state are similar. However, I have to make the additional assumption on how government spending will move towards its new steady
state value. I chose a similar law of motion as for the evolution of the debt ratio target \( \eta_t^B \), namely:

\[
G_t = (1 - \rho_g) G_{\text{new}} + \rho_g G_{t-1}
\]

while \( \rho_g \) is chosen such that \( G_t \) converges after 40 quarters to \( G_{\text{new}} \).\(^{12}\)

### 3.4.2 Government spending

If the consolidation is achieved through a reduction in government expenditures, the spending path evolves according to

\[
G_t - G = \phi_g \left( \frac{B_t^L + B_t}{P_t} - 4Y_{\text{new}} \eta_t^B \right)
\]

(15b)

Tax rates will evolve towards their new steady state value by

\[
\tau_t = (1 - \rho_\tau) \tau_{\text{new}} + \rho_\tau \tau_{t-1}
\]

(16b)

### 3.4.3 Monetary policy rate

Monetary policy is set according to a standard Taylor-rule:

\[
\frac{1 + i_t}{1 + i} = \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\phi_\pi}
\]

(17)

### 3.5 Aggregation and exogenous rules

Finally, the goods market must clear such that

\[
Y_t \Delta_t = A_t N_t
\]

(18)

with

\[
\Delta_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{d}{dt}
\]

and by the Calvo-property

\[
\Delta_t = \theta \Delta_{t-1}(1 + \pi_t)^\epsilon + (1 - \theta) \left( \frac{P_t^*}{P_t} \right)^{-\epsilon}
\]

(19)

The aggregate resource constraint is

\[
Y_t = C_t + G_t
\]

(20)

and aggregate technology evolves according to

\[
\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t^a.
\]

\(^{12}\)I provide robustness results for different transitional specifications of exogenous transition. Overall the results are robust to linear, front loading or back-loading adjustments or when government spending is fixed. The exact adjustment graphs and the corresponding consumption variations are available upon request.
3.6 Model calibration and solution technique

Equations 1 to 21 describe the non-linear model economy. To analyze the transition towards the new steady state I use the perfect foresight solver in Dynare. As this paper determines how the maturity structure affects the macroeconomic implications of fiscal consolidation I set the amount of short term debt $B_t = B = 0$ and thus abstract from any portfolio decision taken by the government. I calibrate the model to the US economy in 2013. The model starts with an initial debt to GDP level of 100% and a debt target of $\eta_{\text{new}} = 90\%$. The simple fiscal feedback rules will lead to an endogenous adjustment until the new steady state is reached.

The model is parameterized at quarterly frequency. The time preference rate $\beta$ is chosen to match an average annual real return of 4%. The inter-temporal elasticity of substitution of private and public goods $\sigma_c, \sigma_g$ as well as the inverse Frish elasticity $\frac{1}{\phi}$ are both set to 1. In the economy there will be a steady state mark up of 20% and the average adjustment of nominal prices will take one year, so $\epsilon = 6$ and $\theta = 0.75$. The policy parameters for the Taylor rule are standard values that satisfy the Taylor principle with $\phi_{\pi} = 1.5$. The adjustment parameters on the fiscal feedback rules were chosen such that the actual debt level will be reduced by 10%—points within 40 quarters. $\gamma$ is equal to 0.055 to match the average maturity of US debt in 2011 of 55 months.

However, that parameter also captures the average amount of debt that matures within one quarter. Figure 3 depicts how that calibration fits the US data quite well. I set government spending equal to 20% of GDP, roughly the average of post WWII levels. The weighting parameter on labor and on government spending are chosen such that with the current level of debt (100%) it would be optimal to spend 20% of GDP on public goods and to work $N = \frac{1}{3}$ hours.

4 Long run implications

With a lower debt to GDP ratio the government can allocate more resources to either public consumption or to reduce the tax rate as the interest payments on the outstanding debt stock are lower. The question is, how these proceeds should be used. The approach usually taken in the literature, as for example in Coenen et al. (2008) or Forni et al. (2010), is to assume

$^{13}$Krause and Moyen (2013) set the real level of debt $\frac{b_t}{P_t} = b_t = b$. However, as there is no steady state inflation in my specification both specifications yield similar results. A complementary approach would be to choose a constant proportion of short relative to long term bonds.

$^{14}$I chose 2011 because I have not been able to get newer data on the amount that matures within the next 10 years so I took the data from Bohn (2011).
Figure 3: Fraction of debt maturing within next 10 years. The dashed line depicts the amount of debt in the US as of June 2011 that matures in years 2012 to 2021. The solid line is the corresponding fraction implied by the model.
Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td><strong>Preferences</strong></td>
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<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Time discount factor</td>
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<tr>
<td>$\sigma_c$</td>
<td>1</td>
<td>Inter-temporal elasticity of substitution private consumption, implies log-utility</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>1</td>
<td>Inter-temporal elasticity of substitution private consumption, implies log-utility</td>
</tr>
<tr>
<td>$1 / \phi$</td>
<td>1</td>
<td>Inverse of the Frish of labor supply</td>
</tr>
<tr>
<td>$\chi_n$</td>
<td>6.67</td>
<td>Weighting parameter of dis-utility of work, targets $N^{old} = \frac{1}{3}$</td>
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<tr>
<td>$\chi_g$</td>
<td>0.2732</td>
<td>Together with $\sigma_g = 1$ implies that optimal to spend 20% of output on public goods</td>
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<tr>
<td><strong>Firms</strong></td>
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<tr>
<td>$\epsilon$</td>
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<td>Price markup of 20%</td>
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<td>$\theta$</td>
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<td>One year price contracts</td>
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<td><strong>Monetary policy</strong></td>
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<tr>
<td>$\phi_\pi$</td>
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<td>Response of interest rate to inflation</td>
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<td><strong>Fiscal policy</strong></td>
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<td>$\phi_r$</td>
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<td>Ensures that debt follows target</td>
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<tr>
<td>$\phi_g$</td>
<td>-0.5</td>
<td>Ensures that debt follows target</td>
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<td>$\rho_b$</td>
<td>0.84</td>
<td>Autocorrelation of debt target</td>
</tr>
<tr>
<td><strong>Long term bonds</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.055</td>
<td>Implies maturity of 4.5 years</td>
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that each fiscal consolidation will imply a different steady state, which will depend on the fiscal strategy in place. More precisely, if government spending (the tax rates) was reduced (increased) during the transition, then all the proceeds would be used to increase government spending (reduce tax rates) in the long-run. This will have a feedback effect on the household behavior, as for instance lower taxes increase the incentive to work.

A disadvantage of such an approach is that comparisons between different fiscal measures might be driven by comparing not the same steady state. As an alternative, I will determine the optimal composition of tax rates, government expenditure and private consumption that maximizes the households welfare for a given (lower) debt level. I thus assume that, for reasons outside of the model, the government decides not only to reduce debt levels but also to converge to a new steady state in which this debt level implies an optimal allocation of the other aggregate variables. The remaining task is then to assess which instrument to use in order to transit from steady state A to steady state B. However, the drawback is to choose a path for the other instrument that is not used for consolidation, although the results do not depend on specific functional forms of the other instrument. I also checked the approach taken by the literature and results did not change in an important way.

To get the optimal allocation of variables for a given debt amount I thus set up a Lagrangian that maximizes the households welfare function given the constraints 4, 5, 7, 14, 18 and 20. As this is a long-run perspective only equations 7, 14, 18 and 20 bind. One can show that it boils down to the following Lagrangian:

\[
L(N, \tau, G; \gamma^1, \gamma^2) = u(N - G) - v(N) + g(G) + \\
\gamma^1 \left[ MC(1 - \tau) - \chi_n N^\phi (N - G)^\eta \right]_{\text{labor-leisure}} + \gamma^2 \left[ MC \tau N - G - i4N \eta_{new} \right]_{\text{gov. budget constraint}}
\]

Maximization leads to 4 equations and 4 unknown \((N, \tau, G, \gamma^2)\) that can only be solved numerically. Table 3 shows the results of private and public consumption, hours worked and the tax rate for debt to GDP ratios of 100% to 90% and 80%. Additionally it shows the allocation at the optimal level of debt. The percentage change is relative to the initial debt level of 100% except for tax rates where the percentage point change is used.

The additional funds from lower debt repayments are used to reduce distortionary tax rates to increase public good provision. As a result of two effects the households will provide more labor: First, lower tax rates increase the incentive to work by reducing the intra-temporal labor-leisure distortions. Second, the increase in permanent government consumption constitutes c.p. a negative wealth effect that induces the agent to work more as in Coenen et al.
<table>
<thead>
<tr>
<th>Priv. cons.</th>
<th>Hours</th>
<th>Public good</th>
<th>Tax rate</th>
<th>CV</th>
</tr>
</thead>
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<td>100% debt to GDP</td>
<td>0.2667</td>
<td>0.3333</td>
<td>0.0667</td>
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<td>90% debt to GDP</td>
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<td>0.3344</td>
<td>0.6700</td>
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<tr>
<td>80% debt to GDP</td>
<td>0.2681</td>
<td>0.3355</td>
<td>0.0674</td>
<td>0.2799</td>
</tr>
<tr>
<td>First best</td>
<td>0.3431</td>
<td>0.4369</td>
<td>0.0938</td>
<td>-0.200</td>
</tr>
<tr>
<td>-943% debt to GDP</td>
<td>28.677%</td>
<td>31.068%</td>
<td>40.6321%</td>
<td>-48.885pp</td>
</tr>
</tbody>
</table>

Table 3: Steady state comparison for different debt levels. CV is the welfare equivalent consumption variation.

(2008). While in their analysis private consumption is crowded out in my setting it rises due to the simultaneous lowering of distortionary tax rates. The table also reports the welfare equivalent consumption variation (CV) that is required every period to make the household in the initial steady state as well off as in the new one.\textsuperscript{15} Households demand 0.1801\% of permanent consumption such that they do not want to have lower debt levels. With a 20 percentage points reduction it is 0.3577\% and one can show that the linear relation persists.\textsuperscript{16} The qualitative result are robust to different CRRA-parameters in the utility function for private or public consumption and for different mark-ups and Frisch elasticities.

## 5 Transition dynamics

While the previous analysis has shown potential welfare gains from lower debt levels in the long-run, this section sheds some light on the costs during the consolidation and whether an equilibrium with lower debt levels is preferable relative to the status quo if the transitional costs are taken into account. I will first present each consolidation separately, compare them and then show the importance of a maturity-structure above one period.

\textsuperscript{15}More precisely, \( V((1 + \zeta)C^{old}, N^{old}, G^{old}) = \sum_{t=0}^{\infty} \beta^t (u((1 + \zeta)C^{old}) + g(G^{old})) = \frac{1}{\beta} (u((1 + \zeta)C^{old}) - v(N^{old}) + g(G^{old})) \equiv \frac{1}{\beta} (u(C^{new}) - v(N^{new}) + g(G^{new})) = V(C^{new}, N^{new}, G^{new}). \) \( \zeta > 0 \) implies that the household asks for a compensation to be indifferent between both states.

\textsuperscript{16}See 4, especially the left side is very linear.
Figure 4: Welfare equivalent consumption variation for different debt to GDP ratios.
5.1 Fiscal consolidation

Figure 5: Tax-based consolidation. Notes: All variables are in percentage deviation from steady state, except tax rates, inflation and interest rates which are in percentage point deviations.

Figure 5 shows the macroeconomic effects for a tax-based consolidation. The fall in the debt target induces the labor tax rate to rise until period 6 and then to gradually convert back until its new (lower) value $\tau_{\text{new}}$. As a result of higher distortionary labor taxes the incentive to supply labor is reduced, leading the economy into a recession. As the previous section has shown government spending will be higher in the new long run equilibrium. Independent of how the transition will be accomplished exactly, the increase in government spending constitutes a negative wealth effect and lowers private consumption while increasing the supply of labor. This effect lowers the output drop, for instance compared to Coenen et al. (2008), to about 2%. The recession lasts for 5 to 6 years until labor supply recovers and converges to its higher long term equilibrium. As taxes reduce c.p. the after tax wage income, households bargain for higher pretax wages which increases the marginal costs for firms. Those firms that can adjust will charge higher prices which leads to inflation. The monetary authority follows the Taylor principle and raises the short term rate more than one-to-one, driving up the real short term interest rates. As can be seen from equation 4 to 3.2, and which is shown in Krause and Moyen (2013), the expected future path of the policy
rate $i_t$ determines the interest rate on newly issued bonds $i_{t,n}^{L,n}$ which determine the average payment of the portfolio $i_t^L$. As the model is solved by perfect foresight agents foresee that the policy rate will be lower in the future and thus ask for a higher interest rate $i_{t,n}^{L,n}$ but lower than $i_t$. Since the average interest rate is a weighted sum of its previous value $i_{t-1}^L$ and $i_{t,n}^{L,n}$ it reacts only sluggishly by a mere 0.4% compared to 2.5% of the short term interest rate. Therefore, even as the policy rate is raised according to the Taylor principle, the interest rate charged on newly issued bonds is raised less as it takes into account a smaller policy rate in the future. Additionally the increased payments $i_{t,n}^{L,n}b_{t,n}$ do not affect the budget that much as it is a relatively small fraction compared to the outstanding stock. Taking into account that inflation is higher than the nominal average long term rate the real average long term rate is reduced and therefore also the real financing needs for a given debt level.\footnote{After a while inflation is below the average long term rate thus increasing financing costs of the outstanding debt stock. However, as the total stock is reduced that effect is only of minor importance.} Additionally, the real value of nominal debt is reduced and thus dampens the financing needs to achieve the target. This mechanism will be important (and taken up again) when compared to a world with short term debt.

Figure 6: Spending-based consolidation. Notes: All variables are in percentage deviation from steady state, except tax rates, inflation and interest rates which are in percentage point deviations.
Figure 6 depicts the same variables for the spending-based scenario. Public good provision will be reduced up to 18% in period 6 and recovers gradually until it reaches its higher long run level $g^{\text{new}}$. The prospect of higher government expenditure coupled with lower tax rates leads to an expansion in labor in the short run. On the other hand, as the government cuts spending during the transition, agents experience a positive wealth effect that increases private consumption and lowers the incentive to work. This short run positive wealth effect is more pronounced and thus drives the economy into a recession. However, compared to Coenen et al. (2008), spending reductions are associated with relatively lower recessions. The importance of future composition of variables on current dynamics is shown, for instance, by Cogan et al. (2013). The inflation rate rises for the first year as a result of higher production but then drops and stays below 0 before it converges slowly back to its steady state value. The cut in government spending reduces production and inflation as firms compete for private demand and thus lower prices. As households supply less labor part of the deflationary pressure is dampened. Similar to the above argument, the short term interest rate follows the inflation pattern due to the Taylor rule but the interest rates on newly issued bonds take into account the whole future path of the policy rates and thus are lowered. This reduces the average interest rate but only to a small degree as the lower rate is paid for a small fraction while the rest still pays the higher rate from steady state. In the periods with positive inflation the real average long term rate is lowered even more which eases to keep real debt close to the target. However, once inflation falls below 0, i.e. there is deflation, that first increases real average long term rates and second, also the real value of debt, thus making the adjustment process more severe. That is the reason why spending has to be cut by that large amount.

5.2 Comparing the fiscal consolidations

Both consolidation strategies lead to the same long run equilibrium, but both entailed different adjustment costs (lower government or private consumption). I use two metrics to compare the relative desirability/associated costs. The first is the “fiscal sacrifice ratio”, a measure, that relates the output loss to the percentage point reduction of debt. For a smoother comparison I use the average output drop rather the exact drop within that period. More precisely, the ratio is defined as $\zeta_T = \frac{1}{T} \sum_{t=1}^{T} \frac{Y_t - \bar{Y}^{\text{old}}}{\bar{Y}^{\text{old}}}$, with $\bar{B}$ denoting total nominal debt. Figure 5 presents the ratio at a two, three

\footnote{In the specification of government spending transition used, there is an initial jump in output in the first period followed by a recession. The reduced long term interest rate leads to a growing demand for private consumption but public goods have not been reduced a lot in the first period. Therefore, labor supply has to increases to satisfy the demand. One can show that with a steeper decline in government spending in the first period there is always a recession so the boom depends on the specification used.}
Figure 5: Fiscal sacrifice ratio $\zeta_T$. Notes: The fiscal sacrifice ratio is calculated as

$$\zeta_T = \frac{1}{T} \sum_{t=1}^{T} \frac{Y_t - Y_{old}}{Y_{old}} / \frac{\bar{B}_t}{\bar{B}_{old}}.$$

and four year horizon. Within two years, both fiscal consolidations reduce the debt/GDP ratio by about 3.5% points while output falls on average about 1%, consistent with a “fiscal sacrifice ratio” of around 1/3. Increasing the time horizon reduces the sacrifice ratio as growth will catch up to its new long run level. Over the whole time span tax-based consolidation is associated with a lower FSR ratio than spending-based adjustments.

A second approach is to evaluate the welfare equivalent consumption variation (CV) associated with each reduction scenario, that is the permanent amount of consumption that makes the household indifferent between staying at the steady state with higher debt and moving to the

\textsuperscript{19}In the first year, both sacrifice ratios are negative, however, spending cuts are preferable as output growth reduces the debt ratio while tax hikes lead to a recession with an initial increase in debt. A quantitative comparison is thus difficult.
lower debt steady state. It is defined as

$$\sum_{t=0}^{\infty} \beta^t (U((1 + \nu)C^{\text{old}}) - v(N^{\text{old}}) + g(G^{\text{old}})) = \sum_{t=0}^{\infty} \beta^t (U(C_t) - v(N_t) + g(G_t)).$$

Positive values imply that agents want to stay at the old steady state only if they get additional permanent consumption and thus prefer the lower debt steady state. The corresponding values are -0.0639 for spending cuts and 0.0422 for tax hikes.

Therefore households are better off to consolidate only when the adjustment is tax-based. They, however, want to stay at the status quo if spending has to be adjusted. This is in line with the previous result of the FSR. A positive and higher CV for tax hikes compared to spending cuts is in contrast to Forni et al. (2010) and depends on the introduction of long term debt, as the next section will make clear.\(^{20}\)

5.3 Comparison to one period debt

![Graphs showing the comparison to one period debt](image)

Figure 8: Tax-based consolidation with short-term debt. Notes: All variables are in percentage deviation from steady state, except tax rates, inflation and interest rates which are in percentage point deviations.

\(^{20}\)See, for instance, Table 8 within their paper.
How important are the channels through which long term debt affect the macroeconomic implications of debt consolidation? To shed more light on this issue I will first evaluate both adjustment when only short term debt is available and then for intermediate values of maturity.

The current framework nests the one period debt model when setting $\gamma = 1$. Figures 8 and 9 depict the dynamics for the same variables as above. As one can see the transition of both fiscal consolidations is similar qualitatively, however, the magnitudes differ:

![Graphs showing dynamics of various macroeconomic variables](image)

Figure 9: Spending-based consolidation with short-term debt. Notes: All variables are in percentage deviation from steady state, except tax rates, inflation and interest rates which are in percentage point deviations.

Tax-based consolidation is associated with an enlarged fiscal distress. The labor tax rate roughly doubles to 5% points, output drops by more than 3% and the recession is more persistent. Inflation rises after an initial drop to as high as 2.5%, however, since all debt is reissued, the only surprise change in real terms is in the first period. All other price changes are already expected and priced into the interest rate. Positive inflation leads to an increase in the short term nominal and real interest rates which affects the budget more pronounced,

\[ \text{First, strong changes in short term rates do not affect the budget position by a lot as only part of the debt is reissued and those rates depend also on future policy rates. Second, inflation/deflation affects the real financing costs and, third, the real value of nominal debt more in case of long-term debt.} \]
increasing the necessary adjustment which explains the prolonged tax rise and the longer time to consolidate. In the spending-based adjustments, public goods still have to be cut by roughly 18% but recover much more quickly. As a result of the steeper drop there is deflation over the whole consolidation period. However, as the central bank cuts its policy rate that deflation results in a lower real interest rate which stimulates private demand and reduces the necessary fiscal adjustment as all of the debt is reissued and pays a lower real amount. The drop in output is more severe, however, it is not so persistent as with long-term debt. Overall, the aggregate variables do not move that much compared to the tax-based consolidation. Figure 10 illustrates the different sacrifice ratios for the model with short term debt. Spending cuts become much more preferable than tax hikes, since in the latter the output drop was much more severe. The FSR is between 2 to 5 times lower when public expenditures are adjusted and the CV is with -0.011% compared to -0.13% still negative but much lower.
Figure 11: Fiscal Sacrifice Ratio and CV evaluated at different maturities for tax- and spending-based consolidation.

The difference for the results lies in the way inflation helps to mitigate fiscal consequences. First, with short term debt only, all outstanding liabilities have to be rolled over so the increase in the real financing costs directly affect the budget by a high margin. On the other hand the increase in the price level reduces the real value of debt and thus the financing costs. Overall, the effect is ambiguous ex ante. With long term debt contracts the government has to issue only part of the outstanding debt stock. Therefore, an increase in real rates will raise total debt servicing costs by less. Second, inflation becomes much more pronounced in reducing the real value of debt. Both effects unambiguously reduce total financing costs relative to one-period debt. This is the key mechanism why in a model with long term debt contracts, tax hikes are less disruptive than spending cuts: they are associated with higher inflation. To be quantitatively important in one period debt models inflation would need to be much larger.

To illustrate how different levels of average maturity change the result, figure 11 summarizes the sacrifice ratio and welfare equivalent consumption variation for both spending cuts and tax hikes for various maturities. In general, for higher average maturities the CV is increasing (decreasing) when consolidation is tax-(spending-)based. The FSR is decreasing for tax hikes but stays relatively constant for spending cuts across different maturities. The threshold after which tax-based consolidation is preferable is 2 years for the CV or 4 years for the FSR. Additionally, for maturities above 3 years households prefer debt consolidation when done via higher taxes. Independent of the maturity, spending-based consolidation is never preferable.

\[22\] In case of a consol with \(\gamma = 0\) the government only pays predetermined interest rates while the stock is rolled over. Changes in the real interest rates would thus not materialize at all.
6 Robustness

TO BE COMPLETED

Insert here robustness results with respect to parameter values, debt amount to be reduced and different time horizon to reduce given debt level. Different monetary policy accomodations?

7 Conclusion

The Great Recession has sent debt levels to a post-WWII high for several advanced economies. It seems to be a consensus that these elevated debt levels have to be reduced at some point - the question remains is when and which instruments to use. In this paper I try to shed some light on the latter and how the relative attractiveness of fiscal consolidation schemes depend on the average maturity of outstanding debt. The main difference between short and long term debt is the inflationary consequence of the fiscal consolidation and more precisely how inflation affects real values: First, any change in the financing costs of bonds transmits only partly on the budget as just a small fraction of bonds is reissued. Second, inflation reduces the real value of debt in proportion to its average maturity. Lower real debt levels mitigate the extent to which other fiscal measures have to be raised. This inflationary impact is of minor importance in one period debt models but it is the key mechanism why in a model with long term debt contracts, tax hikes can become less disruptive than spending cuts if the average maturity is above 2 years: tax hikes are inflationary whereas spending cuts deflationary.

The present analyses clarifies the important role of inflation in the consolidation process, even though raising its target rate directly might either not be desirable because of commitment inconsistencies as in Barro and Gordon (1983) or not feasible as in the case of a small country within a currency union. Therefore, when assessing the relative attractiveness of various debt reduction tools it might be important to also consider their inflationary impact.

References


