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The quality of child care:  
A signaling game with incomplete information

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**Abstract**

This paper analyzes the effects of asymmetric information between parents and child care centers about the quality of child care. In a dynamic game of incomplete information the child care center sends a signal about its child care quality. The parents cannot observe the true quality. By updating the information the parents decide whether they enforce high quality. We consider policy measures to increase the ex-post probability of high quality. Some measures turn out to have negative effects on ex-post quality. Furthermore, we determine the welfare in the perfect Bayesian equilibria and the welfare-maximizing effort to increase the probability of high quality.

*JEL classification:* C73, J13

*Keywords:* Quality of child care, incomplete information, signaling game

# 1 Introduction

A high quality of child care is important for the development of a child. It has been shown that in general it is positively associated with children's well-being (Waldfogel 2002; Love et al. 1996) and it improves the academic outcomes of children (Ramey and Campbell 1991). Furthermore, high-quality child care has positive externalities for a society. For example, by helping to improve the education and labor market opportunities it generates individuals who are less dependent on welfare programs and less prone to crime. There is evidence, however, that on average child care centers do not provide an appropriate level of quality (Whitebook et al.1989; Mocan et al. 1995). This paper addresses one reason for this underprovision: because parents are unable to evaluate or monitor adequately the quality of services their children receive.

The information on the quality of child care services is typically not the same for parents and the child care centers (e.g. kindergarten) which offer those services. Although the children as direct consumers should know in principle how they are treated in the facility there are at least two reasons why the information about the quality of the services is not perfectly transmitted to the parents. First the children may be simply too young to communicate unambiguously the daily events in the child care center. Second children may have a different view on quality of child care than parents. They may consider some events or treatments as good care (e.g. watching television all the day) whereas the parents have other notions of reasonable care (e.g. learning). Hence, children may deliberately mislead the parents in order not to risk changes of the child care. This may play in the hands of the child care center which at given fees prefers to have low costs in providing the services (e.g. television may be cheaper than trained kindergarten teacher). And if there is adverse selection the child care centers could not even charge higher fees. Due to information asymmetry parents would not be willing to pay higher fees

This asymmetric information allows child care institutions to conceal the true quality of their child care. They may pretend a good quality although high costs of providing it make low quality preferable for the them. Then parents are in the situation either to trust the statements of the kindergarten or not to bring children to external child care facilities. Both options are not optimal since parents may be better off to have children

in the kindergarten and to be sure that they are well looked after. A third option is that parents can learn from the signals that child care institutions render about their quality. This option and the consequences for policy and welfare are analyzed in this paper.

There is empirical evidence which confirms information asymmetry on the market for child care. Mocan (2007) uses data of 400 child care centers in the United States and shows that the information asymmetry between parents and child care centers with respect to the quality of services forces parents to try to extract signals from observable center and classroom characteristics. However, these attempts are, for the most part, unsuccessful, as parents associate certain center characteristics with quality when they are not, and they do not read other signals of quality. Mocan concludes that the low average quality of child care can be traced back to information asymmetry between the consumer and the producers.

This paper analyzes theoretically the effects of information asymmetry on the child care market in a signalling game with incomplete information. The quality of child care is given and the kindergarten provides a signal about this quality which may be true or false. The parents cannot observe the true quality. However, they can learn from the signal of the kindergarten by updating the information. Depending on their beliefs they either decide to trust the signal or to enforce the kindergarten to provide high quality if they think the institution only pretends good child care. The resulting Bayesian equilibria are analyzed. Then we determine the ex-post probability to achieve high quality and we discuss policy measures to increase this probability. Finally, we define total welfare in the Bayesian equilibria and derive the optimal policy with respect to the ex-ante probability of high quality.

## 2 The model

Child care can assume two levels of quality:  $q_H$  and  $q_L$  with  $q_H > q_L$ . The kindergarten (KG) provides the quality  $q^c \in (q_H, q_L)$ . The net benefit of the KG is given by  $U(q_i) = P - C(q_i)$ ,  $i = H, L$ , where  $P$  is a constant price for the child care service and  $C(q_i)$  is the cost function of providing child care. The costs are higher for better quality  $C(q_H) > C(q_L)$ . Hence, the net benefit of the KG is higher if it provides low quality,  $U(q_L) > U(q_H)$ . The

parents of the children who are taken care of have the utility  $V(q_j), j = H, L$ . They prefer high quality of the child care,  $V(q_H) > V(q_L)$ . The KG knows the quality provided. However, the parents cannot observe the quality. The reason may be that the children give unreliable or no information about the way they are treated at the KG. Or the children have different attitudes towards high quality of child care than the parents. E.g. they may consider watching TV all day long at the KG as preferable to playing outside or learning with a nursery teacher. In this case, they may want to conceal the true day-to-day routine at the KG.

The parents receive a signal about the quality by the KG,  $\theta \in (\theta_H, \theta_L)$ , that may be truthful or deceptive. So the parents have to form a belief on (probability of) the quality provided by the KG conditional on the signal,  $\beta(q^c|\theta)$ . If they believe that the quality is low they can enforce a high quality by legal action against the KG. In this case they can top up the quality by  $q^p = \Delta q \equiv q_H - q_L$ . Hence, the final quality of child care is high either if the KG initially provides high quality  $q^c = q_H$  or if the parents force the KG to do so eventually:  $q^p = \Delta q$  and  $q^c = q_L$ .

Now we define the probability  $\gamma$  that the parents choose  $q^p = 0$  if the signal is  $\theta_H$ . If the quality of the KG is low this is the probability that deception works: parents rely on the signal of the KG although it is dishonest and do not take legal action against the KG. Furthermore, the a priori probability that the KG cheats, i.e.  $\theta = \theta_H$  if the true quality is  $q^c = q_L$ , is  $\rho$ .

**Assumptions** We assume that the KG has costs in deceiving the parents about the true quality provided. This may be the costs of effort to pretend a high quality of child care and to remain consistent in cheating the parents or it may be moral costs. Still, the costs  $\kappa$  are small enough so that the KG benefits from lying if the parents believe the KG:  $U(q_L) - \kappa > U(q_H)$ .

Furthermore, we assume that parents have costs of taking legal action,  $\varphi$ , in order to enforce high quality if the KG provides low quality. However, the legal expenses are manageable and the parents benefit from enforcing quality in spite of the costs:  $V(q_H) - \varphi > V(q_L)$ .

**Timing** The sequence of events is as follows.

First nature determines the quality of the child care services of the KG such that  $q^c = q_H$  arises with probability  $\pi$  and  $q^c = q_L$  with probability  $1 - \pi$ .

Second the KG sends a signal  $\theta \in (\theta_L, \theta_H)$  on the quality of the child care to the parents so as to maximize its utility net of costs of lying,  $U(q) - \kappa I$ .  $I$  is an index function with  $I = 1$  if  $\theta = \theta_H$  and  $q^c = q_L$  and  $I = 0$  otherwise. If the quality of child care is high,  $q^c = q_H$ , the KG will always send the signal  $\theta = \theta_H$  which yields the net benefit  $U(q_H)$  for the KG. If the KG would signal  $\theta = \theta_L$ , the net benefit is given by  $U(q_H) - \kappa$  which is clearly dominated by sending the true signal,  $\theta = \theta_H$ . Thus the probability of signalling low quality if the quality is high is nil:  $\beta(\theta_L|q_H) = 0$ . The converse probability is  $\beta(\theta_H|q_H) = 1$  and we get from the Bayesian rule:

$$\begin{aligned}\beta(q_H|\theta_L) &= \frac{\beta(q_H)\beta(\theta_L|q_H)}{\beta(q_H)\beta(\theta_L|q_H) + \beta(q_L)\beta(\theta_L|q_L)} \\ &= 0\end{aligned}$$

Third the parents enforce quality,  $q^p \in (0, \Delta q)$  so as to maximize utility net of costs of enforcing quality,  $V(q) - \varphi J$ .  $J$  is an index function with  $J = 1$  if  $q^p = \Delta q$  and  $J = 0$  otherwise.

### 3 Equilibria of the Bayesian game

**Second stage: parents enforce quality** At the second stage of the game the parents maximize the expected net utility by deciding about the enforcement of quality for a given signal of the KG. The parents' objective is given by:

$$\max_{q^p \in (0, \Delta q)} \beta(q_H|\theta)[V(q_H) - \varphi J] + (1 - \beta(q_H|\theta))[V(q) - \varphi J]$$

If the signal of the KG indicates low quality,  $\theta = \theta_L$ , the probability is zero that the quality is high:  $\beta(q_H|\theta_L) = 0$ . This implies that the parents decide to enforce high quality,  $q^p = \Delta q$ .

If the signal of the KG indicates high quality,  $\theta = \theta_H$ , the parents weakly prefer not to

enforce high quality, i.e.,  $q^p = 0$ , if and only if

$$\begin{aligned} & \beta(q_H|\theta_H)V(q_H) + (1 - \beta(q_H|\theta_H))V(q_L) \geq V(q_H) - \varphi \\ \Leftrightarrow & \beta(q_H|\theta_H) \geq 1 - \frac{\varphi}{V(q_H) - V(q_L)} \end{aligned} \quad (1)$$

Under this condition the utility gain of obtaining high quality if the parents do not believe the signal,  $(1 - \beta(q_H|\theta_H)) [V(q_H) - V(q_L)]$ , is smaller than the costs of enforcing high quality  $\varphi$ .

**First stage: KG sends signal:** At the first stage, the KG maximizes benefits net of enforcement costs by choosing the signal about a given quality of the child care which is determined by nature's draw. The KG's objective is given by:

$$\max_{\theta} U(q) - \kappa I$$

If the quality is high,  $q^c = q_H$ , we know by the reasoning above that the KG signals high quality:  $\theta = \theta_H$ . If the quality is low,  $q^c = q_L$ , the KG weakly prefers to deceive the parents, i.e.,  $\theta = \theta_H$ , if and only if

$$\begin{aligned} & \gamma U(q_L) + (1 - \gamma)U(q_H) - \kappa \geq U(q_H) \\ \Leftrightarrow & \gamma \geq \frac{\kappa}{U(q_L) - U(q_H)} \end{aligned} \quad (2)$$

The condition states the KG prefers to lie if and only if the expected utility gain of lying  $\gamma [U(q_L) - U(q_H)]$  is higher than the costs of lying  $\kappa$ . In this case, the probability  $\gamma$  is sufficiently high that the parents do not enforce high quality if the KG signals high quality. Lying is very likely to be successful.

**Perfect Bayesian Equilibria (PBE)** Now we determine the interior perfect Bayesian equilibrium. Using the probabilities  $\beta(q_H) = \pi$ ,  $\beta(\theta_H|q_H) = 1$  and  $\beta(\theta_H|q_L) = \rho$  we can derive the belief of the parents that quality is high if the KG signals so,  $\beta(q_H|\theta_H)$ , by applying the Bayesian rule:

$$\begin{aligned} \beta(q_H^c|\theta_H) &= \frac{\beta(q_H^c)\beta(\theta_H|q_H^c)}{\beta(q_H^c)\beta(\theta_H|q_H^c) + \beta(q_L^c)\beta(\theta_H|q_L^c)} \\ &= \frac{\pi}{\pi + (1 - \pi)\rho} \end{aligned} \quad (3)$$

By condition (2) we know that the KG is indifferent between lying and truth-telling in the case of  $q^c = q_L$  if and only if:

$$\gamma = \gamma^* := \frac{\kappa}{U(q_L) - U(q_H)} \quad (4)$$

Furthermore, equations (3) and (1) imply that parents are indifferent between  $q^p = 0$  and  $q^p = \Delta q$  if and only if:

$$\beta(q_H^c | \theta_H) = \frac{\pi}{\pi + (1 - \pi)\rho} = 1 - \frac{\varphi}{V(q_H) - V(q_L)}$$

Hence, the probability that the KG deceives the parents in an interior equilibrium is given by:

$$\rho = \rho^* := \frac{\pi}{1 - \pi} \left( \frac{\varphi}{V(q_H) - V(q_L) - \varphi} \right) \quad (5)$$

Note that parents prefer not to enforce the high quality:  $q^p = 0$  if the probability of the KG lying about the true low quality is smaller than the equilibrium value,  $\rho < \rho^*$ .

Now we can determine four PBE:

(i) For  $0 < \rho^* < 1$  the interior PBE is given by:  $\rho = \rho^*$ ,  $\gamma = \gamma^*$ . The reaction curves are shown in Figure 1. Assume the probability of the parents not to enforce high quality facing a signal  $\theta_H$  is  $\gamma > \gamma^*$ , then the KG prefers to lie about low quality and  $\rho = 1$ . In the reverse case,  $\gamma < \gamma^*$ , the KG's signal  $\theta_H$  is truthful and  $\rho = 0$ . For  $\gamma = \gamma^*$ , the KG is indifferent between lying and truth-telling and accepts any mix of both strategies:  $\rho = [0, 1]$ . In analogy, we get also the reaction function of the parents:

$$\rho = \begin{cases} 1 & \text{if } \gamma > \gamma^* \\ [0, 1] & \text{if } \gamma = \gamma^* \\ 0 & \text{if } \gamma < \gamma^* \end{cases}, \quad \gamma = \begin{cases} 0 & \text{if } \rho > \rho^* \\ [0, 1] & \text{if } \rho = \rho^* \\ 1 & \text{if } \rho < \rho^* \end{cases}$$

In  $\rho = \rho^*$ ,  $\gamma = \gamma^*$  the reaction curves intersect which gives the equilibrium in mixed strategies. With those probabilities both the parents and the KG cannot be made better off with alternative strategies.

Furthermore, there are three corner equilibria. The reaction curves are illustrated in Figure 2. The intersection area of the functions indicates the corner equilibria.

(ii) At  $\rho^* = 0$  a corner PBE is  $\rho = \rho^* = 0$ ,  $0 \leq \gamma \leq \gamma^*$ . The reaction functions are given by:



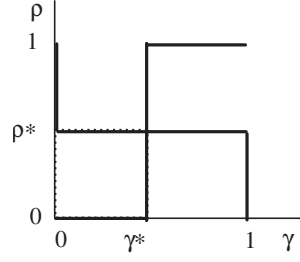


Figure 1: Reaction curves and interior PBE  $(\rho^*, \gamma^*)$

$$\rho = \begin{cases} 1 & \text{if } \gamma > \gamma^* \\ [0, 1] & \text{if } \gamma = \gamma^* \\ 0 & \text{if } \gamma < \gamma^* \end{cases}, \quad \gamma = \begin{cases} 0 & \text{if } \rho > \rho^* \\ [0, 1] & \text{if } \rho = \rho^* \end{cases}$$

In this corner equilibrium the parents are indifferent about enforcing high quality but the probability that the KG lies about its low quality is zero.

(iii) At  $\rho^* = 1$  another corner PBE is  $\rho = \rho^* = 1, \gamma^* \leq \gamma \leq 1$ . The reaction functions are given by:

$$\rho = \begin{cases} 1 & \text{if } \gamma > \gamma^* \\ [0, 1] & \text{if } \gamma = \gamma^* \\ 0 & \text{if } \gamma < \gamma^* \end{cases}, \quad \gamma = \begin{cases} [0, 1] & \text{if } \rho = \rho^* \\ 1 & \text{if } \rho < \rho^* \end{cases}$$

Again the parents are indifferent about legal enforcement but in contrast to the former corner equilibrium the KG deceives the parents with certainty.

(iv) At  $\rho^* > 1$  a third corner PBE is  $\rho = 1, \gamma = 1$ . The reaction functions are given by:

$$\rho = \begin{cases} 1 & \text{if } \gamma > \gamma^* \\ [0, 1] & \text{if } \gamma = \gamma^* \\ 0 & \text{if } \gamma < \gamma^* \end{cases}, \quad \gamma = 1 \text{ if } \rho \leq 1$$

In this corner equilibrium parents decide not to enforce high quality and the KG lies about quality if it is low.

The equilibria are summarized in the following proposition.

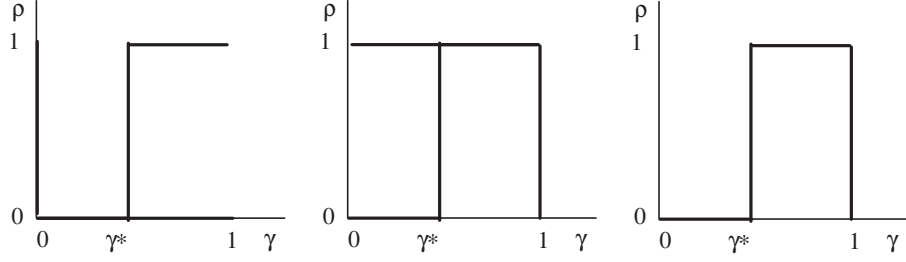


Figure 2: Reaction curves and corner PBE ( $\rho^* = 0, \rho^* = 1, \rho^* > 1$ )

- Proposition 1** (i) If  $0 < \rho^* < 1$ ,  $\rho = \rho^*$  and  $\gamma = \gamma^*$  is a perfect Bayesian equilibrium.  
(ii) If  $\rho^* = 0$ ,  $\rho = 0$  and  $\gamma \leq \gamma^*$  is a perfect Bayesian equilibrium.  
(iii) If  $\rho^* = 1$ ,  $\rho = 1$  and  $\gamma \geq \gamma^*$  is a perfect Bayesian equilibrium.  
(iv) If  $\rho^* > 1$ ,  $\rho = 1$  and  $\gamma = 1$  is a perfect Bayesian equilibrium.

Now we determine the final ex-post probability to achieve high quality in the equilibria and we analyze how policy measures affect this probability.

## 4 Policy effects on ex-post quality

In an interior perfect Bayesian equilibrium the final ex-post probability  $\alpha$  of obtaining high quality of child care can be written as:

$$\begin{aligned}\alpha &= \pi + (1 - \pi)[1 - \rho + \rho(1 - \gamma)] \\ &= \pi + (1 - \pi)(1 - \rho\gamma)\end{aligned}$$

The probability  $\alpha$  is composed of the ex-ante probability  $\pi$  that the KG provides high quality and, in the opposite case,  $1 - \pi$ , of the probability that the KG does not conceal low quality,  $1 - \rho$ , and parents enforce high quality, and the probability that the KG deceives parents about the true quality,  $\rho$ , but parents do not believe the signal,  $1 - \gamma$ .

There are two effects of the ex-ante probability of high quality on ex-post probability. The direct effect is clearly positive:

$$\frac{\partial \alpha}{\partial \pi} = \gamma\rho > 0$$

The indirect effect arises via the impact on probability that the KG lies about the true quality,  $\rho$ . The higher  $\rho$  the lower is the ex-post probability  $\alpha$  because parents abstain from enforcing high quality to a larger extent:

$$\frac{\partial \alpha}{\partial \rho} = -(1 - \pi)\gamma < 0$$

Furthermore, the probability to lie  $\rho$  increases with a the ex-ante probability of high quality. The reason is that lying becomes more credible:

$$\frac{\partial \rho}{\partial \pi} = \frac{\rho}{\pi(1 - \pi)} = \frac{\varphi}{(1 - \pi)^2[V(q_H) - V(q_L) - \varphi]} > 0$$

The total effect of ex-ante on ex-post probability of high quality is negative. The indirect effect overcompensates the direct effect:

$$\begin{aligned} \frac{d\alpha}{d\rho} &= \frac{\partial \alpha}{\partial \pi} + \frac{\partial \alpha}{\partial \rho} \frac{\partial \rho}{\partial \pi} = -\frac{\gamma(1 - \pi)\rho}{\pi} \\ &= -\frac{\kappa\varphi}{[U(q_L) - U(q_H)][V(q_H) - V(q_L) - \varphi]} < 0 \end{aligned}$$

Another effect is the impact of enforcement costs on the probability to lie and the ex-post probability of high quality. If the costs of legal enforcement of high quality rises fewer parents decide to take legal action. This encourages the KG to deceive parents and the probability of lying increases:

$$\frac{\partial \rho}{\partial \varphi} = \frac{\pi[V(q_H) - V(q_L)]}{(1 - \pi)[V(q_H) - V(q_L) - \varphi]^2} > 0$$

Since the effect of  $\rho$  on  $\alpha$  is negative the total effect of quality enforcement costs on ex-post quality is negative:

$$\frac{d\alpha}{d\varphi} = \frac{\partial \alpha}{\partial \rho} \frac{\partial \rho}{\partial \varphi} = -\frac{\pi\kappa[V(q_H) - V(q_L)]}{[U(q_L) - U(q_H)][V(q_H) - V(q_L) - \varphi]^2} < 0$$

A third effect is how the costs of lying affect the ex-post probability of high quality. The impact is via the probability that parents believe the signal of high quality and do not engage in legal enforcement,  $\gamma$ . The higher the costs of lying,  $\kappa$ , the more parents trust in the signal of high quality: the probability  $\gamma$  increases.

$$\frac{\partial \gamma}{\partial \kappa} = \frac{1}{U(q_L) - U(q_H)} > 0$$

However, a higher  $\gamma$  reduces the ex-post probability of high quality because if the KG gives a false signal about its quality parents are more prone to take this signal as truthful and do not enforce high quality. Thus the ex-probability decreases:

$$\frac{\partial \alpha}{\partial \gamma} = -(1 - \pi) \rho < 0$$

In total, higher costs of lying decrease the probability to achieve high quality:

$$\frac{d\alpha}{d\kappa} = -\frac{(1 - \pi) \rho}{U(q_L) - U(q_H)} < 0$$

**Proposition 2** *At the interior PBE  $(\rho, \gamma) = (\rho^*, \gamma^*)$  the ex-post probability to achieve high quality of child care services decreases with the ex-ante probability of high quality:  $d\alpha/d\pi < 0$ . Increasing the costs of enforcing high quality leads to a lower ex-post probability of high quality:  $d\alpha/d\varphi < 0$ . Higher costs of lying reduce the ex-post probability of high quality:  $d\alpha/d\kappa < 0$ .*

This result shows that some measures to improve the high quality of child care services are not effective. Controlling the ex-ante quality of the KG,  $\pi$ , might have the detrimental effect of encouraging the deception of the true quality. Furthermore, while subsidising the enforcement costs of parents increase the probability of better quality the increase of costs of lying (e.g. more intense scrutiny by the parents or more social pressure) might lead parents to be more trustworthy to signals of the KG which can prevent to enforce high quality to a larger extent.

In the corner equilibria the ex-post probability of high quality is  $\alpha = 1$  in case b),  $\alpha = \pi + (1 - \pi)(1 - \gamma)$  in case c), and  $\alpha = \pi$  in case d). Hence, in case c) again the higher costs of lying would decrease the ex-post quality:  $\frac{\partial \alpha}{\partial \kappa} = -\frac{1 - \pi}{U(q_L) - U(q_H)} < 0$ .

## 5 Welfare analysis

For the welfare analysis we have to define the costs  $K(\pi)$  of providing high quality in child care. The cost function is assumed to be strictly convex:  $K'(\pi) > 0$ ,  $K''(\pi) > 0$ .

Then the total welfare consists of the net welfare of the parents,  $W_V$ , which enters total welfare with a weight  $\delta$ , and the welfare of the KG,  $W_U$ , weighted with  $1 - \delta$ , less the costs

of ex-ante probability of high quality,  $K(\pi)$  :

$$W = \delta W_V + (1 - \delta)W_U - K(\pi) \quad (6)$$

The net welfare of the parents is given by

$$W_V = \alpha V(q_H) + (1 - \alpha)V(q_L) - \{(1 - \pi)(1 - \rho) + [\pi + (1 - \pi)\rho](1 - \gamma)\}\varphi \quad (7)$$

The expected net welfare of the parents is the expected utility  $\alpha V(q_H) + (1 - \alpha)V(q_L)$  minus the enforcement costs. Those costs arise if the quality of the KG drawn by nature is low (with prob.  $1 - \pi$ ) and the KG tells the truth (with prob.  $1 - \rho$ ) or if the quality is high ( $\pi$ ) or signalled to be high ( $(1 - \pi)\rho$ ) but the parents choose nevertheless to top up the quality  $(1 - \gamma)$ , not believing the signal.

The net welfare of the KG is given by

$$W_U = \alpha U(q_H) + (1 - \alpha)U(q_L) - \kappa(1 - \pi)\rho \quad (8)$$

The expected net welfare of the KG is the expected utility  $\alpha U(q_H) + (1 - \alpha)U(q_L)$  minus the costs of lying which arise with probability  $\rho$  if the true quality is low (with prob.  $1 - \pi$ ).

Now we analyze the impact of the ex-ante probability of high quality  $\pi$  on welfare. As a benchmark note that at  $\rho^* = 1$  the probability  $\pi$  assumes the value  $\hat{\pi} := 1 - \varphi/[V(q_H) - V(q_L)]$ . First we determine welfare for the range of  $\pi$ .

For  $\pi = 0$ , the probability of parents to be indifferent between trusting the signal and enforcing high quality is  $\rho^* = 0$ . The corresponding PBE (case (ii)) is given by:  $\rho = 0$ ,  $0 \leq \gamma \leq \gamma^*$  and the ex-post probability  $\alpha = 1$ . By (8) the welfare of the KG is  $W_U = U(q_H)$  and by (7) the welfare of the parents is  $W_V = V(q_H) - \varphi$ .

For  $0 < \pi < \hat{\pi}$ , we get an interior solution for  $\rho^* : 0 < \rho^* < 1$  and the equilibrium is the interior PBE (case (i)):  $\rho = \rho^*$ ,  $\gamma = \gamma^*$ . The ex-post probability of high quality is  $\alpha = \pi + (1 - \pi)(1 - \rho^*\gamma^*) = 1 - (1 - \pi)\rho^*\gamma^*$ . Using equation (4) the welfare of the KG is given by

$$\begin{aligned} W_U &= U(q_H) + (1 - \pi)\rho^*\gamma^*[U(q_L) - U(q_H)] - \kappa(1 - \pi)\rho^* \\ &= U(q_H) + (1 - \pi)\rho^*[\gamma^*(U(q_L) - U(q_H)) - \kappa] \\ &= U(q_H) \end{aligned}$$

The welfare of parents can be derived by taking account of equation (5):

$$\begin{aligned}
W_V &= V(q_H) + (1 - \pi)\rho^*\gamma^*[V(q_L) - V(q_H)] - (1 - \gamma^*\pi - (1 - \pi)\rho^*\gamma^*)\varphi \\
&= V(q_H) + (1 - \pi)\rho^*\gamma^*[V(q_L) - V(q_H) + \varphi] - (1 - \gamma^*\pi)\varphi \\
&= V(q_H) - \varphi
\end{aligned}$$

Hence, for  $\pi$  in the range of  $[0, \hat{\pi})$  the total welfare is given by

$$W_{0 \leq \pi < \hat{\pi}} = \delta[V(q_H) - \varphi] + (1 - \delta)U(q_H) - K(\pi) \quad (9)$$

For  $\pi = \hat{\pi}$  follows  $\rho^* = 1$  and the corresponding corner PBE (case (iii)):  $\rho = 1$ ,  $\gamma^* \leq \gamma \leq 1$  with  $\alpha = 1 - (1 - \hat{\pi})\gamma$ . The welfare of the KG can be derived as

$$\begin{aligned}
W_U &= U(q_H) + (1 - \hat{\pi})\gamma[U(q_L) - U(q_H)] - \kappa(1 - \hat{\pi}) \\
&= U(q_H) + (1 - \hat{\pi})[\gamma(U(q_L) - U(q_H)) - \kappa] \\
&\geq U(q_H)
\end{aligned}$$

since  $\gamma \geq \gamma^*$ . Using  $1 - \hat{\pi} = \frac{\varphi}{V(q_H) - V(q_L)}$  we get for the welfare of the parents:

$$\begin{aligned}
W_V &= V(q_H) + (1 - \hat{\pi})\gamma[V(q_L) - V(q_H)] - (1 - \gamma)\varphi \\
&= V(q_H) - \varphi
\end{aligned}$$

In this PBE the total welfare is given by

$$\begin{aligned}
W_{\pi = \hat{\pi}} &= \delta[V(q_H) - \varphi] \\
&\quad + (1 - \delta) \{U(q_H) + (1 - \hat{\pi})[\gamma(U(q_L) - U(q_H)) - \kappa]\} - K(\hat{\pi})
\end{aligned} \quad (10)$$

For  $\pi > \hat{\pi}$  we have  $\rho^* > 1$  and the corner PBE (case (iv)):  $\rho = 1$ ,  $\gamma = 1$  with  $\alpha = \pi$ . The welfare of the KG:  $W_U = \pi U(q_H) + (1 - \pi)U(q_L) - \kappa(1 - \pi)$  and the parents  $W_V = \pi V(q_H) + (1 - \pi)V(q_L)$  is easily derived. Using  $\pi > \hat{\pi}$  we yield  $W_V > V(q_H) - \varphi$  and since by assumption  $U(q_L) - \kappa > U(q_H)$  we find that  $W_U = U(q_H) + (1 - \pi)[U(q_L) - \kappa - U(q_H)] > U(q_H)$ . Total welfare is given by

$$\begin{aligned}
W_{1 > \pi > \hat{\pi}} &= \delta[\pi V(q_H) + (1 - \pi)V(q_L)] + \\
&\quad (1 - \delta)[\pi U(q_H) + (1 - \pi)U(q_L) - \kappa(1 - \pi)] - K(\pi)
\end{aligned} \quad (11)$$

Finally for  $\pi = 1$  results  $\rho^* = \infty$  and the PBE:  $\rho = 1$ ,  $\gamma = 1$  and  $\alpha = 1$ . The welfare is given by  $W_U = U(q_H)$  and  $W_V = V(q_H)$ . Total welfare is

$$W_{\pi=1} = \delta V(q_H) + (1 - \delta) U(q_H) - K(\pi) \quad (12)$$

Total welfare on the range of  $\pi$  is illustrated in Figure 3:

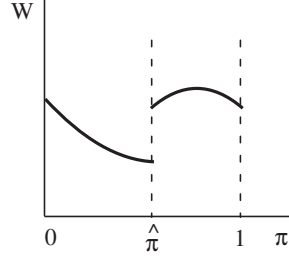


Figure 3:  $\pi$  and welfare

**Optimal policy** In the range  $\pi < \hat{\pi}$ , the derivative of total welfare from equation (9) is  $dW/d\pi = -K'(\pi) < 0$ . Hence, welfare is monotonically decreasing in  $\pi$ . The optimal  $\pi$  in this range is at  $\pi^* = 0$ .

At  $\pi = \hat{\pi}$ , there is a discontinuity upwards of total welfare. If  $\pi$  converges to  $\hat{\pi}$  the limit of welfare from below  $\hat{\pi}$  is smaller than the limit of welfare from above  $\hat{\pi}$ :

$$\begin{aligned} \lim_{\pi \rightarrow \hat{\pi}^-} W &= \delta[V(q_H) - \varphi] + (1 - \delta)U(q_H) - K(\pi) \\ &< \lim_{\pi \rightarrow \hat{\pi}^+} W = \delta[\pi V(q_H) + (1 - \pi)V(q_L)] \\ &\quad + (1 - \delta)[\pi U(q_H) + (1 - \pi)U(q_L) - \kappa(1 - \pi)] - K(\pi) \end{aligned}$$

In the range  $\pi > \hat{\pi}$ , we have  $dW_U/d\pi = U(q_H) - U(q_L) + \kappa < 0$  and  $dW_V/d\pi = V(q_H) - V(q_L) > 0$  so that total welfare is concave. The optimal  $\pi$  in this range is determined by

$$\begin{aligned} dW/d\pi &= \delta[V(q_H) - V(q_L)] - (1 - \delta)[U(q_L) - \kappa - U(q_H)] - K'(\pi) \\ &= 0 \end{aligned}$$

so that  $\pi^* \in [\hat{\pi}, 1]$ . If the (marginal) costs of increasing  $\pi$  are high,  $\pi = 0$  is optimal, otherwise  $\pi^* \in [\hat{\pi}, 1]$  is optimal.

**Proposition 3** *If  $0 \leq \pi < \hat{\pi}$ , welfare is  $W = \delta[V(q_H) - \varphi] + (1 - \delta)U(q_H) - K(\pi)$  and the welfare maximizing  $\pi$  is  $\pi^* = 0$ .*

*If  $\pi > \hat{\pi}$ , welfare is given by*

*$W = \delta[\pi V(q_H) + (1 - \pi)V(q_L)] + (1 - \delta)[\pi U(q_H) + (1 - \pi)U(q_L) - \kappa(1 - \pi)] - K(\pi)$  and the welfare maximizing  $\pi$  is determined by  $\delta[V(q_H) - V(q_L)] = (1 - \delta)[U(q_L) - \kappa - U(q_H)] + K'(\pi)$ .*

*If the costs of increasing the ex-ante probability of high quality are low the optimal  $\pi$  is  $\pi^* \in [\hat{\pi}, 1]$ .*

If high quality is not too expensive it is optimal to increase its probability up to a point where the a priori expected utility gain of parents,  $\pi[V(q_H) - V(q_L)]$ , is larger than the utility gain achieved by enforcement,  $V(q_H) - V(q_L) - \varphi$ .

## 6 Conclusion

In the presence of informational asymmetry on child care markets parents use the signals of the kindergarten to form beliefs about the true quality of child care. This may result in several perfect Bayesian equilibria. In each of these equilibria the chances to achieve high quality and the welfare are different.

Most state interventions on the child care market regulate the inputs of the production of child care services like limiting the number of children per staff in a facility or requiring certain educational standards for nursery teachers. Empirical evidence shows that those regulations while increasing the fraction of centers that are of sufficient quality the imposition of those regulations reduces the number of centers in local markets and thereby reduces the availability of such services to parents (Hotz and Xiao 2011).

Our analysis shows that policies that increase the proportion of high-quality child care centers and so the ex-ante probability for parents to receive high quality may result in a lower ex-post probability to achieve high quality by enforcement. Furthermore, a policy that increases the lying costs of child care centers (e.g. by improved monitoring) may also reduce the chances to yield high quality ex post. So well-intended policy measures may have adverse consequences.



If it is not too costly to increase high quality it is welfare maximizing to reach a probability of high quality where parents expect a utility gain that is larger than the utility gain achieved by enforcement. Hence, complete certainty of high quality is rarely optimal.

Our analysis and results could be translated for other kinds of indirect services where the decider about the demand on the consumer side is not identical with the consumer and information is not transmitted perfectly between decider and consumer. For example, decisions about the demand for long-term care have similar features like the decisions about external child care. A extended theory of services for minor consumers is left for future research.

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