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Quality provision and reporting when health care services are multi-dimensional and quality signals imperfect

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Abstract

We model competition for a multi-attribute health service where patients observe attribute quality imprecisely before deciding on a provider. High quality in one attribute, e.g. medical quality, is more important for ex post utility than high quality in the other attribute. Providers can shift resources to increase expected quality in some attribute. Patients rationally focus on attributes depending on signal precision and beliefs about the providers’ resource allocations. When signal precision is such that patients focus on the less important attribute, any Perfect Bayesian Nash Equilibrium is inefficient. Increasing signal precision can reduce welfare, as the positive effect of better provider selection is overcompensated by the negative effect that a shift in patient focusing has on provider quality choice. We discuss the providers’ strategic reporting incentives and reporting policies. Under optimal reporting, signals about the important attribute are always published. However, banning reporting on less important attributes might be necessary.


Keywords: multi-attribute good, quality signals, focusing, reporting

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1 Introduction

Health care services have multiple relevant quality dimensions. When choosing doctors, hospitals or taking decisions about nursing homes, patients care about medical quality on the one hand, and may take non-medical quality factors such as general appeal of the doctor’s office or hospital environment, short waiting times and interpersonal skills of the staff on the other hand into account. Some of these dimensions are difficult to observe, measure, evaluate and communicate, whereas others can be observed and measured with fairly high precision. For instance, selected mortality rates or Coronary Artery Bypass Graft (CABG) rates provide only an imprecise signal of hospital medical quality.\(^1\) Contrary to that, information brochures with pictures of patient rooms and sample dinner menus provide fairly accurate signals for the hotel attributes of the hospital environment. In Germany, for instance, the public feedback platform Arztnavigator provides detailed information of patient feedback on doctor’s practice rooms, waiting times, and the doctor’s and staff’s friendliness and communication skills.\(^2\)

In this paper, we address the question of which quality dimensions patients rationally focus on when the signals they receive about the qualities of the dimension before deciding on a provider have different precision, and what this focusing implies for the provision of quality and welfare. In particular, we are concerned with settings where patients value quality differences in one attribute, e.g. medical quality of the service, more than quality differences in the other attributes, e.g. the hotel properties of hospitals or nursing homes, but the quality signal in the more important attribute is less precise.

Interestingly, empirical research indicates that public reporting of clinical quality scores has a positive but only weak effect on patients’ provider choice.\(^3\) One reason might be that patients are skeptical about the accuracy of these quality measures. Furthermore, other quality dimensions might play an important role for the choice of health care providers. Goldman and Romley (2008) analyze the role of amenities alongside treatment quality measures on hospital choice for Californian data. They show that various measures of treatment quality of hospitals (e.g. mortality rates) have only a small effect on patient demand while improvements in amenities strongly raise demand. Furthermore, patients’ perceptions of reputation and specialty medical services as well as satisfaction with a prior hospital stay significantly affect hospital choice. Among these, satisfaction with a prior stay may thereby be driven partly by non-medical factors. Fornara, Bonaiuto, and Bonnes (2006) e.g. show that hospital users’ perceived quality of care improves when the humanization degree of the hos-

\(^{1}\)Iezzoni (1997) shows that report card rankings may vary profoundly according to the chosen risk adjusters. Thus, if patients do not have information about the risk adjusters used, there is significant noise. According to Dranove (2012), Medicare Hospital Compare identifies only a small percentage of hospitals as having mortality rates significantly above or below the mean. Thus, although quality reports become increasingly available through e.g. report cards or public feedback platforms, the signals that patients receive through these about medical quality are often still fairly imprecise through an inherent difficulty of observing and measuring and interpreting medical quality accurately.

\(^{2}\)See https://weisse-liste.arzt-versichertenbefragung.aok-arztnavi.de/.

\(^{3}\)See e.g. Dranove (2012) and the discussion therein.
hospital environment increases.\textsuperscript{4} Regarding the demand response, Dafny and Dranove (2008) report that the effect of health plan report cards on Medicare beneficiaries is driven by responses to patient satisfaction scores, while other more objective quality measures did not affect enrollment decisions.

An important concern in this context is whether a potentially strong demand response to non-medical quality attributes such as amenities, interpersonal skills or perceived high quality environment leads to a suboptimal quality of care. This would be the case if medical quality is more important to generate patient welfare than all other dimensions of care - such that quality of care should be high on the clinical quality dimension -, but health care providers do not provide sufficiently high quality in the clinical dimension as patient demand is more responsive to quality differences in other dimensions. However, why should patients respond more to quality differences in other dimensions than medical quality if medical quality is the important dimension in terms of their realized utility? Generally, why would patients focus on an attribute that is less important in terms of consumption utility?

Our starting point is the observation that many quality dimensions can only be observed imperfectly ex ante, and that the precision of information about quality varies across dimensions. In particular, we model provider competition when patients observe attribute quality of a two-attribute health service only imperfectly. Providers can allocate given resources across the attributes in order to increase expected quality in either one or the other attribute. A patient’s utility gain from an increase in quality in one attribute is larger than in the other attribute, thus representing the situation where high quality in the medical treatment dimension is more important for patient welfare than amenities. Patients receive a binary signal about realized quality in each attribute from each provider before deciding on a provider.

We first define rational focusing on attributes: A patient focuses on an attribute if a high quality signal in this attribute drives her provider choice. We say that focusing is strong if this holds for any combination of beliefs that the patient might have about the underlying resource allocation decisions of the providers, whereas there is focusing, but not strong, if this holds for beliefs that are symmetric across providers. With this definition, we can describe a patient’s focus on quality attributes depending on the precision of quality signals in the attributes.

We show that equilibria exist in which providers invest in the less important attribute. This occurs if the quality signal in this attribute is more precise than in the other attribute to the extent that patients focus on this attribute. Equilibrium is unique under strong focusing. If signal precisions are such that patients’ focus is on the less important attribute, all Perfect Bayesian Nash equilibria are inefficient. Increasing signal precision, e.g. by introducing a signal in the less important attribute, can reduce welfare. This occurs if the positive effect of better provider selection due to higher signal precision is overcompensated by the negative effect that the shift in

\textsuperscript{4}For environmental factors, Arneill and Devlin (2002) conducted a study where they showed participants slides of doctors’ waiting rooms and then asked what quality of care participants expected. Arneill and Devlin (2002) find that the perceived quality of care would be significantly higher for waiting rooms that are nicely furnished, light, contain artwork and are warm versus waiting rooms that are dark, have outdated furnishings, contain no artwork or poor quality reproductions and are cold in appearance.
patient focusing, induced by the change in signal precision, has on provider quality choice. We derive conditions under which an increase in signal precision leads to an unambiguous welfare loss.

In the literature on health care reporting, the adverse effect of information that has been emphasized is providers’ patient selection incentives (Dranove, Kessler, McClellan, and Satterthwaite, 2003), i.e., turning away the sickest patients because of providers’ concerns about their ‘ratings’. We point to a further effect that may result from the increase in information on other quality dimensions through e.g. public feedback platforms alongside the increased public reporting of medical quality: If information becomes relatively more precise on less important attributes, patients may focus on these, with adverse consequences for quality provision and welfare.

Feng Lu (2012) analyzes the impact of public reporting of some quality measures on quality in the reported and unreported dimensions. Feng Lu (2012) finds that after the introduction of public reporting, scores of quality measures improve along the reported dimensions, but significantly deteriorate along the unreported dimensions. Feng Lu (2012) furthermore finds no evidence that there was a decrease in quality-related inputs, suggesting a reallocation of resources. Note that in our model, public reporting only has an effect on the resource allocation if it increases the relative precision of quality signals that patients receive in these attributes, and only if the effect is strong enough to shift patient focus.

Our analysis also allows to derive optimal reporting policies. Reporting in our framework is the sending of informative but noisy signals about realized quality with exogenous precision before quality is realized. In order to compare reporting policies including voluntary reporting, we change the baseline model in the following way: Whether patients receive signals (with exogenous precision) in certain attributes now depends on a strategic reporting decision by providers. We show that if the more important attribute is not too important, in the unique equilibrium under strategic reporting providers invest in the less important attribute and only publish signals in this attribute. Thus, not only resource allocation, but also reporting might be inefficient. However, if the more important attribute is sufficiently important, it might also be the case that providers invest in the important attribute and only report in the important attribute although there would be patient focusing on the less important attribute if patients received signals in all attributes. Mandating full reporting might be then be welfare-reducing. Under optimal reporting, signals in the important attribute are always published, however, it might be necessary to control reporting in attribute 2. In particular, a ban on reporting in attribute 2 might have to be imposed.

Contrary to that, Werner, Konetzka, and Kruse (2009) find that overall both unreported and reported care in nursing homes improved following the launch of public reporting. Improvements in unreported care were particularly large among facilities with high scores or that significantly improved on reported measures, whereas low-scoring facilities experienced no change or worsening of their unreported quality of care. In our model, the technology is such that expected qualities in the dimensions are substitutes and not complements.
Related literature

Focusing  We define rational focusing via the precision of signals that patients receive about attributes in an environment with imperfect quality information. A patient evaluates signals according to her expected utility for any given beliefs. We say that she focuses on an attribute if, for given ranges in feasible outcomes, the difference in the precision of signals is such that the difference between signal value and expected outcome in this attribute is, compared to the other attribute, low. Focusing here is thus different from focusing and salience models (Bordalo, Gennaioli, and Shleifer, 2013, Koszegi and Szeidl, 2013) that assume that there is an exogenous difference between decision utility and consumption utility. In Koszegi and Szeidl (2013) e.g., under perfect information, focus weights of attributes in decision utility depend positively on the range of feasible outcomes in attributes.

Multi-attribute goods  The literature on markets with multi-attribute goods and quality investment is scarce. Bar-Isaac, Caruana, and Cunat (2012) analyze monopoly provision of a two-attribute good where quality is imperfectly observable. Contrary to our set-up with exogenous information, they consider active consumers who choose which information to acquire. Customers are heterogeneous in their valuation for attributes and can assess quality at a cost. The monopolist can invest in an increase of the probability of high quality in one attribute. A reduction in the consumers’ costs of acquiring information on the other attribute may then reduce quality investment: The decrease in costs of assessment shifts the consumer that is indifferent between assessing one or the other dimension towards the first attribute, reducing demand and thereby quality investment. The direct positive welfare effect of reduced assessment costs may then be dominated by the negative investment effect leading to a reduction in overall consumer welfare. Closest to our work is Dranove and Satterthwaite (1992). In Dranove and Satterthwaite (1992), competing manufacturers sell goods through retailers where retail price is random and customers are heterogeneous in their valuation for quality. Customers observe prices and quality only with noise and search retailers using an optimal sequential search rule. An increase in the precision of the price observation may then decrease welfare through the indirect effects of a change in the customers’ search: Prices fall, but quality is reduced as well. If the latter effect is stronger, increasing precision of the price observation reduces consumer welfare. In contrast, we model a market with homogeneous consumers that benefit more from high quality in one attribute than in the other. Instead of searching, customers receive signals from all providers. We show under what conditions on signal precision and beliefs the customers’ focus is on the less important attribute and derive the welfare consequences. Furthermore, we discuss strategic reporting by providers and optimal reporting policies.

While the workings in our model show some analogy to the logic of the multitasking literature as in Holmstrom and Milgrom (1991), the modelling and conclusions are however different. In the multitasking literature, effort substitutability implies
complementarity of the optimal (linear) incentive pay for tasks.\textsuperscript{6} Better information in the sense of a reduction in the noise of the performance improves the tailoring of incentive pay and does not have a negative value for the principal. In contrast, we consider a market for a multi-attribute service where consumers receive noisy signals about realized quality by competing providers. The key contractual incompleteness in this market is that attributes cannot be separately priced such that consumers do not separately evaluate expected quality and utility differences in each attribute and that consumers cannot commit to ignore signals. Better information in the sense of increasing signal precision may then decrease welfare, as it is individually rational for customers to focus too strongly on signals in the less important attribute.

Health care quality under imperfect information and quality reporting
Gravelle and Sivey (2010) analyze competition between hospitals under fixed prices where patients receive imperfect signals about quality, which is one-dimensional. Hospitals have different quality cost functions and can set quality. Gravelle and Sivey (2010) show that when patients choose the hospital that sends a higher signal, better information in the sense of a reduction in the variance of the noise term may reduce quality of both hospitals if quality costs are sufficiently different.\textsuperscript{7}

Most of the literature on quality information considers reporting in the form of disclosure of known, realized quality. Sun (2011) analyzes a monopolist’s voluntary disclosure for a multiple-attribute good, where the attributes are a vertical and horizontal quality. When vertical quality is known, horizontal quality might not be disclosed. This is since, to the monopolist, disclosure has the benefit of attracting consumers nearby at the cost of deterring consumers far away. When vertical quality is low, the benefit is crucial and outweighs the cost. As quality becomes higher, consumers are more likely to buy the product without disclosure such that when quality is high enough, the monopolist tries to cover the entire market at a high price without disclosure. Board (2009) analyzes disclosure incentives for a one-dimensional good under competition with heterogeneous firms. If a high-quality firm discloses, competitors must trade off the increase in competition and resulting fall in price if they also disclose with the reduction in perceived quality by consumers, if they do not. Nondisclosure by some high-quality firms thus generates positive externalities for low-quality firms who may pool with them and take advantage of raised consumer expectations. Board (2009) shows that the welfare effects of mandatory disclosure are complex, consumer surplus however rises if firms are sufficiently close in quality that the overall effect is increased competition. Contrary to that, we do not model quality disclosure, but reporting as a decision of publishing signals before quality is realized. Providers voluntarily never report in all attributes, since reporting in their weak attribute, i.e. the one they did not invest in, gives them a competitive disad-

\textsuperscript{6}Kaarboe and Siciliani (2011) analyze optimal contracting between a purchaser and a partly altruistic provider of health services within the multitasking framework where one quality dimension is verifiable whereas the second is not. They show that provider altruism with respect to health benefit can lead to overall complementarity of qualities even if they are substitutes on the effort cost side such that high powered incentives may be optimal.

\textsuperscript{7}Patient demand is however not consistent with that of rational Bayesian agents, see the discussion in Shelegia (2012).
Quality reporting as a policy instrument in the context of healthcare is considered in Glazer and McGuire (2006). Glazer and McGuire (2006) study competition among health plans under adverse selection and fixed prices. They show that averaged quality reports, instead of full reports, can remedy adverse selection incentives, since averaging quality across dimensions and reporting only the average enforces pooling in health insurance. Less information in the form of averaged quality reports thus mitigates the problem of cream-skimming of good patients with tailored quality packages. The right weights for quality averaging may then implement efficient outcomes.

Whereas Glazer and McGuire (2006) consider a common value set-up and fixed prices, Ma and Mak (2014) compare full quality reporting to average quality reporting under private values and price setting by a monopolist. Ma and Mak (2014) show that qualities and prices under an imposed average quality report generate higher consumer welfare than full quality report, as it restrains the firm’s price-quality discrimination strategies. In our model, suppressing quality information in the form of banning reporting in some dimensions might be optimal since this shifts the patients’ demand towards the quality dimensions that matter more to generate welfare.

## 2 Model

We consider a two-attribute health service $q = (q_1, q_2)$ with $q_i \in \{h, l\}$ for $i = 1, 2$ where $h$ stands for high quality and $l$ for standard quality respectively. Two providers $A$ and $B$ provide the service. The provider compensation is a uniform, exogenously set fee $P > 0$ per unit of service provided.\(^8\) Quality cannot be contracted on. Quality is stochastic. Providers can allocate resources in order to achieve high expected quality in either one or the other attribute.\(^9\) In particular, each provider $j \in \{A, B\}$ has fixed resources which are symmetric across providers, and makes a resource allocation decision $a^j \in \{0, 1\}$. For any $a^j \in \{0, 1\}$ the realization probabilities for high quality in one attribute are

<table>
<thead>
<tr>
<th>$a^j$</th>
<th>$\mathbb{P}(q_1 = h)$</th>
<th>$\mathbb{P}(q_2 = h)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1 - p$</td>
<td>$p$</td>
</tr>
<tr>
<td>0</td>
<td>$p$</td>
<td>$1 - p$</td>
</tr>
</tbody>
</table>

with $p \in (0, \frac{1}{2})$. Quality levels are realized independently for each attribute. With this technology, we say provider $j$ invests in attribute 1 (2) if he sets $a^j = 1$ ($a^j = 0$). The lower $p$, the larger is the probability that high quality is realized in the attribute a provider invests in.

The assumptions made about how quality realization depends on the resource allocation incorporates two symmetries: First, a symmetric impact of resource allocation

\(^8\)Fees cannot be set separately for attributes. The fixed, exogenous fee reflects e.g. regulated prices or negotiated prices between health plans and providers for the service in their network.

\(^9\)For a potential split of resources see discussion in section 7.
on quality realization across attributes. This is in order to make attributes perfectly symmetric on the technology side, as our focus is on differences across attributes on the demand side. Second, the modelling implies symmetry across high and low quality realization. The second one is mainly used for simplification. It particularly implies that the probability that high quality is realized in attribute \(i\) if invested in \(i\) equals the probability that low quality is realized if invested in the other attribute.

Both symmetries are discussed in detail in section 7 where we also argue why giving up those symmetries basically preserves our results. Variable costs of providing the service are set to 0. Providers maximize expected profit, which will be equal to maximizing market share since the fee for the service is fixed.

There is a continuum of patients \(C\) in the market with mass 1. Each patient \(c \in C\) receives utility \(u(q)\) from utilizing a health service with quality \(q = (q_1, q_2)\) that is additively separable in attributes, i.e. \(U(q) = \sum_{i=1}^{2} u_i(q_i)\).\(^{10}\) We assume that the utility gain from high quality versus standard quality is higher in the first attribute than in the second attribute, i.e.

\[
\theta \equiv \frac{u_1(q_1 = h) - u_1(q_1 = l)}{u_2(q_2 = h) - u_2(q_2 = l)} > 1.
\]

Thus, high quality in attribute 1 is more important to generate increases in patient utility than high quality in attribute 2, in the following we refer to this property when we say that attribute 1 is the important attribute. In many health care applications, attribute 1 could be thought of as the medical quality, whereas attribute 2 is the friendliness and attentiveness of the staff and comfort of the amenities. Standard quality in the attribute medical quality could then be interpreted as the cure of a health problem with a certain probability of adverse side or medium term effects from the service, whereas high quality is cure of the health problem with a lower associated probability of adverse side or medium term effects from the service. We normalize consumption utility of standard quality in both attribute to zero \((u_1(q_1 = l) = u_2(q_2 = l) = 0)\) and high quality in the second attribute to 1 \((u_2(q_2 = h) = 1)\).\(^{11}\) This implies \(u_1(q_1 = h) = \theta > 1\). Each patient’s utility from abstaining from utilizing the service is \(u < 0\). The fee \(P\) for utilizing the health care service is paid for by a patient’s health insurance such that \(u(q)\) gives the net utility of consuming the health service for the patient.\(^{12}\)

Patients cannot perfectly observe the quality levels \(q^A\) and \(q^B\) of provider \(A\) and \(B\) respectively. They however receive signals about realized quality in the attributes from each provider before deciding on a provider. Each patient receives signals

\(^{10}\)Thus, patients are homogeneous in their valuation of the health care service. We will discuss heterogeneous patients in section 7. Note that \(U(q)\) can be interpreted as an expected utility level patients face once \(q\) is realized. This reflects a setting where providers with quality level \(q\) but might not serve constantly \(q\) but quality levels varying around \(q\) with expectation \(q\).

\(^{11}\)With this normalization we do not loose any generality since for our analysis we will always compare two expected utility levels such that only the size of \(\theta\) will play a role for the provider selection of the patients and net welfare effects.

\(^{12}\)Health insurers here are exogenous to contracting. Alternatively, instead of a health insurer paying the fee we could assume that the utility of not utilizing the health service is sufficiently low.
\[ s^j = (s^j_1, s^j_2) \in \{ll, lh, hl, hh\}, \ j \in \{A, B\} \]. Attribute signals \( s^j_i \) are generated with error \( \epsilon_i \) with
\[ \epsilon_i = P(s_i = h \mid q_i = l) = P(s_i = l \mid q_i = h) < \frac{1}{2} \], we write \( \epsilon = (\epsilon_1, \epsilon_2) \).
For better readability we write \( s^j \) for the signal a patient \( c \) receives instead of \( s^j_c \).
We furthermore might use \( s = s^j \) as long as it is clear from the context. We do not impose any assumptions on the correlation of signals across patients, i.e. we allow signals to be independently distributed as well as to be correlated.\(^{13}\)

Note that we do not model aggregation of signals across patients. One interpretation of the set-up could however be that there is aggregation, e.g. via a feedback platform, and through the aggregation all patients receive a signal in attribute \( i \) with error \( \epsilon_i \) as above. The notion that the signal precisions differ across attributes could then be driven by the fact that, regarding medical quality, there are only few reports about actual medical quality being published, whereas aggregation of patient feedback about amenities, staff and perceived quality leads to a more precise overall signal for these other attributes.

In our basic model, patients do not observe the providers’ resource allocation decisions. To evaluate signals from providers, each patient has beliefs \( b^j \in \{0, 1\} \) about the resource allocation \( a^j \), \( j \in \{A, B\} \). Again, we omit \( c \) as an index for each patient. Given any belief, patients update their belief about the quality of the service from providers according to Bayes’ rule. We denote the expected utility that a patient faces at provider \( j \) when she has belief \( b^j \) about the provider’s resource allocation and receives signal \( s^j = (s^j_1, s^j_2) \) by \( U_s[s^j | b^j, \epsilon] \).\(^{14}\) When receiving signal \( s^A \) from provider \( A \) and signal \( s^B \) from provider \( B \) a patient then chooses provider \( A \) if
\[ U_s[s^A | b^A, \epsilon] > U_s[s^B | b^B, \epsilon] \]
Ties are broken equally. For \( \epsilon \) fixed we write \( (s | b) \succ (s' | b') \) if \( U[s | b, \epsilon] > U[s' | b', \epsilon] \), i.e. when observing signal \( s \) with underlying belief \( b \) a patient faces a higher expected utility than when observing signal \( s' \) with underlying belief \( b' \).

To summarize, the timing of the game is as follows:

* **Stage 1:** Provider \( A \) and provider \( B \) simultaneously decide on their resource allocation \( a^A \) and \( a^B \), respectively. Patients do not observe resource allocations.

* **Stage 2:** For each provider the quality level in both attributes is realized.

* **Stage 3:** Each patient receives identically distributed attribute signals \( s^j_i \in \{h, l\} \) on \( q^j_i \) for all \( i \in \{1, 2\} \) and \( j \in \{A, B\} \) on realized quality.

* **Stage 4:** Each patient chooses a provider.

* **Stage 5:** Patient utility from utilizing the health service is realized.

\(^{13}\)It therefore includes the case that all patients receive the same signals. This shows that with the current set-up, we could also write the model as a representative patient that receives signals generated as above instead of a continuum of patients. We choose the continuum for the discussion of heterogeneous patients in section 7.

\(^{14}\)In this formulation, the belief does not have to be correct. However, in equilibrium we require beliefs to be consistent with actions.
Given the set-up, maximizing profits for providers corresponds to maximizing the probability of being selected as provider. In the following, we analyze perfect Bayesian Equilibria (PBE) in pure strategies and discuss potential mixing strategies in section 7. We require patient beliefs to be consistent with the providers’ resource allocations in equilibrium.

3 Focusing on attributes

A patient receives two signals \( s \), one from each provider. Which provider will the patient choose? Assume that one of the signals, say from provider \( A \), indicates standard quality in the first and high quality in the second attribute, i.e. \( s^A = lh \). The signal from provider \( B \) indicates high quality in the first and standard quality in the second attribute, i.e. \( s^B = hl \). Whether the signal of high quality in the first or in the second attribute is decisive for the patient’s provider choice now does not only depend on \( \theta \), the relative ex-post importance of high quality in attribute 1, but also on the relative attribute signal precisions, for any given beliefs and technology parameter \( p \). Thus, it might well be the case that if signals are \( hl \) for provider \( B \) and \( lh \) for provider \( A \), the patient chooses provider \( A \). This particularly implies that she picks provider \( A \) whenever provider \( A \)’s signal indicates high quality in the second attribute and provider \( B \)’s signal indicates low quality in the second attribute. Then, the signal of high quality in attribute 2 drives patient choice and we say that the patient focuses on attribute 2. This is generalized and formalized in the following definition of focusing.

Definition 1. (Focusing on Attributes) Fix \( \epsilon, p \) and \( \theta \). A patient...

(i) ...focuses on attribute \( i \) if for any two signals \( s^j = (s^j_1, s^j_2) \) and \( s^k = (s^k_1, s^k_2) \) with \( s^j_1 = h \) and \( s^k_1 = l \) and symmetric beliefs \( b^j = b^k \in \{0, 1\} \) signal \( s^j \) yields higher expected utility, i.e. \((s^j|b^j) \succ (s^k|b^k)\) for all \( b^k = b^j \in \{0, 1\} \).

(ii) ...strongly focuses on attribute \( i \) if for any two signals \( s^j = (s^j_1, s^j_2) \) and \( s^k = (s^k_1, s^k_2) \) with \( s^j_1 = h \) and \( s^k_1 = l \) and any beliefs \( b^j, b^k \in \{0, 1\} \) signal \( s^j \) yields higher expected utility, i.e. \((s^j|b^j) \succ (s^k|b^k)\) for all \( b^k, b^j \in \{0, 1\} \).

Since \((hh|b) \succ (s|b)\) for all \( s \neq hh \) and \((s|b) \succ (ll|b)\) for all \( s \neq ll \), the definition implies that focusing on attribute 1 is equivalent to \((hl|b) \succ (lh|b)\) for all beliefs \( b \) and focusing on attribute 2 is equivalent to \((lh|b) \succ (hl|b)\) for all beliefs \( b \). It analogously holds with any beliefs \( b \) and \( b' \) for strong focusing.

Note that for any given \( p \) and \( \theta \), whether patients that maximize their expected utility focus on an attribute or not only depends on the signal technology. This is because the requirements have to hold for all potential (symmetric) beliefs. In particular, the definition of focusing is not linked to equilibrium beliefs. Patient focusing is thus a direct property of the signal technology and not of equilibrium behavior.\(^{15}\)

\(^{15}\)If patients were able to observe \( a' \) we could replace \( b^j \) and \( b^k \) by \( a^j \) and \( a^k \) in the definition of focusing. Again, focusing does not depend on the equilibrium action.
Focusing behavior as defined above is rational in the sense that patients maximize their expected utility given beliefs and update according to Bayes’ rule. Thus, focusing here is different from focusing or salience in the behavioral economics literature (Bordalo et al., 2013, Koszegi and Szeidl, 2013) where there is an exogenous wedge between decision utility and consumption utility. Inefficiency will occur in our model via demand focusing that is nevertheless perfectly rational. Note that the focusing definition could however easily be adjusted to incorporate other, potentially non-rational decision rules where patients update differently or do not maximize expected utility. The focusing definition can also naturally be applied in more general product market settings.

Focusing on attributes depends on the signal error $\epsilon = (\epsilon_1, \epsilon_2)$, the investment technology $p$ and the utility weight $\theta$ of attribute 1. Intuitively, the smaller the signal error in one attribute keeping the signal precision in the other attribute fixed, the more informative the signals are in this attribute and the more likely it is that there is focusing on this attribute. The utility factor $\theta > 1$ implies that high quality provided in attribute 1 is more important than high quality provided in attribute 2. Hence, if signal precision in attribute 1 is not lower than in attribute 2, patients focus on attribute 1. However, conversely, if signal precision in attribute 2 is higher than in attribute 1, patients might focus on attribute 2 if $\theta$ is small enough. Generally, we can divide the attribute signal error space into focusing areas for given $p$ and $\theta$. The following lemma describes the separating lines for the focusing areas.

**Lemma 1.** Fix $p$ and $\theta > 1$. Then there exist continuous and increasing functions $f^{s_1} \leq f^{12} \leq f^{s_2}$ with $f^i : [0, \frac{1}{2}] \to [0, \frac{1}{2}]$, $i \in \{s_1, 12, s_2\}$, that divide the signal error space $[0, \frac{1}{2}]^2$ into focusing areas. A patient...

- …strongly focuses on attribute 2 iff $\epsilon_1 > f^{s_2}(\epsilon_2)$. There is $\epsilon^*_2 < \frac{1}{2}$ such that $f^{s_2}$ strictly increases on $[0, \epsilon^*_2]$ and $f^{s_2}(\epsilon_2) = \frac{1}{2}$ for all $\epsilon_2 \geq \epsilon^*_2$. $\epsilon^*_2 > 0$ iff $\theta < \frac{1}{1-2p}$.
- …focuses on attribute 2 iff $\epsilon_1 > f^{12}(\epsilon_2)$ and focuses on attribute 1 iff $\epsilon_1 < f^{12}(\epsilon_2)$. $f^{12}$ strictly increases in $\epsilon_2$. Furthermore, $0 < f^{12}(0) < f^{12}(\frac{1}{2}) = \frac{1}{2}$.
- …strongly focuses on attribute 1 iff $\epsilon_1 < f^{s_1}$. $f^{s_1}$ strictly increases in $\epsilon_2$ and $0 < f^{s_1}(0) < p < f^{s_1}(\frac{1}{2}) < \frac{1}{2}$.

For $\theta \to 1$ all functions converge to the 45-degree-line . For $\theta \to \infty$ the separating line of strong focusing on attribute 1 converges to $p$ and all other functions converge to $\frac{1}{2}$.

**Proof.** See appendix.

Figure 1 illustrates the separating lines for $p = 0.25$ and $\theta = 2$. Figure 2 illustrates the separating lines for again $p = 0.25$ but $\theta = 1.4$.

The two figures visualize how the focusing areas change when $\theta$ is varied. $\theta > 1$ implies that the area of focusing on attribute 1 is larger than the area of focusing on attribute 2. For large $\theta$ ($\theta > \frac{1}{1-2p}$, which is the case in figure 1), attribute 1 is important enough such that the area of strong focusing on attribute 2 vanishes.
Figure 1: $p = 0.25$ and $\theta = 2$

Figure 2: $p = 0.25$ and $\theta = 1.4$

completely. An area of focusing on attribute 2 exists independent of the magnitude of $\theta$. However, this area becomes arbitrarily small for $\theta$ converging to infinity. For $\theta \to 1$, all separating lines converge to the 45-degree-line.

The lemma shows that for a fixed error in one attribute, lowering the error in the other attribute makes the signals in this attribute more important and might shift the focus of a patient towards this attribute. For any $\theta > 1$ and $p$ we can choose $\epsilon_1$ large enough such that lowering $\epsilon_2$ results in a shift from focusing on attribute 1 to focusing on attribute 2. For the equilibrium and welfare analysis, we will be also interested in the conditions under which there is a shift from strong focusing on attribute 1 to focusing on attribute 2 when lowering $\epsilon_2$. Graphically, this translates to finding a horizontal line such that this line crosses both the area of strong focusing on 1 and the area of focusing on 2. In our examples, for instance, this is the case for $\epsilon_1 = 0.25$. The following corollary provides a sufficient condition on $\theta$ to find such an $\epsilon_1$.

**Corollary 1.** Fix $p$ and $\theta > 1$. There exist errors $\epsilon_1$ such that by varying $\epsilon_2$ the patients’ focus shifts from focusing on attribute 1 to focusing on attribute 2.

For $\theta < \bar{\theta} = \frac{1}{1-2p}$ there exist errors $\epsilon_1$ such that by varying $\epsilon_2$ the patients’ focus shifts from strong focusing on attribute 1 to focusing on attribute 2.

**Proof.** See appendix.

Particularly, by the monotonicity of the separating lines, for $\epsilon = (\epsilon_1, \epsilon_2)$ with $\epsilon_1$ large enough, patients (strongly) focus on attribute 1 for large $\epsilon_2$ and focus on attribute 2 for small $\epsilon_2$. For $\theta$ close enough to 1 it is even possible to find $\epsilon_1$ such that lowering $\epsilon_2$ results in a shift from strong focusing on attribute 1 to strong focusing on attribute 2. However, the weaker conditions presented in the corollary will be sufficient for our further analysis.
4 Provider quality incentives and equilibria

On the basis of the patients’ focusing behavior we can analyze the providers’ incentives to allocate their resources between attributes. We say that a strategy \( a^j \) of a provider \( j \) is dominant if for any patients’ beliefs \((b^A, b^B)\) and any strategy \( a^{-j} \) of the other provider, the strategy \( a^j \) is weakly better than any other strategy and strictly better for at least one combination of beliefs and the other provider’s strategy. We call \( a^j \) strictly dominant if it is strictly better for all combinations of patients’ beliefs \((b^A, b^B)\) and the other provider’s strategy \( a^B \).

In the following we show that once patients focus on an attribute and the signal error in this attribute is lower than the signal error in the other attribute, it is a dominant strategy for a provider to invest in this attribute. If focusing is strong, it is even a strictly dominant strategy to invest in the respective attribute.

**Proposition 1.** Let \( \theta, p \) and \( \epsilon = (\epsilon_1, \epsilon_2) \) be such that patients...

(i) ...(strongly) focus on attribute 2. Then it is a (strictly) dominant strategy for any provider \( j \) to invest in attribute 2, i.e. \( a^j = 0 \).

(ii) ...(strongly) focus on attribute 1 and \( \epsilon_1 < \epsilon_2 \). Then it is a (strictly) dominant strategy for any provider \( j \) to invest in attribute 1, i.e. \( a^j = 1 \).

**Proof.** See appendix.

The main idea of the proof is that for fixed beliefs of patients the resource allocation of the provider does not influence the expected utility of any patient when receiving a specific signal. This is because patients cannot observe the investment but perform the Bayesian updating when receiving the signal based on their belief. What changes when the provider selects a different investment strategy are the probabilities with which the signals are generated. If patients focus on one attribute and the signal error in this attribute is lower than in the other attribute, investing in this attribute generates “better” signals with higher probability than any other strategy. While focusing on attribute 2 already implies \( \epsilon_2 < \epsilon_1 \), we have to additionally condition on \( \epsilon_1 < \epsilon_2 \) when considering focusing on attribute 1.

One might wonder what optimal strategies are in case that there is focusing on attribute 1 but signal errors are such that \( \epsilon_1 > \epsilon_2 \). Focusing implies that for any fixed beliefs, \( hl \) yields higher expected utility than \( lh \). However, investing in attribute 1 instead of investing in attribute 2 does not unambiguously produce better signals with higher probability as it is the case for \( \epsilon_1 < \epsilon_2 \) such that optimal provider strategies then depend on signals errors in more detail.\(^{16}\)

The proposition implies that for strong focusing on attribute 2 it is a strictly dominant strategy for the providers to invest in attribute 2, i.e. it is strictly better for

\(^{16}\)If a provider invests in attribute 1 instead of 2, on the positive side, signal \( hl \) is produced with a higher probability on the cost of signal \( lh \). On the negative side, signal \( ll \) is produced with a higher probability on the cost of signal \( hh \). The closer \( (\epsilon_1, \epsilon_2) \) to the 45-degree line, the large the positive and the smaller the negative effect is, since the difference in expected utilities of \( hl \) and \( lh \) increases and the differences in probabilities of producing \( hh \) compared to \( ll \) decreases.
any strategy of the other provider and any combination of patients’ beliefs. However, if focusing is not strong, providers might be indifferent between different resource allocations. This crucially depends on the beliefs of patients. For symmetric beliefs about the providers’ resource allocations it is strictly better for the providers to invest in attribute 2 when patients focus on attribute 2. However, if patients have asymmetric beliefs, selection of the provider might be based only on the beliefs, ignoring the signals. Then providers are indifferent between different resource allocations. This might occur if patients believe that provider A invested in attribute 1 and provider B in attribute 2 and the parameters are such that patients choose provider A independent of the signals. For instance, \( \epsilon = (\epsilon_1, \epsilon_2) = \left( \frac{1}{2}, 0 \right) \) and \( \theta > \frac{1}{1-2p} \) satisfy \( (ll|b^A = 1) > (hh|b^B = 0) \) from which follows that patients ignore the signals and always select provider A anyway.

Proposition 1 directly implies that if patients focus on one attribute and the signal error in this attribute is lower than in the other attribute, investing in this attribute and corresponding beliefs is a Perfect Bayesian Equilibrium. Strong focusing (and \( \epsilon_1 < \epsilon_2 \) for focusing on attribute 1) implies uniqueness of the respective symmetric equilibrium. However, if focusing is not strong further equilibria might exist. Proposition 2 shows that the only further equilibria that might exist are asymmetric equilibria in which patients select the provider solely based on the beliefs and signals are irrelevant.

**Proposition 2.** Let \( \theta, p \) and \( \epsilon = (\epsilon_1, \epsilon_2) \) be such that patients...

(i) ... focus on attribute 2. Then \( (a^A, a^B) = (b^A, b^B) = (0, 0) \) is a PBE. Any PBE with \( (a^A, a^B) = (b^A, b^B) \neq (0, 0) \) is asymmetric, i.e. \( a^A \neq a^B \) and patients select provider A if and only if \( a^A = 1 \). Strong focusing on attribute 2 implies that the symmetric PBE \( (a^A, a^B) = (b^A, b^B) = (0, 0) \) is unique. Equilibrium is furthermore unique if, for a given \( \epsilon_i \), setting \( \epsilon_{-i} = \frac{1}{2} \) implies strong focusing on attribute \( i \), i.e. either patients strongly focus on attribute 1 once the signal in attribute 2 is uninformative or strongly focus on attribute 2 once the signal in attribute 1 is uninformative.

(ii) ... focus on attribute 1 and \( \epsilon_1 < \epsilon_2 \). Then \( (a^A, a^B) = (b^A, b^B) = (1, 1) \) is a PBE. Any PBE with \( (a^A, a^B) = (b^A, b^B) \neq (1, 1) \) is asymmetric, i.e. \( a^A \neq a^B \) and patients select provider A if and only if \( a^A = 1 \). Strong focusing on 1 implies uniqueness of the symmetric PBE \( (a^A, a^B) = (b^A, b^B) = (1, 1) \).

**Proof.** See appendix. 

For focusing on attribute 2, Proposition 2 shows that the equilibrium is not only unique under strong focusing, but also for signal errors that are such that there would be strong focusing on one attribute if the error for the other attribute would be set to \( \frac{1}{2} \), i.e. if patients were not to receive an informative signal in this attribute. This is because, if an asymmetric equilibrium exists, with consistent beliefs signal \( ll \) from the provider with higher \( a \) is preferred to signal \( hh \) from the other provider. This continues to hold when e.g. increasing \( \epsilon_2 \). Then, however, there is a contradiction with strong focusing, where \( hh \) is preferred to \( ll \) for any symmetric or asymmetric beliefs. The intuition for \( \epsilon_1 \) is the same.
For the cases where multiple equilibria exist, note that only the symmetric equilibrium where both providers invest in the attribute that patients focus on is an equilibrium in dominant strategies of the providers. Therefore, it is robust with respect to perturbation in the patients’ beliefs as the optimal strategy is independent of the beliefs. Furthermore, it is the only equilibrium where signals are informative for the patients such that they matter for their provider choice. Both reasonings might serve as a selection criterion for concentrating on symmetric equilibria.

Corollary 2. Fix $\theta$ and $p$ and consider $\epsilon = (\epsilon_1, \epsilon_2)$ such that patients focus on attribute $i$ and $\epsilon_i < \epsilon_{-i}$. Then the symmetric equilibrium where both providers invest in attribute $i$ is the only equilibrium in dominant strategies. It is furthermore the only equilibrium where signals are informative for patients.

5 Welfare and comparative statics

We can now discuss the welfare consequences of the patients’ focusing on attributes. Note that in the model, total provider surplus is fixed. For the welfare analysis, we will not consider the distribution of producer surplus between providers and henceforth concentrate on patient welfare. Thus, we will use the term welfare synonymous to patient welfare.

Now assume that quality $(q^A, q^B)$ is realized for provider $A$ and $B$ (and is unknown by the patients). We denote by $U_q[(q^A, q^B)|(b^A, b^B), \epsilon]$ the expected utility of quality provision of a patient when quality $(q^A, q^B)$ is realized and the patient, under beliefs $(b^A, b^B)$, chooses providers to maximize her expected utility given signals when signals are generated with errors $\epsilon = (\epsilon_1, \epsilon_2)$. Denote by $W[(a^A, a^B)|(b^A, b^B), (\epsilon_1, \epsilon_2)]$ welfare if providers’ resource allocations are $a = (a^A, a^B)$, patients have beliefs $b = (b^A, b^B)$, receive quality signals with error $\epsilon = (\epsilon_1, \epsilon_2)$ and choose providers maximizing expected utility given signals and beliefs. Then

$$W[(a^A, a^B)|(b^A, b^B), (\epsilon_1, \epsilon_2)] = \sum_{q^A} \sum_{q^B} \mathbb{P}(q^A|a^A)\mathbb{P}(q^B|a^B)U_q[(q^A, q^B)|(b^A, b^B), \epsilon]$$

(1)

where $\mathbb{P}(q^i|a^i)$ is the probability that $q^i$ is realized for resource allocation $a^i$.

There are two key drivers of welfare in the market: Firstly, a pure quality aspect, i.e. the expected consumption utility without considering signals, which is determined by the resource allocations. Secondly, a provider selection effect, i.e. selecting the provider whose quality realizations are high, which works through signal precision. This last one is important when considering the welfare effect of changes in signal precision, where a lower error c.p. improves selection based on true underlying quality. Before analyzing changes in the precision of the signals, we first look at welfare for a given signal precision.

\[^{17}\text{Note that our welfare definition directly incorporates optimal demand side behavior given beliefs. We could of course define } U_q[(q^A, q^B)|(b^A, b^B), \epsilon]\text{ based on patients’ actions more generally. We write welfare in this way to concentrate the analysis on the welfare effect of different provider resource allocations and patients beliefs. Note that in the welfare definition above, patients beliefs do not yet have to be correct, they only have to be correct when comparing welfare in equilibrium.}\]
Lemma 2. Fix $p$ and $\theta$. For all $\epsilon = (\epsilon_1, \epsilon_2)$ investing in attribute 1 and corresponding beliefs yields higher welfare than investing in attribute 2 and corresponding beliefs, i.e.

$$W[(1, 1)|(1, 1), (\epsilon_1, \epsilon_2)] > W[(0, 0)|(0, 0), (\epsilon_1, \epsilon_2)].$$

Proof. See appendix.

Thus, independent of $\epsilon$, if both providers invest in 1 (and patients have corresponding beliefs), welfare is higher than if both provider invest in 2 (and patients have corresponding beliefs). For $\epsilon_1 \leq \epsilon_2$, this is intuitive. For $\epsilon_1 > \epsilon_2$, there are some opposing effects. While, by investing in 1, providers increase the probability of quality $q^j = hl$ at the cost of $q^j = lh$ where $hl$ yields higher utility than $lh$, for high $\epsilon_1$ and low $\epsilon_2$ patients can barely infer information about quality realization in attribute 1 from signals while they reasonably can for attribute 2. In aggregation, however, the quality effect dominates the signal precision effect and welfare is higher when providers invest in attribute 1.

We already know that if $\epsilon$ is such that patients strongly focus on attribute 2, in the unique PBE both providers invest in attribute 2 with corresponding patients beliefs. Thus, when patients strongly focus on attribute 2, the unique PBE is inefficient. Under focusing on attribute 2, from Proposition 2 any equilibrium that is not the equilibrium in which both providers choose $a = 0$ is asymmetric and provider $j$ is chosen if and only if $a^j > a^{-j}$. I.e., except for the symmetric equilibrium with investment in attribute 2, in equilibrium a provider is chosen with probability 1, independently of the signals that the patients receive. Then, welfare in these equilibria is again lower compared to the situation where both providers invest in attribute 1 and patients hold the corresponding belief, as quality provision is partly inefficient, and there is no selection based on signals. This is summarized in Proposition 3 below.

Proposition 3. Fix $p$ and $\theta$. If $\epsilon$ is such that patients focus on attribute 2, any PBE is inefficient.

Proof. See appendix.

The interesting question is whether increasing signal precision increases welfare. For $\epsilon_1$ large enough we saw that by increasing the precision in the second attribute we might move from an equilibrium where both provider invest in attribute 1 to an equilibrium where both invest in attribute 2. From above, the latter is inefficient. The welfare effect when increasing signal precision is however not obvious as there are two effects. On the one hand, increasing signal precision might lead to a “worse” provision of quality. On the other hand, patients can better select the providers with high quality realizations. In the following we show that there exist parameter ranges such that increasing signal precision in attribute 2 for given $\epsilon_1$ unambiguously leads to a reduction in welfare if it induces a shift from both providers investing in attribute 1 to both providers investing in attribute 2.
Proposition 4. Fix $\theta > \frac{1-p-p^2}{1-2p}$, $p$ and $\epsilon_1$. Consider any $\epsilon_2$ and $\epsilon'_2$ such that patients focus on attribute 2 for $\epsilon = (\epsilon_1, \epsilon'_2)$. Then the following holds

$$W[(0,0)|(0,0), (\epsilon_1, \epsilon'_2)] < W[(1,1)|(1,1), (\epsilon_1, \epsilon_2)]$$

Proof. See appendix.

There is a lower bound on $\theta$ which ensures that, even for a maximal improvement in welfare from increasing signal precision – which would be the case for a change from $\epsilon_2 = \frac{1}{2}$ to $\epsilon_2 = 0$ –, the effect of reducing expected quality in attribute 1 with the shift in investment dominates. Proposition 4 implies in particular that if a change in $\epsilon_2$ causes a shift from an equilibrium where both providers invest in attribute 1 to an equilibrium where both providers invest in attribute 2, there is an unambiguous welfare loss. This is made precise in the following corollary.

Corollary 3. Fix $p$ and $\theta > \frac{1-p-p^2}{1-2p}$. Consider $\epsilon = (\epsilon_1, \epsilon_2)$ with $\epsilon_1 < \epsilon_2$ and $\epsilon' = (\epsilon_1, \epsilon'_2)$ such that for $\epsilon$ patients focus on attribute 1 and for $\epsilon'$ patients focus on attribute 2. Then an increase in the signal precision of attribute 2 from $\epsilon_2$ to $\epsilon'_2$ results in a welfare loss in the respective dominant strategy equilibrium.

If, furthermore, $\theta < \theta = \frac{1-p-p^2}{1-2p}$, consider $\epsilon = (\epsilon_1, \epsilon_2)$ with $\epsilon_1 < \epsilon_2$ and $\epsilon' = (\epsilon_1, \epsilon'_2)$ such that for $\epsilon$ patients strongly focus on attribute 1 and for $\epsilon'$ patients focus on attribute 2. Then an increase in the signal precision of attribute 2 from $\epsilon_2$ to $\epsilon'_2$ results in a welfare loss in equilibrium.

For $\epsilon = (\epsilon_1, \epsilon_2)$ and $\epsilon' = (\epsilon_1, \epsilon'_2)$ such that patients focus on attribute 1 for $\epsilon = (\epsilon_1, \epsilon_2)$ and on attribute 2 for $\epsilon' = (\epsilon_1, \epsilon'_2)$ multiple equilibria might exist. Therefore it is a priori not clear which equilibria are selected and thus whether a reduction in welfare occurs when lowering $\epsilon_2$ to $\epsilon'_2$. However, as discussed the symmetric equilibrium stands out as it is the only equilibrium in dominant strategies and robust with respect to perturbations in the beliefs. When only concentrating on equilibria in dominant strategies, for any $\theta > \frac{1-p-p^2}{1-2p}$ the welfare loss occurs when lowering $\epsilon_2$ such that it induces a shift from focusing on attribute 1 to focusing on attribute 2.

From Corollary 1 we know that for $\theta < \theta = \frac{1-p-p^2}{1-2p}$ there exist $\epsilon_1$ such that for $\epsilon = (\epsilon_1, \frac{1}{2})$ patients strongly focus on attribute 1 and for $\epsilon = (\epsilon_1, 0)$ patients focus on attribute 2. Thus, there exists $\epsilon$ and $\epsilon'$ as described above. Furthermore, Corollary 2 and Proposition 2 showed that in this case the equilibria are unique. $\theta > \frac{1-p-p^2}{1-2p}$ ensures that there is a welfare loss.

6 Quality reporting

So far we assumed that patients receive informative signals from each provider for all attributes. However, it might be a strategic choice of providers to send quality signals in attributes, e.g. via participation in evaluations and quality reporting, or establishment of an online feedback platform. From a policy perspective, it is important to understand which reporting policies induce optimal outcomes. When is it necessary to require providers to undertake quality reporting in certain attributes or ban reporting in others? In the following we first discuss strategic reporting of
providers. We then analyze different reporting policies and compare them to strategic reporting by providers.

**Strategic reporting**

To incorporate strategic quality reporting by providers, we change the game in the following way: Whether patients receive signals about attribute quality now depends on a reporting decision by providers. Each provider can decide at the time of resource allocation for each attribute whether to send signals about quality or not.\(^{18}\) We assume that a provider, when deciding about reporting, again cannot influence the precision of the signals. I.e., when reporting in attribute 1, the provider sends a signal about this attribute with error \(\epsilon_1\) and when reporting in attribute 2 he sends a signal about this attribute with error \(\epsilon_2\). The reason that he cannot influence the signal precision is again the general difficulty in observing, measuring and communicating quality in certain attributes. In terms of hospital quality, think of an external report or a platform where patients rate experienced quality in a hospital. While medical quality is rather difficult to evaluate, non-medical quality attributes are fairly easy to rate. Note that not reporting in attribute \(i\) is equivalent to a signal error of \(\frac{1}{2}\) in attribute \(i\).

Providers simultaneously decide on their resource allocation \(a\) and their reporting \(r\), i.e. in which attributes they want to report signals. Patients now might not receive signals in some attribute, but they update their beliefs about resource allocations depending on whether they receive signals in attributes. To keep the game simple, we exploit section 4’s results and restrict attention to strategies\(^{19}\)

\[
(a, r) \in \{(1, s_1), (0, s_2), (x(\epsilon), s_1 s_2), (1, \text{none})\},
\]

where \(s_1\) (\(s_2\)) stands for reporting only on attribute 1 (2) and \(s_1 s_2\) for reporting in both. Furthermore, \(x(\epsilon) \in \{0, 1\}\) with \(x(\epsilon) = 0\) if \(\epsilon\) is such that patient focusing is on attribute 2 and \(x(\epsilon) = 1\) if \(\epsilon_1 < \epsilon_2\) (and therefore patient focusing is on attribute 1 when they receive signals in both attributes). Thus, we consider the cases that (i) no signals are sent (no reporting) and providers invest in attribute 1, (ii) a provider sends the signal in the attribute that he invested in, but not in the other attribute (partial reporting), and (iii) signals in both attributes are sent, and investments are in the attribute that patients focus on when receiving signals in both attributes, given \(\epsilon\) (full reporting). Again we concentrate on pure strategy equilibria.

\(^{18}\)Crucial here is that providers do not know their quality at the time of deciding whether to take part in reporting. Thus, reporting is not signaling on realized quality.

\(^{19}\)Thereby we ensure the exclusion of implausible equilibria. For any combination of patient beliefs when the strategy space is not restricted, i.e. for any combination of reporting and resource allocation, any of the excluded strategies would be weakly dominated. For this note that we know from the results in section 4 that receiving a signal only in one attribute \(i\) implies that investing in attribute \(i\) weakly dominates investing in the other attribute (keeping the signal structure constant). With restricting strategies, we can restrict patient beliefs accordingly and can thereby rule out implausible equilibria where dominated strategies are selected by the providers. If no signals are reported, a provider’s action has no influence on any information the patient receive. In this case we assume that providers invest in 1 to avoid a point of discontinuity when considering receiving no signal in attribute 2 and facing signal errors \(\epsilon_1\) that are close to \(\frac{1}{2}\).
How do providers strategically report and invest? Assume that $\epsilon$ is such that if signals are sent in both attributes, there is focusing on 2. Now consider the situation that both providers report in both attributes and invest in attribute 2. Then, each provider is selected with probability $\frac{1}{2}$. Now assume a provider changes his reporting to only reporting in attribute 2, and not reporting in attribute 1. Then, this provider is selected with probability higher than $\frac{1}{2}$ when playing against the provider who is reporting in both attributes. This is because, since investments are in attribute 2, the provider reporting in both attributes sends a low quality signal in attribute 1 with probability higher than $\frac{1}{2}$, and since the signal is informative, in these cases the provider not reporting in attribute 1 is selected when the signal in the other attribute is the same. Thus, not reporting in the ‘weak’ attribute is a profitable deviation. This logic can be generalized to show that there are no equilibria with reporting in both attributes.

Lemma 3. Fix $p$ and $\theta$ and consider $\epsilon$ such that patients either focus on attribute 2 or they focus on attribute 1 and $\epsilon_1 < \epsilon_2$. Then, an equilibrium in which both providers report in both attributes does not exist.

Proof. See appendix.

To determine equilibria, a crucial consideration is how patients choose providers when one provider sends only a signal in attribute 1 (and invests in attribute 1) and the other provider sends a signal only in attribute 2 (and invests in attribute 2). Although patients do not observe resource allocations directly, they can update their beliefs when receiving, respectively not receiving, signals. Then, if $(h \cdot |1) \succ (-h|0)$ (i.e. a signal of high quality in attribute 1 and no signal in attribute 2 under belief 1 yields higher expected utility than a high quality signal in attribute 2 and no signal in attribute 1 under belief 0), the provider only sending a signal in attribute 1 is selected with probability greater than $\frac{1}{2}$. Then both providers sending a signal only in attribute 2 (with investing in 2) cannot be an equilibrium, as sending a signal only in attribute 1 is a profitable deviation. It is straightforward to show that

$$(h \cdot |1) \succ (-h|0) \forall \epsilon \iff \theta > \theta^c = \frac{1 - p}{1 - 2p}.$$  

This particularly also says that if $\theta < \theta^c$ there exist $\epsilon$, e.g. $\epsilon = (\frac{1}{2}, 0)$ and some neighborhood, such that $(-h|0) \succ (h \cdot |1)$. Note that $\overline{\theta} < \theta^c < \overline{\theta}$ with $\overline{\theta}$ and $\overline{\theta}$ as defined in the previous sections. We can now describe equilibria under strategic reporting.

Proposition 5. (i) Fix $p$ and $\theta$. For any $\epsilon$ such that $\epsilon_1 < \epsilon_2$ (and therefore patients focus on attribute 1), in the unique PBE providers invest in attribute 1 and report only on attribute 1.

(ii) Fix $p$ and $\theta > \theta^c$. Then there exist errors $\epsilon$ such that patients focus on attribute 2 when receiving signals in both attributes, however in the unique PBE providers invest in attribute 1 and report only on attribute 1.

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(iii) Fix $p$ and $\theta < \theta^c$. Then there exist errors $\epsilon$ such that in the unique PBE providers invest in attribute 2 and report only on attribute 2.

Proof. See appendix. □

Proposition 5 states that, under strategic reporting, there exist equilibria in which providers invest in an attribute and only publish quality signals in that respective attribute. Thus, it might be the case that not only resource allocation, but also information provision is inefficient. However, as the second part of Proposition 5 shows, if $\theta > \theta^c$, strategic reporting might even result in providers voluntarily withholding information in attribute 2 and investing in attribute 1, although $\epsilon$ is such that there would be focusing on 2.

To get an intuition for parts (ii) and (iii) of the proof, consider the extreme case of $\epsilon_1 = \frac{1}{2}$, e.g. there is no signal in attribute 1, and $\epsilon_2 = 0$, e.g. signals in attribute 2 are precise. Focusing on 2 when receiving both signals is therefore satisfied as for symmetric beliefs it always yields higher expected utility when receiving signal $h$ in attribute 2 than signal $l$. Since $\epsilon_1 = 0$ only two strategies are relevant: reporting about attribute 2 or not. It is a strictly dominant strategy for a provider to withhold information about attribute 2 if and only if the expected utility for the patient is higher if resources are concentrated on attribute 1 but she receives no signal about the realization, i.e. $(1 - p)\theta + p$, than if resources are concentrated on attribute 2 and she receives an exact signal about the realization in attribute 2, i.e. $p\theta + 1$. This holds if and only if $\theta > \theta^c = \frac{1 - p}{2p}$. The proof in the appendix elaborates some more general conditions on $\epsilon$ for which the claims hold. Particularly, it shows that claim (ii) is not only satisfied in a neighborhood of $\epsilon = \left(\frac{1}{2}, 0\right)$ but also once $p > \frac{1}{3}$ and $\epsilon$ is such that patients focus on attribute 2 and $\epsilon_1 > p$. For claim (iii) it is crucial that $\epsilon$ is such that $\left(h | 0\right) \succ \left(h | 1\right)$.

Comparison of reporting policies

Since not reporting in attribute $i$ is equivalent to a signal error of $\frac{1}{2}$ in attribute $i$, we can use our results from section 5 to determine the welfare of potential outcomes with reporting and thus optimal outcomes.

Recall that $W[a | b, \epsilon]$ denotes expected (patient) welfare if providers’ resource allocations are $a = (a^A, a^B)$, patients have belief $b = (b^A, b^B)$ and receive quality signals with errors $\epsilon = (\epsilon_1, \epsilon_2)$. Keeping the resource allocation constant and only improving signal precision by sending a signal, we have, by the simple selection effect, for any errors $(\epsilon_1, \epsilon_2)$,

$$W[(1, 1) | (1, 1), (\epsilon_1, \epsilon_2)] > W[(1, 1) | (1, 1), (\epsilon_1, \frac{1}{2})],$$

$$W[(0, 0) | (0, 0), (\epsilon_1, \epsilon_2)] > W[(0, 0) | (0, 0), (\frac{1}{2}, \epsilon_2)].$$

Furthermore, it holds that

$$W[(1, 1) | (1, 1), (\epsilon_1, \epsilon_2)] > W[(0, 0) | (0, 0), (\frac{1}{2}, \epsilon_2)].$$
since here the selection and resource allocation effect go in the same direction. For a selection and resource allocation effect going in opposite directions we know from Proposition 4, that if $\theta > \bar{\theta}$ and $\epsilon = (\epsilon_1, \epsilon_2)$ is such that patients focus on attribute 2, signal provision only in attribute 1 (with investing in 1) yields higher welfare than signal provision in both attributes and investing in 2, i.e.

$$W[(1, 1), (1, 1), (\epsilon_1, \frac{1}{2})] > W[(0, 0), (0, 0), (\epsilon_1, \epsilon_2)].$$

Put together, an equilibrium where both providers invest in attribute 2 and report only in attribute 2 is welfare dominated by an equilibrium in which providers invest in attribute 2 but report in both attributes. Whether investing in attribute 1 and reporting only in 1 dominates full reporting and investing in 2 depends on $\theta$ and $\epsilon$. From section 5 we know that for $\theta > \bar{\theta}$ it holds for all $\epsilon$. However, even for $\theta < \bar{\theta}$, as long as $\theta$ is not too small, reporting only in attribute 1 and both providers allocating resources in 1 might still yield higher welfare than full reporting with investment in attribute 2. In particular, for any given $\epsilon$, there exists $\hat{\theta}(\epsilon) \leq \bar{\theta}$ such that for $\theta > \hat{\theta}(\epsilon)$, reporting in 1 and investing in 1 is the optimal outcome, and for $\theta < \hat{\theta}(\epsilon)$, full reporting and investing in 2 is the optimal outcome.\footnote{Consider any $\epsilon$ and $\theta$. If reporting in 1 and investing in 1 and dominates full reporting and investing in 2, it does as well for any $\theta' > \theta$. If full reporting and investing in 2 dominates reporting in 1 and investing in 1 it does as well for any $\theta' < \theta$. Since we face the first case for all $\theta > \bar{\theta}$ and the second one for $\theta$ close enough to one, we can find such $\hat{\theta}(\epsilon)$.}

With the analysis of equilibria under strategic reporting and the welfare considerations above, we can now compare welfare of different reporting policies. A reporting policy describes for each attribute whether signal reporting is voluntary, mandatory or banned. Whenever we call a policy mandatory reporting in attribute $i$ or banning reporting in attribute $i$ it implies that reporting in the other attribute is a voluntary decision of the providers.

**Proposition 6.** (i) Fix $p$ and $\theta$. For any $\epsilon$ such that $\epsilon_1 < \epsilon_2$ (and therefore patients focus on attribute 1), mandatory full reporting is optimal and strictly increases welfare compared to voluntary reporting in both attributes. \(\square\)

(ii) Fix $p$ and $\theta > \theta^c$. Let $\epsilon$ be such that patients focus on attribute 2 when receiving signals in both attributes, and in the unique PBE under voluntary reporting in both attributes there is reporting only in attribute 1. Then, voluntary reporting in both attributes is already optimal. Banning reporting in attribute 2 as well as mandating reporting in attribute 1 are both optimal. Any policy mandating reporting in attribute 2 is not optimal.

(iii) Fix $p$ and $\theta < \theta^c$. Let $\epsilon$ be such that reporting only in attribute 2 is the unique PBE under strategic reporting. For $\theta > \hat{\theta}(\epsilon)$, mandating reporting in attribute 1 strictly increases welfare compared to voluntary reporting in both attributes but is not necessarily an optimal policy. Banning reporting in attribute 2 is optimal. For $\theta < \hat{\theta}(\epsilon)$, mandating full reporting is optimal while voluntary reporting as well as banning reporting in attribute 2 are not optimal. Banning reporting in attribute 2 might even decrease welfare compared to voluntary reporting in both attributes.

**Proof.** See appendix.
Note that, directly implied by the welfare discussion above, reporting in attribute 1 is always part of an optimal reporting policy, even if the signal precision in attribute 1 is very low. The proposition above shows that different policies might lead to optimal reporting in equilibrium. For some parameter constellations it is even not necessary to intervene with a specific policy to reach optimal reporting as providers might already voluntarily withhold information in attribute 2 when desirable from a welfare maximizing perspective. This occurs despite focussing on attribute 2 when patients receive signals in both attributes.

Proposition 6 also shows that mandating reporting in 1 might require at the same time to regulate reporting in attribute 2. Depending on the parameters it might be necessary to ban signals in attribute 2 or to mandate them.

From our discussion about difficulties in measuring and communicating medical quality compared to other attributes of a health care service, a particularly relevant case is the situation where the signal is imprecise on attribute 1 but fairly precise on attribute 2. To emphasize this case, we will summarize the results for high $\epsilon_1$ and low $\epsilon_2$ in the following corollary.

**Corollary 4.** Fix $p$ and $\theta$. Then for all errors $\epsilon = (\epsilon_1, \epsilon_2)$ close enough to $(\frac{1}{2}, 0)$, patients focus on attribute 2 and for

(i) $\theta > \theta^c$, voluntary reporting, mandatory reporting in attribute 1 as well as banning reporting in 2 are optimal policies. Mandatory full reporting is not optimal.
(ii) $\theta < \theta < \theta^c$, banning reporting in attribute 2 is necessary and sufficient for optimal reporting.
(iii) $\theta < \theta$, mandatory reporting on attribute 1 as well as mandatory full reporting are optimal policies, whereas banning reporting on attribute 2 is not.

Thus, considering signals that are very precise in attribute 2 but very imprecise in attribute 1, for high and low $\theta$ mandating reporting in 1 is already optimal. $\theta > \theta^c$ implies for errors close enough to $\epsilon = (\epsilon_1, \epsilon_2)$ providers voluntarily withhold information about attribute 2 which corresponds to optimal reporting. For $\theta < \theta$ optimal reporting is sending both signals. However, for errors close enough to $\epsilon = (\epsilon_1, \epsilon_2)$, providers voluntarily also send information in attribute 2 when information in attribute 1 is mandated. For intermediate $\theta$, i.e. $\theta < \theta < \theta^c$, it is not sufficient to mandate information in attribute 1 to yield optimal reporting. In this case it is necessary to control signals in attribute 2 by banning them.

For medical services, our results then imply that if the information structure is such that medical quality signals are imprecise but signals on amenities precise it depends on how important medical care compared to the other dimensions is whether or not optimal reporting includes attribute 2. Mandating information in attribute 1 yields optimal reporting except for some intermediate $\theta$ where an additional ban in attribute 2 is necessary.

**7 Discussion**

In this section, we will discuss the consequences of relaxing several modeling assumptions as well as extensions. We will first discuss (i) symmetries in the quality...
realization technology and (ii) symmetries across providers before discussing how to (iii) model the technology for splitting the resources or mixing strategies and the consequences of (iv) correlation in quality realizations. Finally, we discuss (v) observability of the providers’ resource allocations for patients and (vi) heterogeneity in $\theta$.

Symmetries in quality realization.
To keep the model tractable, it incorporates two symmetries about how the resource allocation impacts the quality realization, (1) a symmetric impact of the resource allocation on quality realization across attributes and that (2) quality realization probabilities are symmetrically spread around $\frac{1}{2}$. We will shortly discuss both in the following. Both symmetries arise from the assumption

$$P(q_1 = h | a^j) = (1 - p)a^j + p(1 - a^j) = P(q_2 = h | 1 - a^j).$$

We do not need the symmetries for our qualitative results - the symmetries rather shift thresholds but do not change the qualitative claims. In the following we explain how the symmetries can be removed and the implications of allowing for asymmetries.

**Symmetric impact of resource allocation on quality realization across attributes.** We assume that for any resource allocation decision $a^j$ the probability that high quality is realized in attribute 1 equals the probability of high quality realization in attribute 2 if resources are allocated according to $1 - a^j$, i.e. $P(q_1 = h | a^j) = P(q_2 = h | 1 - a^j)$. The parameter $p$ can be interpreted as a measure of how effective resources in both attributes are for quality realization. Our assumption therefore reflects a symmetry across the two attribute meaning that resources have the same impact of quality realization for both attributes.

One way to give up this assumption is to consider different parameters $p_1, p_2 \in (0, \frac{1}{2})$ for the effectiveness of the resource allocation for both attributes, particularly

$$P(q_1 = h | a^j) = (1 - p_1)a^j + p_1(1 - a^j)$$

$$P(q_2 = h | 1 - a^j) = (1 - p_2)a^j + p_2(1 - a^j).$$

Once $p_1$ is smaller than $p_2$ resources are more effective in attribute 1 than in attribute 2 on quality realization and the other way around. This additional asymmetry does not qualitatively change our results but would only add one additional asymmetry across attributes in addition to signal errors $\epsilon$ and relevance $\theta$ to our model. Thus, $p_1 < p_2$ would additionally favor investments in attribute 1 while $p_1 > p_2$ would favor investments in attribute 2. This produces a shift in the borders of focusing as well as when investing in one attribute is a dominant strategy. The smaller the difference between $p_1$ and $p_2$, the closer we come to the presented results. However, since this source of asymmetry across attributes is not the focus of our work we do not include it into our basic model while being aware that technological asymmetries across attributes exist in applications.

**Quality realization probabilities symmetrically spread around $\frac{1}{2}$**. The second symmetry behind our assumption on how $a^j$ impacts quality realization is that the probability
that high (low) quality in an attribute is realized investing in $i$ and the probability that high (low) quality is realized investing in the other attribute add up to one, i.e. $P(q_i = h|a^j) = 1 - P(q_i = h|1-a^j)$. It can be interpreted as a symmetry across low and high quality realization.

This symmetry can be given up by assuming instead

$$P(q_1 = h|a^j) = a^j \bar{p} + (1 - a^j)p = P(q_2 = h|1-a^j)$$

with $p < \bar{p}$

Here, the probability $1 - p$ of high quality realization in the attribute a provider invested in is replaced by $\bar{p}$ and the probability $p$ of high quality realization in the attribute the provider did not invest in is replaced by $p$. Then, resources are still equally effective in both attributes (see discussion point above), but probabilities of high quality realization in one attribute for $a^j$ and $1-a^j$ are not any more symmetrically spread around $\frac{1}{2}$. Particularly, if both $\bar{p}$ and $p$ are rather high, the probability that high quality is realized in one attribute is high independently of whether the provider invested in the attribute or not (and the probability of low quality realization is low). For $\bar{p}$ and $p$ both being rather low, the probability that high quality is realized in one attribute is low independently of the provider’s action.

In the following we argue that our qualitative results do not change but only critical values for $\theta$ or $\epsilon$ might change. For this, we first consider how the error space is divided into focusing areas (see Lemma 1). For any fixed $(\bar{p}, p)$ and $\theta$, we again can describe separating lines by monotonically increasing functions. Again, an area of focusing on attribute 2 and attribute 1 and an area of strong focusing on attribute 1 always exist. The area of strong focusing on attribute 2 exists if and only if $\theta < \frac{1}{\bar{p}-p}$. Thus, the general characteristics of the separating lines for the focusing areas remain the same. The incentives for the providers do not change and thus, Proposition 1 can be formulated in the same way. Particularly, once patients focus on one attribute and the signal error in this attribute is smaller than in the other attribute, it is a weakly dominant strategy to invest in this attribute. Strong focusing implies strict dominance.

Considering welfare implications, what has to be adjusted is the critical value $\bar{\theta}$ above which the negative welfare effect of a shift in resources from attribute 1 to attribute 2 dominates the positive welfare effects from selection improvements when increasing signal precision in attribute 2. Particularly, $\bar{\theta} = \frac{1-(1-p)^2-p}{\bar{p}-p}$.

**Symmetric providers.**

We consider symmetric providers in the sense that both face the same signal errors and the same realization probabilities for a resource allocation decision $a^j$. If we assumed asymmetric provider in the sense that they might differ in $\epsilon$ and $p$ the main drivers of the model are the same. What changes is that the focusing areas of patients might differ across providers. However, if for both providers patients (strongly) focus on the same attribute there is no qualitative difference in the results except that the bounds for the critical $\theta$ might change. If for one provider the patient focuses on one attribute and for the other provider on the other attribute, only asymmetric equilibria might exist.
Splitting resources and mixing strategies.

We let the providers choose among investing their fixed resources either in attribute 1 or in attribute 2, i.e. they choose \( a^j \in \{0, 1\} \). A natural way to extend the set of strategies is to consider divisible resources and allow providers to choose \( a^j \in [0, 1] \). We then interpret \( a^j \) as a share \( a^j \) of the resources being invested in attribute 1 while the other part is invested in attribute 2. The implications of allowing for a budget split crucially depend on how a budget split translates into quality realization probabilities.

Once the probability of high quality realization is concave enough in the share of resources invested in this attribute, splitting resources is not effective enough for high quality realizations and the game we considered in our basic model would basically remain the same. However, there are several other options of how to interpret a budget split in terms of quality realization probabilities, two of which we discuss below. In both cases, the multi-attribute character of the good is important. Furthermore, we show that at least for small \( \theta \) and errors close enough to \((\epsilon_1, \epsilon_2) = (\frac{1}{2}, 0)\) our results continue to hold.

Mixing strategies. One way to interpret the budget split is interpreting it as mixing strategies \( a^j \in \{0, 1\} \). Then, quality realization for any \( a^j \in [0, 1] \) can be denoted as

\[
P(q_1,q_2|a^j) = a^jP(q_1,q_2|a^j = 1) + (1-a^j)P(q_1,q_2|a^j = 0).
\]

Particularly, realization probabilities for an equal budget split of \( a^j = \frac{1}{2} \) are \( P(hl) = P(ll) = p(1-p) \) and \( P(hl) = P(ll) = \frac{1}{2} - p(1-p) \). Since furthermore \( P(q_1 = h) = P(q_2 = h) = \frac{1}{2} \), for \( a^j \not\in \{0, 1\} \) quality realization in the attributes is not any more independent but negatively correlated (see also discussion about correlation).

Now we consider errors \( \epsilon \) that are close enough to \((\frac{1}{2}, 0)\). For the extreme case of \( \epsilon = (\frac{1}{2}, 0) \), signals in attribute 1 are uninformative while signals in attribute 2 are precise. However, in contrast to our basic model where quality realization is always independent in each attribute, a signal of high quality in attribute 2 is not unambiguously good. Whenever patients believed that a provider mixed, i.e. \( a^j \in (0, 1) \) a signal \( h \) in attribute 2 does not only indicate high quality in attribute 2, but, at the same time indicates that the probability of high quality in attribute 1 is lower than the probability of low quality in attribute 1.

As long as \( \theta \) is very small, \( s_2 = h \) yields higher expected utility than \( s_2 = l \). This implies that concentrating resources on 2 is a dominant strategy and our previous results remain valid. However, if \( \theta \) is very large, depending on the beliefs of patients, \( s_2 = l \) might yield higher expected utility than \( s_2 = h \). Then, providers have an incentive to concentrate their resources on attribute 1 and it is not valid any more that for errors close enough to \((\frac{1}{2}, 0)\), concentrating resources on attribute 2 is a dominant strategy.

Budget split with keeping independent realization. An alternative way how a budget split \( a^j \in [0, 1] \) translates into quality realization probabilities is keeping the independent quality realization across attributes and defining the quality realization...
probability in attribute $i$ as

$$P(q_i = h|a^j) = a^j P(q_i = h|a^j = 1) + (1 - a^j) P(q_i = h|a^j = 0).$$

The characteristics of the focusing areas generally remain the same, except that the areas of focusing on 1 and 2 will slightly shrink. For $\epsilon_1 = \frac{1}{2}$, $f^1(\frac{1}{2})$ decreases and $f^2(\frac{1}{2})$ increases compared to our basic model. Particularly, there will be an area where patients neither focus on attribute 1 nor on attribute 2. However, the areas of strong focusing remain exactly the same as before because $a^j \in \{0, 1\}$ will be the extreme cases that define the borders.

What might not remain the same are investment incentives. This is because a budget split makes a quality realization of $hh$ more likely compared to a concentration of resources on 1 or 2. For $a^j = \frac{1}{2}$ all possible quality levels are realized with equal probability. Particularly, the probability that $hh$ (as well as $ll$) is realized is $\frac{1}{2}$ while it is $p(1 - p)$ for investing in 1 or investing in 2. Thus, the probabilities for $hh$ and $ll$ increase when splitting the budget while the sum of the probabilities for $hl$ and $lh$ decrease.

However, as long as the signal errors are close enough to $\epsilon = (\frac{1}{2}, 0)$, it is a dominant strategy to concentrate resources on attribute 2. This is because putting more resources on attribute 1 has only marginal effects on signals in attribute 1 while putting more resources on attribute 2 significantly increases the probability of high quality signals in attribute 2 (when patients’ beliefs are fixed and with it expected utilities of a specific signal). Thus, our results remain the same at least for errors close enough to $\epsilon = (\frac{1}{2}, 0)$.

**Independent quality realization.**

We assume that quality is realized independently for both attributes. One might think of settings where quality realization in both attributes is correlated, i.e. the probability that high quality is realized differs depending on whether high or low quality was realized in the other attribute. This might be either a positive or negative correlation.

Consider the case of a positive correlation. Focusing can be defined analogously and for focusing on 2 it still holds that investing in 2 is a dominant strategy and equilibria are inefficient. The area for focusing on 2 might be even larger than for independent quality realization. However, whether increasing signal precision in attribute 2 results in a welfare loss depends on how strong the correlation is. For strong correlations, the selection effect might always dominate the investment effect.

When there is a negative correlation, the mechanisms differ. Again, we can define focusing on attribute 2 as before. However, a signal $hh$ might yield lower expected utility than a signal $hl$. For a low error in attribute 2, a high error in attribute 1 and a strong negative correlation, a signal $hl$ is an indicator for high quality in attribute 1. Those effects might result in the area of focusing on 2 being smaller than before, particularly, patients might not always focus for $(\epsilon_1, \epsilon_2) = (\frac{1}{2}, 0)$. However, if it is an equilibrium that both providers invest in 2 the welfare loss when varying $\epsilon_2$ might be even larger.
Observable Resource Allocation.

In our model, patients have beliefs about the providers’ resource allocations. In the following, we investigate how our results change if patients can observe the resource allocation, but still do not observe the realization of quality and again receives signals about it. The main difference to the case where the resource allocation is unobservable is that by choosing a particular $a$ the providers now send additional information. This has the following effect: Under unobservable provider choice in Proposition 1, for a certain belief of a patient a change in a provider’s action did not change the expected utility of a signal, but only the probabilities with which the signals are generated. When $a$ is however observable, a change in a provider’s action also changes the expected utility of a particular signal.

Then, for parameter constellations where investing in attribute 2 is a strictly dominant strategy under non-observability of provider choice, investing in attribute 1 might be a strictly dominant strategy once resource allocations are observable, since patients now update with the investment choice and demand shifts more strongly. If this is the case, the inefficiency from low expected quality in attribute 1 in equilibrium disappears once the resource allocations are observable. Whether this change occurs depends on the probability $e_2 = e_2(1 - p) + p(1 - e_2)$ that a low signal for attribute 2 is generated if the provider invests in attribute 2. For low $e_2$, i.e. if the probability that a high signal is generated in attribute 2 remains high, observability of investments does not influence the equilibrium outcome as investing in attribute 2 remains more profitable. However, for large $e_2$ the equilibrium might differ.

Proposition 7. (Observable Resource Allocation) Fix $\theta < \frac{1}{1 - 4p}$. Let $\epsilon = (\epsilon_1, \epsilon_2)$ be such that patients strongly focus on attribute 2. Define $e_2 = \epsilon_2(1 - p) + p(1 - e_2)$. If $e_2 < 1 - \sqrt{\frac{1}{2}}$ investing in attribute 2 is a strictly dominant strategy such that the corresponding symmetric PBE is unique.

If $e_2 > \frac{3 - \sqrt{2}}{2}$, investing in attribute 1 is a strictly dominant strategy such that the corresponding symmetric PBE is unique.

Proof. See appendix.

The intuition behind Proposition 7 is that if $p$ or $e_2$ are rather large (which implies that $e_2$ is rather large), investing in attribute 2 does not payoff for the provider as the probability that only a low signal in attribute 2 is generated is high. On the other hand, for non-observable resource allocations with given patients’ beliefs, investing in attribute 2 might be a dominant strategy as for this only that $e_2$ is small enough is crucial. If $e_2$ is intermediate such that it is not covered by the bounds presented in the proposition, it depends on the specific combination of the parameters whether investing in attribute 1 or investing in attribute 2 is strictly dominant.

From a welfare perspective, observable resource allocations could enhance efficiency in equilibrium as increasing precision in the less important attribute might not induce the negative resource allocation effect under observable resource allocations. However, it requires that $e_2$ is large enough. Applying it to our leading example of attribute 2 representing amenities etc., we rather expect a high probability that investments in this attribute are reflected in the signal, i.e. $e_2$ is low. Furthermore,
it might be difficult for patients to interpret resource allocations directly.

Assumption of homogeneous $\theta$

We set-up the model to particularly look at a situation where patients are homogeneous and all have a higher utility from high quality in one attribute, i.e. where results are not driven by heterogeneous patient valuations for attributes. However, patients might of course differ in the utility $\theta$ of high quality in the first attribute compared to high quality in the second attribute.$^{21}$ For different clinical areas different $\theta$ hold. For instance, $\theta$ for patients suffering from cancer should be rather high as clinical factors are much more important than amenities. On the other hand, for births $\theta$ might be rather low as generally not many complications are expected. Our results than can be applied for each health area separately. In areas with a high $\theta$ investing in attribute 1 is an equilibrium while in areas with a low $\theta$ investing in attribute 2 might be an equilibrium.

Even within one area $\theta$ might differ among patients. Reasons might be differences in individual preferences or the severity of the individual patient’s health case. Consider any signal error $\epsilon = (\epsilon_1, \epsilon_2)$ with $\epsilon_2 < \epsilon_1$. This implies that there is a threshold $\theta_2$ such that for $\theta < \theta_2$ the patients strongly focus on attribute 2. It is clear that if for each patient $c \in C$, $\theta_k < \theta_2$ holds, investing in attribute 2 is a dominant strategy. Analogously, there is a threshold $\theta_1$ such that if for each patient $c \in C$, $\theta_k > \theta_1$ holds, investing in attribute 1 is a dominant strategy. Generally, which effect dominates depends on the distribution of $\theta$ in the population. If the mass of patients whose $\theta$ is below (above) the respective critical thresholds is sufficiently large, then investing in attribute 2 (1) is an equilibrium outcome.

8 Conclusion

We model quality competition among health care providers in a market where health care services have multiple quality attributes and patients observe attribute quality only imperfectly before deciding on a provider. A patient focuses on a particular attribute if a high quality signal in this attribute drives her provider choice. Focusing is strong if this is the case for all combinations of beliefs that the patient has about the underlying resource allocations of providers. We show that, even if high quality in one attribute is less important in terms of patient utility, patients might focus on this attribute such that providers invest in quality improvement in this attribute. If signal precision is such that patients focus on this less important attribute, any equilibrium is inefficient. An increase in signal precision can then lead to a welfare reduction as the positive effect of a better provider selection from an increase in signal precision might be overcompensated by the negative effect that a shift in patient focusing has on provider quality choice. When providers can choose reporting in the

$^{21}$Note that ex-post differences in $\theta$, i.e. differences that occur after the decision for a provider, can be considered as being already incorporated in $\theta$ when interpreting utilities for each quality state as expected utilities. Reasons for ex-post heterogeneity includes e.g. differences in quality perception.
form of sending informative signals strategically, we furthermore show that providers
do not report in all attributes such that not only resource allocations, but also re-
porting might be inefficient.
In health care, there has been an increase in the availability of information about
provider quality via e.g. quality reporting requirements or public feedback platforms.
For hospital report cards, most empirical literature finds positive but small patient
reactions to publicized quality information. Our model is fully consistent with the
positive demand effect: if quality reporting reduces signal error only in the medical
attribute, it unambiguously increases welfare if the effect is strong enough. However,
reporting requirements or the increasing availability of public feedback platforms of-
ten also improve the precision of information about other dimensions. Better overall
information about health care providers might however imply a higher relative pre-
cision of information in the less important quality attributes like the hotel properties
of hospitals, with adverse effects on quality. For overall welfare, the quality reporting
policy is crucial. While under optimal reporting signals in the more important at-
tributes are always published, banning reporting in less important attributes might
be necessary.

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Appendix

Preliminaries

Before turning to the proofs we introduce a notation that will be helpful to calculate the expected utilities \( U_s[s|1, \epsilon] \) and \( U_s[s|0, \epsilon] \) when receiving a signal \( s \), facing signal errors \( \epsilon \) and having a belief \( b = 1 \) or \( b = 0 \).

Quality realizes independently for each attribute. Therefore, we can calculate the expected utilities separately for each attribute for \( b \in \{0, 1\} \). To calculate and compare expected utilities the following function will be useful to us.

\[
f(y, z) := \frac{yz}{yz + (1 - y)(1 - z)} \quad \text{for} \quad y \in [0, 1], z \in (0, 1)
\]

The function \( f(y, z) \) has the following properties

- \( f(y, z) = f(z, y) \) and \( f(y, z) \) is increasing in \( y \) and in \( z \)
- \( f(y, z) + f(1 - y, 1 - z) = 1 \)
- \( f(y, 1 - z) - f(y, z) = f(1 - y, 1 - z) - f(1 - y, z) \) is decreasing in \( z \) and symmetrically spread around \( y = \frac{1}{2} \). For \( z < \frac{1}{2} \) it is increasing in \( y \in (0, \frac{1}{2}) \) and decreasing in \( y \in (\frac{1}{2}, 1) \), analogously for \( z > \frac{1}{2} \) it is decreasing in \( y \in (0, \frac{1}{2}) \) and increasing in \( y \in (\frac{1}{2}, 1) \).

To see how the function is related to expected utilities when observing signals consider any signal \( s_i \) about quality in attribute 1, any corresponding signal error \( \epsilon_i \) and any belief \( b \in \{0, 1\} \) the patients might have. The expected utility \( U_s[s_i] \) when observing \( s_i \in \{l, h\} \) in attribute \( i \) then is

\[
U_s[s_i|b, \epsilon_i] = \mathbb{P}(q_i = h|s_i, b)u_i(q_i = h) = \frac{\mathbb{P}(s_i|q_i = h)\mathbb{P}(q_i = h|b)}{\mathbb{P}(s_i|b)}u_i(q_i = h) = \frac{\mathbb{P}(s_i|q_i = h)\mathbb{P}(q_i = h|b)}{\mathbb{P}(s_i|q_i = h)\mathbb{P}(q_i = h|b) + \mathbb{P}(s_i|q_i = l)\mathbb{P}(q_i = l|b)}u_i(q_i = h) = f(y(s_i), z(i))u_i(q_i = h)\quad \text{with} \quad y = \mathbb{P}(s_i|q_i = h) \quad \text{and} \quad z = \mathbb{P}(q_i = h|b) \quad (3)
\]

\( y(s_i = h) = 1 - \epsilon_i \) and \( y(s_i = l) = \epsilon_i \). \( z_i \) is the probability that high quality is served in attribute \( i \), therefore \( z_1 = 1 - z_2 = 1 - p \) if \( b = 1 \) and \( z_1 = 1 - z_2 = p \) if \( b = 0 \). Thus, whenever patients receive a signal \( s = s_1s_2 \) from any provider and have a belief \( b \in \{0, 1\} \) the expected utility if choosing this provider is as follows.

\[
U_s[s_1s_2|1, \epsilon] = f(y(s_1), 1 - p)\theta + f(y(s_2), p) \quad \text{with} \quad y(s_i = h) = 1 - \epsilon_i = 1 - y(s_i = l) \quad (2)
\]

\[
U_s[s_1s_2|0, \epsilon] = f(y(s_1), p)\theta + f(y(s_2), 1 - p) \quad \text{with} \quad y(s_i = h) = 1 - \epsilon_i = 1 - y(s_i = l) \quad (3)
\]

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Proofs

Proof of Lemma 1. To define the separating lines for the areas of focusing, the difference in expected utilities when observing signal lh with underlying belief \( b \in \{0, 1\} \) and signal hl with underlying belief \( b' \in \{0, 1\} \) is crucial. It will be convenient to use beliefs about the probability \( x \) of high quality realization in attribute 1 instead of beliefs \( b \) about the resource allocation. Then, \( x = 1 - p \) for \( b = 1 \) and \( x = 0 \) for \( b = 0 \). In the following, when we use \( b \) we refer to the beliefs \( b \in \{0, 1\} \) about the actions of the providers and when we use \( x \) we refer to corresponding beliefs \( x \in \{p, 1 - p\} \) about the high quality realization in attribute 1. We define

\[
g(x, x', \epsilon_1, \epsilon_2) = U_s[hl, b, \epsilon_1, \epsilon_2] - U_s[lh, b', \epsilon_1, \epsilon_2]
\]

(4)

\[
g = [f(1 - \epsilon_1, x) - f(\epsilon_1, x')]\theta - [f(1 - \epsilon_2, 1 - x') - f(\epsilon_2, 1 - x)]
\]

(5)

\[
g = [f(1 - \epsilon_1, x) - f(\epsilon_1, x')]\theta - [f(1 - \epsilon_2, x) - f(\epsilon_2, x')]
\]

(6)

where \( f \) is defined in the preliminaries and the last inequality is implied by the characteristics of \( f \).

The sign of \( g \) is important for the focusing of the patients since \( (hl|b) \succ (lh|b) \Leftrightarrow g(b, b', \epsilon_1, \epsilon_2) > 0 \) and \( (lh|b') \succ (hl|b) \Leftrightarrow g(b, b', \epsilon_1, \epsilon_2) < 0 \). Equation (6) together with the fact that \( f(y, z) \) is strictly increasing in \( y \) for \( z \neq 0 \) we can deduce that \( g(b, b', \epsilon_1, \epsilon_2) \) is strictly decreasing in \( \epsilon_1 \) and strictly increasing in \( \epsilon_2 \). Therefore, if for \( (\epsilon_1^*, \epsilon_2^*) \) a patient (strictly) focuses on attribute 2 he (strictly) focuses on attribute 2 for all \( (\epsilon_1, \epsilon_2) \) with \( \epsilon_1 > \epsilon_1^* \) and \( (\epsilon_1^*, \epsilon_2) \) with \( \epsilon_2 < \epsilon_2^* \) as well. The same holds for (strict) focusing on attribute 1 with reversed signs.

Definition of the separating lines We use the function \( g \) to describe the separating lines of the four focusing areas. For a fixed \( \epsilon_2 \) define \( \epsilon_1^*(x, x') \) as the unique root of \( g(\cdot, \epsilon_2, x, x') \) if existent and \( \frac{1}{2} \) otherwise. If existent, the root is unique because of the monotonicity characteristics.

\[
f^2(\epsilon_2) = \max_{x,x'}\{\epsilon_1^*(x, x')|x, x' \in \{p, 1 - p\}\}
\]

\[
f^2(\epsilon_2) = \max_x\{\epsilon_1^*(x, x)|x \in \{p, 1 - p\}\}
\]

\[
f^1(\epsilon_2) = \min_x\{\epsilon_1^*(x, x)|x \in \{p, 1 - p\}\}
\]

\[
f^1(\epsilon_2) = \min_{x,x'}\{\epsilon_1^*(x, x')|x, x' \in \{p, 1 - p\}\}
\]

Once we show that \( f^2 = f^1 \) and define \( f^{12} = f^1 = f^2 \), the focusing behavior as described in the lemma follows by the definitions of the functions. For this note that \( \epsilon_1 = 0 \) implies \( g = \theta - [f(1 - \epsilon_2, x) - f(\epsilon_2, x')] > 0 \) independent of the beliefs.

Characteristics of the separating lines Since \( g \) is continuous and monotonically decreasing in \( \epsilon_1 \) and increasing in \( \epsilon_2 \) the functions \( f^i \) are continuous and increasing in \( \epsilon_2 \). The more specific characteristics are as follows.

Focusing on 1 or 2: For any symmetric beliefs, \( g \) can be described by

\[
g(x, x, \epsilon_1, \epsilon_2) = [f(1 - \epsilon_1, x) - f(\epsilon_1, x)]\theta - [f(1 - \epsilon_2, x) - f(\epsilon_2, x)]
\]
\[ g(b, b, 0\epsilon_2) = \theta - [f(1 - \epsilon_2, x) - f(\epsilon_2, x)] > 0 \] and \[ g(b, b, \frac{1}{2}\epsilon_2) = 0 - [f(1 - \epsilon_2, x) - f(\epsilon_2, x)] \leq 0. \] Strict monotonicity of \( g \) in \( \epsilon_1 \) therefore implies that there exists a unique root \( \epsilon^*_1(x, \epsilon) \) such that \( g(x, x, \epsilon^*_1) = 0 \). Furthermore, since \( f(1 - \epsilon_i, x) - f(\epsilon_i, x) = f(1 - \epsilon_i, 1 - x) - f(\epsilon_i, 1 - x) \) the unique root \( \epsilon^*_1 \) of \( g(b, b, \epsilon_1\epsilon_2) \) is the same for \( b = 0 \) and \( b = 1 \). Thus, we can define \( f^{12} = f^1 = f^2 = \epsilon^*_1(1 - p, 1 - p) \). For \( \epsilon_2 = \frac{1}{2} \) we have \( f^{12}(\frac{1}{2}) = \frac{1}{2} \).

Since \( \theta > 1 \), for any \( \epsilon_2 \) the function \( g(x, x, \epsilon_1\epsilon_2) \) can only be 0 if \( \epsilon_1 \geq \epsilon_2 \). Thus, \( f^1 = f^2 \) lies above the 45-degree line and patients focus on 1 for any errors with \( \epsilon_1 \leq \epsilon_2 \).

**Strong focusing on 2:** Consider \( \theta = \frac{1}{1 - 2p} \). Then, for any beliefs \( x, x' \) we get \( g(x, x', \frac{1}{2}0) = (x - x')\frac{1}{2} - 1 \leq 0 \) with equality for \( x = 1 - 2p \) and \( x' = p \). Therefore, for \( \theta > \frac{1}{1 - 2p} \) there always exist beliefs such that no root exist and therefore \( f^{12}(\epsilon_2) = \frac{1}{2} \) for all \( \epsilon_2 \).

No assume \( \theta < \frac{1}{1 - 2p} \). Particularly, \( 0 < f^{21}(0) < \frac{1}{2} \), and \( f^{22}(\frac{1}{2}) = \frac{1}{2} \). Define \( \epsilon^*_2 \) such that \( g(1 - p, p, \frac{1}{2}\epsilon_2) = 0 \). Then the following holds: \( 0 < \epsilon^*_2 < \frac{1}{2} \) and for all \( \epsilon_2 > \epsilon^*_2 \) no root of \( g(1 - p, p, \epsilon_2) \) exists and therefore \( f^{22}(\epsilon_2) = \frac{1}{2} \) for all \( \epsilon_2 > \epsilon^*_2 \).

**Strong focusing on 1:** For \( \epsilon_1 = 0 \) the function \( g \) is always larger than zero (independent of the belief and \( \epsilon_2 \)). For \( \epsilon_1 = \frac{1}{2} \) we have \( g(p, 1 - p, \frac{1}{2}\epsilon_2) < 0 \) independent of \( \epsilon_2 \).

Therefore, the minimum root of \( g(x, x', \epsilon_2) \) is always larger than zero and smaller than \( \frac{1}{2} \) which shows \( 0 < f^{12}(\epsilon_2) < \frac{1}{2} \) for all \( \epsilon_2 \). For \( \epsilon_2 = 0 \) the function \( g \) has the form

\[ g(x, x', \epsilon_1, \epsilon_2) = [f(1 - \epsilon_1, x) - f(\epsilon_1, x')]\theta - 1 \]

The smallest root \( \epsilon^*_1 \) occurs for beliefs that minimize \( g \). This is the case for \( x = p \) and \( x' = 1 - p \). Therefore, \( f^{12}(0) = \epsilon^*_1 \) with \( \epsilon^*_1 \) being the root of \( g = f(1 - \epsilon_1, p) - f(\epsilon_1, 1 - p)\theta - 1 \). This shows that \( f^{12}(0) < p \) because for \( \epsilon_1 = p \) it is still negative. The same argument holds to show that \( f^{12}(\frac{1}{2}) > p \).

**Remark.** Comparable to focusing on 1 or 2 we can more explicitly specify the separating lines for strong focusing by defining \( f^{22} = \epsilon^*_1(1 - p, p) \) if the root exists and \( f^{12} = \frac{1}{2} \) otherwise and \( f^{11} = \epsilon^*_1(p, 1 - p) \).

For strong focusing on 2 note that we already know that strong focusing on 2 implies that \( \epsilon_1 \leq \epsilon_2 \). However, for all \( \epsilon_1 \leq \epsilon_2 \), \( U_\epsilon[s, \epsilon] \leq U_\epsilon[s, \epsilon] \) which is equivalent to \( (s|1) = (s|0) \). This is because

\[ (s|1) \succ (s|0) \iff [f(\epsilon_1, 1 - p) - f(\epsilon_1, p)]\theta > f(\epsilon_2, 1 - p) - f(\epsilon_2, p) \tag{7} \]

Here, we again exploited the characteristics of \( f \) described in the preliminaries. Therefore, \( g(x, x', \epsilon) \) is maximal for \( x = 1 - p \) and \( x = p \) and it is sufficient to find the root for this combination of beliefs.

For strong focusing on 1 errors can be such that \( \epsilon_1 \leq \epsilon_2 \) and for any fixed \( \epsilon_2 \) we have \( (s|1) \succ (s|0) \) for high \( \epsilon_1 \) and \( (s|0) \succ (s|1) \) for low \( \epsilon_1 \). However, as (7) shows for any \( \epsilon_2 \) fixed such that this ambivalence exists, there is a unique \( \epsilon_1 \) such that for \( \epsilon = (\epsilon_1, \epsilon_2) \), \( (hl|1) = (hl|0) \) which is equivalent to \( (lh|1) = (lh|0) \). Furthermore, for \( \epsilon = (\epsilon_1, \epsilon_2) \) patients focus on 1 because the error in attribute 1 has to be smaller than the error in attribute 2. Then, for \( \epsilon = (\epsilon_1, \epsilon_2) \) the patient also strongly focuses on attribute 1 since \( (hl|1) \succ (lh|1) = (lh|0) \) and \( (hl|0) \succ (lh|0) = (lh|1) \). This implies that \( f^{12}(\epsilon_2) \)
lies above \( \hat{\epsilon}_i \) and that the line of strong focusing can be defined as \( f^{s1} = \epsilon_1^i(0, 1) \).

**Dependence on \( \theta \).** First, consider \( \theta \to 1 \). Then the function \( g \) converges to

\[
g(\epsilon_1, \epsilon_2, x, x') = [f(1 - \epsilon_1, x) - f(\epsilon_1, x')] - [f(1 - \epsilon_2, 1 - x') - f(\epsilon_2, 1 - x)]
\]

For any beliefs \( x \) and \( x' \) the function \( g \) is zero if and only if \( \epsilon_1 = \epsilon_2 \) (it can be easily seen that it holds for \( x = x' \)). Analogously, it holds for \( x = p \) and \( x' = 1 - p \) as well as \( x = 1 - p \) and \( x' = p \). Thus

\[
(hl|0) = (lh|1) = (hl|1) = (lh|0).
\]

Therefore, for \( \theta \to 1 \) the expected utilities when observing \( hl \) or \( lh \) are the same independent of the underlying beliefs and thus all separating functions converge to

\[
f^{s2}(\epsilon_2) = f^{12}(\epsilon_2) = f^{s1}(\epsilon_2) = \epsilon_2.
\]

Second, consider \( \theta \to \infty \). For \( f^{s2} \) we have already seen that \( f^{s2} = \frac{1}{2} \) for all \( \theta > \frac{1}{1 - 2p} \). If \( x = x' \) and \( \theta \) is arbitrary high, the function \( g \) is always positive except for the case that \( \epsilon_1 = \frac{1}{2} \). Therefore, \( f^{12} = \frac{1}{2} \) as well. For other beliefs, the minimum \( \epsilon_1 \) for which the function \( g \) with \( \theta \to \infty \) is zero, is \( \epsilon_1 = p \). Therefore, \( f^{s1} \) converges to \( f^{s1}(\epsilon_1) = p \).

**Proof of Corollary 1.** The first part of the corollary is directly implied by the characteristics of the separating lines discussed in the previous lemma: If \( \epsilon_1 \) is large enough, patients focus on 2 for \( \epsilon_2 = 0 \). For \( \epsilon_2 = \frac{1}{2} \) they anyway focus on 1.

For the second part of the corollary is sufficient to show that for \( \epsilon = (p, 0) \) and \( \theta < \frac{1}{1 - 2p} \) the patient focuses on attribute 2. This is sufficient because the Lemma implies that for \( \epsilon = (p, \frac{1}{2}) \) the patient strongly focuses on attribute 1.

For \( \epsilon = (p, 0) \) and any belief \( x \) we have to show that \( (lh|x) > (hl|x) \) for \( \theta < \frac{1}{1 - 2p} \). \( (lh|1) > (hl|1) \) is equivalent to \( f(p, 1 - p)\theta + 1 < f(1 - p, 1 - p)\theta \). This is equivalent to \( \theta < \frac{(1-p)^2+p^2}{1-2p} \). As for all \( p, (1-p)^2+p^2 > 1 \) it is sufficient to choose \( \theta < \frac{1}{1-2p} \).

**Proof of Proposition 1.** First, we show that for focusing on attribute \( i \) in combination with errors \( \epsilon_i < \epsilon_{-i} \) and any beliefs and the other provider’s strategy, investing in \( i \) is a weakly better strategy than investing in the other attribute. Second, we show that this implies weak dominance of investing in \( i \), i.e. with the first part it remains to show that there is at least one combination of beliefs and the other providers' strategy such that investing \( i \) is strictly better than investing in the other attribute. Third, we show that strong focusing on \( i \) and \( \epsilon_i < \epsilon_{-i} \) imply strict dominance, i.e. investing in attribute \( i \) is strictly better for all beliefs and the other provider’s strategy.

**Investing in \( i \) is weakly better than investing in \( -i \).** The main idea is that, independent of whether the provider invests in attribute 1 or attribute 2, the same signals are generated. For given beliefs, the expected utility of each possible signal does not depend on the allocation decision. What does depend on the allocation decision is the probability of each signal. For focusing on attribute \( i \) investing in \( i \) generates ”better signals” (i.e. they yield a higher expected utility for patients) with
higher probability compared to investing in the other attribute.

Focusing on attribute 1 and $\epsilon_1 < \epsilon_2$ : Assume each patient has any belief $(b^A, b^B)$ about the providers’ strategy (possibly not the same for each provider and beliefs might differ across patients). Let provider $B$ have any strategy (possibly not known to provider $A$). We have to show that it is a weakly dominant strategy for provider $A$ to invest in attribute 1, i.e. $a^A = 1$.

Each patient either receives signal $ll$, $lh$, $hl$ or $hh$ from provider $A$. Independent of her belief $b^A$ about provider $A$’s resource allocation, focusing on 1 implies that each patient faces the following ordering of signals with respect to expected utilities if received from provider $A$:

$$(hh|b^A) \succ (hl|b^A) \succ (lh|b^A) \succ (ll|b^A).$$

The expected utility of a patient receiving $s$ from provider $A$ and having belief $b^B$ is

$$U_s[s|b^A, \epsilon] = \sum_q u(q)P(q|s, b^A, \epsilon).$$

Importantly, the allocation decision of the providers does not influence the expected utilities that patients with a belief $b^A$ are facing when receiving a signal $s$. However, the probabilities of the signals depend on the allocation decision of the provider. For $a^A = 1$ and $\epsilon_1 \leq \epsilon_2$ the ordering is as follows

$$P(s = lh|1) < P(s = ll|1) \leq P(s = hh|1) < P(s = hl|1)$$

For $a^A = 0$ this ordering is reversed with $P(s = lh|0) = P(s = hl|1)$, $P(s = hh|0) = P(ll|1)$, $P(s = ll|0) = P(s = hh|1)$ and $P(s = hl|0) = P(s = lh|1)$. Thus, for choosing $a^A = 1$ instead of $a^A = 0$ some part of the probability of $s = ll$ is shifted to $hh$, and from $s = lh$ to $s = hl$ (better signals have more weight). Therefore, for any allocation strategy of $B$, provider $A$ is selected by any patient with weakly higher probability when choosing $a^A = 1$ instead of $a^A = 0$. This holds independent of the beliefs $b$ about the allocation decision of $A$ and $B$.\footnote{Note that for focusing on attribute 1 the order of the signals $\epsilon_1 \leq \epsilon_2$ was needed to conclude that more preferred signals are generated with higher probability. For $\epsilon_1 > \epsilon_2$ the effect is ambiguous. $a = 1$ still makes $hl$ more probable on the cost of $lh$ and leads to an increase in the expected profit of the provider. However, at the same time, $ll$ is more probable on the cost of $hh$ and therefore leads to a decrease in expected profit for the provider.}

Focusing on attribute 2: The approach is the same as above. Note that focusing on attribute 2 immediately implies that $\epsilon_1 > \epsilon_2$. If patients focus on attribute 2 the signal ordering for any belief $b^A$ is the following

$$(hh|b^A) \succ (lh|b^A) \succ (hl|b^A) \succ (ll|b^A).$$

The signal probabilities for playing $a^A = 1$ have the ordering

$$P(s = lh|1) < P(s = hh|1) \leq P(s = ll|1) < P(s = hl|1)$$
Here we used that $\epsilon_1 > \epsilon_2$ holds. For choosing $a^A = 0$ the ordering reverses with $\mathbb{P}(s = lh|0) = \mathbb{P}(s = hl|1)$, $\mathbb{P}(s = hh|0) = \mathbb{P}(ll|1)$, $\mathbb{P}(s = ll|0) = \mathbb{P}(s = hh|1)$ and $\mathbb{P}(s = hl|0) = \mathbb{P}(s = lh|1)$. Thus, in this case $a^A = 0$ influences the signal probabilities such that better signals have higher probabilities.

**Focusing implies weak dominance.** Take any symmetric belief $(b^A, b^B) = (b, b)$ of the patients and any strategy $a^B$ of provider $B$. We assume that parameters are such that patients focus on 2. It is then sufficient to show that it is strictly better for $A$ to invest in 2 than to invest in 1.

Assume that $B$ sends a signal $s^B = lh$. If $A$ sends $lh$ as well, $A$ is selected with probability $\frac{1}{2}$. However, if $A$ sends $lh$ he is selected with probability 1. As choosing $a^j = 0$ instead of $a^j = 1$ shifts some of the probability of sending $lh$ to sending $ll$ (see first part of the proof), $A$ can strictly increase his probability of being selected by choosing $a^j = 0$ instead of $a^j = 1$.

Arguments for focusing on 1 and $\epsilon_1 < \epsilon_2$ are the same.

**Strong focusing implies strict dominance.** We now show that for strong focusing on attribute 2 provider $A$ strictly prefers to invest in attribute 2, independent of the beliefs and the resource allocation of provider $B$. The same arguments hold for strong focusing on attribute 1 and $\epsilon_1 < \epsilon_2$.

Assume that patients have any beliefs $b^A$ and $b^B$ and that provider $B$ has chosen any $a^B$. Strong focusing implies that provider $A$ is selected when sending signal $ll$ while $B$ sends $hh$. $B$ is selected when sending $hh$ while $A$ sends $ll$. Now assume that both send $ll$ or both send $hh$. We show that the probability that $A$ is selected is the same in both cases and then show that this is sufficient to show that $A$ is selected with strictly higher probability for $a^j = 0$ instead of $a^j = 1$.

Consider any beliefs $x, x' \in \{p, 1 - p\}$ about the quality realization where $x = 1 - p$ corresponds to a belief $b = 1$ and $x = p$ corresponds to a belief $b = 0$ (as discussed in the beginning of the proof of Lemma 1).

$(ll|x) \succeq (ll|x')$ is equivalent to $[f(\epsilon_1, x) - f(\epsilon_1, x')]\geq [f(\epsilon_2, x) - f(\epsilon_2, x')]$. For $(hh|x) \succeq (hh|x')$ we just have to replace $\epsilon$ by $1 - \epsilon$. If $x = x'$, the inequality is satisfied both for $ll$ and $hh$. For asymmetric $x$ and $x'$ the inequality for $ll$ is equivalent to the one for $hh$. Therefore, the probability that $A$ is selected if both providers send the signal $hh$ equals the probability that $A$ is selected if both providers send the signal $ll$.

This implies that $A$ is strictly better off when choosing $a^A = 0$ instead of $a^A = 1$: First, assume that $A$ is selected with probability 1 if both send $hh$ or $ll$. Assume that $B$ signals $hh$. By the proof of Proposition 1 we know that selecting $a^A = 0$ instead of $a^A = 1$ shifts probabilities from sending worse signals to better signals. Particularly, from sending $ll$ to sending $hh$. Since $\epsilon_2 < \epsilon_1$ the amount of probability shifted is not zero. If $A$ sends $ll$, $B$ is selected, if $A$ sends $hh$, $A$ is selected. Therefore, the shift in probabilities results in strict increase of the probability to be selected. No assume that $A$ is selected with probability less than 1 if both send $hh$ or $ll$ and assume that $B$ signals $ll$. Then, $A$ is selected with probability 1 when signaling $hh$ but is selected with probability less than 1 when signaling $ll$. Here again, the shift in probabilities from $ll$ to $hh$ results in strict increase of the probability to be selected. □
Proof of Proposition 2. We first show the parts of the proposition that claim that strong focusing on an attribute implies uniqueness of the PBE. Then we show that any further equilibria are asymmetric and who is selected in asymmetric equilibria. Finally we discuss the further conditions for uniqueness.

Strong focusing implies uniqueness. The uniqueness for strong focusing and corresponding errors is directly implied by the proof of the strict dominance of Proposition 1. For both providers it is - independent of the beliefs and the other provider’s strategy - strictly better to invest in the attribute the patients strongly focus on.

Asymmetric equilibria. Assume that patients focus on attribute 2 and \((a^A, a^B) = (b^A, b^B) \neq (0, 0)\) is an equilibrium. First, the equilibrium is not symmetric, i.e. \(a^A \neq a^B\). This is because for symmetric beliefs investing in attribute 2 is strictly preferred by any provider to investing in attribute 1 (see above). Second, we want to show that provider \(A\) is selected with probability one if and only if \(a^A > a^B\). Assume that \(a^A > a^B\), i.e. \(a^A = 1\) and \(a^B = 0\). We want to show that this implies already \((ll|x^A) \succ (hh|x^B)\) which means that provider \(A\) is selected independent of the signal. Assume the contrary, i.e. \((hh|a^B)\) yields at least the same expected utility as \((ll|a^A)\). Then provider \(A\) has an incentive to deviate by choosing \(a^A = 0\) instead of \(a^B = 1\): From the proof of Proposition 1 we know \(a^A = 0\) is weakly better than \(a^A = 1\). If \((ll|x^A) \succ (hh|x^B)\) does not hold, it is also strictly better because if \(B\) sends \(hh\) and \(A\) sends \(ll\), provider \(B\) is selected with strictly positive probability. If \(A\) sends \(hh\) and \(B\) send \(hh\), on the other hand, provider \(B\) is never chosen because \(a^A = 1\) and \(b^B = 0\). As a shift from \(a^A = 1\) to \(a^A = 0\) generates signal \(hh\) with higher probability on the cost of sending signal \(ll\) and all other shifts in probabilities are weakly better as well it is strictly dominant for \(A\) to invest in attribute 2. This is a contradiction to the assumption that \(a^A > a^B\) is the providers’ strategy in equilibrium. Thus, if \((a^A, a^B) = (b^A, b^B) \neq (0, 0)\) is a PBE and \(a^A > a^B\), provider \(A\) is selected with probability one. On the other hand, if provider \(A\) is selected with probability one, \(a^A > a^B\) has to hold.

The part for focusing on attribute 1 follows by the same arguments.

Further conditions for uniqueness. Assume that for \(\epsilon = (\epsilon_1, \epsilon_2)\) patients focus on attribute 2 and strictly focuses on attribute 1 for \(\epsilon' = (\epsilon_1, \frac{1}{2})\). Assume that for \(\epsilon = (\epsilon_1, \epsilon_2)\) the equilibrium is not unique. Particularly, this implies that \((ll|1) \succ (hh|0)\) which is equivalent to

\[
[f(\epsilon_1, 1 - p) - f(1 - \epsilon_1, p)]\theta > f(1 - \epsilon_2, 1 - p) - f(\epsilon_2, p) = 2f(1 - \epsilon_2, 1 - p) - 1.
\]

The right hand side is decreasing in \(\epsilon_2\), therefore if \((ll|x^A) \succ (hh|x^B)\) holds for \(\epsilon = (\epsilon_1, \epsilon_2)\) it holds as well when \(\epsilon_2\) increases and particularly for \(\epsilon' = (\epsilon_1, \frac{1}{2})\). If the patient strongly focuses on attribute 1 for \(\epsilon' = (\epsilon_1, \frac{1}{2})\) it is a contradiction because then \((hh|0) \succ (ll|1)\).

Now assume that \(\epsilon'_1\) is such that for \((\epsilon'_1, \epsilon_2)\) patients strongly focus on attribute 2. Now assume that for \((\epsilon_1, \epsilon_2)\) the equilibrium is not unique. This implies particularly \(\epsilon_1 < \epsilon'_1\) and that \((ll|1) \succ (hh|0)\) which is again equivalent to

\[
[f(\epsilon_1, 1 - p) - f(1 - \epsilon_1, p)]\theta < f(1 - \epsilon_2, 1 - p) - f(\epsilon_2, p)
\]
The left hand side is increasing in $\epsilon_1$. Thus, if it holds for any $\epsilon_1$, it also holds for $\epsilon'_1 > \epsilon_1$. This contradicts that for $(\epsilon'_1, \epsilon_2)$ the patient strongly focuses on attribute 2 since then $(hh|0) > (ll|1)$.

\[ \text{Proof of Lemma 2.} \] We fix any $\epsilon = (\epsilon_1, \epsilon_2)$ and therefore omit it in the following. We will first show that for given symmetric patients’ beliefs with $b^A = b^B = b$ about the providers’ resource allocation,

\[ W[(1, a^B)|(b, b)] > W[(0, a^B)|(b, b)] \]

for any $a^B \in \{0, 1\}$, i.e. $W[(1, 0)|(b, b)] > W[(0, 0)|(b, b)]$ and $W[(1, 1)|(b, b)] > W[(0, 1)|(b, b)]$. From symmetry of $W[.]$ with respect to providers it then follows that $W[(1, 1)|(b, b)] > W[(0, 0)|(b, b)]$. Since for any symmetric beliefs patients make the very same selection of providers based on signals they receive, this then also implies that $W[(1, 1)|(1, 1)] > W[(0, 0)|(0, 0)]$.

Note that the only variables in

\[ W[(a^A, a^B)|(b, b)] = \sum_{q^A} \sum_{q^B} \mathbb{P}(q^A|a^A)\mathbb{P}(q^B|a^B)U_q[q^A, q^B|b] \quad (8) \]

that depend on the resource allocation decision of provider $A$ are $\mathbb{P}(q^A|a^A)$ for $q^A = hl$ and $q^A = lh$. This is because $\mathbb{P}(hh|a^A) = \mathbb{P}(ll|a^A) = (1 - p)p$ for all $a^A$. Thus, we need to show that

\[ \sum_{q^B} \mathbb{P}(q^B|a^B)[\mathbb{P}(hl|1)U_q[hl, q^B|b] + \mathbb{P}(lh|1)U_q[lh, q^B|b]] \quad (9) \]

\[ > \sum_{q^B} \mathbb{P}(q^B|a^B)[\mathbb{P}(hl|0)U_q[hl, q^B|b] + \mathbb{P}(lh|0)U_q[lh, q^B|b]] \quad (10) \]

\[ \iff \sum_{q^B} \mathbb{P}(q^B|a^B)[(1 - p)^2U_q[hl, q^B|b] + p^2U_q[lh, q^B|b]] \quad (11) \]

\[ > \sum_{q^B} \mathbb{P}(q^B|a^B)[p^2U_q[hl, q^B|b] + (1 - p)^2U_q[lh, q^B|b]] \quad (12) \]

\[ \iff \sum_{q^B} \mathbb{P}(q^B|a^B)U_q[hl, q^B|b] > \sum_{q^B} \mathbb{P}(q^B|a^B)U_q[lh, q^B|b] \quad (13) \]

For $q^B = lh$ and $q^B = hl$ we have $U_q[hl, q^B|b] \geq U_q[lh, q^B|b]$. Furthermore, $\mathbb{P}(hh|a^B) = \mathbb{P}(ll|a^B) = p(1 - p)$ independent of $a^B$. It thus remains to show that

\[ U_q[hl, hh|b] + U_q[hl, ll|b] > U_q[lh, hh|b] + U_q[lh, ll|b] \quad (14) \]

Note that $U_q[q^A, q^B|b] = u(q^A)\mathbb{P}(q^A|q^A, q^B, b) + u(q^B)(1 - \mathbb{P}(q^A|q^A, q^B, b))$ where $\mathbb{P}(q^A|q^A, q^B, b)$ is the probability that $q^A$ is chosen by the patient if quality levels $q^A$ and $q^B$ are realized, patient has belief $b$ and the signal error is $\epsilon$. Thus the previous inequality is
equivalent to
\[
(u(hl) - u(hh))P(hl|hl, hh, b) + (u(hl) - u(ll))P(hl|hl, ll, b) = (15)
\]
\[
> (u(lh) - u(hh))P(lh|lh, hh, b) + (u(lh) - u(ll))P(lh|lh, ll, b) = (16)
\]
\[
\Leftrightarrow \theta P(lh|lh, hh, b) + \theta P(hl|hl, ll, b) > P(hl|hl, hh, b) + P(lh|lh, ll, b) = (17)
\]

As \( b = b^A = b^B \) is the belief for both providers,
\[
P(hl|hl, hh, b) = P(ll|ll, lh, b) = 1 - P(lh|lh, ll, b) = (18)
\]
\[
P(hl|hl, ll, b) = P(hh|hh, hl) = 1 - P(hl|hl, hh, b) = (19)
\]

Inserting this into the above inequality reduces the inequality to \( \theta > 1 \) which holds by definition of \( \theta \) in our model.

\[\Box\]

**Proof of Proposition 3.** It is to show that for focusing on attribute 2, any PBE is inefficient. For strong focusing, this follows directly by the discussion above. For focusing, first consider the symmetric BNE where both providers invest in attribute 2. Then quality provision is inefficient by Proposition 3. Second, consider any other BNE \( (a^A, a^B) = (b^A, b^B) \) with \( a^A > a^B \). Proposition 2 showed that patients then choose provider \( A \) ignoring the signals sent. Thus, expected utility is \((1 - p)\theta + p \) since \( a^A = 1 \). If both providers invest in attribute 1 and patients have corresponding beliefs, welfare is strictly higher as signals are then valuable to patients and by selection based on the signals they receive an expected utility higher than \((1 - p)\theta + p \).

\[\Box\]

**Proof of Proposition 4.** First note that \( W[a|b, (\epsilon_1, \epsilon_2)] \) is decreasing in both \( \epsilon_1 \) and \( \epsilon_2 \) (the more precise signals the better the patient can select). Therefore, for any \( \epsilon_1 \) fixed it is sufficient to show the inequality for \( \epsilon_2 = \frac{1}{2} \) and \( \epsilon_2' = 0 \) because this then implies that the inequality holds for any other \( \epsilon_2 \) and \( \epsilon_2' \).

Denote
\[
\Delta W_{10}(\epsilon_1) = W[(1, 1)|(1, 1), (\epsilon_1, 1/2)] - W[(0, 0)|(0, 0), (\epsilon_1, 0)].
\]

We first show that \( \Delta W_{10}(\frac{1}{2}) > 0 \) and then show that this implies the inequality for all other \( \epsilon_1 \).

To show that \( \Delta W_{10}(\frac{1}{2}) > 0 \) holds we explicitly calculate the expected utilities. For \( a = (1, 1), \epsilon_2 = \frac{1}{2} \) and corresponding beliefs the signals are of no value for patients and therefore
\[
W[(1, 1)|(1, 1), (\frac{1}{2}, 1/2)] = (1 - p)\theta + p.
\]
For \( a = (0, 0), \epsilon_2 = 0 \) and corresponding beliefs \( b = (0, 0) \) the patient receives no signal in the first attribute and a precise signal in the second attribute. Thus, in the first attribute high quality is realized with probability \( p \) while in the second attribute high quality is realized with probability \( 1 - p^2 \) (the patient focuses on attribute 2 and therefore she only picks low quality in the second attribute if both providers realize
low quality). Therefore

\[ W[(0, 0)|(0, 0), \left(\frac{1}{2}, 0\right)] = p\theta + 1 - p^2. \]

This implies that \( \Delta W_{10}(\frac{1}{2}) > 0 \) is equivalent to \( \theta > \frac{1-\epsilon_1}{1-2p} \).

Now we show that for all \( \epsilon_1 \) such that patients focus on attribute 2 for \((\epsilon_1, 0)\), the welfare difference \( \Delta W_{10}(\epsilon_1) \) decreases in \( \epsilon_1 \), i.e. \( \frac{\partial}{\partial \epsilon_1} \Delta W_{10}(\epsilon_1) < 0 \). This then implies that \( \Delta W_{10}(\epsilon_1) > 0 \) for all \( \epsilon_1 \) such that patients on attribute 2 for \((\epsilon_1, 0)\). The intuition of \( \Delta W_{10}(\epsilon_1) \) decreasing in \( \epsilon_1 \) is as follows: An improvement of the signal quality in the first attribute has a larger effect on expected utility if there is no signal in the second attribute \((\epsilon_2 = \frac{1}{2})\) compared to a precise signal \((\epsilon_2 = 0)\). Thus, the welfare difference increases when \( \epsilon_1 \) decreases.

For explicit calculation we calculate the partial derivative of the expected utilities separately. First, consider \( W[(1, 1)|(1, 1), (\epsilon_1, \frac{1}{2})] \). Signals in the second attribute have no value for the patient. As quality is realized independently for both attributes the patient’s expected utility in the second attribute is \( p \). For the first attribute there are four different combinations of quality realization of the two providers. The patient faces high quality in the first attribute if both providers realize high quality (occurs with probability \((1 - p)^2\)) or if one of the providers realizes high quality and the other one standard quality (occurs with probability \(2(1 - p)p\)) and the patient chooses correctly the provider with the high quality realization (which she does with probability \((1 - \epsilon_1)\)).\(^{23}\) Thus, for the expected utility the following holds

\[ W[(1, 1)|(1, 1), (\epsilon_1, \frac{1}{2})] = [2(1 - \epsilon_1)(1 - p)p + (1 - p)^2]p. \]

Second, consider \( W[(0, 0)|(0, 0), (\epsilon_1, 0)] \). For this we consider all possible realizations of quality in the second attribute separately. \( q_2 = (h, l) \) or \( q_2 = (l, h) \) is realized with probability \(2p(1 - p)\). In both cases the signal of attribute 1 is irrelevant as the patient focuses on attribute 2 and has a precise signal in attribute 2. Thus the expected utility given realizations \( q_2 = (h, l) \) or \( q_2 = (l, h) \) is \( \theta p + 1 \) as utility in the first attribute is realized independent of quality in the second attribute.

If \( q_2 = (h, h) \) or \( q_2 = (l, l) \) is realized the selection of the provider is only based on the signal in the first attribute. If \( q_1 = (h, l) \) or \( q_1 = (l, h) \) high quality is selected with probability \((1 - \epsilon_1)\). For \( q_1 = (h, h) \) the patient selects high quality in attribute 1 with probability 1 and for \( q_1 = (l, l) \) standard quality is selected.

Consolidation of those considerations gives

\[
W[(0, 0)|(0, 0), (\epsilon_1, 0)] = 2(1 - p)p(\theta p + 1) \\
+ (1 - p)^2(\theta(\theta p^2 + 2(1 - p)p(1 - \epsilon_1)) + 1) \\
+ p^2(\theta(\theta p^2 + 2(1 - p)p(1 - \epsilon_1)))
\]

where the first term represents expected utility of the patient if \( q_2 = (h, l) \) or \( q_2 = (l, l) \). If \( A \) realizes \( h \) and \( B \) realizes \( l \), \( A \) is chosen with probability \( \frac{1}{2} \) if both send the same signal and with probability 1 if \( A \) sends \( h \) and \( B \) sends \( l \). The overall probability that \( A \) is choose is the \( 2\frac{1}{2}\epsilon_1(1 - \epsilon_1) + (1 - \epsilon_1)^2 = 1 - \epsilon_1 \)

\(^{23}\)If \( A \) realizes \( h \) and \( B \) realizes \( l \), \( A \) is chosen with probability \( \frac{1}{2} \) if both send the same signal and with probability 1 if \( A \) sends \( h \) and \( B \) sends \( l \). The overall probability that \( A \) is choose is the \( 2\frac{1}{2}\epsilon_1(1 - \epsilon_1) + (1 - \epsilon_1)^2 = 1 - \epsilon_1 \)
(l, h) is realized, the second if \( q_2 = (h, h) \) is realized and the third if \( q_2 = (l, l) \) is realized.

Now we can calculate \( \frac{\partial}{\partial \epsilon_1} \Delta W_{10}(\epsilon_1) \)

\[
\frac{\partial}{\partial \epsilon_1} \Delta W_{10}(\epsilon_1) = -2(1-p)p + 2(1-p)^3p + 2(1-p)p^3
\]

This is always negative as \(-2(1-p)p + 2(1-p)^3p + 2(1-p)p^3 < 0\) is equivalent to \( p - 1 < 0 \) which always holds. Therefore, we showed that \( \Delta W_{10}(\epsilon_1) \) decreases in \( \epsilon_1 \).

Proof of Lemma 3. For the first part, consider \( \epsilon \) such that patients focus on attribute 2 when they receive informative signals in both attributes. To show that in equilibrium providers never disclose information on both attributes, assume to the contrary that an equilibrium exists with reporting in both attributes by both providers. Both providers then invest in attribute 2. We show that if one provider deviates by disclosing information only in attribute 2 and investing in attribute 2, he is selected with a probability higher than \( \frac{1}{2} \). This makes the deviation profitable.

Assume that provider \( A \) reports only in attribute 2 while \( B \) reports in both attributes (which implies that for both the belief is investing in attribute 2). \( A \) either sends signal \( h \) or signal \( l \) in attribute 2, \( B \) either sends \( ll, hl, lh \) or \( hh \). \( A \) is selected when signaling \( h \) in attribute 2 while \( B \) sends \( lh, ll \) or \( hl \) (the last two are due to focusing on 2). Furthermore, if \( A \) signals \( l \) and \( B \) signals \( ll \), \( A \) is selected as well. Summing up the probabilities with which the signals are sent, provider \( A \) is selected with probability \( (1-e^2_1) - (1-e^2_2) + e^2_2(1-e^1_1) \) (with \( e_i = \epsilon_i(1-p) + (1-\epsilon_i)p \)). The term is strictly decreasing in \( e_1 \). \( e_1 \) is always smaller or equal to \( \frac{1}{2} \) and the term is \( \frac{1}{2} \) for \( e_1 = \frac{1}{2} \). Therefore, the probability that \( A \) is selected is larger than \( \frac{1}{2} \).

If \( \epsilon \) is such that \( \epsilon_1 < \epsilon_2 \) and it can be shown with the same arguments that if one provider is reporting only in attribute 1 and investing in attribute 1 and the other reports in both attributes and invests in attribute 1, the first is selected with a higher probability than the latter.

Proof of Proposition 5. (i) Assume that \( \epsilon_1 < \epsilon_2 \) and both providers invest in 1 and report only in 1, patients have corresponding beliefs. We show that none of the providers has an incentive to deviate.

In the Lemma above we already showed that nobody has an incentive to deviate to report in both attributes. Furthermore, reporting only in attribute 2 and investing in attribute 2 is not a profitable deviation. This is because if \( A \) reports and invests in 2 and \( B \) reports and invest in 1, \( B \) is selected whenever sending signal \( h \). This is because \( \epsilon_1 < \epsilon_2 \) and \( \theta > 1 \). However, he sends \( h \) with probability \( (1-\epsilon_1) \) which is greater than \( \frac{1}{2} \) (with \( \epsilon_i = \epsilon_i(1-p) + (1-\epsilon_i)p \)). The same argument holds, if \( A \) does not send any signal and \( B \) sends a signal only in attribute 1. Then, \( B \) is selected as well whenever sending \( h \) which occurs with a probability \( \frac{1}{2} \). Therefore, there is no profitable deviation if both invest and report in attribute 1 which shows that this is an equilibrium.

\footnote{Except for \( \epsilon_1 = \frac{1}{2} \), but then there is no point in deciding about reporting in both attribute as there is anyway no signal to report in attribute 1.}
It remains to show that this equilibrium is unique. Assume another equilibrium exists. If there is one provider that is selected with probability smaller than $\frac{1}{2}$ he has an incentive to deviate by copying the other provider’s strategy. Therefore, in equilibrium both have to be selected with probability $\frac{1}{2}$. However, at least one of the providers, say $B$, necessarily has another strategy than investing in 1 and reporting in 1 (as we consider an equilibrium different to the one where both invest in 1 and report in 1). Then, due to the considerations above, $A$ is selected with probability greater than $\frac{1}{2}$ when investing in 1 and reporting in 1 and therefore has an incentive to deviate. This shows the uniqueness.

(ii) Consider any $\epsilon$ such that patients focus on attribute 2 when receiving both signals. Once $\theta > \theta^c$, $(h \cdot |1) \succ (\cdot |0)$ holds (see considerations previous to the Proposition).

$(h \cdot |1) \succ (\cdot |0)$ implies that reporting only in attribute 1 and investing this attribute yields a selection probability greater than $\frac{1}{2}$ if the other provider reports only in attribute 2 and invests in 2. The same holds if the other provider does not report since then the one reporting and investing in 1 is as well always selected when signaling $h$ in attribute 1. If, furthermore, it holds that if the other provider reports in both attributes and invests in attribute 2, investing in 1 and reporting in 1 yields a selection probability greater than $\frac{1}{2}$, we showed that investing and reporting in 1 is a PBE.

To show that there exist $\epsilon$ such that this holds, assume that provider $A$ invests and reports only in attribute 1, and provider $B$ reports in both attributes and invests in 2. We want to know for which $\epsilon$ the probability that $A$ is selected is greater than $\frac{1}{2}$.

Note that $A$ is always selected when sending $h$ in attribute 1 and, on the same time, $B$ either sends $ll$, $hl$ or $lh$. For $\epsilon = (\frac{1}{2}, 0)$ provider $A$ is also selected when $B$ sends $hh$ and $A$ send $h$ in attribute 1. Therefore, for all $\epsilon$ close enough to $\epsilon = (\frac{1}{2}, 0)$ provider $A$ is selected with probability greater than $\frac{1}{2}$. Note that there are several other $\epsilon$ for which this holds. For instance, once $\epsilon_1 > p$, provider $A$ also is selected when sending $l$ in attribute 1 and provider $B$ sends $hl$ or $ll$. Then, once $p > \frac{1}{3}$ the total probability that $A$ is selected is greater than $\frac{1}{2}$ which can be shown by explicit calculation.

It remains to show that investing and reporting only in 1 is a unique PBE. The arguments for this are exactly the same we saw in (i) for uniqueness.

(iii) First, we show that if $\theta < \theta^c$ we can choose $\epsilon = (\epsilon_1, \epsilon_2)$ such that $(\cdot |0) \succ (h \cdot |1)$. Second, we show that $(\cdot |0) \succ (h \cdot |1)$ is sufficient such that reporting only on attribute 2 and investing in attribute 2 with corresponding beliefs is a PBE. The uniqueness of the equilibrium then again follows by the same arguments as seen in (i).

1. Choice of $\epsilon$: For $(\cdot |0) \succ (h \cdot |1)$ the following holds

$$(\cdot |0) \succ (h \cdot |1) \iff p\theta + f(1 - \epsilon_2, 1 - p) > f(1 - \epsilon_1, p)\theta + (1 - p)$$

The left hand side is decreasing in $\epsilon_2$ and the right hand side is decreasing in $\epsilon_1$. So the error for which the inequality is the easiest to fulfill is $\epsilon = (\frac{1}{2}, 0)$. For this error the inequality transfers to $\theta < \frac{1 - p}{1 - 2p}$. Thus, only if $\theta < \frac{1 - p}{1 - 2p} = \theta^c$ there exist an
\( \epsilon = (\epsilon_1, \epsilon_2) \) such that \( \langle -h|0 \rangle \succ \langle h \cdot |1 \rangle \). We just showed that at least for \( \epsilon = (\frac{1}{2}, 0) \) it is the case which implies that there exists a neighborhood of \( \epsilon = (\frac{1}{2}, 0) \) such that it holds for all \( \epsilon \) in this neighborhood.

2. **Investing and reporting only in attribute 2 with corresponding beliefs form a PBE.** Consider \( \epsilon \) such that \( \langle -h|0 \rangle \succ \langle h \cdot |1 \rangle \) holds and assume that both providers invest in attribute 2 and report only in attribute 2. Then both providers are selected with probability \( \frac{1}{2} \). In the following we show that for any provider there is no incentive to deviate.

Assume that provider \( A \) deviates by not reporting in 2 but only in 1. If \( A \) discloses information on 1 and \( B \) on 2 then by \( \langle -h|0 \rangle \succ \langle h \cdot |1 \rangle \), \( B \) wins whenever generating a signal \( h \) in the second attribute the probability of which is larger than \( \frac{1}{2} \) since \( B \) invests in 2. Thus, provider \( A \) does not have any incentive to deviate in 1.

Now assume that provider \( A \) deviates by reporting in both signals and investing in 2. Again, \( B \) wins whenever generating signal \( h \) in the second attribute - except for \( A \) generating \( hh \). On the other hand, \( B \) also is selected when generating \( l \) in the second attribute and \( A \) generates \( ll \). Thus, \( B \) wins with at least a probability of \( (1 - e_2) - (1 - e_2)^2 + e_2^2 = \frac{1}{2} \). Therefore \( A \) has no incentive to deviate.

Finally, assume that provider \( A \) deviates by not reporting at all. Then again, \( B \) wins whenever \( B \) sends signal \( h \) in the second attribute by \( \langle -h|0 \rangle \succ \langle h \cdot |1 \rangle \). Therefore, \( A \) has no incentive to deviate.

**Proof of Proposition 6.** The proof combines the results of Proposition 5 and the welfare discussion.

For part (i) note that it is optimal if signals in both attributes are reported (and with it providers then invest in 1). Voluntary reporting leads to reporting only in attribute 1.

To discuss parts (ii) and (iii) we only consider signal errors \( \epsilon \) such that if receiving both signals, patients focus on attribute 2. Optimal reporting is then such that it induces that in equilibrium either both providers invest in attribute 1 and report only in attribute 1, or both providers invest in attribute 2 and report in both attributes. The first is desired if

\[
W[(1, 1); (1, 1), (\epsilon_1, \frac{1}{2})] > W[(0, 0); (0, 0), (\epsilon_1, \epsilon_2)],
\]

the latter if the reverse holds.

First, consider \( \theta > \theta^c \) and \( \epsilon \) such that in the unique PBE there is reporting only in attribute 1. From Proposition 5 we already know that this is the case if \( \epsilon_1 \) is high enough and \( \epsilon_2 \) is low enough since it holds for \( \epsilon = (\frac{1}{2}, 0) \) (furthermore, it holds for all \( \epsilon \) with focusing on 2 as long as \( \epsilon_1 > p \) and \( p > \frac{1}{3} \)). \( \theta > \theta^c \) implies \( \theta > \theta \) and therefore the optimal policy has to induce an equilibrium where both providers invest in 1 and report only on 1. Thus, voluntary reporting is already optimal, while any policy mandating reporting in attribute 2 is not optimal. Mandating reporting only
in 1 or banning reporting in 2 yields the same outcome.
Second, consider \( \theta < \theta^c \) and \( \epsilon \) such that disclosing only in attribute 2 is the unique PBE. Again, by the proposition above, this holds if \( \epsilon_1 \) is high enough and \( \epsilon_2 \) is low enough. For \( \hat{\theta}(\epsilon) < \theta < \theta^c \), it is desirable that signals are sent only in attribute 1. This can be achieved by banning reporting on attribute 2. For \( \epsilon_1 \) high enough and \( \epsilon_2 \) low enough, mandatory reporting in 1 is not an optimal policy since then providers would additionally report about attribute 2. However, mandatory reporting in 1 yields higher welfare than voluntary reporting since voluntary reporting in both attributes leads to reporting only in 2 in equilibrium. Banning reporting in 2 leads in equilibrium to only reporting in 1, therefore it is optimal.
For \( \theta < \hat{\theta}(\epsilon) \) it is desirable that information about both attributes is available. For \( \epsilon_1 \) high enough and \( \epsilon_2 \) low enough, mandating reporting in attribute 1 is already an optimal policy since providers voluntarily report about attribute 2. In this case, banning reporting in attribute 2 is not optimal. Banning reporting in 2 might even decrease welfare compared to voluntary reporting in both attributes. This occurs whenever voluntary reporting yields reporting only in attribute 2 and, on the same time, reporting only in attribute 1 is associated with lower welfare than reporting only in attribute 2. This occurs if \( \theta \) is close enough to 1 because then, the better selection effect in the second attribute dominates any potentially better resource allocation effect such that receiving information only on attribute 2 and investment in 2 yields higher welfare than receiving information only about attribute 1 but providers invest in attribute 1.

Proof of Proposition 7. Fix \( p, \theta \) and \( \epsilon \) as considered in the proposition. We are interested in the winning probability of provider A if B invests in 1 and A invests in attribute 2 when investments are observable (i.e. the patient has also the corresponding beliefs). If the probability of A winning is larger than one half, it is a strict dominant strategy to invest in attribute 2. If it is smaller than one half it is a strict dominant strategy to invest in attribute 1.

To assess the winning probabilities we explicitly consider for which signal combinations A wins. If B sends a signal with \( s_2 = l \) and A sends a signal with \( s_2 = h \) (which occurs with probability \((1-\epsilon_2)^2\)) the patient selects provider A as she strongly focuses on attribute 2. The only other cases where A might win are the signal combinations \((s^A, s^B) = (hh, lh)\) and \((s^A, s^B) = (hl, ll)\) (whether or not A is selected depends again on the parameters). In all other cases B is selected. This follows by the fact that if the same signals are generated provider B is selected and all other remaining signal combinations are implied either by strong focusing or by B winning for the same signals.

Therefore, provider A is selected at least with probability \((1-\epsilon_2)^2\) and at most with probability \((1-\epsilon_2)^2 + 2\epsilon_1^2\epsilon_2(1-\epsilon_2)\).

Thus, if \((1-\epsilon_2)^2 > \frac{1}{2}\) investing in attribute 2 is a strictly dominant strategy which holds for all \( \epsilon_2 < 1 - \sqrt{\frac{1}{2}} \).

If \((1-\epsilon_2)^2 + 2\epsilon_1^2\epsilon_2(1-\epsilon_2) < \frac{1}{2}\) investing in attribute 1 is a strictly dominant strategy which is equivalent to \( \epsilon_2 > \frac{3-\sqrt{5}}{2} \).

\( \square \)